Small-*x* Helicity Phenomenology With the Valence Quark Model

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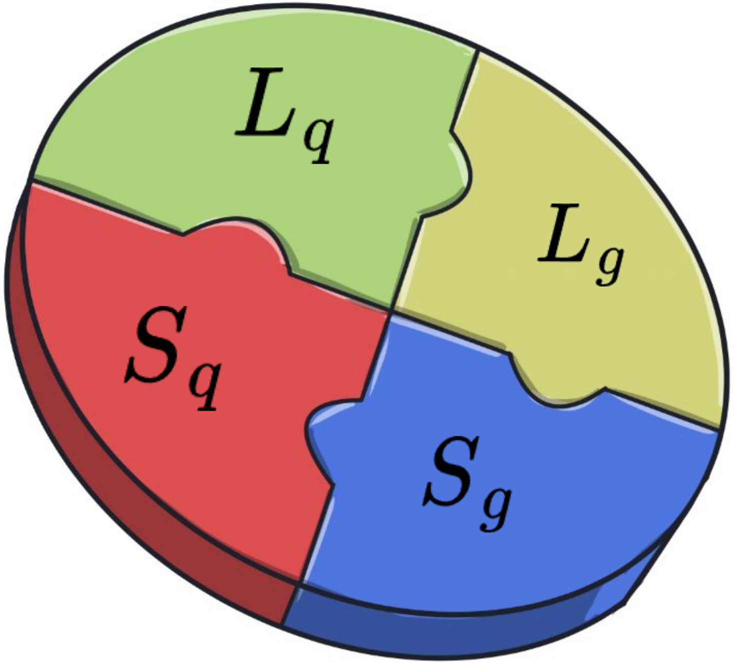
Proton Spin Puzzle

Jaffe-Manohar Spin Sum Rule: $\frac{1}{2} = S_q + L_q + S_g + L_g$

$$\frac{1}{2} =$$



$S_{q,g}$ = Helicity of quarks and gluons

$L_{q,g}$ = Orbital angular momentum

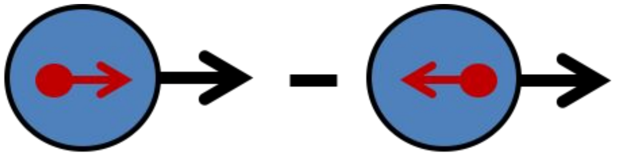
Quark Helicity Parton Distribution Functions (hPDFs)

Net quark spin Quark hPDF Anti-quark hPDF Singlet hPDF

$$S_q(Q^2) = \frac{1}{2}\int_0^1 dx \sum_q \left(\Delta q(x, Q^2) + \Delta\bar{q}(x, Q^2)\right) = \frac{1}{2}\int_0^1 \Delta\Sigma(x, Q^2)$$

Helicity PDFs:

$$\Delta q =$$

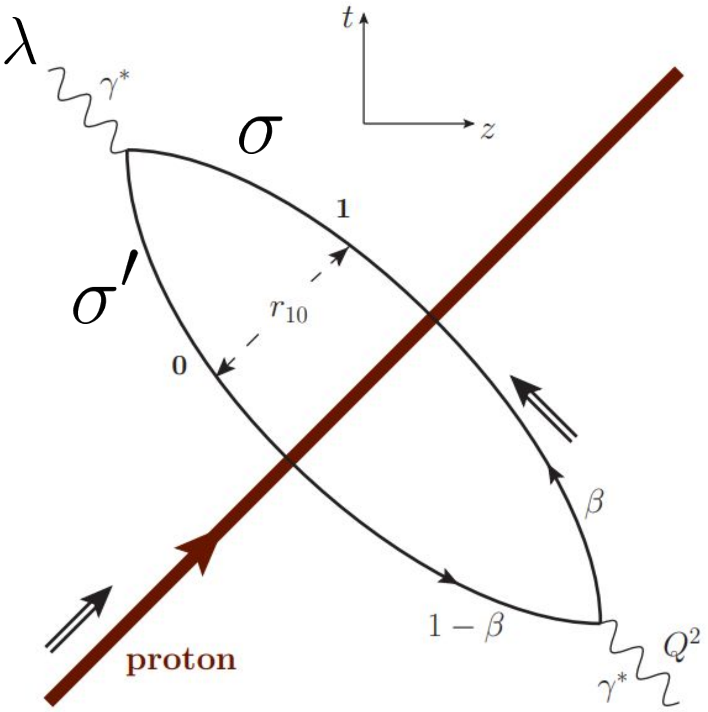


- Q^2 = resolution at which we probe the proton
- Longitudinal momentum fraction, Bjorken $x \sim \frac{1}{s}$. We need theory to extrapolate to x=0

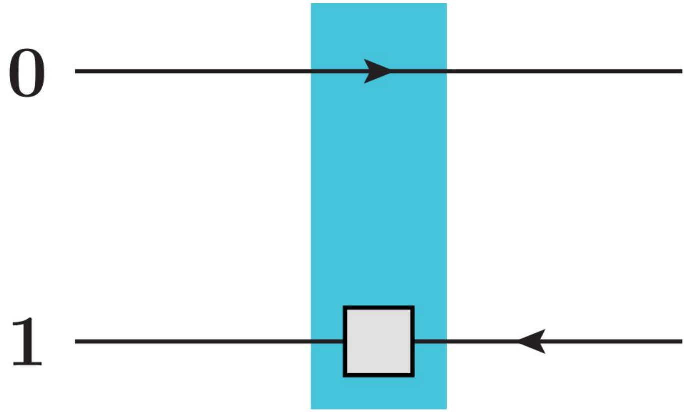
(Polarized) DIS in the (Polarized) Dipole Picture

