Quark TMDs from back-to-back dijet production, at forward rapidities in pA collisions beyond eikonal accuracy in the CGC

Etienne Blanco[a]

in collaboration with : T. Altinoluk [a], G. Beuf [a]

and S. Mulani [a]

QCD Evolution 2025

[a] *Theoretical Physics Division, National Centre for Nuclear Research (Poland)*
supported by NCN grant No. 2020/38/E/ST2/00122.

NCBJ

CONTEXT

Color Glass Condensate

- Effective theory describing high energy gluons in hadron
- Non-linear evolution equations BK-JIMWLK
- Eikonal approximation

Edmond Iancu, Andrei Leonidov, and Larry D. McLerran. Nonlinear gluon evolution in the color glass condensate. 1. ***Nucl. Phys. A*, 692:583–645, 2001**

Elena Ferreiro, Edmond Iancu, Andrei Leonidov, and Larry McLerran. Nonlinear gluon evolution in the color glass condensate. 2. ***Nucl. Phys. A*, 703:489–538, 2002**

Edmond Iancu, Andrei Leonidov, and Larry McLerran. The Color glass condensate: An Introduction. **In *Cargese Summer School on QCD Perspectives on Hot and Dense Matter*, pages 73–145, 2 2002**

Francois Gelis, Edmond Iancu, Jamal Jalilian-Marian, and Raju Venugopalan. The Color Glass Condensate. ***Ann. Rev. Nucl. Part. Sci.*, 60:463–489, 2010**

Color Glass Condensate

- Effective theory describing high energy gluons in hadron
- Non-linear evolution equations BK-JIMWLK
- Eikonal approximation

pA collisions

- Hybrid factorization
- Dense-dilute system

Color Glass Condensate

- Effective theory describing high energy gluons in hadron
- Non-linear evolution equations BK-JIMWLK
- Eikonal approximation

pA collisions

Projectile

- Dilute
- Described by PDF
- DGLAP evolution

Target

- Dense
- MV model
- Described by classical color field $\mathcal{A}_a^\mu(x)$

$$\mathcal{A}_a^{\mu}(x^-, x^+; \mathbf{x}) = \delta^{\mu -}\delta(x^+)\mathcal{A}_a^-(\mathbf{x})$$

Eikonal approximation

High energy $\Rightarrow$ boost of the target along x^- direction
Consequences for $\mathcal{A}_a^{\mu}(x)$:

- Localized around $x^+ = 0$
- Leading "-" component
- Independence on x^-

$$\mathcal{A}_a^\mu(x^-, x^+; \mathbf{x}) = \delta^{\mu -}\delta(x^+)\mathcal{A}_a^-(\mathbf{x})$$

Eikonal approximation

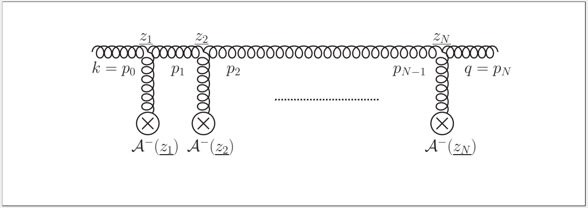
High energy $\Rightarrow$ boost of the target along x^- direction
Consequences for $\mathcal{A}_a^\mu(x)$:

- Localized around $x^+ = 0$
- Leading "-" component

$\Rightarrow$ Shockwave limit

- Independence on x^-

$\Rightarrow$ Static limit

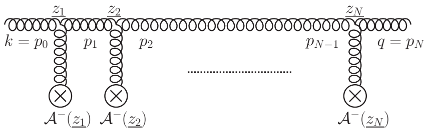


Eikonal interaction

- p^+ preserved
- Negligible transverse momentum exchange

$\Rightarrow$ Straight trajectory

- Arbitrary number of interaction with $\mathcal{A}^-$



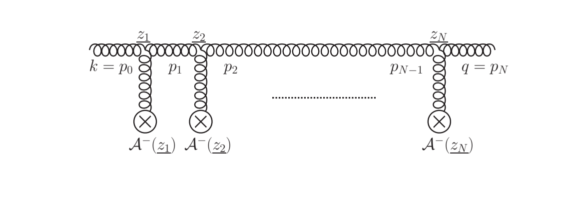
Eikonal interaction

- p^+ preserved
- Negligible transverse momentum exchange

$\Rightarrow$ Straight trajectory

- Arbitrary number of interaction with $\mathcal{A}^-$

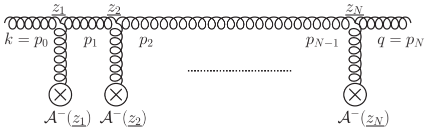
Ressumation of $(g\mathcal{A}^-(x^+, \mathbf{x}))^n \Rightarrow$ Wilson lines along x^+ direction



Wilson Line

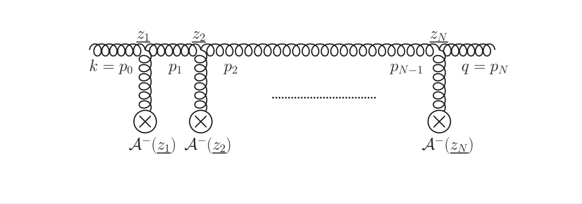
$$\mathcal{U}_{\rm R}(x^+, y^+; \mathbf{z}) \equiv \mathcal{P}_+ e^{\left\{-ig\int_{y^+}^{x^+} {\rm d}z^+ T_{\rm R}\cdot\mathcal{A}^-(z^+;\mathbf{z})\right\}}$$

$$= 1 + \sum_{N=1}^{+\infty} \frac{1}{N!}\mathcal{P}_+ \left(-ig\int_{y^+}^{x^+} {\rm d}z^+ T_{\rm R}\cdot\mathcal{A}^-(z^+;\mathbf{z})\right)^N$$



Gluon Propagator

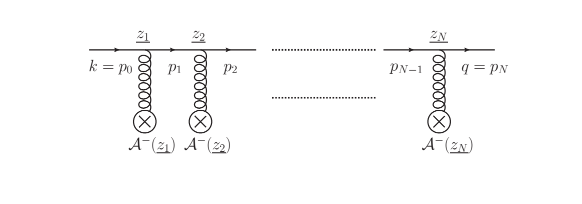
$$G_{\mathrm{F}}^{\mu\nu}(x,y)\big|_{\mathrm{Eik.}} = \sum_{N=1}^{+\infty}\left[\prod_{n=1}^{N-1}\int\frac{d^4p_n}{(2\pi)^4}\right]\int\left[\prod_{n=1}^{N} d^3\underline{z_n}\, e^{i\underline{z_n}\cdot(p_n-p_{n-1})}2\pi\delta(p_n^+ - p_{n-1}^+)\right]$$
$$\times\left\{\mathcal{P}_n\prod_{n=1}^{N}\left[-igT\cdot\mathcal{A}^-(\underline{z_n})\right]\right\}\left[\prod_{n=1}^{N}\frac{i}{p_n^2+i\epsilon}\left[-g^{\mu_{n+1}\mu_n}+\frac{p_n^{\mu_{n+1}}\eta^{\mu_n}+\eta^{\mu_{n+1}}p_n^{\mu_n}}{p_n^+}\right]\right]$$
$$\times\left[-(p_n^+ + p_{n-1}^+)g_{\mu_n\mu_{n-1}} + (2p_{n-1_{\mu_n}} - p_{n_{\mu_n}})g^+_{\mu_{n-1}} + (2p_{n_{\mu_{n-1}}} - p_{n-1_{\mu_{n-1}}})g^+_{\mu_n}\right]$$



Gluon Propagator

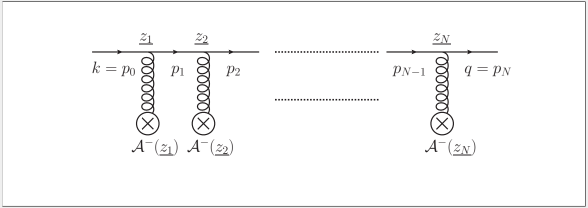
$$G_{\mathrm{F}}^{\mu\nu}(x,y)\big|_{\mathrm{Eik.}} = \int \frac{d^3\underline{k_1}}{(2\pi)^3}\,\theta(k_1^+)\,e^{-ix\cdot\check{k}_1}\int \frac{d^3\underline{k_2}}{(2\pi)^3}\,\frac{\theta(k_2^+)}{k_1^+ + k_2^+}\,e^{iy\cdot\check{k}_2}$$

$$\times\left[-g^{\mu\nu} + \frac{\check{k}_2^\mu\eta^\nu}{k_2^+} + \frac{\eta^\mu\check{k}_1^\nu}{k_1^+} - \frac{\eta^\mu\eta^\nu}{k_1^+k_2^+}\check{k}_1\cdot\check{k}_2\right]\int_{\mathbf{z}} e^{-i\mathbf{z}\cdot(\mathbf{k}_1-\mathbf{k}_2)}\int_{z^-} e^{iz^-(k_1^+-k_2^+)}\,\mathcal{U}_{\mathrm{A}}(x^+,y^+;\mathbf{z})$$



Quark Propagator

$$S_{\mathrm{F}}(x,y)\big|_{\mathrm{Eik.}} = \sum_{N=1}^{+\infty} \int \left[\prod_{n=1}^{N-1} \frac{d^4 p_n}{(2\pi)^4}\right] \int \left[\prod_{n=1}^{N} d^3 \underline{z_n} e^{i\underline{z_n}\cdot(p_n - p_{n-1})} 2\pi\delta(p_n^+ - p_{n-1}^+)\right]$$
$$\times \left\{\mathcal{P}_n \prod_{n=1}^{N} \left[-igt\cdot\mathcal{A}^-(\underline{z_n})\right]\right\}_{\beta\alpha} \frac{i(\not{q}+m)}{q^2 - m^2 + i\epsilon} \left\{\mathcal{P}_n \prod_{n=0}^{N-1} \left[\gamma^+ \frac{i(\not{p}_n + m)}{p_n^2 - m^2 + i\epsilon}\right]\right\}$$



Quark Propagator

$$S_{\rm F}(x,y)\big|_{\rm Eik.} = \int \frac{d^3\underline{k_1}}{(2\pi)^3}\frac{\theta(k_1^+)}{2k_1^+}e^{-ix\cdot\check{k}_1}\int \frac{d^3\underline{k_2}}{(2\pi)^3}\frac{\theta(k_2^+)}{2k_2^+}e^{iy\cdot\check{k}_2}\int_{\mathbf{z}} e^{-i\mathbf{z}\cdot(\mathbf{k}_1-\mathbf{k}_2)}$$
$$\times\int_{z^-} e^{iz^-(k_1^+-k_2^+)}(\check{\not{k}}_1+m)\gamma^+(\check{\not{k}}_2+m)\,\mathcal{U}_{\rm F}(x^+,y^+;\mathbf{z})$$

Motivations

Motivations

Why subeikonal ?

