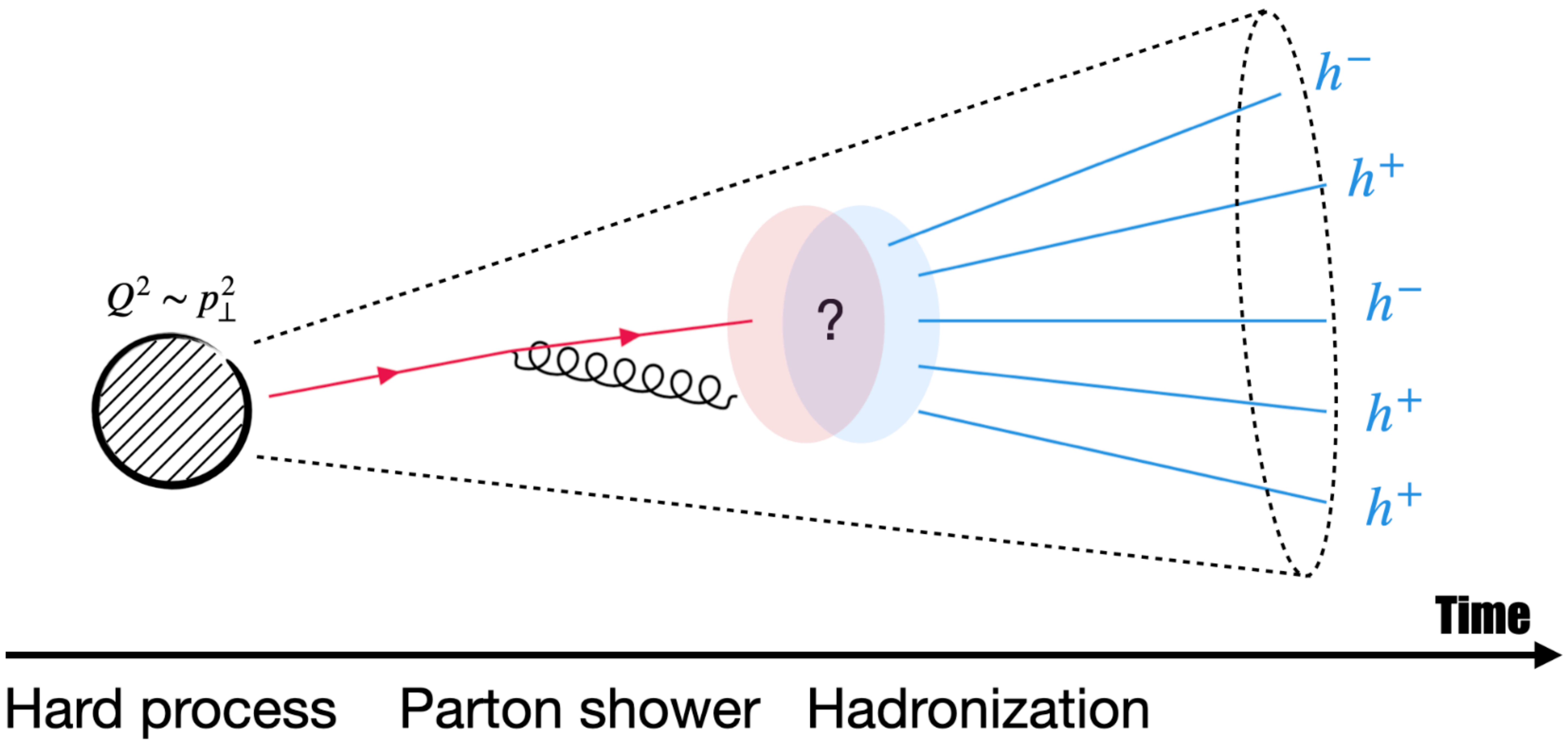
Entanglement as a Probe of Hadronization

Datta, Deshpande, Kharzeev, Naim, Tu PRL.134.111902

Jets Probe QCD Dynamics Across Multiple Time Scales



- Hard process in ep or pp collisions: $Q^2 \sim p_\perp^2$
- Jet evolution from parton shower to hadronization
- <u>Observable:</u> **hadron multiplicity distribution** $P(N)$

Jet Production in Proton-Proton Collisions

$$\frac{d\sigma_{pp\to(h,jet)+X}}{dp_\perp^{jet}\,d\eta^{jet}\,dz} = \sum_{i,j,k} \mathscr{H}_{ij\to k} \otimes f_i \otimes f_j \otimes D_k^h(z)$$

- Quark- or gluon-initiated jet from gg, qq or qg
- The hadron's momentum fraction within a jet: $z = p_\perp^i / p_\perp^{jet}$
- Fragmentation Functions (FFs) are universal and non-perturbative

$\boxed{\text{FFs are extracted using a global QCD fit}}$

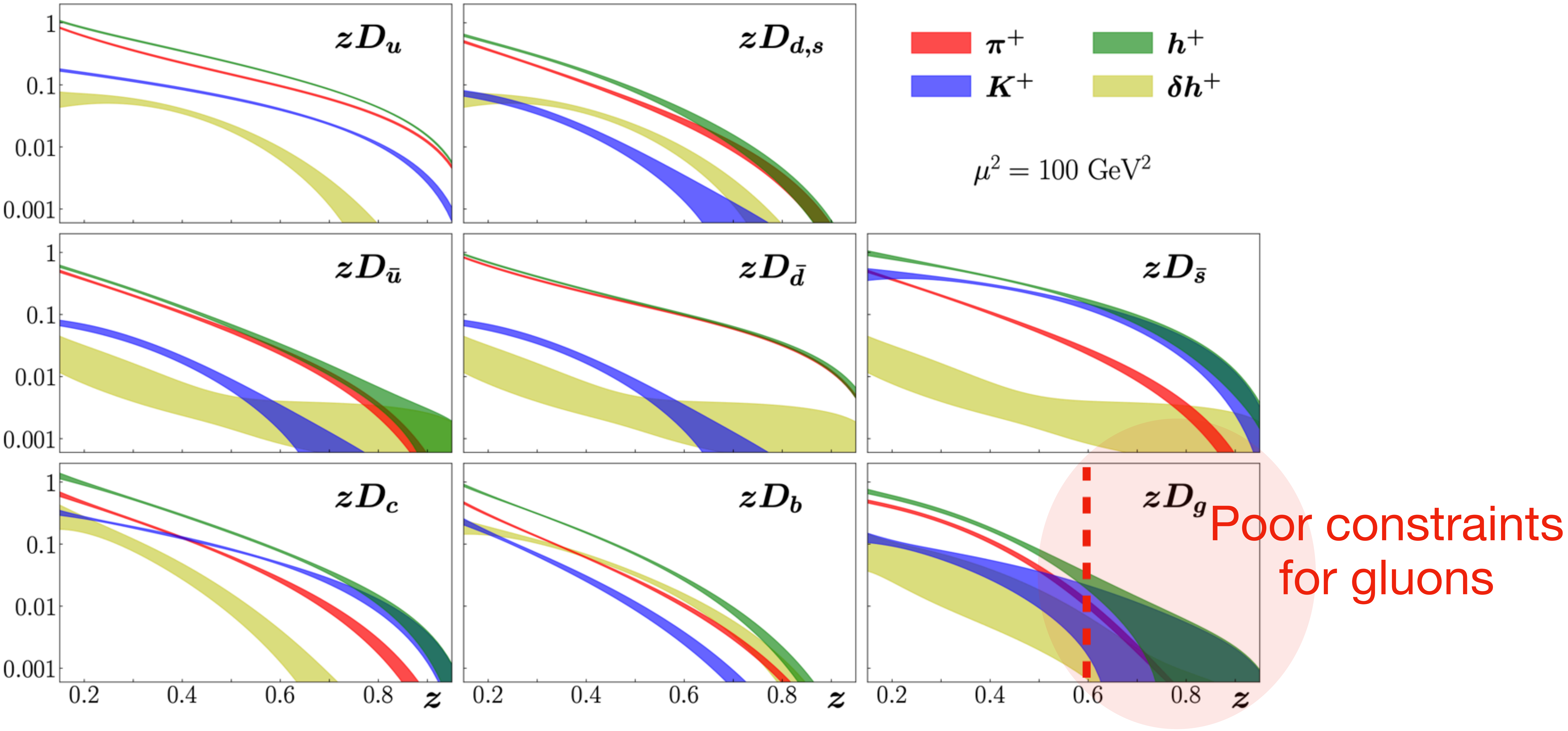
Fragmentation Functions

JAM and NNFF use different methods / datasets to extract FFs

- **NNFF** : Neural network–based fitting
  ⇨ using Single Inclusive Annihilation (SIA) and LHC data

- **JAM** : Parametrization with genetic algorithm optimization
  ⇨ using SIA and SIDIS data

