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Better operators for boosted hadrons

Michael Wagman

QCD Evolution 2025

Jefferson Lab

May 21, 2025

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Kinematically-enhanced interpolating operators for boosted hadrons

Rui Zhang,[1,*] Anthony V. Grebe,[2] Daniel C. Hackett,[2] Michael L. Wagman,[2] and Yong Zhao[1]

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arXiv:2501.00729

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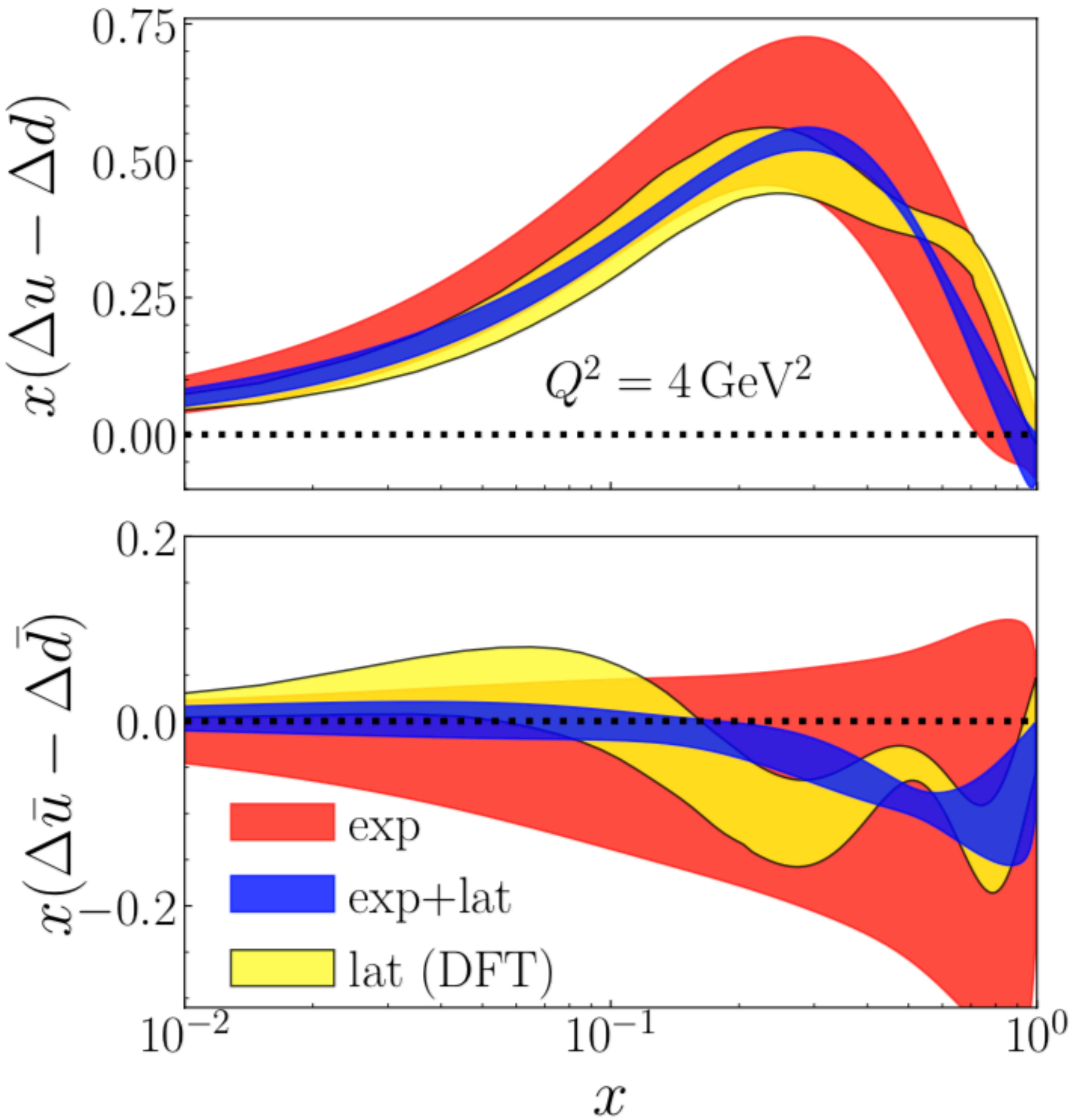
Argonne NATIONAL LABORATORY → Massachusetts Institute of Technology

Quasi PDFs

Large momentum effective theory (LaMET) connects light-cone PDFs to Euclidean matrix elements that can be calculated using lattice QCD

Review: Ji et al, Rev. Mod. Phys. 93, 35005 (2021)

Quasi PDF: $$\widetilde{q}(x,P_z) = \int_{-\infty}^{\infty} \frac{dz}{4\pi}\, e^{-ixzP_z}\, \langle h(P_z)|\overline{q}(z)\gamma_4 W(z,0)q(0)|h(P_Z)\rangle$$



Bringewatt et al [JAM], PRD 103 (2021)

For large P_z, quasi PDFs can be related to light-cone PDFs by perturbative matching coefficients

Increasingly refined quasi PDF calculations are being actively pursued

See Snowmass white paper arXiv:2202.07193

For e.g. isovector polarized nucleon PDFs, LQCD results can already improve global fits

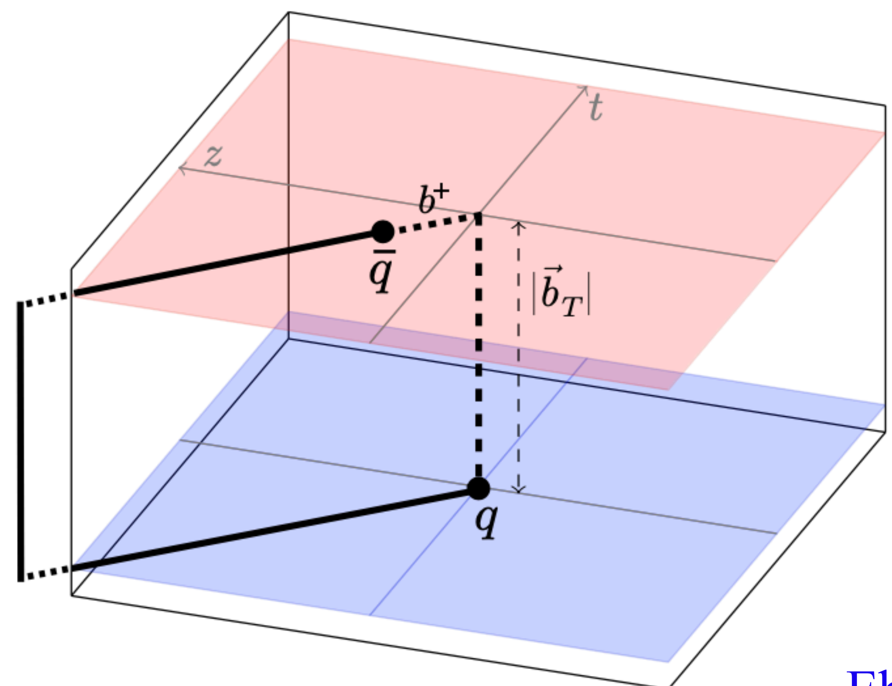
Quasi TMDPDFs

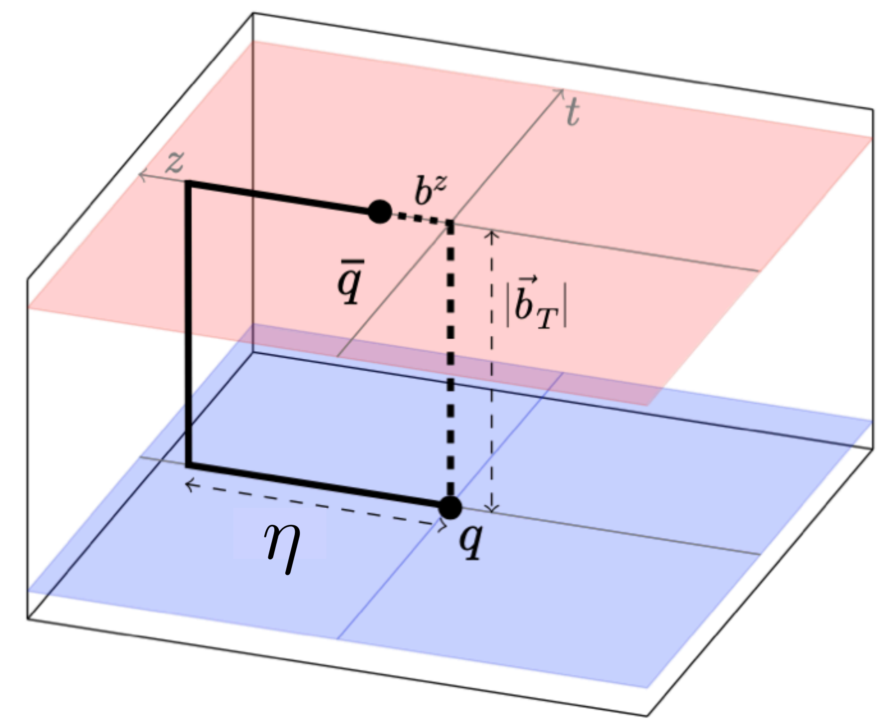
The construction of quasi TMDPDFs is more complicated than collinear PDFs