- EIC and RHIC : sizable power-suppressed corrections
- Spin physics
- Understanding of CGC domain of validity
- Study link between CGC and other framework

TMD

- $Q_s \sim k_t \ll P_t$
- On-shell matrix elements
- TMDs

HEF

- $k_t \sim P_t$
- Off-shell matrix elements
- uPDF

John Collins. *Foundations of Perturbative QCD*, volume 32 of *Cambridge Monographs on Particle Physics, Nuclear Physics and Cosmology*. **Cambridge University Press, 7 2023**

TMD

- $Q_s \sim k_t \ll P_t$
- On-shell matrix elements
- TMDs

HEF

- $k_t \sim P_t$
- Off-shell matrix elements
- uPDF

S. Catani, M. Ciafaloni, and F. Hautmann. High-energy factorization and small x heavy flavor production. ***Nucl. Phys. B*, 366:135–188, 1991**

M. Deak, F. Hautmann, H. Jung, and K. Kutak. Forward Jet Production at the Large Hadron Collider. ***JHEP*, 09:121, 2009**

TMD

- $Q_s \sim k_t \ll P_t$
- On-shell matrix elements
- TMDs

HEF

- $k_t \sim P_t$
- Off-shell matrix elements
- uPDF

iTMD

Interpolates both approaches

- $\forall k_t \in [Q_s, P_t]$
- Off-shell matrix elements
- Several gluon TMDs

P. Kotko, K. Kutak, C. Marquet, E. Petreska, S. Sapeta, and A. van Hameren. Improved TMD factorization for forward dijet production in dilute-dense hadronic collisions. ***JHEP*, 09:106, 2015**

TMD

- $Q_s \sim k_t \ll P_t$
- On-shell matrix elements
- TMDs

HEF

- $k_t \sim P_t$
- Off-shell matrix elements
- uPDF

CGC

Small-x TMD $\leftrightarrow$ collinear limit of CGC

Elena Petreska. TMD gluon distributions at small x in the CGC theory. ***Int. J. Mod. Phys. E*, 27(05):1830003, 2018**

Fabio Dominguez, Cyrille Marquet, Bo-Wen Xiao, and Feng Yuan. Universality of Unintegrated Gluon Distributions at small x. ***Phys. Rev. D*, 83:105005, 2011**

TMD

- $Q_s \sim k_t \ll P_t$
- On-shell matrix elements
- TMDs

HEF

- $k_t \sim P_t$
- Off-shell matrix elements
- uPDF

CGC

Small-x HEF $\leftrightarrow$ dilute limit of CGC for forward jets

Elena Petreska. TMD gluon distributions at small x in the CGC theory. ***Int. J. Mod. Phys. E*, 27(05):1830003, 2018**

Fabio Dominguez, Cyrille Marquet, Bo-Wen Xiao, and Feng Yuan. Universality of Unintegrated Gluon Distributions at small x. ***Phys. Rev. D*, 83:105005, 2011**

Approach

Boost along x^- direction

- Lorentz boost parameter : γ_t

$\mathcal{F}_{+j}(x) = \mathcal{O}\left(\gamma_t\right)$	$\mathcal{F}_{ij}(x) \sim \mathcal{F}_{+-}(x) = \mathcal{O}\left(1\right)$	$\mathcal{F}_{-j}(x) = \mathcal{O}\left(\frac{1}{\gamma_t}\right)$

- Width of the target : $L^+ = \mathcal{O}\left(\frac{1}{\gamma_t}\right)$

Fermionic current

$$J^-(x) \sim \overline{\Psi}(x)\gamma^-\Psi(x) = \overline{\Psi^{(-)}}(x)\gamma^-\Psi^{(-)}(x) = \mathcal{O}\left(\gamma_t\right)$$

$$J^j(x) \sim \overline{\Psi}(x)\gamma^j\Psi(x) = \overline{\Psi^{(-)}}(x)\gamma^j\Psi^{(+)}(x) + \overline{\Psi^{(+)}}(x)\gamma^j\Psi^{(-)}(x) = \mathcal{O}\left(1\right)$$

$$J^+(x) \sim \overline{\Psi}(x)\gamma^+\Psi(x) = \overline{\Psi^{(+)}}(x)\gamma^-\Psi^{(+)}(x) = \mathcal{O}\left(\frac{1}{\gamma_t}\right)$$

Boost along x^- direction

- Lorentz boost parameter : γ_t

$\mathcal{A}^-(x) = \mathcal{O}(\gamma_t)$	$\mathcal{A}^j(x) = \mathcal{O}(1)$	$\mathcal{A}^+(x) = \mathcal{O}\left(\frac{1}{\gamma_t}\right)$

- Width of the target : $L^+ = \mathcal{O}\left(\frac{1}{\gamma_t}\right)$

Fermionic current

$$J^-(x) \sim \overline{\Psi}(x)\gamma^-\Psi(x) = \overline{\Psi^{(-)}}(x)\gamma^-\Psi^{(-)}(x) = \mathcal{O}(\gamma_t)$$

$$J^j(x) \sim \overline{\Psi}(x)\gamma^j\Psi(x) = \overline{\Psi^{(-)}}(x)\gamma^j\Psi^{(+)}(x) + \overline{\Psi^{(+)}}(x)\gamma^j\Psi^{(-)}(x) = \mathcal{O}(1)$$

$$J^+(x) \sim \overline{\Psi}(x)\gamma^+\Psi(x) = \overline{\Psi^{(+)}}(x)\gamma^-\Psi^{(+)}(x) = \mathcal{O}\left(\frac{1}{\gamma_t}\right)$$

Boost along x^- direction

- Lorentz boost parameter : γ_t

$\mathcal{A}^-(x) = \mathcal{O}(\gamma_t)$	$\mathcal{A}^j(x) = \mathcal{O}(1)$	$\mathcal{A}^+(x) = \mathcal{O}\left(\frac{1}{\gamma_t}\right)$

- Width of the target : $L^+ = \mathcal{O}\left(\frac{1}{\gamma_t}\right)$

Quark background field

$\Psi^{(-)}(x) = \mathcal{O}\left(\gamma_t^{\frac{1}{2}}\right)$	$\Psi^{(+)}(x) = \mathcal{O}\left(\gamma_t^{-\frac{1}{2}}\right)$

$$\Psi^{(-)}(x) = \frac{\gamma^+\gamma^-}{2}\Psi(x)$$

$$\Psi^{(+)}(x) = \frac{\gamma^-\gamma^+}{2}\Psi(x)$$

$$\mathcal{A}_a^\mu(x^-, x^+; \mathbf{x}) = \delta^{\mu -}\delta(x^+)\mathcal{A}_a^-(\mathbf{x})$$

Eikonal approximation relaxation

$$\mathcal{A}^{\mu}_{a}(x^-, x^+; \mathbf{x}) = \delta^{\mu -}\delta(x^+)\mathcal{A}^-_a(\mathbf{x})$$

Eikonal approximation relaxation

- Finite longitudinal width of the target L^+

$$\mathcal{A}_a^{\mu}(x^-, x^+; \mathbf{x}) = \delta^{\mu -}\delta(x^+)\mathcal{A}^-{}_a(\mathbf{x})$$

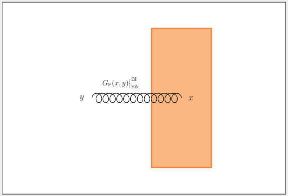
Eikonal approximation relaxation

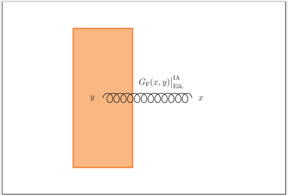
- Finite longitudinal width of the target L^+
- Interaction with transverse component of the background field

$$\mathcal{A}_a^{\mu}(x^-, x^+; \mathbf{x}) = \delta^{\mu -}\delta(x^+)\mathcal{A}_a^-(\mathbf{x})$$

Eikonal approximation relaxation

- Finite longitudinal width of the target L^+
- Interaction with transverse component of the background field
- Dynamics of the gluon background field

