Quantum algorithms for high energy evolution

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Motivation

JIMWLK: Jalillian-Marian,Iancu,Mclerran,Weigert,Leonidov and Kovner

- JIMWLK evolution equation: small-x observables
- Important to understand gluon saturation, high energy collisons
- Current method: Map to Langevin equation

J.P.Blaizot,E.Iancu,H.Weigert
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Problems

- Computationally expensive for better statistics
- Higher order JIMWLK lacks a Langevin formulation
- Not all observables can be evolved

Need a new method to simulate JIMWLK!

J.P.Blaizot,E.Iancu,H.Weigert
K.Rummukainen and H.Weigert

Lindblad-JIMWLK evolution

We propose a new algorithm to compute JIMWLK on quantum computers

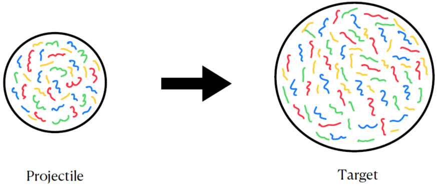
- Uses open quantum system methods
- Uses Lattice gauge theory methods
- Computationally much faster
- Can compute the evolution of the entanglement entropy and other "off-diagonal" observables

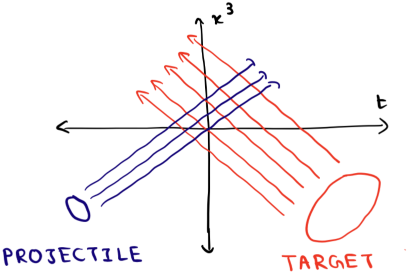
Toy model of JIMWLK for SU(2)

Lin, Lin (2024)
N.Kleo, J.Stryker, M.Savage (2019)

The setup

Projectile hadron colliding with target hadron

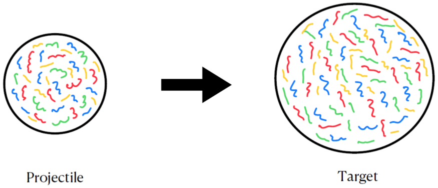


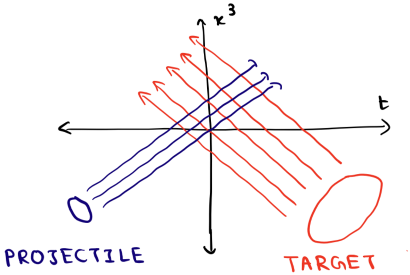


Color charges ρ_P interact with color charges ρ_T

The setup

Projectile hadron colliding with target hadron





Color charges distributed with a color charge density $W_Y[\rho]$

The setup

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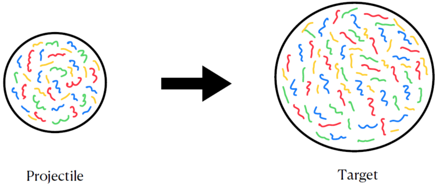
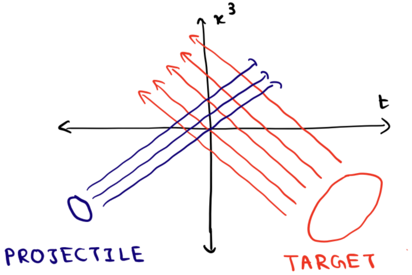
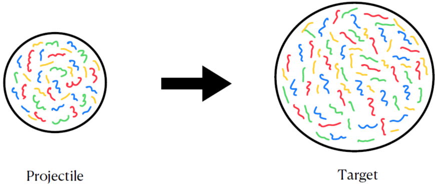


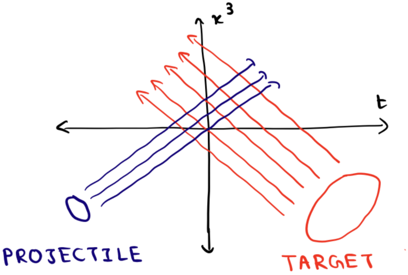


The projectile and the target have a rapidity gap Y

The setup

Projectile hadron colliding with target hadron

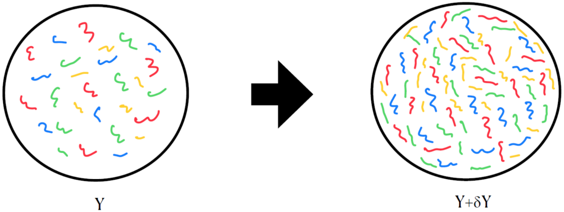




What if we increase the rapidity gap Y by boosting the projectile or the target?