Ji, PRL 110 (2013)

Quasi beam functions can be constructed that are related to light-cone beam functions by a Lorentz boost

$$\widetilde{q}(x, b_T, P_z) = \lim_{\eta \to \infty} \int_{-\infty}^{\infty} \frac{dz}{4\pi}\, e^{-ixzP_z} \left\langle h(P_z) | \overline{q}(b_T) \gamma_4 W(b_T, \eta + b_T) W_T^\dagger(\eta + b_T, \eta) W_z^\dagger(\eta, 0) q(0) | h(P_Z) \right\rangle$$





Ebert, Stewart, Zhao, JHEP 1909 (2019)

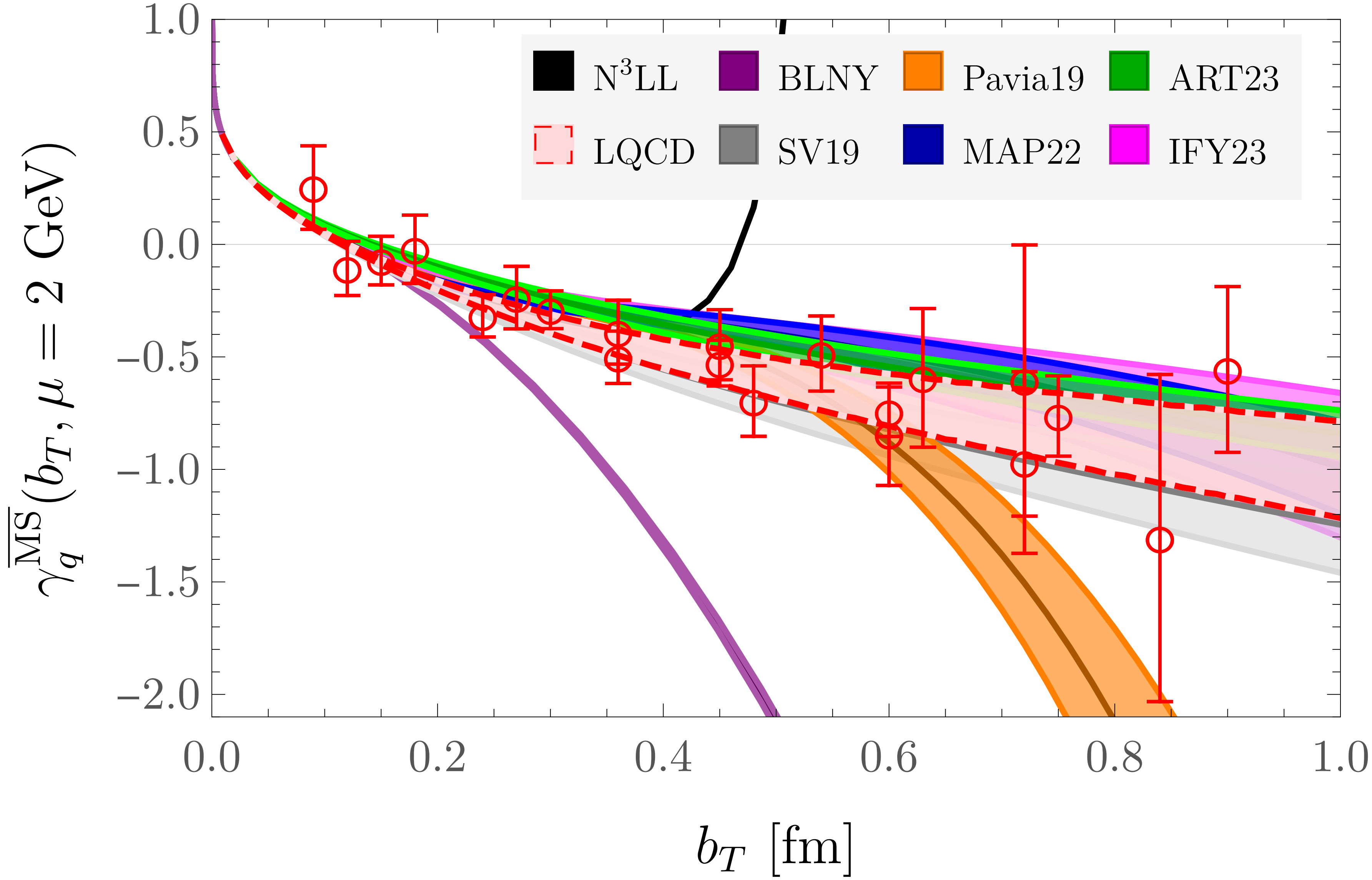
Rapidity evolution of TMDPDFs governed by Collins-Soper kernel can be computed from ratios of quasi TMDPFs where many complications cancel

The CS kernel from LQCD

Collins-Soper kernel from LQCD with fully quantified systematic uncertainties:

- Continuum extrapolation from 3 lattice spacings
- All ensembles use physical pion masses
- "Brute force" DFT from b_z to x space, truncation effects explicitly studied

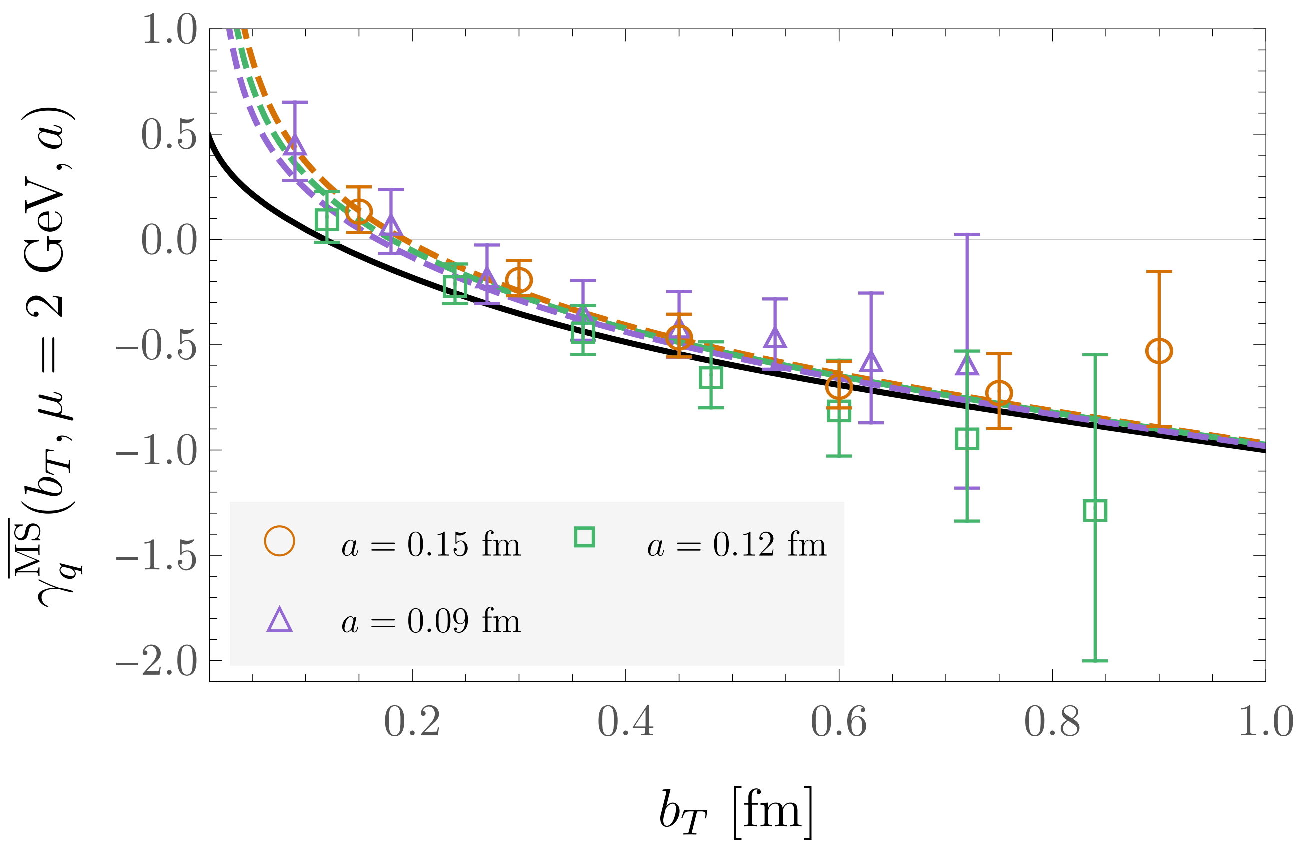
Avkhadiev, Shanahan, MW, Zhao, PRL 132 (2024)

