Two-Photon Exchange Effects for the Azimuthal Asymmetries of SIDIS Cross Section

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Coordination with Harut Avakian (JLab) is acknowledged

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- Motivation & Introduction
- Background
- Assumptions & Calculations
- Results
- Conclusions

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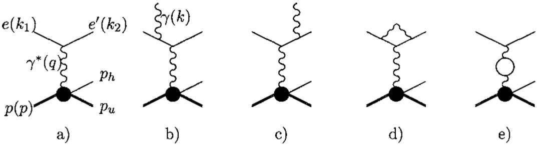
- Photon emission is a part of any electron scattering process: accelerated charges radiate;
- Exclusive electron scattering processes such as $p(e,e'h_1)h_2$ are actually inclusive $p(e,e'h_1)h_2\ n\gamma$, where an infinite number of low-energy photons can be generated
- Low-energy photons do not affect polarization observables, thanks to Low theorem

QED Corrections for Electroproduction of Pions

Afanasev, Akushevich, Burkert, Joo, Phys.Rev.D**66**, 074004 (2002)

- Conventional RC, precise treatment of phase space, no peaking approximation, no dependence on hard/soft photon separation; extension to DVMP is straightforward;
- Can be used for any exclusive electroproduction of 2 hadrons, e.g., d(e,e'p)n (EXCLURAD code)



- Fortran code EXCLURAD is available at www.jlab.org/RC
- Used for data analysis at JLab, COMPASS, MAMI,...

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QED Corrections for (Exclusive) Electroproduction of Pions

Sample results from EXCLURAD

- QED corrections to unpolarized cross sections reach tens of per cent
- Corrections are dependent on both polar and azimuthal angles of outgoing hadron (pion), which affects extraction of resonance parameters in the resonance region and GPDs in the deep-virtual region

AFANASEV *et al.* PHYSICAL REVIEW D **66**, 074004 (2002)

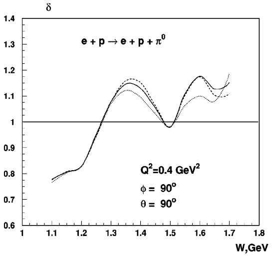

FIG. 3. *W* dependence of RC to the cross section of neutral pion

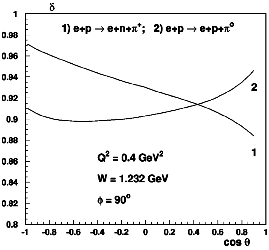

FIG. 5. RC to the cross section as a function of cos θ.

- QED corrections due to real-photon emission are smaller for polarization asymmetries

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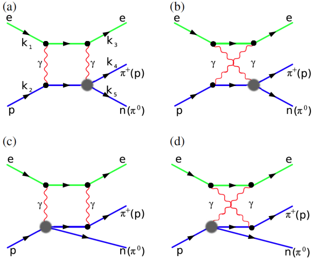
- Ge/Gm polarization vs Rosenbluth discrepancy is agreed to be partly due to two-photon exchange (resulting from about 5 per cent missing systematic correction at high momentum transfers (see for review A Afanasev, PG Blunden, D Hasell, BA Raue, Prog. Part. Nucl. Phys., 2017
- JLab experiment Katich et al., Phys.Rev.Lett. 113 (2014)022502 reveals about 5 per cent polarization asymmetries in DIS on 3He that are zero in one-photon exchange approximation
- Proposed positron beamline at JLab will provide a direct probe for two-photon effects via measurements of electron-positron asymmetries

Two-Photon Exchange Corrections for Electroproduction of Pions

Afanasev, Aleksejevs, Barkanova, Phys.Rev. D88: 053008, 2013

- Calculated previously neglected QED corrections from two-photon exchange
- Used a soft-photon approximation, results expressed in terms of Passarino-Veltman integrals



- Computed corrections result in about 5 per cent variation of cross section from backward to forward scattering angles
- Conclusion: Important for the analysis of angular dependences, $\cos(\phi)$ moments in particular