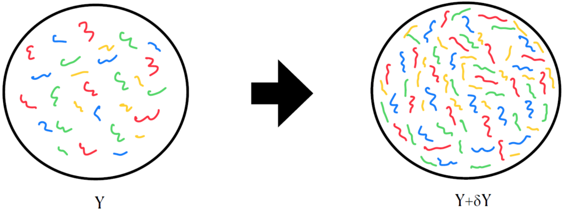
JIMWLK

Boosting $\Longrightarrow$ gluon radiation $\Longrightarrow$ change in target charge density



Boosting $\Longrightarrow$ gluon radiation $\Longrightarrow$ corrections to projectile

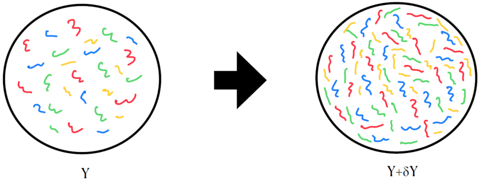
Change in color charge density $W_Y[\rho]$ or equivalently the density matrix of the target is **JIMWLK evolution**



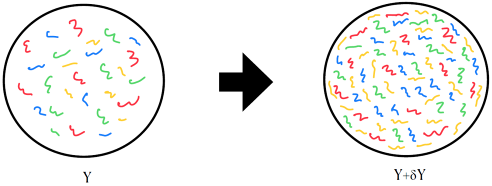
Can equivalently be understood as the “evolution” of how the projectile scatters on the target

Usual implementation: Random walk/ Langevin formulation

JIMWLK can be thought of as a random walk of the color charge distribution in the configuration space



Random walk 1

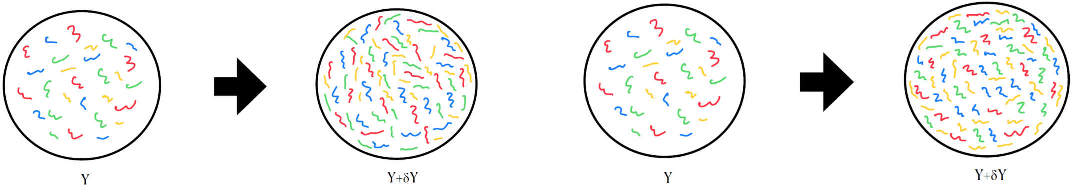


Random walk 2

Average over all random walks!

Usual implementation: Random walk/ Langevin formulation

Typically written as a random color rotation of wilson lines



Random walk 1

Random walk 2

$$V \longrightarrow S_1(\eta).V.S_2(\eta)$$

Average over gaussian parameter η to get evolved V

T.Altinoluk, G.Beuf, M.Lublinsky, V.V.Skokov (2024)

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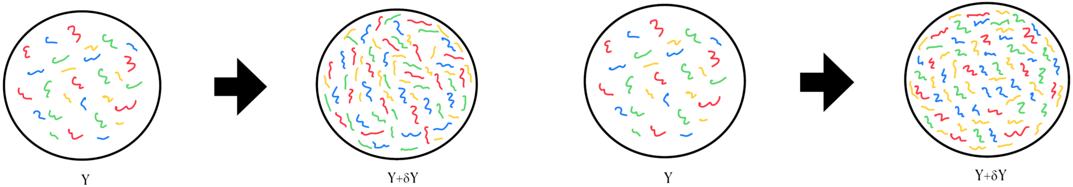


