Thoughts on Photons, from JLab to EIC

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An overview, and where can 'soft' photons come from?

1. Where we stand

2. Low's theorem and its decendents

3. Beyond-Low soft photons?

4. A surprise at three loops (using QCD calculations)

5. Soft photons and correlations

1. Where we stand

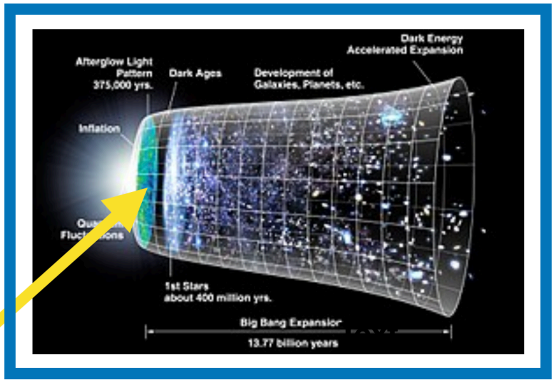
- Over a decade ago, the last of the great Standard Model discovery machines, the Tevatron, closed up shop, following LEP and HERA, while RHIC continued its second fruitful decade (now coming to a close). The past fifteen years have seen the historic LHC Runs I – III, as CEBAF transitioned from 6 to 12 GeV at Jefferson Lab.

- Starting with RHIC, many accelerator capabilities have been designed with QCD in mind, at JLab of course, and in the decade unfolding, the EIC. The LHC wasnt built for QCD, but the insightful designs of its detectors make it (of necessity) a powerful QCD machine.

- Over the past twenty years, QCD has brought nuclear and particle physics (back) together. Roads from Newport News and Upton lead to Geneva (and back).

- The specifically QCD experimental capabilities that will link the 2020s and the 2030s, including fixed target experiments at Fermilab, JLab, CERN and Brookhaven, have already paved the way for the Electron Ion Collider project, based on the demonstrated need for high statistics to reveal the structure of the nucleon, and high energy to unlock the dense gluonic matter from which the mass of the visible universe is generated.

- **That same energy is needed to provide a window into the emergence of hadronic from partonic matter.**
- **What our machines reveal:**



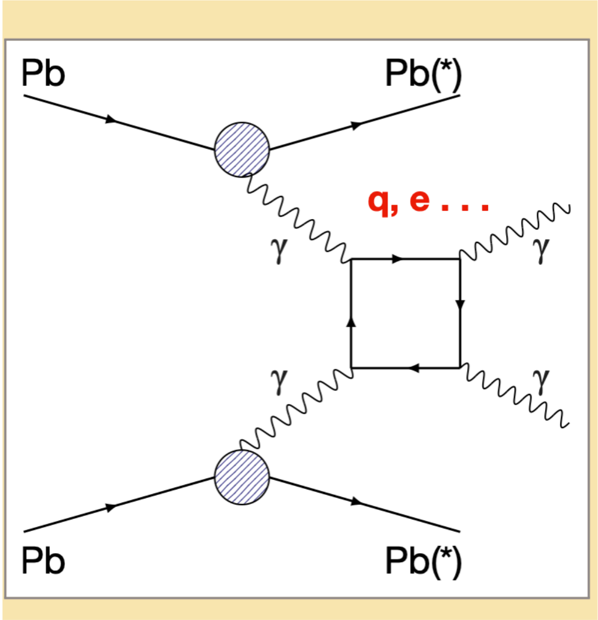
Very early on, quarks and gluons secluded themselves to a nearly vanishingly small, and ever-decreasing, proportion of space, occupying something like

one 10^{-45} th

of the volume of the observable universe.

- **From inside nuclei, the quarks speak to the outside world through the rest of the Standard Model. Nucleons and nuclei give electrons a reason to stick around and form the world we can see. We'll explore such "signals" in soft photon radiation.**

But the QCD degrees of freedom are always available, lurking in the vacuum, ready to lend a hand and work alongside the rest of the Standard Model, whenever enough energy arrives in the neighborhood.