Rapidity factorization between a quark-splitting wave function and a quark-antiquark dipole:

$$g_1 \propto |\psi|^2 \otimes (Q + 2G_2)$$



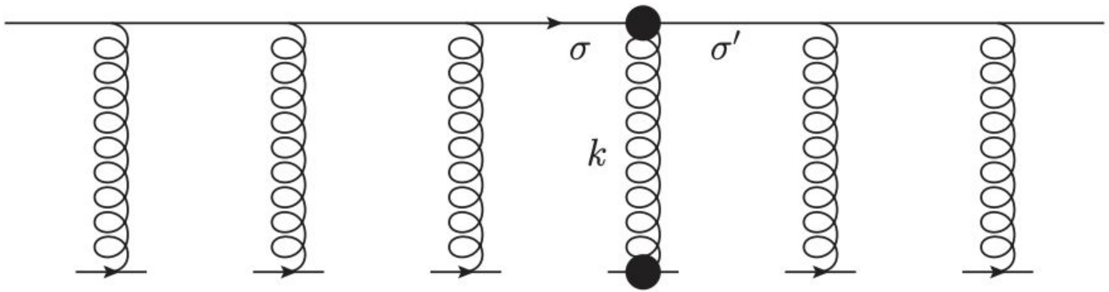
(Polarized) DIS in the (Polarized) Dipole Picture



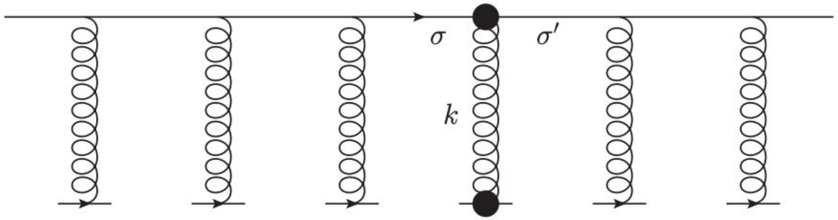
- In pDIS, the electron and proton have their helicity specified
- Cross-section now dependent on **Polarized Dipole Amplitudes**:

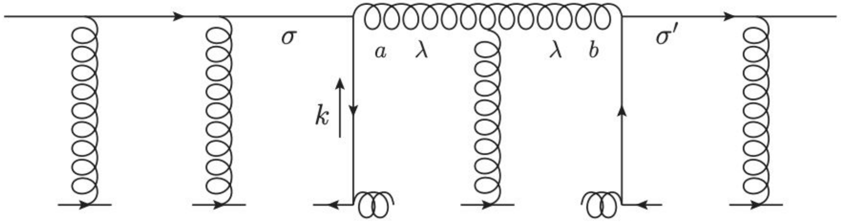
$$\left\langle \mathrm{tr}\left[V_{\underline{1}} V_{\underline{0}}^{\mathrm{pol}\dagger}\right]\right\rangle(s)$$

- Quark line undergoes one extra exchange, containing helicity information, which is energy, or *x* suppressed, a.k.a. **sub-eikonal**



Polarized Wilson Lines





$$V_{\mathbf{x}}^{\text{pol}} = \int dz^- V_{\mathbf{x}}(\infty, z^-)\Gamma V_{\mathbf{x}}(z^-, -\infty)$$

Γ :

$\bar{\psi}\gamma^+\gamma^5\psi$

- Axial Current

$e^{-ix^-\frac{k_\perp^2}{2k^-}} \approx 1 - ix^-\frac{k_\perp^2}{2k^-}$

- Sub-eikonal phase expansio

$\sim F_{12}$

- Chromo-magnetic field

$A_\perp$

- Polarized gluon vertex

Operator definition of quark helicity TMD

$$g_{1L}^{q}(x,k_T^2) = \frac{1}{(2\pi)^3}\frac{1}{2}\sum_{S_L} S_L \int d^2r dr^- e^{ik\cdot r} \times \langle pS_L|\bar{\psi}(0)\mathcal{U}[0,r]\frac{\gamma^+\gamma^5}{2}\psi(r)|pS_L\rangle_{r^+=0}$$

- The gauge link in $A^-=0$ gauge can be written as an infinite Wilson line
- The contraction of the quark fields gives a quark propagator which contains a Wilson line
- Expand one of the Wilson lines in eikonality and contract against Dirac matrix to pick out relevant polarized dipoles

$$\left\langle \mathrm{tr}\left[V_{\underline{1}} V_{\underline{0}}^{\mathrm{pol}\dagger}\right]\right\rangle(s)$$

Helicity Distributions

At small-*x*, the operator definitions of the gluon, singlet and non-singlet helicity distributions coincide with the operator definitions of the polarized dipole amplitudes

$$\Delta\Sigma(x,Q^2) = -\frac{N_c N_f}{2\pi^3} \int\limits_{\Lambda^2/s}^{1} \frac{dz}{z} \int\limits_{\frac{1}{zs}}^{\min\left\{\frac{1}{zQ^2},\frac{1}{\Lambda^2}\right\}} \frac{dx_{10}^2}{x_{10}^2} \left[Q(x_{10}^2, zs) + 2\,G_2(x_{10}^2, zs)\right]$$

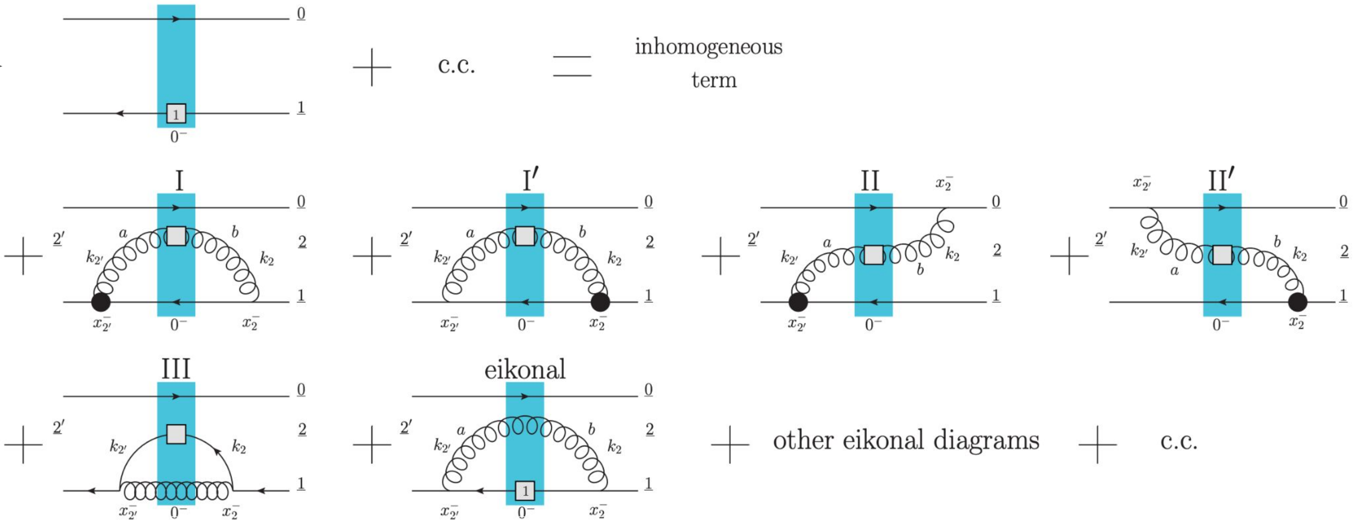
$$\Delta G(x,Q^2) = \frac{2N_c}{\alpha_s(Q^2)\pi^2}\, G_2\left(x_{10}^2 = \frac{1}{Q^2}, zs = \frac{Q^2}{x}\right)$$

