Accessing the Nucleon Gluon Momentum Fraction using the Gradient Flow on the Lattice

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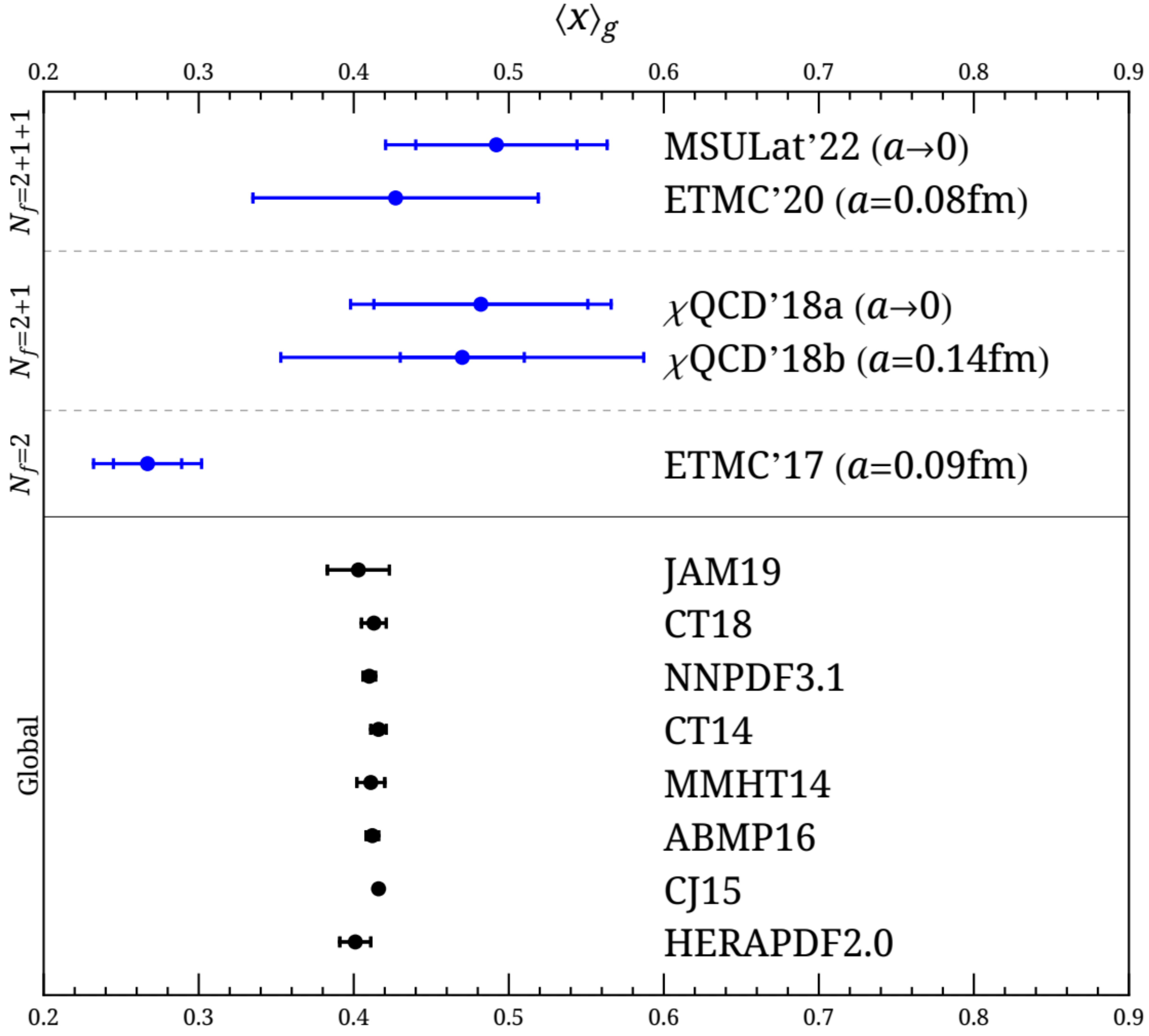
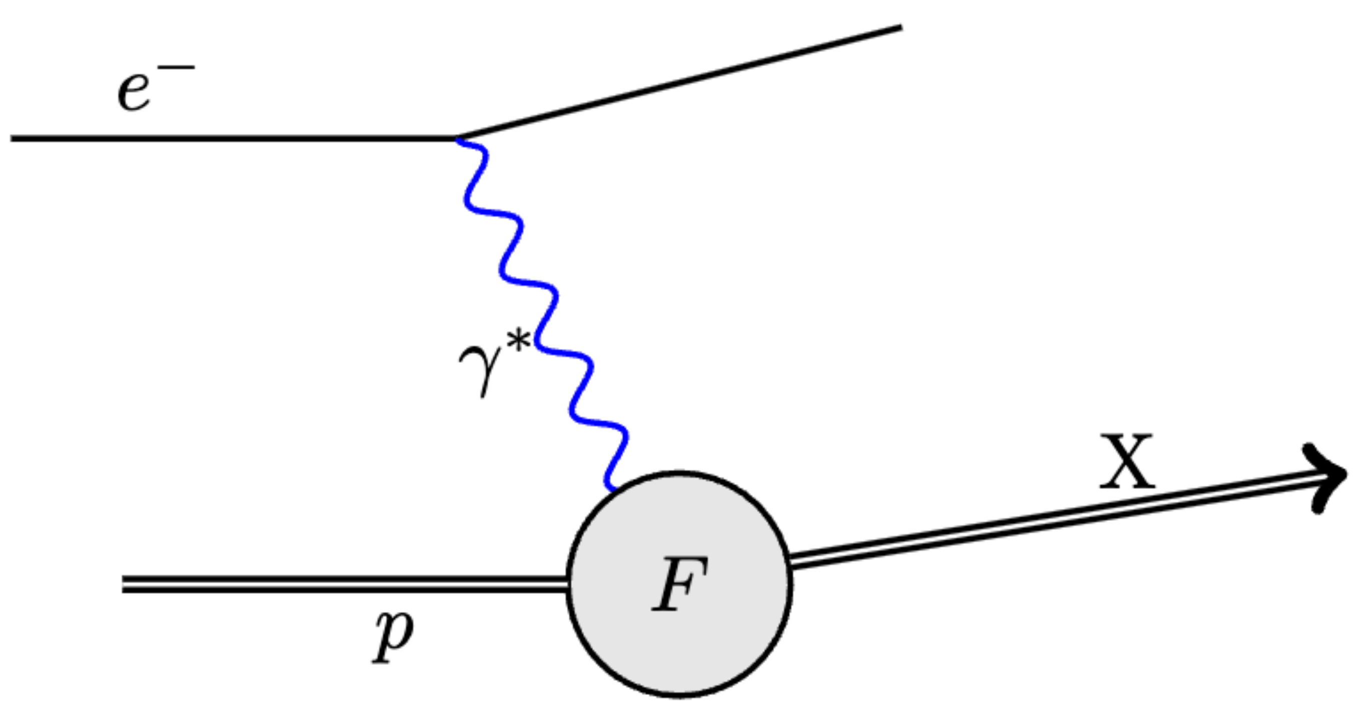
PHENOMENOLOGY LATTICE QCD THEORY

Jefferson Lab

Hadronic Structure





Group	N_f	a (fm)	M_π^{val} (MeV)	Fermion	N_{meas}	Renorm.	G-smearing	$\langle x \rangle_g$
ETMC16 [21]	2+1+1	0.08	370	TM	34,470	1-loop	2-stout	$0.284(27)_{\text{stat.}}(17)_{\text{ES}}(24)_{\text{PT}}$
ETMC16 [21]	2	0.09	131	TM	209,400	1-loop	2-stout	$0.267(22)_{\text{stat.}}(19)_{\text{ES}}(24)_{\text{PT}}$
ETMC17 [22]	2	0.09	131	TM	209,400	1-loop	2-stout	$0.267(12)_{\text{stat.}}(10)_{\text{ES}}$
MIT18 [23]	2 + 1	0.12	450	clover	572,663	RI-MOM	Wilson flow	$0.54(8)_{\text{stat.}}$
χQCD18a [24]	2+1	0.114	[135, 372][a]	overlap	81 cfgs	RI-MOM	1-HYP	$0.47(4)_{\text{stat.}}(11)_{\text{NPR+mixing}}$
χQCD18b [26]	2+1	[0.08, 0.14]	[140,400]	overlap	[81, 309] cfgs	RI-MOM	1-HYP	$0.482(69)_{\text{stat.}}(48)_{\text{cont.}}$
ETMC20 [25]	2 + 1 + 1	0.08	139.3	TM	48,000	1-loop	10-stout	$0.427(92)_{\text{stat.}}$
χQCD21 [27]	2 + 1	0.14	[171, 391][b]	overlap	8,200	RI-MOM	1-HYP	$0.509(20)_{\text{stat.}}(23)_{\text{cont.}}$
MSULat22 (this work)	2 + 1 + 1	[0.09,0.15]	[220,700][c]	clover	10^5–10^6	RI-MOM	5-HYP	$0.492(52)_{\text{stat.+NPR}}(49)_{\text{mixing}}$