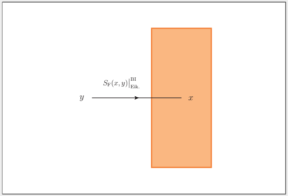


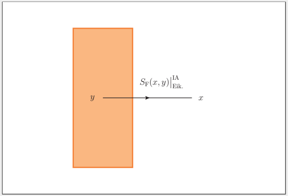


Gluon propagators

$$G_F^{\mu\nu}(x,y)\big|^{\text{BI}}_{\text{Eik.}ab} = \int \frac{d^3\underline{k}}{(2\pi)^3}\frac{\theta(k^+)}{2k^+}e^{iy\cdot\check{k}}e^{-ix\cdot\underline{k}}$$
$$\times\left[-g^\mu_j g^{j\nu} + g^\mu_j\eta^\nu\frac{k^j}{k^+} + i\left(\frac{\eta^\mu g^\nu_j}{k^+} - \eta^\mu\eta^\nu\frac{k^j}{(k^+)^2}\right)\left(\overrightarrow{\mathcal{D}}^A_{x^j} + ik^j\right)\right]\mathcal{U}_A(x^+,y^+,\mathbf{x})_{ab}$$

$$G_F^{\mu\nu}(x,y)\big|^{\text{IA}}_{\text{Eik.}ab} = \int \frac{d^3\underline{k}}{(2\pi)^3}\frac{\theta(k^+)}{2k^+}e^{-ix\cdot\check{k}}\mathcal{U}_A(x^+,y^+,\mathbf{y})_{ab}$$
$$\times\left[-g^\mu_j g^{j\nu} + \eta^\mu g^\nu_j\frac{k^j}{k^+} + i\left(\frac{g^\mu_j\eta^\nu}{k^+} + \eta^\mu\eta^\nu\frac{k^j}{(k^+)^2}\right)\left(\overleftarrow{\mathcal{D}}^A_{y^j} - ik^j\right)\right]e^{iy\cdot\underline{k}}$$

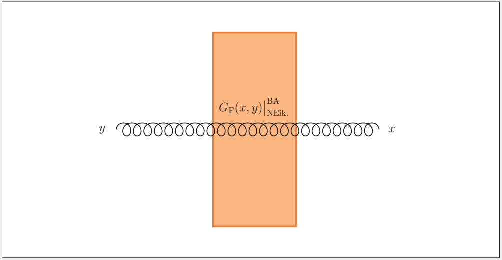




Quark propagators

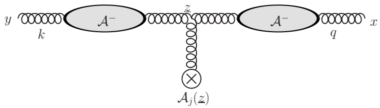
$$S_{\mathrm{F}}(x,y)\big|^{\mathrm{BI},q}_{\mathrm{Eik.}} = \int \frac{d^3\underline{k}}{(2\pi)^3}\frac{\theta(k^+)}{2k^+} e^{-ix\cdot\underline{k}} e^{iy\cdot\check{k}} \left[1 - i\frac{\gamma^+\gamma^j}{2k^+}\overrightarrow{\mathcal{D}}^{F}_{x^j}\right](\check{\not{k}} + m)\mathcal{U}_{\mathrm{F}}(x^+, y^+, \mathbf{x})$$

$$S_{\mathrm{F}}(x,y)\big|^{\mathrm{IA},q}_{\mathrm{Eik.}} = \int \frac{d^3\underline{k}}{(2\pi)^3}\frac{\theta(k^+)}{2k^+} \mathcal{U}_{\mathrm{F}}(x^+, y^+, \mathbf{y})(\check{\not{k}} + m)\left[1 - i\frac{\gamma^+\gamma^j}{2k^+}\overleftarrow{\mathcal{D}}^{F}_{y^j}\right] e^{-ix\cdot\check{k}} e^{iy\cdot\underline{k}}$$



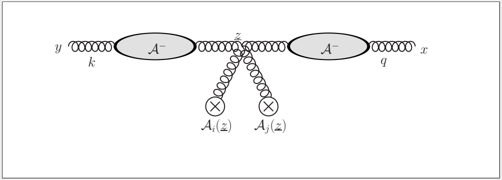
Gluon propagator

$$G_{\rm F}^{\mu\nu}(x,y)\Big|^{\rm NEik.} = G_{\rm F}^{\mu\nu}(x,y)\Big|^{\rm Eik.} + \delta G_{\rm F}^{\mu\nu}(x,y)\Big|^{\rm NEik.}_{\text{pure } \mathcal{A}^-} + \delta G_{\rm F}^{\mu\nu}(x,y)\Big|^{\rm NEik.}_{\text{single } \mathcal{A}_\perp} + \delta G_{\rm F}^{\mu\nu}(x,y)\Big|^{\rm NEik.}_{\text{double } \mathcal{A}_\perp,\text{loc.}} + \delta G_{\rm F}^{\mu\nu}(x,y)\Big|^{\rm NEik.}_{\text{double } \mathcal{A}_\perp,\text{non-loc.}}$$



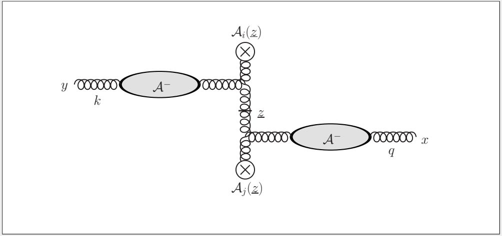
Gluon propagator

$$G_{\mathrm{F}}^{\mu\nu}(x,y)\Big|^{\mathrm{NEik.}} = G_{\mathrm{F}}^{\mu\nu}(x,y)\Big|^{\mathrm{Eik.}} + \delta G_{\mathrm{F}}^{\mu\nu}(x,y)\Big|^{\mathrm{NEik.}}_{\text{pure } \mathcal{A}^-} + \delta G_{\mathrm{F}}^{\mu\nu}(x,y)\Big|^{\mathrm{NEik.}}_{\text{single } \mathcal{A}_\perp}$$
$$+ \delta G_{\mathrm{F}}^{\mu\nu}(x,y)\Big|^{\mathrm{NEik.}}_{\text{double } \mathcal{A}_\perp,\text{loc.}} + \delta G_{\mathrm{F}}^{\mu\nu}(x,y)\Big|^{\mathrm{NEik.}}_{\text{double } \mathcal{A}_\perp,\text{non-loc.}}$$



Gluon propagator

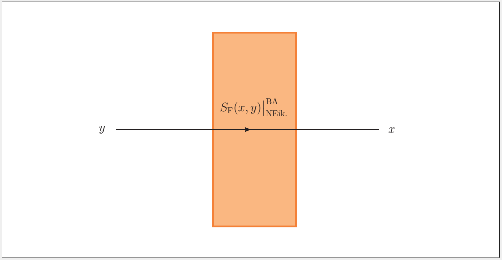
$$G_{\mathrm{F}}^{\mu\nu}(x,y)\Big|^{\mathrm{NEik.}} = G_{\mathrm{F}}^{\mu\nu}(x,y)\Big|^{\mathrm{Eik.}} + \delta G_{\mathrm{F}}^{\mu\nu}(x,y)\Big|^{\mathrm{NEik.}}_{\text{pure } \mathcal{A}^-} + \delta G_{\mathrm{F}}^{\mu\nu}(x,y)\Big|^{\mathrm{NEik.}}_{\text{single } \mathcal{A}_\perp}$$
$$+ \delta G_{\mathrm{F}}^{\mu\nu}(x,y)\Big|^{\mathrm{NEik.}}_{\text{double } \mathcal{A}_\perp,\text{loc.}} + \delta G_{\mathrm{F}}^{\mu\nu}(x,y)\Big|^{\mathrm{NEik.}}_{\text{double } \mathcal{A}_\perp,\text{non-loc.}}$$



Gluon propagator

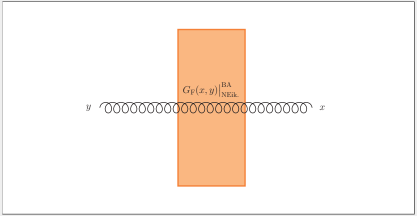
$$G_{\mathrm{F}}^{\mu\nu}(x,y)\Big|^{\mathrm{NEik.}} = G_{\mathrm{F}}^{\mu\nu}(x,y)\Big|^{\mathrm{Eik.}} + \delta G_{\mathrm{F}}^{\mu\nu}(x,y)\Big|^{\mathrm{NEik.}}_{\text{pure } \mathcal{A}^-} + \delta G_{\mathrm{F}}^{\mu\nu}(x,y)\Big|^{\mathrm{NEik.}}_{\text{single } \mathcal{A}_\perp}$$

$$+ \delta G_{\mathrm{F}}^{\mu\nu}(x,y)\Big|^{\mathrm{NEik.}}_{\text{double } \mathcal{A}_\perp,\text{loc.}} + \delta G_{\mathrm{F}}^{\mu\nu}(x,y)\Big|^{\mathrm{NEik.}}_{\text{double } \mathcal{A}_\perp,\text{non-loc.}}$$



Quark propagator

$$S_{\rm F}(x,y)\Big|^{\rm NEik.} = S_{\rm F}(x,y)\Big|^{\rm Eik.} + \delta S_{\rm F}^{\mu\nu}(x,y)\Big|^{\rm NEik.}_{\text{pure } \mathcal{A}^-} + \delta S_{\rm F}(x,y)\Big|^{\rm NEik.}_{\text{single } \mathcal{A}_\perp} + \delta S_{\rm F}(x,y)\Big|^{\rm NEik.}_{\text{double } \mathcal{A}_\perp,\text{non-loc.}}$$

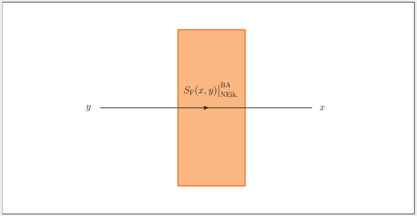


Gluon propagator

$$G_{\rm F}^{\mu\nu}(x,y)\Big|^{\rm BA}_{{\rm NEik.}ab} = \int \frac{d^3\underline{p}}{(2\pi)^3}\frac{\theta(p^+)}{2p^+}e^{-ix\cdot\check{p}} \int \frac{d^3\underline{k}}{(2\pi)^3}\frac{\theta(k^+)}{2k^+}e^{iy\cdot\check{k}} \int_{\mathbf{z}} e^{-i\mathbf{z}\cdot(\mathbf{p}-\mathbf{k})}$$

$$\times \int_{z^-} e^{iz^-(p^+-k^+)}\left[g^{\mu\rho}-\frac{\eta^\mu\check{p}^\rho}{p^+}\right]\left[g^{\rho'\nu}-\frac{\check{k}^{\rho'}\eta^\nu}{k^+}\right]$$

$$\times\left\{g^{\rho\rho'}\left[\mathcal{U}_{\rm A}(+\infty,-\infty;\mathbf{z},z^-)_{ab}-\frac{p^j+k^j}{2}\mathcal{U}^{(1)}_{{\rm A};j}(z)_{ab}-i\mathcal{U}^{(2)}_{\rm A}(z)_{ab}\right]-2g^{\rho i}g^{\rho' j}\mathcal{U}^{(3)}_{{\rm A};ij}(z)_{ab}\right\}$$