• **Among our standard tools for interpreting such phenomena . . .**

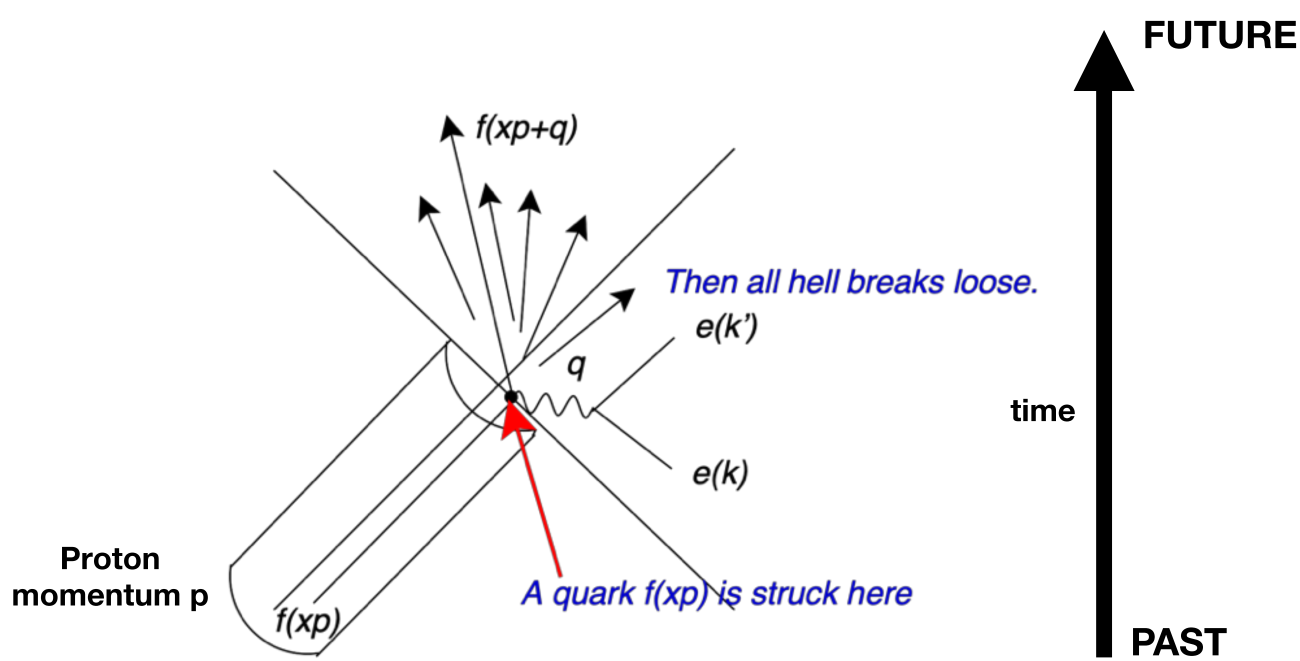
partonic process
(perturbative)