[a] partially quenched calculation on domain-wall fermion $M_\pi^{\text{sea}} = 140$-MeV lattice
[b] partially quenched calculation on domain-wall fermion $M_\pi^{\text{sea}} = 171$-MeV lattice
[c] clover-on-HISQ mixed action with valence pion masses tuned to lightest sea-quark ones

Fan, Lin, Zeilbeck; 2022

Energy-Momentum Tensor

$c^{-2}\cdot$(energy density) — momentum density

$$T^{\mu\nu} = \begin{bmatrix} T^{00} & T^{01} & T^{02} & T^{03} \\ T^{10} & T^{11} & T^{12} & T^{13} \\ T^{20} & T^{21} & T^{22} & T^{23} \\ T^{30} & T^{31} & T^{32} & T^{33} \end{bmatrix} = T_q^{\mu\nu} + T_g^{\mu\nu}$$

shear stress

pressure

energy flux — momentum flux

$$T_g^{\mu\nu} = G^{\mu\rho} G_\rho{}^{\nu} + \frac{1}{4}\delta^{\mu\nu} G^{\rho\sigma} G_{\rho\sigma}$$

$$= \mathcal{O}_1^{\mu\nu} + \mathcal{O}_2^{\mu\nu}$$

Gluon momentum fraction

$$\langle P | \mathcal{O}_1^{44} - \frac{1}{3}\mathcal{O}_1^{jj} | P \rangle = \left(\frac{2}{3}\boldsymbol{P}^2 + 2E^2 \right) \langle x \rangle_g$$

$$\begin{aligned} \langle P | \mathcal{O} | P \rangle = & |\mathcal{M}_0|^2 \langle P_0 | \mathcal{O} | P_0 \rangle e^{-E_0 t} \\ & + |\mathcal{M}_0||\mathcal{M}_1| \langle P_0 | \mathcal{O} | P_1 \rangle e^{-E_0(t-t_g)} e^{-E_1 t} \\ & + |\mathcal{M}_1||\mathcal{M}_0| \langle P_1 | \mathcal{O} | P_0 \rangle e^{-E_1(t-t_g)} e^{-E_0 t} \\ & + |\mathcal{M}_1|^2 \langle P_1 | \mathcal{O} | P_1 \rangle e^{-E_1 t} + \ldots \end{aligned}$$

$$\langle P | P \rangle = |\mathcal{M}_0|^2 e^{-E_0 t} + |\mathcal{M}_1|^2 e^{-E_1 t} + \ldots$$

Göckeler, Horsley, Ilgenfritz, et. al. 1996

Alexandrou et. al. 2017

$$\langle x \rangle_g \Big|_{\vec{p}=0} = \frac{1}{2m_N} \frac{\langle P | \mathcal{O}_1^{44} - \frac{1}{3}\mathcal{O}_1^{jj} | P \rangle}{\langle P | P \rangle}$$

Moments of PDFs

$$\langle x \rangle_g = \int_0^1 dx \; x \; g(x)$$

$$\nu = -p \cdot z$$

$$x \; g(x) = \int \frac{d\nu}{2\pi} \cos(x\nu) \; \mathfrak{M}(\nu, z^2)$$

$$\mathfrak{M}(\nu, z^2) = \frac{1}{4p_+^2} \langle P \,|\, G^{+j}(z^-) W[z^-, 0] G^+_{\;j}(0) \,|\, P \rangle$$

Radyushkin; 2020

Gluon momentum fraction

Can be extracted from matrix elements of the form

$z = 0$

$$M^{j}_{\alpha\beta;\,\gamma\delta}(z,P) = Z(z)\langle P\,|\,G_{\alpha\beta}(z)W[z,0]\Gamma^{j}G_{\gamma\delta}(0)\,|\,P\rangle$$

$$M^{j}(z,P) = P^{j}\mathcal{M}(\nu,z^2) + z^{j}\mathcal{M}_{z}(\nu,z^2)$$

Necessary for Ioffe-time distributions

$$\mathfrak{M}(\nu,z^2) = \left(\frac{\mathcal{M}(\nu,z^2)}{\mathcal{M}(\nu,0)\,|_{z=0}}\right) \Bigg/ \left(\frac{\mathcal{M}(0,z^2)\,|_{p=0}}{\mathcal{M}(0,0)\,|_{p=0,\,z=0}}\right)$$

T. Khan et. al. 2021

Monahan, Orginos 2014

Lattice QCD:

- Only systematically improvable, non-perturbative approach to QCD built from first-principles
- Monte-Carlo integration of path-integral formalism
  - Calculation of correlation functions for a statistical ensemble
- Ultimately, aim to control systematics:
  - Finite lattice spacings
  - Finite volumes
  - Simulation strategy
  - Renormalization approach
- Signal-to-noise can be controlled through various techniques

- All of the analysis was conducted using the new 24s clusters here at JLab

Lattice details:

- Isotropic
- 2+1 stout-link (1-iteration) smeared clover Wilson fermions
- Tree-level tadpole improved Symanzik gauge action
- Configurations generated by rational HMC
- 64 temporal sources

a (fm)	M_π (MeV)	$L^3 \times N_t$	N_{cfg}
0.094(1)	358(3)	$32^3 \times 64$	1121

JLAB/W&M Collaboration

Computed 2-pt functions and disconnected gluon loops

Data improvement techniques

- Gradient Flow

$$A_\mu(x) \to B_\mu(\tau, x) \qquad B(0, x) = A_\mu(x)$$

$$\dot{B}_\mu = D_\nu G_{\nu\mu} \quad , \qquad D_\mu = \partial_\mu + [B_\mu, \cdot\,]$$

