Brookhaven National Laboratory

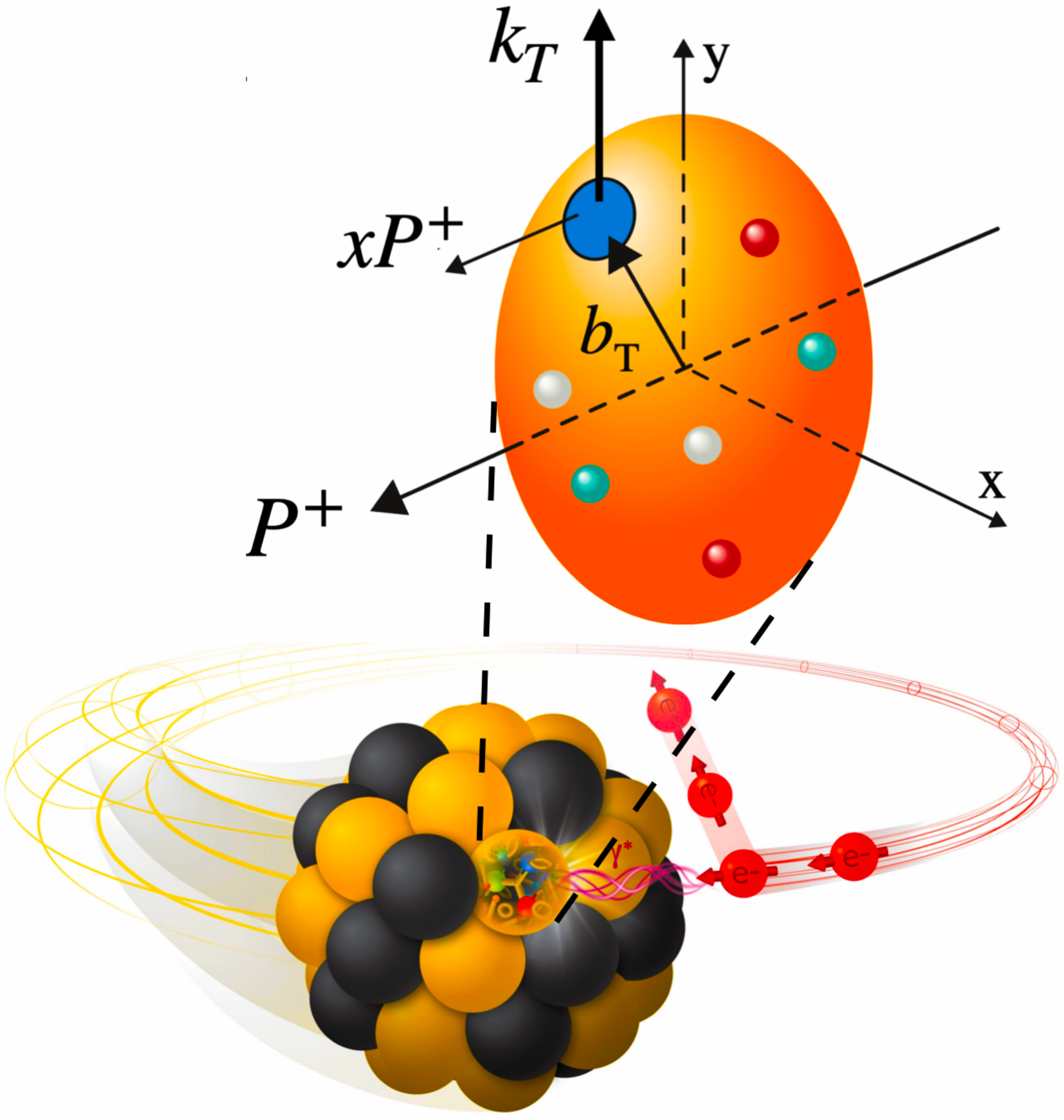
U.S. Department of ENERGY

Parton Distributions from Boosted Fields in the Coulomb Gauge

Xiang Gao

QCD Evolution 2025 @ Jefferson Lab, May 19- May 23, 2025.

The internal structure of hadron



$f(x, \vec{k}_T)$ **TMDs**

$f(x, \vec{b}_T)$ **IPDs**

$b_T \leftrightarrow t$ F.T.

$f(x, t)$ **GPDs**

3D

$\int d^2k_T$ $\int d^2b_T$ $\int dx x^{n-1}$

$f(x)$ **PDFs**

$A_{n,0}(t) + \ldots$ **Form factors**

1D

$e^- + q$ **Hard interaction**

$$\sigma(y) \sim c(y, x, \mu) \otimes f(x, \mu)$$

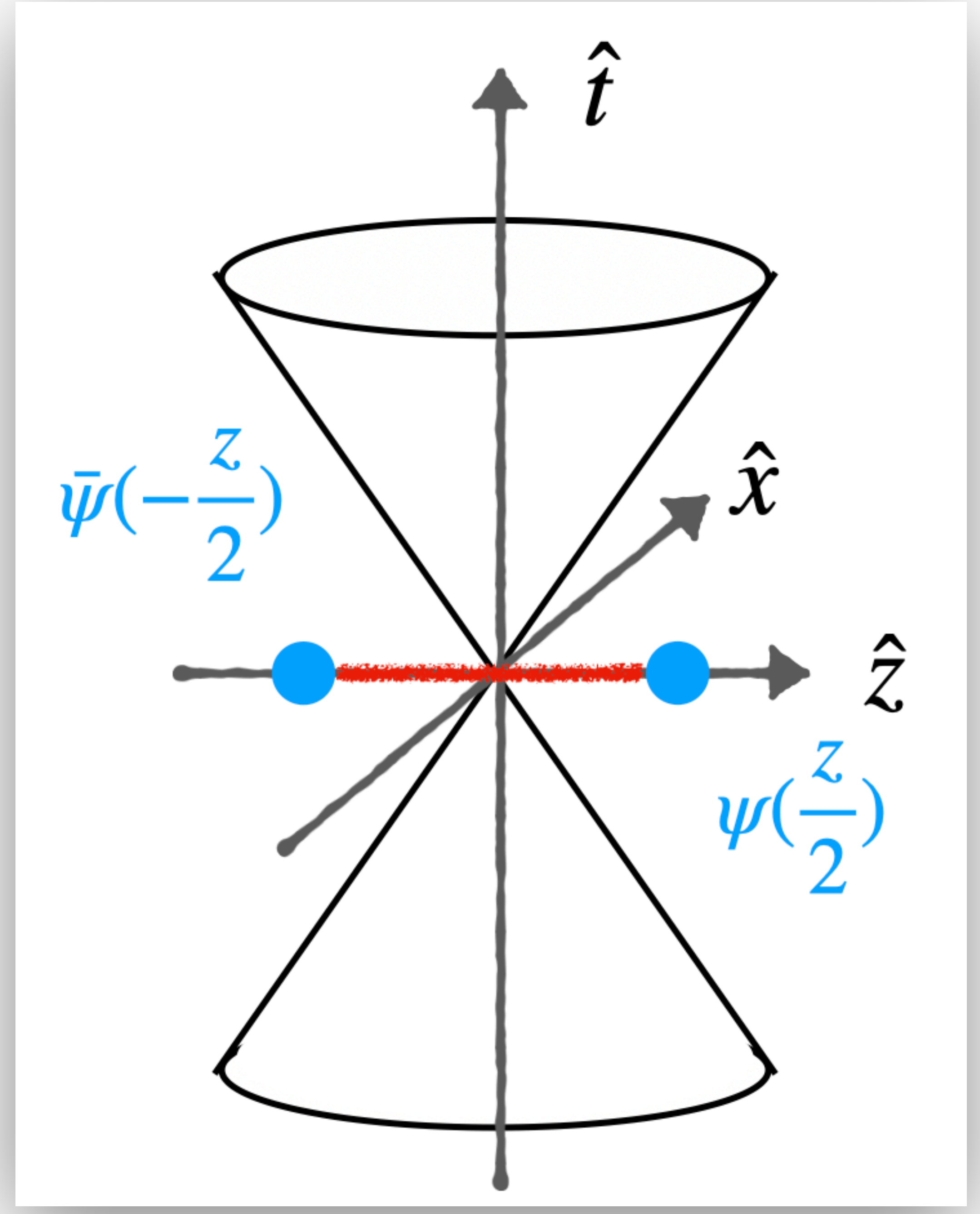
Nonperturbative parton distribution

Parton distributions from lattice QCD

- X. Ji, PRL 110 (2013); SCPMA57 (2014);
- X. Xiong, X. Ji, et al, 90 PRD (2014);
- Y.-Q. Ma, et al, PRD98 (2018), PRL 120 (2018);
- T. Izubuchi, X. Ji, et al PRD98 (2018).
- X. Ji, Y. Zhao, et al, RMP 93 (2021).

- Equal-time correlators with spacial separation.
- Approaching light-cone parton distribution in the large momentum limit.

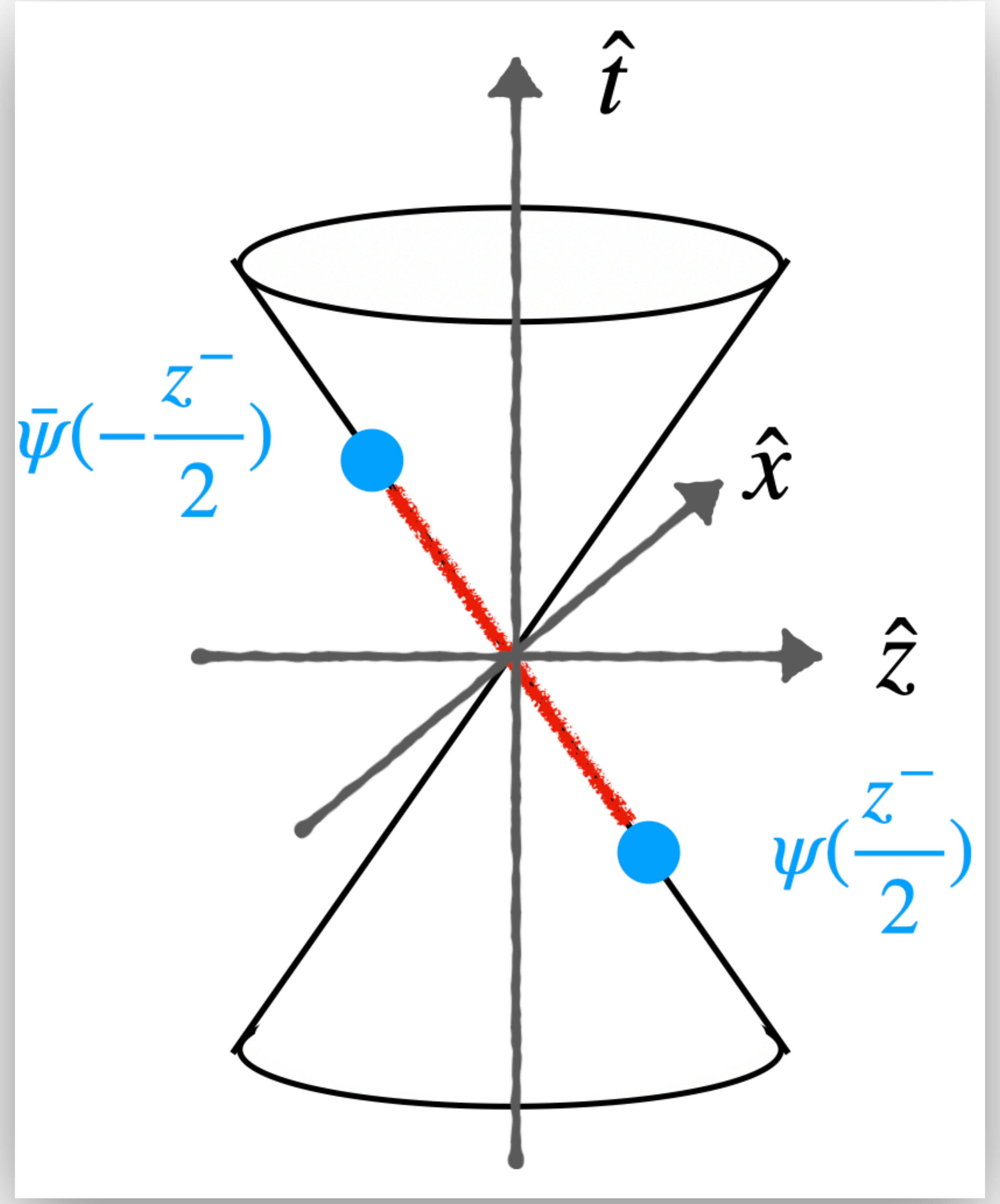
Equal-time correlators



Quasi PDF

$\langle P \to \infty | \bar{\psi}(0)\Gamma W(0,z)\psi(z) | P \to \infty \rangle$

$P_z \to \infty$



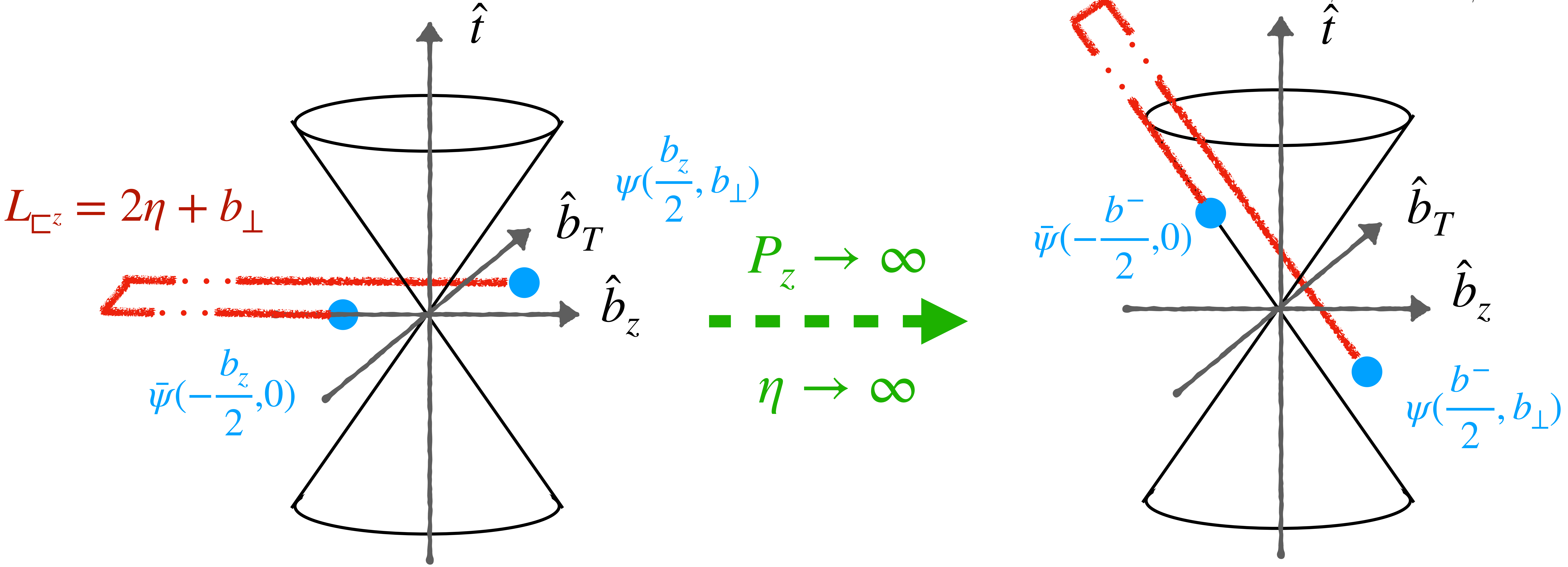
Physical PDF

$\langle P | \bar{\psi}(0)\Gamma W(0,z^-)\psi(z^-) | P \rangle$

Light-cone correlators: sign problem for lattice

TMDs from lattice QCD

- Ji, Liu and Liu, NPB 955 (2020), PLB 811 (2020);
- A. Vladimirov, A. Schäfer Phys.Rev.D 101 (2020), 074517
- I. Stewart, Y. Zhao et al., JHEP 09 (2020) 099
- X. Ji et al., Phys.Rev.D 103 (2021) 7, 074005
- I. Stewart, Y. Zhao et al., JHEP 08 (2022) 084



Quasi TMD Beam function

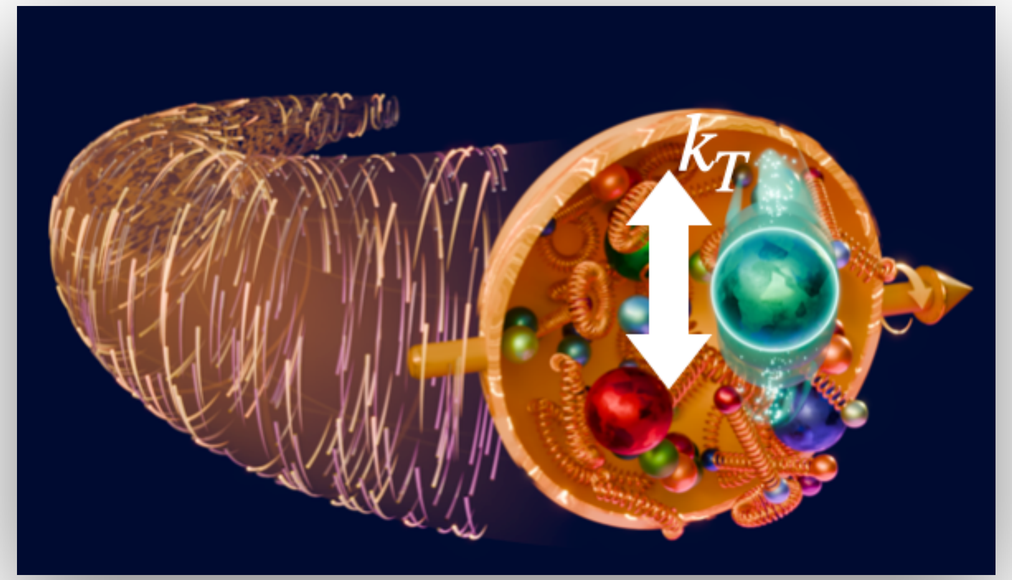
$$\langle P \to \infty \,|\, \bar{\psi}(\frac{b_z}{2}, b_\perp)\Gamma W_{\sqsupset^z}\psi(-\frac{b_z}{2},0) \,|\, P \to \infty \rangle$$

Light-cone TMD Beam function

$$\langle P \,|\, \bar{\psi}(\frac{b^-}{2}, b_\perp)\Gamma W_{\sqsupset^+}\psi(-\frac{b^-}{2},0) \,|\, P \rangle$$

TMDs from lattice QCD

- Computable from lattice QCD with $P_z \ll 1/a$.
- Large P_z expansion of quasi TMDs made of beam function and soft factor.



