Meson electroproduction at very high transverse momentum

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QCD EVOLUTION 2025
Hosted by Jefferson Lab

19 December—23 December 2025, Newport News, VA, USA

Based on old and new work with Andrei Afanasev, Christian Wahlquist, and others

Meson electroproduction at very high p

- Part I: Intro for $e + p \to e + \text{meson} + X$
- Part II: Generalities
- Part III: Direct pion production
- Part IV: Other processes
- Part V: What might we learn
- Part VI: The end

Semi-Exclusive Deep Inelastic Scattering

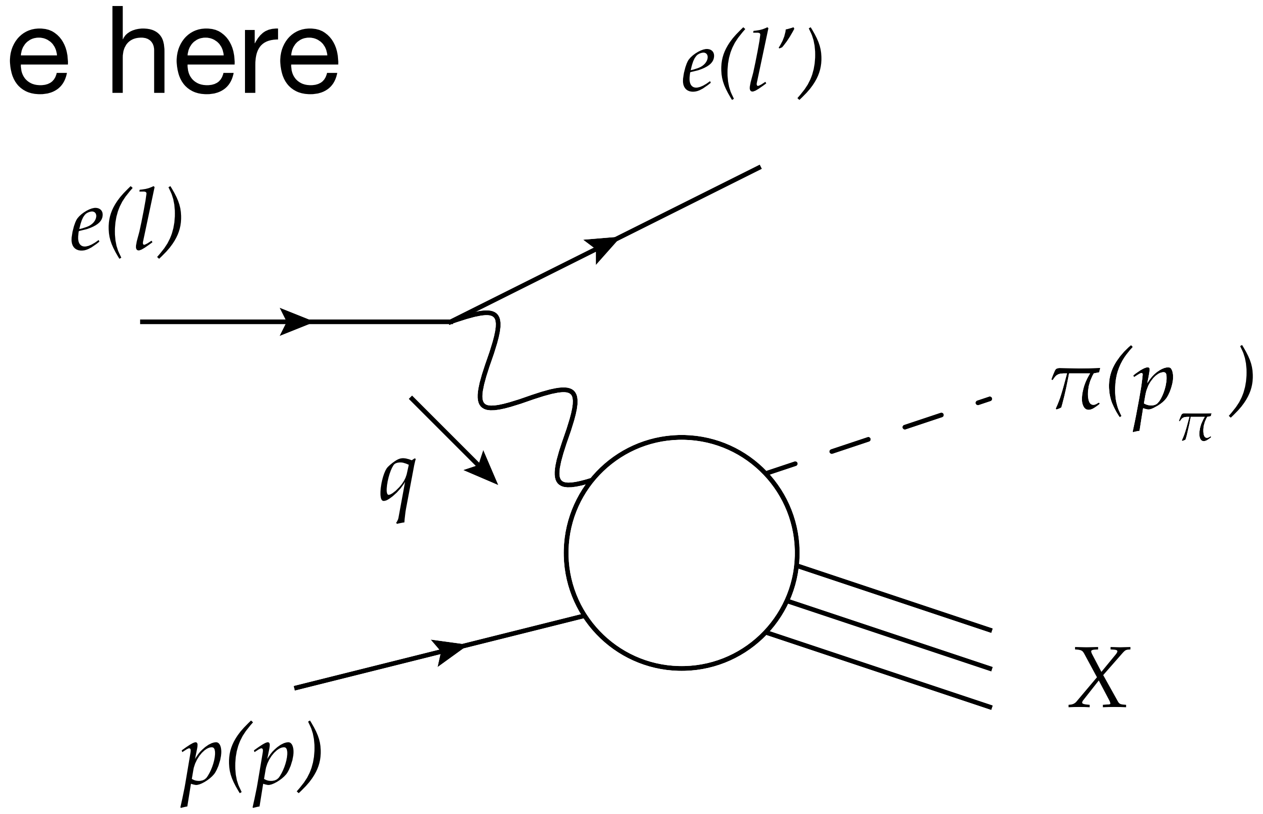
- $\boxed{e + p \to e + \text{meson} + X}$ with mesons, mostly pions, at high $p_{\pi\perp}$
- Especially: Mesons produced in isolation (i.e., not part of a jet)
  - There are other processes, including fragmentation and vector meson dominated (VMD) processes, but the isolated pion processes give the highest $p_{\pi\perp}$
- Mostly: Isolated meson production perturbatively calculable.
- Educational: May learn about target quark pdf's at high x, and about the pion's distribution amplitude.

Some earlier work

- Baier and Grozin, 1980
- Milana and CC, 1991; Wakely and CC, 1993
- Brandenburg, Khoze, and Müller, 1995
- Afaasev, Wahlquist, and CC, 2000
- Afanasev and CC, 2003
- Liu and Qiu, 2020
- Afanasev and CC, 2505.xxxxx
- Apologies for omissions

PartII: Generalities (Xsctn formulas)

- Will be calculating some (unpolarized) cross sections, so define here
- Full cross section is flux factor time cross section for virtual photon semi-inclusive scattering, $\gamma^* + p \to \pi + X$,



$$E'\omega_\pi \frac{d^6\sigma}{d^3l' d^3p_\pi} = \frac{\alpha}{2\pi^2}\frac{|\vec{q}\,|}{EQ^2}\frac{1}{1-\epsilon}$$

$$\times\, \omega_\pi \frac{d}{d^3p_\pi}\left\{\sigma_T + \epsilon\sigma_L + \epsilon\cos(2\phi_h)\sigma_{TT} + \sqrt{2\epsilon(1+\epsilon)}\cos\phi_h\,\sigma_{LT} + (2\lambda_e)\sqrt{2\epsilon(1-\epsilon)}\sin\phi_h\,\sigma'_{LT}\right\}$$

- ϵ is the usual

$$\epsilon = \left(1 + 2\tau(1+\tau)\tan^2(\theta_e/2)\right)^{-1} \quad \text{with} \quad \tau = \nu^2/Q^2$$

- Some modern people may prefer structure functions version ➡

Or in terms of structure functions

- Without target polarization, $e + p \to e + \pi + X$ has 5 structure functions,

$$\frac{d^6\sigma}{dx\,dy\,d\psi\,dz\,d\phi_h\,dp^2_{\pi\perp}} = \frac{\alpha^2}{2x_B Q^2(1-x_B)}\frac{y}{1-\epsilon}\left(1+\frac{m_p}{\nu}\right)$$

$$\times\left\{F_{UU,T} + \epsilon F_{UU,L} + \epsilon\cos(2\phi_h)F_{UU}^{\cos 2\phi_h} + \sqrt{2\epsilon(1+\epsilon)}\cos\phi_h\,F_{UU}^{\cos\phi_h} + h\sqrt{2\epsilon(1-\epsilon)}\sin\phi_h\,F_{LU}^{\sin\phi_h}\right\}$$

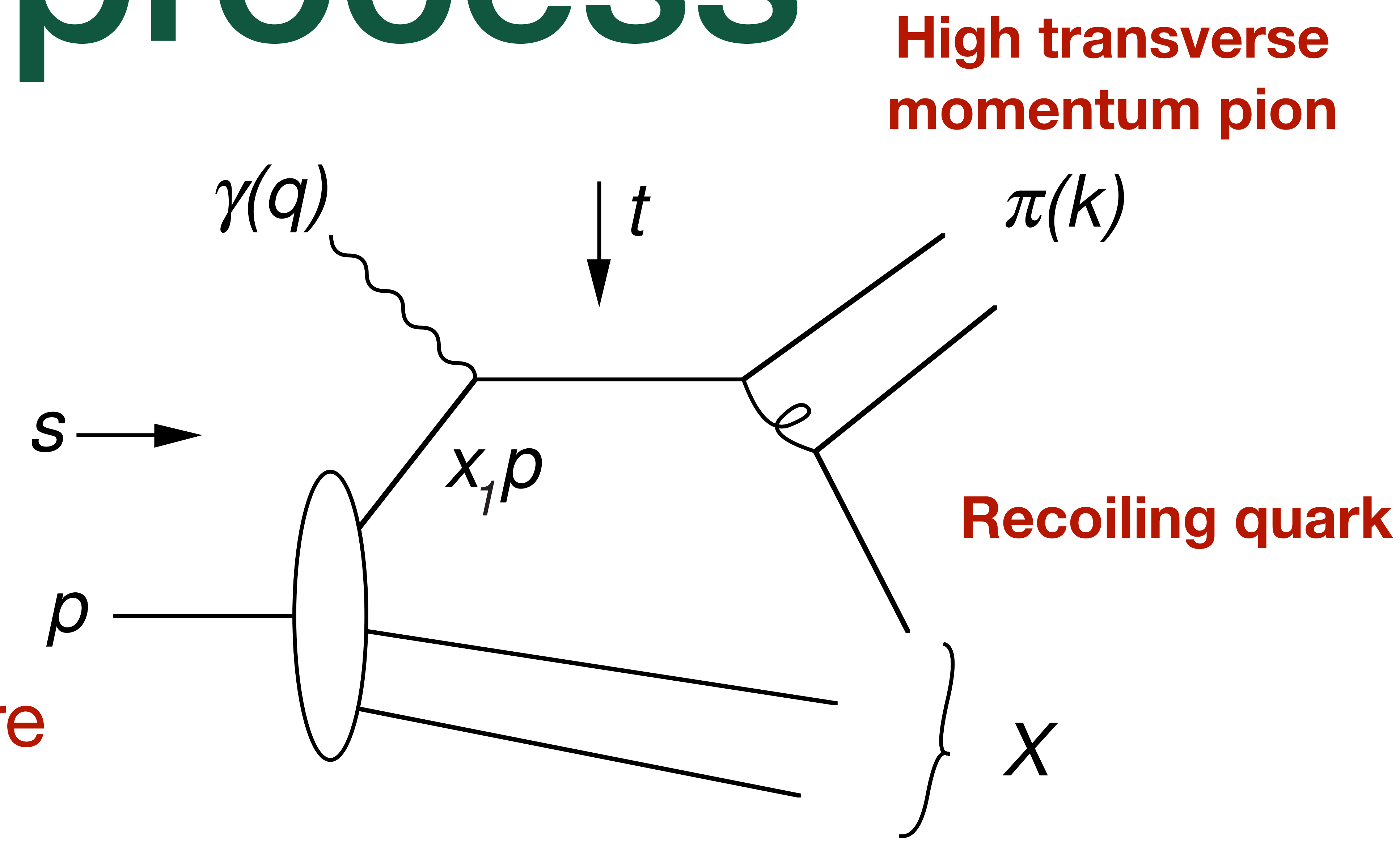
- Easy to match to cross sections on last slide.

- (θ = electron scattering angle in lab, ν = electron energy loss in lab, ϕ = azimuthal angle of pion-photon plane relative to electron scattering plane. Other new notation is exercise for viewer or reader.)