Q, G_2 are Polarized Dipole Amplitudes defined in terms of the quark axial current and covariant derivative operators respectively.

Small-x / Rapidity Evolution

- Relate Polarized Dipole Amplitude to themselves at higher energies by resumming emission diagrams - resumming Double Log (DLA) contributions: $\alpha_s \ln^2(1/x)$

$Q_q(x_{10}^2, zs) \longrightarrow$



Double Logarithm Approximation

Using Light-Cone Operator Treatment, we need to resum all gluon exchanges that exchange helicity information

$\Rightarrow$ Resumming all terms containing: $$\alpha_s \int\limits_x^1 \frac{dz}{z} \int\limits_{1/s}^{1/Q^2} \frac{d^2 x_{21}}{x_{21}^2}$$

Longitudinal part. Present in un-polarized evolution

Transverse part. UV exactly cancelled in un-polarized evolution

Resum double log (DLA) terms:

$$\alpha_s \ln^2(1/x)$$

$\Rightarrow$ Small-*x* is *x*<0.1

Large Nc&Nf, Linearized Helicity Evolution

In the large Nc&Nf, Nc/Nf fixed limit, the evolution equations for the polarized dipole amplitudes close:

$$Q_q(s_{10},\eta) = Q_q^{(0)}(s_{10},\eta) + \int_{s_{10}+y_0}^{\eta} d\eta' \int_{s_{10}}^{\eta'-y_0} ds_{21}\, \alpha_s(s_{21}) \Big[Q_q(s_{21},\eta') + 2\tilde{G}(s_{21},\eta') + 2\tilde{\Gamma}(s_{10},s_{21},\eta') \\ - \bar{\Gamma}_q(s_{10},s_{21},\eta') + 2G_2(s_{21},\eta') + 2\Gamma_2(s_{10},s_{21},\eta') \Big] \\ + \frac{1}{2}\int_{y_0}^{\eta} d\eta' \int_{\max\{0,s_{10}+\eta'-\eta\}}^{\eta'-y_0} ds_{21}\, \alpha_s(s_{21}) \Big[Q_q(s_{21},\eta') + 2G_2(s_{21},\eta') \Big]$$

+ 9 more

- 5 Polarized dipole amplitudes mix under evolution: $Q_{u,d,s}, \tilde{G}, G_2$
- With 5 auxiliary dipoles: $\Gamma_{u,d,s}, \tilde{\Gamma}, \Gamma_2$ - which impose lifetime ordering
- Small-*x* cutoff, $y_0 \propto \ln 1/x_0$

Recap:

$$\frac{1}{2} = S_q + L_q + S_g + L_g$$

$$S_q(Q^2) = \frac{1}{2}\int_0^1 dx \, \sum_q (\Delta q(x, Q^2) + \Delta \bar{q}(x, Q^2))$$

$$S_g(Q^2) = \int_0^1 dx \Delta G(x, Q^2)$$

$$\Delta q + \Delta \bar{q} = -\frac{1}{\pi} \int_0^{\eta_{max}} d\eta \int_{s_{10}^{min}}^{\eta} ds_{10} \left[Q_q(s_{10}, \eta) + 2G_2(s_{10}, \eta) \right]$$

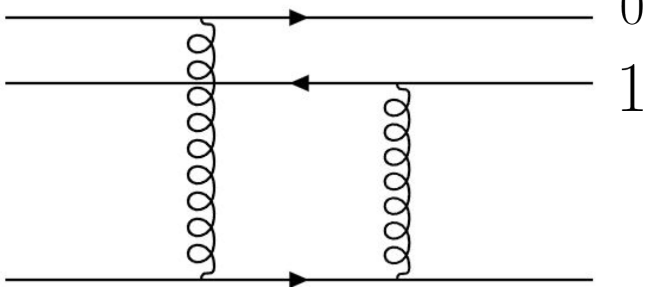
$$\Delta G(x, Q^2) = \frac{2N_c}{\alpha_s(Q^2)\pi^2} G_2 \left(x_{10}^2 = \frac{1}{Q^2}, zs = \frac{Q^2}{x} \right)$$

Large $N_c \& N_f$ Helicity Evolution

$$Q_q^{(0)}, \tilde{G}^{(0)}, G_2^{(0)}$$

Inhomogeneous term

The inhomogeneous term is given by a **Born-inspired** ansatz:



$$\propto \int_0^s \frac{dk_\perp^2}{k_\perp^2}\left(1 - e^{-\underline{k}\cdot\underline{x}_{10}}\right) = \pi \ln(s x_{10}^2)$$

$$\propto \eta - s_{10}$$

$$\Gamma_q^{(0)} = Q_q^{(0)} = a_q \eta + b_q s_{10} + c_q$$

η - Log of longitudinal momentum fraction

s_{10} - Log of transverse dipole size

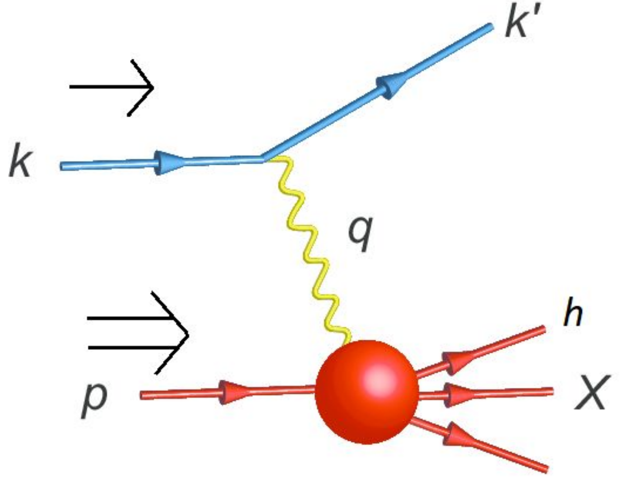
- Same form of the other Dipole Amplitudes
- Parameters a,b,c need to be extracted from data
- **24** parameters for singlet+non-singlet hPDFS