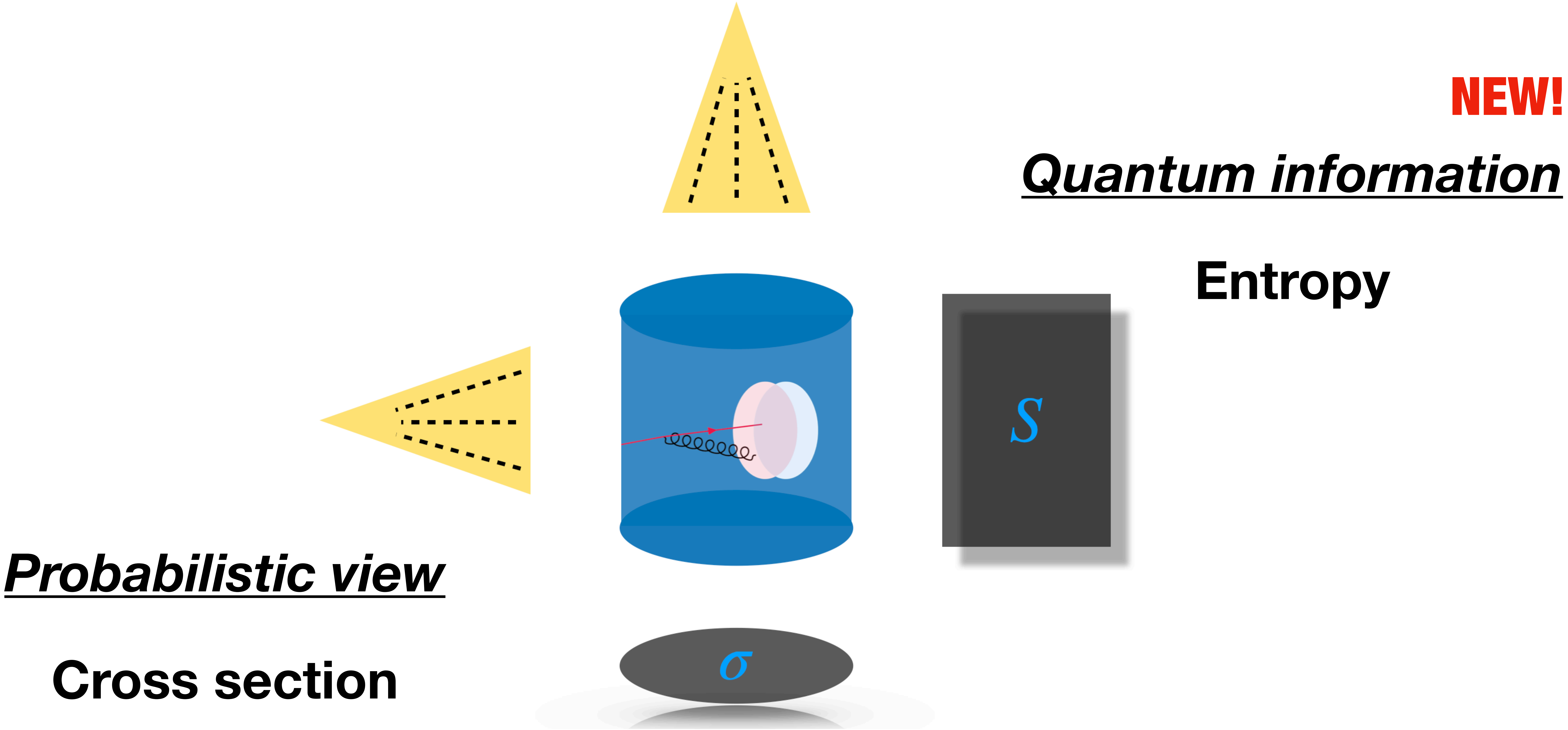
Fragmentation Functions: JAM



- FF of **the sum of hadrons** are available

Hadronization in the Spotlight

Can we describe the same phenomenon from different perspectives?



Entanglement Principle

Entanglement is a **key feature** of **quantum mechanics**

It represents **non-local correlations** between **subsystems**

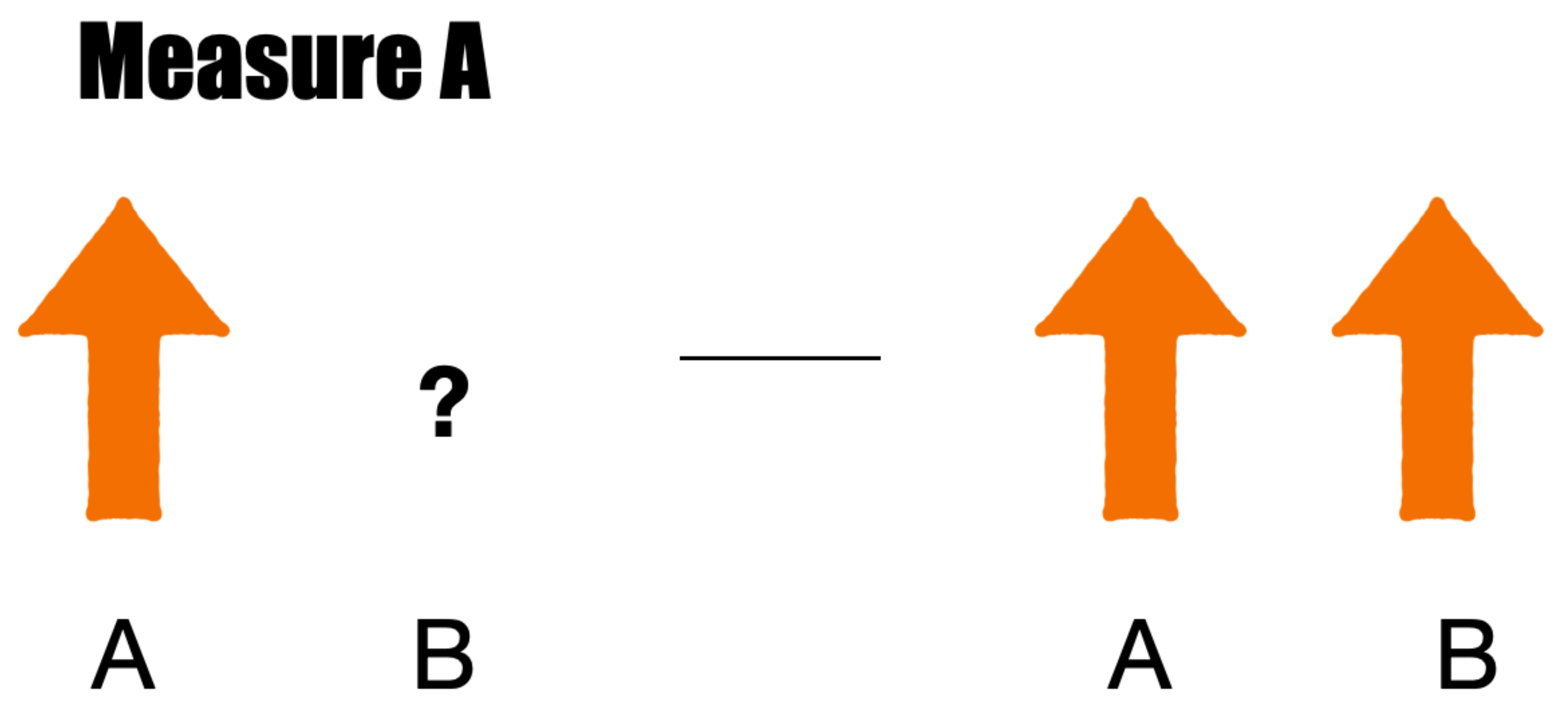
- A pure state $|\psi\rangle$ of a bipartite system **A** $\otimes$ **B** is entangled

if it cannot be written as a tensor product:

$$|\psi\rangle \neq |\phi_a\rangle \otimes |\chi_\beta\rangle$$

- **Example: Bell state**

$$|\Phi^+\rangle = (1/\sqrt{2})\,(\,|00\rangle + |11\rangle\,)$$



Maximum Entanglement Principle

The most entangled states maximize the entanglement entropy

- **Von Neumann entropy** for a bipartite state:

$$S_A = -\mathrm{Tr}(\rho_A \log \rho_A), \quad \rho_A = \mathrm{Tr}_B(|\psi\rangle\langle\psi|)$$