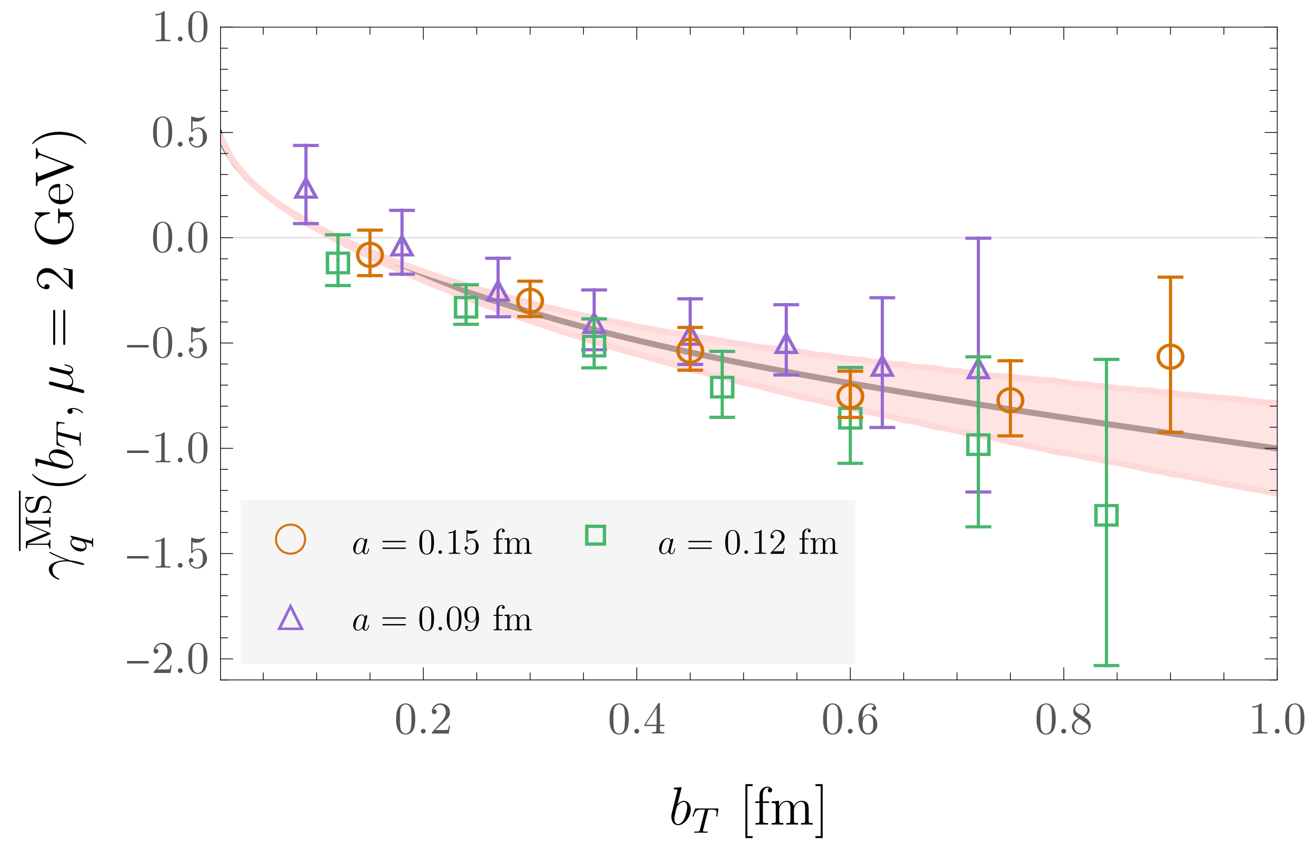


The continuum limit

Discretization effects subtracted by fitting to parameterization of continuum CS kernel + lattice artifacts

$$\hat{\gamma}_q^{\overline{\rm MS}}(b_T, \mu, a) = \gamma_q^{\overline{\rm MS}}(b_T, \mu) + k_1 \frac{a}{b_T}$$





- Variety of other parameterizations, e.g. with $(a/b_T)^2$ terms, explored
- AIC used for data-driven model selection

Avkhadiev, Shanahan, MW, Zhao, PRL 132 (2024)

LQCD results

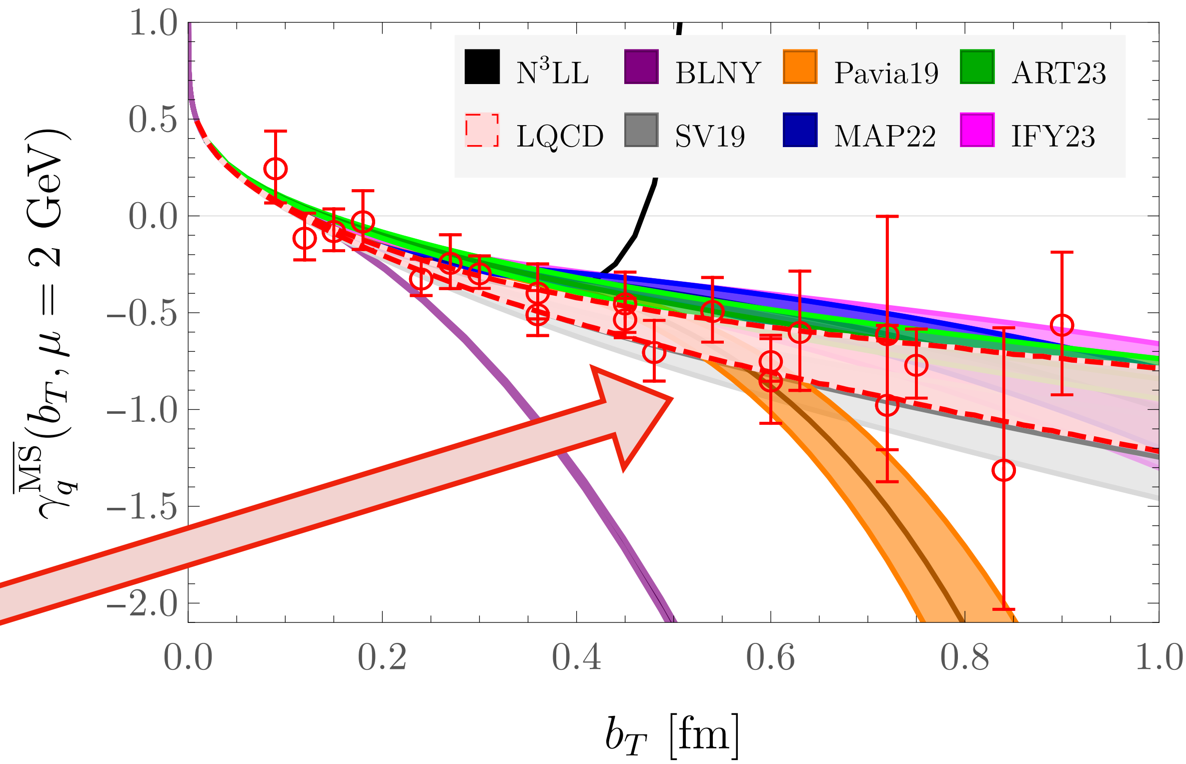
Continuum-limit LQCD results can be directly included in future global fits by fixing CS kernel to pQCD+LQCD parameterization

$$\gamma_q^{\overline{\rm MS}}(b_T,\mu) = -2\mathcal{D}_{\rm res}(b^*(b_T),\mu) - 2\mathcal{D}_{\rm NP}(b_T)$$

$$b^*(b_T) = b_T/\sqrt{1+b_T^2/(2\ {\rm GeV})^2}$$

$$\mathcal{D}_{\rm res}(b^*,\mu) = \int_{\mu_{b^*}}^{\mu} \frac{d\mu'}{\mu'}\Gamma_{\rm cusp}[\alpha_s(\mu')] + d[\alpha_s(\mu_{b^*})]$$