Phenomenology

[DA et al 2308.07461]

Observables - Double Spin Asymmetries in (SI)DIS

$$A_{||}(z) = \frac{\sigma^{\uparrow\Downarrow} - \sigma^{\uparrow\Uparrow}}{\sigma^{\uparrow\Downarrow} + \sigma^{\uparrow\Uparrow}} \propto g_1^h(z)$$

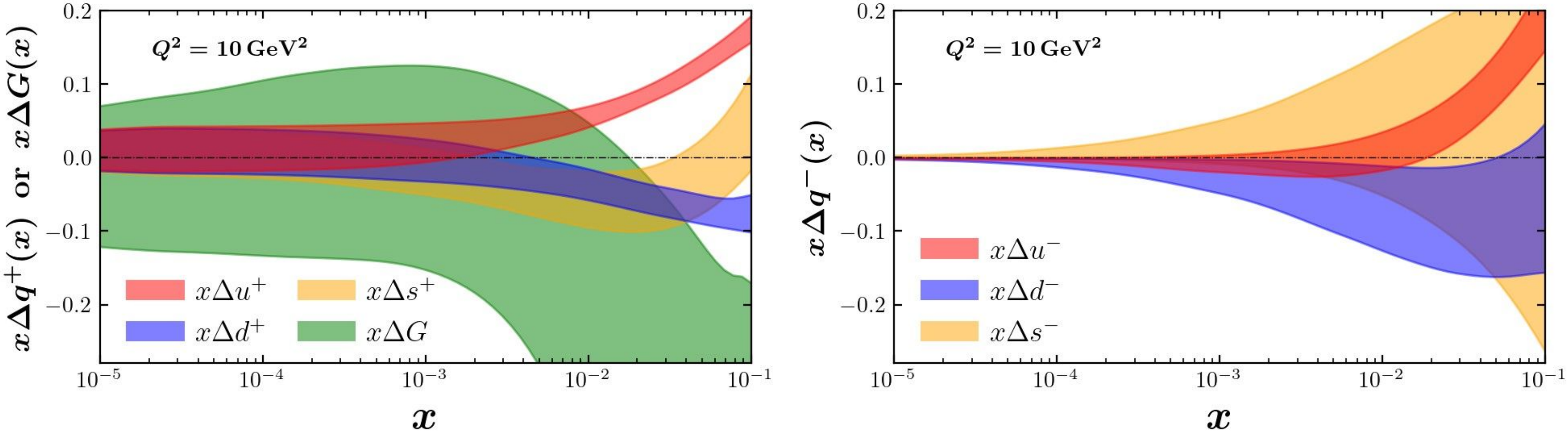


- *h* is the tagged hadron
- *z* is the momentum fraction of the virtual photon carried by the tagged hadron

χ^2 and Data Cuts

- First simultaneous fit of small-*x* theory to polarized DIS & SIDIS data
- Cut of 0.005< x < x_0 =0.1, given by $\alpha_s \ln^2 \frac{1}{x_0} \sim \mathcal{O}(1)$
- Cut of $1.69\ GeV^2 < Q^2 < 10.5\ GeV^2$
- Cut of 0.2 < z <0.8
- 24 fit parameters
- Describing 226 data points
- With a $\chi^2/npts$ = 1.03

Extraction of Helicity Parton Distribution Functions



Contribution to spin of the proton from spin of quarks and gluons

Quark helicity:

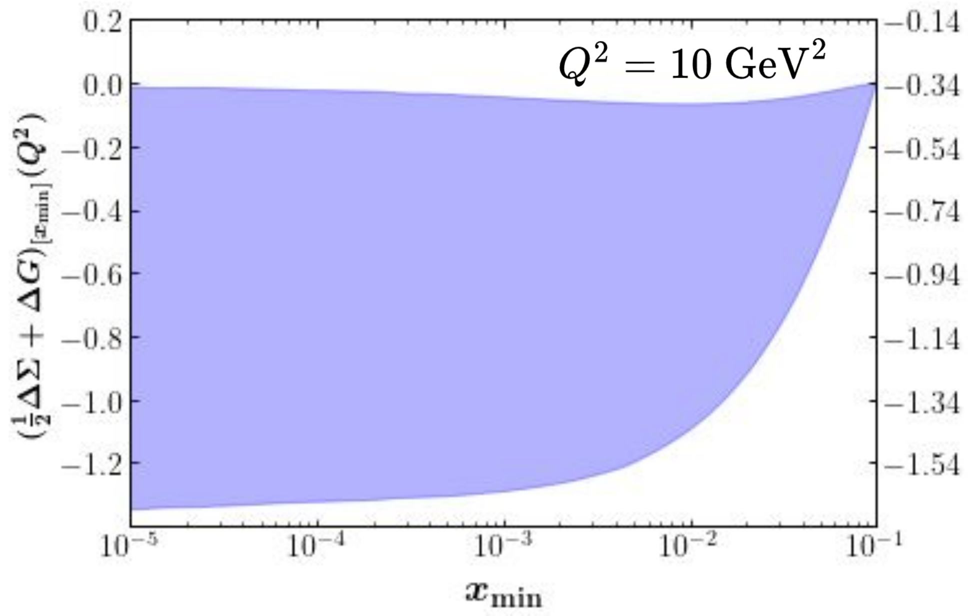
$$\Delta\Sigma(x,Q^2) \equiv \Delta u^+(x,Q^2) + \Delta d^+(x,Q^2) + \Delta s^+(x,Q^2)$$