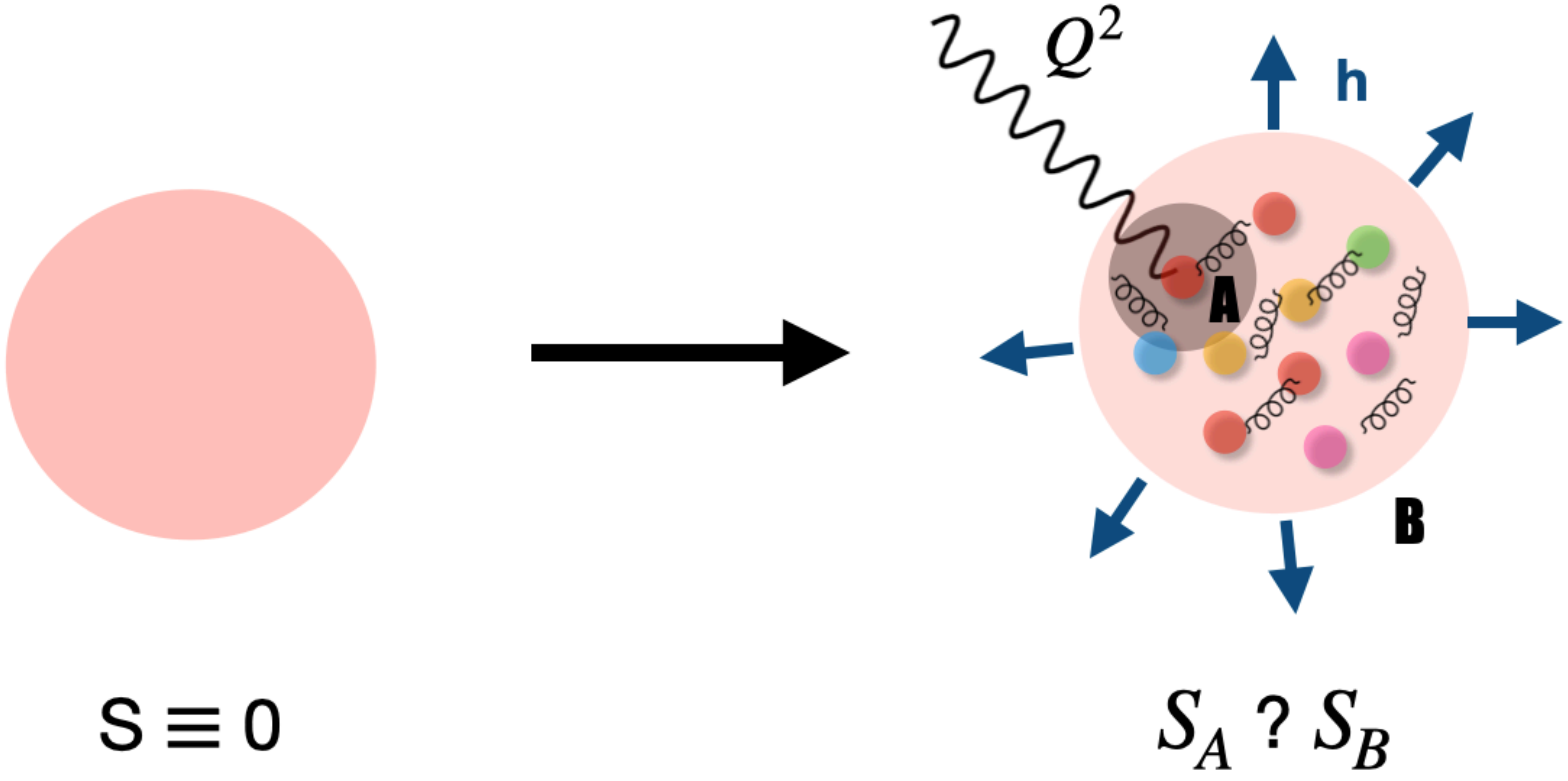
- **Bell state entropy**:

$$\rho_A = \frac{1}{2}I \Rightarrow S_A = \log 2$$

Strong condition for maximum entanglement: $S_A = S_B$

Recent Developments in High-Energy Collisions

Deep-Inelastic Scattering (DIS) : $ep \to e + h^{\pm} + X$



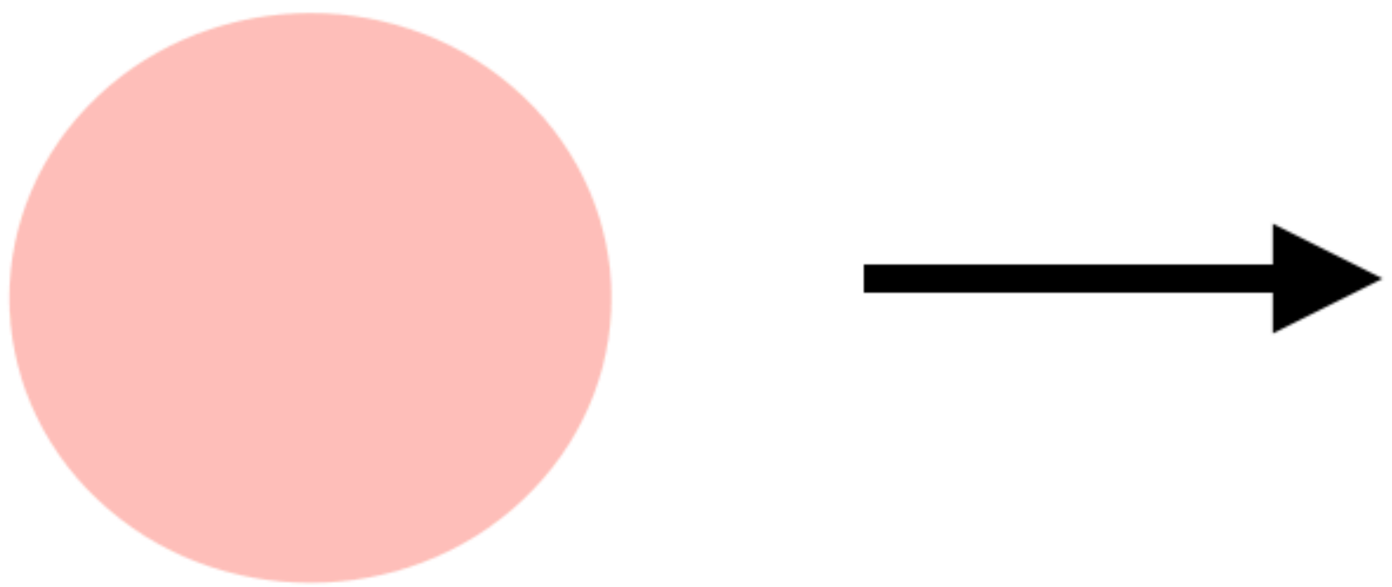
- Entanglement entropy relations: $S_A \sim \log\left[xG(x)\right]; \quad S_B = -\sum P(N) \log P(N)$

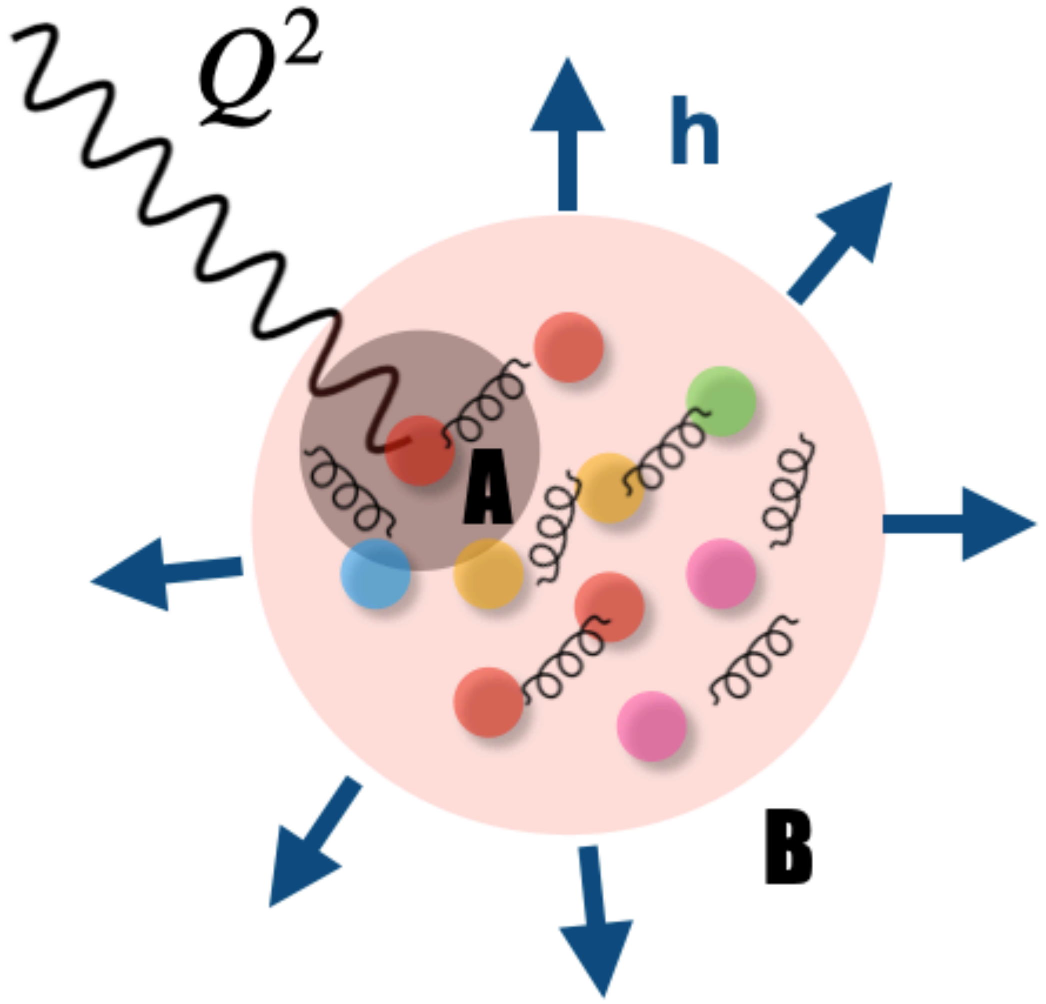
Recent Developments in High-Energy Collisions