Quark propagator

$$S_{\rm F}(x,y)\big|^{{\rm BA},q}_{\rm NEik.} = \int \frac{d^3\underline{p}}{(2\pi)^3}\frac{\theta(p^+)}{2p^+}e^{-ix\cdot\check{p}} \int \frac{d^3\underline{k}}{(2\pi)^3}\frac{\theta(k^+)}{2k^+}e^{iy\cdot\check{k}} \int_{\mathbf{z}} e^{-i\mathbf{z}\cdot(\mathbf{p}-\mathbf{k}))} \int_{z^-} e^{iz^-(p^+-k^+)}$$

$$\times\, (\check{\not p}+m)\Big\{\mathcal{U}_{\rm F}(+\infty,-\infty,\mathbf{z},z^-) - \frac{p^j+k^j}{2(p^++k^+)}\mathcal{U}^{(1)}_{{\rm F};j}(z)$$

$$+\frac{i}{p^++k^+}\mathcal{U}^{(2)}_{\rm F}(z) - \frac{[\gamma^i,\gamma^j]}{4(p^++k^+)}\mathcal{U}^{(3)}_{{\rm F};ij}(z)\Big\}\gamma^+(\check{\not k}+m)$$

$$\mathcal{U}^{(1)}_{\mathrm{R};j}(z) = \int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dz^+ (2z^+) \mathcal{U}_{\mathrm{R}}(\frac{L^+}{2}, z^+; \mathbf{z}, z^-)\; ig\mathcal{F}_j{}^-(z)\, \mathcal{U}_{\mathrm{R}}(z^+, -\frac{L^+}{2}; \mathbf{z}, z^-)$$

$$\mathcal{U}^{(2)}_{\mathrm{R}}(z) = \int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dz^+ \int_{-\frac{L^+}{2}}^{z^+} dz'^+ (z^+ - z'^+) \mathcal{U}_{\mathrm{R}}(\frac{L^+}{2}, z^+; \mathbf{z}, z^-)\; ig\mathcal{F}_j{}^-(z)$$
$$\times\, \mathcal{U}_{\mathrm{F}}(z^+, z'^+; \mathbf{z}, z^-)\; ig\mathcal{F}_j{}^-(z'^+; \mathbf{z}, z^-)\, \mathcal{U}_{\mathrm{R}}(z'^+, -\frac{L^+}{2}; \mathbf{z}, z^-)$$

$$\mathcal{U}^{(3)}_{\mathrm{R};ij}(z) = \int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dz^+ \mathcal{U}_{\mathrm{R}}(\frac{L^+}{2}, z^+; \mathbf{z}, z^-)\; g\mathcal{F}_{ij}(z)\, \mathcal{U}_{\mathrm{R}}(z^+, -\frac{L^+}{2}; \mathbf{z}, z^-)$$

Calculation

Dijet production in pA collision induced by the quark background field of the target

- Processes with interaction with the quark background field

$\Rightarrow$ Hard processes of the form $p \to p_1 p_2$

- Strictly Neik. contributions
- Order $\mathcal{O}\left(\sqrt{\gamma_t}\right)$ at amplitude level
- Energy suppressed at cross-section level : $\mathcal{O}\left(\frac{1}{s}\right)$
- Only Eik. propagators needed

$$g \to gq$$

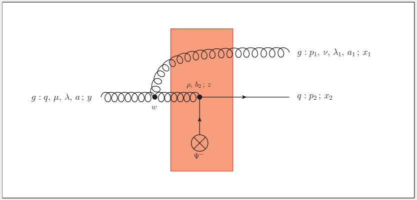


Diagram 1

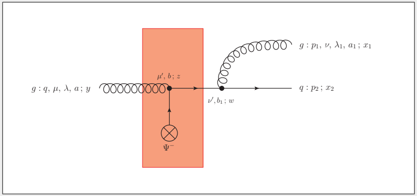


Diagram 2

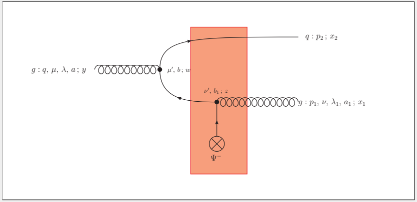


Diagram 3

Calculation \ Neik. contribution to dijet production

$$g \to gq$$

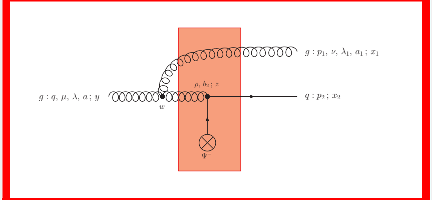


Diagram 1

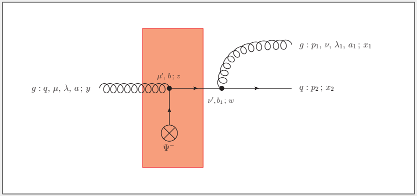


Diagram 2

$$\begin{aligned}\mathcal{S}_{g\to gq,\,1} =& \lim_{y^+\to-\infty}\lim_{x_1^+,x_2^+\to\infty}\int_{\mathbf{y},\mathbf{x_1},\mathbf{x_2}}\int_{y^-,x_1^-,x_2^-}\int_{\mathbf{w},\mathbf{z}}\int_{w^-,z^-}\int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dz^+\int_{-\infty}^{-\frac{L^+}{2}} dw^+\\ &\times e^{ix_1\cdot\check{p}_1}\, e^{ix_2\cdot\check{p}_2}\, e^{-iy\cdot\check{q}}(-2q^+)\,\epsilon_\lambda^\mu(q)\,(-2p_1^+)\,\epsilon_{\lambda_1}^{\nu}(p_1)\\ &\times\left[G_{0,\mathrm{F}}^{\mu'\mu}(w,y)\right]_{a'a}\left[G_{\mathrm{F}}^{\nu\nu'}(x_1,w)\Big|_{\mathrm{Eik.}}^{\mathrm{BA}}\right]_{a_1b_1}\left[G_{\mathrm{F}}^{\rho\rho'}(z,w)\Big|_{\mathrm{Eik.}}^{\mathrm{BI}}\right]_{b_2b} V_{\mu'\nu'\rho'}^{a'b_1b}\\ &\times\overline{u}(\check{p}_2,h)\gamma^+\left[S_{\mathrm{F}}(x_2,z)\Big|_{\mathrm{Eik.}}^{\mathrm{IA},q}(-ig)t^{b_2}\right]_{\alpha_2\beta}\gamma_\rho\Psi_\beta^-(z)\end{aligned}$$

$$g \to gq$$

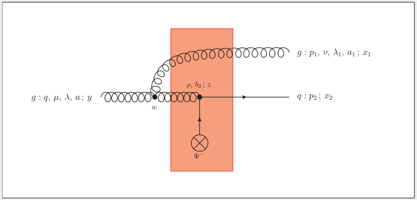


Diagram 1

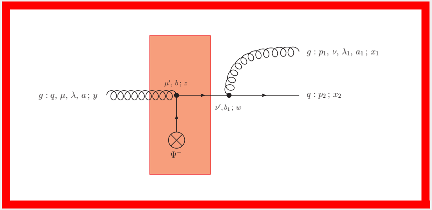


Diagram 2

$$\begin{aligned}
\mathcal{S}_{g\to gq,\,2} = & \lim_{y^+\to-\infty}\ \lim_{x_1^+,x_2^+\to\infty} \int_{\mathbf{y},\mathbf{x_1},\mathbf{x_2}} \int_{y^-,x_1^-,x_2^-} \int_{\mathbf{w},\mathbf{z}} \int_{w^-,z^-} \int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dz^+ \int_{\frac{L^+}{2}}^{\infty} dw^+ \\
& \times e^{ix_1\cdot\check{p}_1}\, e^{ix_2\cdot\check{p}_2}\, e^{-iy\cdot\check{q}}\, (-2q^+)\epsilon_\mu^\lambda(q)\, (-2p_1^+)\, \epsilon_\nu^{\lambda_1}(p_1)^* \\
& \times \Big[G_{\mathrm{F}}^{\mu'\mu}(z,y)\Big|_{\mathrm{Eik.}}^{\mathrm{BI}}\Big]_{ba} \Big[G_{0,\mathrm{F}}^{\nu\nu'}(x_1,w)\Big]_{a_1b_1} \\
& \times \overline{u}(\check{p}_2,h)\,\gamma^+ \Big[S_{0,\mathrm{F}}(x_2,w)(-ig)t^{b_1}\gamma_{\nu'} S_{\mathrm{F}}(w,z)\Big|_{\mathrm{Eik.}}^{\mathrm{IA},q}(-ig)t^b\Big]_{\alpha_2\beta} \gamma_{\mu'}\Psi_\beta^-(z)
\end{aligned}$$

$$g \to gq$$

$$\mathcal{S}_{g\to gq,\,3} = \lim_{y^+\to-\infty}\ \lim_{x_1^+,x_2^+\to\infty} \int_{\mathbf{y},\mathbf{x_1},\mathbf{x_2}} \int_{y^-,x_1^-,x_2^-} \int_{\mathbf{w},\mathbf{z}} \int_{w^-,z^-} \int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dz^+ \int_{-\infty}^{-\frac{L^+}{2}} dw^+$$
$$\times\, e^{ix_1\cdot\check{p}_1}\, e^{ix_2\cdot\check{p}_2}\, e^{-iy\cdot\check{q}}\, (-2q^+)\, \epsilon_\mu^\lambda(q)^*\, (-2p_1^+)\, \epsilon_\nu^{\lambda_1}(p_1)^*$$
$$\times \left[G_{0,\mathrm{F}}^{\mu'\mu}(w,y)\right]_{ba} \left[G_{\mathrm{F}}^{\nu\nu'}(x_1,z)\Big|_{\mathrm{Eik.}}^{\mathrm{IA}}\right]_{a_1b_1}$$
$$\times\, \overline{u}(\check{p}_2,h)\,\gamma^+ \left[S_{\mathrm{F}}(x_2,w)\Big|_{\mathrm{Eik.}}^{\mathrm{BA},q}(-ig)\,t^b\,\gamma_{\mu'}\,S_{\mathrm{F}}(w,z)\Big|_{\mathrm{Eik.}}^{\mathrm{BI},\bar{q}}(-ig)\,t^{b_1}\right]_{\alpha_2\beta}\gamma_{\nu'}\,\Psi_\beta^-(z)$$