Quasi-TMD

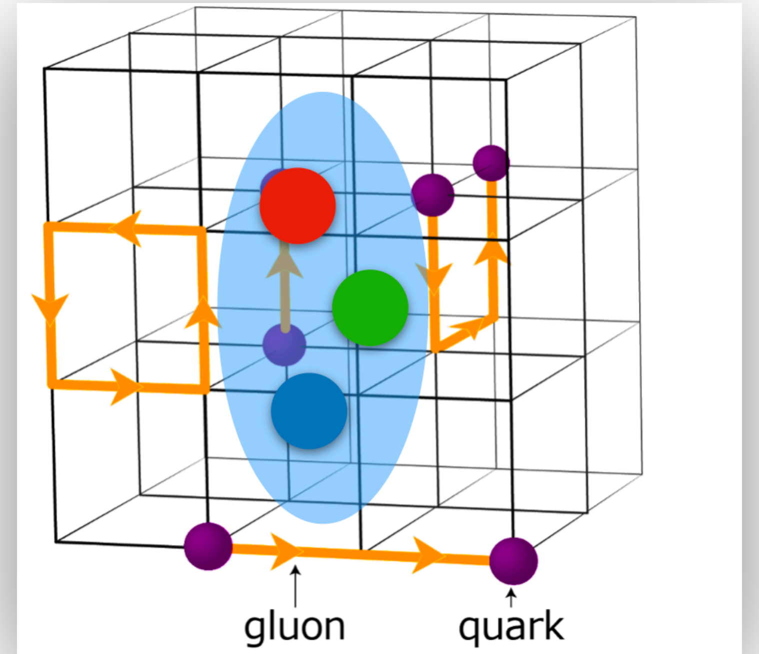
Collins-Soper kernel

$$\frac{\tilde{f}(x, \vec{b}_T, \mu, P_z)}{\sqrt{S_r(\vec{b}_T, \mu)}} = C(\mu, xP_z) e^{\frac{1}{2}\gamma_\zeta(\mu, b_T)\ln\frac{(2xP_z)^2}{\zeta}} f(x, \vec{b}_T, \mu, \zeta)\{1 + \mathcal{O}[\frac{1}{(xP_z b_T)^2}, \frac{\Lambda^2_{\rm QCD}}{(xP_z)^2}]\}$$

Hard kernel

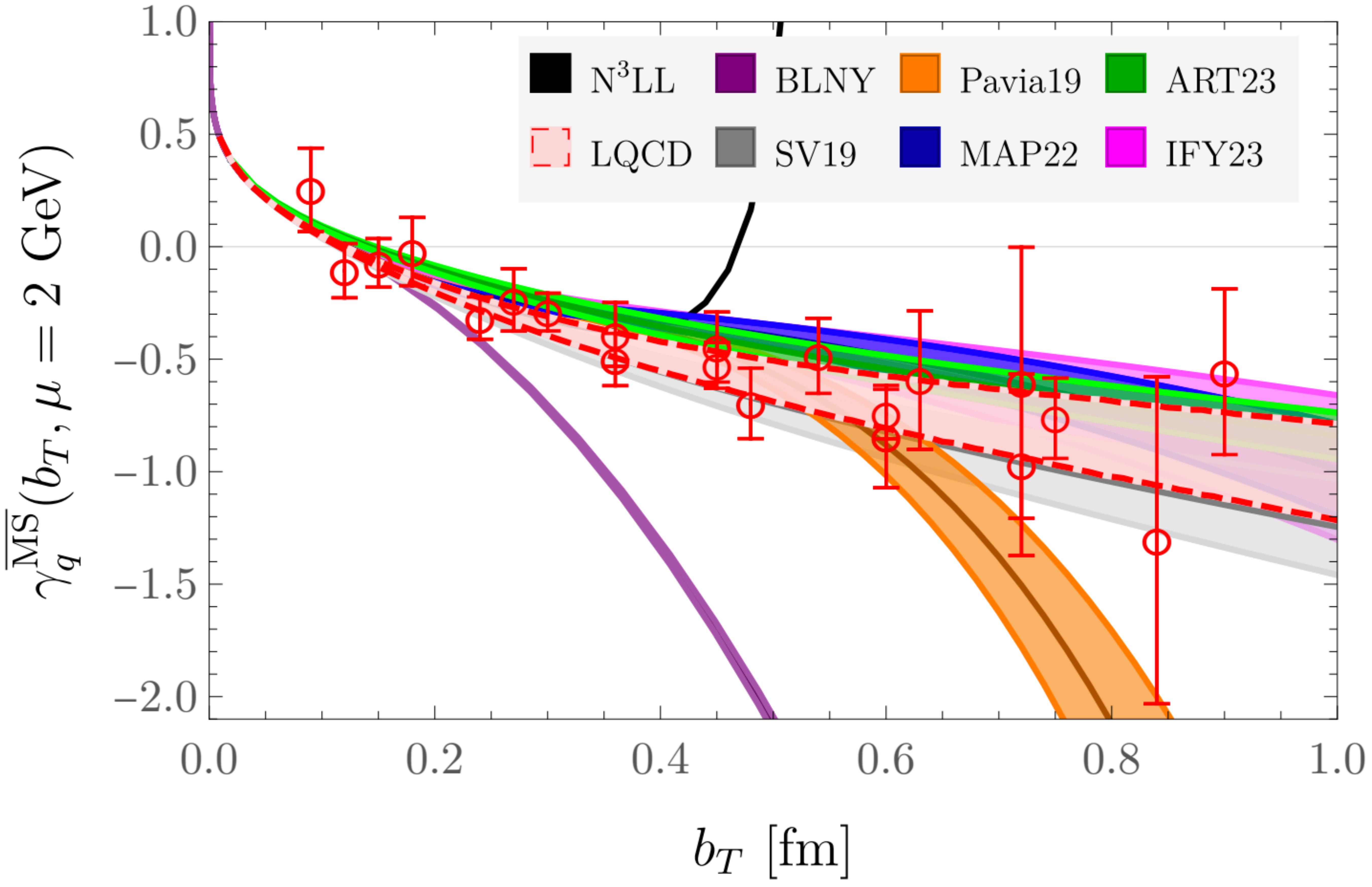
Physical TMD

$$P_z < a^{-1}$$



TMDs from lattice QCD

Encouraging progress has been reported recently, e.g., the Collins-Soper kernel.



Quasi-TMDWF

$$\gamma_q(\mu, b_T) = \frac{d}{d\ln P_z}\ln\frac{\tilde{f}(x, \vec{b}_T, \mu, P_z)}{C(\mu, xP_z)}$$

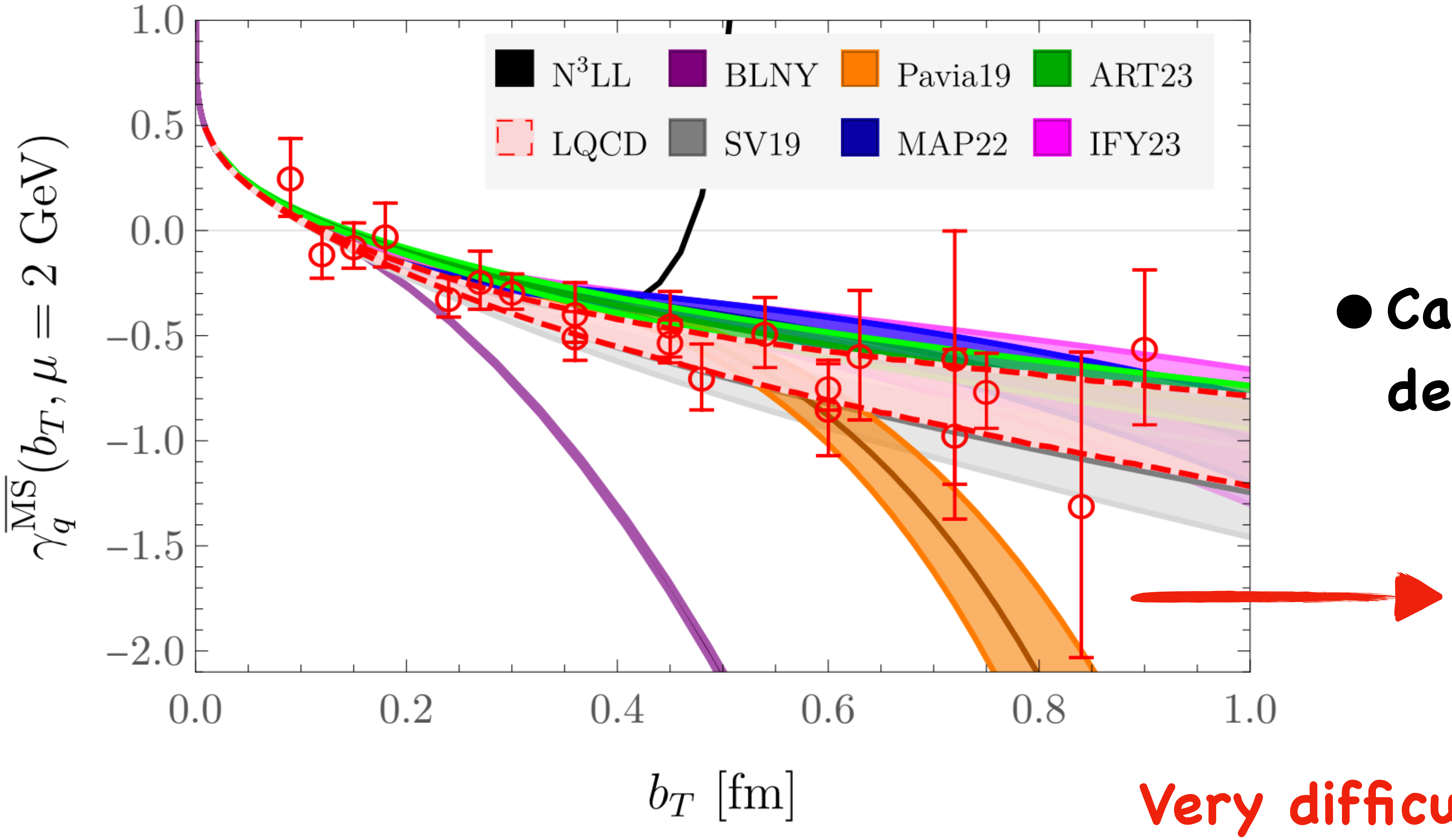
NNLL perturbative matching

- Begin to discriminate different global analysis at nonperturbative b_T when experimental data are sparse.

• A. Avkhadiev, P. Shanahan, M. Wagman, Y. Zhao, PRL 132 (2024) 23, 231901

TMDs from lattice QCD

Encouraging progress has been reported recently, e.g., the Collins-Soper kernel.

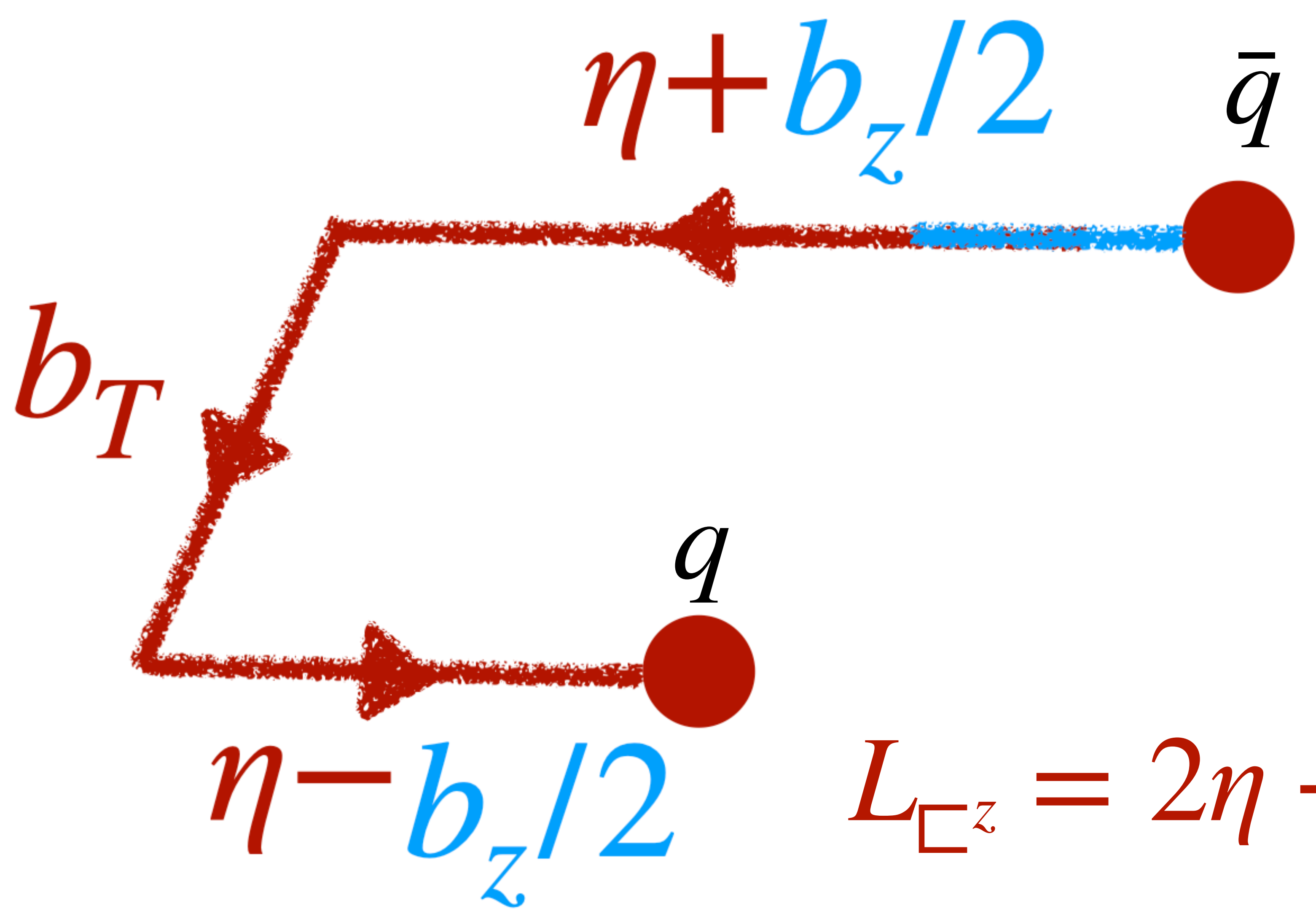


- **Can lattice QCD push further in deep nonperturbative region?**

Very difficult: errors grow rapidly!

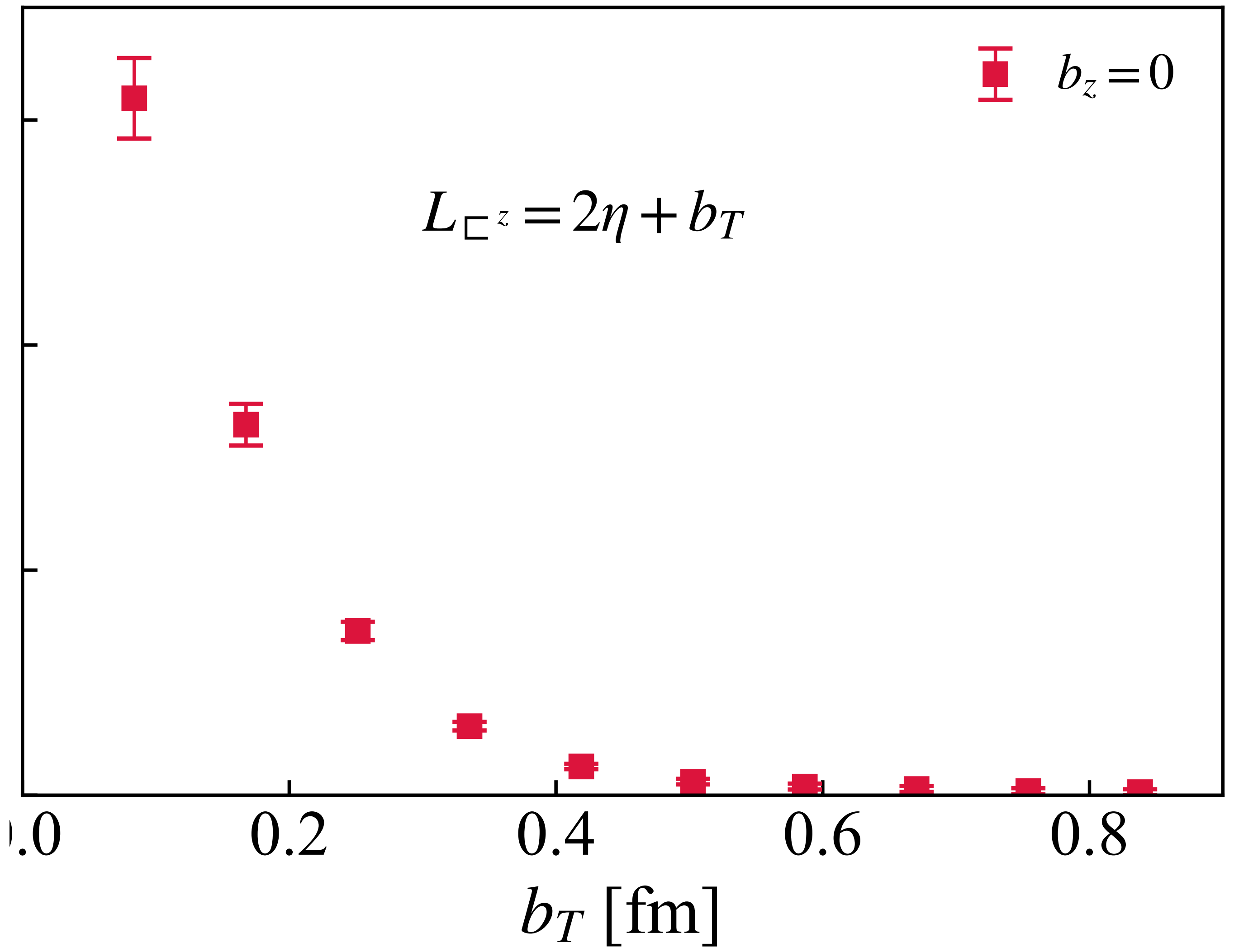
- A. Avkhadiev, P. Shanahan, M. Wagman, Y. Zhao, PRL 132 (2024) 23, 231901