Lüscher, Weisz; 2011

$$G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu + [B_\mu, B_\nu]$$

- Distillation

$$\square_{xy}(t) = \sum_{k=1}^{N_D} v_x^{(k)}(t) v_y^{(k)\dagger}(t)$$

Peardon, et. al; 2009

- Summed GEVP

$$C_{3\text{pt}}^{i,s}(t) = \sum_{t_g=1}^{t-1} C_{3\text{pt}}^{i}(t, t_g)$$

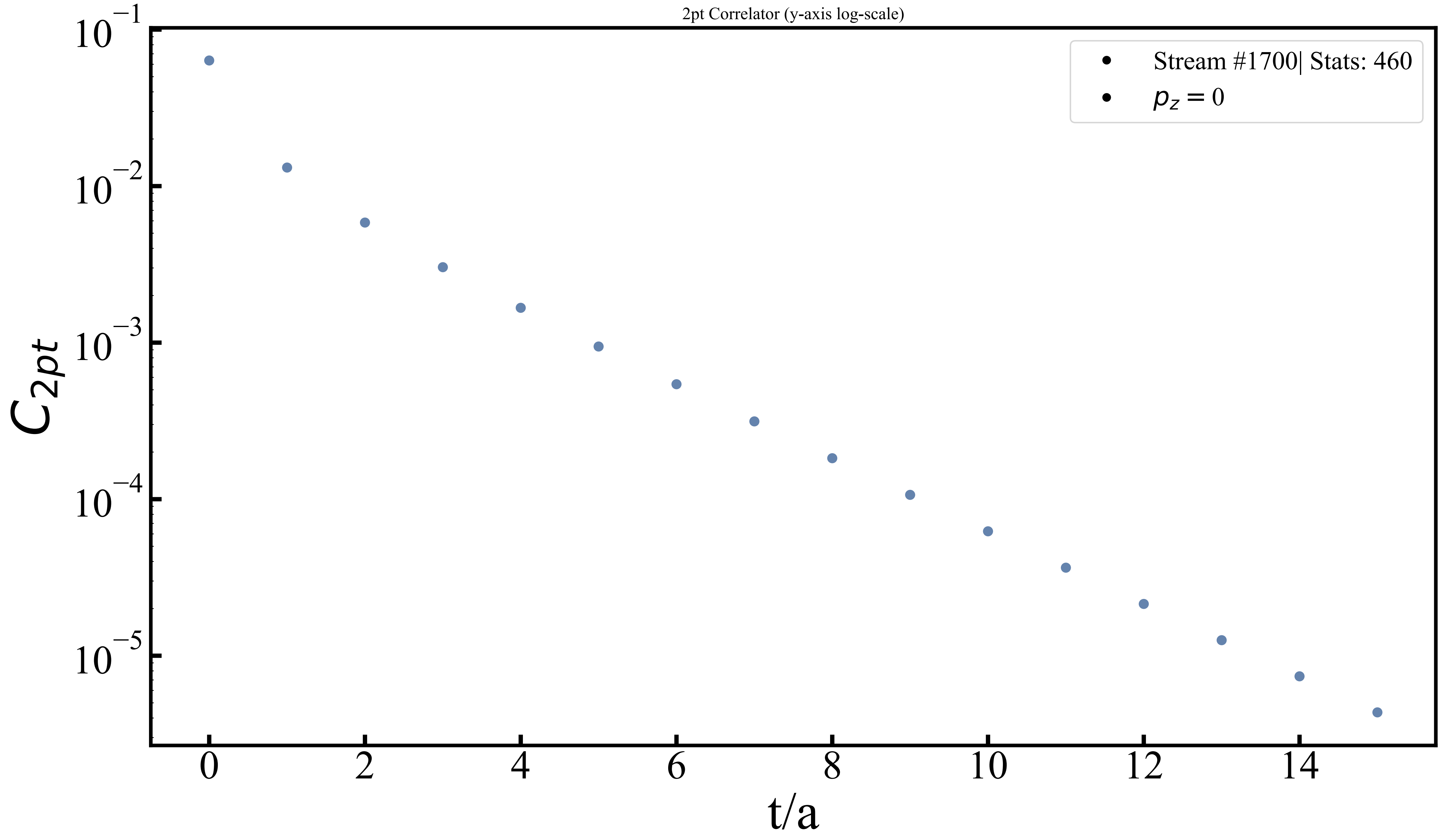
Blossier, et. al; 2009

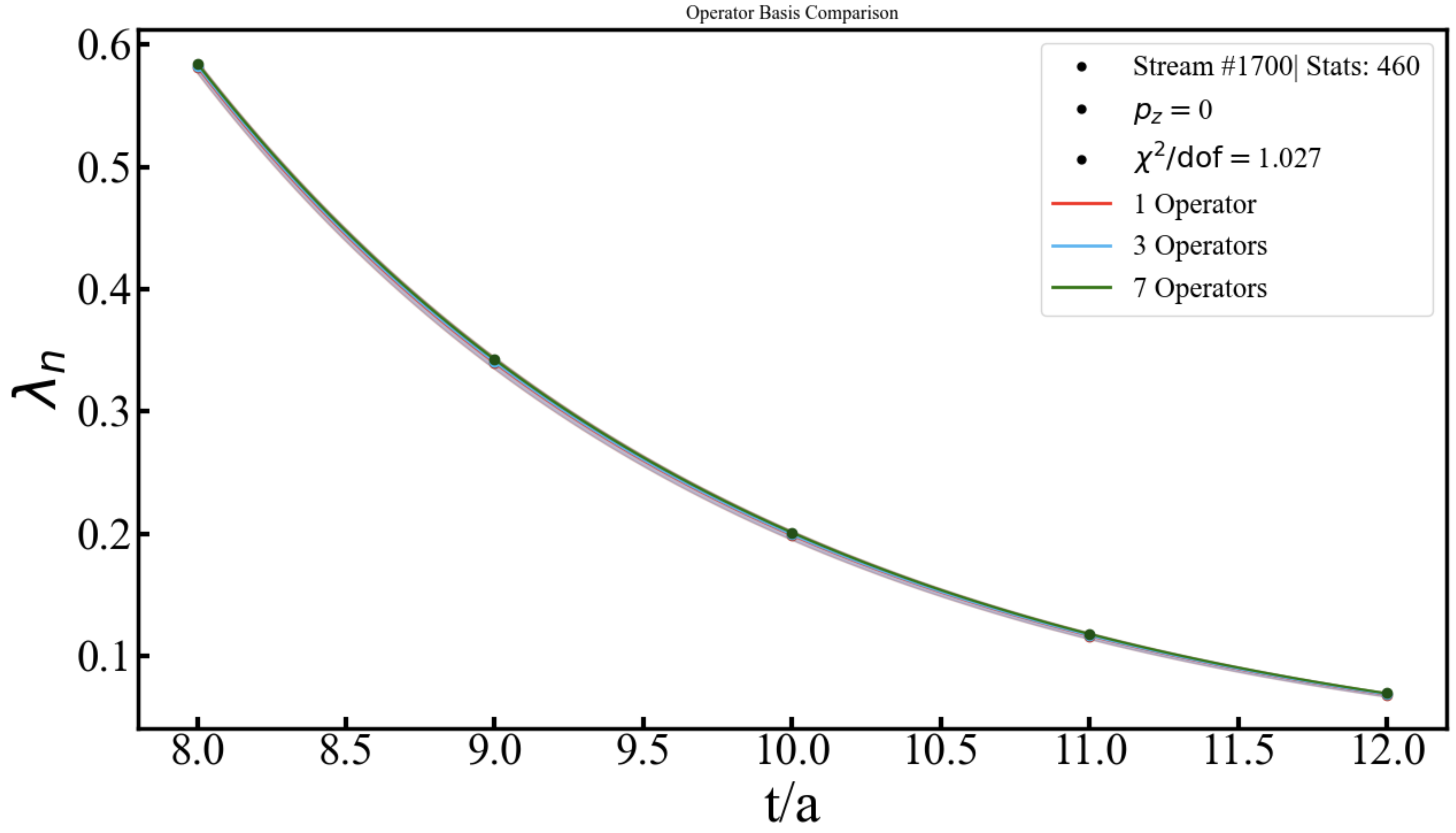
- Bayesian Model Averaging

Jay, Neil; 2019

$$pr(M|D) \approx \exp\left[-\frac{1}{2}(\chi^2_{\text{aug}}(\mathbf{a}^\star) + 2k + 2N_{\text{cut}})\right]$$