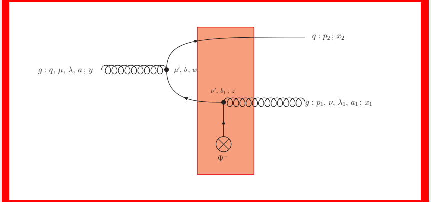


Diagram 3

$$q \to gg$$

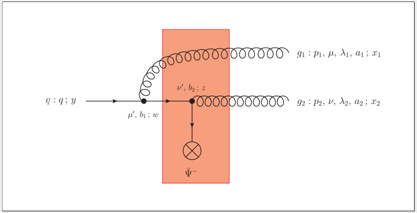


Diagram 1

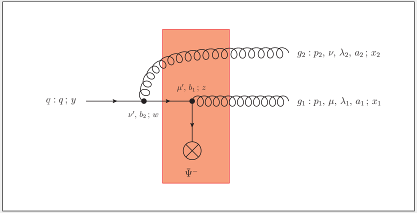


Diagram 2

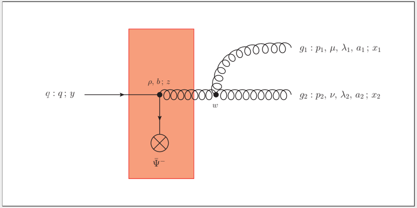


Diagram 3

Calculation \ Neik. contribution to dijet production

$$q_f \to q_{f_1} q_{f_2}$$

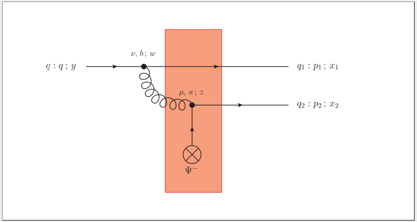


Diagram 1

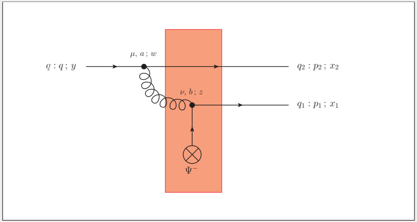


Diagram 2

$$q_f \to q_{f_1} \bar{q}_{f_2}$$

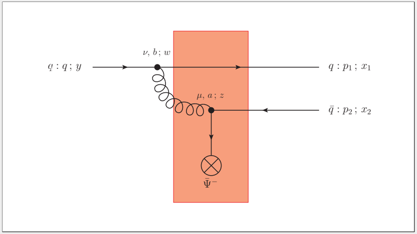


Diagram 1

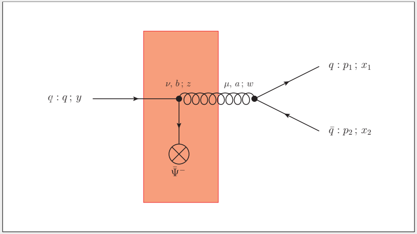


Diagram 2

Dijet variables

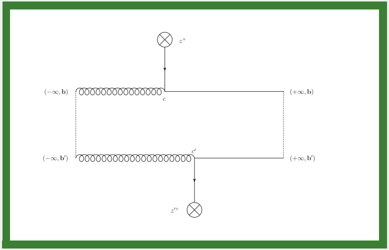
- Relative dijet momentum : $\mathbf{P} \equiv (1-z)\mathbf{p_1} - z\mathbf{p_2}$
- Dijet momentum imbalance : $\mathbf{k} = \mathbf{p_1} + \mathbf{p_2}$
- with $z \equiv \frac{p_1^+}{p_1^+ + p_2^+}$, $(1-z) \equiv \frac{p_2^+}{p_1^+ + p_2^+}$
- Conjugate variables :

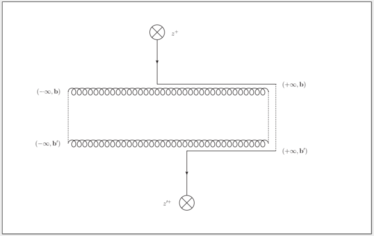
$$\mathbf{r} \equiv \mathbf{x}_1 - \mathbf{x}_2 \,, \qquad \mathbf{b} \equiv z\mathbf{x}_1 + (1-z)\mathbf{x}_2$$

Back-to-back limit

Corresponds to $|\mathbf{P}| \gg |\mathbf{k}|$, or equivalently $|\mathbf{r}| \ll |\mathbf{b}|$

$$g \to gq$$

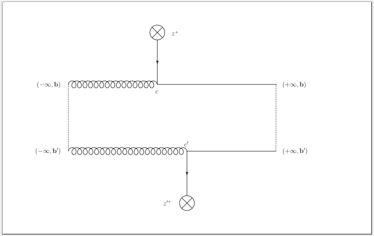


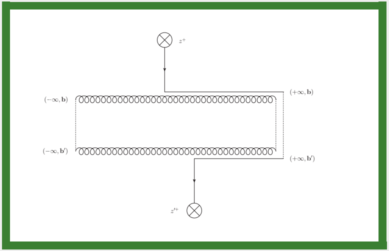


$\mathcal{C}^{+g}$

$$\mathcal{C}^{+g} \equiv \Big\langle \overline{\Psi}(z'^+; \mathbf{b}')\, \gamma^-\, t^{c'}\, \mathcal{U}_{\mathrm{F}}^{\dagger}(\infty, z'^+; \mathbf{b}')\, \mathcal{U}_{\mathrm{F}}(\infty, z^+; \mathbf{b})\, t^c\, \Psi(z^+; \mathbf{b}) \\ \times \mathcal{U}_{\mathrm{A}}(z'^+, -\infty; \mathbf{b}')_{c'a}\, \mathcal{U}_{\mathrm{A}}(z^+, -\infty; \mathbf{b})_{ca} \Big\rangle$$

$$g \to gq$$

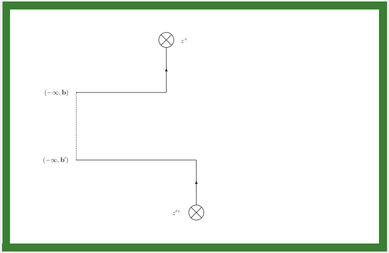


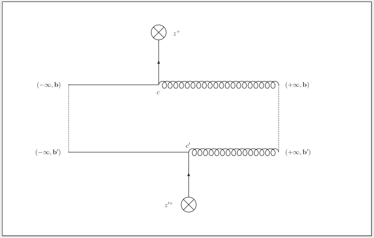


$\mathcal{C}^{+\square_g}$

$$\mathcal{C}^{+\square_g} \equiv \Big\langle \overline{\Psi}(z'^+;\mathbf{b}')\, \gamma^- \,\mathcal{U}_{\mathrm{F}}^\dagger(\infty, z'^+;\mathbf{b}')\,\mathcal{U}_{\mathrm{F}}(\infty, z^+;\mathbf{b})\; \Psi(z^+;\mathbf{b}) \times \mathcal{U}_{\mathrm{A}}(\infty,-\infty;\mathbf{b}')_{ba}\,\mathcal{U}_{\mathrm{A}}(\infty,-\infty;\mathbf{b})_{ba} \Big\rangle$$

$$q \to gg$$

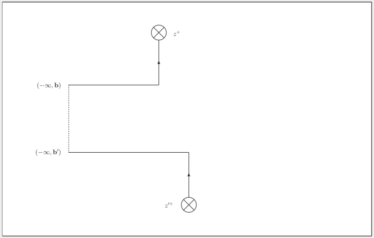


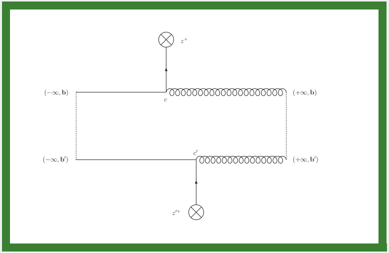


$\overline{\mathcal{C}}^-$

$$\overline{\mathcal{C}}^- \equiv \left\langle \mathrm{Tr} \left\{ \mathcal{U}_{\mathrm{F}}^\dagger(z'^+, -\infty; \mathbf{b}')\Psi(z'^+; \mathbf{b}')\overline{\Psi}(z^+; \mathbf{b})\gamma^- \mathcal{U}_{\mathrm{F}}(z^+, -\infty; \mathbf{b}) \right\} \right\rangle$$