Avkhadiev, Shanahan, MW, Zhao, PRL 132 (2024)



Nonperturbative effects can summarized by one parameter fit to LQCD data:

$$\mathcal{D}_{\rm NP}(b_T) = c_0 b_T b^*(b_T)$$

$$c_0 = 0.32(12)$$

$$\chi^2/{\rm dof} = 0.4$$

Other parameterizations also give acceptable fits, e.g. hadron structure oriented (HSO)

Aslan et al, PRD 110 (2024)

$$b_K = 0.63(19)$$

$$\chi^2/{\rm dof} = 0.4$$

LQCD for LaMET is hard

CS kernel is relatively easy; still took 5 years and insights from multiple collaborations

Shanahan, MW, Zhao, PRD 102 (2020)

Zhang et al [LPC], PRL 125 (2020)

Schlemmer et al, JHEP 08 (2021)

Li et al [ETMC+], PRL 128 (2022)

Shanahan, MW, Zhao, PRD 104 (2021)

Chu et al [LPC], PRD 106 (2022)

Shu et al, PRD 108 (2023)

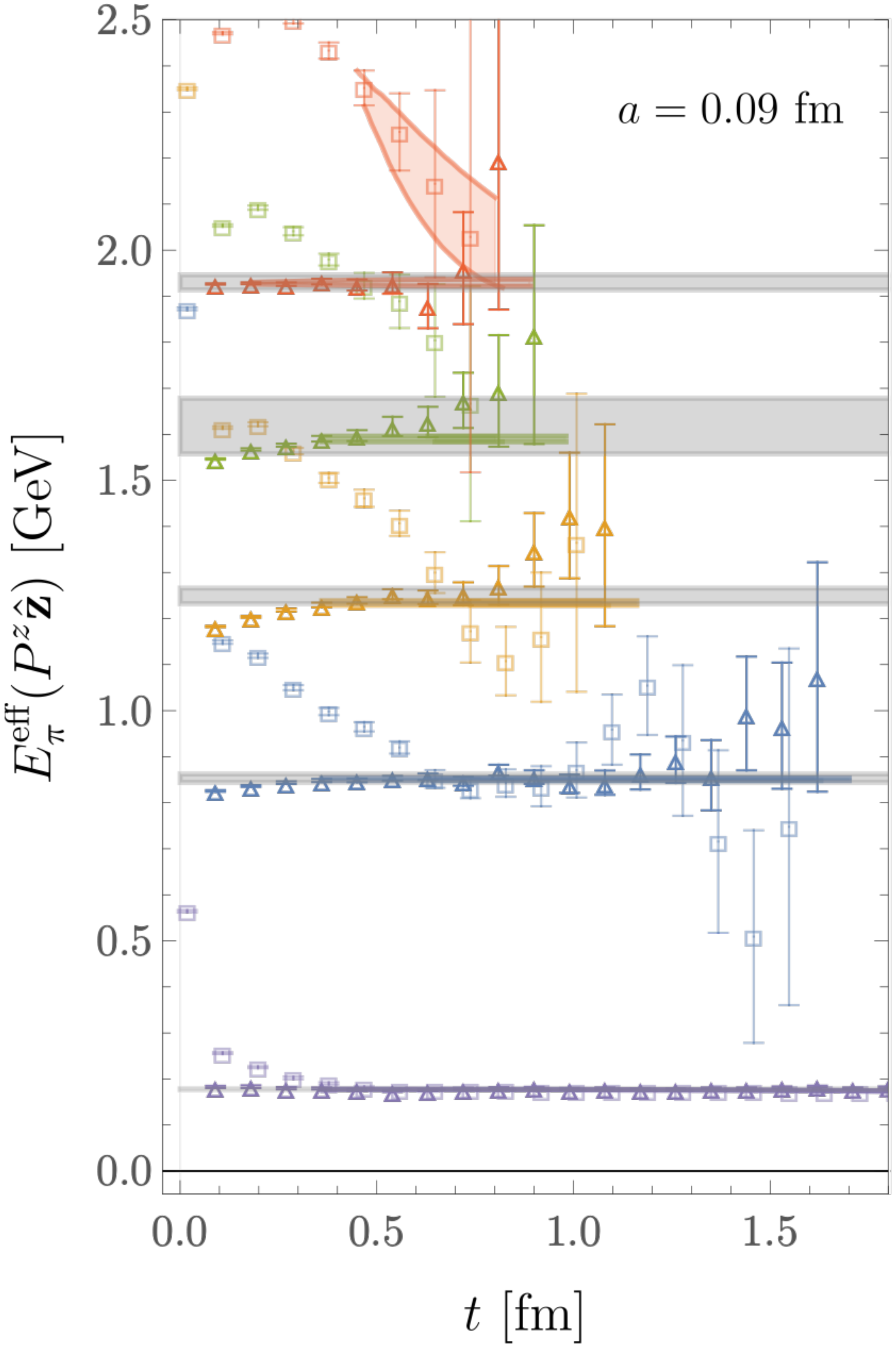
ASWZ, PRD 108 (2023)

ASWZ, PRL 132 (2024)

- Universality lets CS kernel calculations use pion states instead of nucleon states
- Renormalization effects largely cancel in ratios governing rapidity evolution
- Computable from TMD wavefunctions — 2-point vs 3-point functions

What's so hard about large momentum LQCD calculations?

Boosted states are noisy



Exponential signal-to-noise degradation common to all large-momentum correlation functions used for LaMET

e.g. pion state

$$\langle C_\pi \rangle \sim e^{-E(P)t}$$

$$\mathrm{SNR}(C_\pi) \sim e^{-[E(P)-m_\pi]t}$$

Avkhadiev, Shanahan, MW, Zhao, PRL 132 (2024)

Parisi-Lepage for boosted states

Boosted pion correlation functions are complex in background gauge fields

$$C_\pi = \sum_{\vec{x}} \pi(\vec{x},t)\pi^\dagger(0)e^{i\vec{P}\cdot\vec{x}} \qquad \langle C_\pi \rangle \sim e^{-E(P)t}$$

Variance of real part includes distinct contributions:

$$\mathrm{Var}(C_\pi) = \left\langle [\mathrm{Re}(C_\pi)]^2 \right\rangle - \langle C_\pi \rangle^2$$

$$= \frac{1}{2}\left\langle |C_\pi|^2 \right\rangle + \frac{1}{2}\left\langle C_\pi^2 \right\rangle - \langle C_\pi \rangle^2$$

Parisi, Phys. Rept. 103 (1984)

Lepage, TASI (1989)