Two-point correlation functions

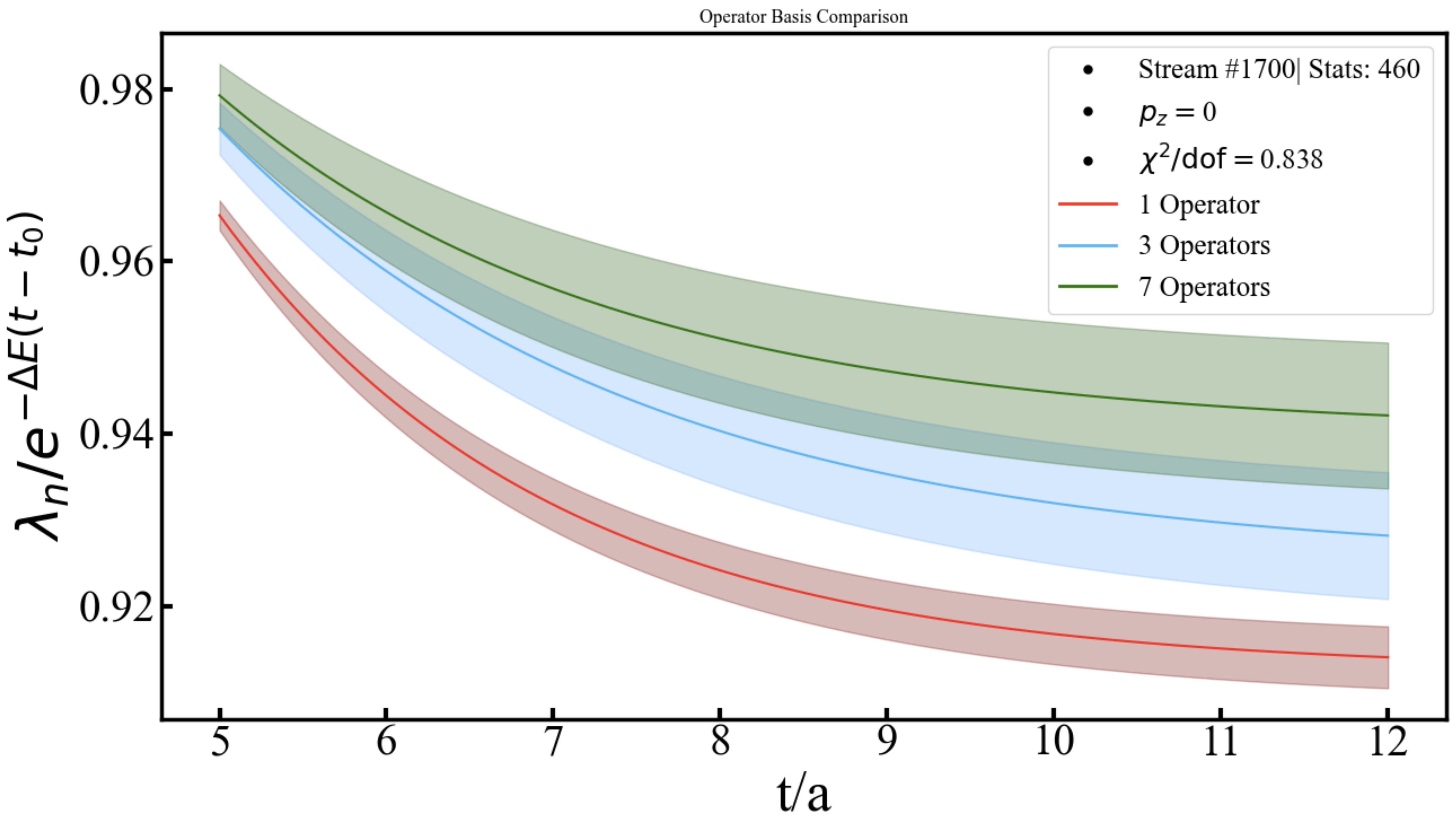


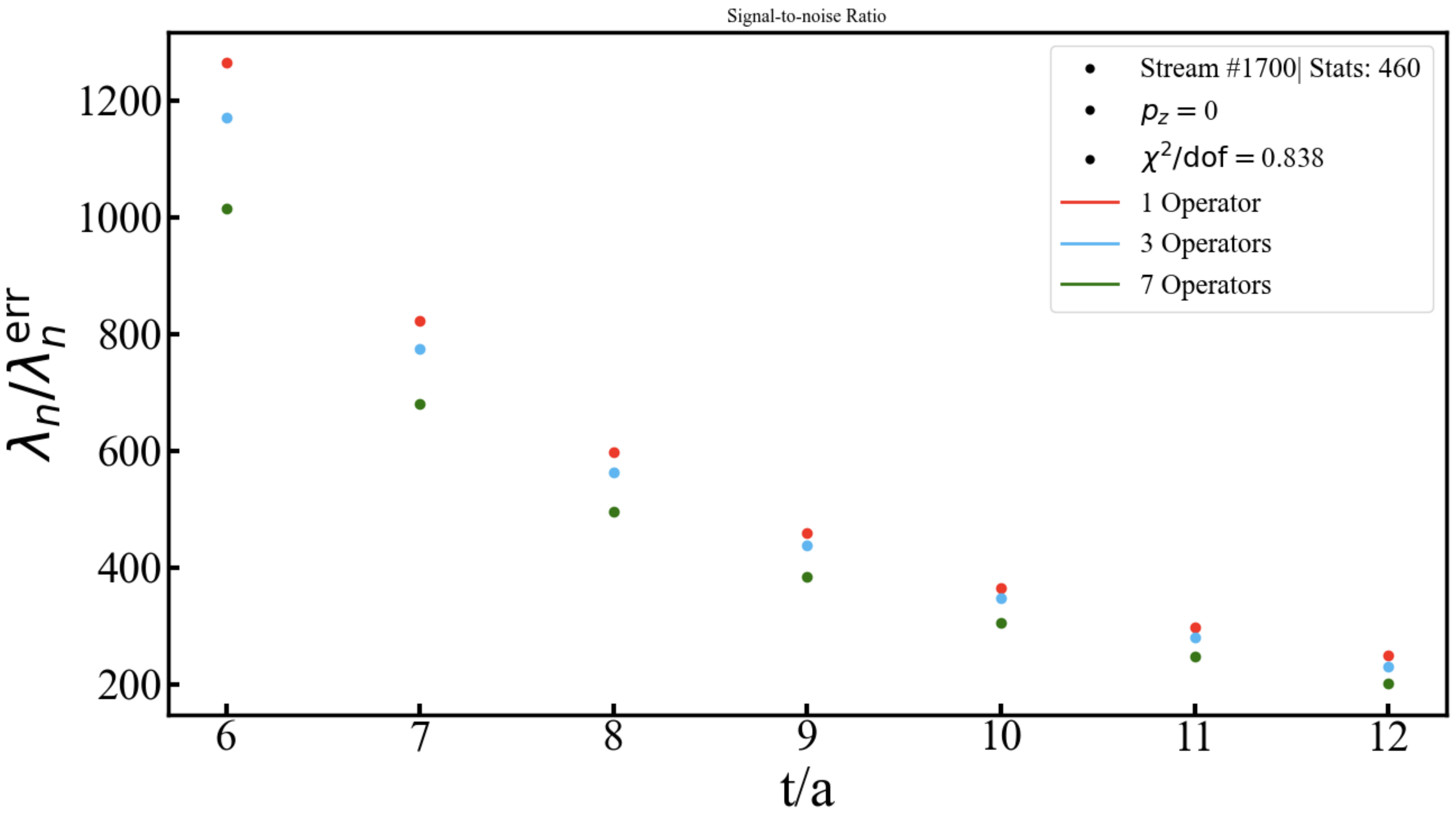


$$\Delta E(t) = \ln \frac{C_{2\text{pt}}(t)}{C_{2\text{pt}}(t+1)}$$

$$\lambda_n(t, t_0) = (1 - A)\, e^{-\Delta E(t - t_0)} + A e^{-\Delta E'(t - t_0)}$$