Deep-Inelastic Scattering (DIS) : $ep \to e + h^{\pm} + X$

Multiplicity of hadrons

$$S_B = -\sum P(N)\log P(N)$$





$\mathrm{S} \equiv 0$

$S_A \sim \log\left[xG(x)\right]$

Maximum entanglement condition : $S_A = S_B$

First demonstration of maximum entanglement experimentally

Recent studies

Kharzeev & Levin: 1702.03489

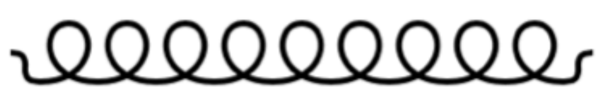
Tu, Kharzeev & Ullrich: 1904.11974

Hentschinski, Kharzeev, Kutak & Tu 2305.03069, 2408.01259

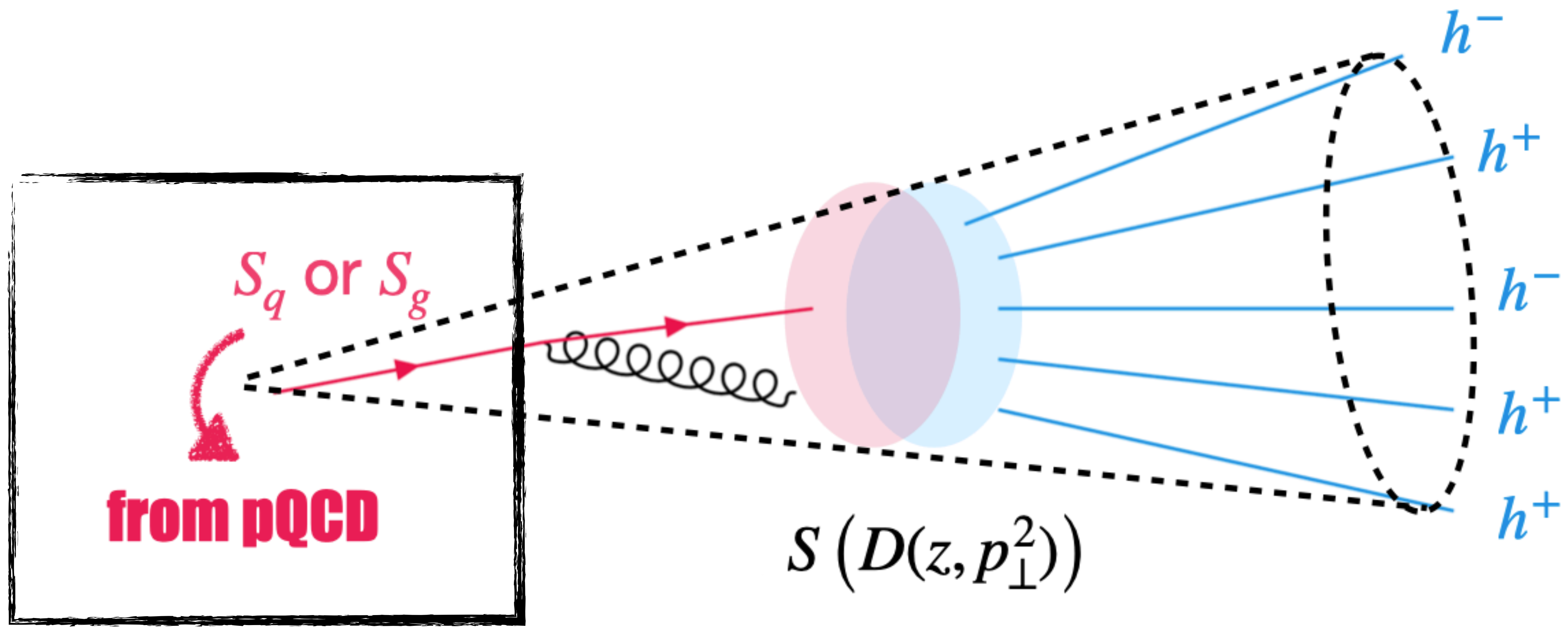
Maximum Entanglement in Jet

- Jet initiated by an entangled quark/gluon pair
- A jet is not a pure quantum state

$S_q = \log\left(N_c\right)$

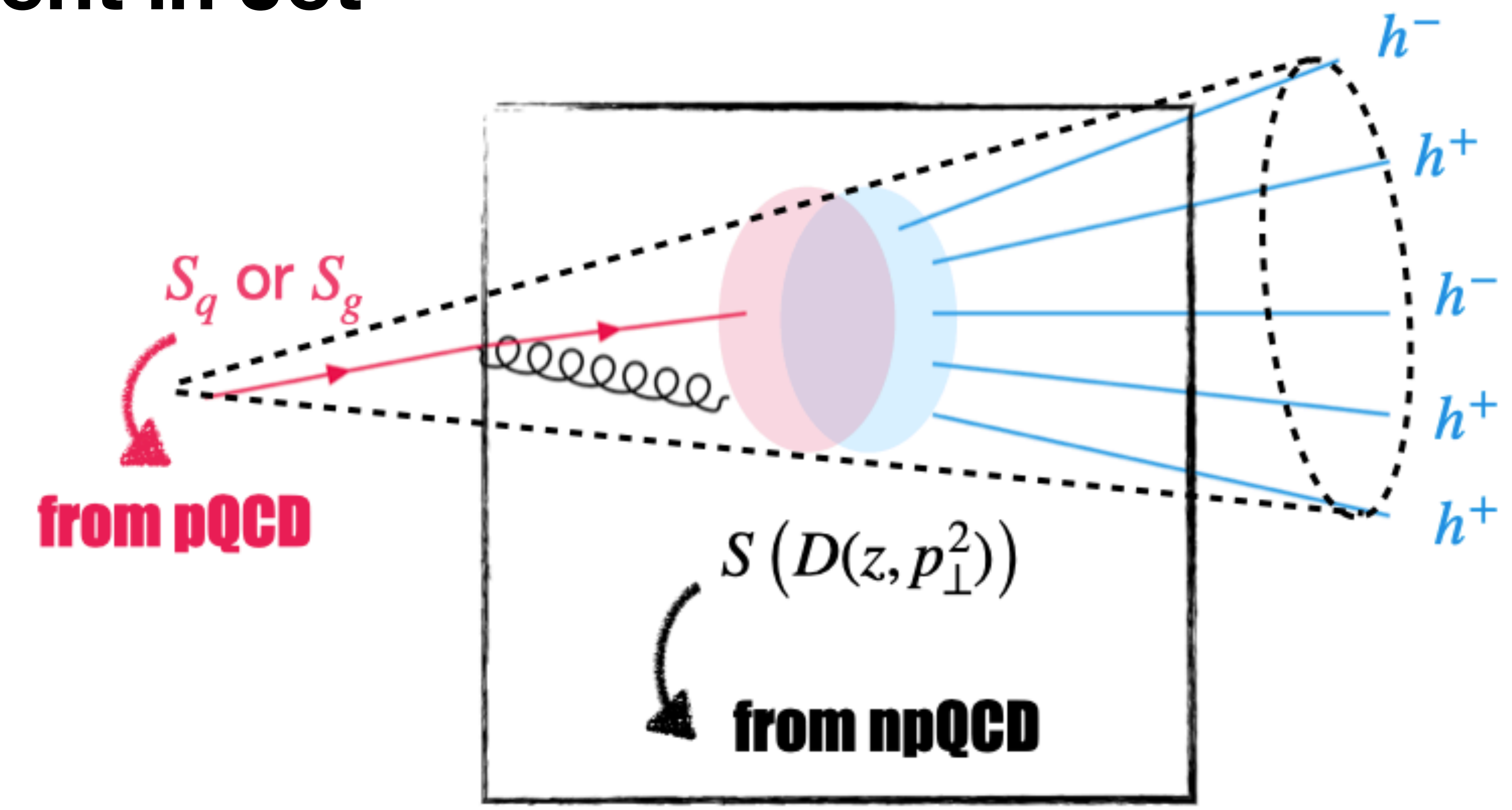


$S_g = \log\left(N_c^2 - 1\right)$



Gluon jets have higher maximal entanglement entropy than quark jets

Maximum Entanglement in Jet



- Derived from **maximal entanglement in the proton:** $S_A \sim \log\left[xG(x,Q^2)\right]$
- By crossing symmetry, **applies to fragmentation:** $S_A \sim \log\left[D(z,p_\perp^2)\right]$

Crossing symmetry: relates PDFs and FFs, e.g., $f_{i/p}(x) \longleftrightarrow D_{j/h}(z)$

Maximum Entanglement in Jet

The model:

$$S_{FF} = S_{q/g} + \log \left[\int_{z_{min}(p_\perp^{jet})}^{1} D(z, p_\perp^h)\, dz \right]$$