Gluon helicity:

$$\Delta G(x,Q^2)$$

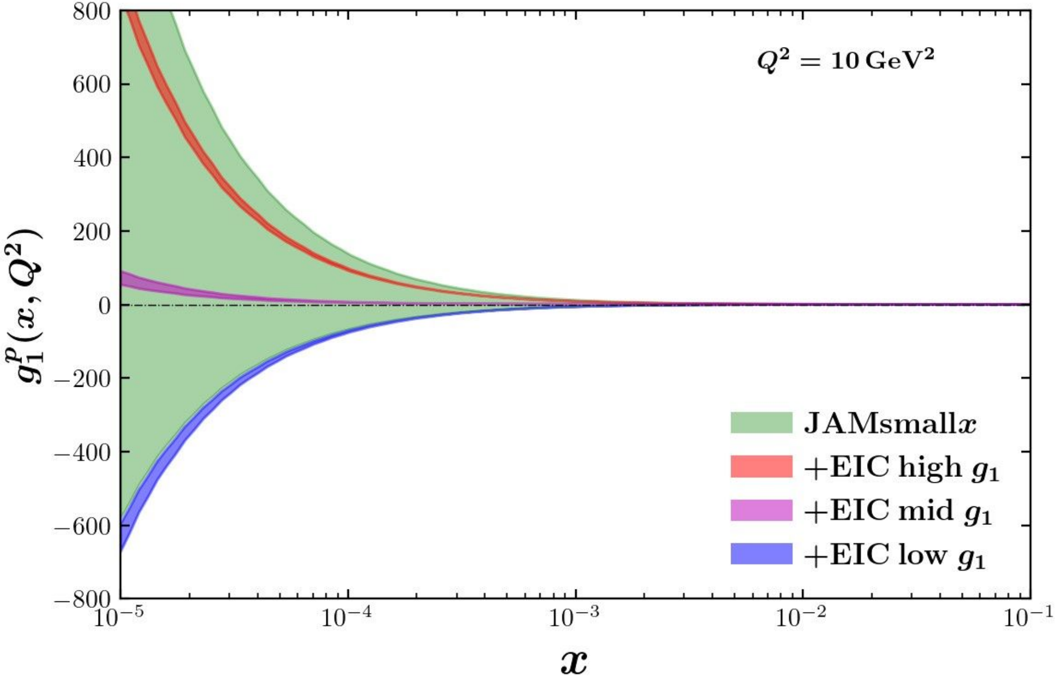
Partial moment of parton helicity:

$$\left(\tfrac{1}{2}\Delta\Sigma + \Delta G\right)_{[x_{\min}]} \equiv \int_{x_{\min}}^{0.1} dx\ \left(\tfrac{1}{2}\Delta\Sigma + \Delta G\right)(x,Q^2)$$

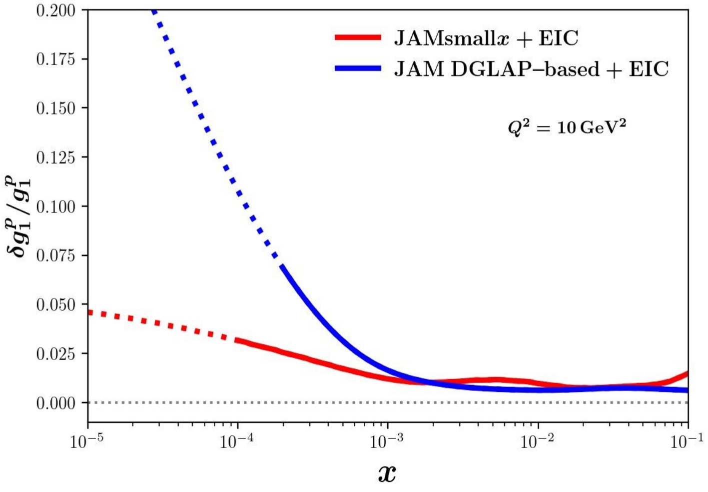


$$\int_{10^{-5}}^{0.1} dx\ \left(\tfrac{1}{2}\Delta\Sigma + \Delta G\right)(x,Q^2) = -0.64 \pm 0.60$$

Electron Ion Collider Impact



Uncertainty ratios



EIC pseudo data consistent with detector design in Yellow Report

Ways to Reduce Uncertainty

- EIC data: Large impact, but we prefer to predict instead of postdict
- New observables: proton-proton data more sensitive to gluon helicity than SIDIS, see [Baldonado et al 2503.21006]
- Improve our model of the initial condition:

  Valence Quark Model of the Proton

Valence Quark Model

[Dumitru, Miller, Venugopalan, 1808.02501]
[Dumitru, Skokov, Stebel, 2001.04516]
[Dumitru, Paatelainen, 2010.11245]

Valence Quark Model

- Polarized dipole amplitudes involve helicity-dependent averaging over (target) proton state:

$$\langle \cdots \rangle = \frac{1}{2} \sum_{\mathcal{S}_L} \mathcal{S}_L \frac{\langle P^+, \underline{P}, \mathcal{S}_L | \cdots | P^+, \underline{P}, \mathcal{S}_L \rangle}{\langle P^+, \underline{P}, \mathcal{S}_L | P^+, \underline{P}, \mathcal{S}_L \rangle}$$

with $\langle K^+, \underline{K}, \mathcal{S}'_L | P^+, \underline{P}, \mathcal{S}_L \rangle = \delta_{\mathcal{S}_L \mathcal{S}'_L} \, 2P^+ \, 2\pi \delta(P^+ - K^+) \, (2\pi)^2 \delta^2(\underline{P} - \underline{K})$