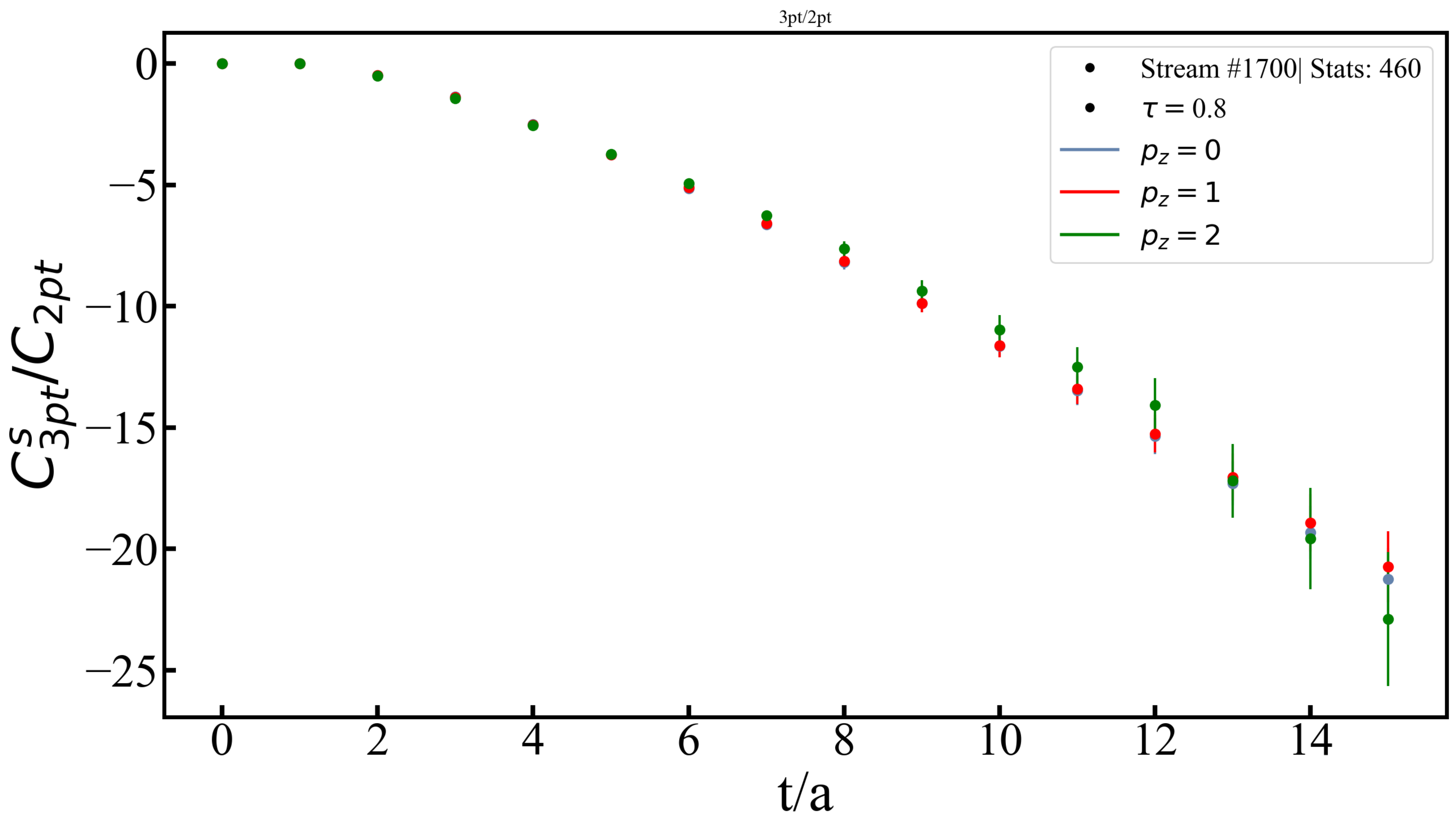
Two-point correlation functions

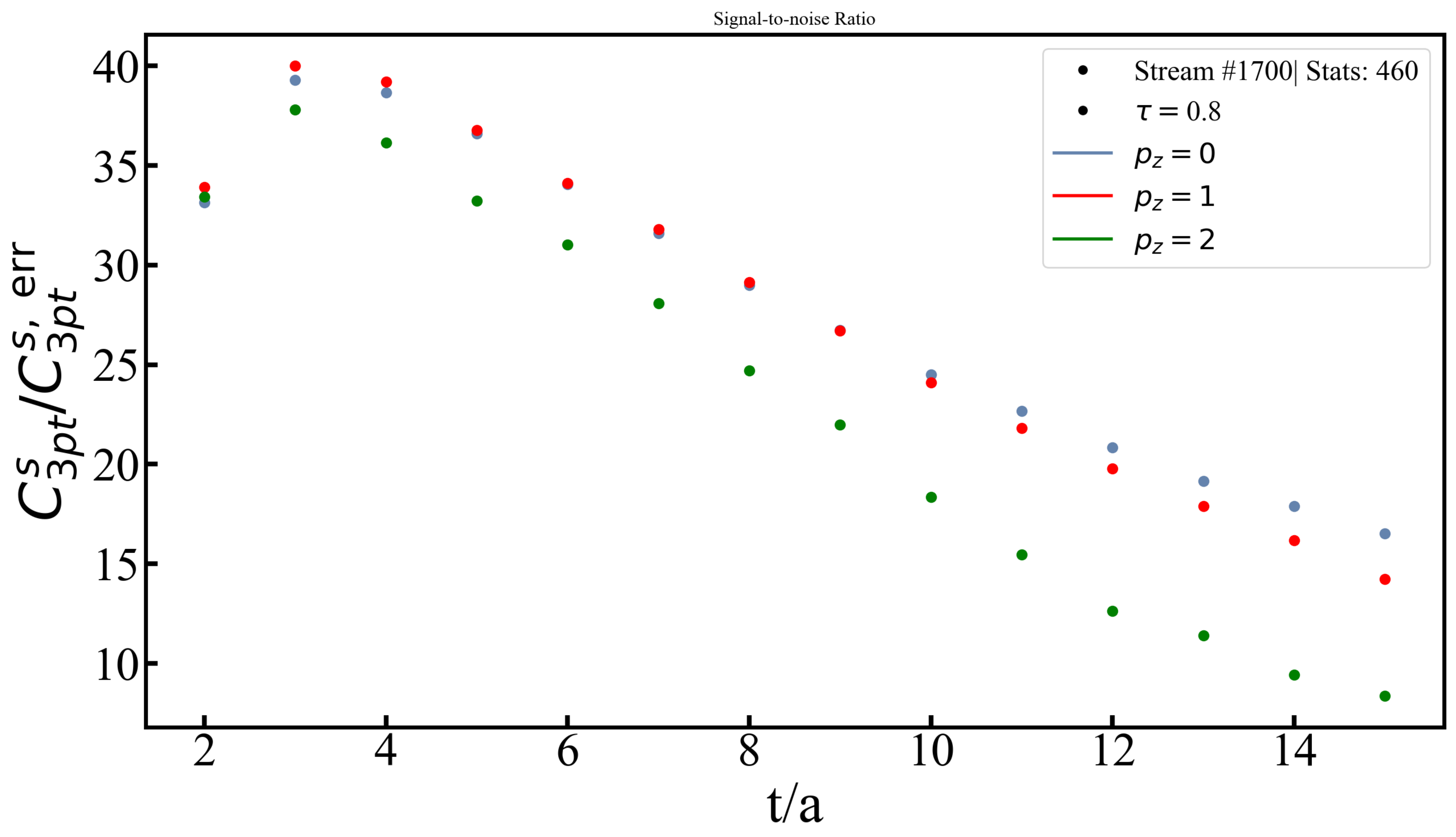




$$\lambda_n(t, t_0) = (1 - A)\, e^{-\Delta E(t-t_0)} + A e^{-\Delta E'(t-t_0)}$$

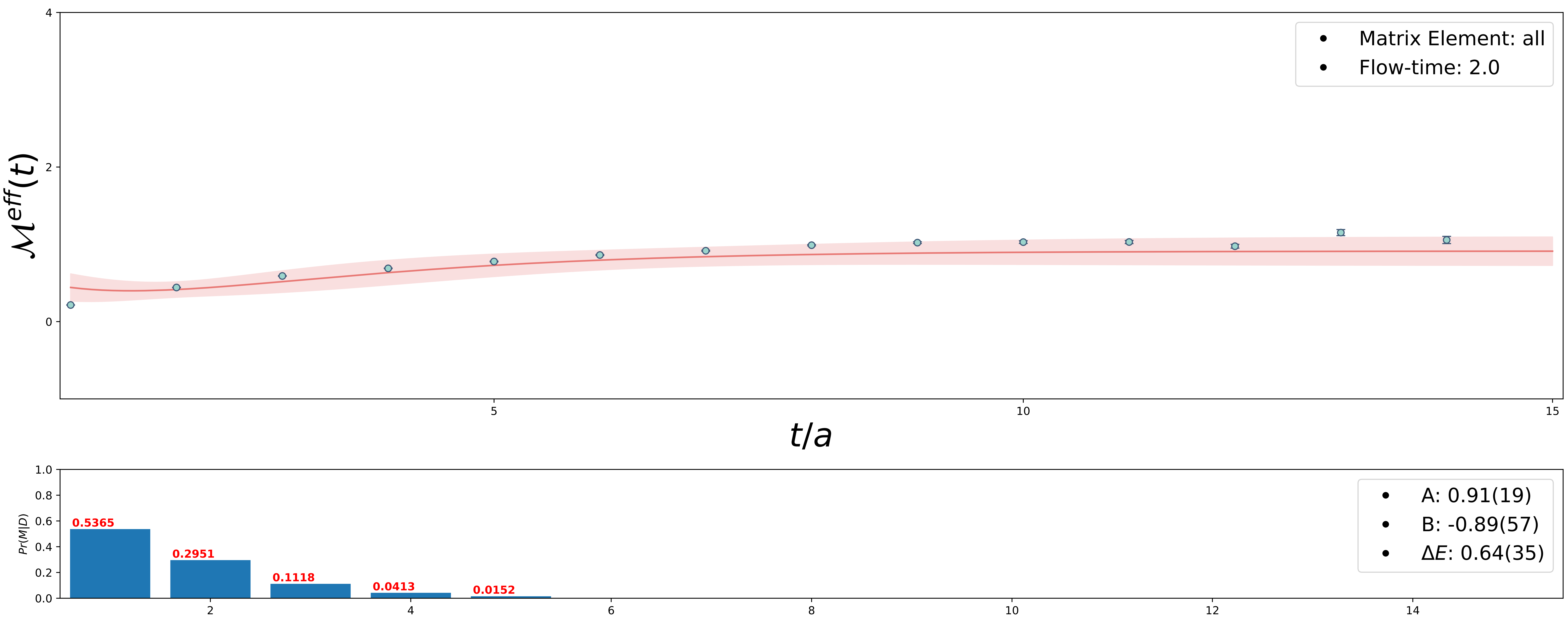
Three-Point Correlators





Effective Matrix Elements Fits

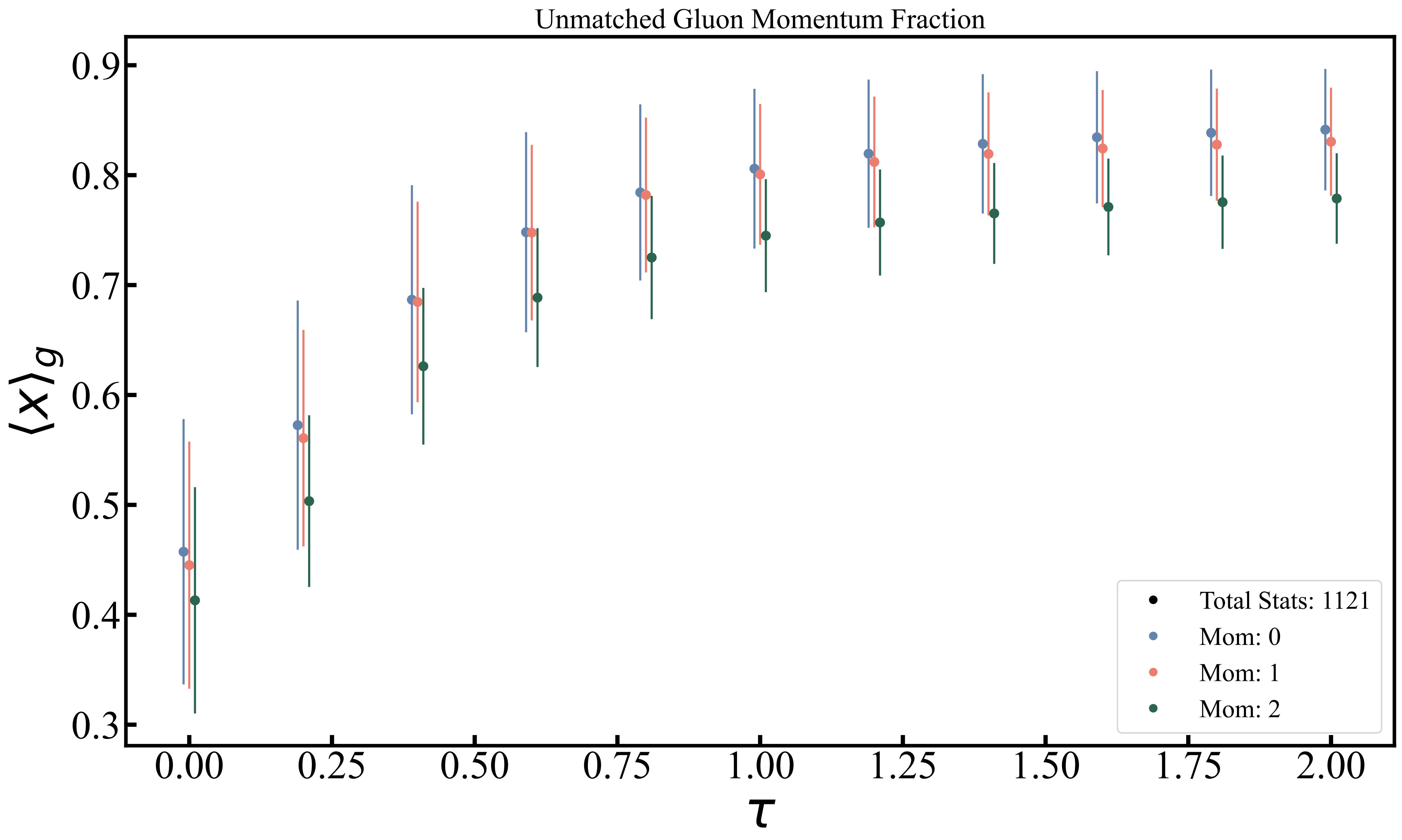
$$\mathscr{M}(t) = A + Bte^{-t\Delta E}$$



Gradient-flow Matching

$$\tilde{\mathcal{O}}_i^{\mu\nu}(\tau, x) = \zeta_{i,j}(\tau)\mathcal{O}_j^{\mu\nu}(x) + \ldots$$

$$\mathcal{O}_i^{\mu\nu}(x) = \zeta_{i,j}^{-1}(\tau)\tilde{\mathcal{O}}_j^{\mu\nu}(\tau, x) + \ldots$$



Harlander, Kluth, Lange; 2019
Makino, Suzuki; 2015

Summary

- Computed Two-Point Correlators
  - Using distillation
  - Implemented variational method to extract hadron masses
- Computed Gluon-Loops
  - Smoothed configurations with Gradient-Flow
- Combined data to generate Three-Point Correlators
  - Used sGEVP to extract three-point correlators
- Extracted gluon momentum fraction from combined data
  - Used BAIC method to fit correlator ratios
  - Recovered MS-bar results using matching coefficients

Gradient-flow

$$A_\mu(x) \to B_\mu(\tau, x) \qquad B(0, x) = A_\mu(x)$$

$$\dot{B}_\mu = D_\nu G_{\nu\mu} \quad , \qquad D_\mu = \partial_\mu + [B_\mu, \cdot\,]$$

$$G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu + [B_\mu, B_\nu]$$

$$\dot{B}_\mu(\tau, x) = -\, g_0^2 \left\{ \partial_{x,\mu}\, S_W(B) \right\} B_\mu(\tau, x)$$

Lüscher, Weisz; 2011

Lüscher 2009, 2013, 2014