S = C x F

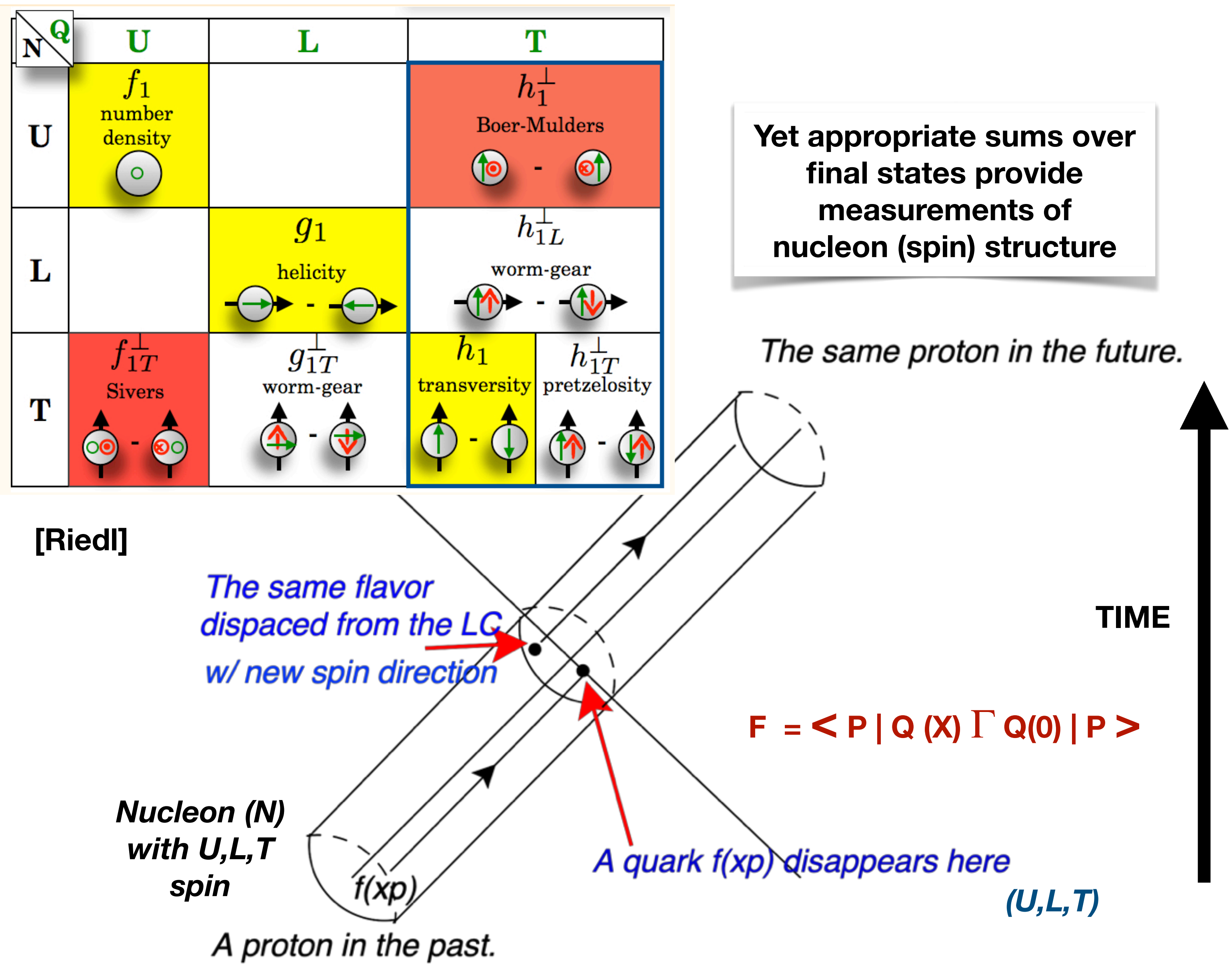
hadronic
matrix element
(nonperturbative)

The “universal” form of hard-scattering observables
“Factorization” for cross sections and amplitudes.

Picturing a typical “deep inelastic” ep event, in the usual variables



• **But unitarity comes to the rescue, leaving us with parton distributions . . .**