Difficulties in the conventional quasi-TMDs



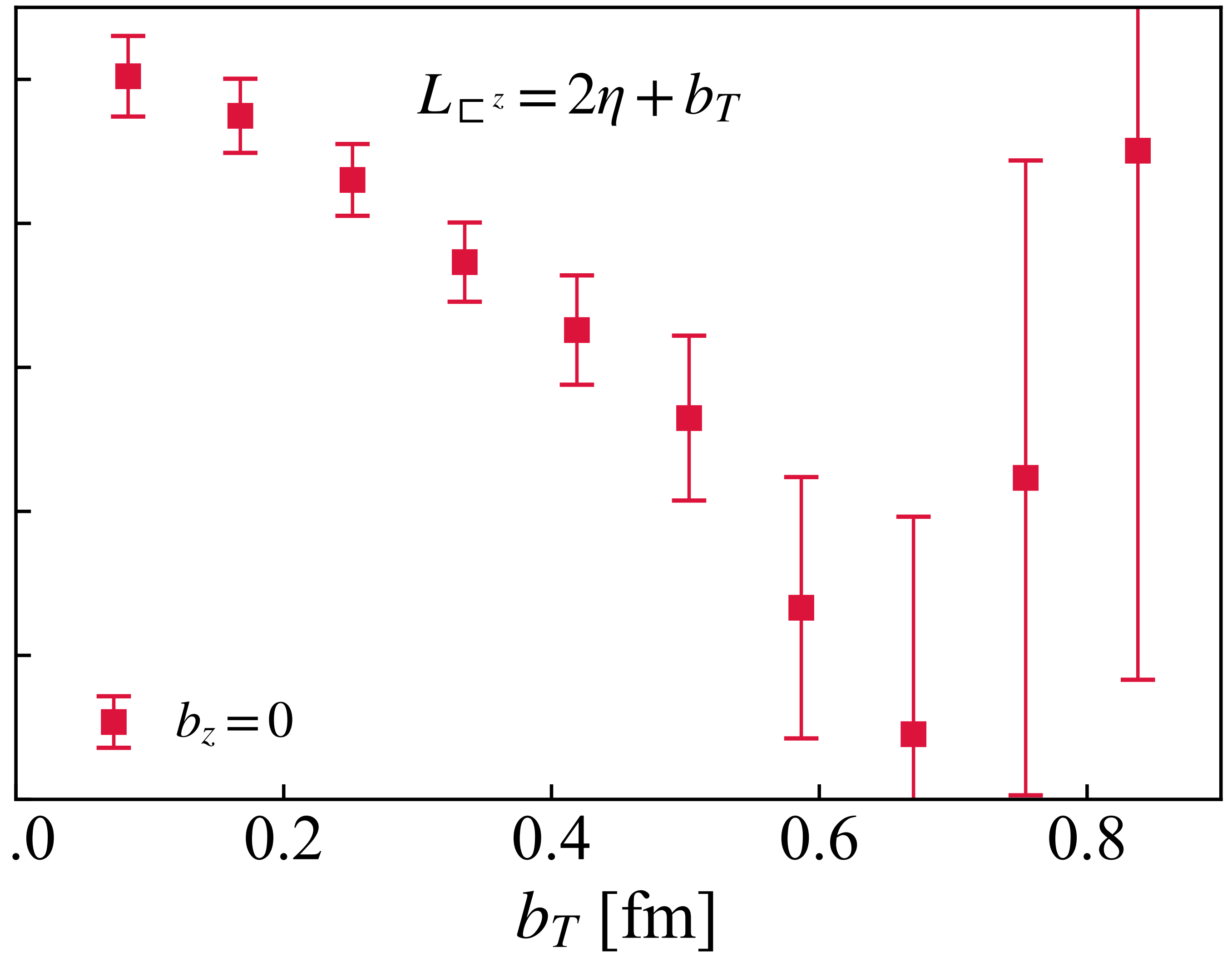
$L_{\sqsubset z} = 2\eta + b_T$

Bare matrix elements

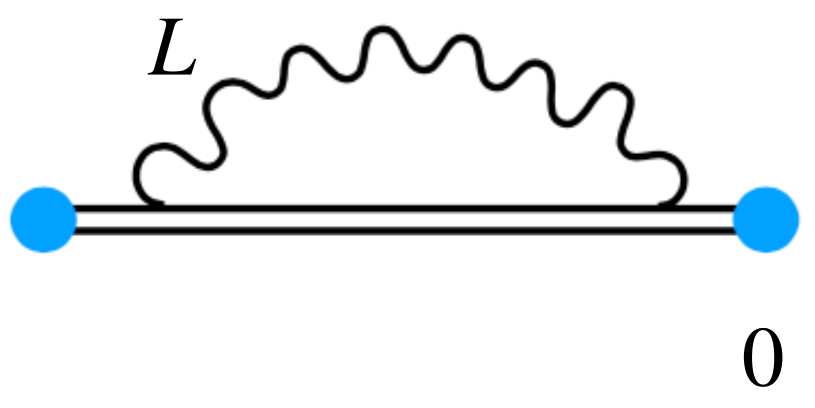


$\times\, e^{\delta m(a) \cdot L_{\sqsubset z}}$

Renormalized matrix elements



- Exponential decaying signal and complicated renormalization due to the Wilson line artifacts.



$\sim e^{-\delta m(a) \cdot L_{\sqsubset z}}$

Linear divergence from Wilson line self energy

The non-local operator in gauge theory

- The non-local operator in gauge theory:

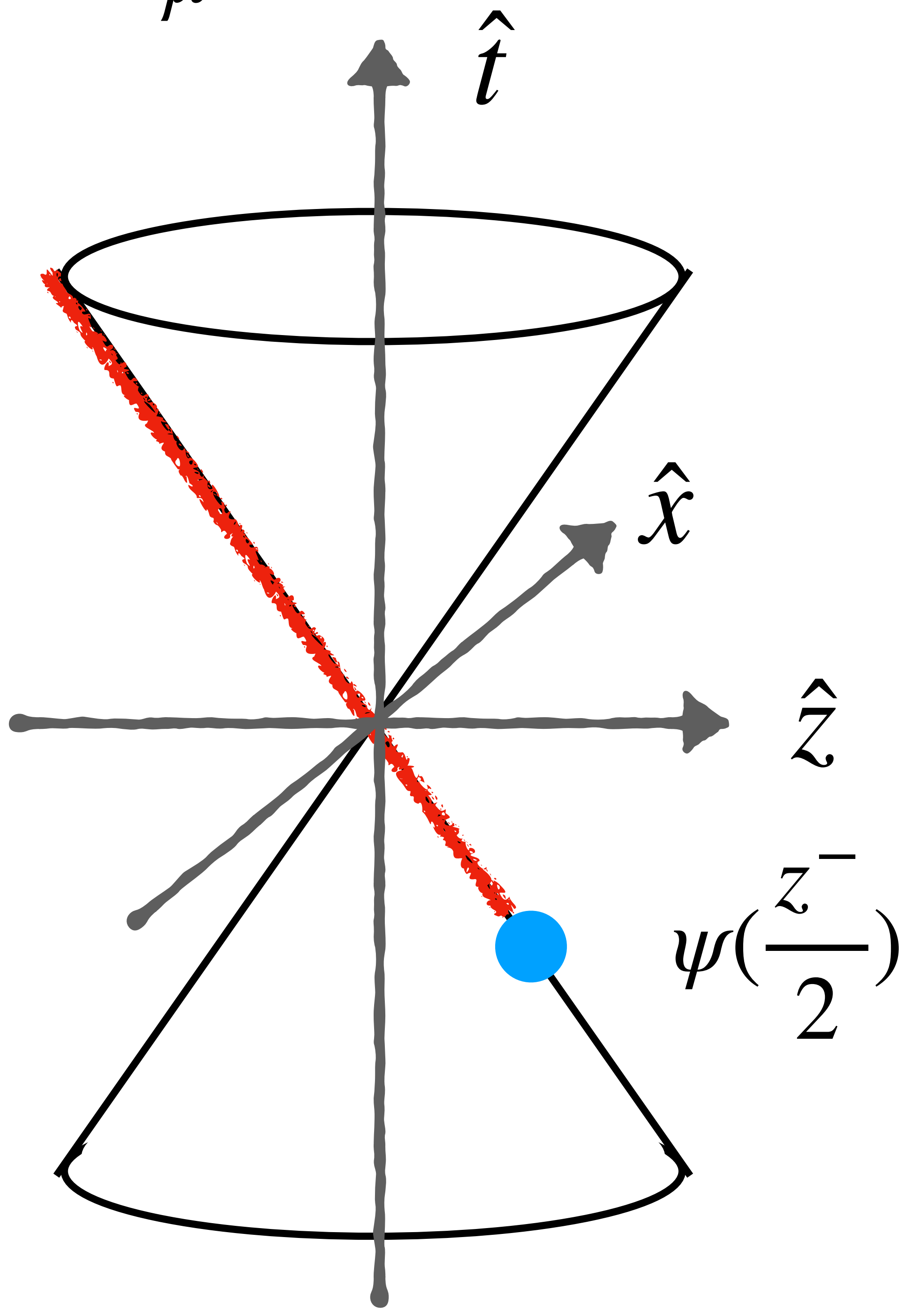
$$\bar{\psi}^*(-\frac{z}{2})\Gamma\psi^*(\frac{z}{2})$$

• P. A. M. Dirac, Can.J.Phys. 33 (1955) 650

with $\psi^*(z) = \psi(z)e^{iC(z)}$ and $C(z)$ is a linear function of A_μ .

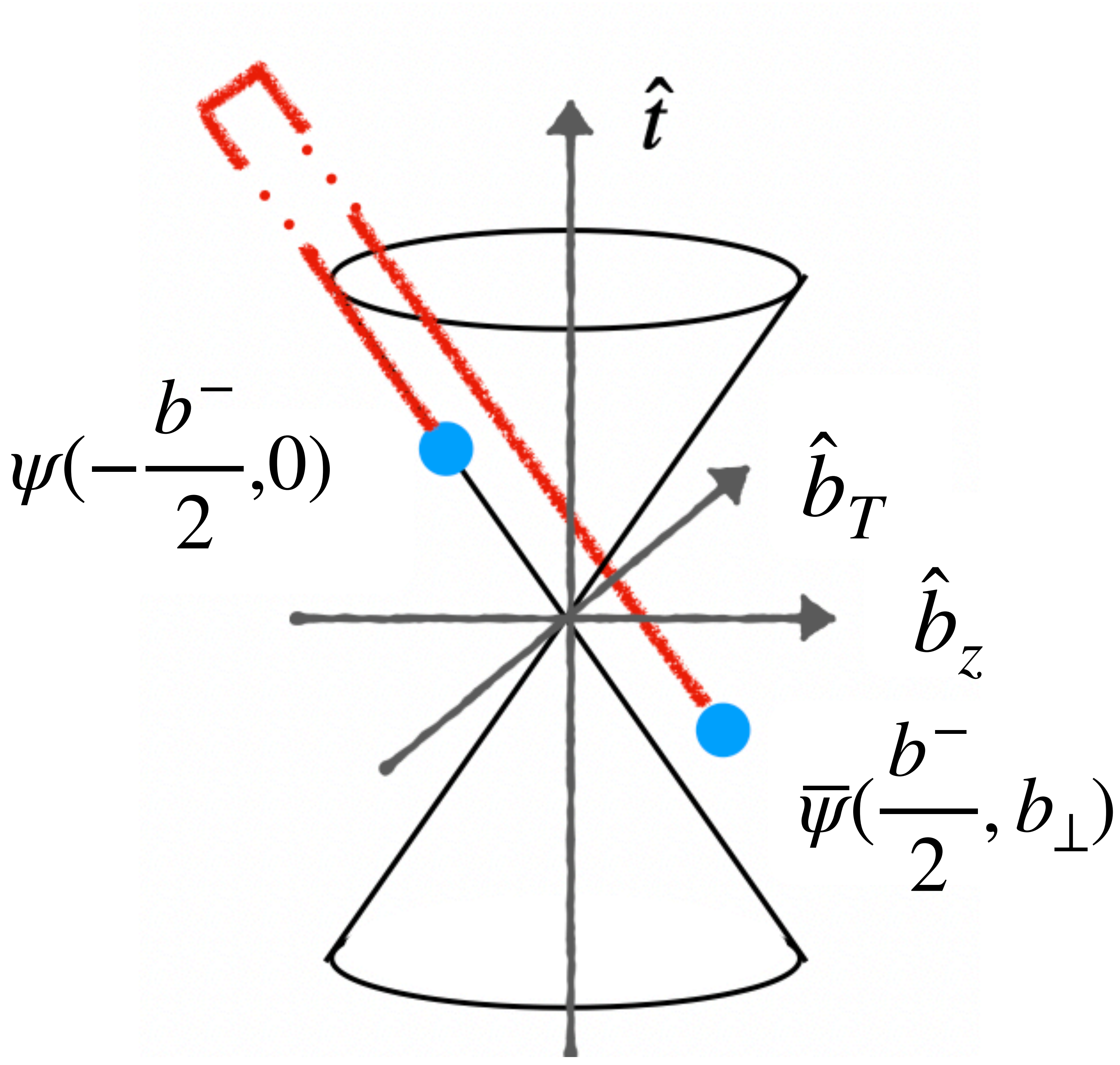
- In DIS, the physical quark $\psi(z)e^{iC(z)}$ represents a gauge-invariant object with a gauge link extended to infinity along the light-cone direction.

$$\psi^*(z) = \psi(z)e^{\left[-ig\int_{-z^-/2}^{\infty} dz\ A^+(z)\right]}$$



Parton distributions in the light-cone gauge

Light-cone TMD

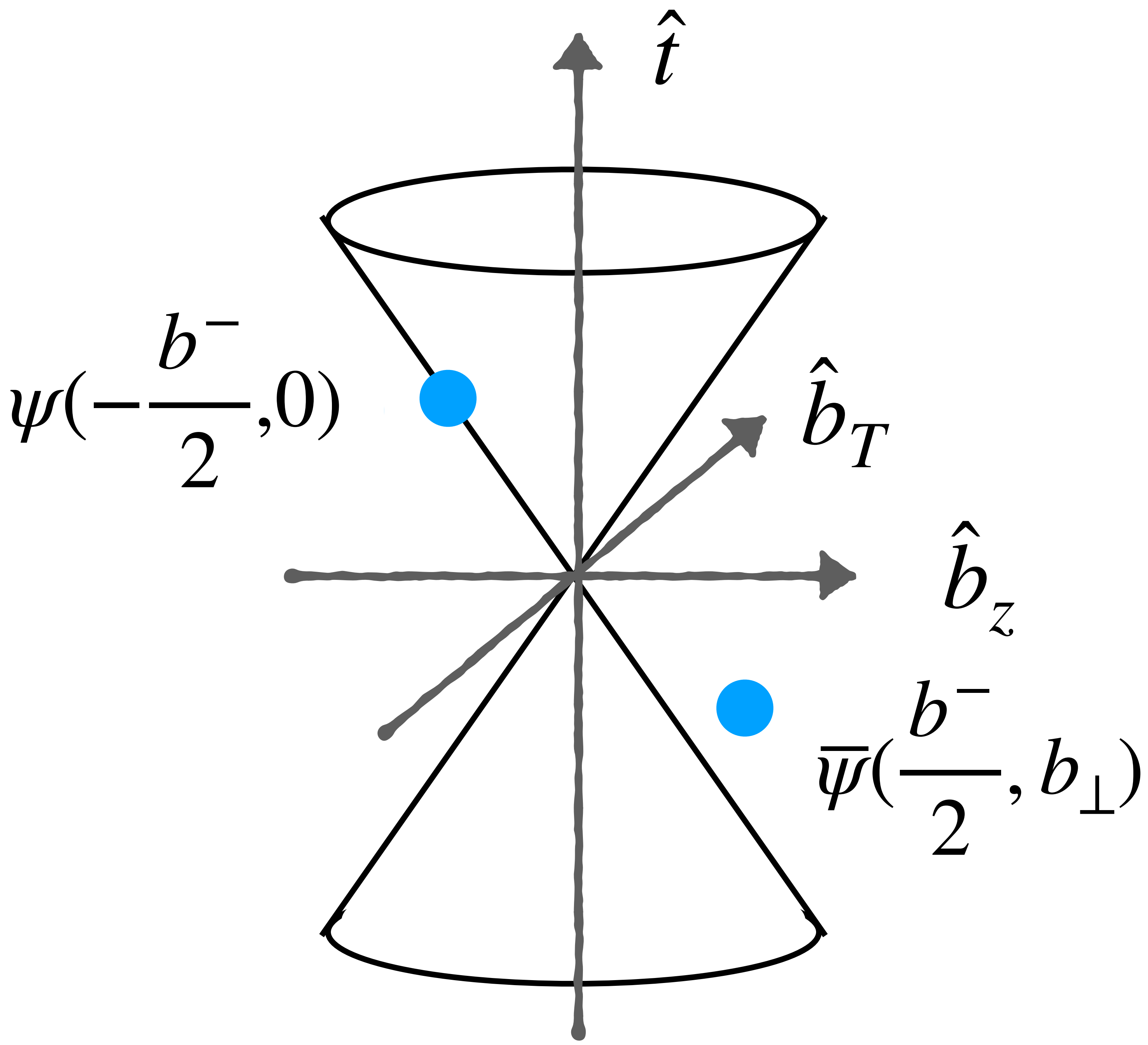


$$\overline{\psi}(\frac{b^-}{2},b_\perp)\Gamma {\color{red}W_{\sqsupset^+}}\psi(-\frac{b^-}{2},0)$$