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Two-Photon Exchange Corrections for Electroproduction of Pions

Afanasev, Aleksejevs, Barkanova, Phys.Rev. D88: 053008, 2013

- Angular dependencies of two-photon corrections affect σ_L/σ_T extraction

AFANASEV, ALEKSEJEVS, AND BARKANOVA PHYSICAL REVIEW D **88,** 053008 (2013)

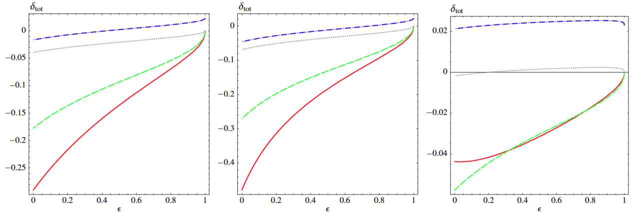

FIG. 5 (color online). π^0 electroproduction two-photon box correction (for detected proton) dependencies on virtual photon degree of polarization parameter ϵ for momentum transfers $Q^2 = 3.0\ \mathrm{GeV}^2$ (left plot), $Q^2 = 7.0\ \mathrm{GeV}^2$ (middle plot), and $Q^2 = 0.4\ \mathrm{GeV}^2$ (right plot). All plots are given for $\phi_4 = 90°$ and $\theta_4 = 90°$ and $W = 1.232$ GeV. Dot-dashed curve, SPT; dotted curve, SPT with $\alpha\pi$ subtracted; dashed curve, SPMT; solid curve, FM approach.

- These effects can be directly measured with proposed positron beamline at JLab
- Two-photon correction times two = electron-positron scattering asymmetry

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Theory Challenges

- Both soft and hard photons are present
- Soft photons do not resolve the quark/parton structure
- Soft/hard scale separation is necessary in the loop integral
- We used Grammer-Yennie procedure for soft/hard separation - as in AV Afanasev, SJ Brodsky, CE Carlson, YC Chen, M Vanderhaeghen, PRD72, 013008 (2005)
- The results become dependent on soft-hard separation scheme, QED and QCD have to be consistently combined
- Not all of the contributions are factorizable in terms of GPDs

Semi-Inclusive electroproduction and TMD studies

x-section for eN→e'hX assuming one-photon exchange

from Bacchetta et al, 1703.10157

$$\frac{d\sigma}{dx\,dy\,d\phi_S\,dz\,d\phi_h\,dP_{h\perp}^2} = \frac{\alpha^2}{x\,y\,Q^2}\,\frac{y^2}{2\,(1-\varepsilon)}\Big\{F_{UU,T} + \varepsilon\,F_{UU,L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h} + \varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h}$$
$$+ \lambda_e\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{LU}^{\sin\phi_h} + S_L\Big[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\,F_{UL}^{\sin\phi_h} + \varepsilon\sin(2\phi_h)\,F_{UL}^{\sin 2\phi_h}\Big]$$
$$+ S_L\,\lambda_e\Big[\sqrt{1-\varepsilon^2}\,F_{LL} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_h\,F_{LL}^{\cos\phi_h}\Big]$$
$$+ S_T\Big[\sin(\phi_h-\phi_S)\Big(F_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon\,F_{UT,L}^{\sin(\phi_h-\phi_S)}\Big) + \varepsilon\,\sin(\phi_h+\phi_S)\,F_{UT}^{\sin(\phi_h+\phi_S)}$$
$$+ \varepsilon\,\sin(3\phi_h-\phi_S)\,F_{UT}^{\sin(3\phi_h-\phi_S)} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\,F_{UT}^{\sin\phi_S}$$
$$+ \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\,F_{UT}^{\sin(2\phi_h-\phi_S)}\Big] + S_T\lambda_e\Big[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)}$$
$$+ \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\,F_{LT}^{\cos\phi_S} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos(2\phi_h-\phi_S)\,F_{LT}^{\cos(2\phi_h-\phi_S)}\Big]\Big\}$$

SIDIS phenomenology based on several assumptions[1], including:

- One-photon exchange dominates;
- Transverse photon cross section dominates, and F_{UU}^{L} can be ignored.