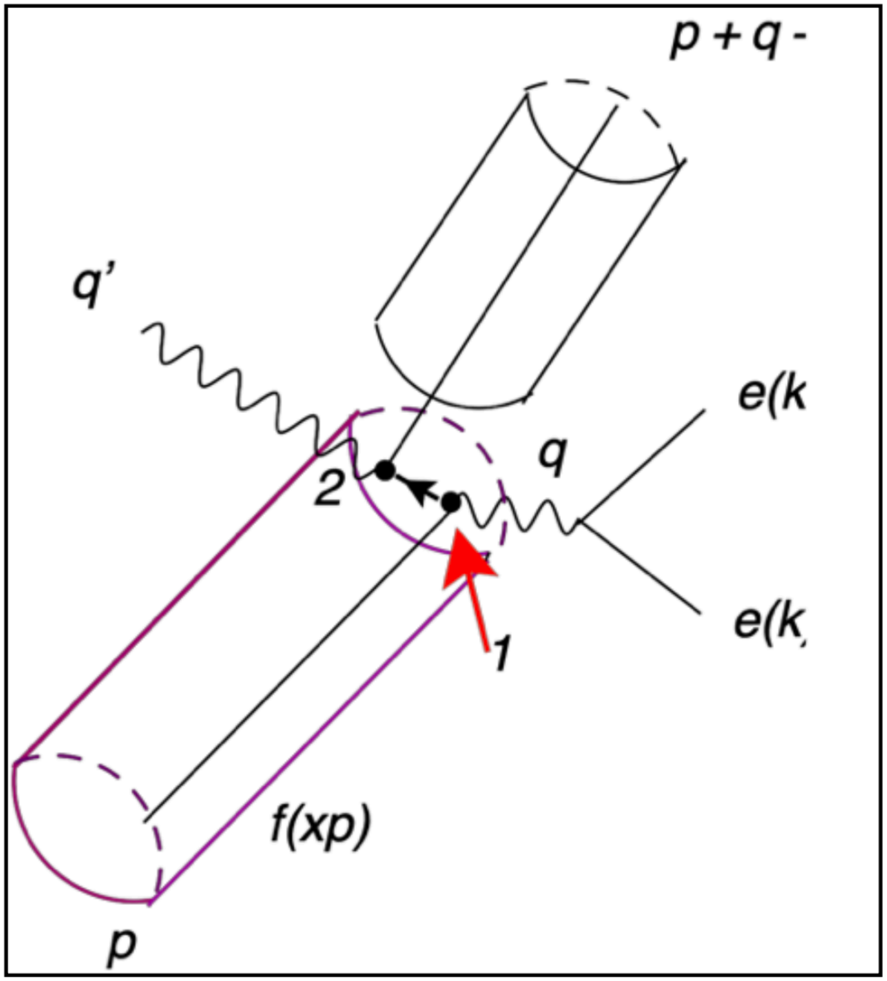


- **More and more these distributions are amenable to independent lattice calculations.**

Exclusive final states provide even more direct measurements.

If the quark is almost free between points 1 and 2 we measure

$F = \langle P + q \mid Q(x)\, \Gamma\, Q(0) \mid P \rangle$



- **Factorization depends in general on the choice of events**
  - **Very inclusive (DIS exemplary) washes out (in a well-defined way) potentially IR-sensitive corrections in the final state.**
  - **Very exclusive (Elastic form factors and DVCS exemplary) can have factorization "built in" for its external states**

- **In factorized cross sections or amplitudes, the dynamics of hadronization either cancels (inclusive) or itself factorizes (exclusive).**

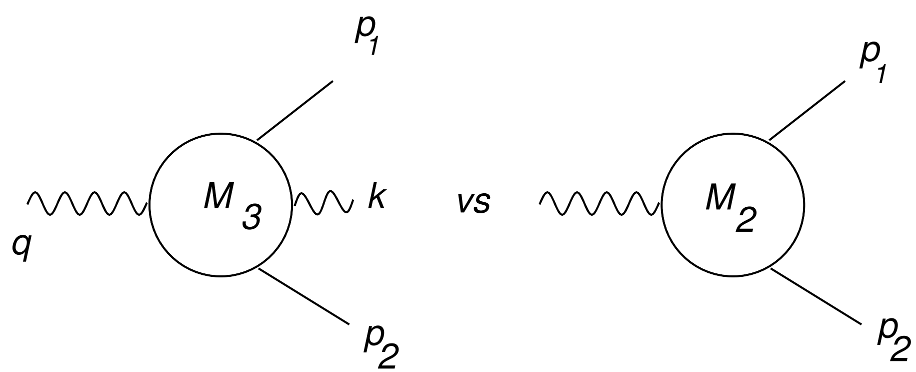
- **Is there any experimental access to the hadronization stage of evolution?**