- The **entanglement entropy of the partonic jet**, $S_{q/g}$

  ⇨ which depends on the **quark or gluon origin**

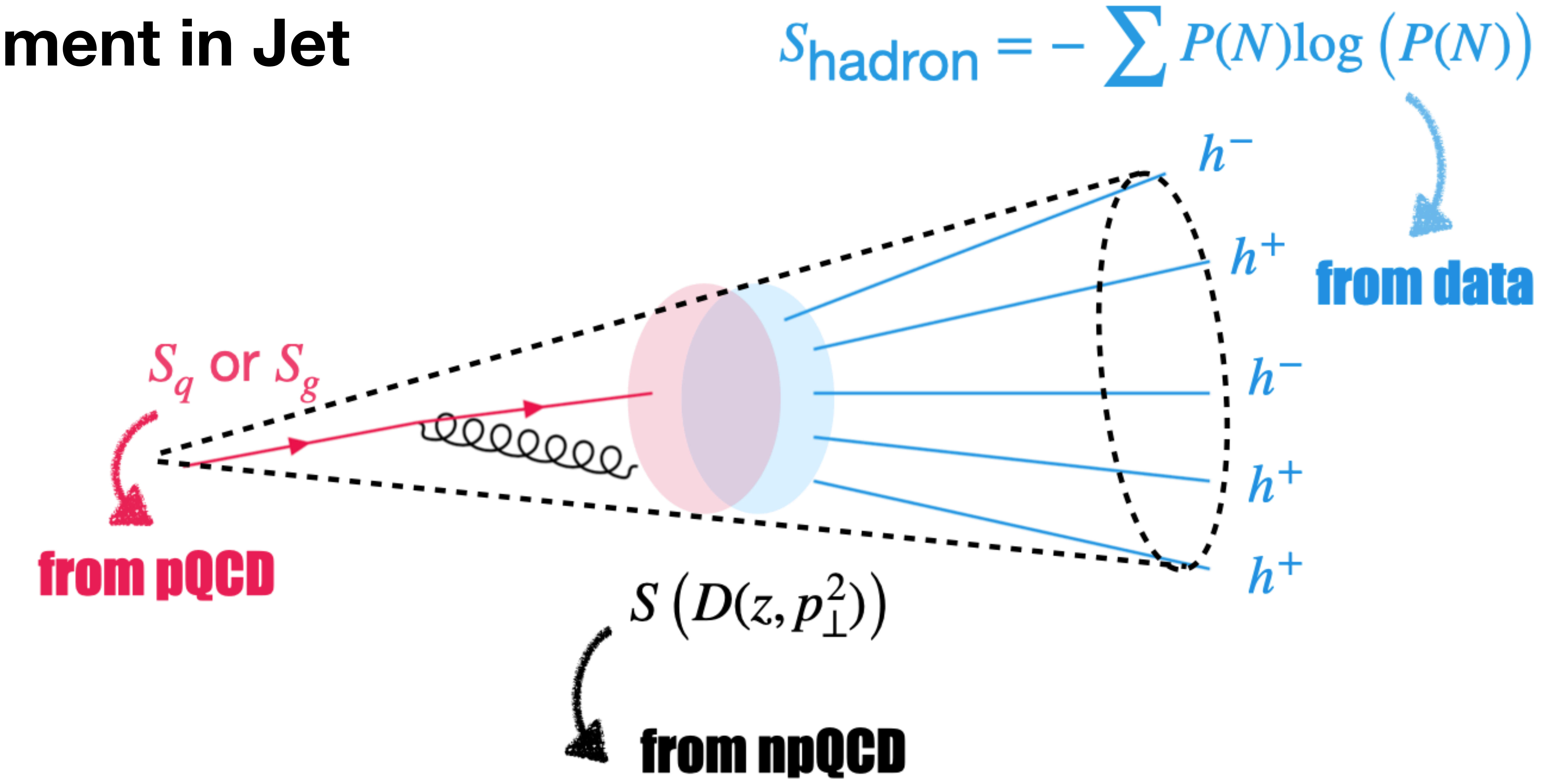
- The integral represents the FF: **parton-to-hadron transition**

  ⇨ from JAM and NNPDF

- The lower limit z_{min} depends on the jet's transverse momentum

  ⇨ ensuring **only relevant fragmentation processes are included**

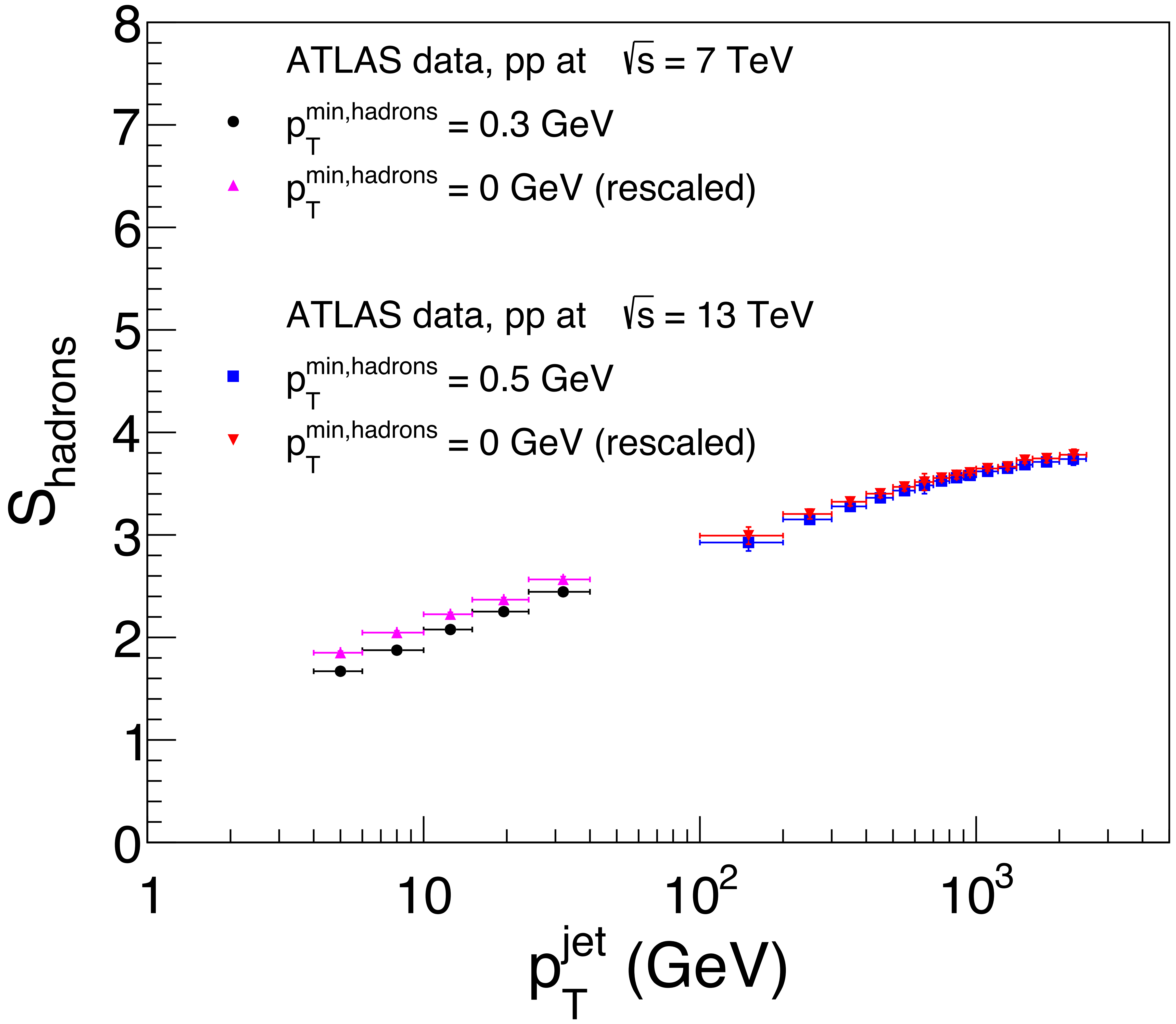
Maximum Entanglement in Jet

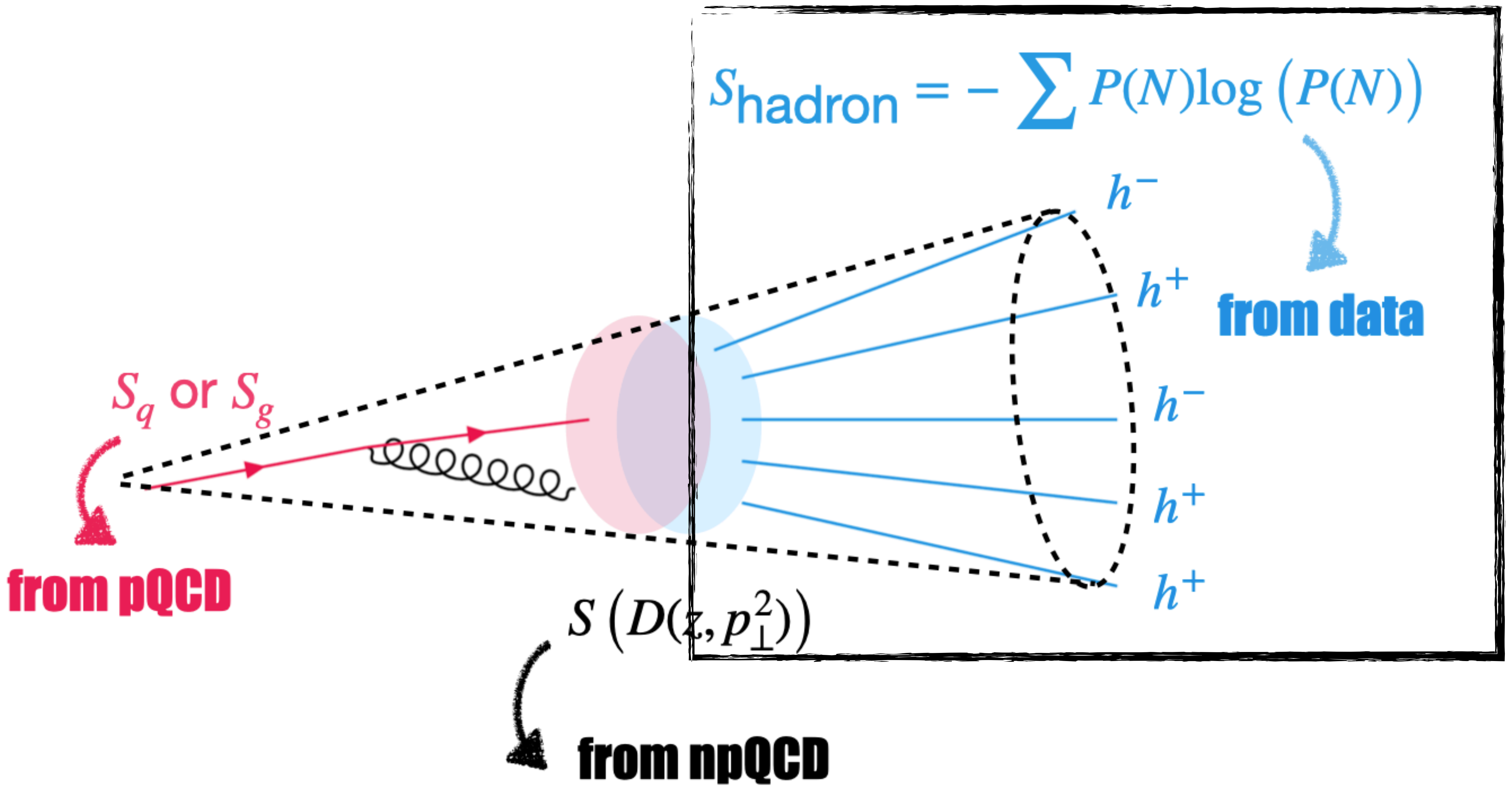


- The partonic jet flavor is determined by pQCD
- Hadronization is described by fragmentation functions (FF)

