Disconnected Contribution in Pseudo-PDF Reconstruction

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Outline

Lattice QCD in a Nutshell

- **Goal:** Nonperturbative, first-principles calculation of QCD observables.[1]
- **Method:** Discretize Euclidean space-time on a 4D lattice.
  - Gauge fields: SU(3) links on lattice edges.
  - Fermions: Quarks on lattice sites, path-integral evaluated numerically.
- **Key Outputs:**
  - Correlation functions $\rightarrow$ hadron masses, matrix elements, PDFs.
  - Systematic control over $a \rightarrow 0$, $m_\pi \rightarrow m_\pi^{\text{phys}}$, $V \rightarrow \infty$.
- **Relevance to this talk:**
  - Disconnected loops arise from sea quark effects.
  - Pseudo-PDFs allow access to partonic structure in Euclidean space.

[1] Achenbach et al., "The present and future of QCD".

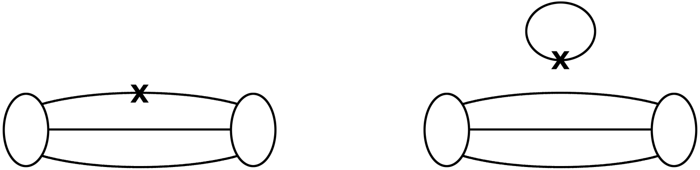
Motivation

- Full flavor decomposition of the proton requires **both connected and disconnected** diagrams
- Disconnected contributions
  - Appear in all isoscalar and flavor singlet matrix elements
  - Are **essential** for the strange quark, no connected contribution exists[2]
  - significantly more noisy than corresponding connected pieces
- Ignoring them leads to biased observables

[2] Constantinou et al., “Parton distributions and lattice-QCD calculations: Toward 3D structure”.

Disconnected Diagrams in Lattice QCD

- Disconnected: quark loops not directly connected to valence propagators.



$$\langle N|\ \bar{\psi}^f(z)\Gamma W(z,0)\psi^f(0)|N\rangle = \text{Connected} + \text{Disconnected}$$

- Appear only in isoscalar channels or flavor-singlet observables
- Strange quark: Entire signal is disconnected
- Computational Challenges: Noise-reduction

Pseudo-PDF Approach

- Nonlocal operator on lattice[3][4]

$$\mathcal{M}(\nu, z^2) = \langle p|\bar{\psi}(z)\Gamma W(z,0)\psi(0)|p\rangle \quad \text{with } \nu = -p\cdot z$$

- Key Points
  - Defined at finite z
  - Allows access to parton distributions from Euclidean lattice.
  - Disconnected loops are computationally expensive and notoriously noisy, requiring advanced techniques like hierarchical probing[5] to obtain a usable signal.

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[3] Ji, "Parton Physics on a Euclidean Lattice".

[4] A. V. Radyushkin, "Quasi-parton distribution functions, momentum distributions, and pseudo-parton distribution functions".

[5] Chambers et al., "Disconnected contributions to the spin of the nucleon".

Renormalization

Double Ratio Method:

$$\mathfrak{M}(\nu, z^2) = \frac{\langle P|\mathcal{O}(z)|P\rangle}{\langle P|\mathcal{O}(0)|P\rangle} \Bigg/ \frac{\langle 0|\mathcal{O}(z)|0\rangle}{\langle 0|\mathcal{O}(0)|0\rangle}$$

- Removes power divergences from Wilson line[6]
- Avoids need for nonperturbative renormalization constants[7]
- Need corresponding connected piece to form ratio
  - Disconnected piece is 0 at $\nu = 0$.

[6] Orginos et al., *Lattice QCD exploration of pseudo-PDFs*.

[7] Ishikawa et al., "Renormalizability of quasiparton distribution functions".

ITD to Pseudo-PDF

- The Renormalized Matrix elements are related to Pseudo-PDFs by:

$$\mathfrak{M}(\nu, z^2) = \int_{-1}^{1} dx\, e^{i\nu x} \mathcal{P}(x, z^2)$$

- $P(x, z^2)$ is the lattice calculable pseudo-PDF. $z^2 \neq 0$,
- $x < 0$ corresponds to anti-particles
- Matching to $\overline{MS}$-ITD, $Q(\nu, \mu^2)$ is possible up to NLO.[8]

[8] A. Radyushkin, "One-loop evolution of parton pseudo-distribution functions on the lattice".