- **Could QCD processes "signal" us through electromagnetic radiation?**

- **If so, how should we quantify this signal? It has been noticed that such a signal might exist, and that there is evidence for it. To see how, we need to go back to a famous result.**

2. Low's theorem and its descendents (the classic result)

$$M_{a+1}(\{p_i\}, k, \epsilon(k)) \;=\; \sum_{i=1}^{a} \delta_i \, e_i \, \frac{p_i^\mu}{p_i \cdot k} \; \left[\epsilon_\mu(k) \;-\; (k_\mu \epsilon^\nu(k) - \epsilon_\mu(k) k^\nu) \; \frac{\partial}{\partial p_i^\nu}\right] M_a(\{p_i\})$$



- **First term $\sim 1/\omega_k$, "soft photon theorem" (Identified in full generality by Weinberg 1965)**

- **Second term: Low (1954), Burnett & Kroll (1968). "Lorentz force".**
  This analysis required $\omega_k \ll m^2/\omega_{p_i}$.

Here, we'll mainly concern ourselves with the what the leading term predicts.

- **Low's theorem says that the soft radiative amplitude can be predicted directly from the non-radiative amplitude and its derivatives.**

- **Both the "leading power" $(p_i \cdot \epsilon / p_i \cdot k)$ *and* the first correction (ω_k^0) are determined by correspondence to classical radiation. [Liénard-Weichert potential and Lorentz force recoil, respectively.]**

- **A beautiful result, but it says soft photons tell us "nothing new". Really, that's not surprising – arbitrarily soft photons can't resolve short times or distances.**

- **Still, if charges propagate independently of each other over time scale τ, they can radiate photons of energies $\omega_k > 1/\tau$. At energies $\omega_k < 1/\tau$, this propagation becomes invisible.**

- **There may be several time scales τ_i, corresponding to charged particles of different masses (quarks!).**

- **The soft photon theorem really only applies for energies $\omega_k < 1/\tau_{\rm min}$.**

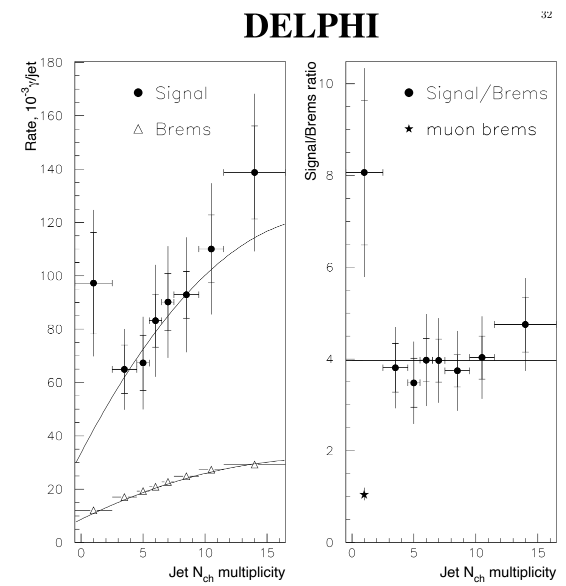
3, Beyond-Low soft photons? What the data say, and what it suggests

- **Low's analysis can be extended to multiparticle processes, and these predictions have been tested (for the leading power) in several high energy experiments involving hadrons at fixed-target energies, and by DELPHI at LEP1 [ex/0604038, 0901.4488, 1004.1587].** *Many* **have found an excess compared to the soft photon theorem.**
- **Cross section in the notation of DELPHI [ex/0604038] (for $e^+e^- \to Z^0 \to \mathrm{hadrons}+\gamma$):**

$$\frac{dN_\gamma}{d^3\vec{k}} = \frac{\alpha}{(2\pi)^2}\frac{1}{E_\gamma}\int d^3\vec{p}_1 ... d^3\vec{p}_N \sum_{i,j}\eta_i\eta_j\frac{-(P_iP_j)}{(P_iK)(P_jK)}\frac{dN_{hadrons}}{d^3\vec{p}_1...d^3\vec{p}_N}$$