Equivalent

=

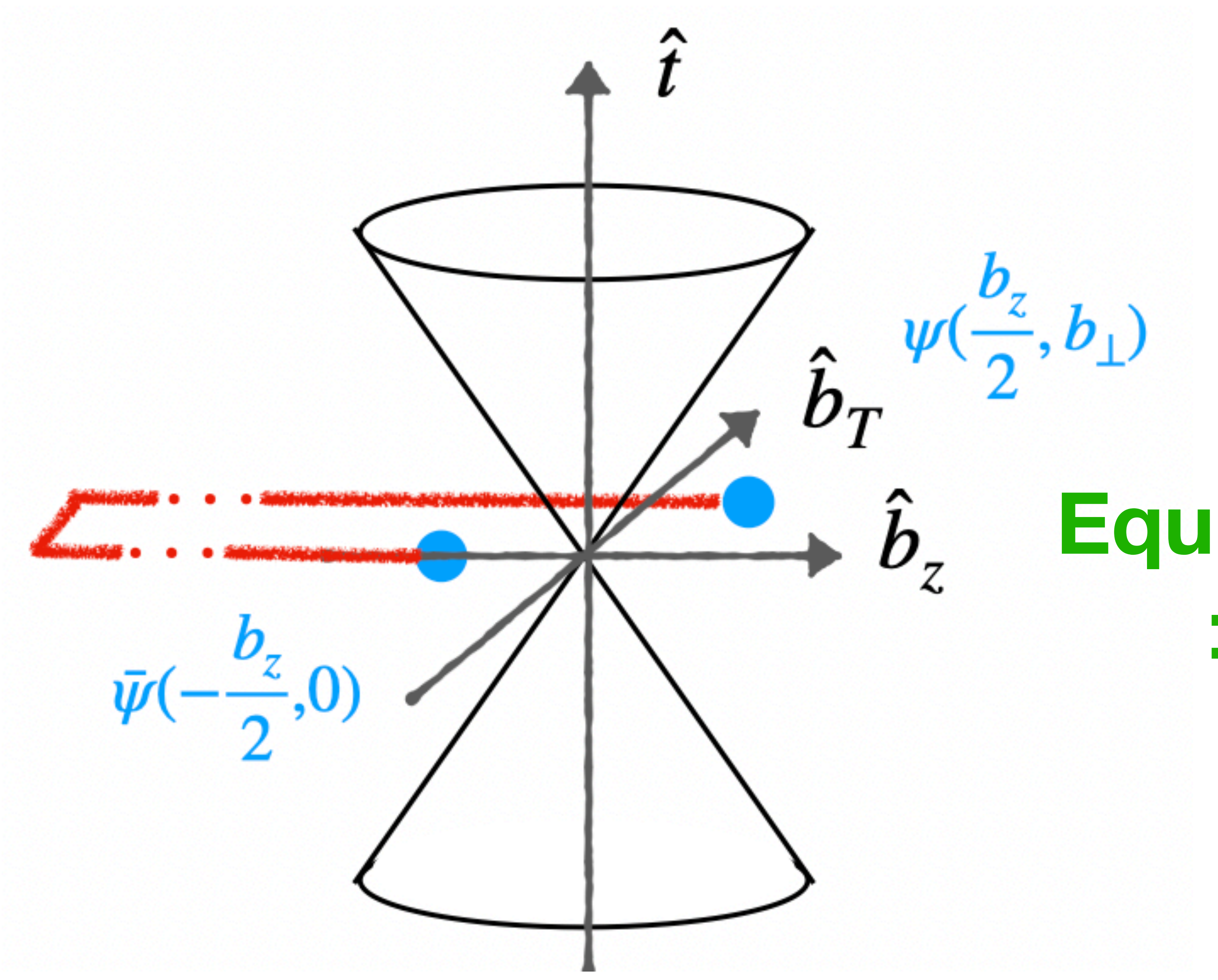
TMD in light-cone gauge $A^+=0$



$$\overline{\psi}(\frac{b^-}{2},b_\perp)\Gamma\psi(-\frac{b^-}{2},0)\,|_{A^+=0}$$

Conventional quasi distributions in axial-gauge

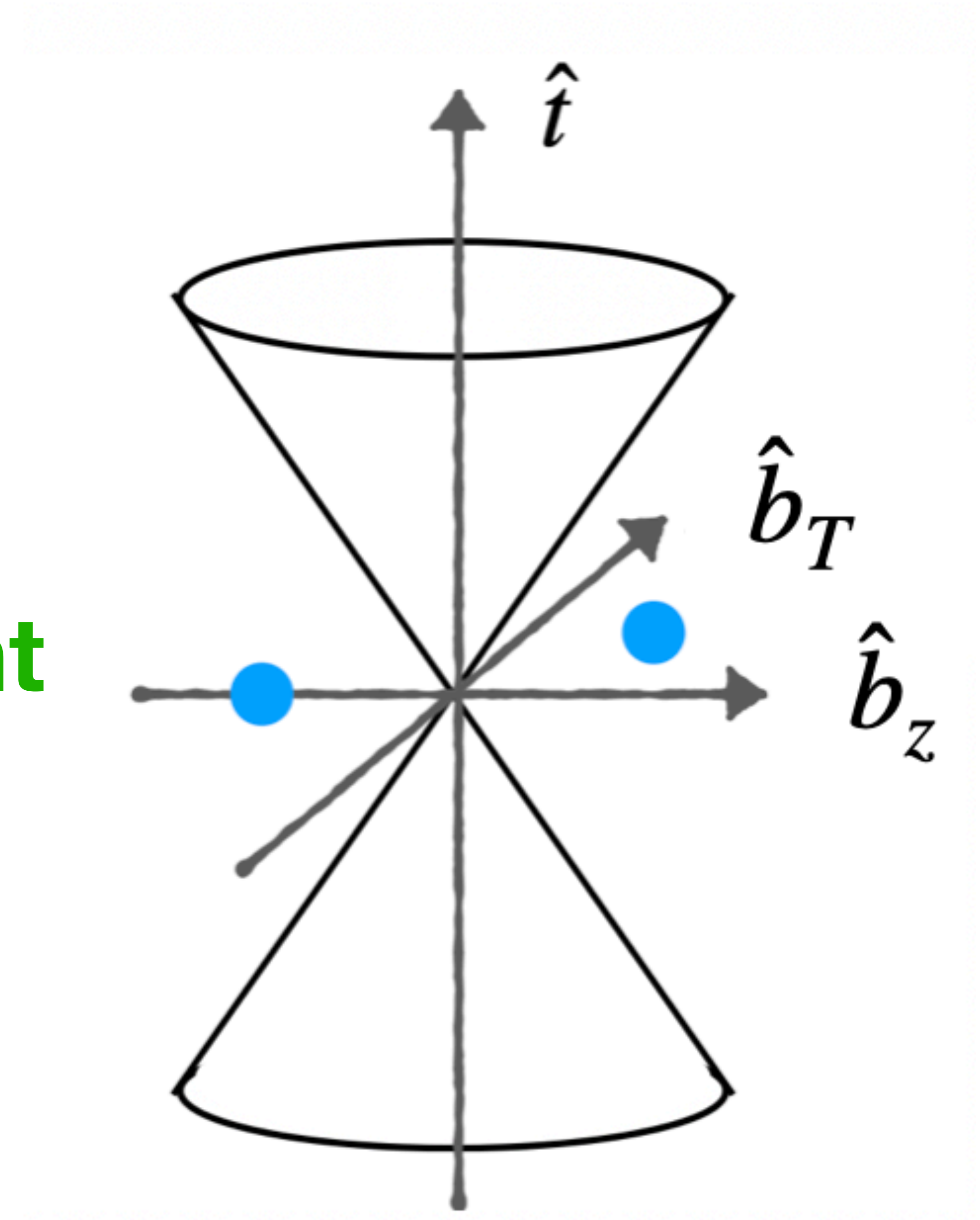
Gauge invariant Quasi-TMD



Equivalent

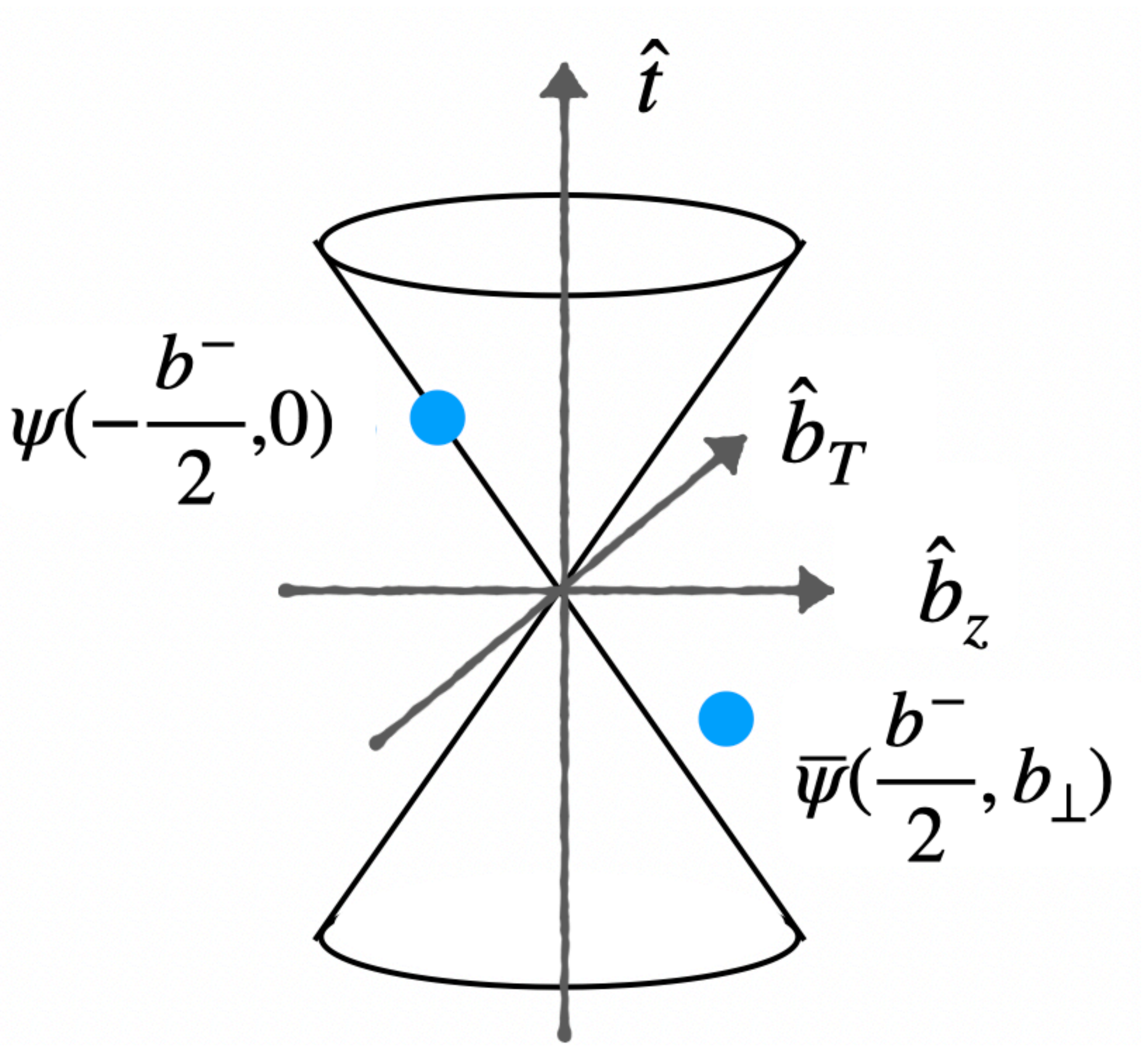
=

Quasi-TMD in axial gauge $A^z = 0$



$P_z \to \infty$

TMD in light-cone gauge $A^+ = 0$



$\overline{\psi}(\frac{b_z}{2}, b_\perp)\Gamma W_{\sqsupset z}\psi(-\frac{b_z}{2}, 0)$

$\overline{\psi}(\frac{b^z}{2}, b_\perp)\Gamma\psi(-\frac{b^z}{2}, 0)|_{A^z=0}$

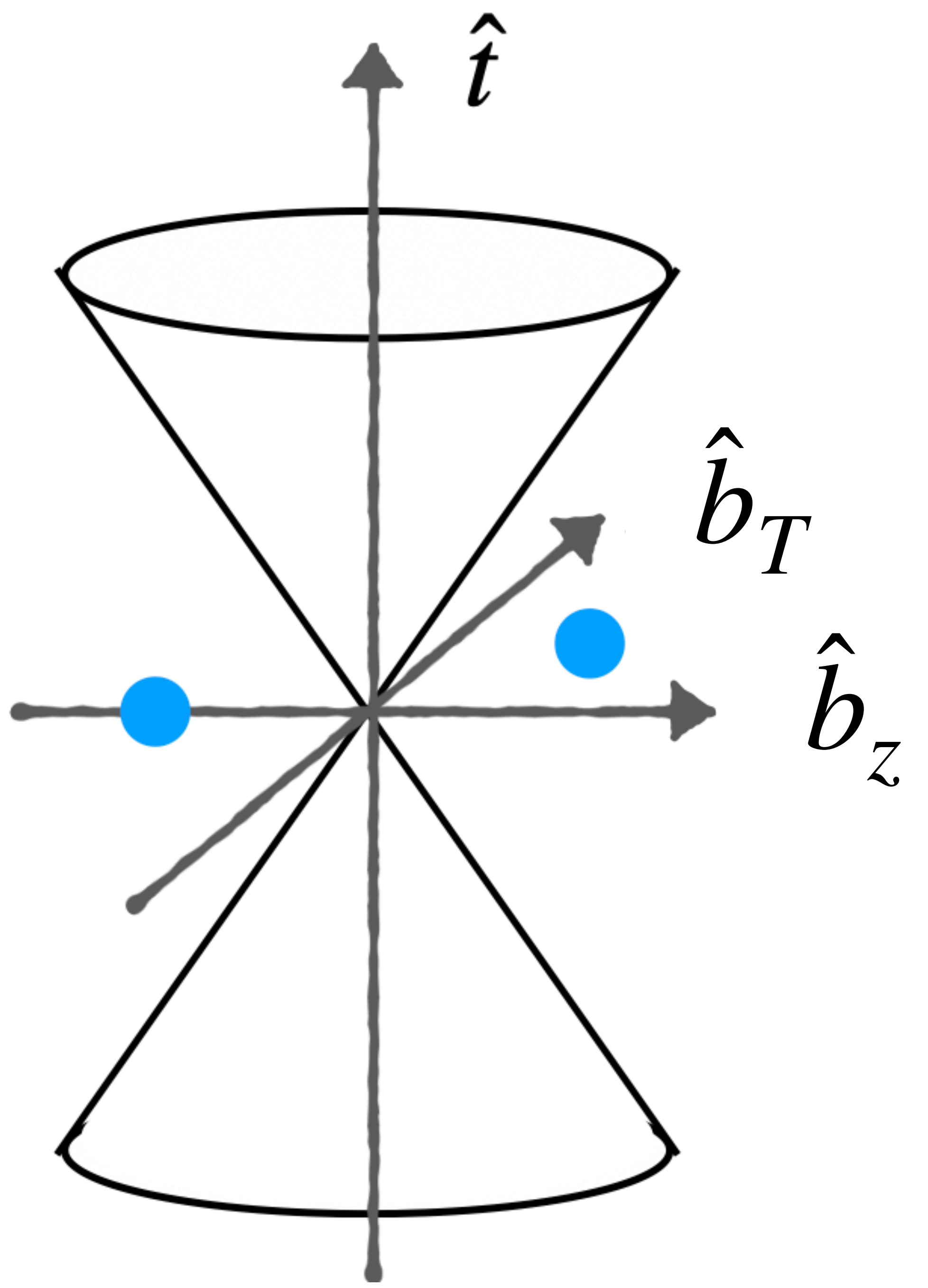
$\overline{\psi}(\frac{b^-}{2}, b_\perp)\Gamma\psi(-\frac{b^-}{2}, 0)|_{A^+=0}$

Universality in LaMET quasi distributions

Quasi-TMD in physical gauge:

$A^z = 0,\ A^0 = 0,$ **Coulomb gauge**

$\vec{\nabla} \cdot \vec{A} = 0$



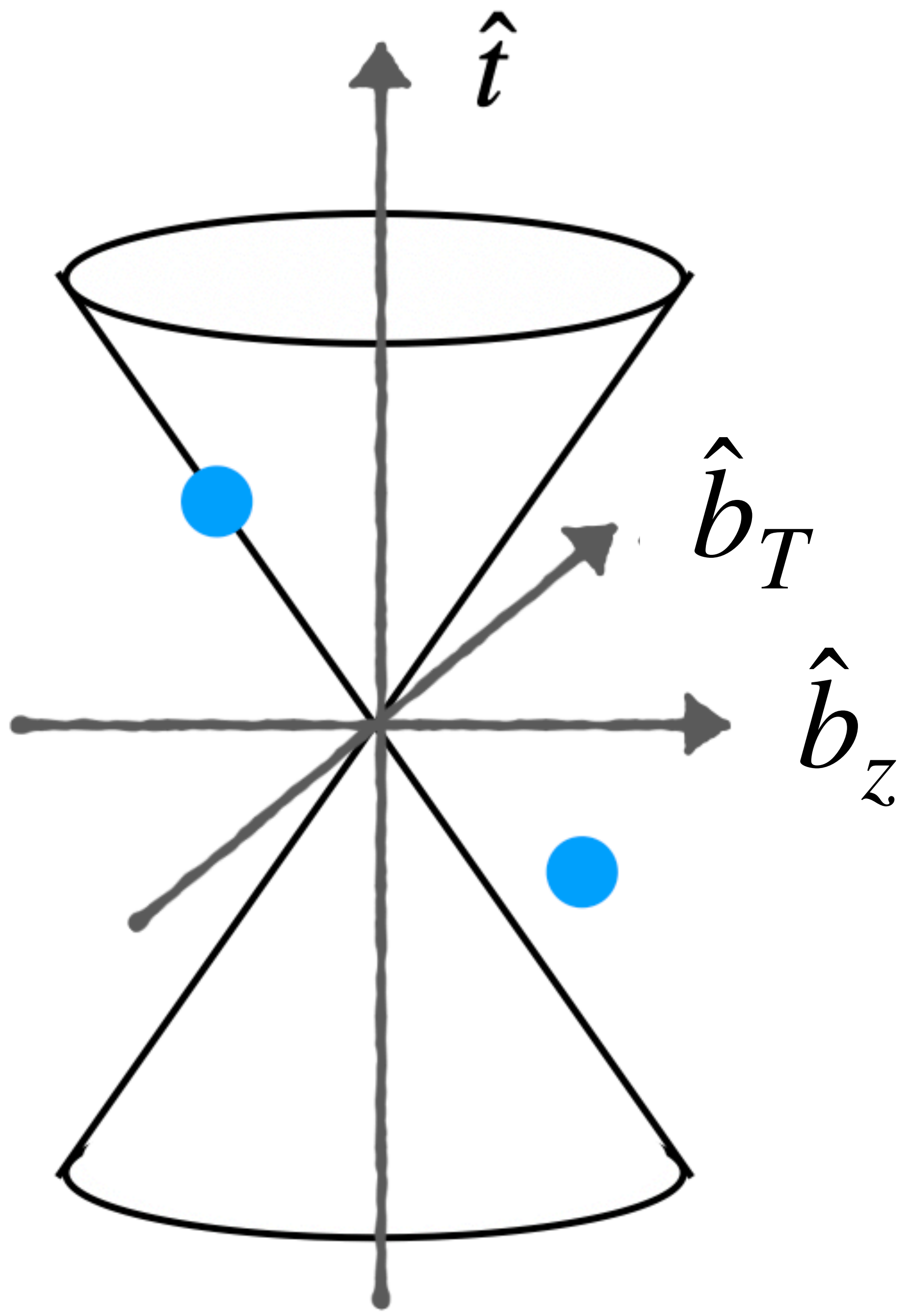
$\bar{\psi}(\frac{b^z}{2}, b_\perp)\Gamma\psi(-\frac{b^z}{2},0)\,|_{\vec{\nabla}\cdot\vec{A}=0}$