At large t, variance dominated by first term describing two pions at rest

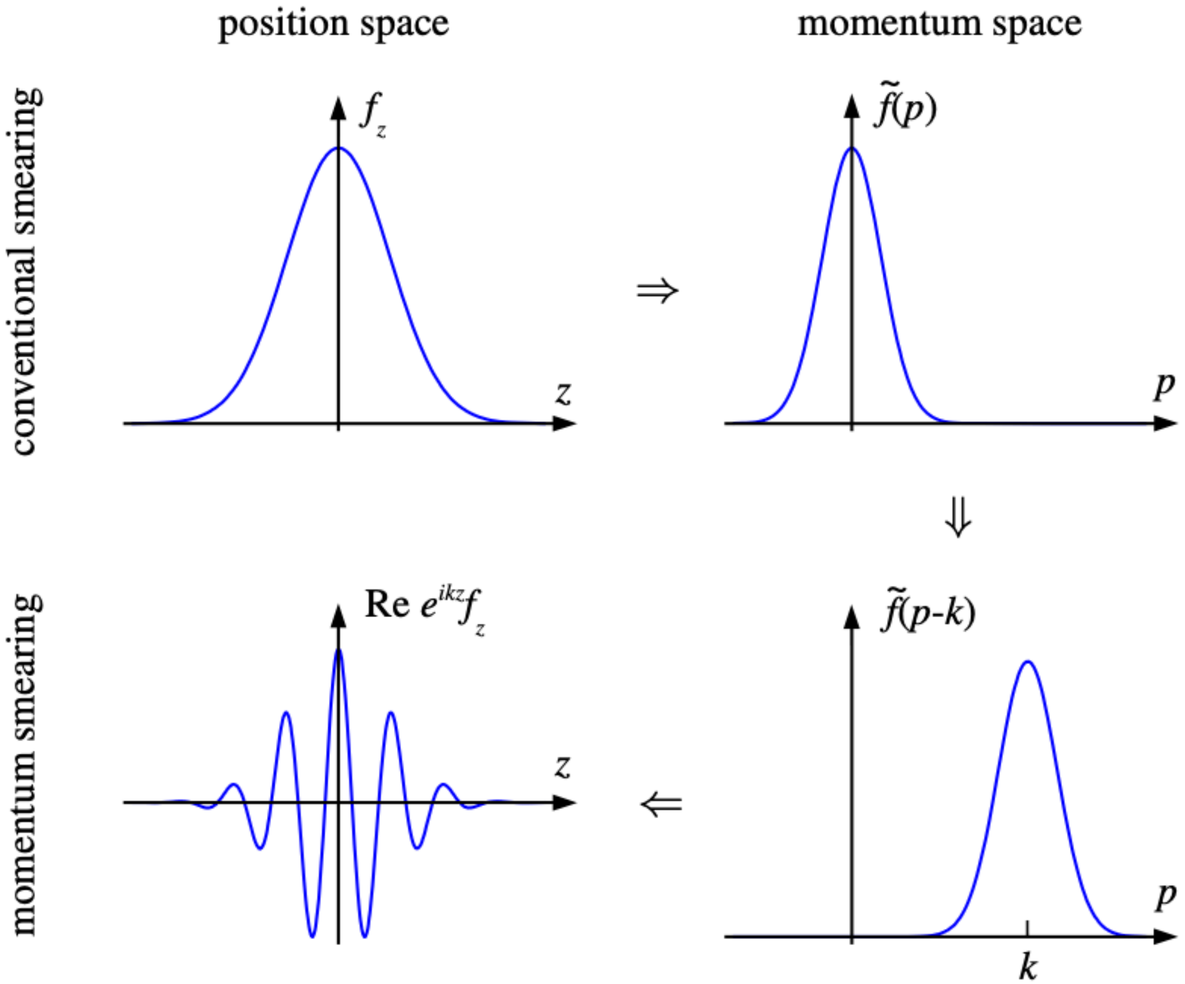
$$|C_\pi|^2 = \sum_{\vec{x},\vec{y}} \pi(\vec{x},t)\pi^\dagger(\vec{y},t)\pi^\dagger(0)\pi(0)e^{i\vec{P}\cdot(\vec{x}-\vec{y})}$$

$$\mathrm{Var}(C_\pi) \sim e^{-2m_\pi t}$$

Momentum smearing

Precision of highly boosted states in LQCD greatly enhanced by using operators with momentum smearing

Bali, Lang, Musch, Schäfer [RQCD], PRD 93 (2016)



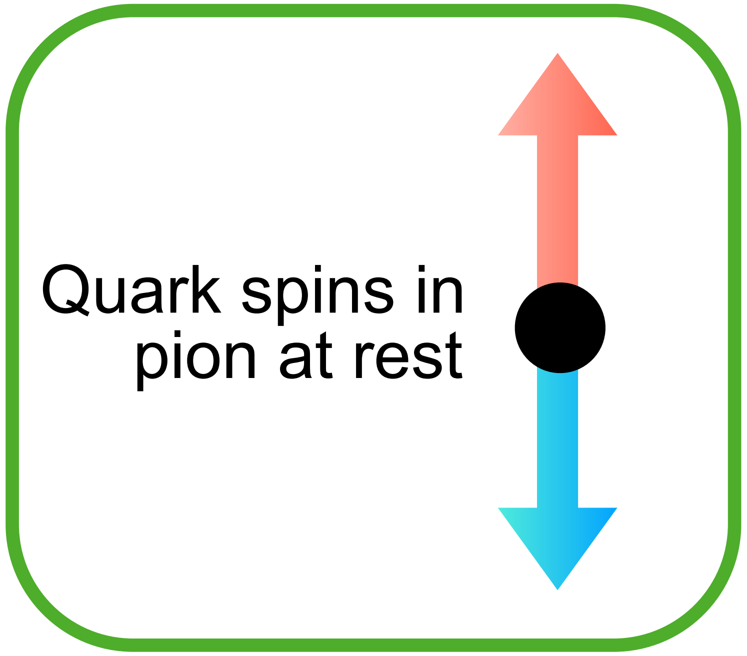
Gaussian wavefunctions for quarks at rest have poor overlap with high-momentum quark states

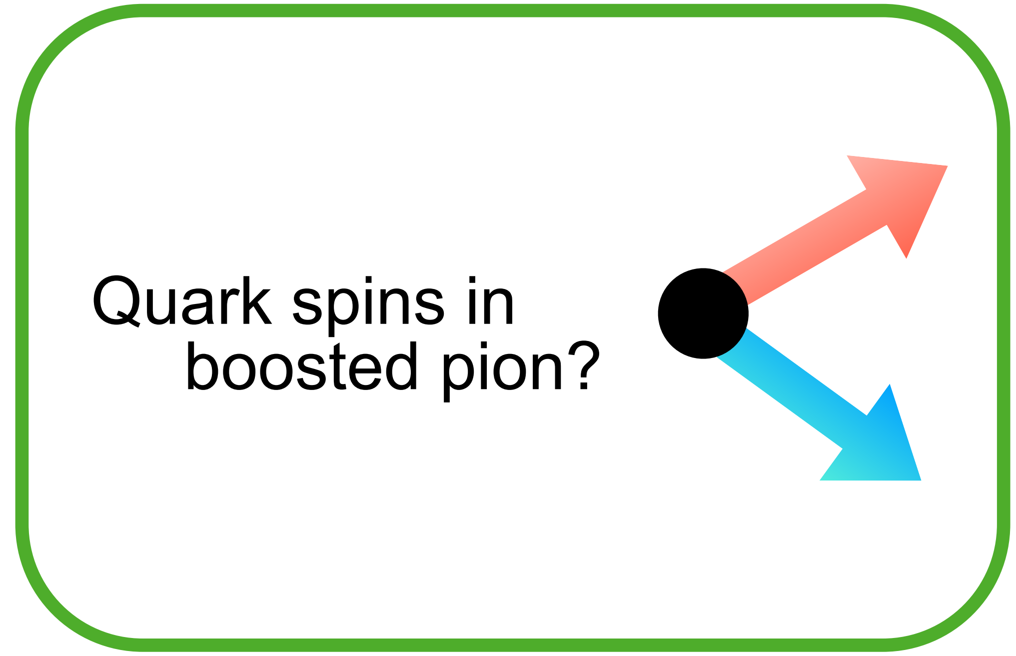
Adding non-zero mean to momentum-space Gaussian wave function leads to much larger overlap and better SNR for highly boosted hadron states

Outperforms other physically motivated wavefunctions previously explored such as Lorentz contracted “pancake” wavefunctions

Roberts et al, PRD 86 (2012) Della Morte, Jaeger, Rae, Wittig, Eur. Phys. J. A 48 (2012)

What about spin?





Formally, light-cone dynamics dominated by ψ_+ spinor components

Burkardt, Ji, Yuan, Phys. Lett. B 545 (2002)

Xi, Ma, Yuan, Eur. Phys. J. C 33 (2004)

$$\psi = \psi_+ + \psi_-$$

$$\psi_\pm = \frac{1}{\sqrt{2}}\gamma_\mp\gamma_\pm\psi$$

Pion operator constructed from ψ_+ components:

$$u_+^\dagger \gamma_5 d_+ = \sqrt{2}\,\overline{u}\gamma_+\gamma_5 d$$

Lepage, Brodsky, Phys. Lett. B 87 (1979)