Maximal entanglement condition : $S_{\text{FF}} = S_{\text{hadrons}}$

Charged-Hadrons Multiplicity in Jet





The large $p_\perp^{jet}$ probes a wide z phase space

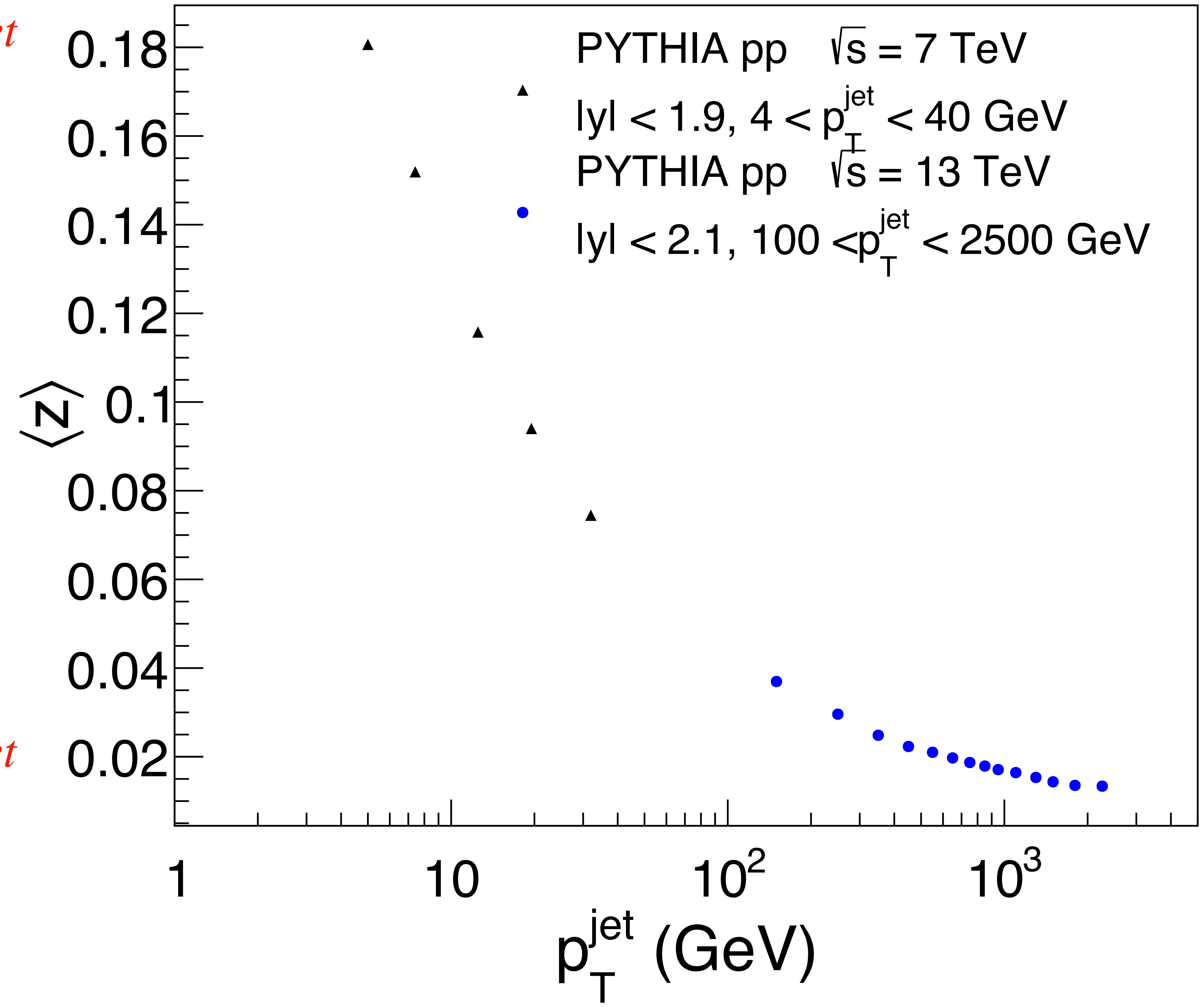
$$z \propto 1/p_\perp^{jet}$$

Charged-Hadrons Multiplicity in Jet

Experimental data are published as a function of $p_{\perp}^{jet}$ instead of z

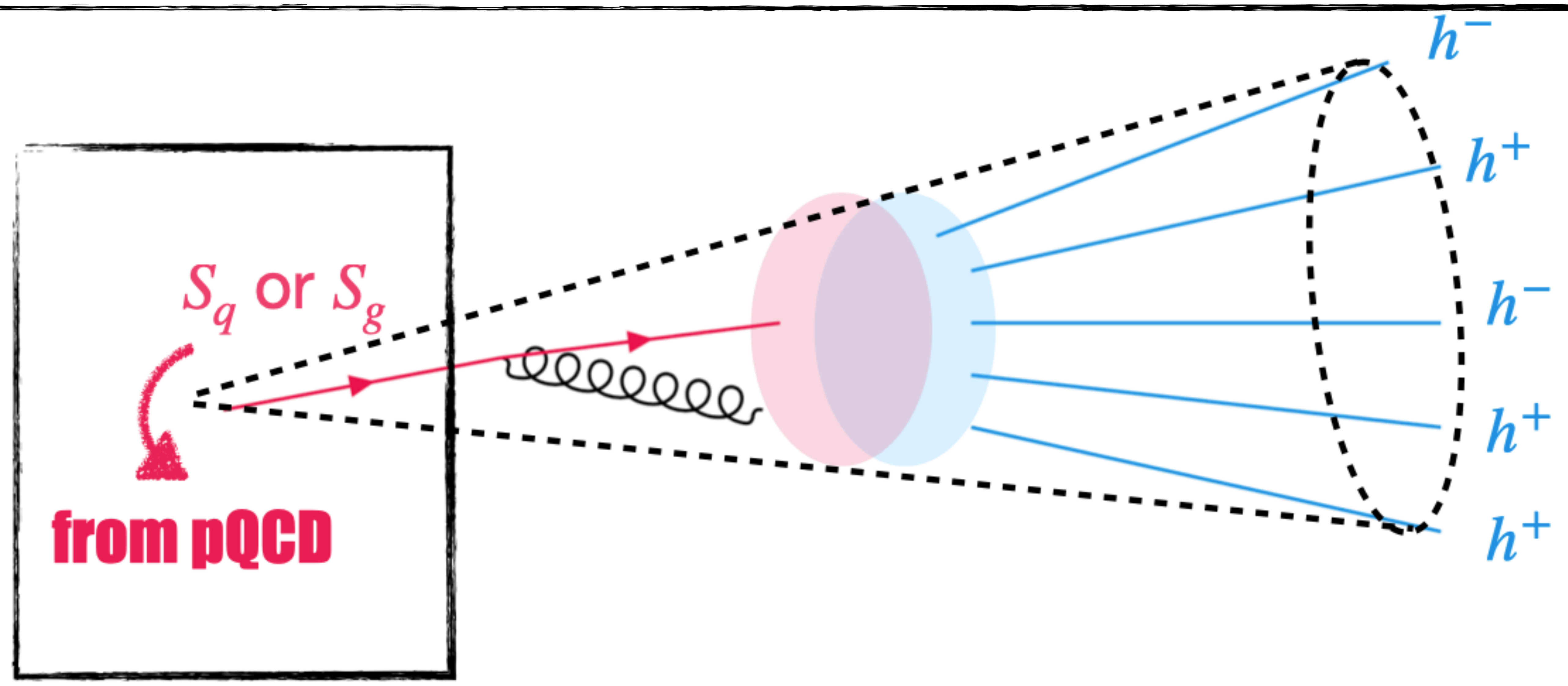
Small $p_{\perp}^{jet}$

Large $p_{\perp}^{jet}$



Parton-Initiated Jet

The processes $qq \to qq, gg \to gg,$ and $gq \to gq$ evolve with $p_\perp^{jet}$