Distillation

Low-rank approximation to Jacobi smearing kernel

$$-\nabla^2(t)\, v^{(k)}(t) = \lambda^{(k)}(t)\, v^{(k)}(t), \qquad \square_{xy}(t) = \sum_{k=1}^{N_D} v_x^{(k)}(t) v_y^{(k)\dagger}(t)$$

$$\Phi^{(i,j,k)}_{r,\,\alpha\beta\gamma}(t) = \epsilon^{abc} S_{r,\,\alpha\beta\gamma} \left(\mathscr{D}_{1r} v^{(i)}\right)^a \left(\mathscr{D}_{2r} v^{(j)}\right)^b \left(\mathscr{D}_{3r} v^{(k)}\right)^c (t)$$

$$\tau^{(l,k)}_{\alpha\beta}\left(T_f,\, T_0\right) = v^{(l)\dagger}(T_f)\, M^{-1}_{\alpha\beta}\left(T_f,\, T_0\right) v^{(k)}(T_0)$$

$$\langle \mathscr{O}_p(m) \overline{\mathscr{O}}_q(n) \rangle = \Phi^{(i,j,k)}_{p,\,\alpha\beta\gamma}(t_m) \left[\tau^{(i,\bar{i})}_{\alpha\bar{\alpha}} \tau^{(j,\bar{j})}_{\beta\bar{\beta}} \tau^{(k,\bar{k})}_{\gamma\bar{\gamma}} - \tau^{(i,\bar{i})}_{\alpha\bar{\alpha}} \tau^{(j,\bar{k})}_{\beta\bar{\gamma}} \tau^{(k,\bar{j})}_{\gamma\bar{\beta}} \right] \left(t_m,\, t_n\right) \Phi^{(\bar{i},\bar{j},\bar{k})}_{q,\,\bar{\alpha}\bar{\beta}\bar{\gamma}}(t_n)$$

Peardon, et. al; 2009

Summed-GEVP

$$C_{ij}^{2\text{pt}}(t) = \langle \mathcal{O}_i(t)\overline{\mathcal{O}}_j(0)\rangle$$

$$C^{2\text{pt}}(t)\ u_n = \lambda_n(t, t_0)C^{2\text{pt}}(t_0)u_n$$

$$C_{ij}^{3\text{pt},\,s}(t) = \sum_{t_i=1}^{t-1} C_{ij}^{2\text{pt}}(t)\ \mathcal{O}(t_i)$$

$$\mathcal{M}_{nn}(t) = -\,\partial_t \left\{ \frac{u_n^\dagger \left[C^{3\text{pt},\,s}(t)\lambda_n^{-1}(t) - C^{3\text{pt},\,s}(t_0)\right] u_n}{u_n^\dagger C^{2\text{pt}}(t_0)u_n} \right\}$$