[1] Bacchetta et al. JHEP06(2017)081

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TMDs in SIDIS

Assuming the one-photon exchange and dominance of the transverse photon. SIDIS phenomenology for last decades was extracting the underlying transverse momentum dependent (TMD) distribution and fragmentation functions from multiplicities and single spin asymmetries in SIDIS.

Analysis of multiplicities was done based on factorization of the x-section from transverse part.

TMD Parton Distribution Functions

TMD Parton Fragmentation Functions

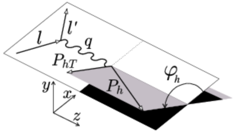
$$F_{UU,T}(x, z, \boldsymbol{P}_{hT}^2, Q^2) = x \sum_q \underbrace{\boxed{\mathcal{H}_{UU,T}^q(Q^2, \mu^2)}}_{\text{hard scattering}} \int d^2\boldsymbol{k}_\perp \, d^2\boldsymbol{P}_\perp \, f_1^a(x, \boldsymbol{k}_\perp^2; \mu^2) \, D_1^{a\to h}(z, \boldsymbol{P}_\perp^2; \mu^2) \, \delta(z\boldsymbol{k}_\perp - \boldsymbol{P}_{hT} + \boldsymbol{P}_\perp) + Y_{UU,T}(Q^2, \boldsymbol{P}_{hT}^2) + \mathcal{O}(M^2/Q^2)$$

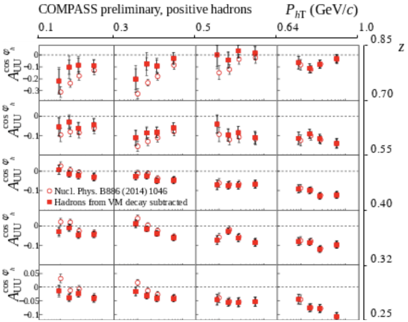
from Bacchetta et al, 1703.10157

Several JLab proposals focus on extraction of the longitudinal photon contributions.

No measurement so far available for evaluation of systematics from two-photon exchange.

TMDs in SIDIS, Azimuthal modulation





COMPASS data

Considering the correction δ^{TPE},

$$\frac{d\sigma_{tot}}{dxdzdQ^2d^2P_T} \equiv d\sigma_{tot} = d\sigma_{exp}/(1+\delta^{TPE})$$
$$\sim (1-\delta^{TPE})\{K(y)[(1+\epsilon\frac{F_{UU,L}}{F_{UU,T}}) + \sqrt{2\epsilon(1+\epsilon)}\cos 2\phi\frac{F_{UU}^{\cos(2\phi)}}{F_{UU,T}}$$
$$+\epsilon\cos\phi\frac{F_{UU}^{\cos\phi}}{F_{UU,T}}]\} \tag{1}$$

with x is Bjorken-x, transverse momentum of the detected meson P_T, Q^2 relates to the momentum transfer of the virtual photon.

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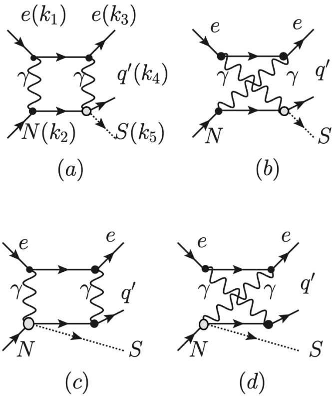
Assumptions & Calculations

$$e(k_1) + N(k_2) \to e(k_3) + q'(k_4) + S(k_5),$$



For quark-diquark model, q' represents quark and S represents diquark.