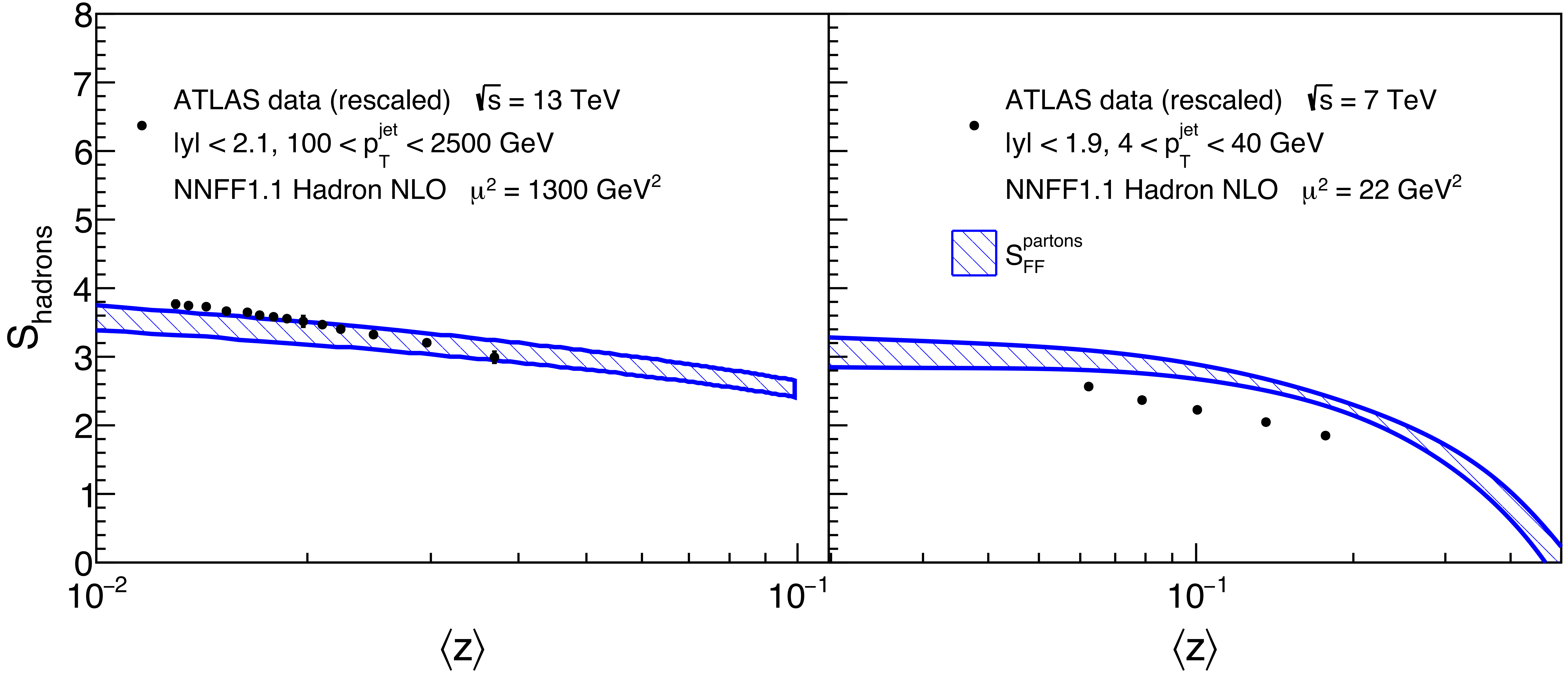


- At large $p_\perp^{jet}$, large Bjorken-x : $x \sim 2p_\perp^{jet}/\sqrt{s}$, $gq \to gq$ process dominates