- The proton state can be written as

$$|P^+, \underline{P}, \mathcal{S}_L\rangle = \frac{1}{\sqrt{6}} \int \frac{dx_1 dx_2 dx_3}{(4\pi)^3 \sqrt{x_1 x_2 x_3}} \, 4\pi \delta(1 - x_1 - x_2 - x_3) \int \frac{d^2 q_1 d^2 q_2 d^2 q_3}{(2\pi)^6} \, (2\pi)^2 \delta^2(\underline{q}_1 + \underline{q}_2 + \underline{q}_3)$$
$$\times \sum_{\{f_1, f_2, f_3\} = \{u, u, d\}} \sum_{\sigma_1, \sigma_2, \sigma_3} \Phi(x_1, \underline{q}_1; x_2, \underline{q}_2; x_3, \underline{q}_3) \, S(\sigma_1, f_1; \sigma_2, f_2; \sigma_3, f_3)$$
$$\times \sum_{i_1, i_2, i_3} \epsilon_{i_1 i_2 i_3} \, |x_1 P^+, x_1 \underline{P} + \underline{q}_1, i_1, \sigma_1, f_1\rangle \, |x_2 P^+, x_2 \underline{P} + \underline{q}_2, i_2, \sigma_2, f_2\rangle \, |x_3 P^+, x_3 \underline{P} + \underline{q}_3, i_3, \sigma_3, f_3\rangle$$

Valence Quark Model

Valence quark model is three quarks bound by a harmonic potential (momentum space wave function is Gaussian) tuned (via width of Gaussian) to give a realistic proton radius.

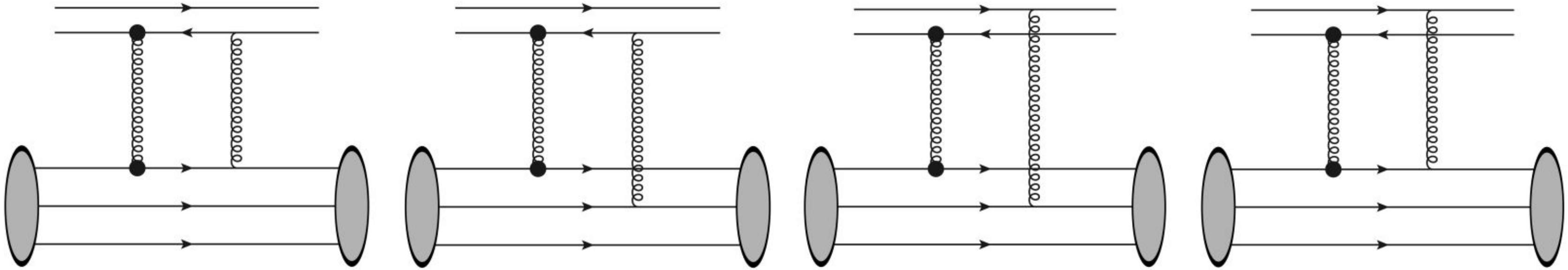
$$\Phi(x_i, q_{i\perp}) = N \exp\left[-\frac{1}{2\beta^2}\sum_{i=1}^{3}\frac{q_{i\perp}^2 + M^2}{x_i}\right]$$

Once you have to proton wave function, you can explicitly compute the expectation value of the polarized dipole amplitude

Valence quark diagram

Amounts to computing diagrams of the form:

[Dumitru et al. 2407.08893]



$$\Gamma_q^{(0)} = Q_q^{(0)} = \boxed{a_q}\eta + \boxed{b_q}s_{10} + c_q$$

16 out of 24 parameters fully determined!

η - Log of longitudinal momentum fraction

s_{10} - Log of transverse dipole size

Domain of Applicability of the Valence Quark Model

- We are modeling a bridge between large and small *x*
- The valence quark model is a large-*x* model
- Previous attempts at phenomenology in the unpolarized sector failed because the gap between small-*x* and the valence region was too large, [2303.16339]:

Unpolarized fit: $\alpha_s \ln \frac{1}{x} \sim \mathcal{O}(1) \Rightarrow x < 0.01, \qquad \chi^2 = 2.3$

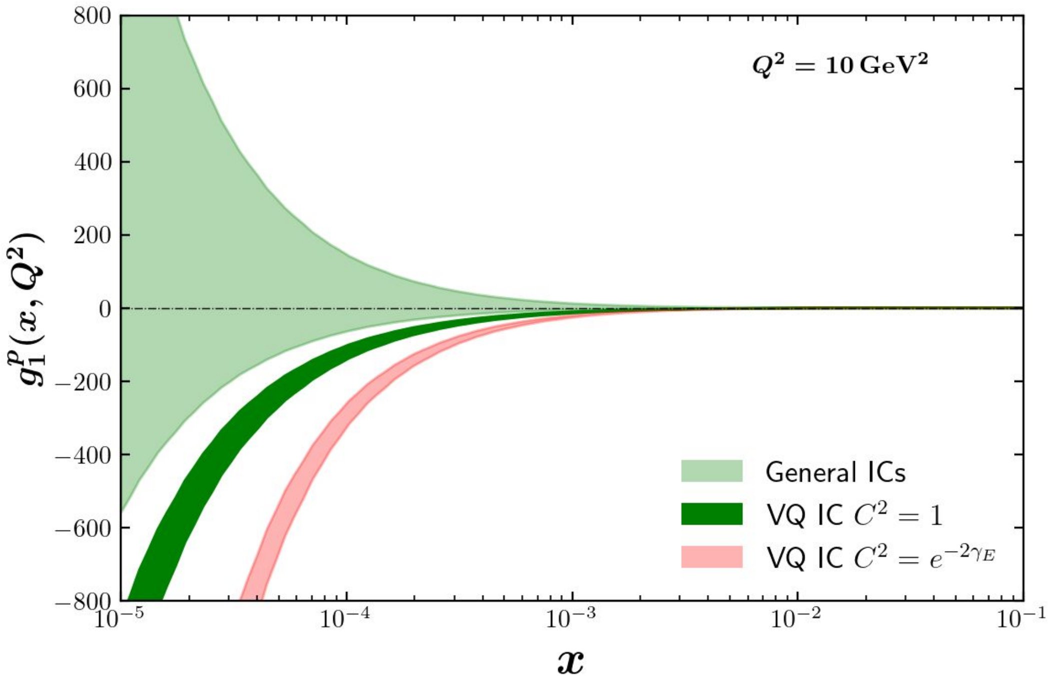
Polarized fit: $\alpha_s \ln^2 \frac{1}{x} \sim \mathcal{O}(1) \Rightarrow x < 0.1$