Random walk 1

Random walk 2

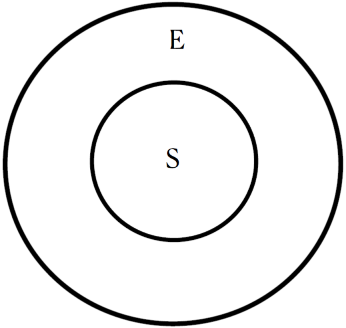
Shortcomings

- Cannot compute all observables. Ex: Dense-dense scattering, 2 gluon production with large rapidity separation
- Cannot evolve Next to leading log JIMWLK

T.Altinoluk, G.Beuf, M.Lublinsky, V.V.Skokov (2024)

An alternative

Write JIMWLK as a Lindblad evolution for open quantum systems



Lindblad evolution: Evolution of system density matrix ρ_S in the presence of environment

Interaction parameterized by **jump operators** Q_α

N.Armesto, F. Dominguez, A.Kovner, M.Lublinsky, V.V.Skokov (2019)

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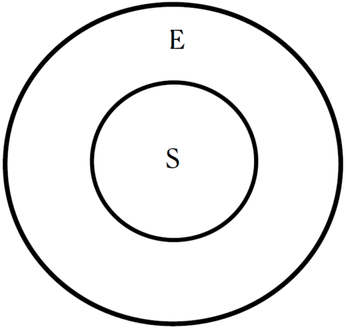


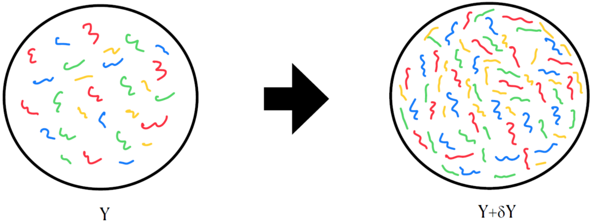
Lindblad evolution: Evolution of system density matrix ρ_S in the presence of environment

Interaction parameterized by **jump operators** Q_α

$$\partial_t \rho_S(t) = -[H, \rho_S(t)] + \boxed{\sum_\alpha Q_\alpha \rho_S(t) Q_\alpha^\dagger - \frac{1}{2}\{Q_\alpha^\dagger Q_\alpha, \rho_S(t)\}}$$

JIMWLK as Lindblad equation

Write JIMWLK as a Lindblad evolution for open quantum systems



time → rapidity

system → target

environment → QCD vacuum

$$\frac{d}{dY}\rho_T(Y) = \int \frac{d^2 z_\perp}{2\pi}\Big[Q_i^a[z_\perp],\Big[Q_i^a[z_\perp],\rho_T(Y)\Big]\Big]$$

N.Armesto, F. Dominguez, A.Kovner, M.Lublinsky, V.V.Skokov (2019)

What do we want?

Ultimate goal: Simulate the full JIMWLK equation starting with an initial density matrix

- Impossible on a quantum computer
- Need a discretization/truncation of field space
- Work in reduced dimensions
- Reduce color space to SU(2)

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Reduced goal: Simulate the reduced JIMWLK equation in 0 dimensions (2 points) with SU(2), starting with an initial density matrix.

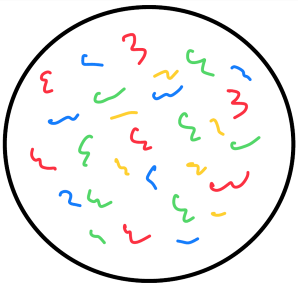
Two types of basis

Reduced goal: Simulate the reduced JIMWLK equation in 0 dimensions (2 points) with SU(2), starting with an initial density matrix.