Real vs Imaginary Parts

$Q(\nu, \mu^2)$ is the $\overline{\text{MS}}$-ITD given by:

$$\mathfrak{M}(\nu, z^2) = \int_0^1 du\, K(u, z^2\mu^2, \alpha_s) Q(\mu\nu, \mu^2)$$

- The real (CP even) component describes the valence quark distribution[9]

$$\text{Re } Q(\nu, \mu^2) = \int_0^1 dx \cos(\nu x)\Big(q(x, \mu^2) - \bar{q}(x, \mu^2)\Big)$$

- The imaginary (CP odd) component describes the sum of the quark and anti-quark distributions

$$\text{Im } Q(\nu, \mu^2) = \int_0^1 dx \sin(\nu x)\Big(q(x, \mu^2) + \bar{q}(x, \mu^2)\Big)$$

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[9] Joó et al., "Parton distribution functions from Ioffe time pseudo-distributions".

Physical Impact of Disconnected Contributions

- Isoscalar observables receive meaningful corrections
  - Total quark spin contribution: $\Delta\Sigma$
  - Momentum fractions: $\langle x \rangle_{u+d}$
  - Nucleon Charges[10]
- **Strange Quark PDFs** are entirely disconnected:
  - $s(x), \quad \Delta s(x), \quad \delta s(x)$
  - Direct input for global fits (e.g. JAM[11], NNPDF[12])
- **Experimental relevance**:
  - SIDIS, W/Z production
  - EIC: sensitivity to sea quark flavor separation

[10] Alexandrou et al., *Nucleon charges and σ-terms in lattice QCD*.

[11] Bringewatt et al., "Confronting lattice parton distributions with global QCD analysis".

[12] Ball et al., *The intrinsic charm quark valence distribution of the proton*.

Lattice Parameters

Lattice Parameters

- **Lattice Volume**: $32^3 \times 64$
- **Lattice Spacing**: $a = 0.094\,\mathrm{fm} \Rightarrow a^{-1} \approx 2.1\,\mathrm{GeV}$
- **Pion Mass**: $m_\pi \approx 358\,\mathrm{MeV}$
- **Clover-improved Wilson fermions**

Kinematics and Statistics:

- Proton momentum: up to $p = 6 \times \frac{2\pi}{L}$
- Multiple z separations (up to $12a$)
- 1834 configurations for disconnected loops
- 484 configurations for connected ($\approx 50\times$ less statistics)

Computational Methodology

$$C^{\text{disc}}(T,\tau) = \Big(C_{\text{2pt}}(T) - \langle C_{\text{2pt}}(T)\rangle\Big) \times \Big(D(t) - \langle D(t)\rangle\Big)$$

- **Disconnected Loop** corresponds to the inner quark bilinear

$$D = \sum_{s,c} \bar{\psi}^{sc}(z)W(z,0)\psi^{sc}(0) \rightarrow Tr[WD_{0z}^{-1}]$$

  - Coloring scheme[13], deflation[14], trace estimator[15]
  - Average over x, y, z, Only select (evenly spaced) time slices
  - Compute several z, p to extract Ioffe-time, $\nu = -p \cdot z$.
- **2 Point Functions**: $C_{2pt}(T) = \langle 0|\mathcal{O}_N(T)\overline{\mathcal{O}}_N(0)|0\rangle$

[13] Switzer et al., *Probing for the Trace Estimation of a Permuted Matrix Inverse Corresponding to a Lattice Displacement*.

[14] Romero, Stathopoulos, and Orginos, "Multigrid deflation for Lattice QCD".

[15] Alberico, Gervino, and Lavagno, "Phenomenological approach to baryon resonance damping in nuclei".

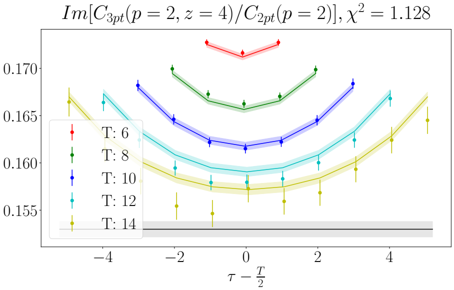
From Lattice to Matrix Element

- Example 2 state extraction

$$C_{3\mathrm{pt}}(T,\tau) = \mathcal{M}_{00}e^{-E_0 T} + \mathcal{M}_{11}e^{-E_1 T} + \mathcal{M}_{01}\left(e^{-E_0\tau}e^{-E_1(T-\tau)} + e^{-E_0(T-\tau)}e^{-E_1\tau}\right)$$