Part III: Direct process

- Notes: For $\gamma^* p \to \pi X$ directly,

$$\frac{d\sigma}{dx_1 dt} = \sum_a G_{a/p}(x_1)\frac{d\hat{\sigma}}{dt} \text{ or } \sum_a f_a(x_1)\frac{d\hat{\sigma}}{dt}$$



- Special note: the "internal-external miracle" works here also, that is, x_1 is fixed by observable quantities

- Subprocess Mandelstam variables
$\hat{s} = (p_1 + q)^2, \quad \hat{t} = t = (q-k)^2, \quad \hat{u} = (p_1 - k)^2$

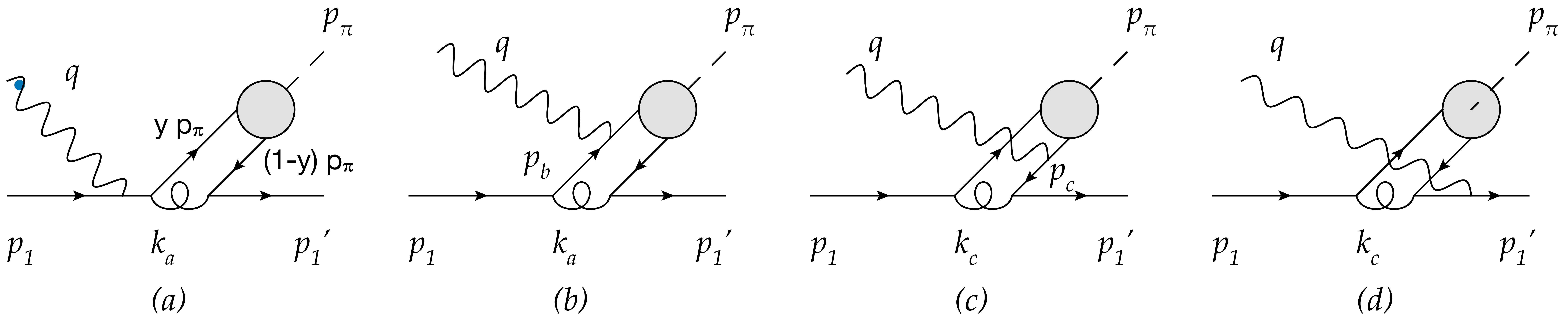
- Overall and observable Mandelstam variables
$s = (p+q)^2, \qquad t = (q-k)^2, \qquad u = (p-k)^2$

  - with $p_1 = x_1 p$, find $x_1 = -t\,/\,(s + u + Q^2)$

- Some references: Berger, Brodsky;
Baier, Grozins;
Brandenberg, Khoze, Müller;
Hyer;
Milana, Wakely, Wahlquist, Afanasev, me

Direct process, page 2

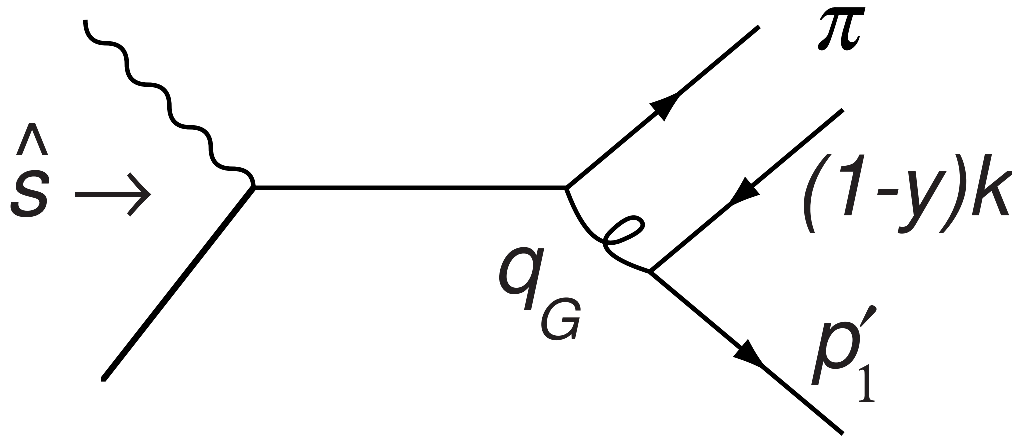
- Subprocess calculable using pQCD and (arguably) known pion $q\bar{q}$ Fock component wave function. Given by "distribution amplitude" $\phi_\pi(y)$.



For $Q^2 = 0$, need $I_\pi = \int_0^1 dy\, \frac{\phi_\pi(y)}{y}$; For $Q^2 \neq 0$, need $I'_\pi = \int_0^1 dy\, \frac{\phi_\pi(y)}{y - (1-y)Q^2/t}$

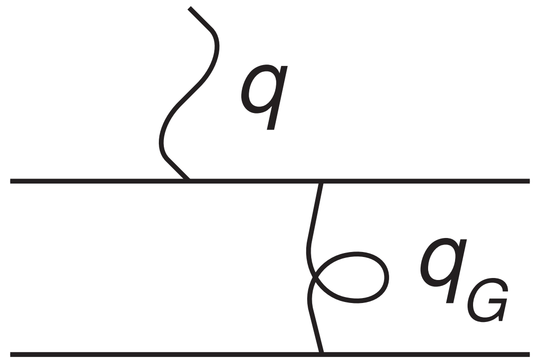
- For π^+ (e.g.), initial up quark dominates, so $\sum_{\text{quarks}}$ needs only one term.

Direct process, side remarks

- 

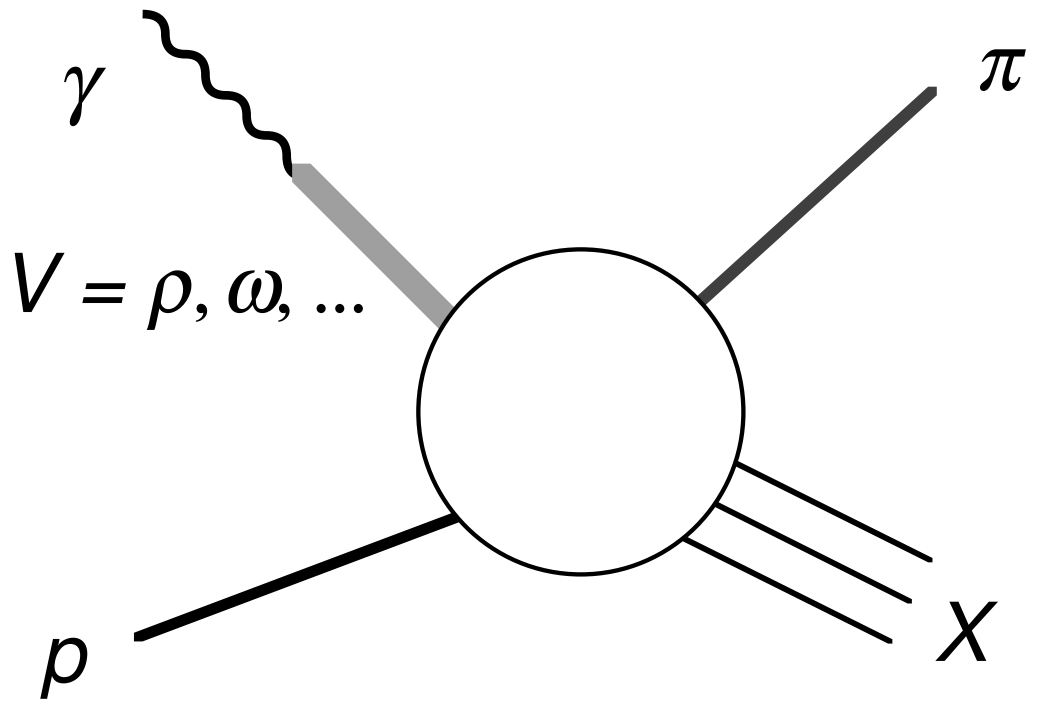
$q_G^2 = (1-y)\hat{s} = x_1(1-y)s \approx \frac{1}{6}s \approx 8\,\mathrm{GeV}^2$ (for 22 GeV JLab)

Pion form factor uses same distribution amplitude and same I_π integral, and



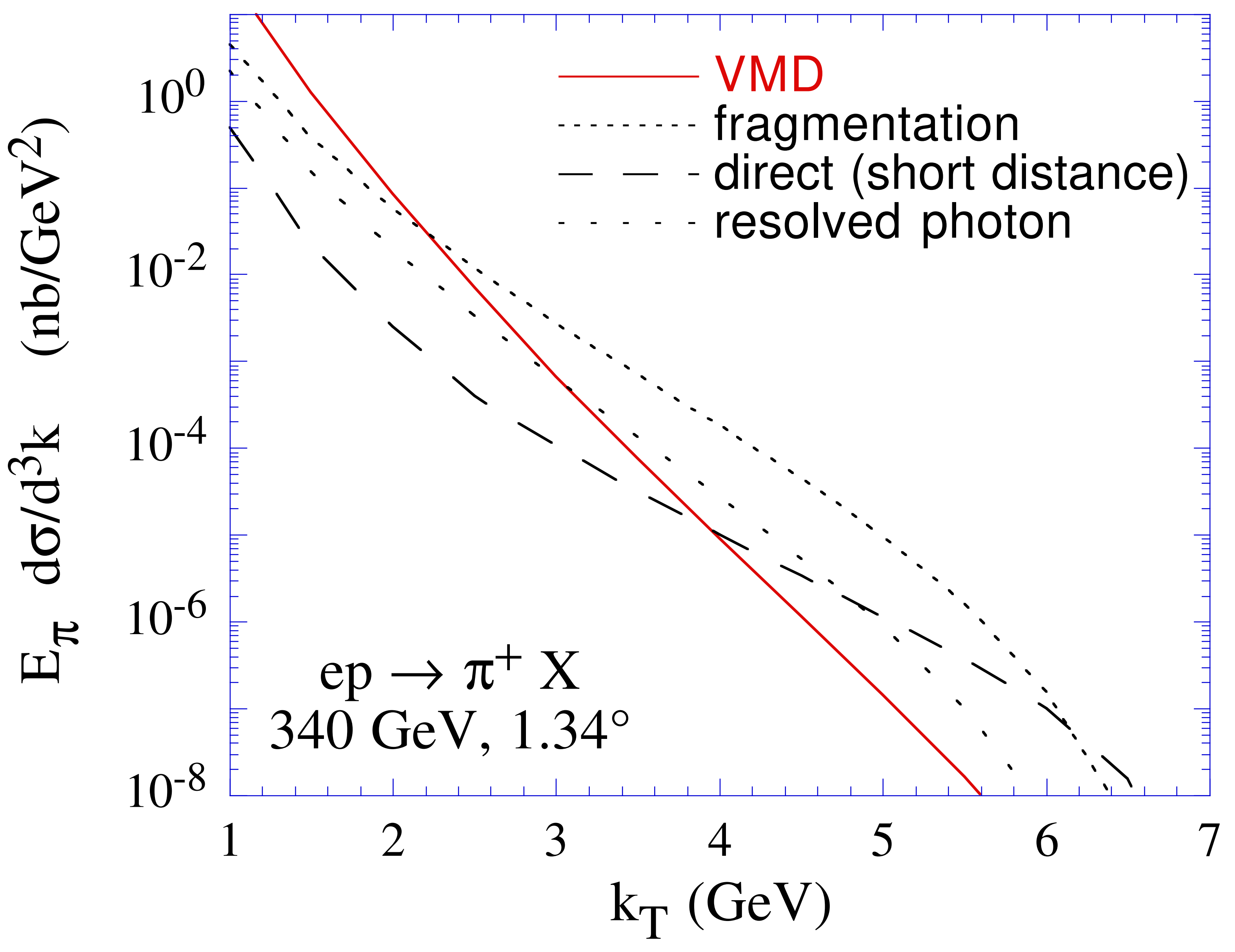
$q_G^2 \approx \frac{1}{9}q^2, \;\Rightarrow$ JLab SEDIS as good as pion FF at $Q^2 = 72$ GeV^2

- For VMD at $Q^2 \neq 0$, propagator suppresses ρMD contributions by $(m_\rho^2/(m_\rho^2 + Q^2))^2 < 1/7$ for $Q^2 \approx 1$ GeV^2.