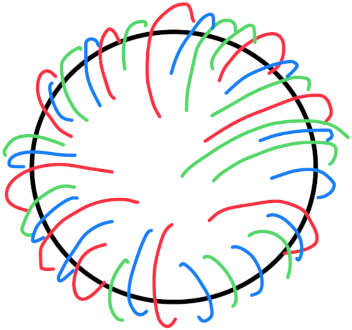
- Field value basis
- Representation/Angular momentum basis

We would like to see which of these basis choices give us results consistent with the langevin evolution of dipole expectation value

Field value basis



Charge density

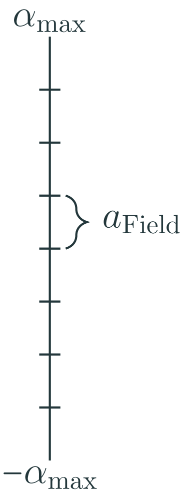


Gluon fields

Since the hadron is moving along x^- with large rapidity, $A^- \equiv \alpha$ is the dominant component

Field value basis

- Truncate at α_{max}
- Discretize with spacing a_{Field}
- 3 colors
- Simplest case: two field values

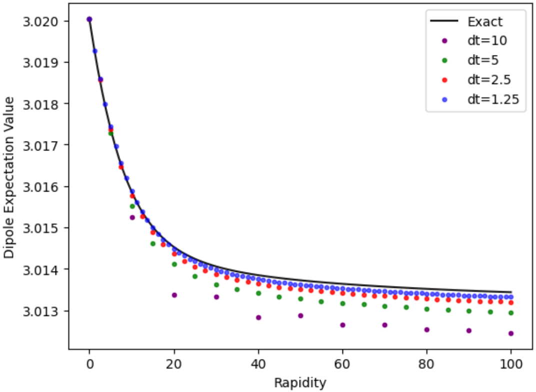


Hilbert space :

$$|\alpha\rangle = |\underbrace{\alpha^1(1_\perp), \alpha^2(1_\perp), \alpha^3(1_\perp)}_{\text{1st position}}, \underbrace{\alpha^1(2_\perp), \alpha^2(2_\perp), \alpha^3(2_\perp)}_{\text{2nd position}}\rangle$$

Evolution

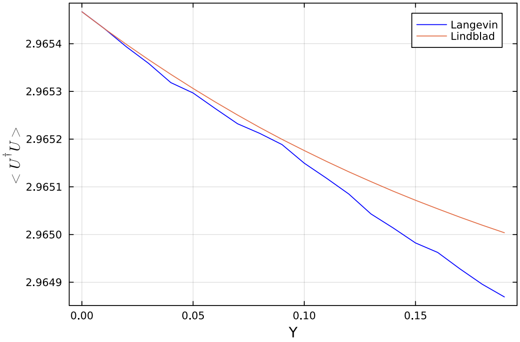
- Compute jump operators in the dilute limit
- Choose an arbitrary initial condition
- Use Lin-Lin scheme to evolve Lindblad (non-unitary)

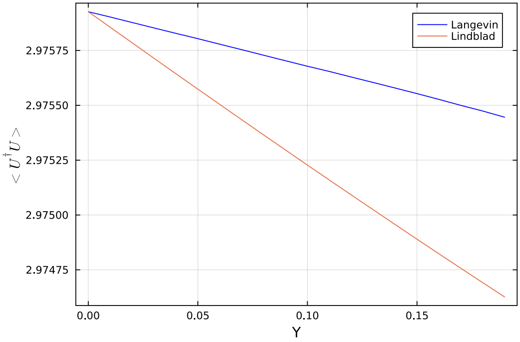


Lin, Lin (2024)

Issues with field basis

But we have problems!

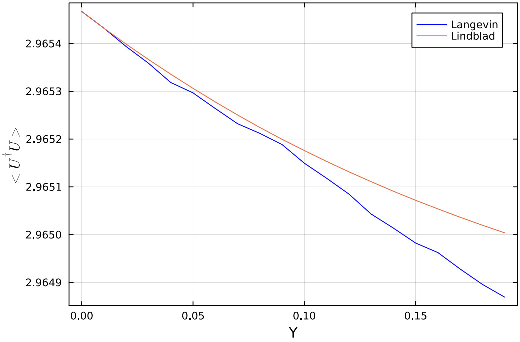


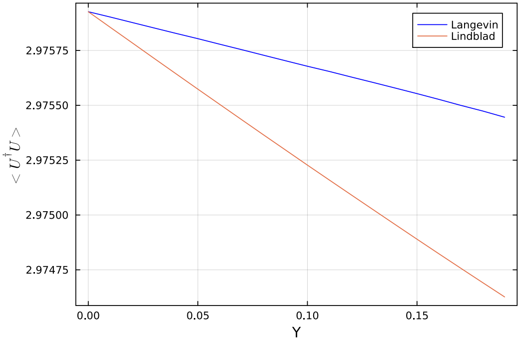


Match not perfect: Discretization effects!!