$q \to gg$



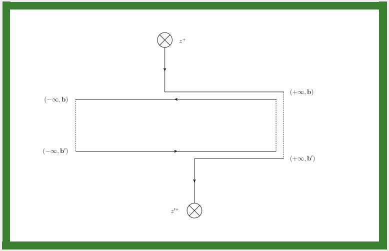


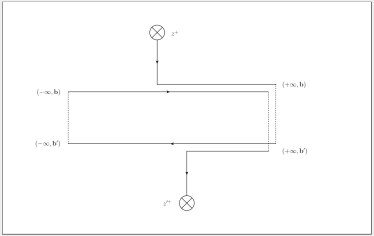
$\overline{\mathcal{C}}^{-g}$

$$\overline{\mathcal{C}}^{-g} \equiv \Big\langle \mathcal{U}_{\mathrm{A}}(\infty, z'^+; \mathbf{b}')_{dc'}\, \mathcal{U}_{\mathrm{A}}(\infty, z^+; \mathbf{b})_{dc}$$

$$\times \operatorname{Tr}\Big\{ \mathcal{U}_{\mathrm{F}}^\dagger(z'^+, -\infty; \mathbf{b}')\, t^{c'}\ \Psi(z'^+; \mathbf{b}')\overline{\Psi}(z^+; \mathbf{b})\, \gamma^-\, t^c\, \mathcal{U}_{\mathrm{F}}(z^+, -\infty; \mathbf{b}) \Big\} \Big\rangle$$

$$q \to qq$$

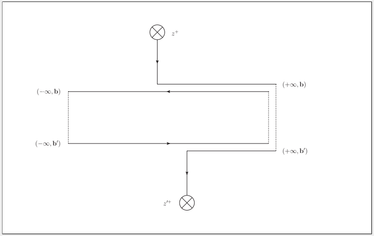


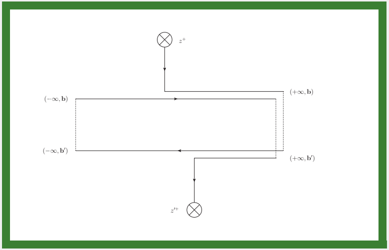


$\mathcal{C}^{+\Box}$

$$\mathcal{C}^{+\Box} \equiv \Big\langle \operatorname{Tr}\Big[\mathcal{U}_{\mathrm{F}}^{\dagger}(\infty,-\infty;\mathbf{b}')\mathcal{U}_{\mathrm{F}}(\infty,-\infty;\mathbf{b})\Big] \\ \times \overline{\Psi}(z'^{+};\mathbf{b}')\gamma^{-}\mathcal{U}_{\mathrm{F}}^{\dagger}(\infty,z'^{+};\mathbf{b}')\mathcal{U}_{\mathrm{F}}(\infty,z^{+};\mathbf{b})\Psi(z^{+};\mathbf{b})\Big\rangle$$

$$q \to qq$$

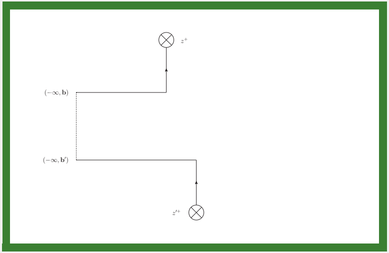


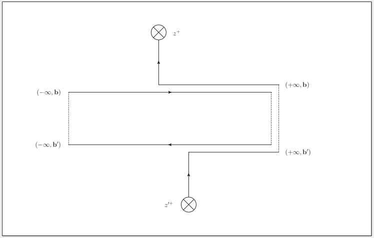


$\mathcal{C}^{+-+}$

$$\mathcal{C}^{+-+} \equiv \Big\langle \overline{\Psi}(z'^+; \mathbf{b})\, \gamma^-\, \mathcal{U}_{\mathrm{F}}^\dagger(\infty, z'^+; \mathbf{b}')\, \mathcal{U}_{\mathrm{F}}(\infty, -\infty; \mathbf{b}) \\ \times\ \mathcal{U}_{\mathrm{F}}^\dagger(\infty, -\infty; \mathbf{b}')\, \mathcal{U}_{\mathrm{F}}(\infty, z^+; \mathbf{b})\ \Psi(z^+; \mathbf{b}) \Big\rangle$$

$$q \to q\bar{q}$$

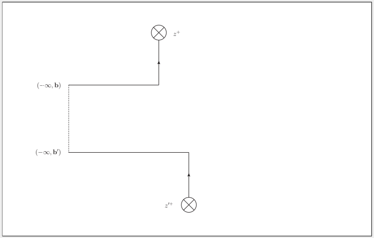


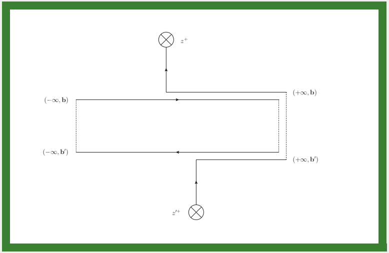


$\overline{\mathcal{C}}^-$

$$\overline{\mathcal{C}}^- \equiv \left\langle \operatorname{Tr}\left\{\mathcal{U}_{\mathrm{F}}^\dagger(z'^+, -\infty; \mathbf{b}')\Psi(z'^+; \mathbf{b}')\overline{\Psi}(z^+; \mathbf{b})\gamma^-\mathcal{U}_{\mathrm{F}}(z^+, -\infty; \mathbf{b})\right\}\right\rangle$$

$$q \to q\bar{q}$$





$\overline{\mathcal{C}}^{+\square}$

$$\overline{\mathcal{C}}^{+\square} \equiv \Big\langle \mathrm{Tr}\left\{\mathcal{U}_{\mathrm{F}}(\infty, z'^{+}; \mathbf{b}')\Psi(z'^{+}; \mathbf{b}')\overline{\Psi}(z^{+}; \mathbf{b})\gamma^{-}\mathcal{U}_{\mathrm{F}}^{\dagger}(\infty, z^{+}; \mathbf{b})\right\}$$
$$\times\, \mathrm{Tr}\left[\mathcal{U}_{\mathrm{F}}(\infty, -\infty; \mathbf{b}')\mathcal{U}_{\mathrm{F}}^{\dagger}(\infty, -\infty; \mathbf{b})\right] \Big\rangle$$

Color Structure

$$\int_{\mathbf{b},\mathbf{b}'} e^{-i\mathbf{k}\cdot(\mathbf{b}-\mathbf{b}')} \int_{z^+,z'^+} \left\langle \mathcal{C}^{(\cdots)} \right\rangle$$

Target Average

$$\langle \mathcal{O} \rangle = \lim_{P'_{tar}\to P_{tar}} \frac{\langle P'_{tar}|\hat{\mathcal{O}}|P_{tar}\rangle}{\langle P'_{tar}|P_{tar}\rangle}$$

Unpolarized quark TMD

$$f_q^+(\mathsf{x},\mathbf{k}) = \frac{1}{(2\pi)^3}\int_{\mathbf{b}} e^{i\mathbf{k}\cdot\mathbf{b}} \int_{z^+} e^{-i\mathsf{x}P^-_{tar}z^+} \left\langle P_{tar} \middle| \overline{\Psi}(z^+;\mathbf{b})\frac{\gamma^-}{2}\mathcal{U}^\dagger_{\mathrm{F}}(\infty,z^+;\mathbf{b})\mathcal{U}_{\mathrm{F}}(\infty,0;\mathbf{0})\Psi(0;\mathbf{0}) \middle| P_{tar} \right\rangle$$

Relation with quark TMDs

$$\int_{\mathbf{b},\mathbf{b}'} e^{-i\mathbf{k}\cdot(\mathbf{b}-\mathbf{b}')} \int_{z^+,z'^+} \frac{\mathcal{C}^{(\cdots)}}{\mathcal{N}^{(\cdots)}} = \frac{(2\pi)^3}{P^-_{tar}} f_q^{(\cdots)}(\mathsf{x}=0,\mathbf{k})$$

$g \to gq$

$$\frac{d\sigma^{\mathrm{b2b},\, m=0}_{g\to gq}}{d\phi} = \frac{q^+\delta(k^+-q^+)\; 2\alpha_s^2}{z(1-z)(2q^+P^-_{tar})}\left[C_F\,\mathcal{H}^{+g}_{g\to gq}\; f^{+g}_{\bar{q}}(\mathsf{x}=0,\mathbf{k}-\mathbf{q}) + (N_c^2-1)\,\mathcal{H}^{+\square_g}_{g\to gq}\; f^{+\square_g}_{\bar{q}}(\mathsf{x}=0,\mathbf{k}-\mathbf{q})\right]$$

$q \to gg$

$$\frac{d\sigma^{\mathrm{b2b},\, m=0}_{q\to gg}}{d\phi} = \frac{q^+\delta(k^+-q^+)\; 2\alpha_s^2}{z(1-z)(2q^+P^-_{tar})}\left[\mathcal{H}^{-}_{q\to gg}\; f^{-}_{q}(\mathsf{x}=0,\mathbf{k}-\mathbf{q}) + C_F\,\mathcal{H}^{-g}_{q\to gg}\; f^{-g}_{q}(\mathsf{x}=0,\mathbf{k}-\mathbf{q})\right]$$

$q \to qq$

$$\frac{d\sigma^{\mathrm{b2b},\,m=0}_{\bar{q}_f\to\bar{q}_{f_1}q_{f_2}}}{d\phi} = \frac{q^+\delta(k^+-q^+)\;2\alpha_s^2}{z(1-z)(2q^+P^-_{tar})}\left[\mathcal{H}^{-}_{q_f\to q_{f_1}\bar{q}_{f_2}}\;f^{-}_{q}(\mathsf{x}=0,\mathbf{k}-\mathbf{q}) + N_c\,\mathcal{H}^{+\Box}_{q_f\to q_{f_1}\bar{q}_{f_2}}\;f^{+\Box}_{q}(\mathsf{x}=0,\mathbf{k}-\mathbf{q})\right]$$