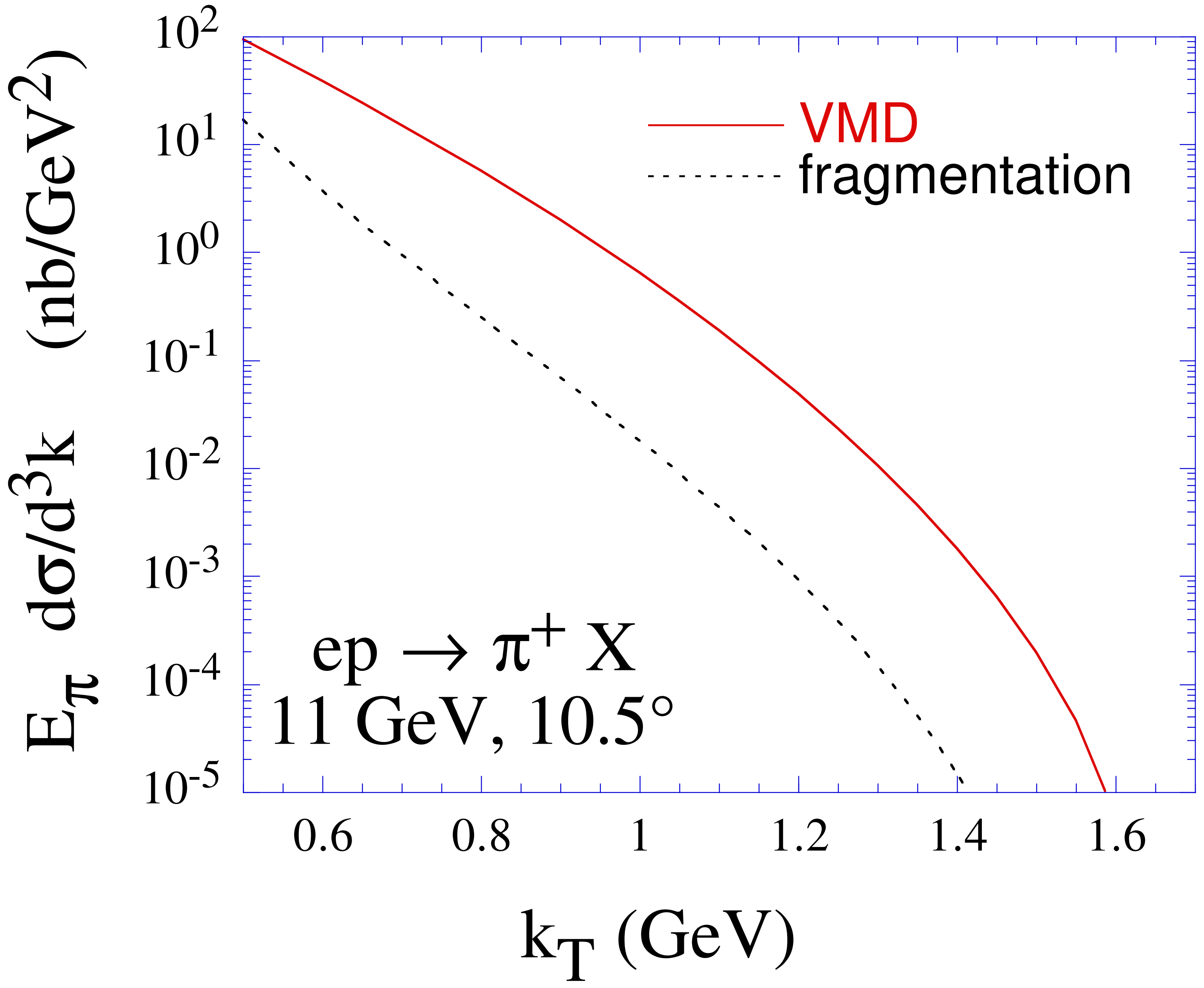


A plot of things so far

- Some plots with final electron not observed (i.e., photons generally very close to on-shell).



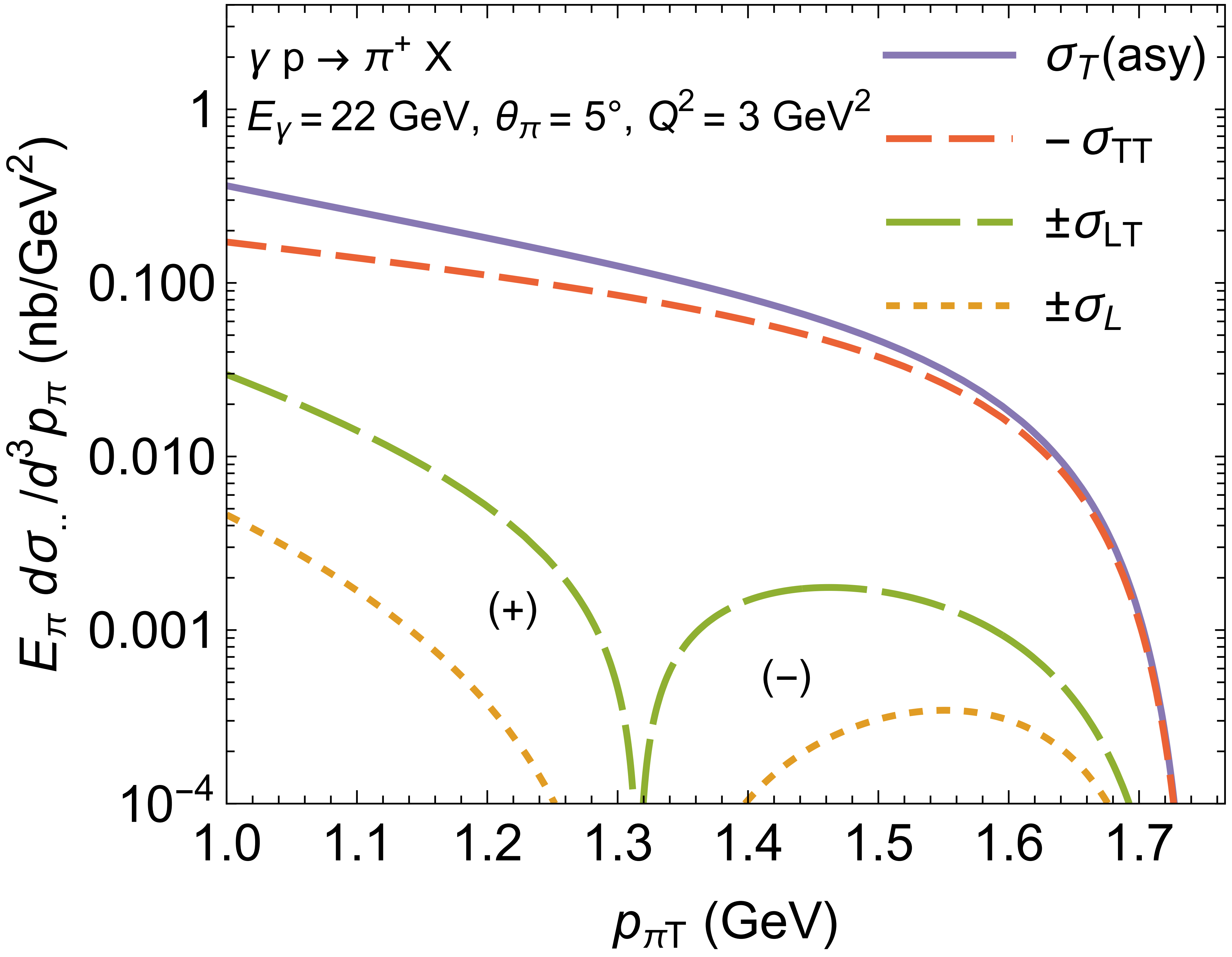
(Ignore long dash and dotted curves for now)



- Soft (i.e., VMD) pretty big for almost real photons.

Plot of “sub-cross sections”