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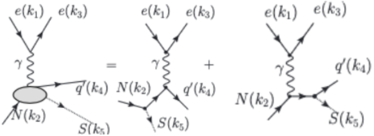
Assumptions & Calculations



Born-level one photon models, which equals to the sum of the "quark graph" and the "proton pole graph". q' and S stand for quark and diquark.[2]

[2] Afanasev and Carlson, Phys. Rev. D 74.114027 (2004)

Using soft-photon approximation (SPT [3]) by neglecting the momentum for one of the photon while calculating the amplitude, such that

$$\begin{aligned} M^{2\gamma} =& M^{1\gamma} \cdot \sum_{l} [\frac{-e^2}{2\pi} \cdot \sum_{i,j} (2k_i \cdot k_j) \\ & \cdot C_0(\{k_i, m_i\}, \{\mp k_j, m_j\}) \\ =& \sum_{l=N,q',s} \sum_{i=a,b,c} M^{1\gamma} M_{l,i,box}, \end{aligned} \quad \begin{matrix} (2) \\ \\ (3) \end{matrix}$$

where the Passarino-Veltman three-point scalar integral

$$C_0(\{k_i, m_i\}, \{k_j, m_j\}) = \frac{1}{i\pi^2} \int d^4q \frac{1}{q^4} \cdot \frac{1}{(k_i - q)^2 - m_i^2} \cdot \frac{1}{(k_j - q)^2 - m_j^2}. \quad (4)$$

The correction

$$\delta_{box} = \frac{2Re[M^{2\gamma} M^{1\gamma\dagger}]}{|M^{1\gamma}|^2} = 2Re[\sum_{l;i} M_{l,i,box}]. \quad (5)$$

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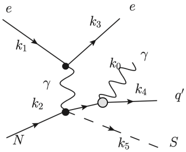
Assumptions & Calculations

Infrared divergence of two-photon exchange is canceled by interference between emission from electron lines and hadron lines (hadron Bremsstrahlung)



One of the possibilities for the hadron Bremsstrahlung process [4].

[4] Afanasev et al. Phys. Rev. D 88, 053008 (2013)

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Results

- $E_{lab} = 10.6$ GeV;
- $Q^2 \approx 2.5$ GeV^2;
- $y < 0.75$ to avoid the region most susceptible to radiative effects and lepton-pair symmetric background;
- $x = 0.31$ (the invariant mass $W \approx 2.7$ GeV);
- $z = 0.5$;
- The polar angle of the detected meson is $\cos\theta = 0.8$ ($P_T \approx 0.35$) for P_T independent figures;
- The azimuthal angle of the detected meson is defined as $\phi = \pi/6$ for the figures that are ϕ independent; SF from Lund model
- $F_{UU,L}/F_{UU,T} \approx 0.2$;
- $F_{UU}^{\cos\phi}/F_{UU,T} \approx -0.05$;
- $F_{UU}^{\cos(2\phi)}/F_{UU,T} \approx 0.1$.

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Results (see arXiv:2504.17123)

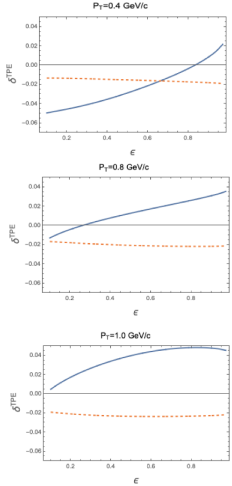

FIG. 5. TPE correction δ^{TPE} as a function of ϵ for fixed values of transverse momentum $P_T = 0.4, 0.8, 1.0$ (GeV/c) at $z = 0.7$, $Q^2 = 2.5$ GeV^2, $x = 0.31$, and $\phi = \pi/6$. The blue solid line and the orange dashed line indicate the detected meson is π^+ and ρ^+, respectively.