- **The results, where "Brem" is the soft photon theorem expectation & "Signal" is data after subtracting hadronic decays, for various analyses show a factor of 3-4 above the prediction.**

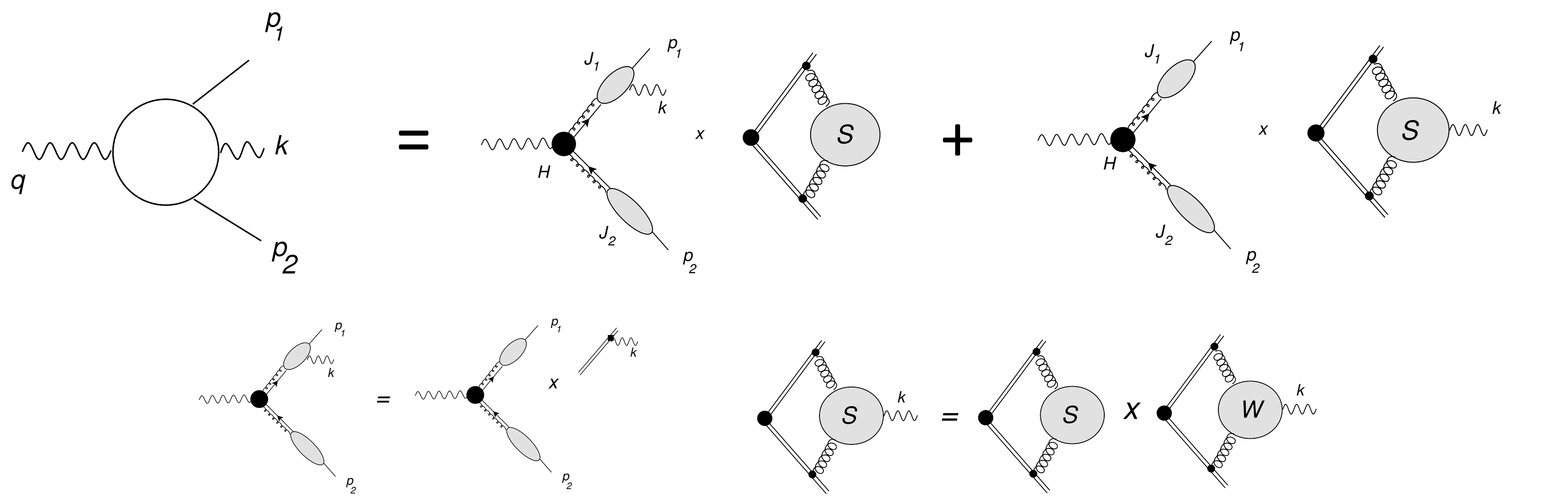
	Selection conditions	Signal	Brems
1	General selection	1.170±0.062	0.340±0.001
2	General selection, DURHAM	1.060±0.067	0.351±0.001
3	General selection, JADE	1.070±0.074	0.332±0.001
4	The zero experiment	0.069±0.048	0.0750±0.0002
5	No rejection of jets containing e^+, e^-	1.170±0.061	0.339±0.001
6	No rejection of jets containing e^+, e^-, DURHAM	1.050±0.066	0.348±0.001
7	Strong rejection of jets with e^+, e^-	1.150±0.062	0.326±0.001
8	Strong rejection of jets with e^+, e^-, DURHAM	1.050±0.067	0.336±0.001
9	General selection + anti-B tag	1.240±0.167	0.363±0.002
10	General selection + B tag	1.390±0.159	0.326±0.002
	General selection, signal corrected for efficiency	69.1±4.5	17.10±0.01