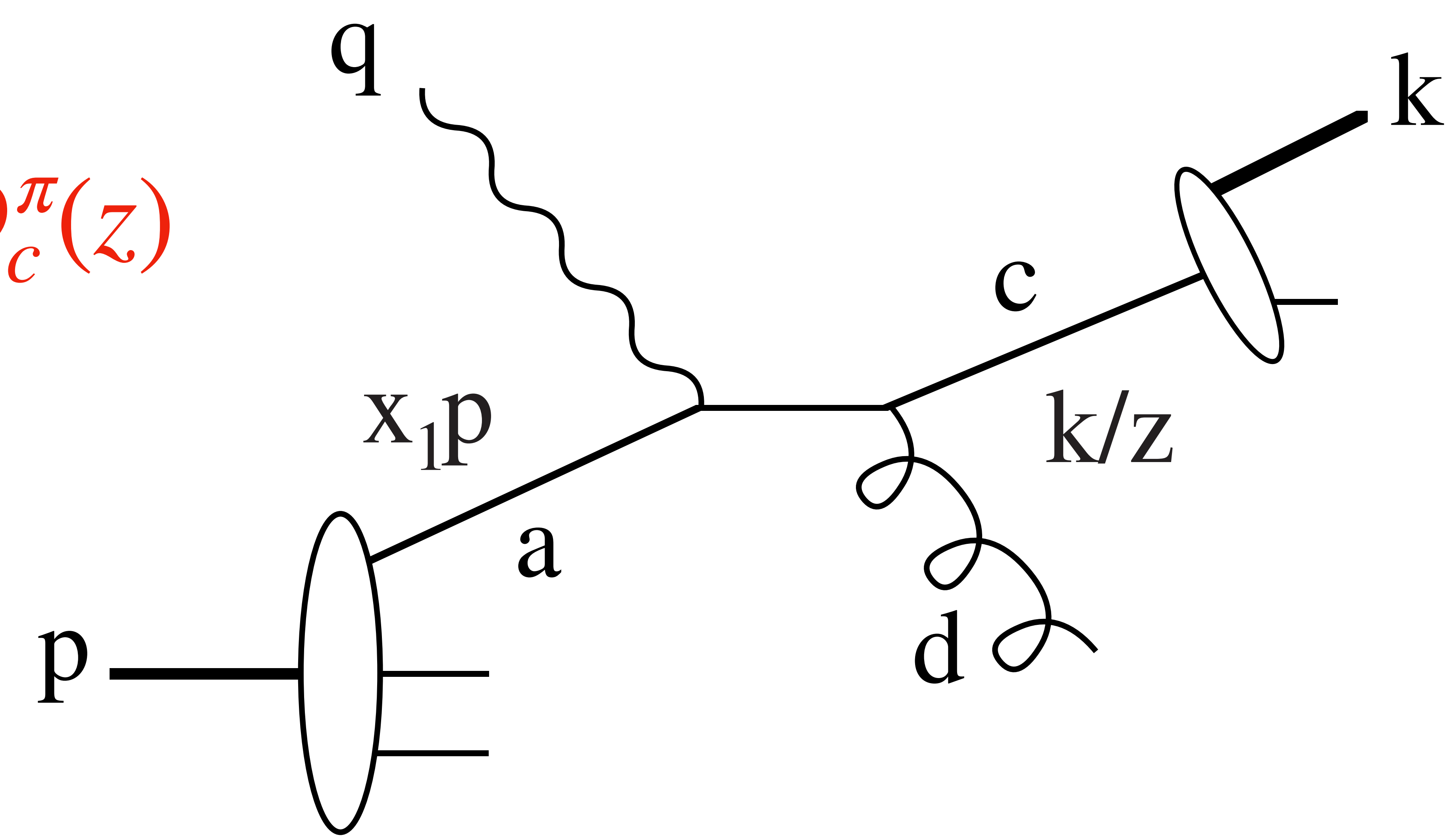
- (For direct process)



Part IV: Fragmentation

- Basic cross section calculation,

$$\sigma = \int dx_1 d\hat{t} dz\ G_{a/p}(x_1)\,\frac{d\hat{\sigma}}{d\hat{t}}(\gamma^* + a \to c + d)\, D_c^{\pi}(z)$$



- Where
  - $G_{a/p}$ = distribution function for a in p
  - $d\hat{\sigma}/d\hat{t}$ = subprocess cross section
  - $D_c^{\pi}(z)$ = fragmentation function

- Two generic subprocesses,
  - "QCD Compton," $\gamma^* + q \to q + g$
  - "Gluon fusion," $\gamma^* + g \to q + \bar{q}$

more fragmentation

- Part is easily calculable perturbatively, e.g., for QCD Compton,

$$\frac{d\hat{\sigma}}{d\hat{t}} = \frac{8\pi\alpha\alpha_s}{3(\hat{s}+Q^2)^2}\left\{-\frac{\hat{u}}{\hat{s}} - \frac{\hat{s}}{\hat{u}} + 2\frac{Q^2}{\hat{s}\hat{u}}(\hat{t} - \hat{k}_\perp^2)\right\}$$

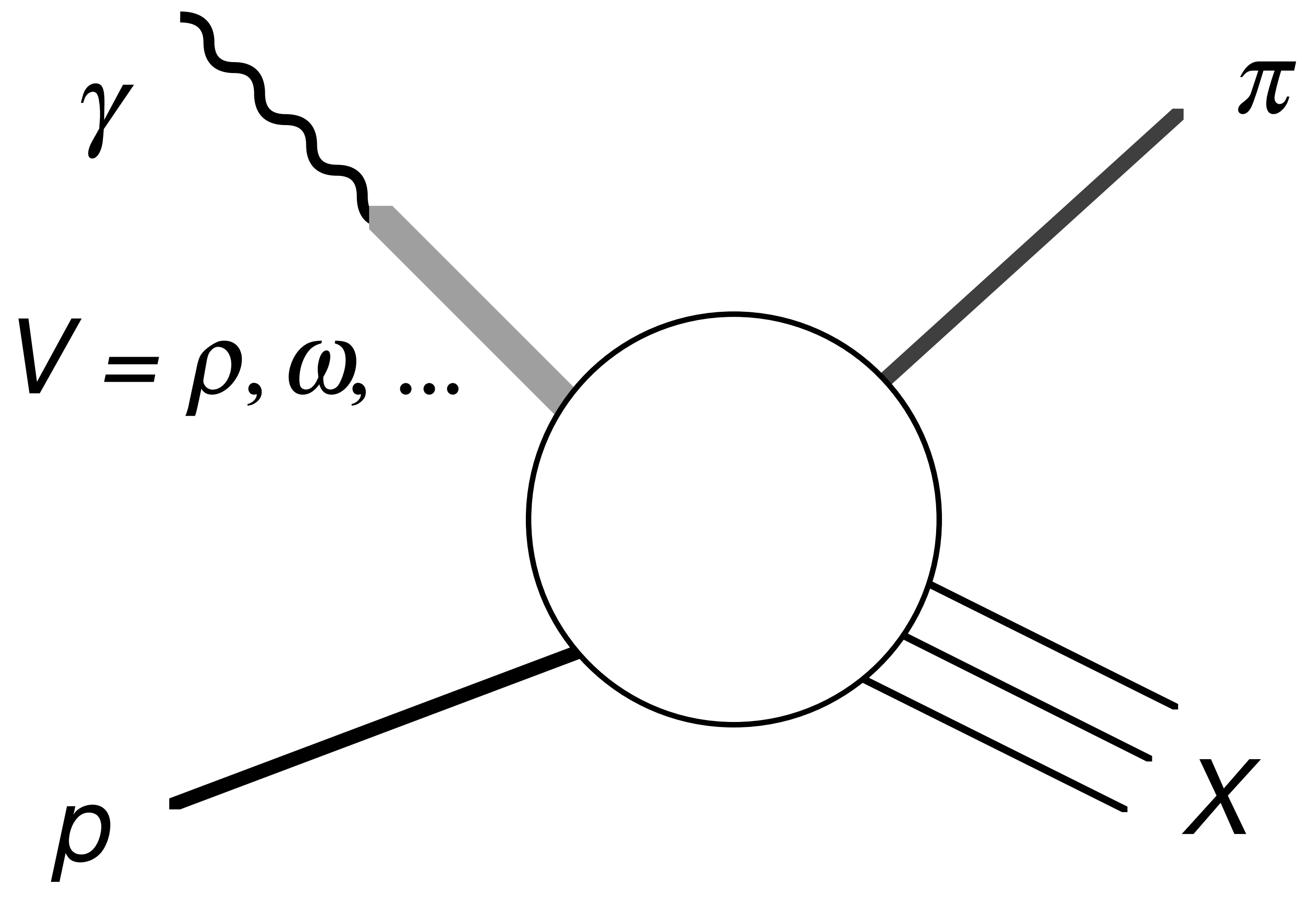
- Then: Get $G(x)$ from analyses of DIS,

  Get Get $D(z)$ from analyses of $e^+e^- \to$ hadrons

- Show results after discussion of soft processes.

more Part IV: soft processes, a.k.a. VDM

- Approximate soft processes by vector meson dominance: Photon enters but fluctuates to a rho or omega or phi or excitations thereof.



- Interacts as hadron. Not calculable ab initio. Amplitude obtained using various relations.

- E.g.,

$$f(\gamma p \to \pi^+ X)\Big|_{\rho\mathrm{MD}} = \frac{e}{f_\rho} f(\rho^0 p \to \pi^+ X), \qquad (\text{for } q^2 = 0)$$

and rho decay constant f_ρ got from $\Gamma(\rho \to e^+ e^-)$.

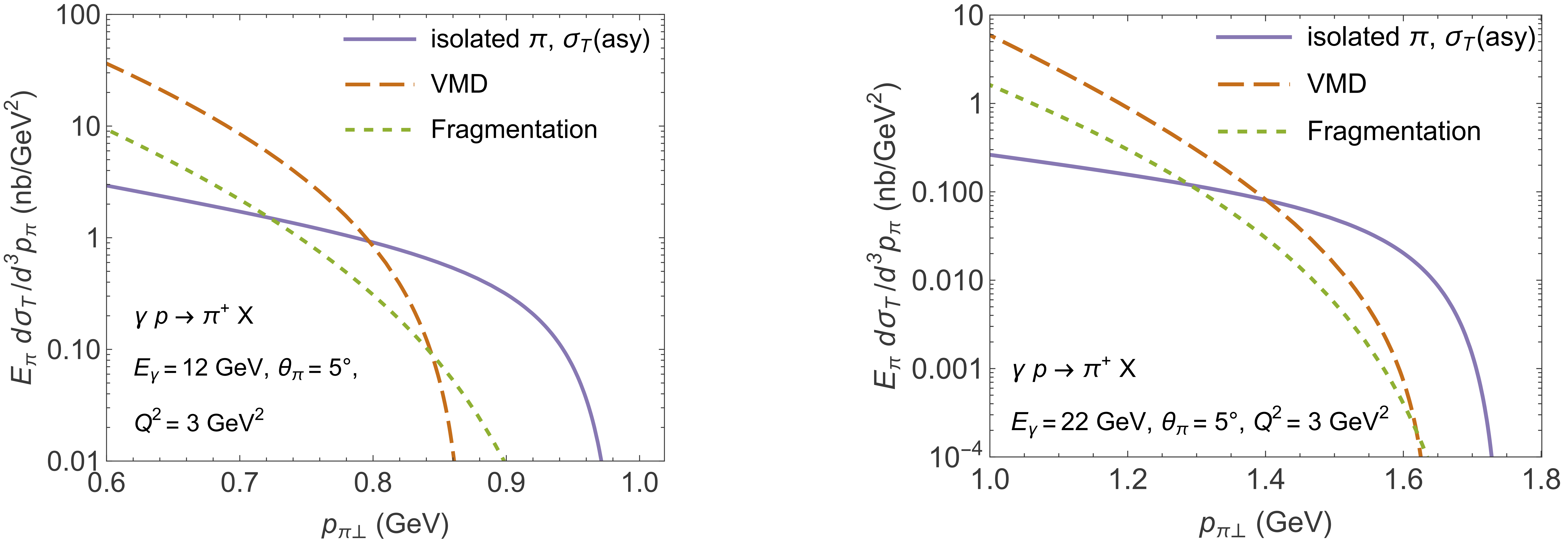
Parameterization of hadronic process.