Results (see arXiv:2504.17123)

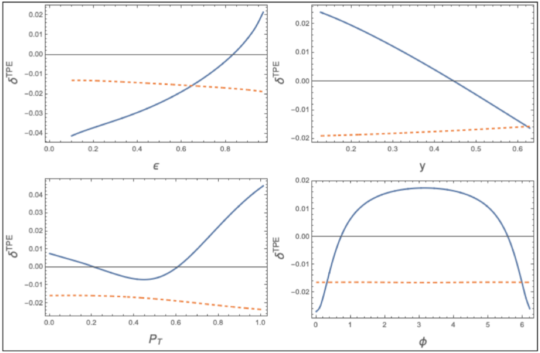

FIG. 6. Dependence of TPE correction δ^{TPE} on the virtual photon ϵ, electron's relative energy loss y, transverse momentum P_T, and azimuthal angle ϕ with $E_{lab} = 10.6$ GeV, $Q^2 = 2.5$ GeV^2, the mean value $\langle x_{BJ} \rangle = 0.31$, $z = 0.7$, using kinematics for projected experiments [29, 30]. The blue solid line and the orange dashed line represent the detected mesons are π^+ and ρ^+ meson, respectively.

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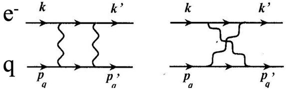
Next Step: Hard Two-Photon Exchange

- Hard TPE on a parton
- Remove Soft-Photon Exchange at parton level and include it at hadronic level following AA, Brodsky, Carlson, Chen, Vanderhaeghen, PRL 93:122301,2004; PRD 92:013008,2005
- Two-photon amplitude for a (massless) quark



$$A^{2\gamma}_{eq\to eq} = \frac{e_q^2}{t}\frac{\alpha_{em}}{2\pi}(V_\mu^e \otimes V_\mu^q \times f_V + A_\mu^e \otimes A_\mu^q \times f_A),$$

$$V_\mu^{e,q} = \bar{u}_{e,q}\gamma_\mu u_{e,q},\ A_\mu^{e,q} = \bar{u}_{e,q}\gamma_\mu\gamma_5 u_{e,q}$$

$$f_V = -2[\log(-\frac{u}{s}) + i\pi]\log(-\frac{t}{\lambda^2}) - \frac{t}{2}[\frac{1}{s}(\log(\frac{u}{t}) + i\pi) - \frac{1}{u}\log(-\frac{s}{t})] +$$
$$+\frac{(u^2-s^2)}{4}[\frac{1}{s^2}(\log^2(\frac{u}{t}) + \pi^2) + \frac{1}{u^2}\log(-\frac{s}{t})(\log(-\frac{s}{t}) + i2\pi)] + i\pi\frac{u^2-s^2}{2su}$$

$$f_A = -\frac{t}{2}[\frac{1}{s}(\log(\frac{u}{t}) + i\pi) + \frac{1}{u}\log(-\frac{s}{t})] +$$
$$+\frac{(u^2-s^2)}{4}[\frac{1}{s^2}(\log^2(\frac{u}{t}) + \pi^2) - \frac{1}{u^2}\log(-\frac{s}{t})(\log(-\frac{s}{t}) + i2\pi)] + i\pi\frac{t^2}{2su}$$

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Soft+Hard Two-Photon Exchange: Results

- Addition of "hard" TPE appears to *reduce* the total effect; ($Q^2 \approx 3.7$ GeV^2; $x_{BJ} = 0.31$; $z = 0.7$; $P_T \approx 0.3$ for P_T independent figures; $\phi = \pi/6$ for ϕ independent figures. Notice stronger dependence on electron's variables ϵ, y)

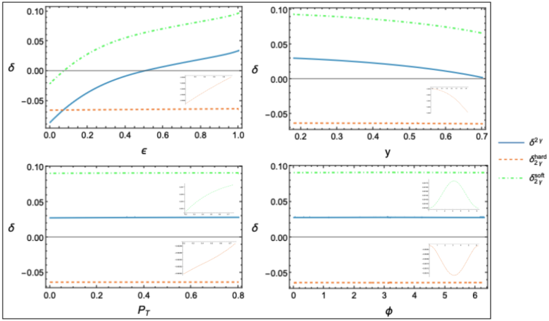

FIG. 5: The orange dashed curves represent calculations based on the hard-photon exchange contribution, $\delta_{2\gamma}^{hard}$. The dot-dashed green curves indicate the soft-photon contribution, $\delta_{2\gamma}^{soft}$. The blue solid curves show the total corrections, representing the full TPE correction $\delta^{2\gamma}$.