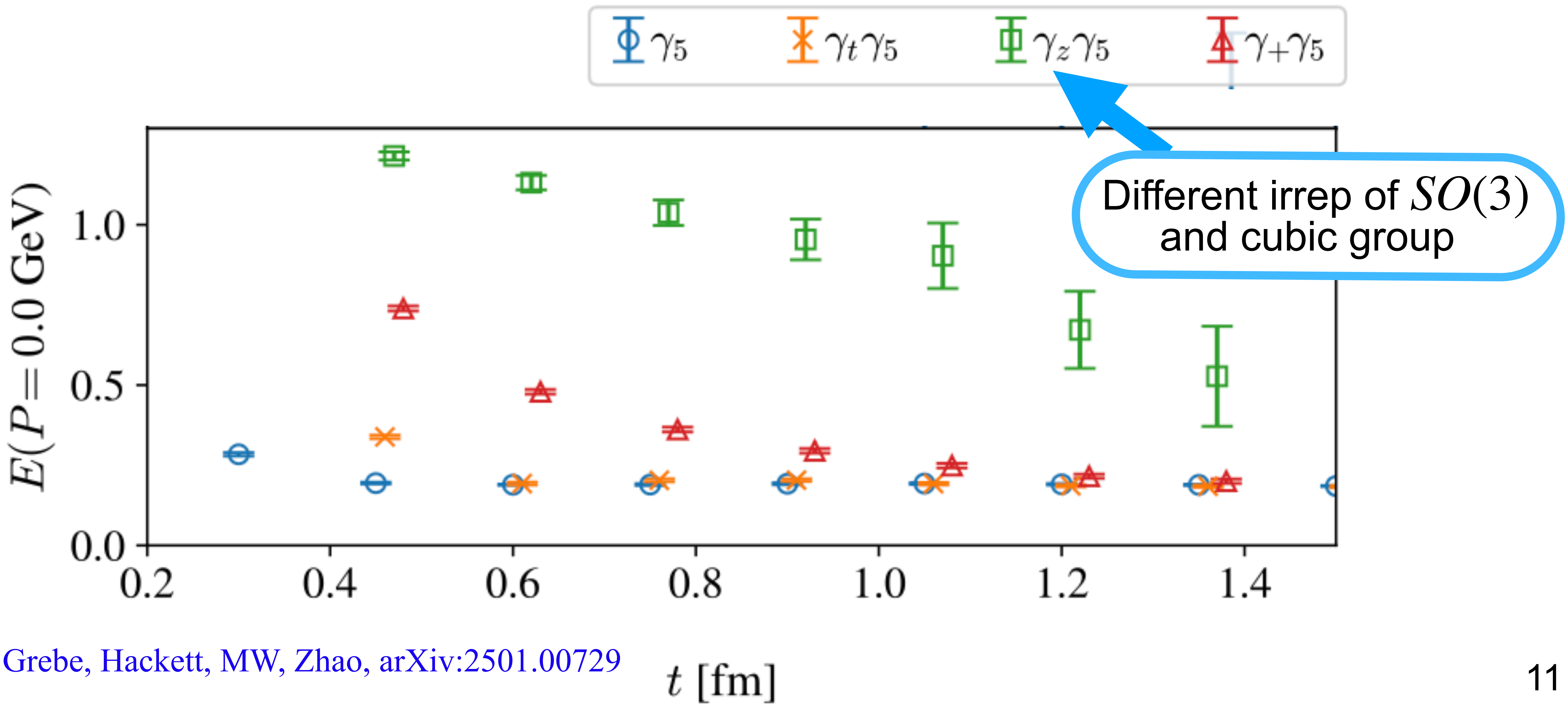
Efremov, Radyushkin, Phys. Lett. B 94 (1980)

ψ_+ pions, Euclidean version

Wick rotation to Euclidean spacetime straightforward for ψ_+ pion operator

$$\overline{u}\gamma_+\gamma_5 d \propto \overline{u}(\gamma_z + \gamma_t)\gamma_5 d$$

For a pion at rest, this is a much worse operator than the usual $\overline{u}\gamma_5 d$



Zhang, Grebe, Hackett, MW, Zhao, arXiv:2501.00729

Boosted ψ_+ pions

Wick rotation to Euclidean spacetime straightforward for ψ_+ pion operator

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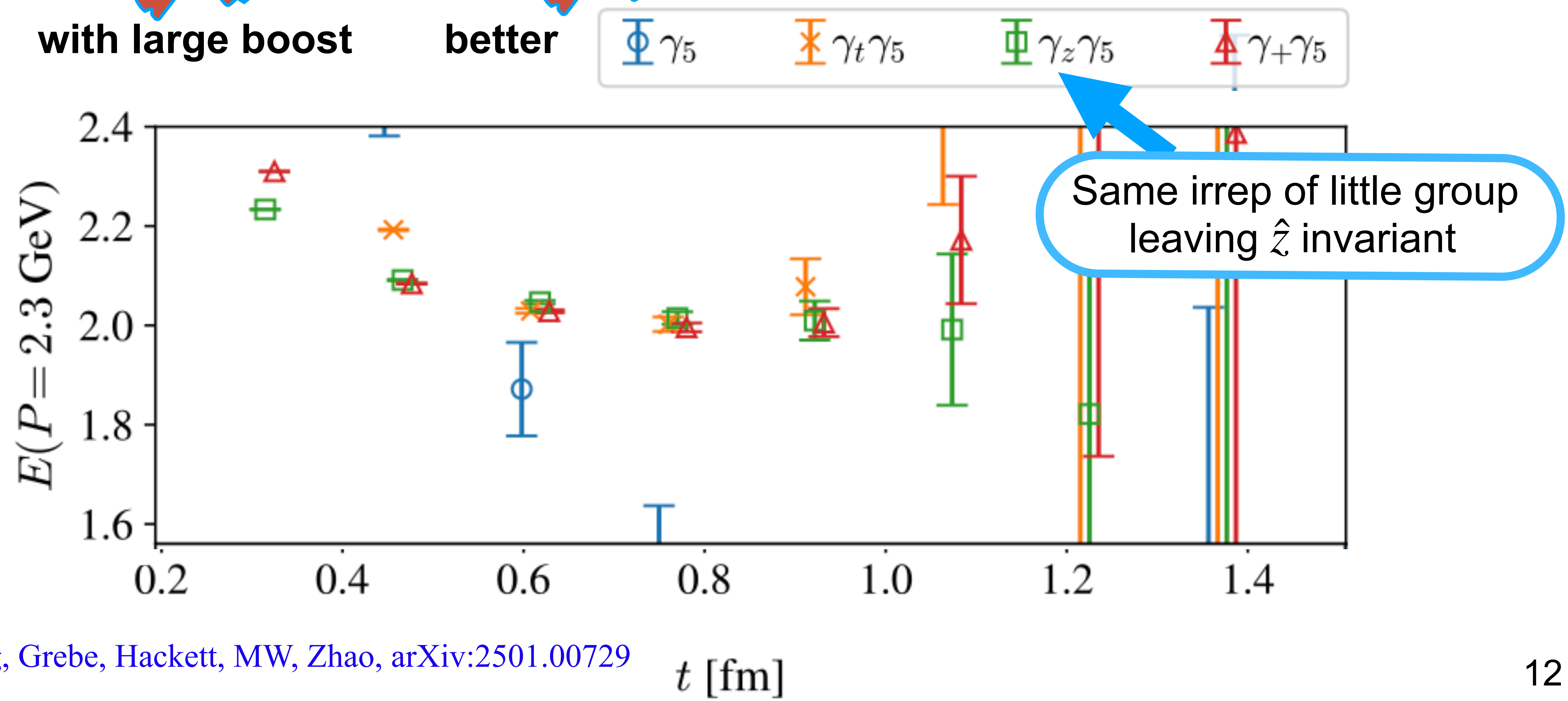
For a pion at rest, this is a much worse operator than the usual $\overline{u}\gamma_5 d$

with large boost **better**



Zhang, Grebe, Hackett, MW, Zhao, arXiv:2501.00729

Signal enhancement

Lorentz covariance of axial-vector opertors:

$$\langle \pi | \overline{d}\gamma_\mu \gamma_5 u | \Omega \rangle \approx iZ f_\pi P_\mu$$

Partially conserved axial current (PCAC) relation:

$$\langle \pi | \overline{d}\gamma_5 u | \Omega \rangle \approx \frac{1}{m_q} \partial^\mu \langle \pi | \overline{d}\gamma_\mu \gamma_5 u | \Omega \rangle \approx iZ f_\pi m_\pi^2 / m_q$$

Together imply $O(P^2/m_\pi^2)$ enhancement of ground-state overlap (="signal") for axial-vector over pseudoscalar operators

$$\frac{\mathrm{Signal}[\overline{u}\gamma_\mu \gamma_5 d]}{\mathrm{Signal}[\overline{u}\gamma_5 d]} \approx \frac{P_\mu^2 m_q^2}{m_\pi^4} \propto \frac{P_\mu^2}{m_\pi^2}$$

Zhang, Grebe, Hackett, MW, Zhao, arXiv:2501.00729

SNR enhancement

Same logic leading to signal enhancement applies to variance correlator…but variance dominated by pions at rest!