- Still more stuff:
  - Don't have ρ^0 beams. Use π^+ or π^- data instead, for example $\pi^+ \ldots \to \pi^0 \ldots$
  - Bosetti et al. (e.g.) have semi-exclusive π in, π out data at many angles but limited energy range.
  - Lots of data on $pp \to \pi X$ at $90°$ CM.

    Where data overlap, pion σ about 2/3 proton σ
  - So get angular distribution from pion data, and energy dependence of pp data. Also estimate contributions from other VM (ϕ, ω, excitations).

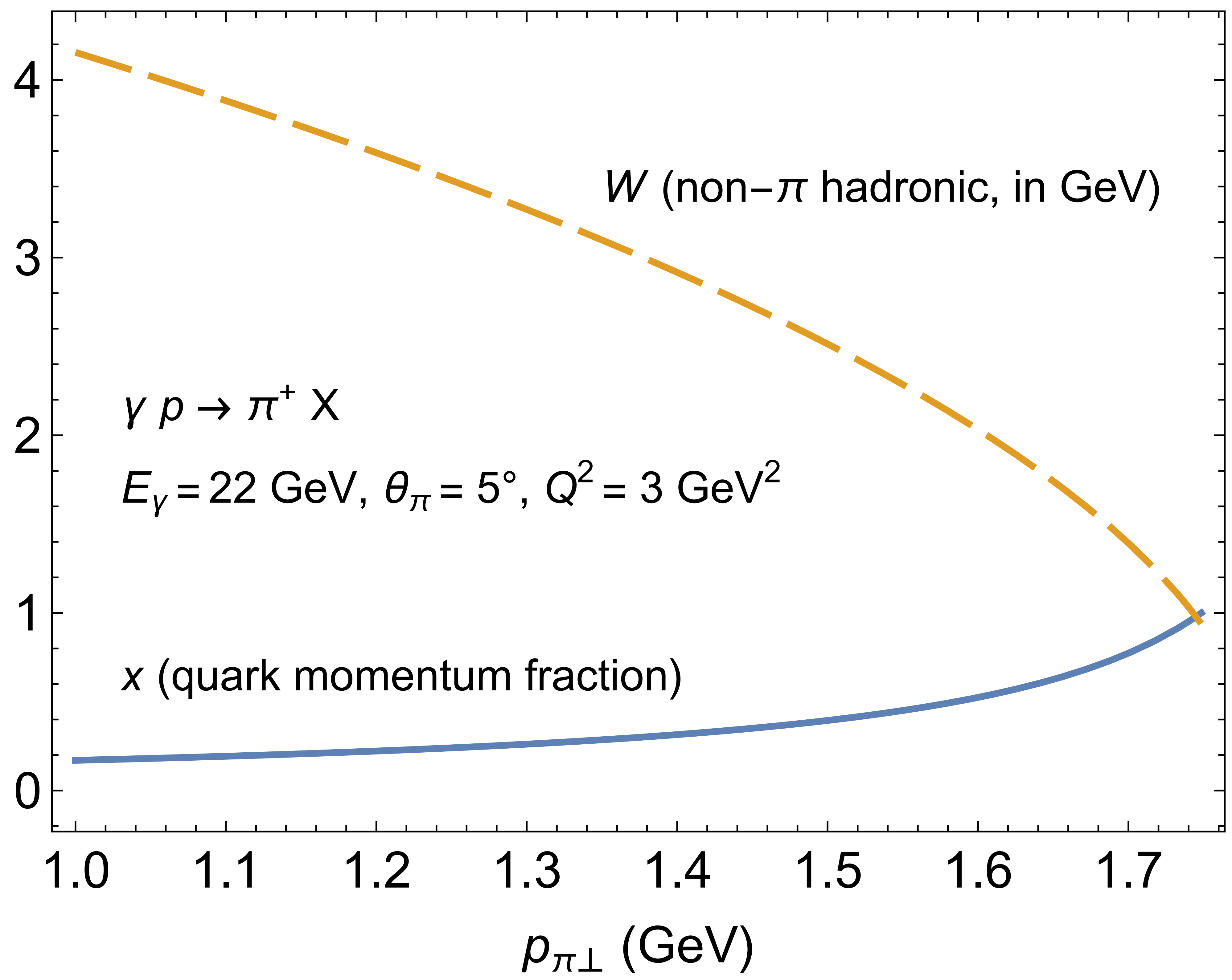
    (Formulas in ACW 2000. See also parameterization by Szczurek, Uleshchenko, and Speth.)

Comparative results



- For $E_\gamma = 22$ GeV (especially), there is window where isolated pion production dominates at high $p_{\pi\perp}$

Interesting double plot



- Note: for the range where direct pions dominate, x is large, as $x \gtrsim 0.3$
- Note: In most of this range, W (unobserved hadron invariant mass) > 2 GeV.

Part V: possible uses of direct pions

- The 22 GeV white paper emphasizes that data will be taken over a extensive kinematic range.

- Generically, for the direct pions,

  $d\sigma \propto$ (target quark pdf) $\times$ (pion DA term) $\times$ (known kinematic terms)

  or

  $$d\sigma \propto f_u(x) \times \int dy \frac{\phi_\pi(y)}{y + (1-y)\, q^2/t} \times \text{known stuff}$$

- May hope to measure the pdf at large x, and discriminate different π DA's

Selected distribution amplitudes

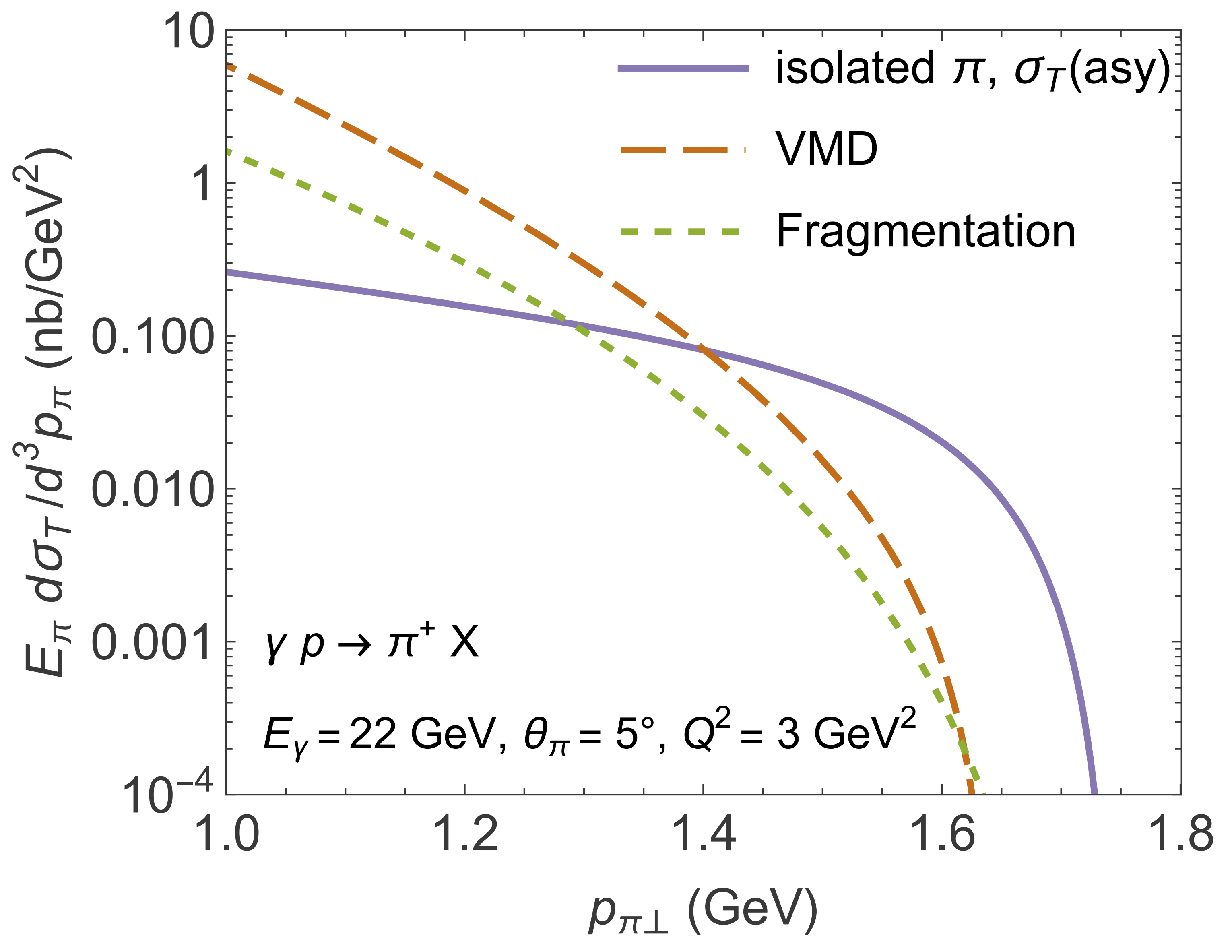
- Two sample π DA's, asymptotic and square root (SR)

$$\phi_\pi(\text{asy}) = \sqrt{3} f_\pi y(1-y) \quad \text{and} \quad \phi_\pi(\text{SR}) = \frac{4}{\pi\sqrt{3}} f_\pi \sqrt{y(1-y)}$$