$q \to q\bar{q}$

$$\frac{d\sigma^{\mathrm{b2b},\,m=0}_{\bar{q}_f\to\bar{q}_{f_1}\bar{q}_{f_2}}}{d\phi} = \frac{q^+\delta(k^+-q^+)\;2\alpha_s^2}{z(1-z)(2q^+P^-_{tar})}\left[N_c\,\mathcal{H}^{+\Box}_{q_f\to q_{f_1}q_{f_2}}\;f^{+\Box}_{\bar{q}}(\mathsf{x}=0,\mathbf{k}-\mathbf{q}) + \mathcal{H}^{+-+}_{q_f\to q_{f_1}q_{f_2}}\;f^{+-+}_{\bar{q}}(\mathsf{x}=0,\mathbf{k}-\mathbf{q})\right]$$

$q \to qq$

$\frac{d\sigma^{b2}_{\bar{q}_f}}{}$ ... $q)\Big]$

q —

$\frac{d\sigma^{b2}_{\bar{q}_f}}{}$ ... $q)\Big]$

Results in agreement with the literature

√ Hard factors

Jian-Wei Qiu, Werner Vogelsang, and Feng Yuan. Single Transverse-Spin Asymmetry in Hadronic Dijet Production. *Phys. Rev. D*, 76:074029, 2007

√ Gauge link

C. J. Bomhof, P. J. Mulders, and F. Pijlman. The construction of gauge-links in arbitrary hard processes. *Eur. Phys. J. C*, 47:147–162, 2006

x dependence

NLP order calculation needed ($\mathcal{O}\left(\frac{\mathbf{k}}{\mathbf{P}}\right)$)

Higher twist TMDs

- NEik. propagator needed
- $\mathcal{U}^{(1)}_{\mathrm{R};j}, \mathcal{U}^{(2)}_{\mathrm{R}}, \mathcal{U}^{(3)}_{\mathrm{R};ij}$ needed

$\Rightarrow$ NNEik. calculation

- Finite longitudinal width effects

Tolga Altinoluk, Néstor Armesto, Guillaume Beuf, Mauricio Martinez, and Carlos A. Salgado. Next-to-eikonal corrections in the CGC: gluon production and spin asymmetries in pA collisions. ***JHEP*, 07:068, 2014**

Tolga Altinoluk, Néstor Armesto, Guillaume Beuf, and Alexis Moscoso. Next-to-next-to-eikonal corrections in the CGC. ***JHEP*, 01:114, 2016**

- Finite longitudinal width effects
- Quark and scalar propagators

Tolga Altinoluk, Guillaume Beuf, Alina Czajka, and Arantxa Tymowska. QUARKS AT NEXT-TO-EIKONAL ACCURACY IN THE CGC: FORWARD QUARK-NUCLEUS SCATTERING. ***Phys. Rev. D*, 104(1):014019, 2021**

Tolga Altinoluk and Guillaume Beuf. QUARK AND SCALAR PROPAGATORS AT NEXT-TO-EIKONAL ACCURACY IN THE CGC THROUGH A DYNAMICAL BACKGROUND GLUON FIELD. ***Phys. Rev. D*, 105(7):074026, 2022**

Pedro Agostini. SCALAR PROPAGATOR IN A BACKGROUND GLUON FIELD BEYOND THE EIKONAL APPROXIMATION. ***JHEP*, 11:099, 2023**

- Finite longitudinal width effects
- Quark and scalar propagators
- Gluon propagator with all Neik. corrections

Tolga Altinoluk, Guillaume Beuf, and Swaleha Mulani. Forward parton-nucleus scattering at next-to-eikonal accuracy in the CGC. **11 2024**

- Finite longitudinal width effects
- Quark and scalar propagators
- Gluon propagator with all Neik. corrections
- Application to DIS dijet production

Tolga Altinoluk, Guillaume Beuf, Alina Czajka, and Arantxa Tymowska. DIS dijet production at next-to-eikonal accuracy in the CGC. ***Phys. Rev. D***, **107(7):074016, 2023**

Pedro Agostini, Tolga Altinoluk, and Néstor Armesto. Next-to-eikonal corrections to dijet production in Deep Inelastic Scattering in the dilute limit of the Color Glass Condensate. ***JHEP***, **07:137, 2024**

- Finite longitudinal width effects
- Quark and scalar propagators
- Gluon propagator with all Neik. corrections
- Application to DIS dijet production
- Quark TMDs from DIS dijet production

Tolga Altinoluk, Nestor Armesto, and Guillaume Beuf. PROBING QUARK TRANSVERSE MOMENTUM DISTRIBUTIONS IN THE COLOR GLASS CONDENSATE: QUARK-GLUON DIJETS IN DEEP INELASTIC SCATTERING AT NEXT-TO-EIKONAL ACCURACY. ***PHYS. REV. D***, **108(7):074023, 2023**

- Finite longitudinal width effects
- Quark and scalar propagators
- Gluon propagator with all Neik. corrections
- Application to DIS dijet production
- Quark TMDs from DIS dijet production
- Gluon TMDs from DIS dijet production

Tolga Altinoluk, Guillaume Beuf, Alina Czajka, and Cyrille Marquet. Back-to-back dijet production in DIS at next-to-eikonal accuracy and twist-3 gluon TMDs. **10 2024**

Thanks for your attention!

Find more at arXiv:2412.08485

Backup \ Hard Factors for $g \to gq$

$$k^+ \frac{d\sigma_{g\to gq}^{\mathrm{b2b},\, m=0}}{dk^+\, d^2\mathbf{k}\, d^2\mathbf{P}\, dz} = q^+\delta(k^+ - q^+)\, \frac{2\alpha_s^2}{\mathbf{P}^2} \Big[C_F\, \mathcal{H}_{g\to gq}^{+g}\, \mathsf{x} f_q^{+g}(\mathsf{x}, \mathbf{k}-\mathbf{q}) + (N_c^2-1)\, \mathcal{H}_{g\to gq}^{+\Box_g}\, \mathsf{x} f_q^{+\Box_g}(\mathsf{x}, \mathbf{k}-\mathbf{q}) \Big]$$

Hard Factors

$$\mathcal{H}_{g\to gq}^{+\Box_g} = \frac{1}{4\mathbf{P}^2} \frac{1}{(N_c^2-1)} \frac{(1+z^2)}{(1-z)}$$

$$\mathcal{H}_{g\to gq}^{+g} = \frac{1}{\mathbf{P}^2} \frac{\left[N_c^2 z^2 - (1-z)^2\right]}{2N_c(N_c^2-1)} \frac{(1+z^2)}{(1-z)}$$

$$k^+ \frac{d\sigma_{q\to gg}^{\mathrm{b2b},\,m=0}}{dk^+\, d^2\mathbf{k}\, d^2\mathbf{P}\, dz} = q^+ \delta(k^+ - q^+)\, \frac{2\alpha_s^2}{\mathbf{P}^2} \Bigg[\mathcal{H}_{q\to gg}^{-}\, \mathsf{x} f_{\bar{q}}^{-}(\mathsf{x}, \mathbf{k}-\mathbf{q}) + C_F\, \mathcal{H}_{q\to gg}^{-g}\, \mathsf{x} f_{\bar{q}}^{-g}(\mathsf{x}, \mathbf{k}-\mathbf{q}) \Bigg]$$

Hard Factors

$$\mathcal{H}_{q\to gg}^{-} = \frac{(N_c^2-1)}{4\, N_c^3 \mathbf{P}^2} \left[z^2 + (1-z)^2 \right]$$

$$\mathcal{H}_{q\to gg}^{-g} = \frac{\left[z^2 + (1-z)^2 \right]}{2\mathbf{P}^2} \left[z^2 + (1-z)^2 - \frac{2}{N_c^2} \right]$$

$$k^+ \frac{d\sigma^{\text{b2b},\, m=0}_{q_f \to q_{f_1} q_{f_2}}}{dk^+\, d^2\mathbf{k}\, d^2\mathbf{P}\, dz} = q^+ \delta(k^+ - q^+)\, \frac{2\alpha_s^2}{\mathbf{P}^2} \Big[N_c\, \mathcal{H}^{+\square}_{q_f \to q_{f_1} q_{f_2}}\ \mathsf{x} f_q^{+\square}(\mathsf{x}, \mathbf{k} - \mathbf{q}) + \mathcal{H}^{+-+}_{q_f \to q_{f_1} q_{f_2}}\ \mathsf{x} f_q^{+-+}(\mathsf{x}, \mathbf{k} - \mathbf{q}) \Big]$$

Hard Factors

$$\mathcal{H}^{+\square}_{q \to qq} = \frac{1}{4N_c\, \mathbf{P}^2} \left[2 \left(1 + \frac{1}{N_c^2} \right) \left(z(1-z) - 3 + \frac{1}{z(1-z)} \right) - \frac{4}{N_c} \right]$$