- More applicable for the polarized sector

Phenomenology 2

[DA et al 2502.16604]

g1 Structure Function



Running coupling prescription:

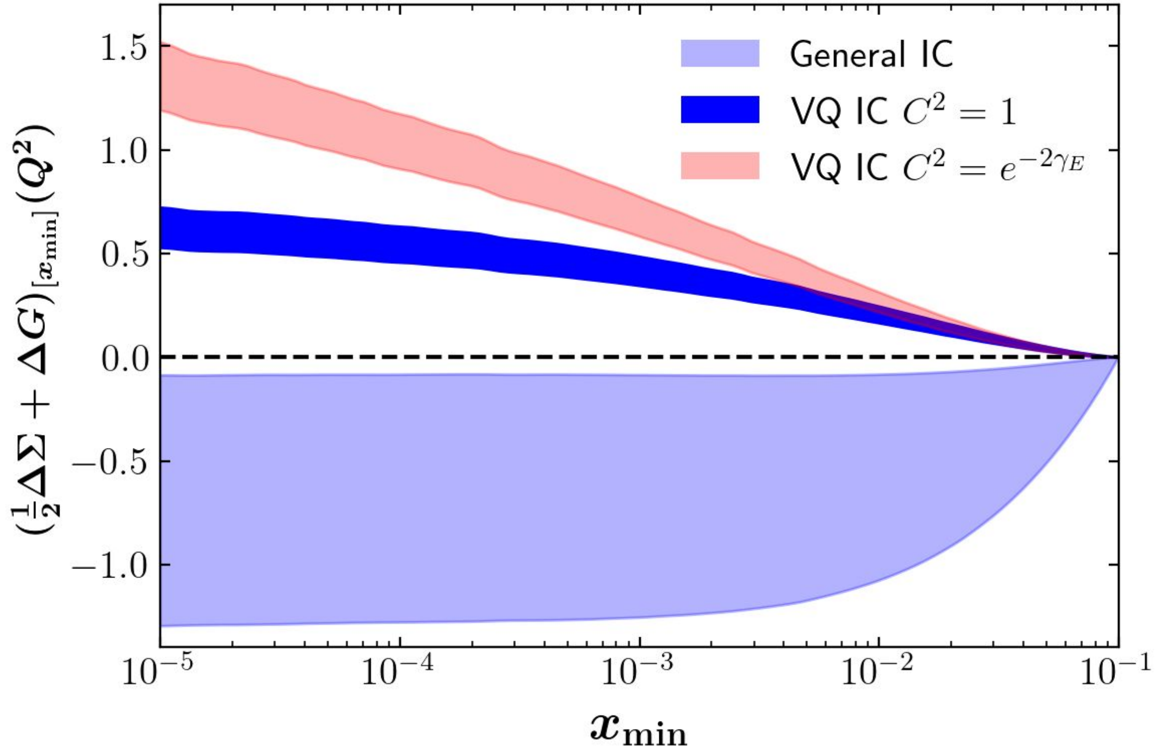
$$\alpha_s(r_\perp) = \frac{12\pi}{(33 - 2N_f)\,\ln\left(\frac{4C^2}{r_\perp^2 \Lambda_{QCD}^2}\right)}$$

- C^2 - Usually a free parameter
- $C^2 = 1$ - Original Pheno work
- $C^2 = e^{-2\gamma_E}$ - Theoretical prediction [Kovchegov, Weigert 0609090]

8 free parameters

$\chi^2/npts$ = 1.3

Net helicity



$C^2 = 1:$

$$\int_{10^{-5}}^{0.1} dx \left(\tfrac{1}{2}\Delta\Sigma + \Delta G\right) = 0.76 \pm 0.13$$

$C^2 = e^{-2\gamma_E}:$

$$\int_{10^{-5}}^{0.1} dx \left(\tfrac{1}{2}\Delta\Sigma + \Delta G\right) = 1.7 \pm 0.2$$

Conclusions

- The quark valence model **reduces uncertainty** dramatically
- Slight reduction in fit quality at the benefit using only **8 out of 24** parameters
- There is a **large uncertainty** associated with the **running coupling prescription**
- A systematic treatment of running coupling requires incorporating the **Single-logarithmic** contributions

Backup Slides

For helicity, 3 polarized dipoles enter:

G_2 "Gluon dipole". Contains $e^{-ix^- \frac{k_\perp^2}{2k^-}}$, $A_\perp$

Q_q "Quark dipole". Contains $\bar{\psi}\gamma^+\gamma^5\psi$, $\sim F_{12}$

$\widetilde{G}$ "Adjoint dipole". Contains the same operators as Q, but inserted into adjoint Wilson lines instead of fundamental

+ neighbour dipoles, $\Gamma_2, \Gamma_q, \widetilde{\Gamma}$, that enforce lifetime ordering

Describing Observables - pDIS

What enters into observables are linear combinations of hPDFs

$$\Delta q^+ = \Delta q + \Delta \bar{q}$$
$$\Delta q^- = \Delta q - \Delta \bar{q}$$

- Three relevant hPDFs in DIS: $\Delta u^+, \Delta d^+, \Delta s^+$
- Data exist for two observables that contain these hPDFs in linearly independent combinations: g_1^p and g_1^n