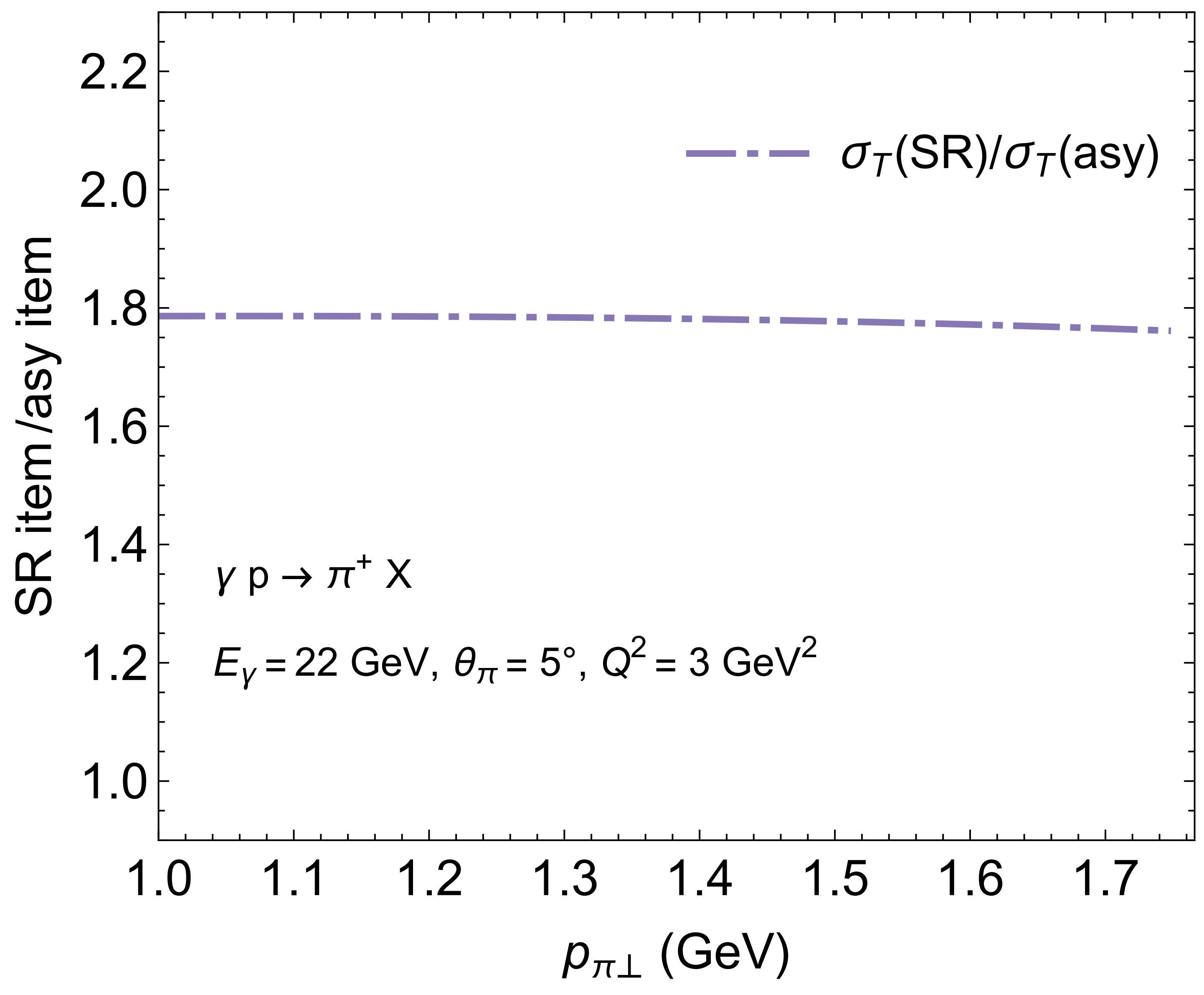
- Both normalized to give correct pion decay constant f_π .

Effect of DA and f_u on Xsctn shape

Regarding plot like



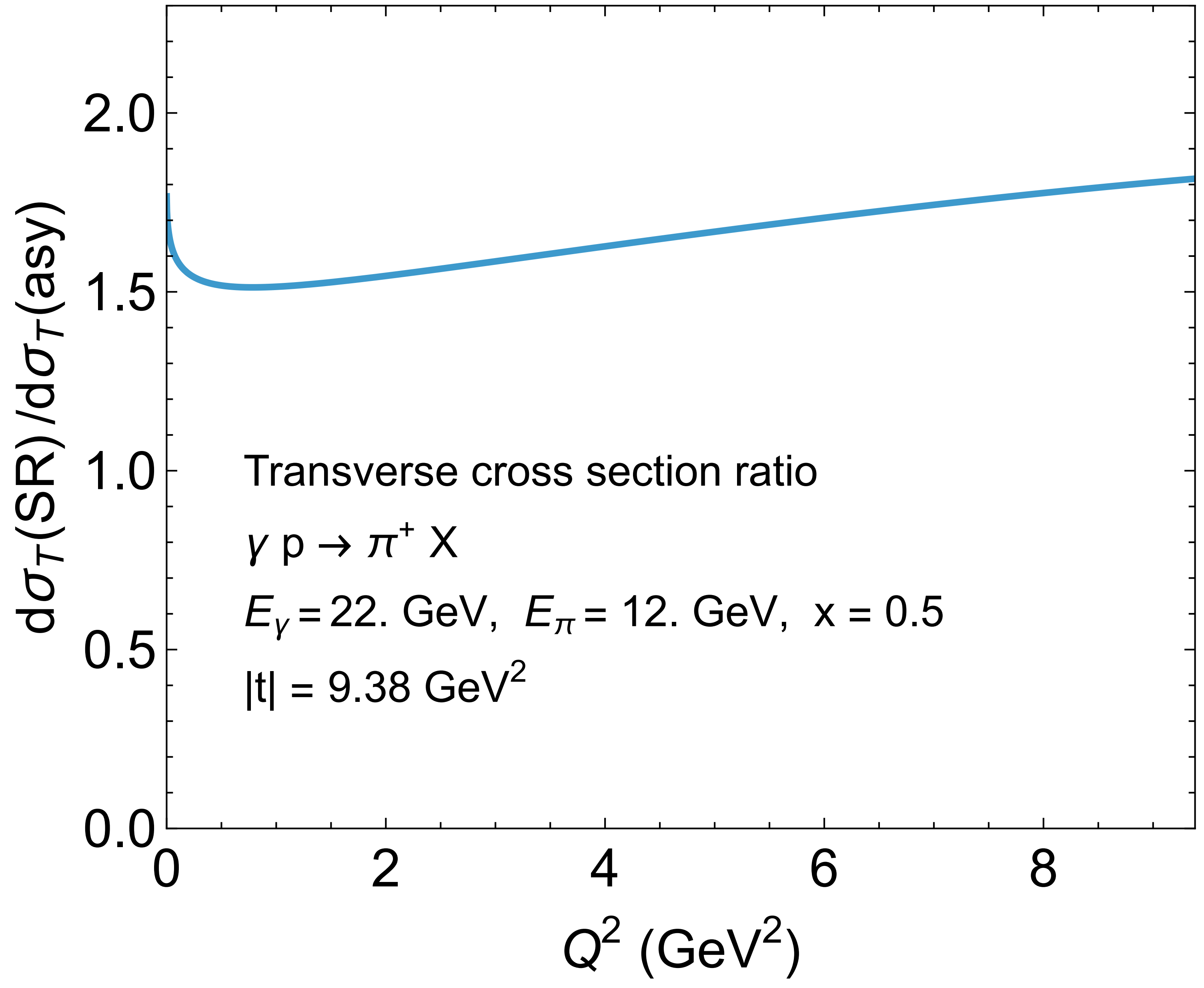
Changing the DA hardly matters (for shape of curve)

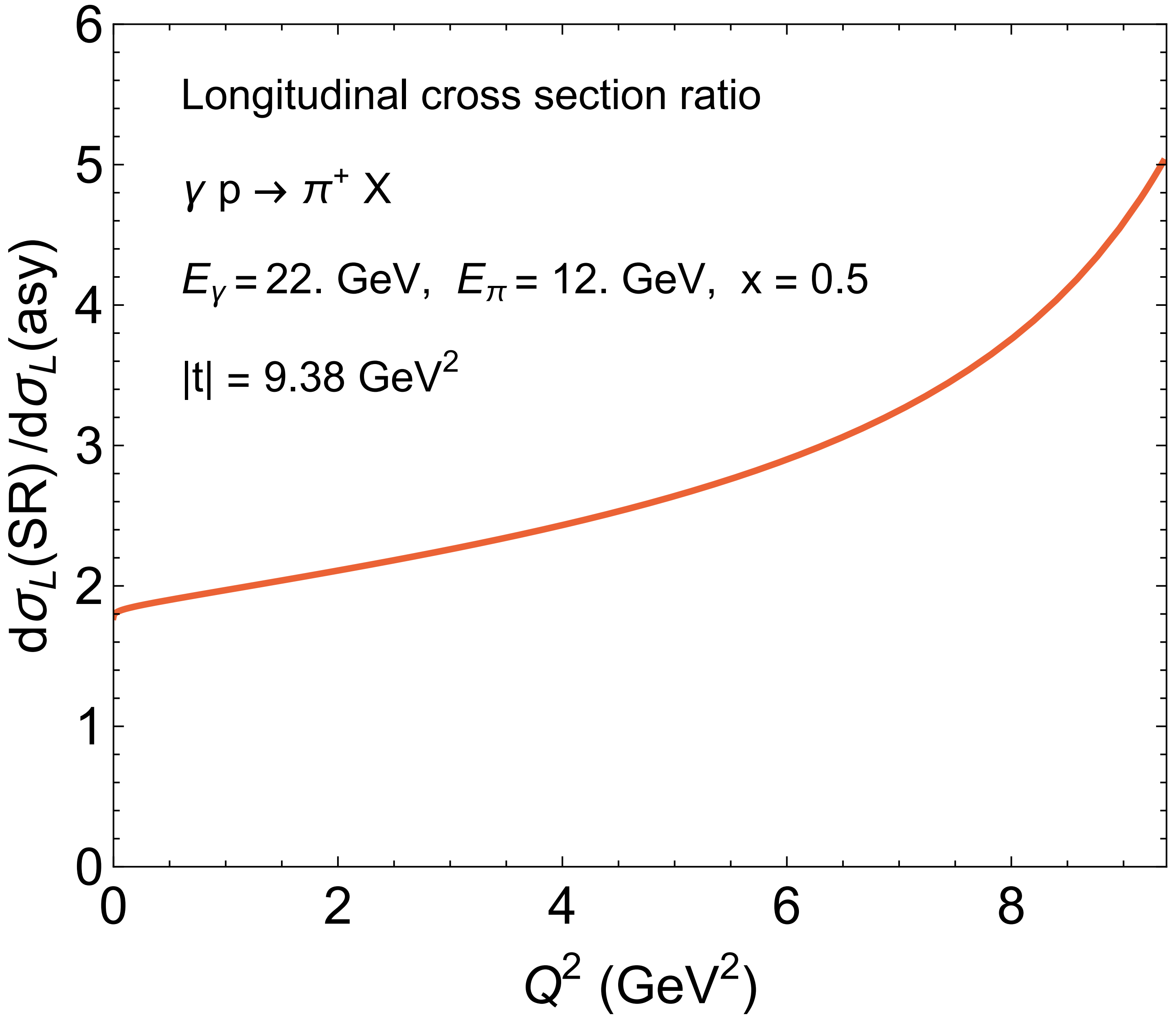


- For $E_\gamma, \theta_\pi,$ and Q^2 fixed, choice of DA does not affect shape.
- $\therefore$ Can get shape of $f_u(x)$ at high x from measurements (plot was made using GRSV pdf).

Another kinematic choice

- Get more variation from different DA's by choosing to fix $E_\gamma, E_\pi,$ and x (thereby also fixing t) and varying Q^2. Not so interesting for transverse cross section, but remarkable for longitudinal.





- Fixed x means quark pdf not changing. Cross section measurements here, especially for the longitudinal case, determine the pion DA.

Part VI: Final remarks

- It appears that direct pion production, a hard higher twist process, can be seen above the soft "background" at high transverse momentum at JLab energies.

- Could be used to learn high-*x* form of quark pdfs, with ability to select flavor of quark by choosing flavor of pion. Could also learn the actual physical pion distribution amplitude, ϕ_π .