Blossier, et. al; 2009
Bulava, et. al; 2011

Bayesian Model-Averaging

$$pr(M|D) \approx \exp\left[-\frac{1}{2}(\chi^2_{\text{aug}}(\mathbf{a}^\star) + 2k + 2N_{\text{cut}})\right]$$

Model 1: $\{C^{2\text{pt}}(t_0 = 0), C^{2\text{pt}}(1), C^{2\text{pt}}(2), C^{2\text{pt}}(3), \ldots\}$

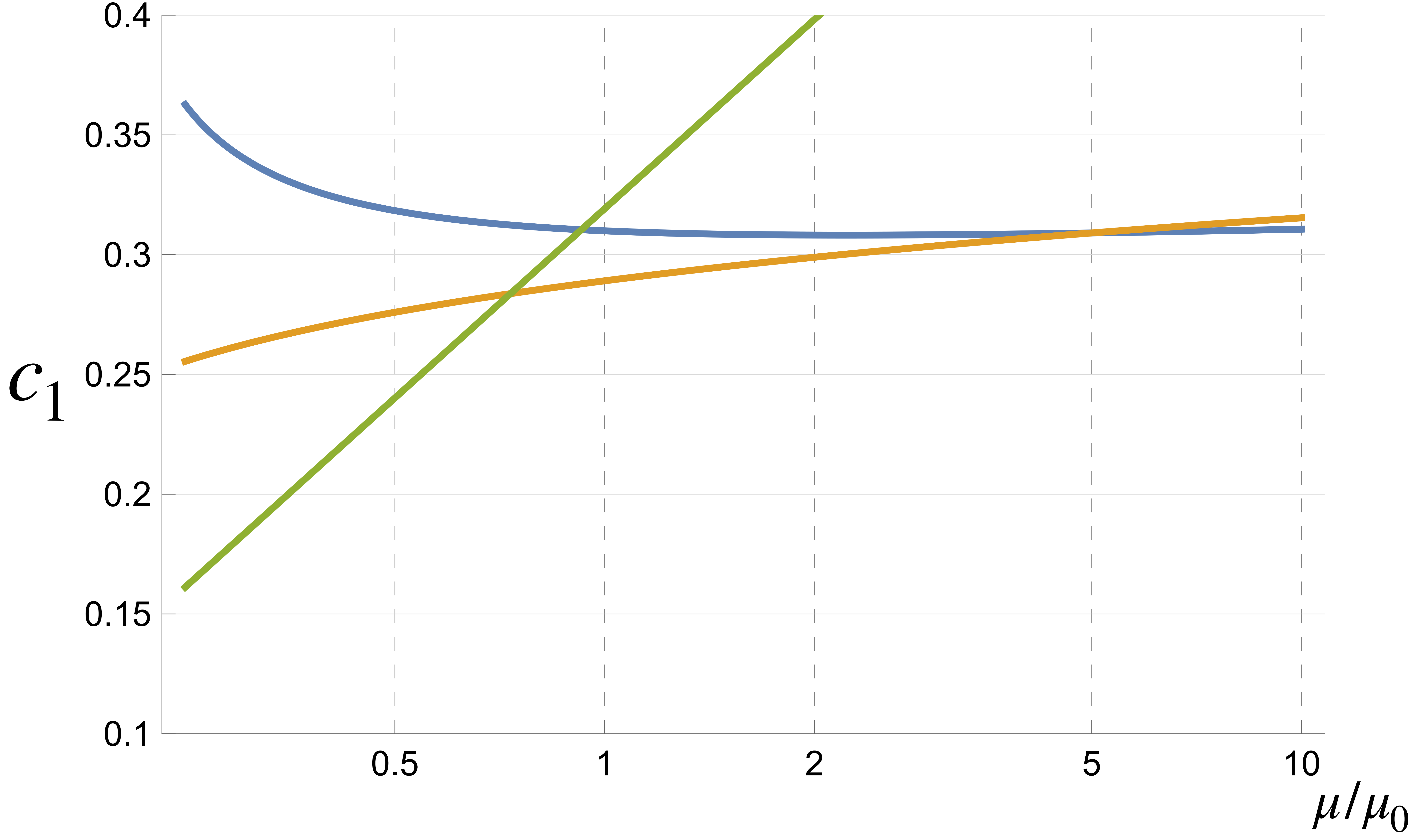
Model 2: $\{C^{2\text{pt}}(t_0 = 0), C^{2\text{pt}}(1), C^{2\text{pt}}(2), C^{2\text{pt}}(3), \ldots\}$

Model 3: $\{C^{2\text{pt}}(t_0 = 0), C^{2\text{pt}}(1), C^{2\text{pt}}(2), C^{2\text{pt}}(3), \ldots\}$