Issues with field basis

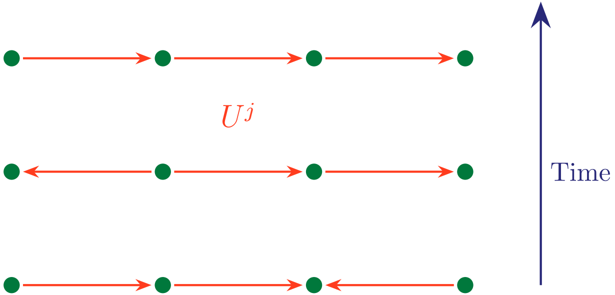
But we have problems!





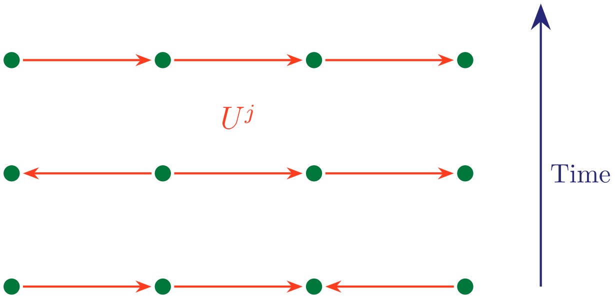
Change the basis!!

Hamiltonian lattice gauge theory



Space: Discretized, Time: Continuous

Hamiltonian lattice gauge theory

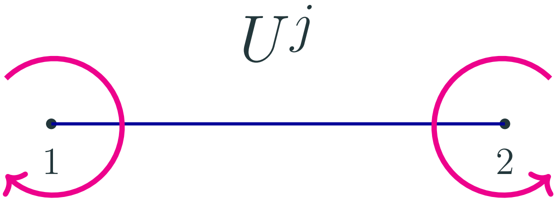


Consider pure Yang Mills

T. Brynes, Y. Yamamoto (2006)

Hilbert space: Angular momentum basis

- j: Angular momentum
- m: Left rotation
- n: Right rotation



$$|j, m, n\rangle = \mathbf{U}^j_{mn} |0\rangle$$

$$\mathrm{U}^j \rightarrow V(1)\, U^j\, V^{-1}(2)$$

T. Brynes, Y. Yamamoto (2006)

Hilbert space: Angular momentum basis

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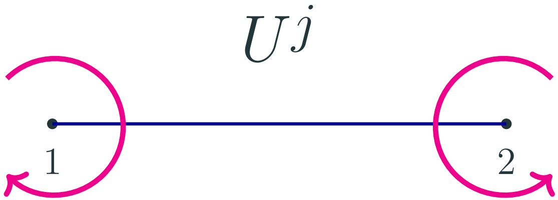
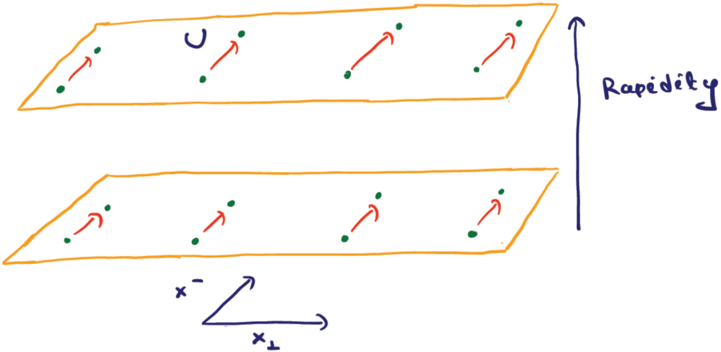