- What's going on?
- DELPHI tested for soft photons for $e^+e^- \to Z^0 \to \mu^+\mu^- + \gamma$ and the theorem works fine [0901.4488]. So it "must" be due to the presence of hadrons.
- Returning to the question of "how soft is soft"? Low's original analysis applied for $p_i \cdot k \ll m^2 \to \omega_k \ll m^2/\sqrt{s}$, with $m \sim m_\pi$. For Low, scattering was relativistic, but not yet "ultra". For analyses at really high energy, we had to wait for:
  - Gribov (1967), who showed that the theorem applies at leading power in a much larger region, where $k_T \ll m$ (with ω_k in the large range: $\sqrt{s} \gg \omega_k \gg m^2/\sqrt{s}$). He emphasized that at high energy
  $$p \cdot \epsilon / p \cdot k \sim 1/k_T.$$
  - Then Del Duca (1990) generalized the next-to-leading part to the same region. For more recent analyses at high energy, see van Beekveld et al. (2019).
- All of this is still for $k_T \ll m$. In this regime, virtual fermion loops behave like k_T/m, and contribute neither to leading nor to next-leading power in k. This is a big part of the analysis – only the external lines (charges e_i) contribute.
- But isn't $k_T \gg m_{u,d}$ when $k_T \sim$ MeV? How can we get a better idea?
- How about looking at massless QCD"?

4. A surprise at three loops (Yao Ma, GS, Aniruddha Venkata [2311.06912], PRL.)

- **Pair production for quark f with charge e_f in QCD with n_0 massless fermions.**
- **Schematic steps in the analysis (separate radiation from external lines (in J) and internal loops (in S)):**

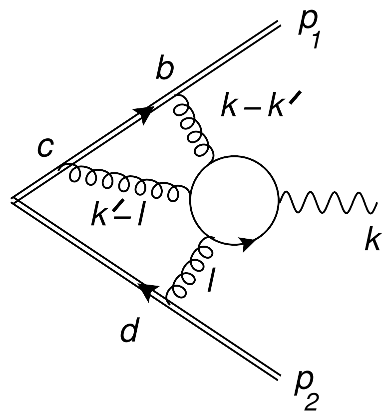


- **The result at leading power:**

$$M_3^{(f)}(\{p_i\},k,\epsilon(k)) \;=\; \left[e_f \;+\; \Gamma_{\rm EM}^{(f)}(\alpha_s)\left(\sum_{n=1}^{n_0} e_n\right)\right]\left[\frac{p_1\cdot\epsilon(k)}{p_1\cdot k} \;-\; \frac{p_2\cdot\epsilon(k)}{p_2\cdot k}\right] M_2^{(f)}(\{p_i\})\,.$$

- **Where the new quantity, $\Gamma_{\rm EM}$ is finite and real to all orders, even though individual diagrams are IR divergent. It's multiplied by the charges in the loops.**
- **It's gauge invariant, as it must be, and behaves like $1/k_T$.**

- **The very lowest order for Γ_{EM} is at α_s^3, and is given by the sum of diagrams with three gluons and one photon connected to a quark loop, like:**



- **Each diagram behaves like $1/(4-D)^5$ in dimensional regularization, but all the poles cancel in the sum. A strong confirmation of the all-orders claim.**
- **The result in massless QCD can be abstracted from the calculations for gluon emission ("soft gluon current") in Chen et al. [23309.03832] & Herzog et al. [2309.07884].**

$$\Gamma_{\mathrm{EM}}^{(F)} = -\left(\frac{\alpha_s}{\pi}\right)^3 C_F^{(3)} \left(-\frac{\zeta_2}{2} + \frac{\zeta_3}{6} + \frac{5\zeta_5}{6}\right) \sim \left(\frac{\alpha_s}{\pi}\right)^3 C_F^{(3)} \times 0.2$$

- **$C_F^{(3)} = 10/9$ is the "cubic Casimir", replaced by 4 in massless QED.**

5. Soft Photons and Correlations: What all this might mean for real QCD

- For k_T "low enough" $(\omega_k < m^2/\sqrt{s})$, we should still expect the "pure soft photon theorem". But experiments to date may not have been "low enough" in photon energy $(\sim 0.2 - 1GeV)$, given the masses of light quarks.