- Higher order processes could affect high $p_{\pi\perp}$ production, for example radiative corrections or initial transverse momentum effects upon rapidly falling cross section. See comments by J. Qiu.

- Side note: The basic subprocess for direct meson production is the same as for quasi-elastic production of mesons, in the region where that production can be described by generalized parton distributions.

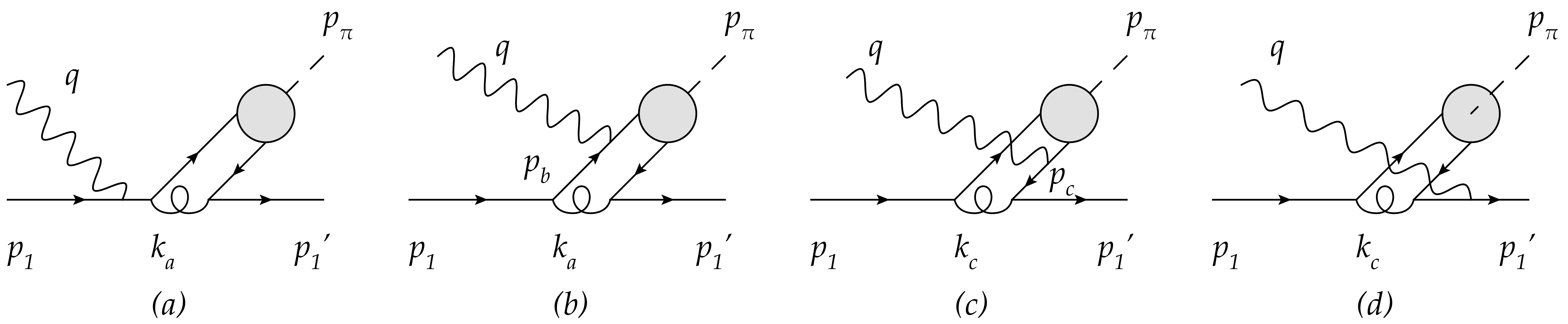
The end

Past the end

Further things to do

- Other structure functions, dependent on target polarization
- Radiative corrections
- Direct (isolated) ρ production
- π^0 plots