$$\langle \alpha | j, m, n \rangle \propto U^j_{mn}(\alpha)$$

$$\mathrm{U}^j \to V(1)\, U^j\, V^{-1}(2)$$

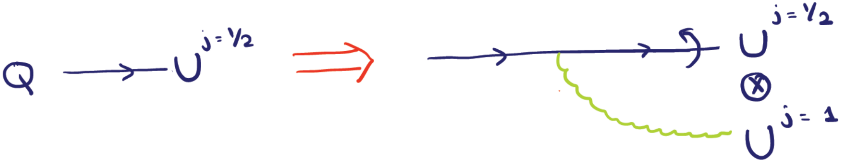
T. Brynes, Y. Yamamoto (2006)

Mapping to space of JIMWLK evolution



$$|q\rangle = \bigotimes_{x_\perp} |j_q^x, n_{Lq}^x, n_{Rq}^x\rangle = \bigotimes_{x_\perp} U^{j_q^x}_{n_{Lq}^x, n_{Rq}^x} |0\rangle$$

Jump operators



$$U^{j'}_{m_L m_R}(\alpha, x)\, U^{j}_{n_L\, n_R}(\alpha, x) \;=\; \sum_{J=|j-j'|}^{|j+j'|} C^{J,M_L}_{j,m_L\, j' n_L}\, C^{J,M_R}_{j m_R\, j, m_R}\, U^{J}_{M_L\, M_R}(\alpha, x)$$

Can compute $\langle q|Q^a|p\rangle$

What do we want?

- Work with a maximum j: j_{max}
- Compute lindblad-JIMWLK evolution
- Compare with Langevin evolution

Problem 1: They both need to have the same initial condition

What do we want?

- Work with a maximum j: j_{max}
- Compute lindblad-JIMWLK evolution
- Compare with Langevin evolution

Problem 2: Langevin evolves "$|\alpha\rangle$" basis only

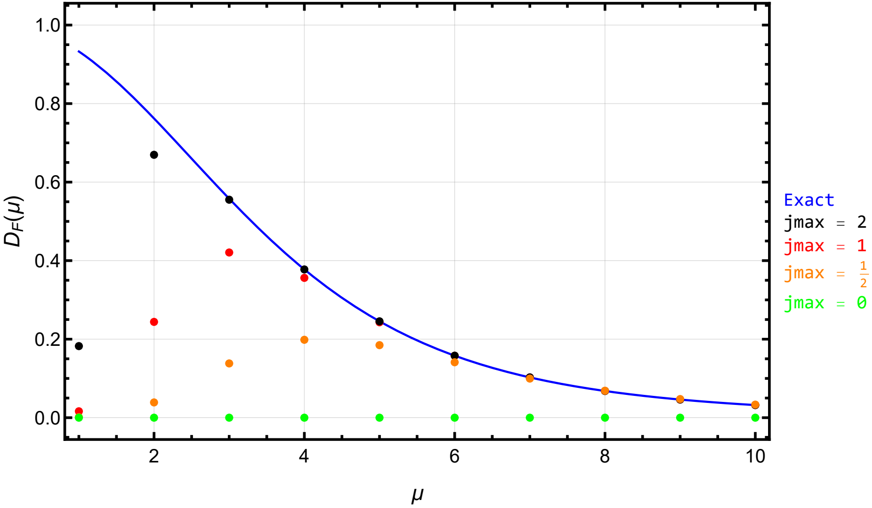
What do we want?