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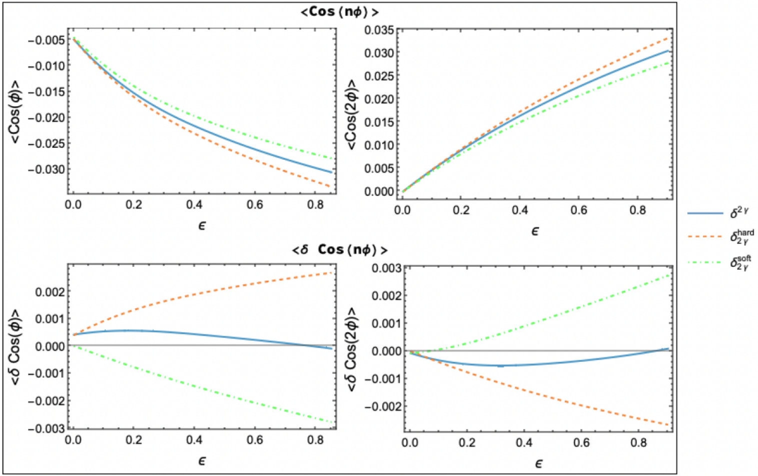
Soft+Hard Two-Photon Exchange: Results

- Addition of "hard" TPE *reduces* the total effect for the azimuthal asymmetries, as well



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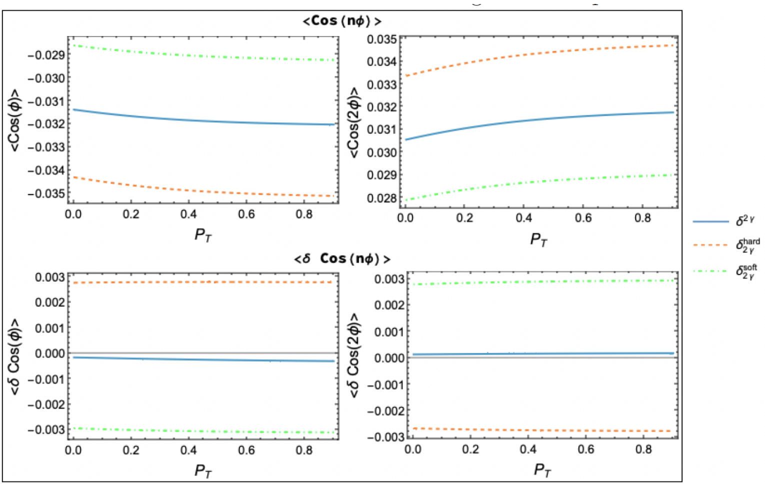
Soft+Hard Two-Photon Exchange: Results

- Negligible variation with P_T



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Conclusion

- Two-photon exchange (TPE) effects alter angular dependence of cross sections in DVMP and SIDIS
- Their measurement is necessary for extracting GPDs, TMDPDFs and FFs, can be done with *positron beams* at CEBAF
- TPE corrections computed for projected JLab measurements of L/T separation and azimuthal asymmetries, TPE corrections of the two-photon exchange are in the range of $\sim \pm 5\%$ for y, ϵ & P_T dependence; $\cos(\phi), \cos(2\phi)$ are affected by $\leq$0.5%

- Addition of hard two-photon exchange at a parton level *reduces* the total correction
- Next steps: Spin asymmetries in SIDIS; "hard" TPE for (exclusive) DVMP

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