- Rainbow Fits - Connected

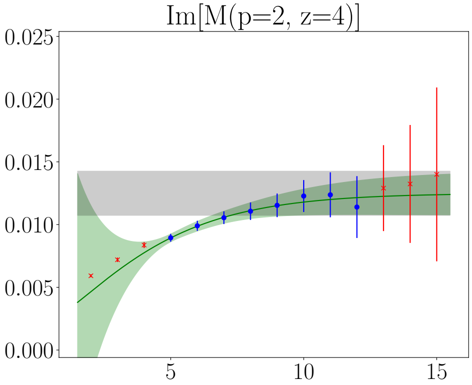
$$\frac{C_{3pt}(T,\tau)}{C_{2pt}(T)} \approx \mathcal{M}_{00} + \tilde{\mathcal{M}}_{11}e^{-\Delta E_p T} + \tilde{\mathcal{M}}_{01}\left(e^{-\Delta E_p\tau} + e^{-\Delta E_p(T-\tau)}\right)$$



From Lattice to Matrix Element (cont.)

- Summation Fits - Disconnected

$$\Delta S = S(T) - S(T-1) = M_{00} + Ae^{-\Delta E_p T} + BTe^{-\Delta E_p T}$$



where $S(T) = \sum_{\tau} \frac{C_{3pt}(T,\tau)}{C_{2pt}(T)}$

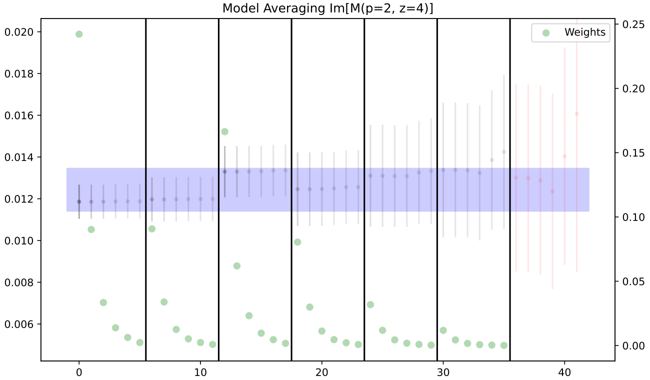
- Provides better control over excited state contamination
- Fit results are more stable

Model Averaging

- Model Average over Fit choices (Akaike Information Criterion)
- Provides more consistent results across $\mathcal{M}(\nu, z^2)$ values

$$w_i = \frac{e^{-\text{AIC}_i}}{\sum_i e^{-\text{AIC}_i}},$$

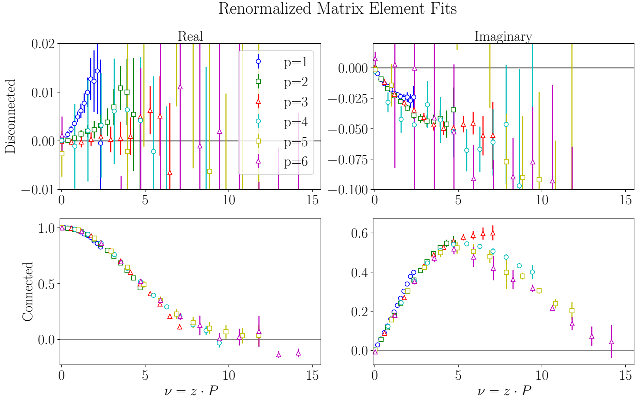
where $\text{AIC}_i = \chi_i^2 + 2k_i$



Building the Double Ratio (1/3)

$$\mathfrak{M}(\nu, z^2) = \left(\frac{\mathcal{M}(\nu, z^2)}{\mathcal{M}(\nu, 0)\big|_{z=0}} \right) \Bigg/ \left(\frac{\mathcal{M}(0, z^2)\big|_{p=0}}{\mathcal{M}(0, 0)\big|_{z=0, p=0}} \right)$$

- Use model averaged fit results to form double ratio
- Disconnected contribution is 0 at $\nu = 0$
- Use connected contribution for those terms
- Still provides appropriate renormalization



Building the Double Ratio (2/3)

$$\mathfrak{M}(\nu, z^2) = \left(\frac{\mathcal{M}(\nu, z^2)}{\mathcal{M}(\nu, 0)\big|_{z=0}} \right) \Big/ \left(\frac{\mathcal{M}(0, z^2)\big|_{p=0}}{\mathcal{M}(0, 0)\big|_{z=0,p=0}} \right)$$