- For $k_T > m_f$ we do expect radiation from virtual quark EM currents involving flavor f.

- We haven't yet generalized the very simple example above to real QCD experiments, but the example is strong evidence that soft photons from virtual currents can be important in addition to those from external particles in the original soft photon theorem.

- If this is the case, the soft photons seen in DELPHI and previous experiments could have been messengers from the era of hadronization (Kharzeev & Loshaj (2013), Wong (2014)), or products of the influence of vaccum fluctuations (Botz, Haberal, Nachtman (1994)). The massless QCD analysis above provides a perturbative analog, perhaps a step toward a unified picture.

- Can we measure this in electron-proton scattering? Doesn't seem to have been done in DIS. The JLab to EIC energy range is open to discovery.

- To quantify the correlations of soft photons with hadrons of any (including no) charge, we might perhaps use a variant of QCD energy flow operators:

$$\mathcal{E}\left(\hat{n}\right)\,|X\rangle \;=\; \sum_{i\in X} k_i^0\,\delta^2\left(\Omega_{\vec{k}_i}-\hat{n}\right)\,|X\rangle$$

Correlations based on this energy-weighted operator have has the nice property of infrared safety; insensitivity to the nonperturbative hadronization.

- For a photon, we don't have IR problems if the energy is nonzero, and we can define a "number" operator at fixed momentum without energy weighting:

$$\mathcal{G}_\gamma\left(\hat{n}'\right)\,|X,k_\gamma\rangle \;=\; \sum_{\gamma\in X} \delta^2\left(\Omega_{\vec{k}_\gamma}-\hat{n}'\right)\,|X,k_\gamma\rangle$$

- Putting them together in a cross section, we get a QCD-energy flow/electromagnetic radiation correlation:

$$\begin{aligned}\sigma(p,p',\hat{n},\hat{n}') \;\;\equiv\;\; & \sum_X \int \frac{d^4k_\gamma}{(2\pi)^3}\,\delta_+(k_\gamma^2)\,\delta^2\left(\Omega_{\vec{k}_\gamma}-\hat{n}'\right) \\ & \times \sum_{i\in X} k_i^0\,\delta^2\left(\Omega_{\vec{k}_i}-\hat{n}\right)\,|\langle X,\gamma|p,p'\rangle|^2\,(2\pi)^4\delta^4(p+p'-p_X-k_\gamma)\end{aligned}$$

- The usual summing over final states with translation invariance and unitarity gives

$$\begin{aligned}\sigma(p,p',\hat{n},\hat{n}') &= \int \frac{d^4k_\gamma}{(2\pi)^3}\,\delta_+(k_\gamma^2)\,\delta^2\left(\Omega_{\vec{k}_\gamma}-\hat{n}'\right)\\ &\quad\times\int d^4y\, e^{-ik_\gamma\cdot y}\,\langle p,p'|\, j_{\rm em}(y)\cdot\epsilon_\gamma\,\mathcal{E}(\hat{n})\, j_{\rm em}(0)\cdot\epsilon_\gamma^*\,|p,p'\rangle\end{aligned}$$

- This could apply to any initial state $|a,b>$ including DIS.

- For photon energies of the order of $\sqrt{s}$, $\sigma(p,p',\hat{n}.\hat{n}')$ is IR safe. The observation of a soft photon, however, could lead to sensitivity to long-distance effects, which may (or may not) be predicted by the soft photon theorem using only the charged particles in the initial and final states.

- This is just an example. Perhaps data on such correlations can provide benchmarks against which models of hadronization can be tested.