$\psi^{\rm CG}(z) \sim \psi(z) e^{i\frac{1}{\vec{\nabla}^2}\vec{\nabla}\cdot\vec{A}}$

$P_z \to \infty$

Physical gauge
$\to A^+$ **gauge**

Physical TMD in
light gauge $A^+ = 0$



$\bar{\psi}(\frac{b^-}{2}, b_\perp)\Gamma\psi(-\frac{b^-}{2},0)\,|_{A^+=0}$

- **XG**, W.-Y. Liu, Y. Zhao, PRD 109 (2024) 9, 094506
- Y. Zhao, PRL 133 (2024) 24, 241904

CG quasi distribution without Wilson lines

- **The quark field in the Coulomb gauge**

$$\psi_C(z) = U_C(z)\psi(z)$$

satisfying,

$$\vec{\nabla}\cdot\left[U_C\vec{A}U_C^{-1} + \frac{i}{g}U_C\vec{\nabla}U_C^{-1}\right] = 0$$

order by order in g, the solution:

$$U_C = \sum_{n=0}^{\infty}\frac{(ig)^n}{n!}\omega_n$$

$$\omega_1 = -\frac{1}{\nabla^2}\vec{\nabla}\cdot\vec{A},$$

$$\omega_2 = \frac{1}{\nabla^2}\left(\vec{\nabla}\cdot(\omega_1^\dagger\vec{\nabla}\omega_1) - [\vec{\nabla}\omega_1, \vec{A}]\right)$$

$\ldots$

▸ $P \to \infty$ **limit boost**

$$-\frac{1}{\nabla^2}\vec{\nabla}\cdot\vec{A} = i\int\frac{d^4k}{(2\pi)^4}e^{-ik\cdot z}\frac{1}{k_z^2+k_\perp^2}[k_zA_z(k) + k_\perp A_\perp(k)]$$

$$\approx i\int\frac{d^4k}{(2\pi)^4}e^{-ik\cdot z}\frac{k^+}{(k^+)^2+\epsilon^2}A^+(k)$$

$$= \frac{1}{2}\left[\int_{-\infty^-}^{z^-} + \int_{+\infty^-}^{z^-}\right]d\eta^-A^+ \equiv \frac{1}{\partial^+_{\text{P.V.}}}A^+(z)$$

Principle value prescription (P.V.) averaging over past and future.

Path-ordered integral

$$\frac{\omega_n}{n!} \to \left(\ldots\left(\frac{1}{\partial^+_{\text{P.V.}}}\left((\frac{1}{\partial^+_{\text{P.V.}}}A^+)A^+\right)A^+\right)\ldots A^+\right)$$

$$U_C \to \mathscr{P}\exp\left[-ig\int_{z^-}^{\mp\infty^-}dzA^+(z)\right] \equiv W(z^-, \mp\infty^-)$$

Infinite light-cone Wilson link

CG quasi distribution without Wilson lines

Quasi-TMDs in CG

Collins-Soper kernel

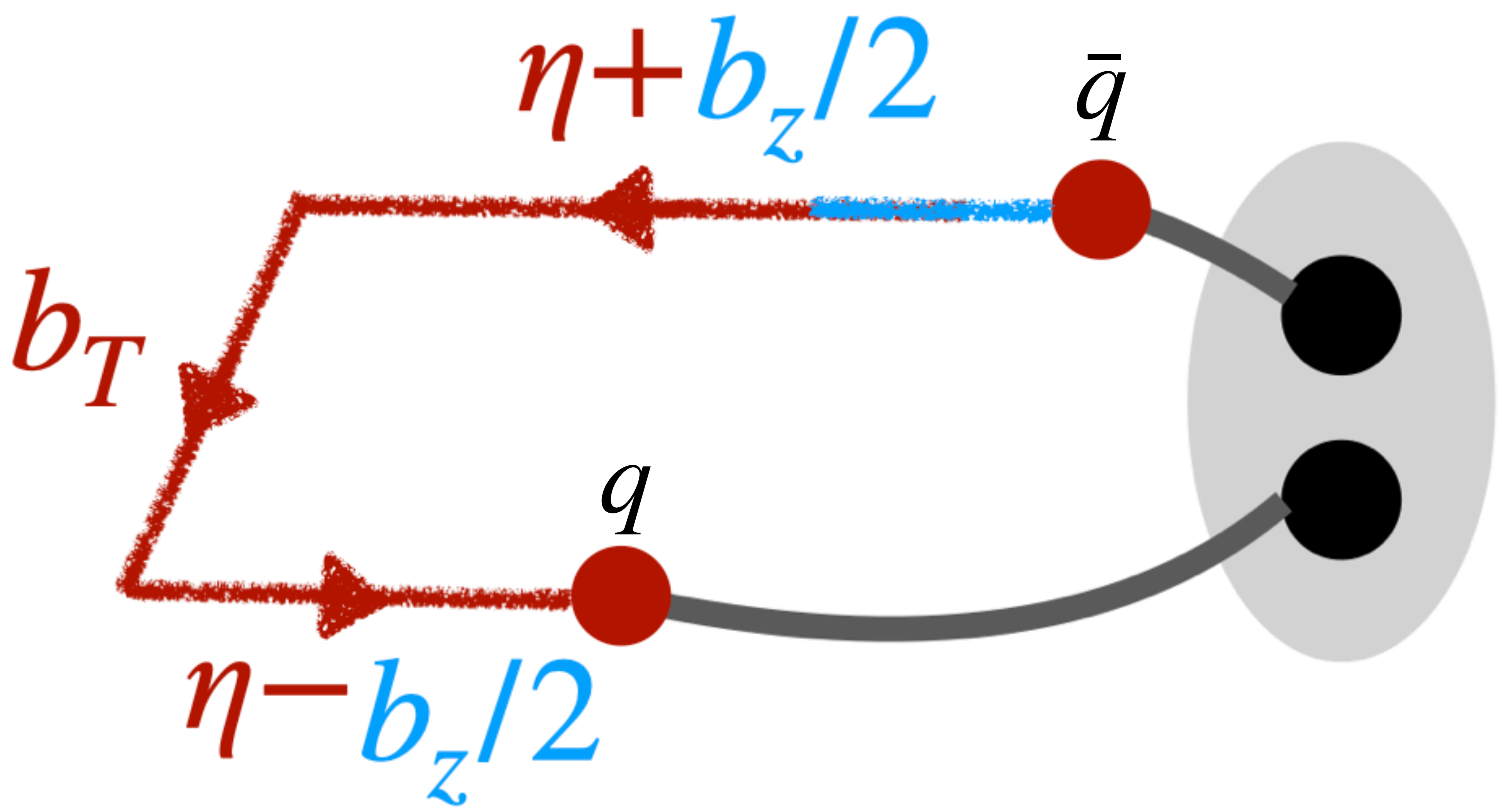
$$\frac{\tilde{f}_{\rm CG}(x,\vec{b}_T,\mu,P_z)}{\sqrt{S_{\rm CG}(\vec{b}_T,\mu)}} = C(\mu,xP_z)e^{\frac{1}{2}\gamma_\zeta(\mu,b_T)\ln\frac{(2xP_z)^2}{\zeta}}f(x,\vec{b}_T,\mu,\zeta)\{1+\mathcal{O}[\frac{1}{(xP_zb_T)^2},\frac{\Lambda_{\rm QCD}^2}{(xP_z)^2}]\}$$

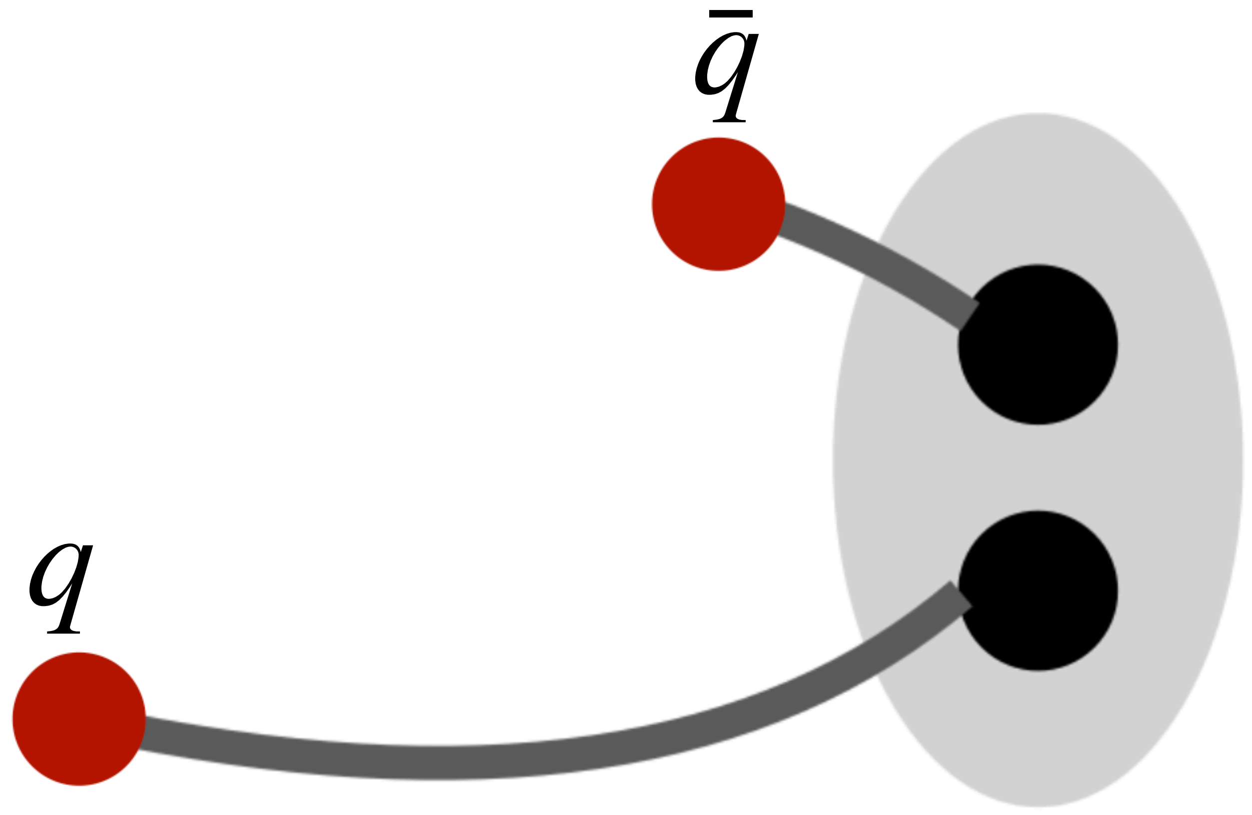
Physical TMD

- The same form of factorization formula as the conventional gauge invariant (GI) approach.
- Different perturbative and power corrections.

- **XG**, W.-Y. Liu, Y. Zhao, PRD 109 (2024) 9, 094506
- Y. Zhao, PRL 133 (2024) 24, 241904
- Y.-Z. Liu, Y.-S. Su., JHEP 02 (2024) 204

Quasi-TMDs: GI v.s. CG





$$\langle\Omega|\overline{\psi}(\frac{b_z}{2},b_\perp)\Gamma W_{\sqsupset^z}\psi(-\frac{b_z}{2},0)|\pi^+,P_z\rangle$$

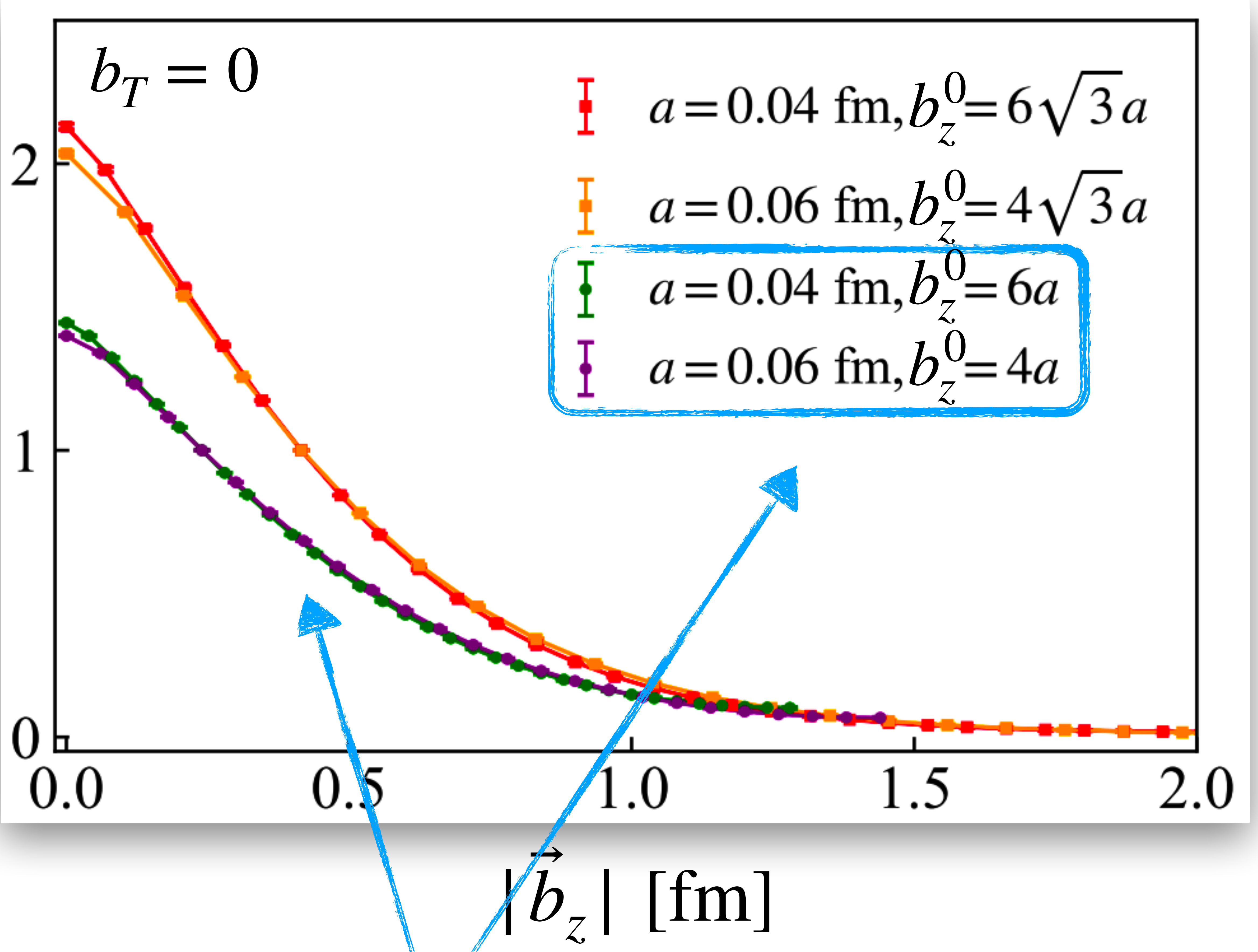
Gauge-invariant (GI)
quasi-TMDWF

$$\langle\Omega|\overline{\psi}(\frac{b_z}{2},b_\perp)\Gamma\psi(-\frac{b_z}{2},0)|_{\vec{\nabla}\cdot\vec{A}=0}|\pi^+,P_z\rangle$$

Coulomb gauge (CG)
quasi-TMDWF

CG quasi-TMDs: simplified renormalization

Renormalized matrix elements



Two lattice spacings: excellent continuum limit!