- Use model averaged fit results to form double ratio
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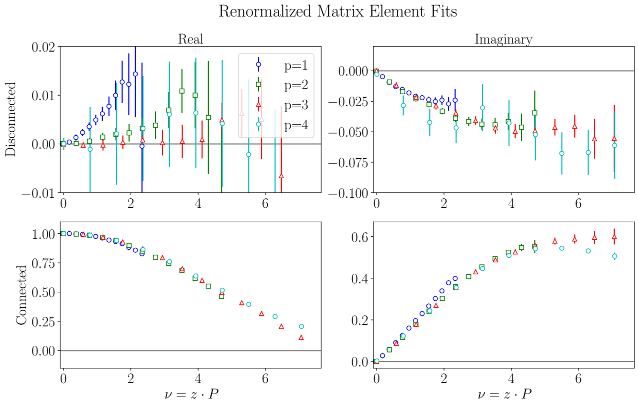
- Still provides appropriate renormalization



Building the Double Ratio (3/3)

$$\mathfrak{M}(\nu, z^2) = \left(\frac{\mathcal{M}(\nu, z^2)}{\mathcal{M}(\nu, 0)\big|_{z=0}} \right) \Big/ \left(\frac{\mathcal{M}(0, z^2)\big|_{p=0}}{\mathcal{M}(0, 0)\big|_{z=0, p=0}} \right)$$

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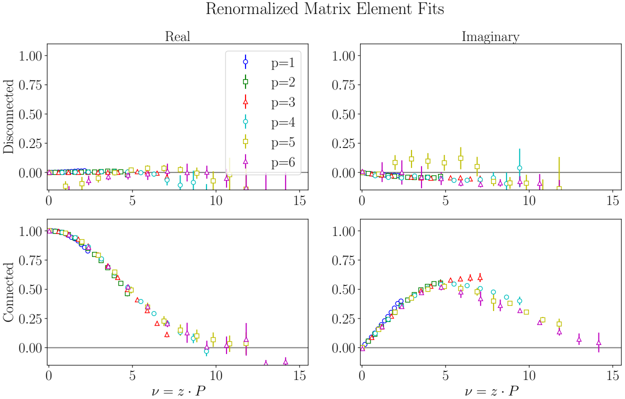
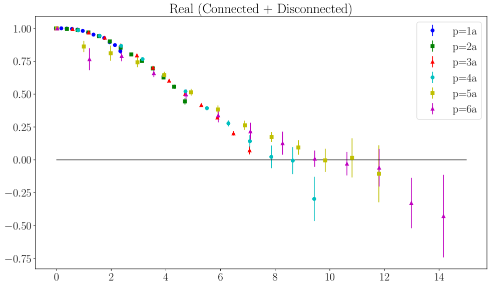
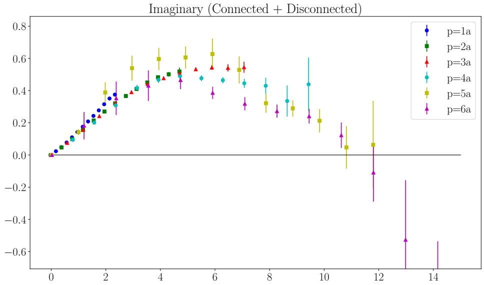


Full Ioffe-time Distribution

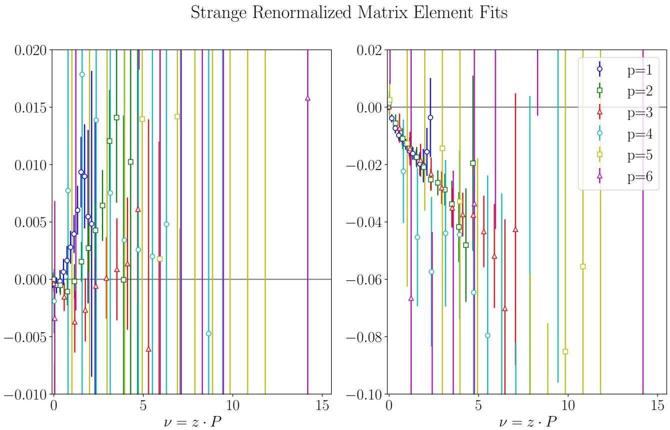




- Even with extremely increased sampling the disconnected contributions significantly more noisy than connected

Strange Quark Results

- Use the same light quark connected to renormalize with the Double Ratio prescription.