- Work with a maximum j: j_{max}
- Compute lindblad-JIMWLK evolution
- Compare with Langevin evolution

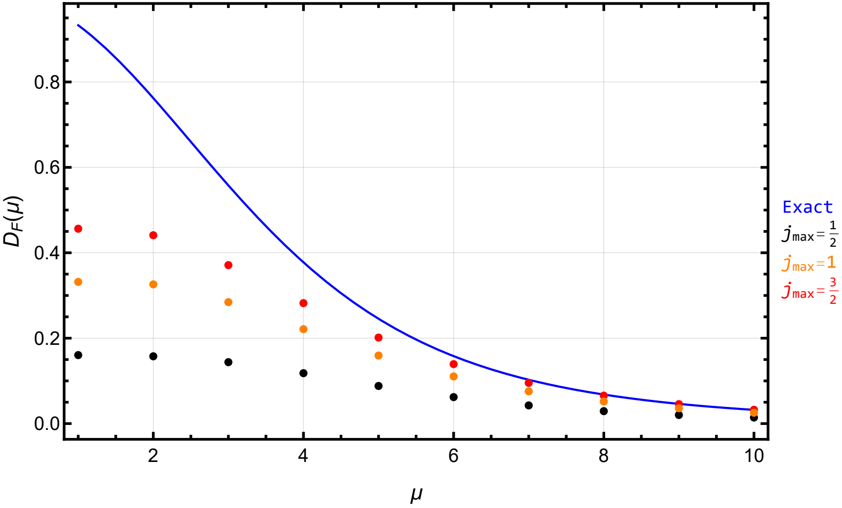
Solution: Work with a gaussian density matrix in $|\alpha\rangle$ and do basis transformation

$$\langle\alpha|\rho|\alpha\rangle \approx e^{-\frac{4\,\alpha^2}{\mu^2}}$$

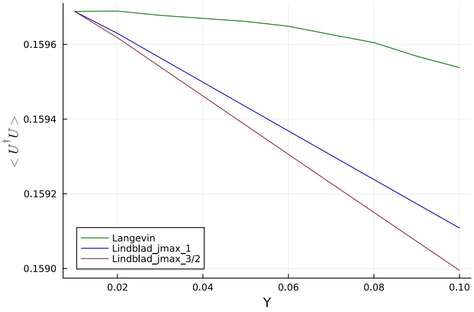
Convergence of initial condition: Pure State



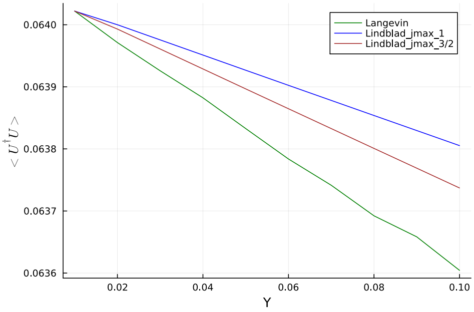
Convergence of initial condition: Mixed State



Evolution

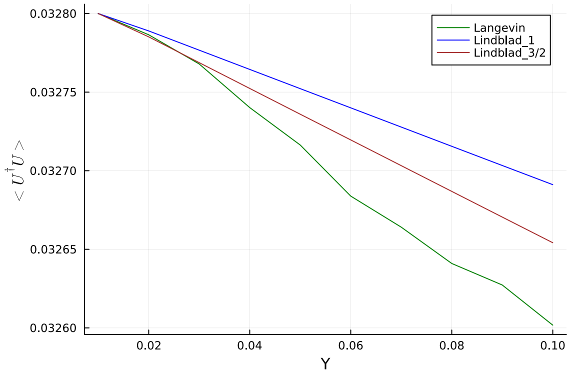


$\mu = 6$



$\mu = 8$

Evolution



$\mu = 10$

Higher μ is well-approximated by the angular momentum basis

Summary

JIMWLK in 2-dimensions (SU(3))

Reduce dimensions $\downarrow$ 2 points

JIMWLK in 0-dimensions (SU(3))

Reduce generators $\downarrow$ 3 colors

JIMWLK in 0-dimensions (SU(2))

Truncate field space $\downarrow$ α_{max}/j_{max}

QM JIMWLK in 0-dimensions (SU(2))

Outlook

- Find ways to decrease μ_{cut}: Gauge invariant basis?
- Compute other observables like entanglement entropy
- Generalize to 1+1 dimensions and ultimately higher dimensions
- Carry out NLO JIMWLK evolution and helicity-dependent JIMWLK evolution