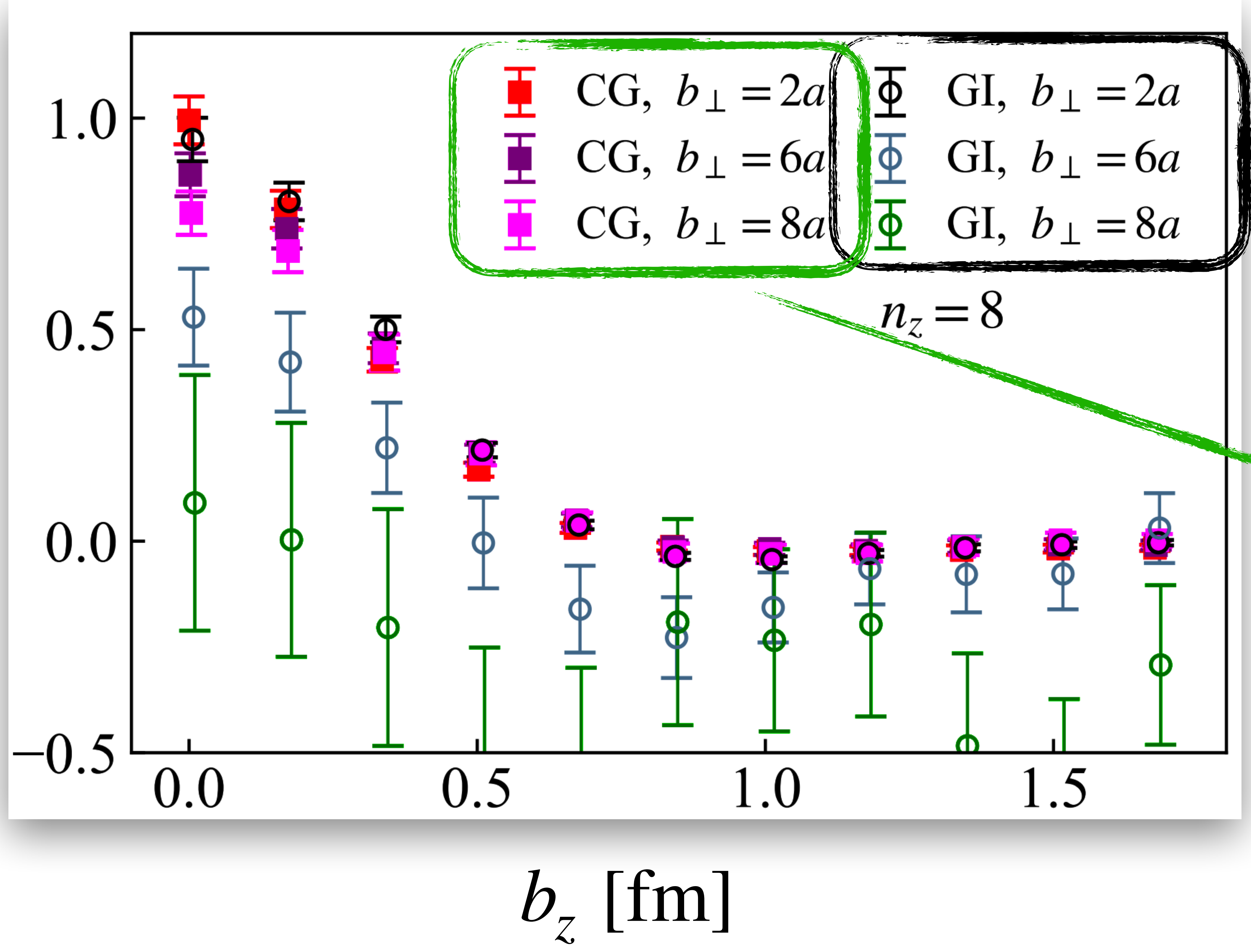
- No linear divergence: the renormalization is an overall constant.

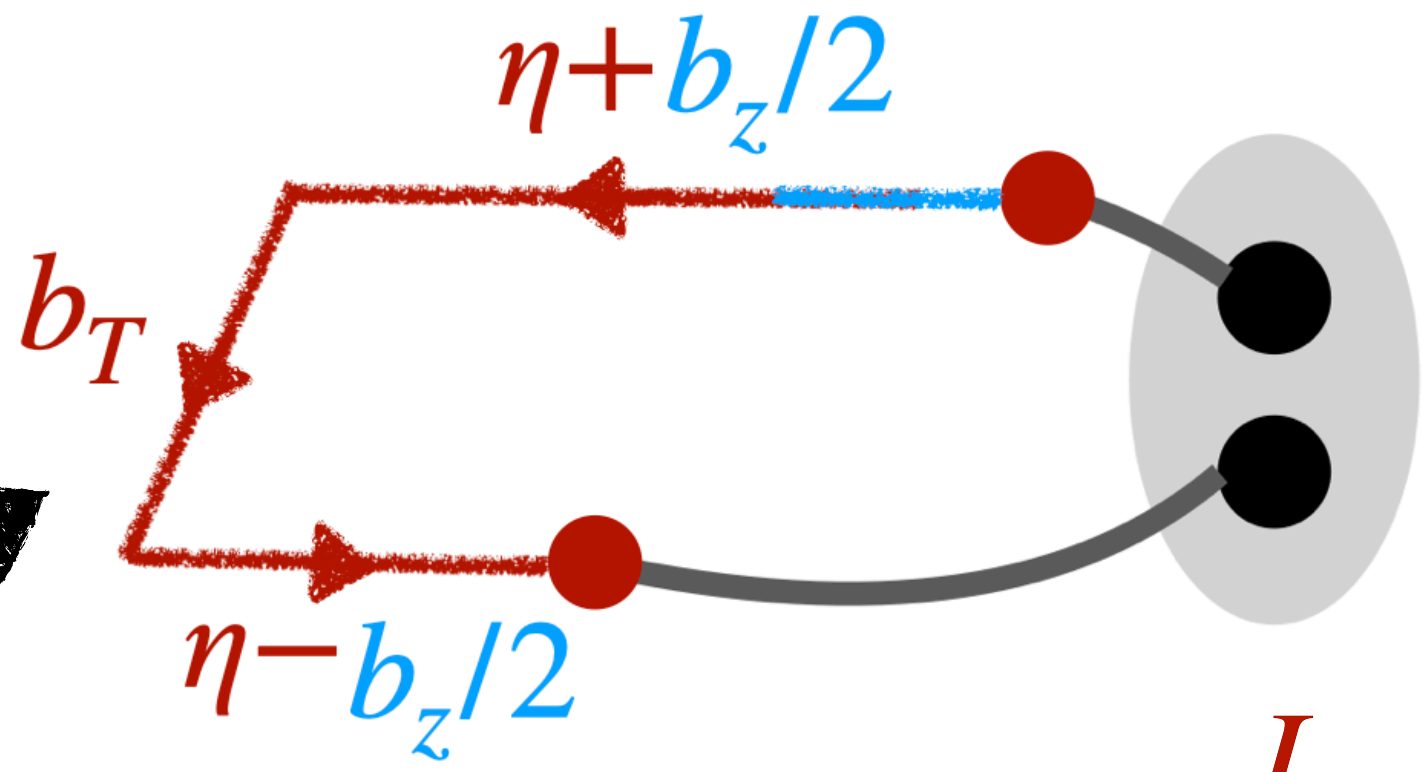
$$[\bar{\psi}(-\frac{\vec{b}}{2})\Gamma\psi(\frac{\vec{b}}{2})]_B = Z_\psi(a)[\bar{\psi}(-\frac{\vec{b}}{2})\Gamma\psi(\frac{\vec{b}}{2})]_R$$

• **XG**, W.-Y. Liu, Y. Zhao, PRD 109 (2024) 9, 094506

CG quasi-TMDs: enhanced long-range signal

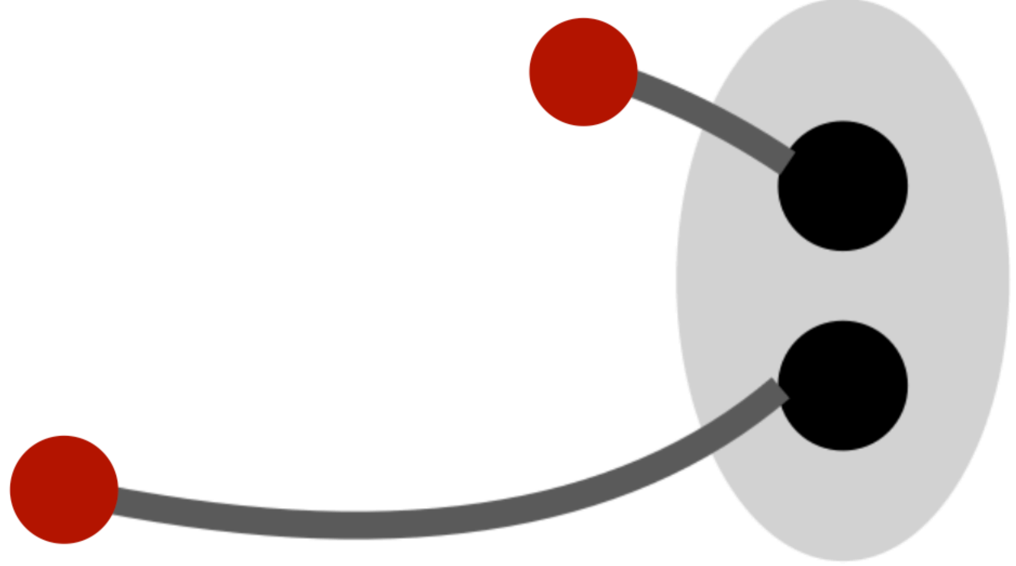
Renormalized matrix elements





$L_{\sqsubset^z} = 2\eta + b_T$

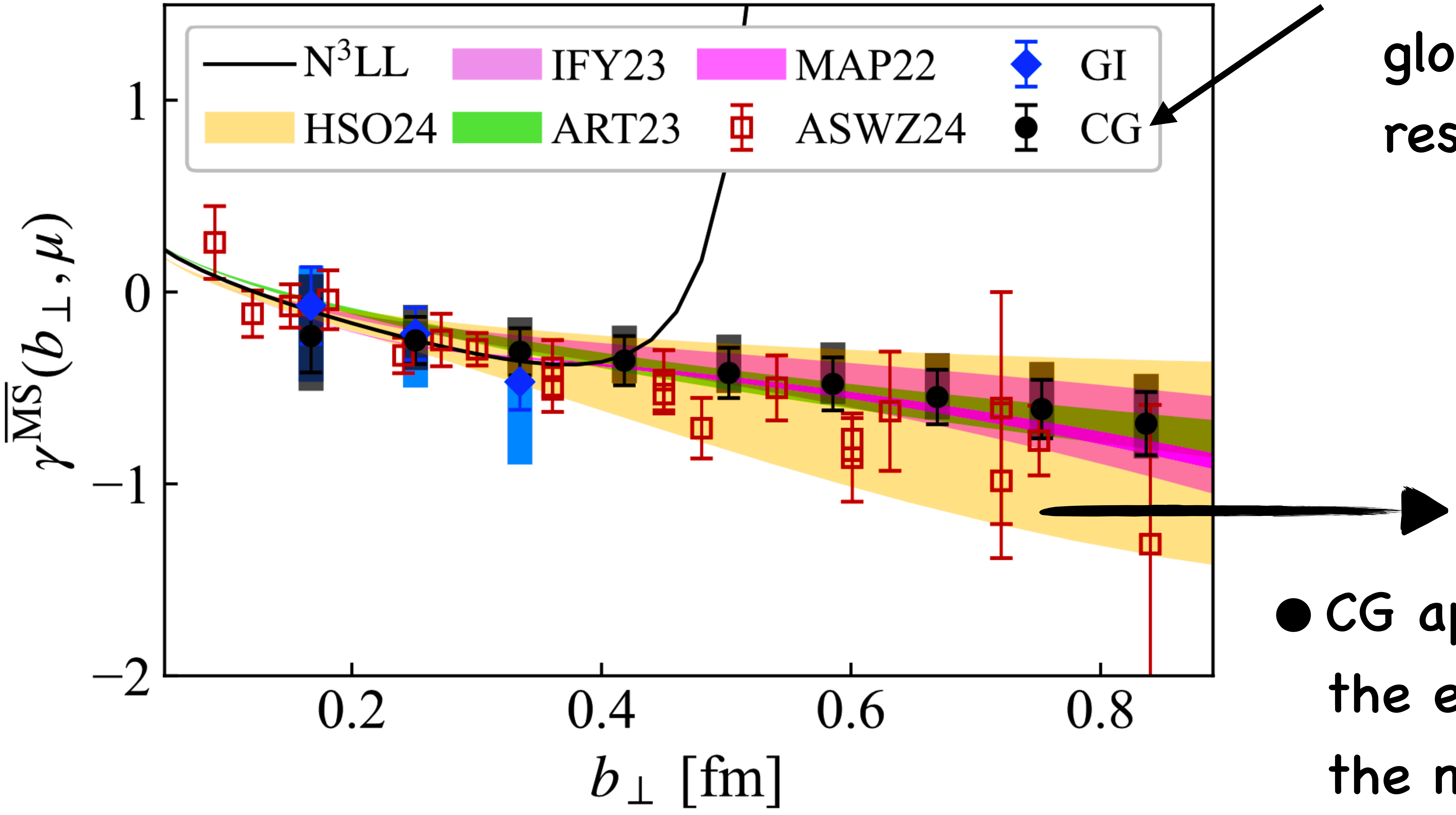
- b_T: $2a \to 6a \to 8a$, rapid signal decay.



- Much slower signal decay compared to the GI cases.

• D. Bollweg, **XG**, S. Mukherjee, Y. Zhao, PLB 852 (2024) 138617

The Collins-Soper kernel from CG quasi-TMDWF



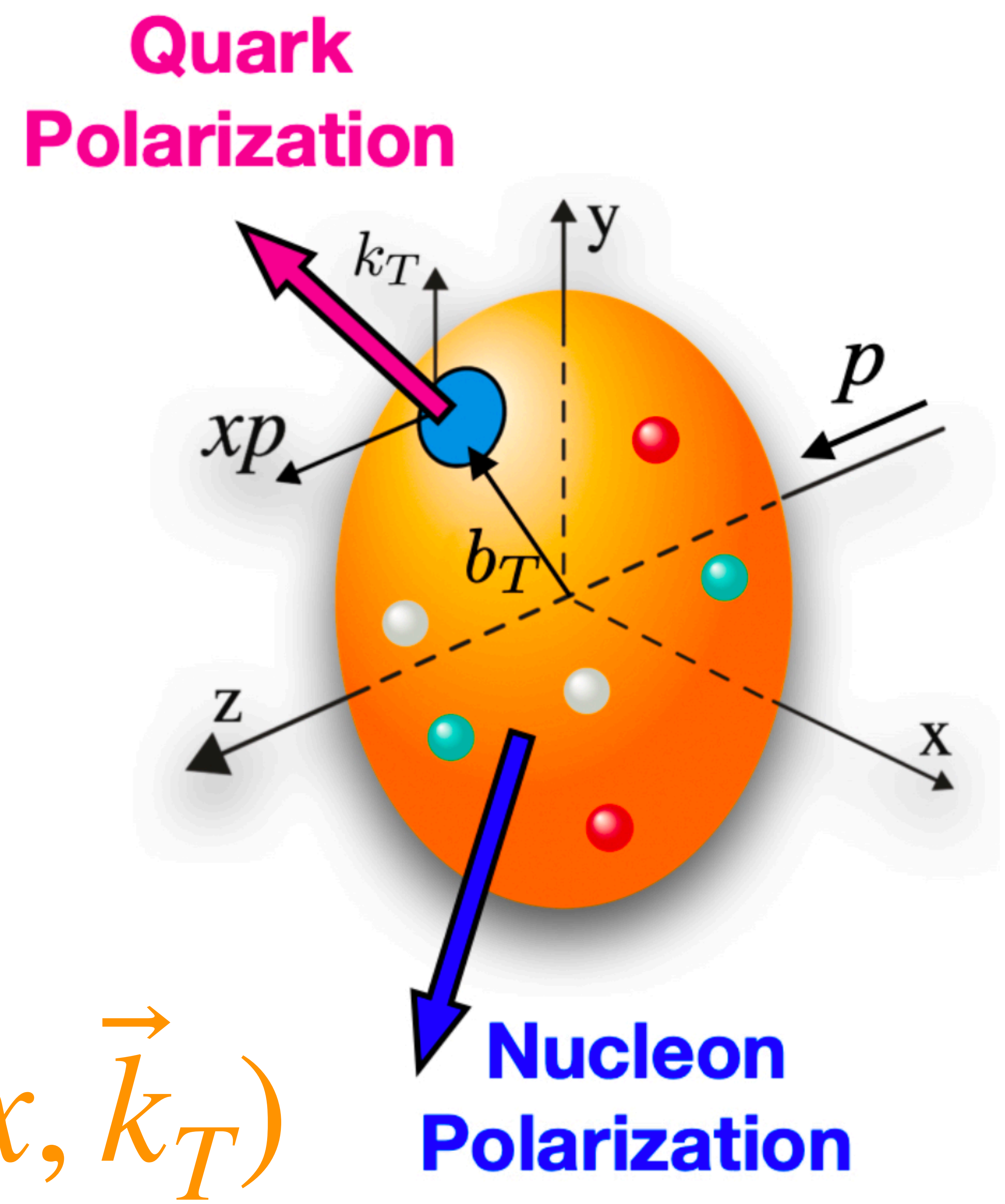
- CS kernel from CG quasi-TMDs: consistent with recent global fits, and lattice results from GI operators.
- CG approach greatly improve the efficiency: broader use in the nonperturbative regime of TMDs!