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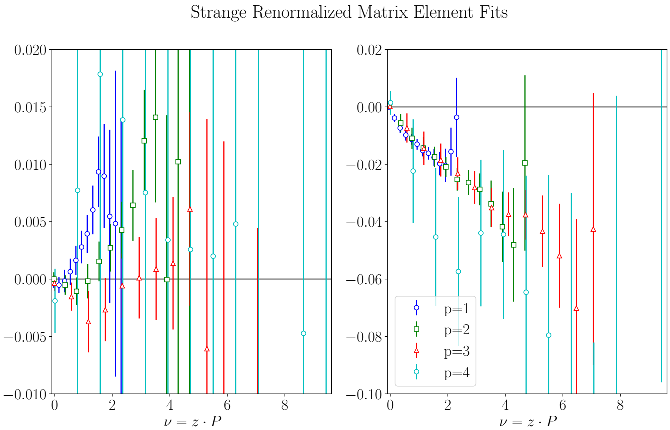
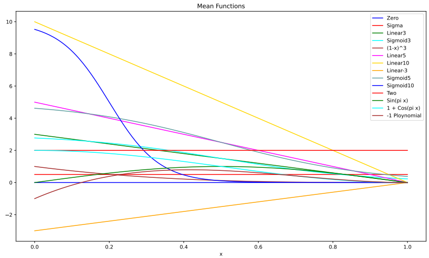


Fits to Pseudo-PDFs

Gaussian Processes

- Non-parametric regression method
- Does not assume a specific functional form
- Provides a mean and uncertainty estimates



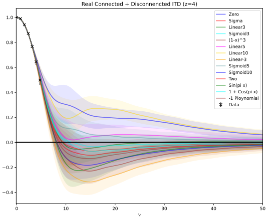
There are a handful of **Hyperparameters**[a]

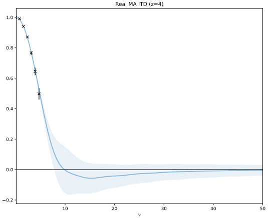
- $K, \quad \sigma, \quad l^2, \quad g(x)$

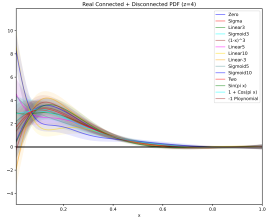
$$K(x,x') = \sigma^2 \exp\left(- \frac{(\ln(x) - \ln(x'))^2}{2l^2} \right)$$

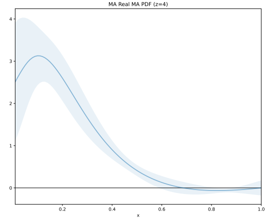
[a] Dutrieux et al., *A simple non-parametric reconstruction of parton distributions from limited Fourier information.*

Light Quark Results - Real

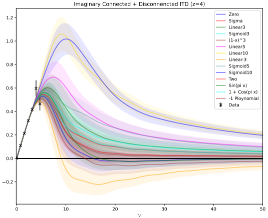


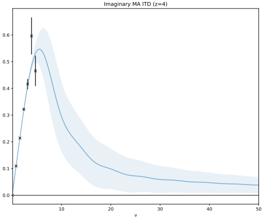


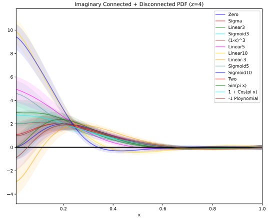


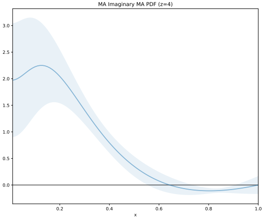


Light Quark Results - Imaginary

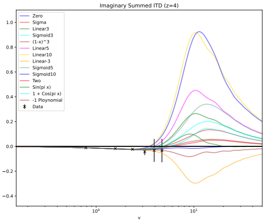


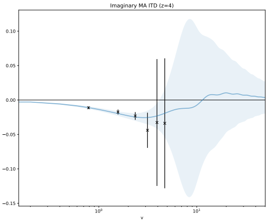


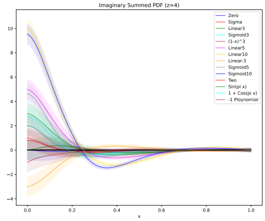


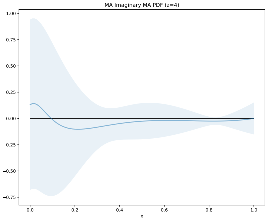


Strange Quark Results - Imaginary









Conclusion and Outlook

- **Summary:**
  - Extracted disconnected contributions to pseudo-ITDs for light and strange quarks.
  - Applied double ratio renormalization to remove power divergences.
  - Used Gaussian Process regression to reconstruct PDFs without model bias.
- **Key Findings:**
  - Disconnected contributions are small but nonzero.
  - Strange PDFs are entirely disconnected and show consistent structure.
  - Renormalized ITDs and PDFs are qualitatively similar to connected ones but noisier.
- **Next Steps:**
  - Improved statistics and variance reduction.
  - Full matching to $\overline{\text{MS}}$ scheme PDFs for phenomenological input.