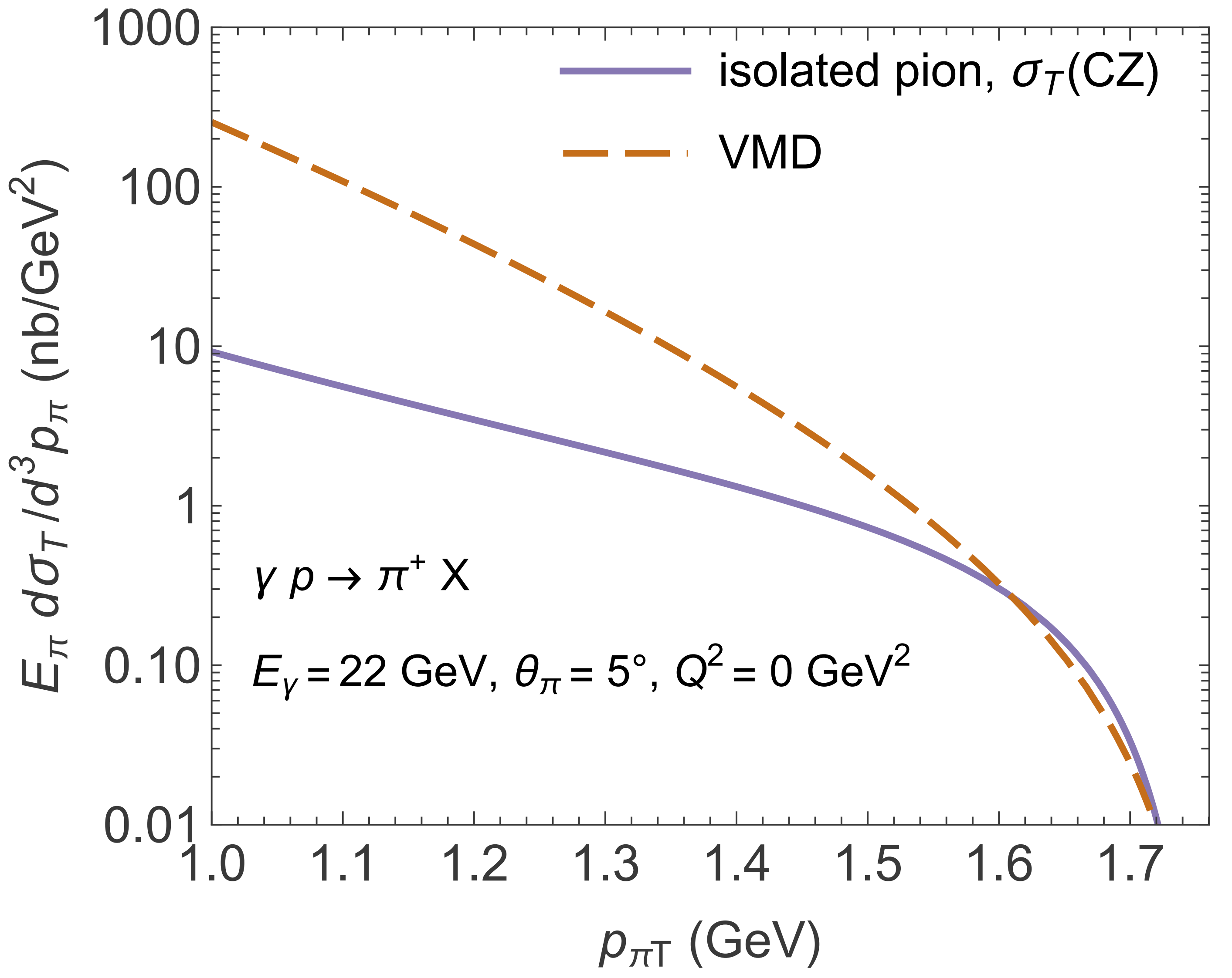
Subprocess cross sections

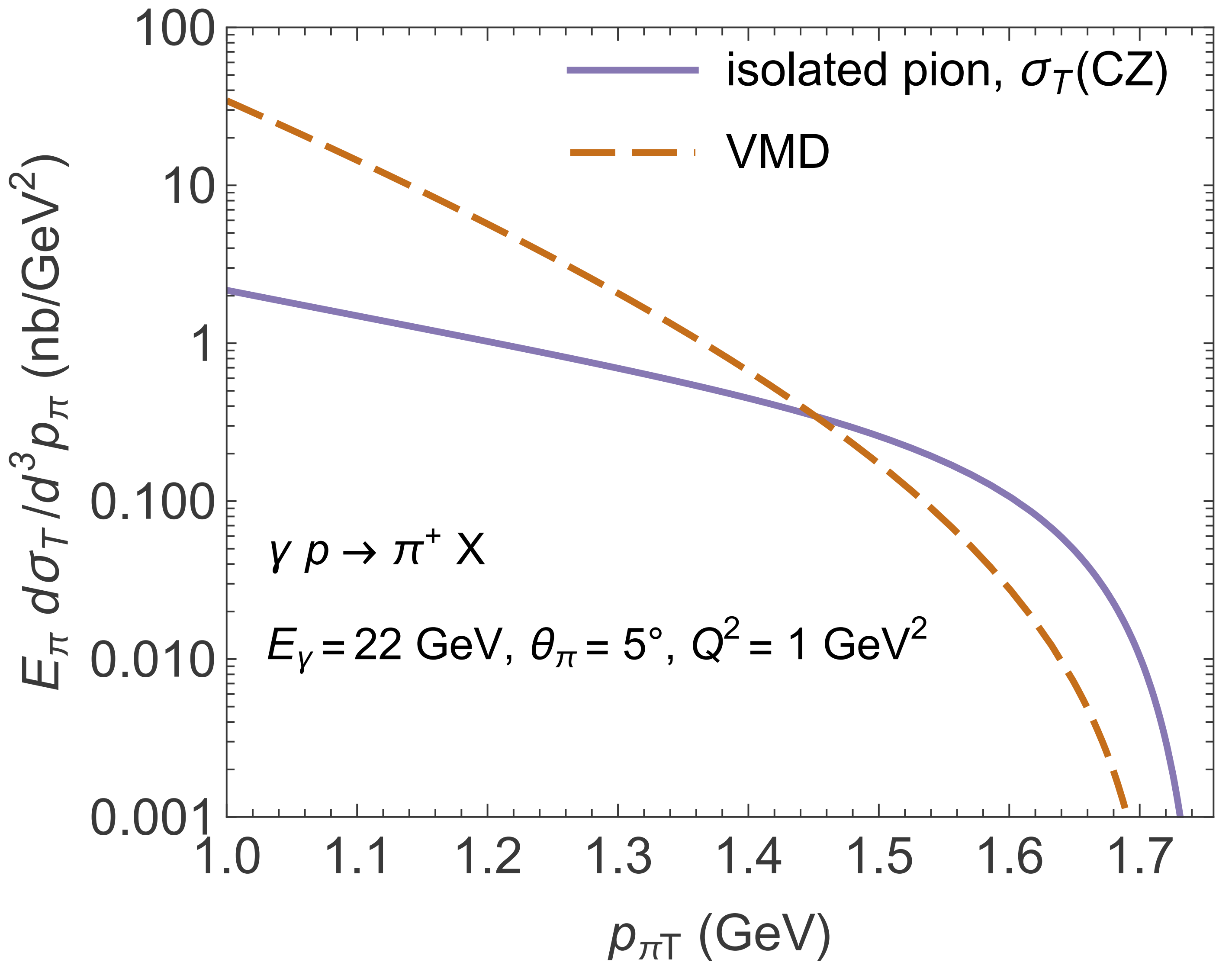


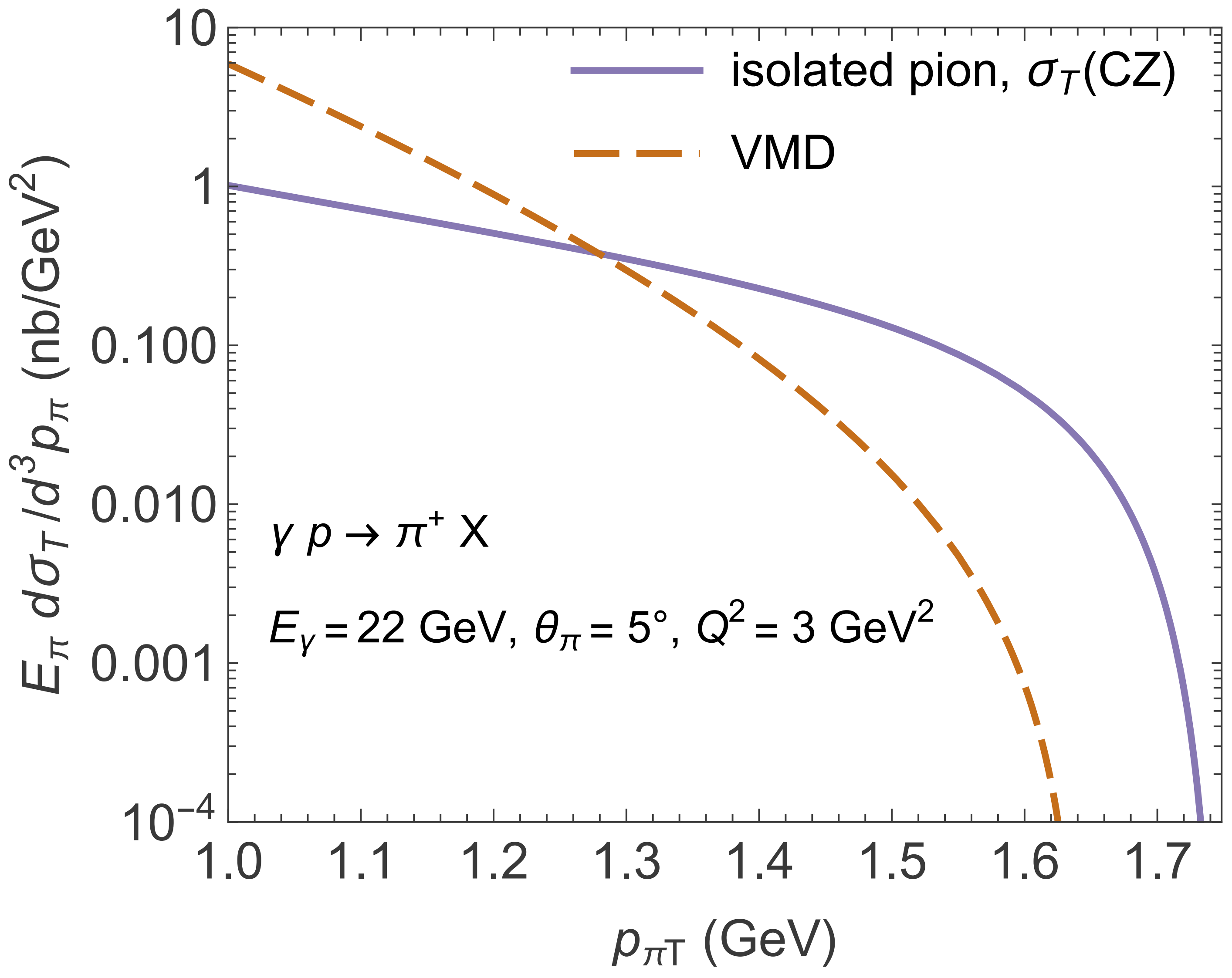
Real photons:

$$\frac{d\hat{\sigma}}{dt} = \frac{128}{27}\pi^2 \alpha \alpha_S^2 I_\pi^2 \left(\frac{e_1}{\hat{s}} + \frac{e_2}{\hat{u}} \right)^2 \frac{\hat{s}^2 + \hat{u}^2}{\hat{s}^2(-t)}$$

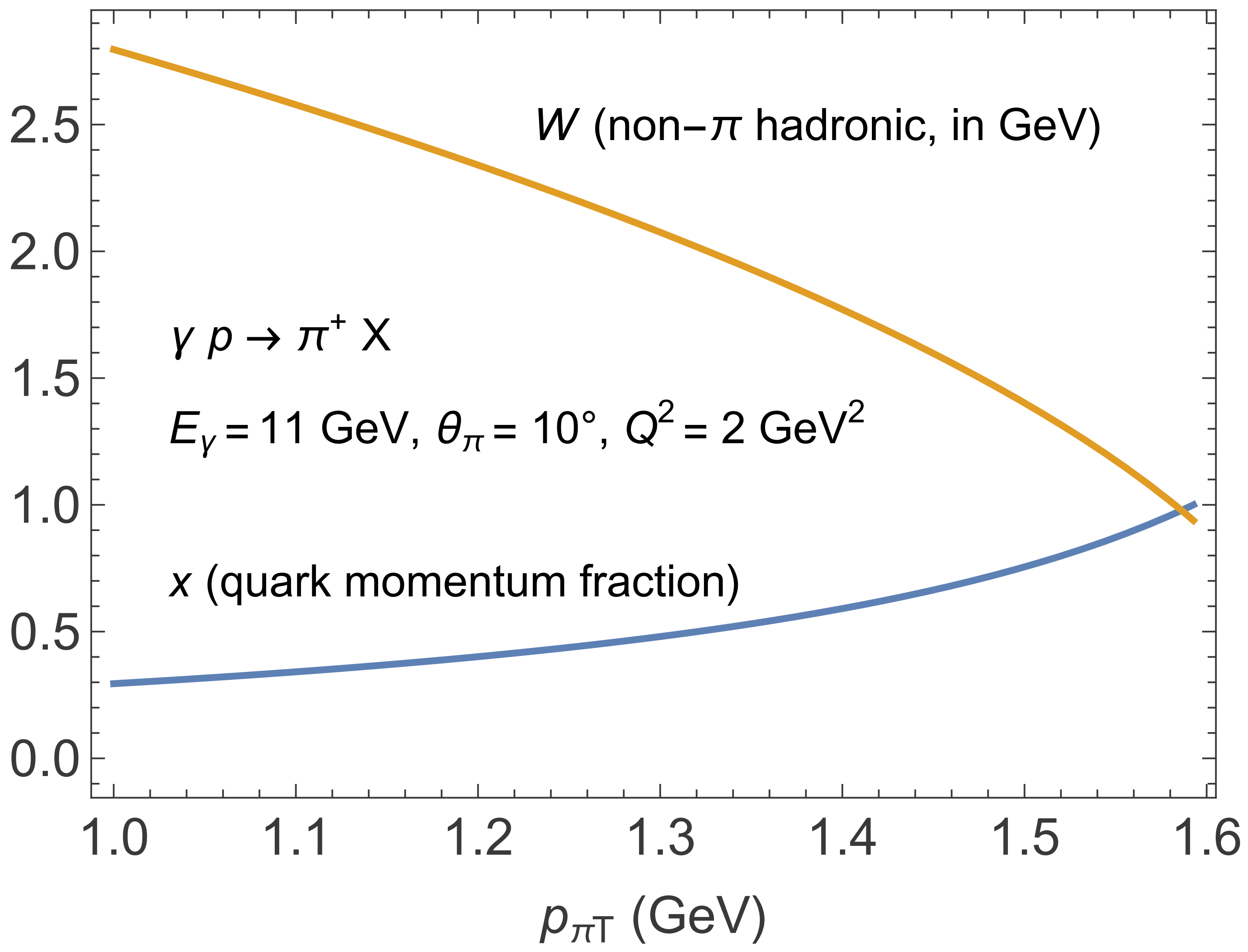
Comparison plots, at several Q^2

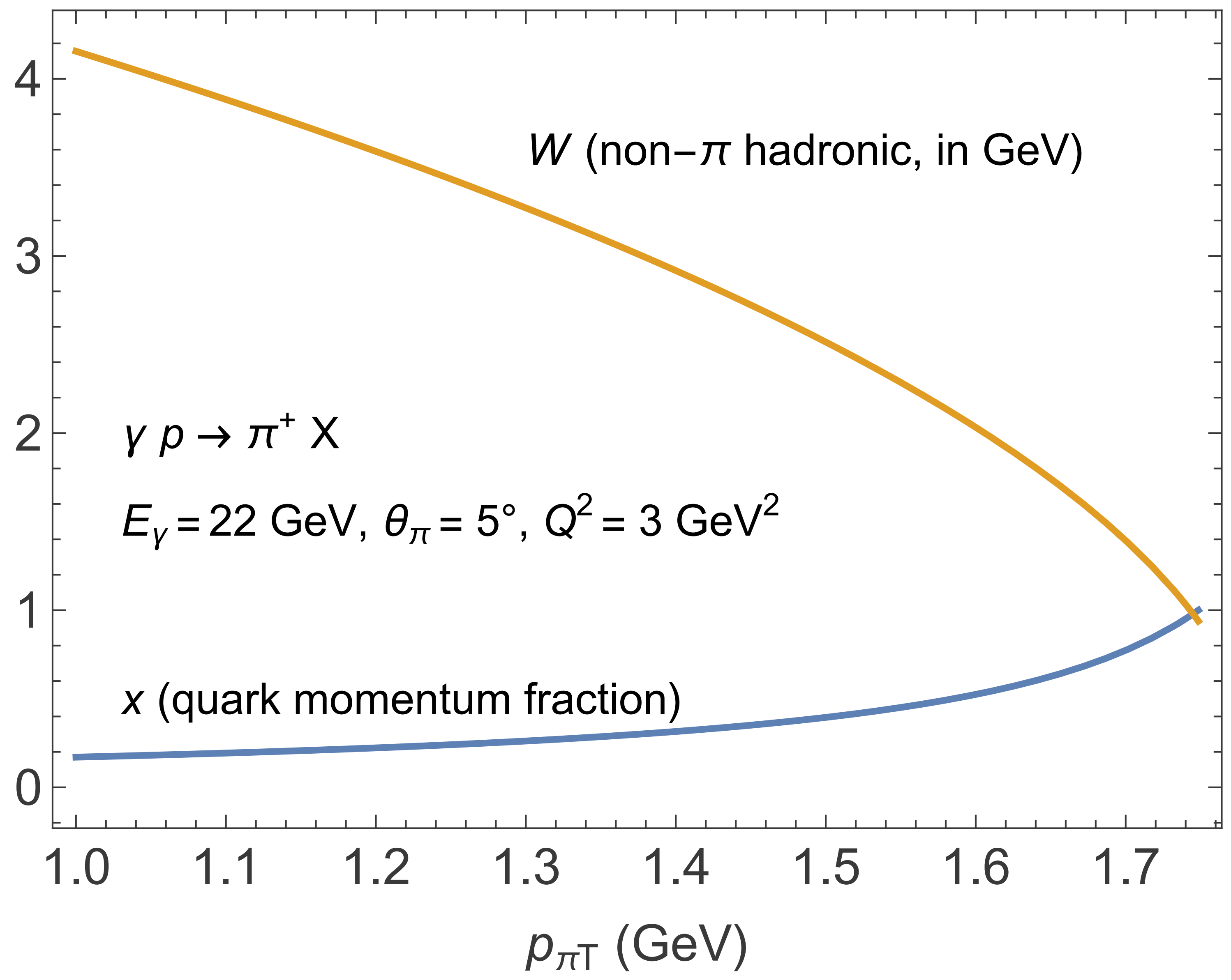






Recoiling mass and x plots





- Plots show recoiling (without the isolated pion) hadronic mass, and also x.
- For 22 GeV, significant window where we are out of the resonance region.