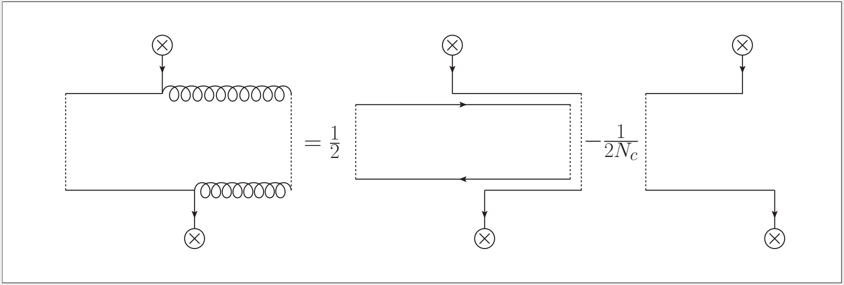
$$\mathcal{H}^{+-+}_{q \to qq} = \frac{1}{4N_c\, \mathbf{P}^2} \left[-\frac{4}{N_c} \left(z(1-z) - 3 + \frac{1}{z(1-z)} \right) + 2 \left(1 + \frac{1}{N_c^2} \right) \right]$$

$$k^+ \frac{d\sigma^{\mathrm{b2b},\, m=0}_{q_f \to q_{f_1} \bar{q}_f \mathsf{x}_2}}{dk^+\, d^2\mathbf{k}\, d^2\mathbf{P}\, dz} = q^+ \delta(k^+ - q^+)\, \frac{2\alpha_s^2}{\mathbf{P}^2} \Bigg[\mathcal{H}^{-}_{q_f \to q_{f_1} \bar{q}_{f_2}}\, \mathsf{x} f^{-}_{\bar{q}}(\mathsf{x}, \mathbf{k} - \mathbf{q}) + N_c\, \mathcal{H}^{+\square}_{q_f \to q_{f_1} \bar{q}_{f_2}}\, \mathsf{x} f^{+\square}_{\bar{q}}(\mathsf{x}, \mathbf{k} - \mathbf{q}) \Bigg]$$

Hard Factors

$$\mathcal{H}^{-}_{q \to q\bar{q}} = \frac{z(1-z)}{4N_c\, \mathbf{P}^2} \left[\frac{1+z^2}{(1-z)^2} \left(N_c - \frac{2}{N_c} \right) - \frac{1}{N_c} \left(z^2 + (1-z)^2 \right) - \frac{2}{N_c^2} \frac{z^2}{1-z} \right]$$

$$\mathcal{H}^{+\square}_{q \to q\bar{q}} = \frac{z(1-z)}{4N_c\, \mathbf{P}^2} \left[z^2 + (1-z)^2 + \frac{2}{N_c} \frac{z^2}{1-z} + \frac{1}{N_c^2} \frac{1+z^2}{(1-z)^2} \right]$$

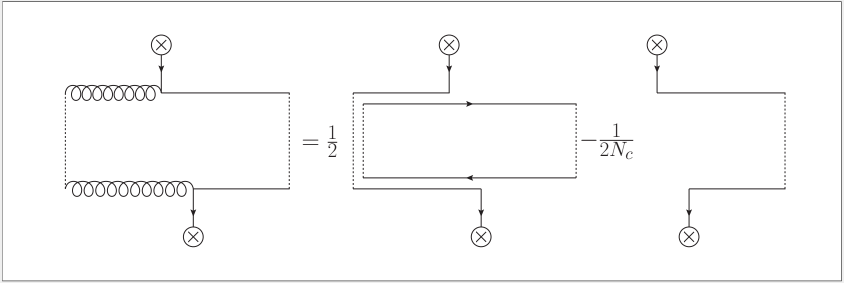


$\mathcal{C}^{-g}$

$$\mathcal{C}^{-g} = \frac{1}{2}\mathcal{C}^{+\Box} - \frac{1}{2N_c}\mathcal{C}^{-}$$

$$\Longrightarrow (N_c^2 - 1)\, f_q^{-g}(\mathsf{x}, \mathbf{k}) = N_c^2\, f_q^{+\Box}(\mathsf{x}, \mathbf{k}) - f_q^{-}(\mathsf{x}, \mathbf{k})$$

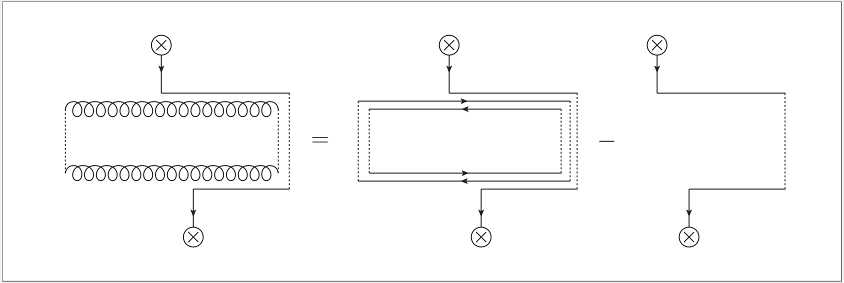
Backup \ Fundamental Gauge links



$\mathcal{C}^{+g}$

$$\mathcal{C}^{+g} = \frac{1}{2}\mathcal{C}^{-\Box} - \frac{1}{2N_c}\mathcal{C}^{+}$$

$$\Longrightarrow (N_c^2 - 1)\, f_q^{+g}(\mathsf{x}, \mathbf{k}) = N_c^2\, f_q^{-\Box}(\mathsf{x}, \mathbf{k}) - f_q^{+}(\mathsf{x}, \mathbf{k})$$



$\mathcal{C}^{+\Box_g}$

$$\mathcal{C}^{+\Box_g} = \mathcal{C}^{+\Box^2} - \mathcal{C}^{+}$$

$$\Longrightarrow (N_c^2 - 1)\, f_q^{+\Box_g}(\mathrm{x}, \mathbf{k}) = N_c^2\, f_q^{+\Box^2}(\mathrm{x}, \mathbf{k}) - f_q^{+}(\mathrm{x}, \mathbf{k})$$

References I

Pedro Agostini. **Scalar propagator in a background gluon field beyond the eikonal approximation.** *JHEP*, 11:099, 2023.

Pedro Agostini, Tolga Altinoluk, and Néstor Armesto. **Next-to-eikonal corrections to dijet production in Deep Inelastic Scattering in the dilute limit of the Color Glass Condensate.** *JHEP*, 07:137, 2024.

Tolga Altinoluk, Nestor Armesto, and Guillaume Beuf. **Probing quark transverse momentum distributions in the color glass condensate: Quark-gluon dijets in deep inelastic scattering at next-to-eikonal accuracy.** *Phys. Rev. D*, 108(7):074023, 2023.

Tolga Altinoluk, Néstor Armesto, Guillaume Beuf, Mauricio Martinez, and Carlos A. Salgado. **Next-to-eikonal corrections in the CGC: gluon production and spin asymmetries in pA collisions.** *JHEP*, 07:068, 2014.

Tolga Altinoluk, Néstor Armesto, Guillaume Beuf, and Alexis Moscoso. **Next-to-next-to-eikonal corrections in the CGC.** *JHEP*, 01:114, 2016.

Tolga Altinoluk and Guillaume Beuf. **Quark and scalar propagators at next-to-eikonal accuracy in the CGC through a dynamical background gluon field.** *Phys. Rev. D*, 105(7):074026, 2022.

- Tolga Altinoluk, Guillaume Beuf, Alina Czajka, and Cyrille Marquet. **Back-to-back dijet production in DIS at next-to-eikonal accuracy and twist-3 gluon TMDs.** 10 2024.
- Tolga Altinoluk, Guillaume Beuf, Alina Czajka, and Arantxa Tymowska. **Quarks at next-to-eikonal accuracy in the CGC: Forward quark-nucleus scattering.** *Phys. Rev. D*, 104(1):014019, 2021.
- Tolga Altinoluk, Guillaume Beuf, Alina Czajka, and Arantxa Tymowska. **DIS dijet production at next-to-eikonal accuracy in the CGC.** *Phys. Rev. D*, 107(7):074016, 2023.
- Tolga Altinoluk, Guillaume Beuf, and Swaleha Mulani. **Forward parton-nucleus scattering at next-to-eikonal accuracy in the CGC.** 11 2024.
- C. J. Bomhof, P. J. Mulders, and F. Pijlman. **The Construction of gauge-links in arbitrary hard processes.** *Eur. Phys. J. C*, 47:147–162, 2006.
- S. Catani, M. Ciafaloni, and F. Hautmann. **High-energy factorization and small x heavy flavor production.** *Nucl. Phys. B*, 366:135–188, 1991.
- John Collins. ***Foundations of Perturbative QCD*, volume 32 of *Cambridge Monographs on Particle Physics, Nuclear Physics and Cosmology*.** Cambridge University Press, 7 2023.
- M. Deak, F. Hautmann, H. Jung, and K. Kutak. **Forward Jet Production at the Large Hadron Collider.** *JHEP*, 09:121, 2009.

References III

Fabio Dominguez, Cyrille Marquet, Bo-Wen Xiao, and Feng Yuan. **Universality of Unintegrated Gluon Distributions at small x.** *Phys. Rev. D*, 83:105005, 2011.

Elena Ferreiro, Edmond Iancu, Andrei Leonidov, and Larry McLerran. **Nonlinear gluon evolution in the color glass condensate. 2.** *Nucl. Phys. A*, 703:489–538, 2002.

Francois Gelis, Edmond Iancu, Jamal Jalilian-Marian, and Raju Venugopalan. **The Color Glass Condensate.** *Ann. Rev. Nucl. Part. Sci.*, 60:463–489, 2010.

Edmond Iancu, Andrei Leonidov, and Larry McLerran. **The Color glass condensate: An Introduction.** In *Cargese Summer School on QCD Perspectives on Hot and Dense Matter*, pages 73–145, 2 2002.

Edmond Iancu, Andrei Leonidov, and Larry D. McLerran. **Nonlinear gluon evolution in the color glass condensate. 1.** *Nucl. Phys. A*, 692:583–645, 2001.

P. Kotko, K. Kutak, C. Marquet, E. Petreska, S. Sapeta, and A. van Hameren. **Improved TMD factorization for forward dijet production in dilute-dense hadronic collisions.** *JHEP*, 09:106, 2015.

Elena Petreska. **TMD gluon distributions at small x in the CGC theory.** *Int. J. Mod. Phys. E*, 27(05):1830003, 2018.

Jian-Wei Qiu, Werner Vogelsang, and Feng Yuan. **Single Transverse-Spin Asymmetry in Hadronic Dijet Production.** *Phys. Rev. D*, 76:074029, 2007.