- D. Bollweg, **XG**, S. Mukherjee, Y. Zhao, PLB 852 (2024) 138617

Towards TMDPDFs of nucleon

- Leading-power ("twist-2") quark TMDs

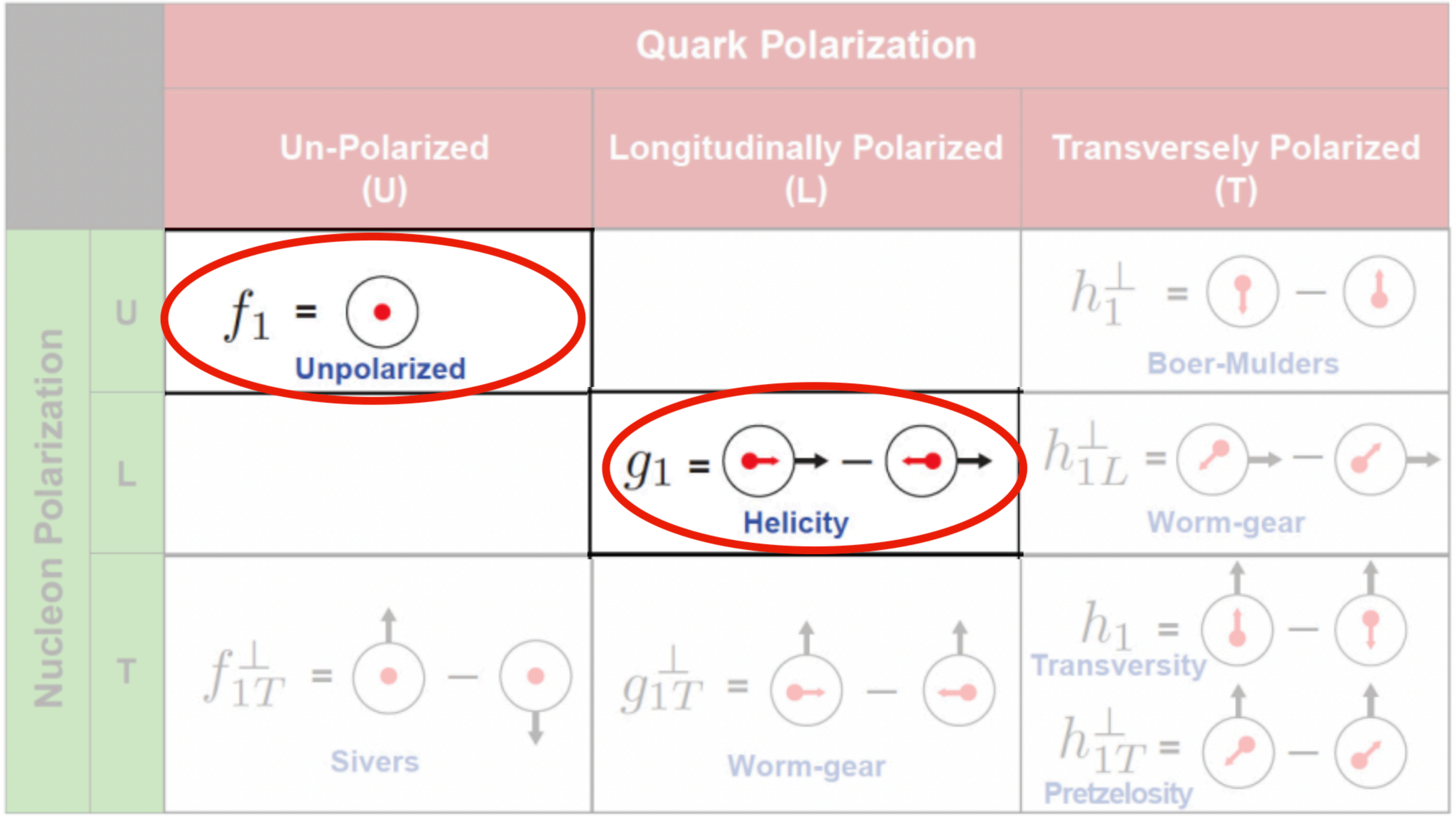
		Quark Polarization		
		Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	f_1 = Unpolarized		$h_1^\perp$ = − Boer-Mulders
	L		g_1 = − Helicity	$h_{1L}^\perp$ = − Worm-gear
	T	$f_{1T}^\perp$ = − Sivers	$g_{1T}^\perp$ = − Worm-gear	h_1 = − Transversity; $h_{1T}^\perp$ = − Pretzelosity



$f(x, \vec{k}_T)$

- 3D image: longitudinal momentum fraction x and confined motion $\vec{k}_T$.
- Nucleon spin structure: orbital motion, Spin-orbit correlations…

Unpolarized and helicity TMDPDFs from lattice



Quasi-TMD Beam functions

$$\tilde{h}(b_T, b_z, P_z, \mu)$$

$$= \langle \lambda; P_z | \bar{\psi}(b_T, \frac{b_z}{2}) \Gamma \psi(0, -\frac{b_z}{2}) |_{\vec{\nabla} \cdot \vec{A}=0} | \lambda; P_z \rangle$$

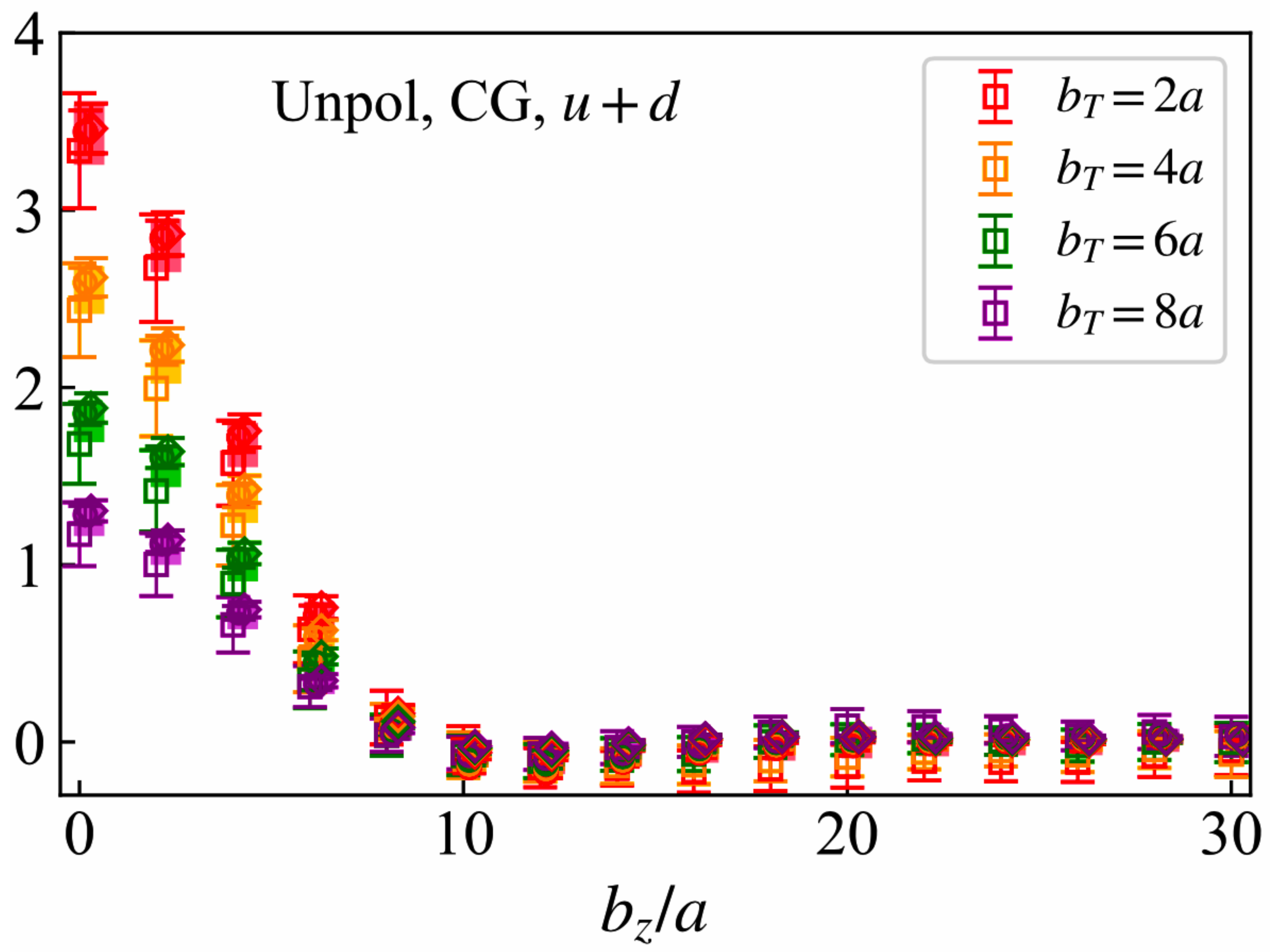
- Unpolarized TMDs: $\Gamma = \gamma^t$, $\psi = u - d$ and $u + d$ (disconnected diagrams ignored)
- Helicity TMDs: $\Gamma = \gamma^z \gamma^5$, $\psi = u - d$

Lattice setup:

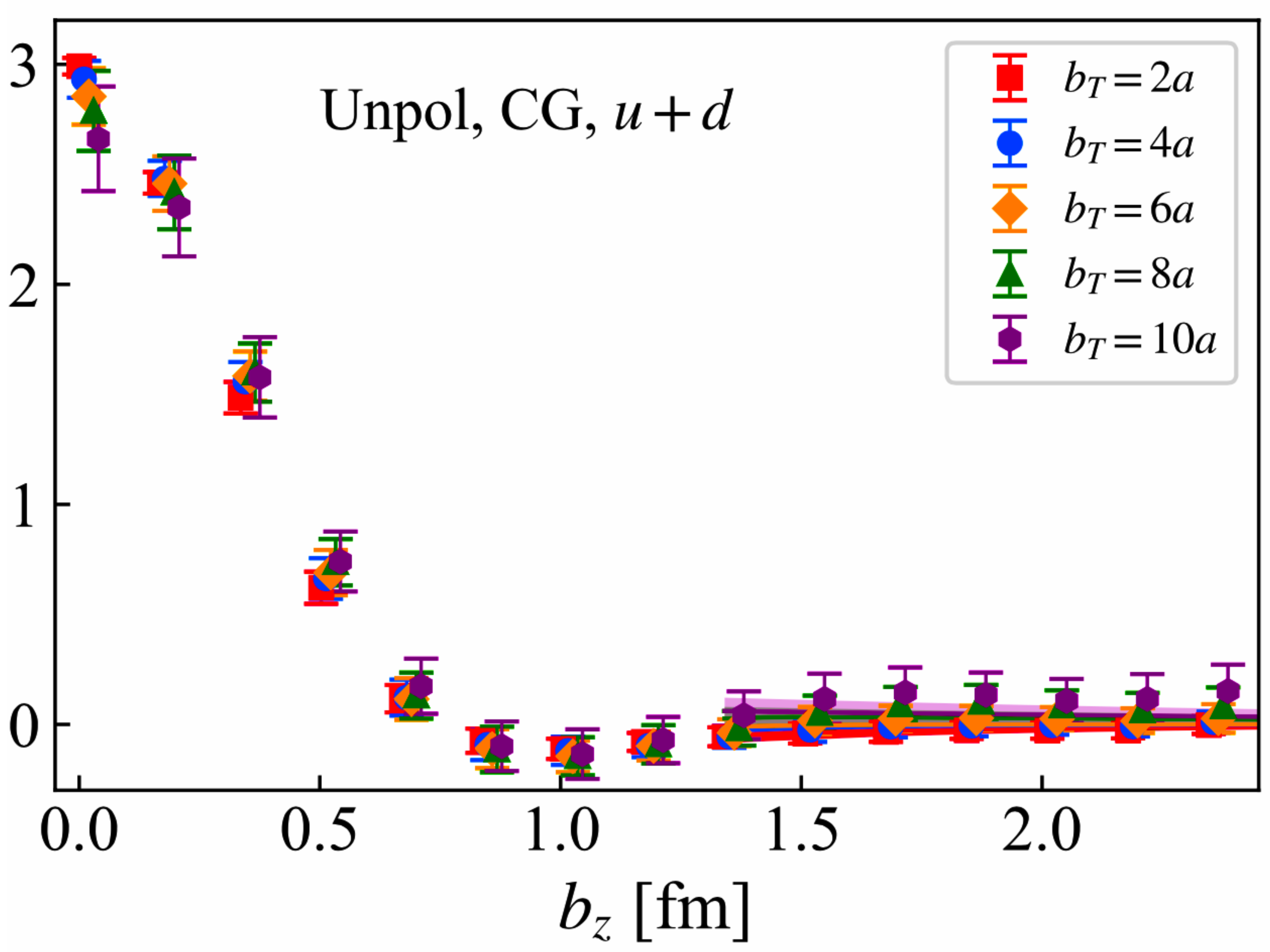
- 2+1 flavor Domain-wall (chiral) fermion discretization.
- Physical quark masses, $64^3 \times 128$ lattice with spacing $a = 0.084$ fm.
- Nucleon momentum up to $P_z = 1.62$ GeV, b_T up to 1 fm.

Quasi-TMD beam functions from lattice

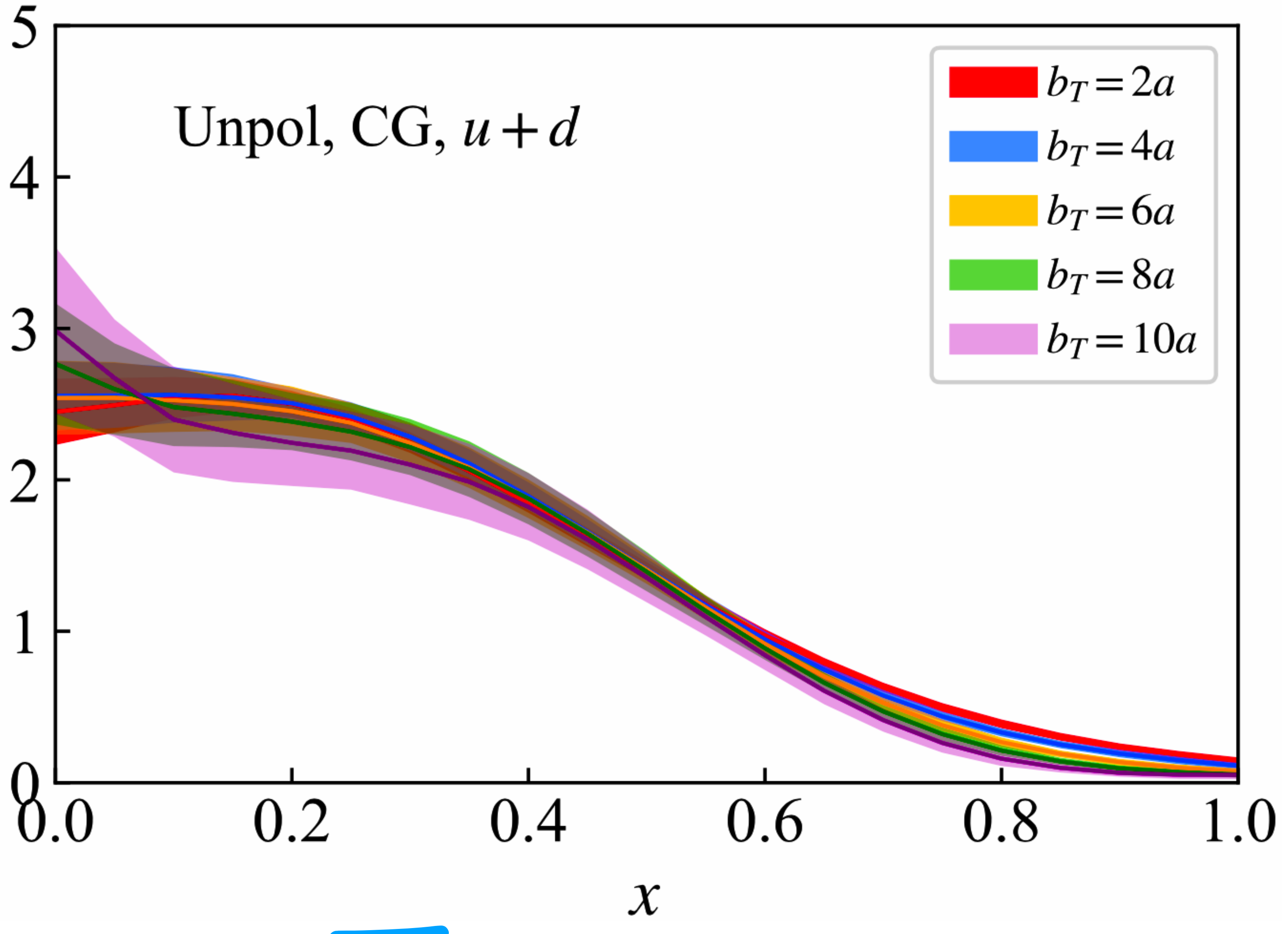
- Bare matrix elements



- Renormalized matrix elements



- Quasi-TMD $\tilde{f}_1(x, b_T, P_z)$



Renormailzation

$$b_z \xrightarrow{\text{F.T.}} x$$

Ratio of TMDPDFs from quasi-TMD beam functions

Quasi-TMD   Physical TMD

$$\frac{\tilde{f}(x, \vec{b}_T, \mu, P_z)}{\sqrt{S_r(\vec{b}_T, \mu)}} = C(\mu, xP_z) e^{\frac{1}{2}\gamma_\zeta(\mu, b_T)\ln\frac{(2xP_z)^2}{\zeta}} f(x, \vec{b}_T, \mu, \zeta)\{1 + \mathcal{O}[\frac{1}{(xP_z b_T)^2}, \frac{\Lambda^2_{\mathrm{QCD}}}{(xP_z)^2}]\}$$

Constructing ratios cancels soft factor.