$$g_1^p(x, Q^2) = \frac{1}{2} \sum_q Z_q^2 \Delta q^+(x, Q^2)$$

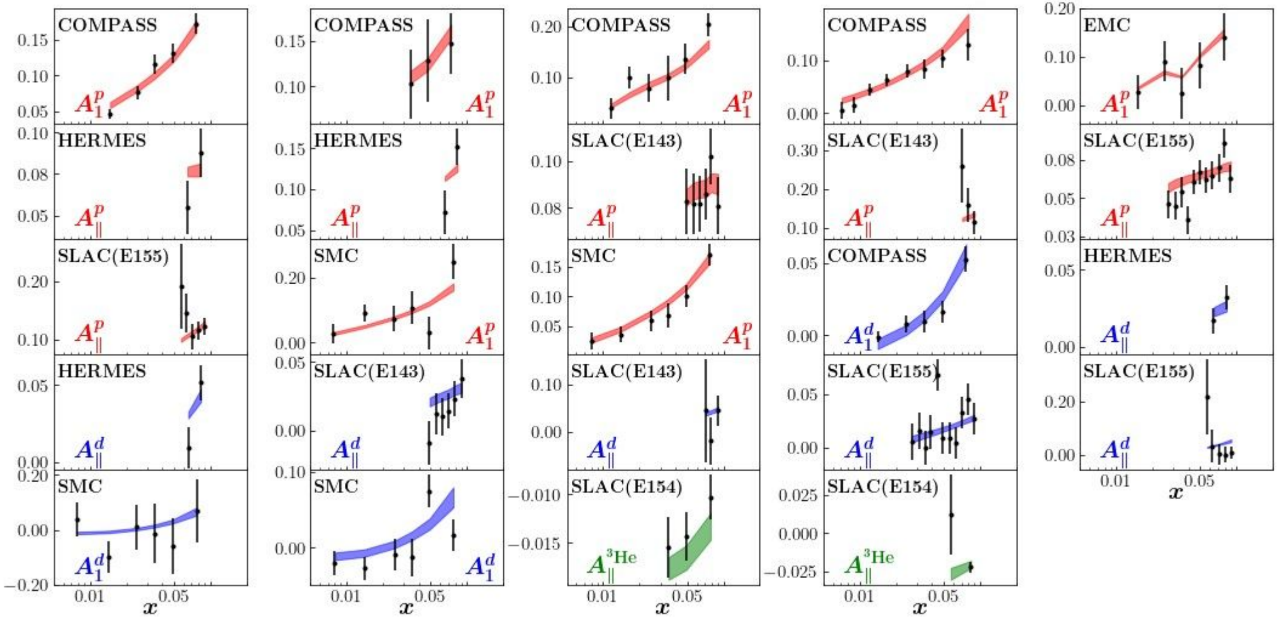
- Z_q is the quark charge fraction

g_1^h Structure Functions

$$g_1^h(x, z, Q^2) = \frac{1}{2}\sum_q Z_q^2 \Delta q(x, z, Q^2) D_q^h(z, Q^2)$$

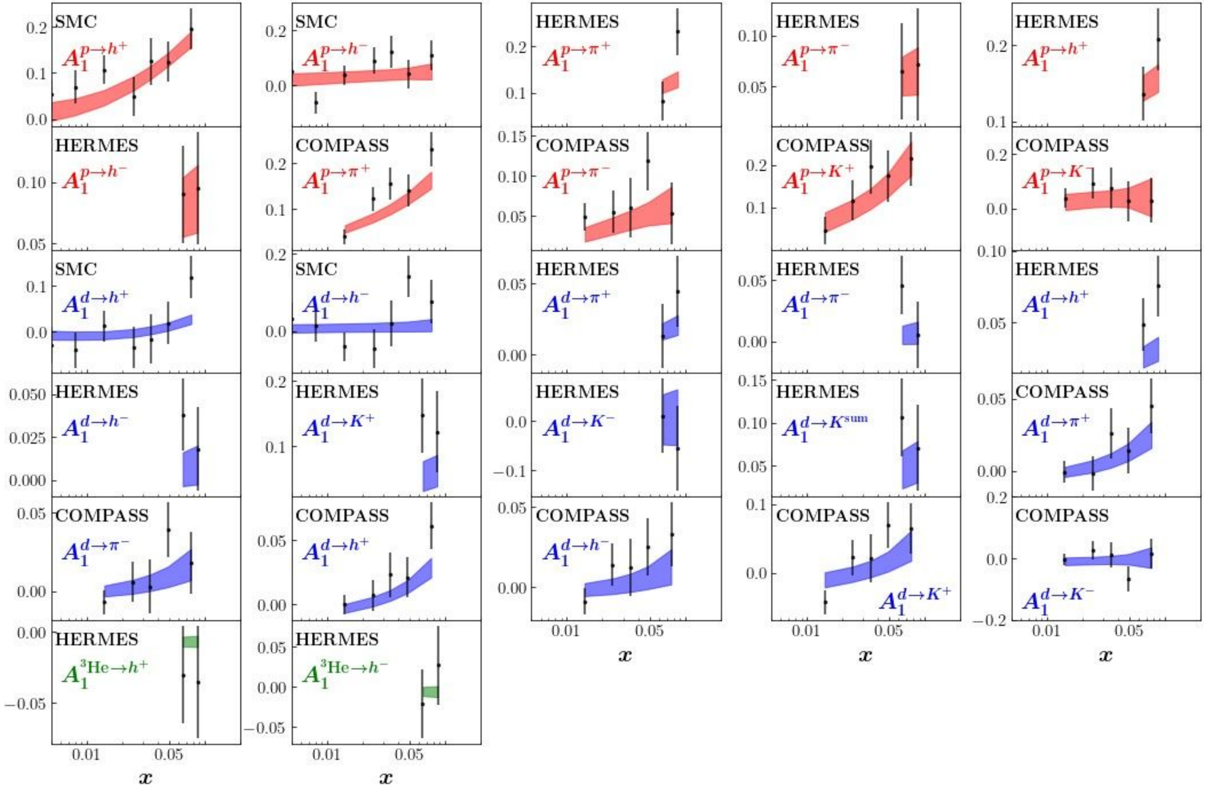
- D_q^h are fragmentation functions - giving the probability quark *q* fragments into hadron *h*
- z Is the fraction of the virtual photons momentum carried by the hadron
- The flavour hPDF is obtained via $\Delta q = \frac{1}{2}(\Delta q^+ + \Delta q^-)$
- Δq^- is obtained from non-singlet evolution

Global fit of DIS - Data vs Theory



- Red curves - our theory
- Black dots - data
  - COMPASS
  - EMC
  - SMC
  - SLAC
  - HERMES

Global fit of SIDIS - Data vs Theory



Gluon production in proton-proton scattering

- Gluon production in *pp* scattering provides an independent channel to probe gluon helicity
- See Ming Li's talk for more details

- Preliminary fit to *pp* data, performed by Nick Baldonado
- Using modified formalism, describes data and leads to a reduction in uncertainty