$$\frac{\mathrm{Signal}[\overline{u}\gamma_\mu\gamma_5 d]}{\mathrm{Signal}[\overline{u}\gamma_5 d]} \sim \frac{P_\mu^2}{m_\pi^2}$$

$$\frac{\mathrm{Noise}[\overline{u}\gamma_\mu\gamma_5 d]}{\mathrm{Noise}[\overline{u}\gamma_5 d]} \sim \frac{m_\pi^2}{m_\pi^2} \sim 1$$

Signal enhancement translates into SNR enhancement

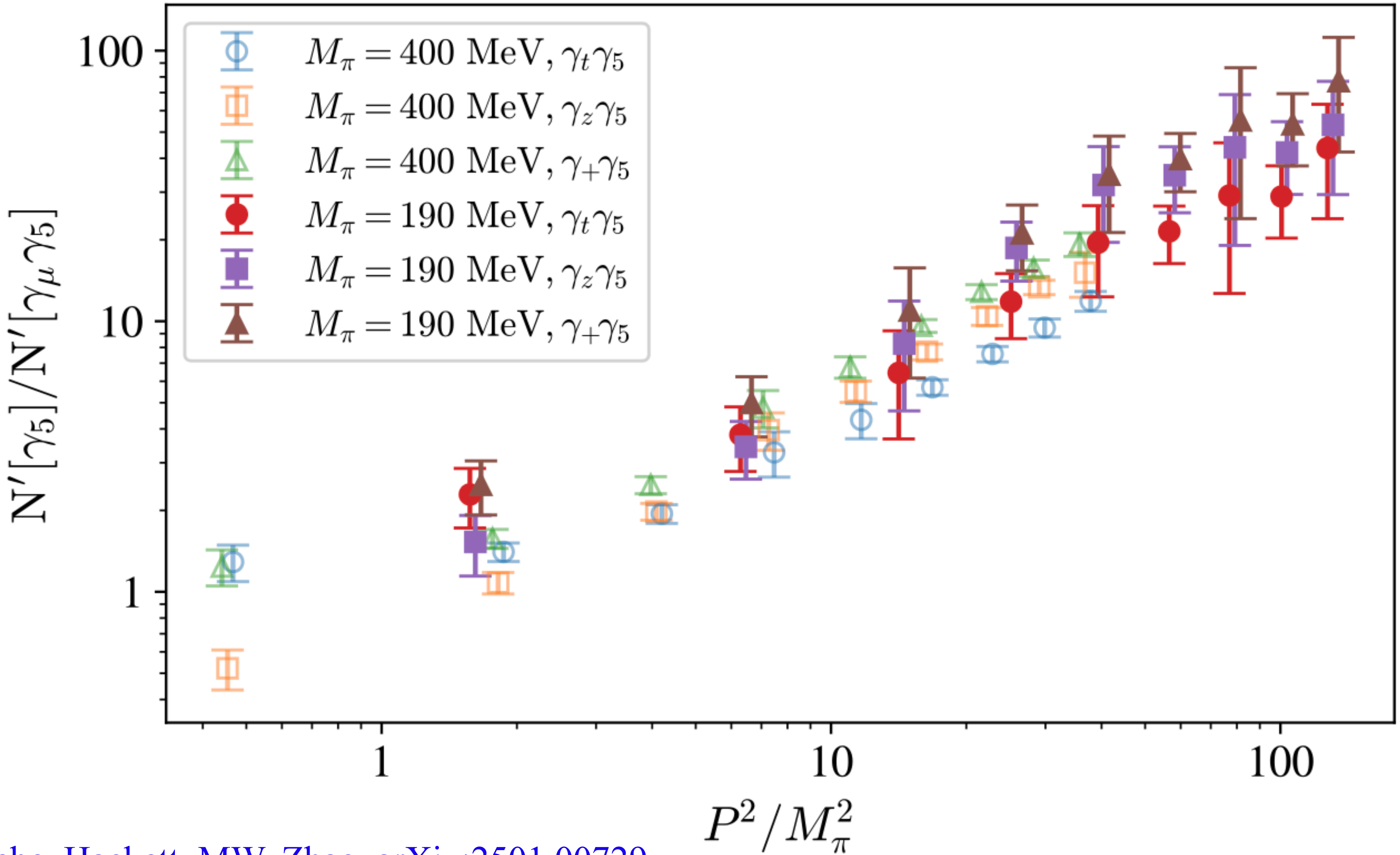
$$\frac{\mathrm{SNR}[\overline{u}\gamma_\mu\gamma_5 d]}{\mathrm{SNR}[\overline{u}\gamma_5 d]} \sim \frac{P_\mu^2}{m_\pi^2}$$

Zhang, Grebe, Hackett, MW, Zhao, arXiv:2501.00729

Scaling

Numerically observed SNR enhancement consistent with theoretical expectation

$$\frac{\mathrm{SNR}[\overline{u}\gamma_\mu\gamma_5 d]}{\mathrm{SNR}[\overline{u}\gamma_5 d]} \sim \frac{P_\mu^2}{m_\pi^2}$$



Zhang, Grebe, Hackett, MW, Zhao, arXiv:2501.00729

Ground-state energy improvement

$$\overline{u}\gamma_+\gamma_5 d \propto \overline{u}(\gamma_3 + \gamma_4)\gamma_5 d$$

Ground-state energies extracted using filtered Lanczos (= Prony = GPOF = Rayleigh-Ritz)

MW, arXiv:2406.20009, *accepted by* PRL

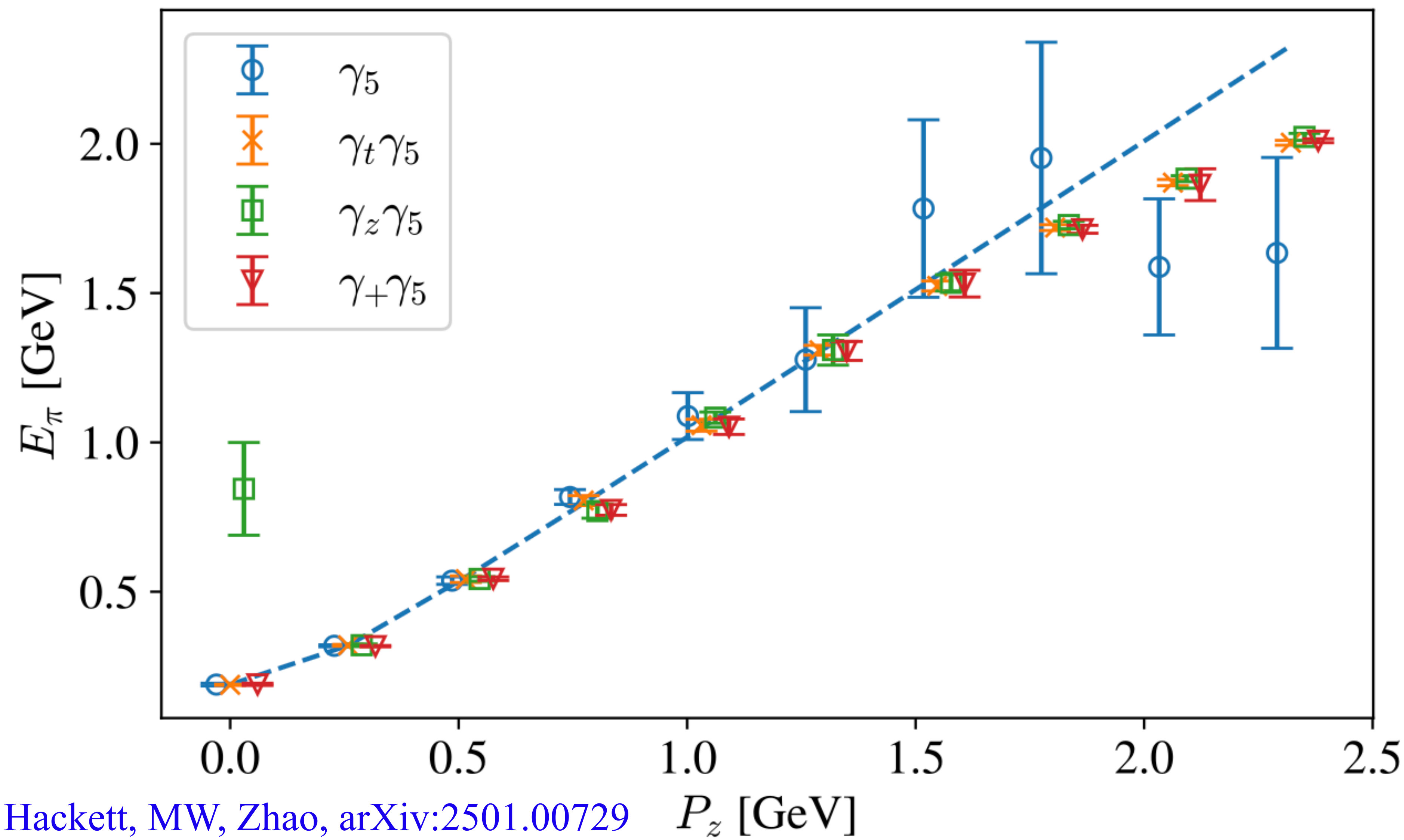
Hackett, MW, arXiv:2407.21777

Ostmeyer et al, Eur. Phys. J. A 61 (2025)