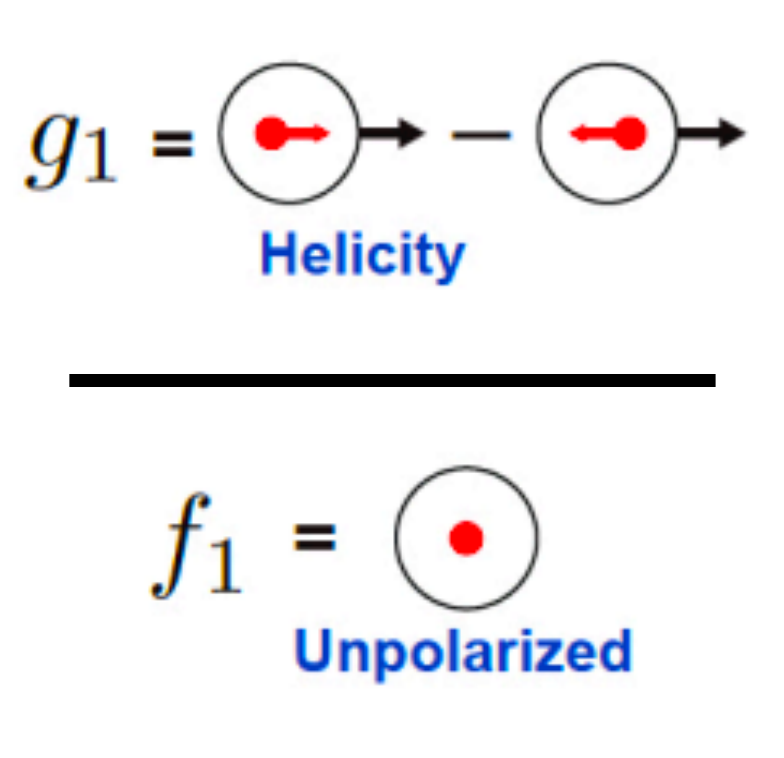
$$\frac{\tilde{f}_1(x, b_T, P_z, \mu)}{\tilde{f}_2(x, b_T, P_z, \mu)} = \frac{f_1(x, b_T, \zeta, \mu)}{f_2(x, b_T, \zeta, \mu)} + \mathrm{p.c.}$$

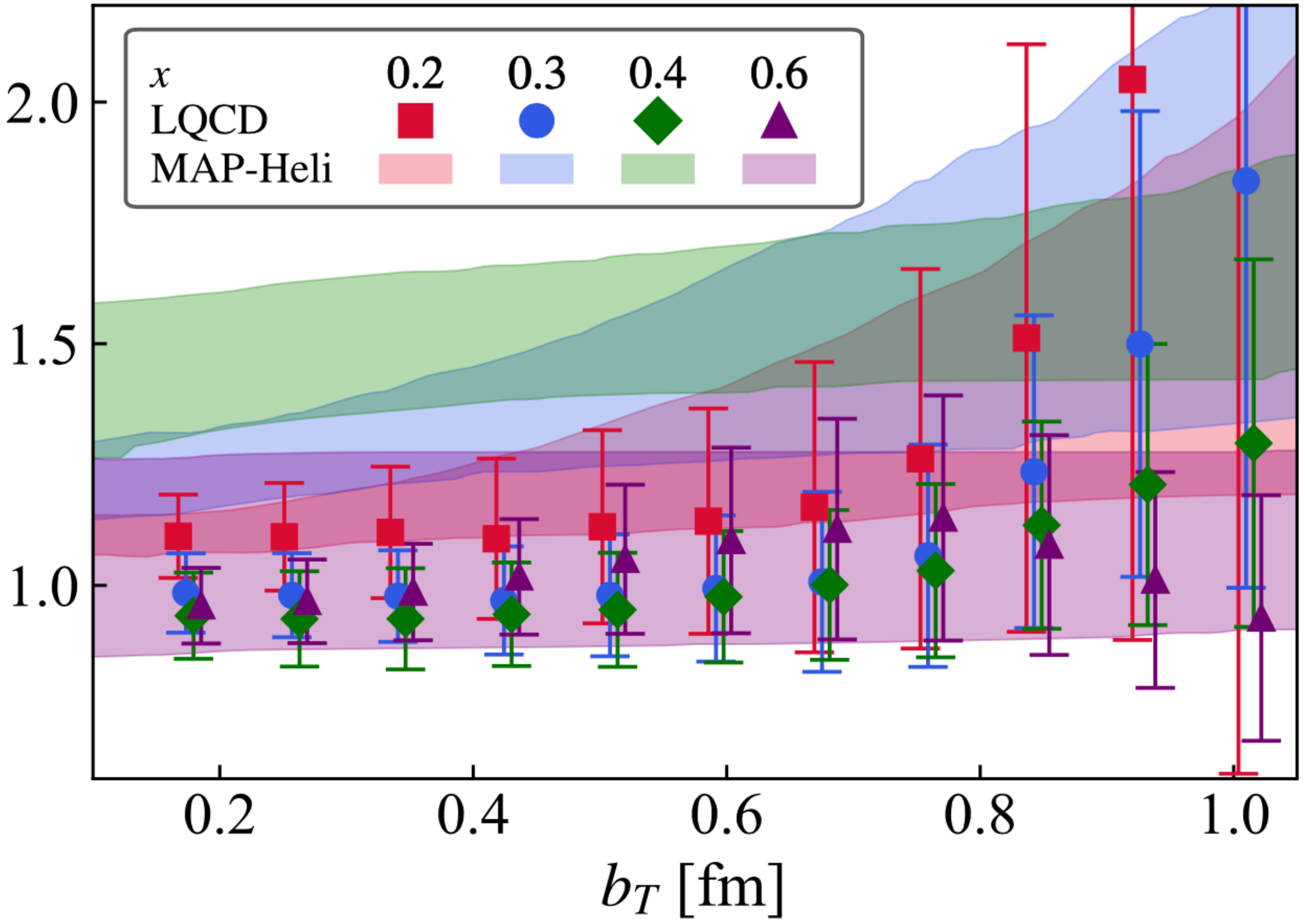
Quasi-TMD beam functions   Physical TMDPDFs

- Perturbative corrections and scale evolution also cancels:
  - renormalization-group-invariant (RGI) ratios.
  - valid to all orders in perturbation theory.

Ratios between $u-d$ heli. and unpol. TMDPDFs

$$\frac{g_{1L}^{\Delta u^+-\Delta d^+}(x,b_T)}{f_1^{u_v-d_v}(x,b_T)}\frac{1}{g_A}=\frac{\tilde{g}_{1L}^{\Delta u^+-\Delta d^+}(x,b_T)}{\tilde{f}_1^{u_v-d_v}(x,b_T)}\frac{1}{g_A}$$



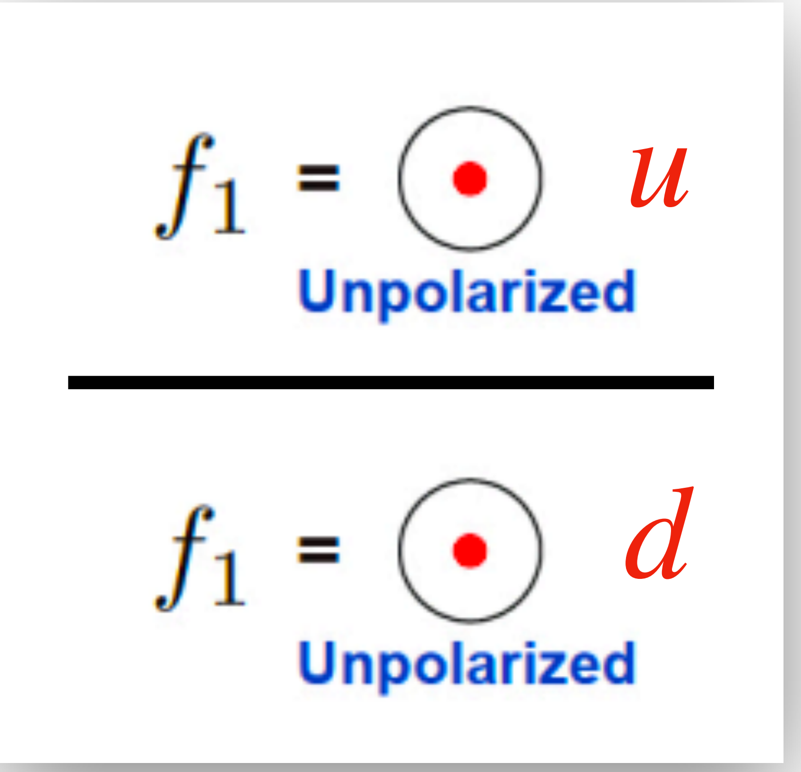


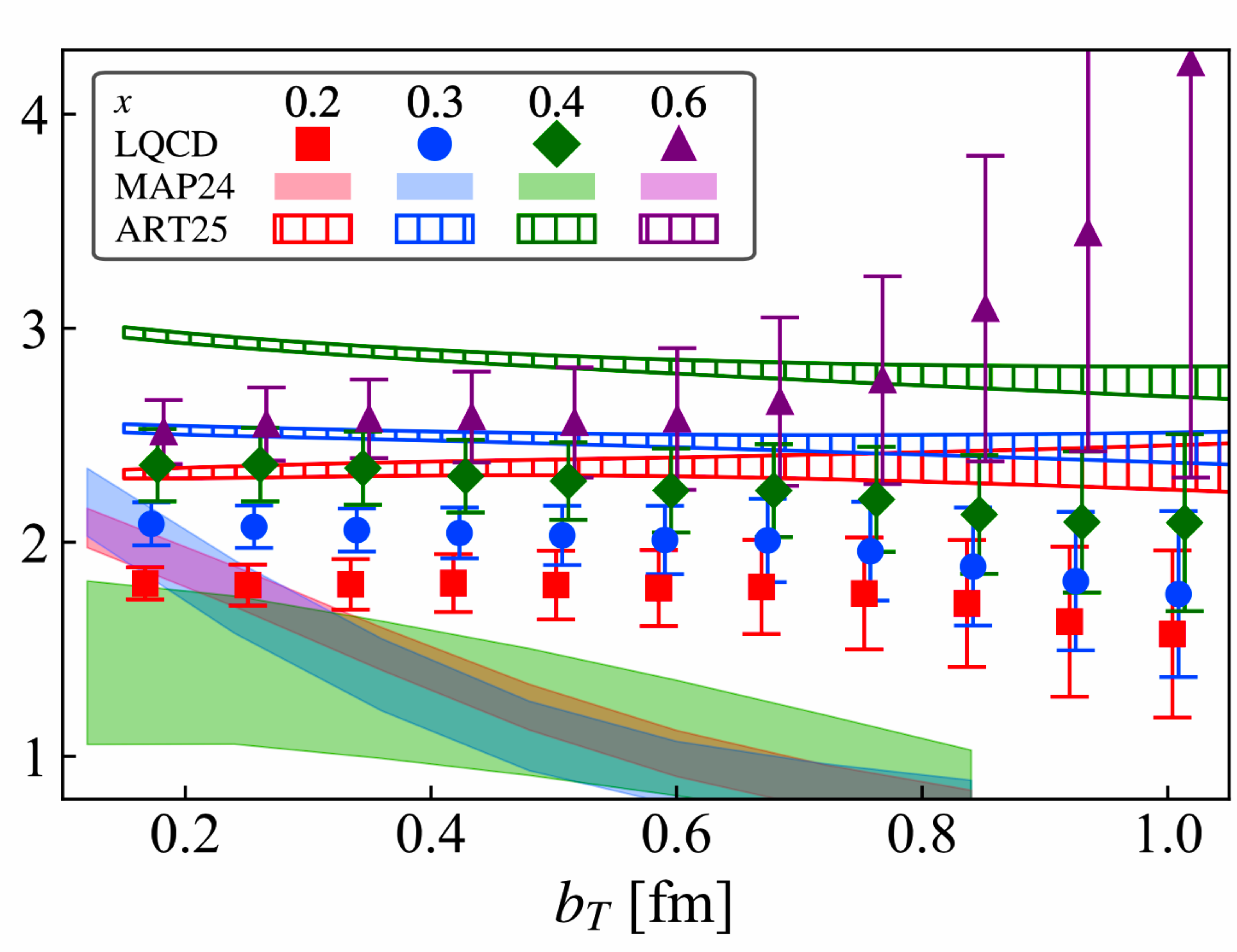
- No strong dependence on b_T: longitudinal spin polarization has limited impact on the intrinsic transverse motion of quark inside nucleon.

• D. Bollweg, **XG**, S. Mukherjee, Y. Zhao, arXiv: 2505.xxxxx

Ratios between valence $u-$ and $d-$unpolarized TMDs

$$\frac{f_1^{u_v}(x,b_T)}{f_1^{d_v}(x,b_T)} = \frac{\tilde{f}_1^{u_v}(x,b_T)}{\tilde{f}_1^{d_v}(x,b_T)}$$



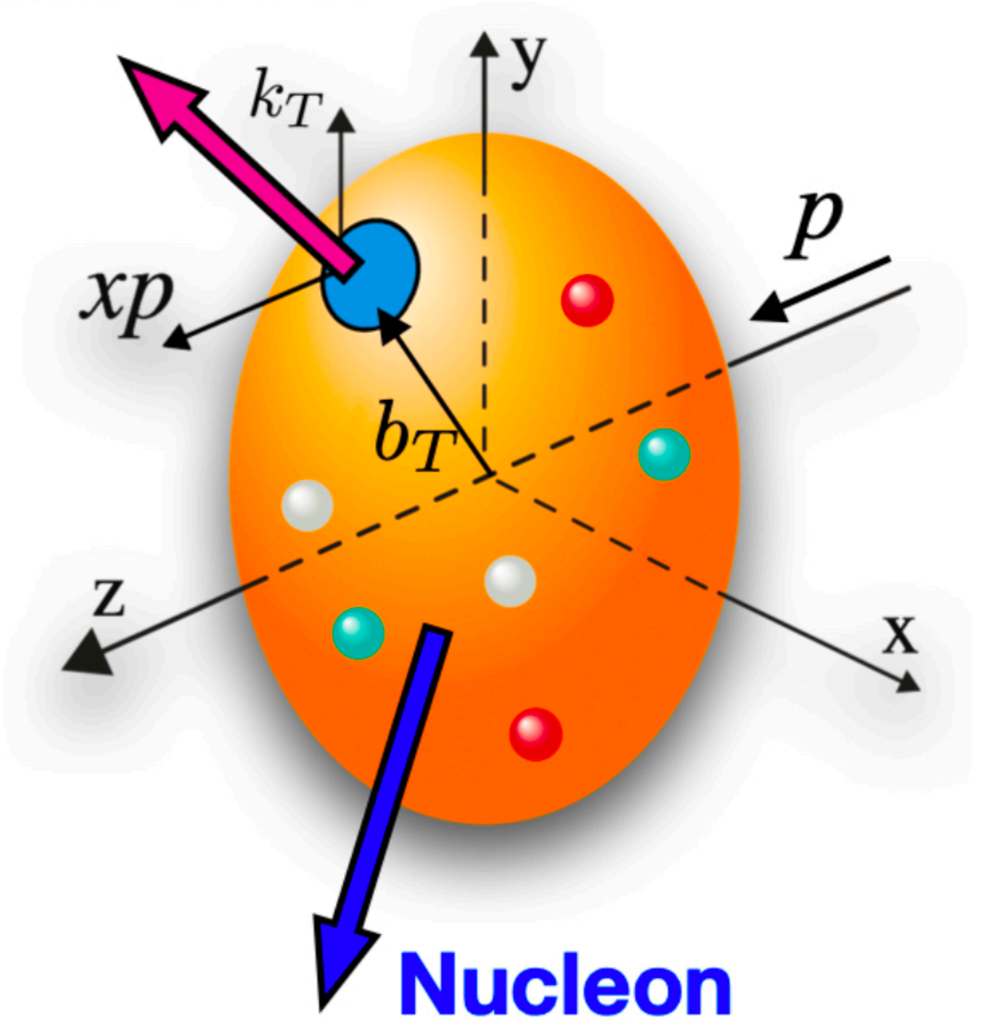


- Weak b_T dependence also observed.
- Lattice results could provide a first-principles benchmark for global fit in less constrained regions

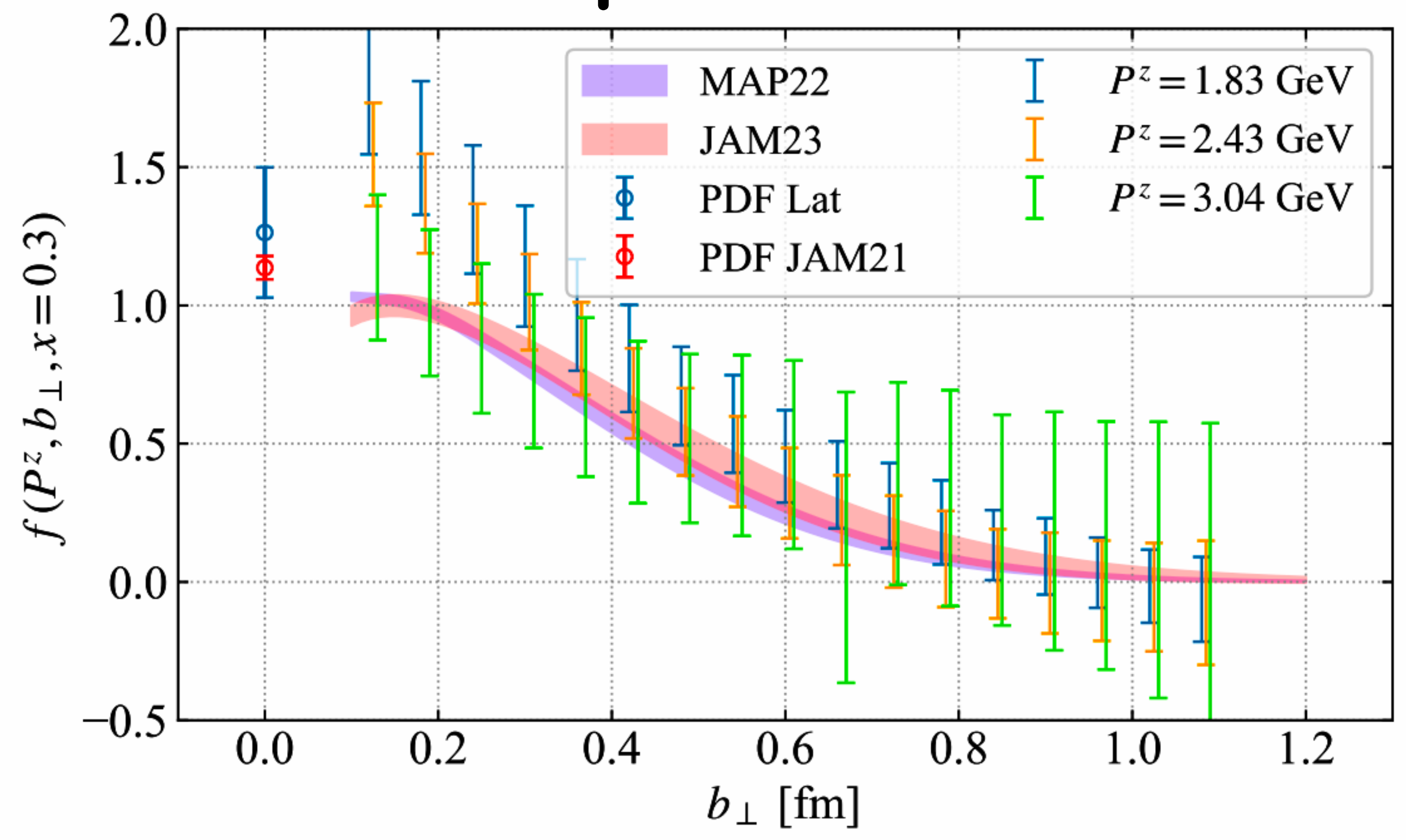
• D. Bollweg, **XG**, S. Mukherjee, Y. Zhao, arXiv: 2505.xxxxx

Summary & outlook

- The parton distributions can be extracted from boosted quasi distributions in the Coulomb gauge.
- The CG methods have great advantages in enhanced long-range precision.
- We have extracted the CS kernels from the evolution of quasi-TMDs.
- We calculated the ratios between several nucleon TMDs.
- When combined with the soft factor, the full TMDs can be determined.



Pion unpolarized TMD



Thanks for your attention!

• D. Bollweg, **XG**, Jinchen He, S. Mukherjee, Y. Zhao, arXiv: 2504.04625