S_{FF} depends on quark and gluon jet

Data-Model Comparison

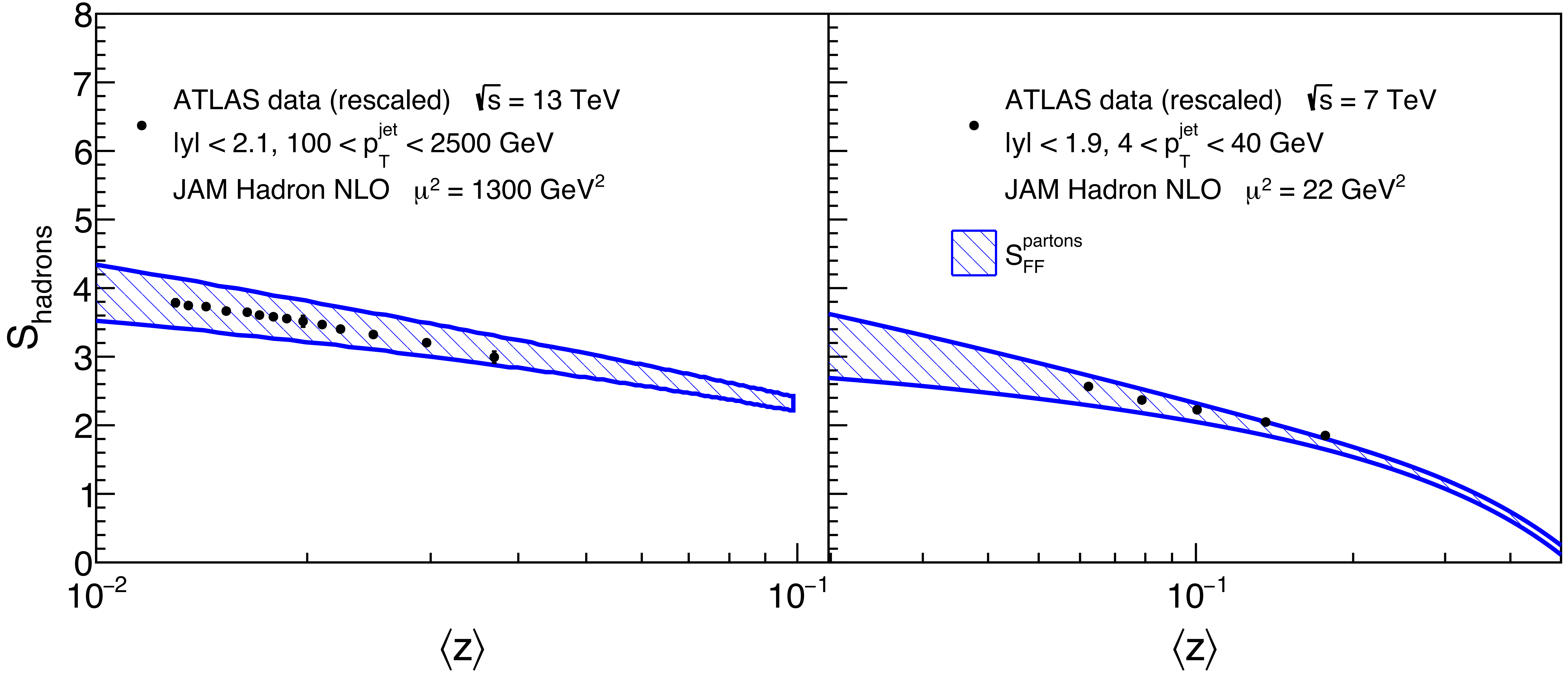
Testing: S_{FF} ? $S_{hadrons}$



Excellent agreement between data and the model using **NNFF**

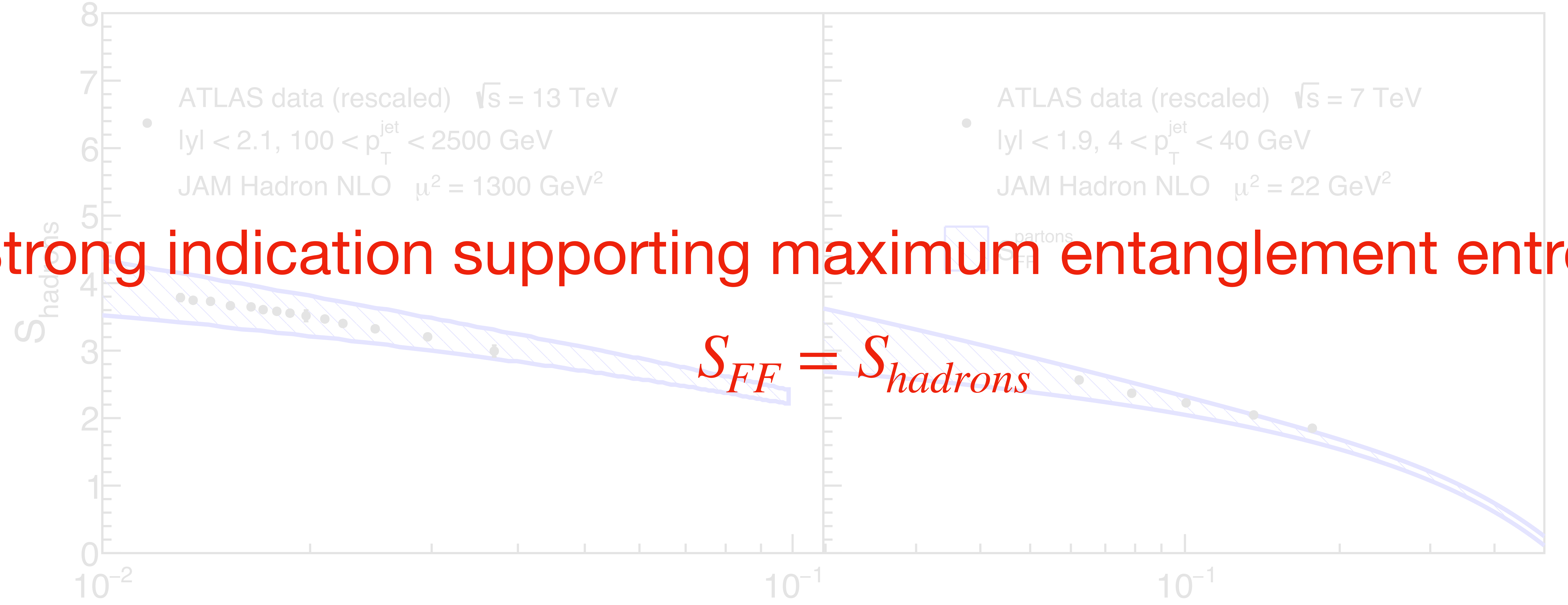
Data-Model Comparison

Testing: S_{FF} ? $S_{hadrons}$



Remarkable agreement between data and the model using **JAM**

Data-Model Comparison



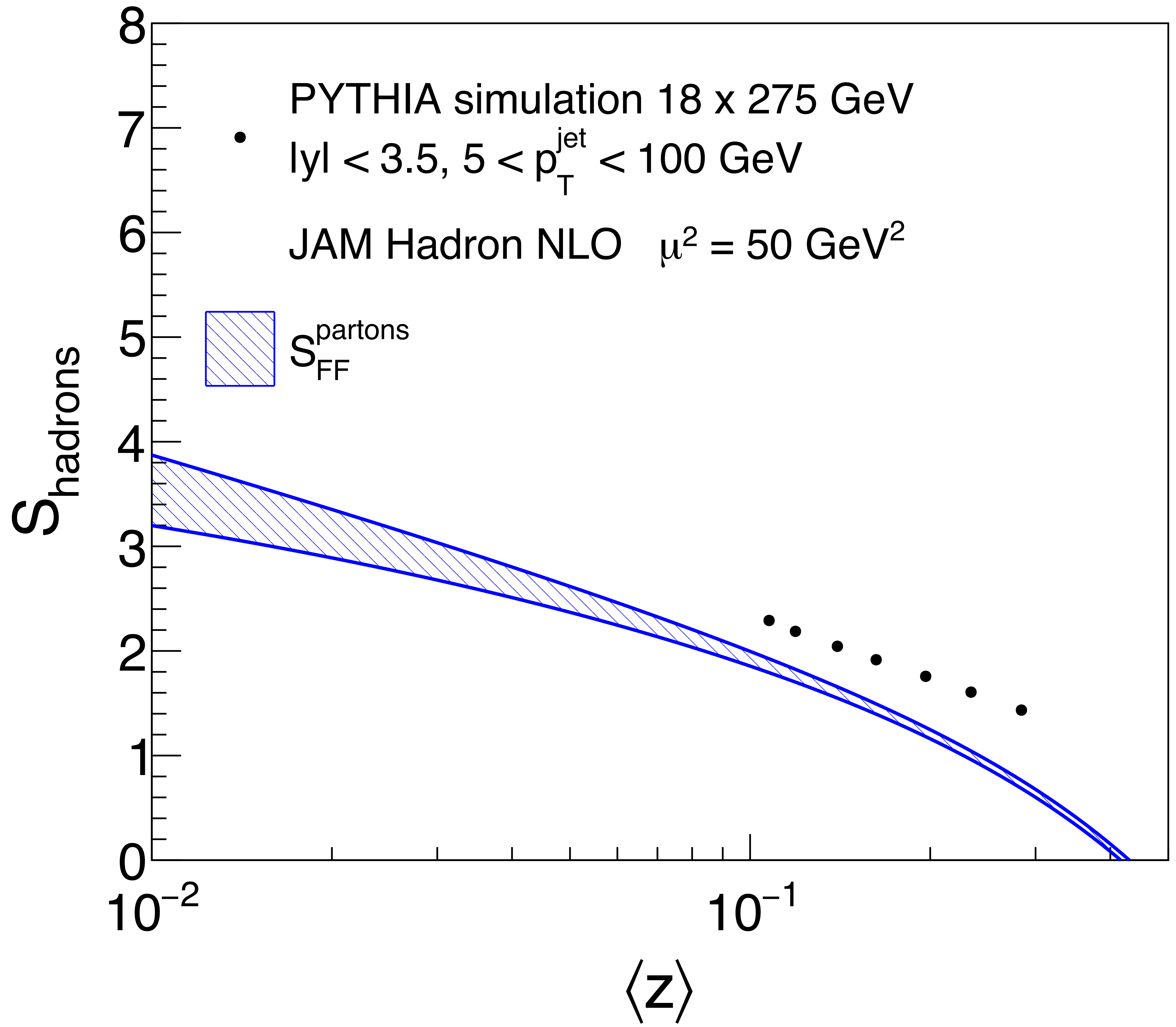
Strong indication supporting maximum entanglement entropy!

$$S_{FF} = S_{hadrons}$$

Excellent agreement between data and the model using **NNFF**

Remarkable agreement between data and the model using **JAM**

EIC Predictions



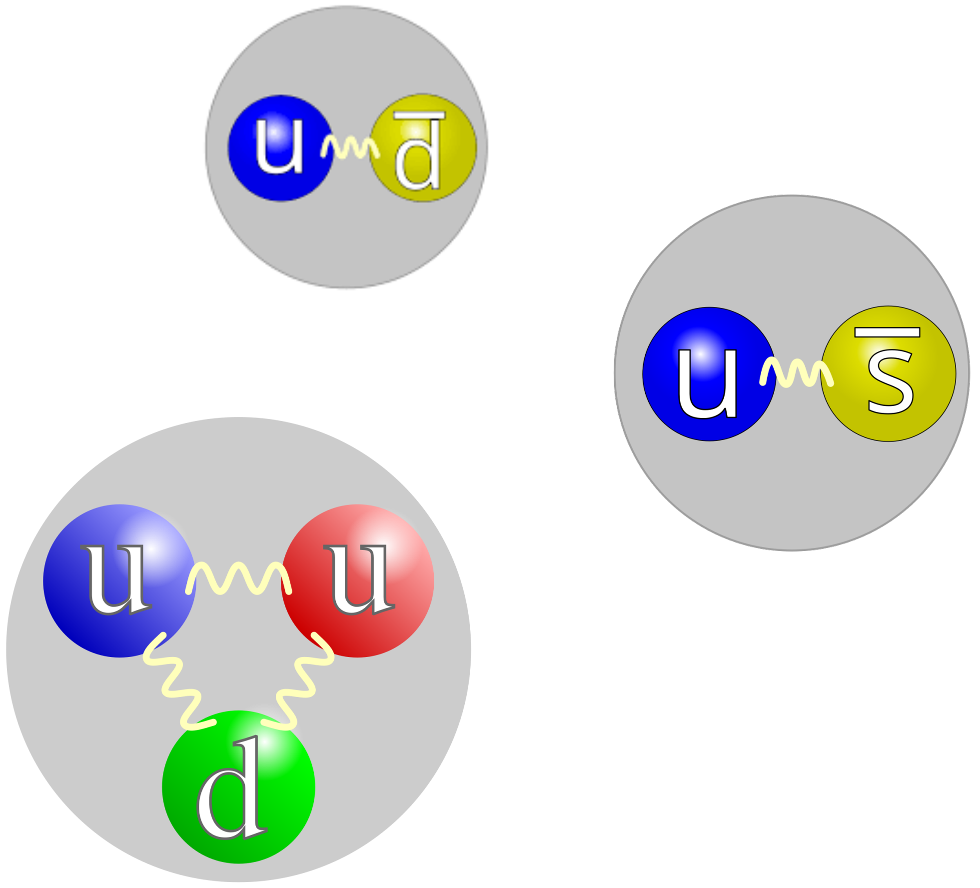
- Charged hadron multiplicity is overestimated in PYTHIA
- Future EIC data will provide key insights for $z \gtrsim 0.3$

Conclusion

- **New approach to hadronization based on quantum entanglement**
- **Excellent agreement between data and model**

- New method for **studying hadronization**
- At EIC, calculate $S_{hadrons}$ for pion, kaon, and proton

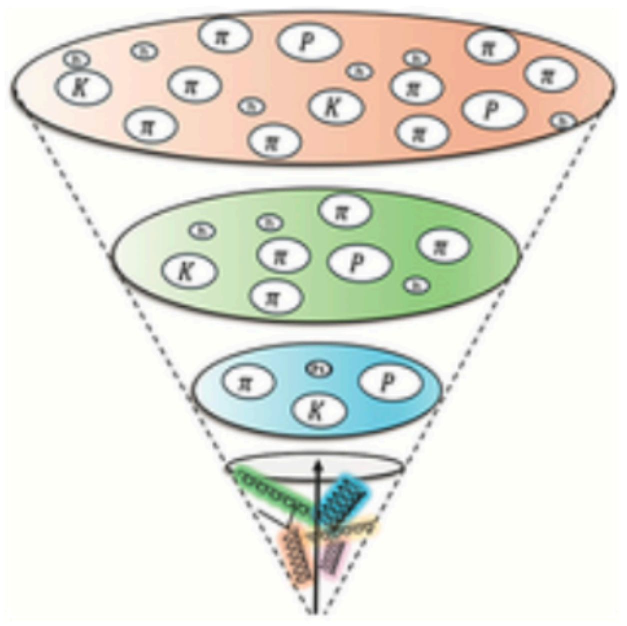
- Concept extendable to **eA/pA data**
- Study the impact of **nuclear medium on entropy**
- Future measurements: P(N) vs z directly



Maximum entanglement entropy as a (simple) bridge between pQCD and npQCD

Editor's suggestion

BNL highlight



EDITORS' SUGGESTION

Entanglement as a Probe of Hadronization

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The entropy of hadrons produced within highly energetic jets can be related to the fragmentation function if the initial quarks and gluons in the jets are maximally entangled.

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