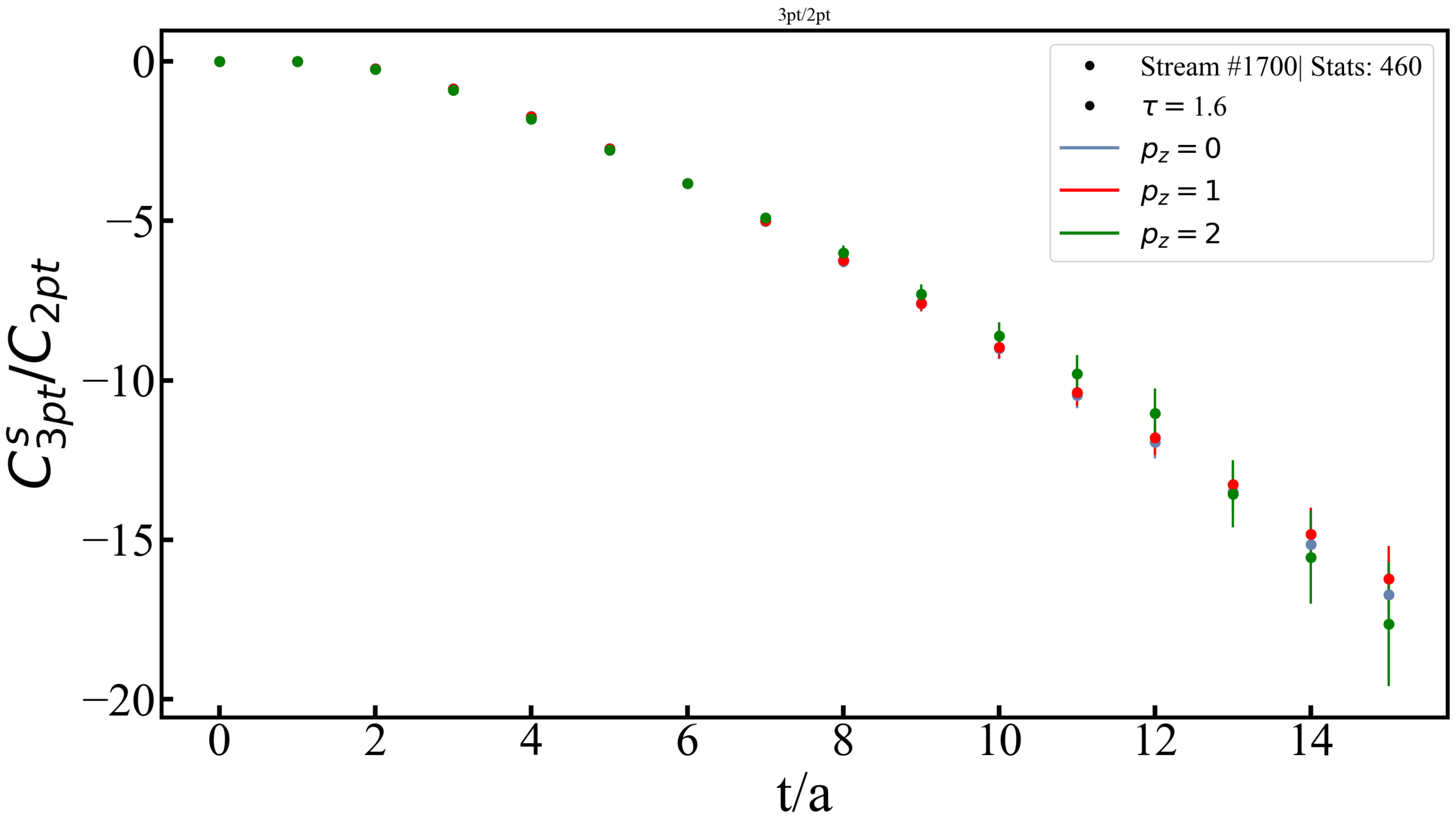
⋮

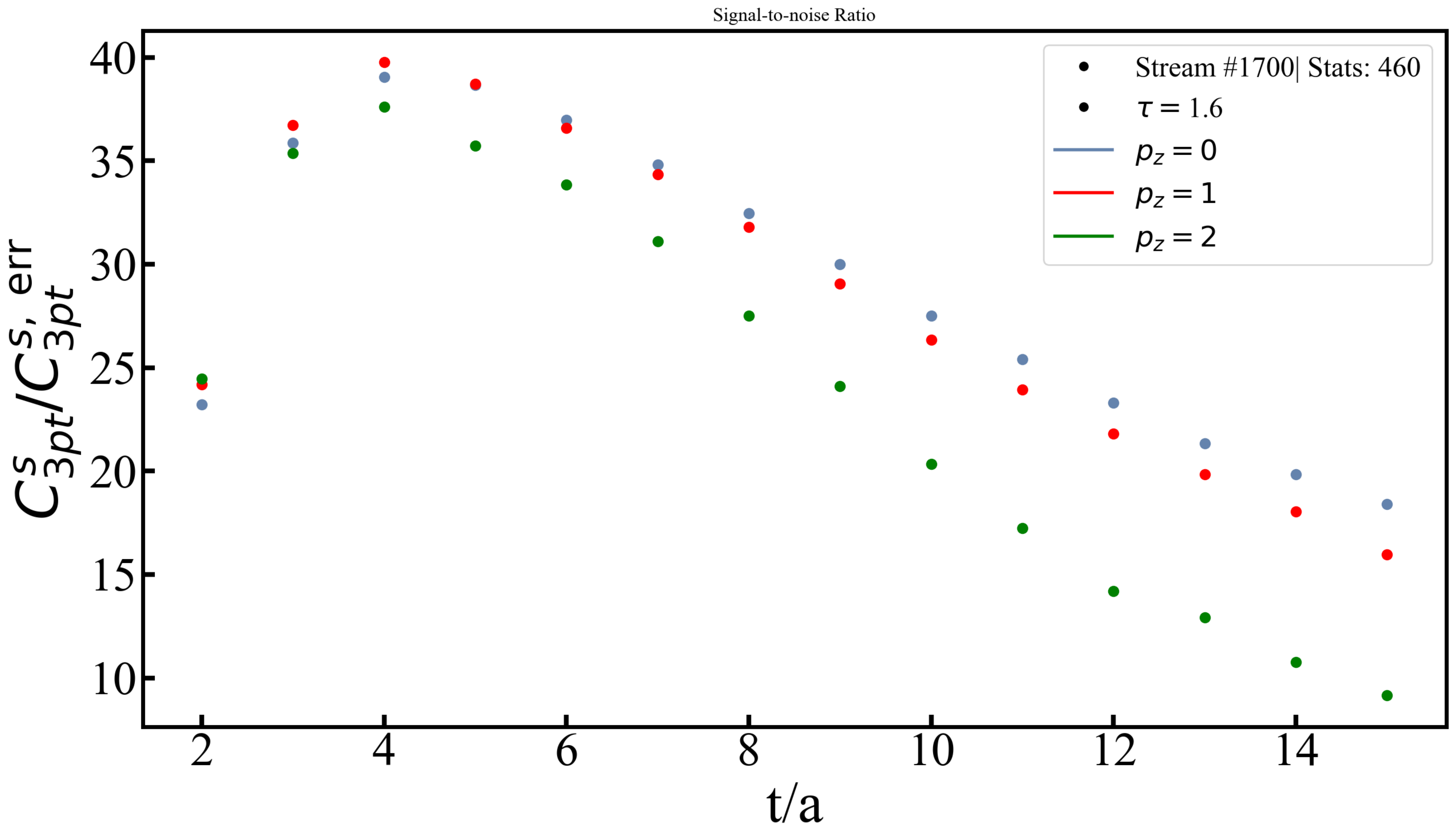
Cut

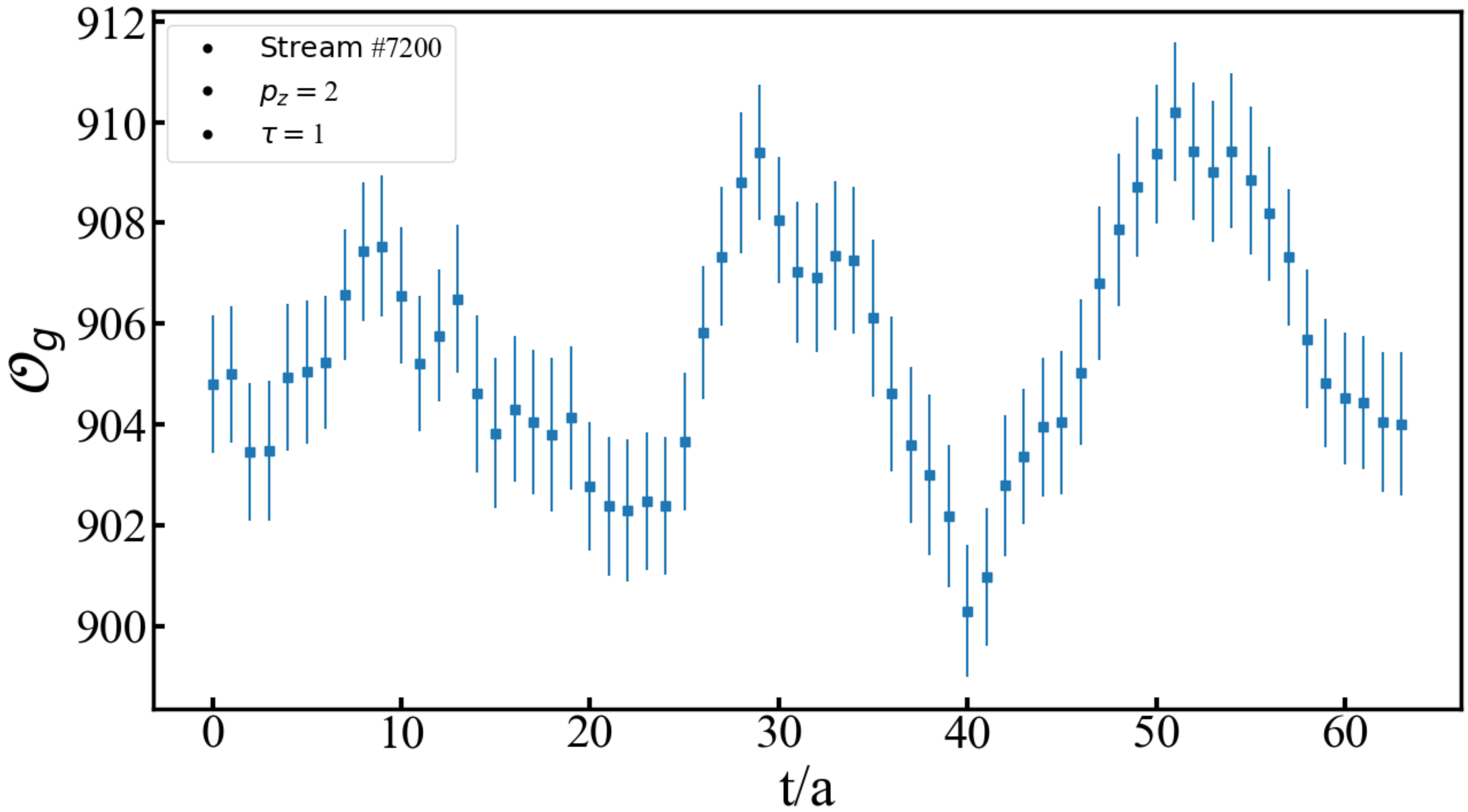
Jay, Neil; 2019, 2022. 2023



c_1
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.5
1
2
5
10
μ/μ_0


3pt/2pt
Stream #1700| Stats: 460
$\tau = 1.6$
$p_z = 0$
$p_z = 1$
$p_z = 2$
C^s_{3pt}/C_{2pt}
t/a


Signal-to-noise Ratio
Stream #1700| Stats: 460
$\tau = 1.6$
$p_z = 0$
$p_z = 1$
$p_z = 2$
$C^s_{3pt}/C^{s,\ err}_{3pt}$
t/a


Stream #7200
$p_z = 2$
$\tau = 1$
912
910
908
906
904
902
900
$\mathcal{O}_g$
0
10
20
30
40
50
60
t/a