Chakraborty et al, arXiv:2412.01900

Hackett, MW, arXiv:2412.04444, *accepted by* PRD

Abbott, Hackett, Fleming, Pefkou, MW, arXiv:2503.17357



Zhang, Grebe, Hackett, MW, Zhao, arXiv:2501.00729

KPS convergence theory

Lanczos converges exponentially faster than power iteration (= effective mass) for transfer matrices with small gaps (e.g. for small a)

Kaniel, Mathematics of Computation 20, 369 (1966)

Paige, PhD thesis 1971

Saad, SIAM 17 (1980)

$$\delta = a(E_1 - E_0)$$

$$\left|E_0 - E_0^{(m)}\right| \propto e^{-2t\sqrt{\delta}}$$

Lanczos

$$\left|E_0 - E_0^{\text{eff}}(t)\right| \propto e^{-t\delta}$$

Power iteration

- Approximate form valid near continuum limit where $1 \gg \sqrt{\delta} \gg \delta$

- Prony / GPOF have identical convergence, but the rate wasn't known before

Block Lanczos converges exponentially faster than GEVP

$$\delta_r = a(E_r - E_0)$$

Saad, SIAM 17 (1980)

$$\left|E_0 - E_0^{(m)}\right| \propto e^{-2t\sqrt{\delta_r}}$$

Block Lanczos

$$\left|E_0 - E_0^{\text{GEVP}}(t)\right| \propto e^{-t\delta_r}$$

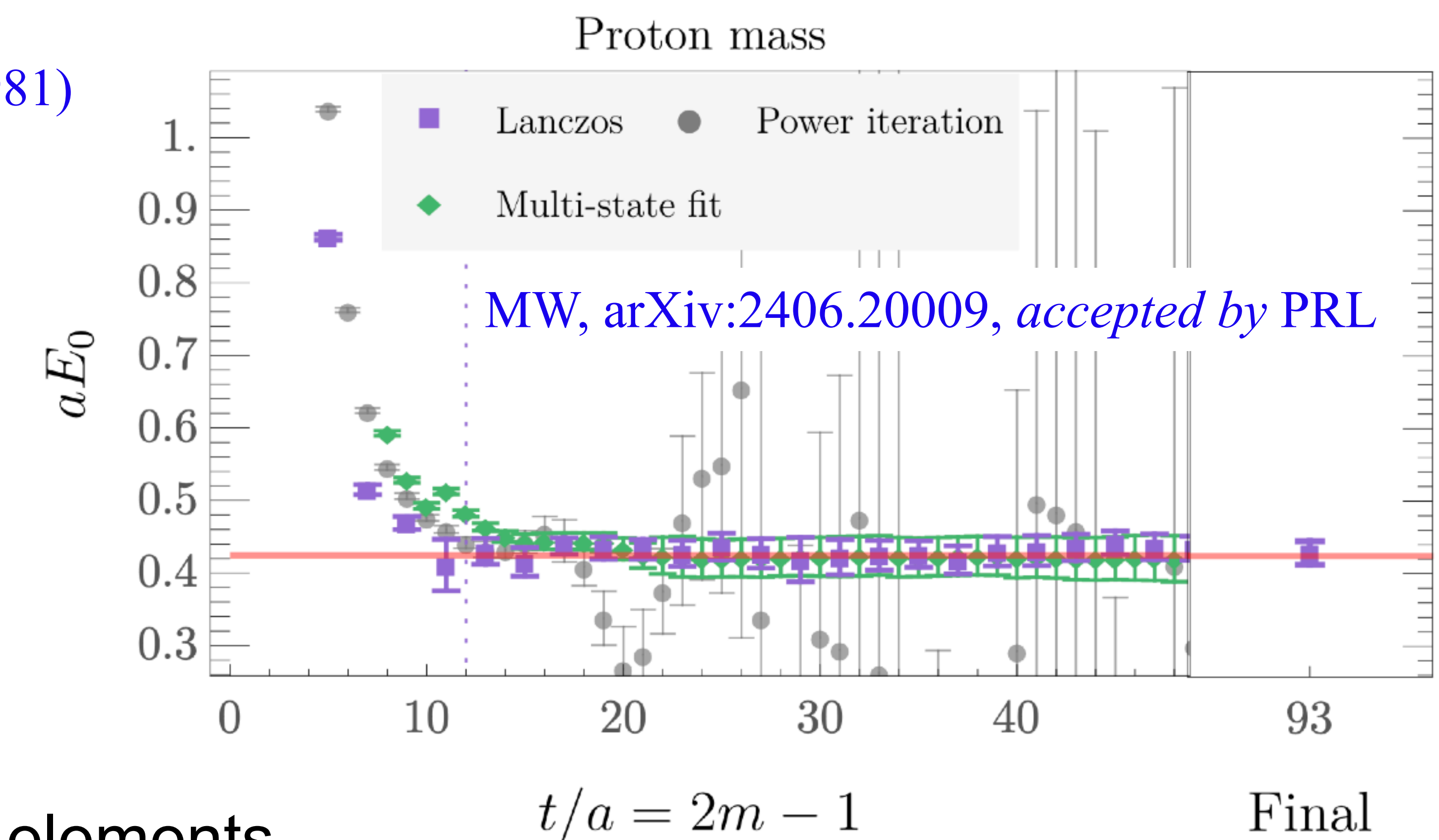
GEVP

Physical noise filtering

"Spurious states" arising from noise can be removed using arcane mathematics or simple physics — demand real energies and non-zero overlap with initial state

Cullum and Willoughby, J. Comp. Phys. 44, 329 (1981)

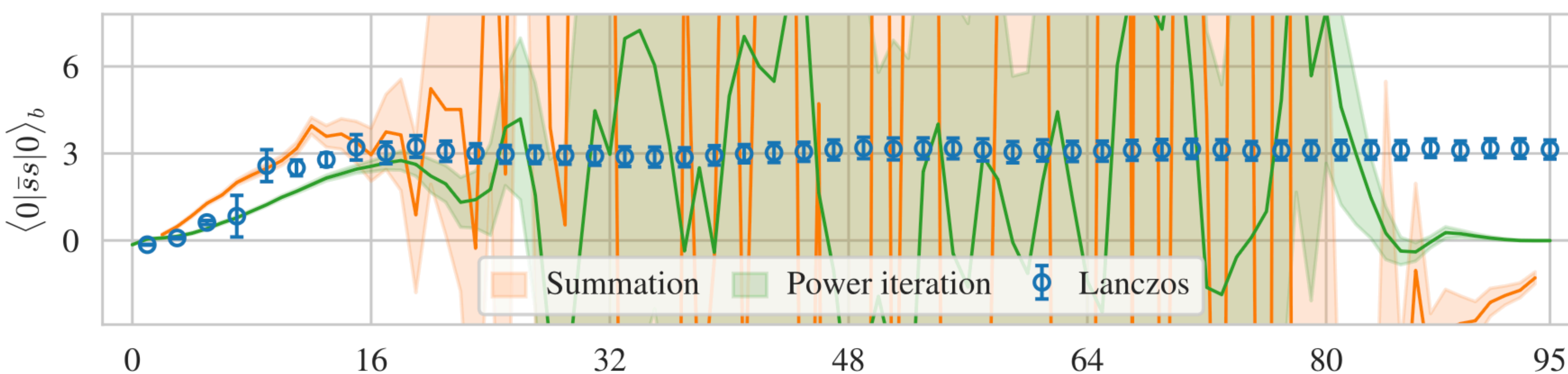
Hackett, MW, arXiv:2412.04444, *accepted by* PRD



Using bootstrap median estimators, results become highly correlated with constant SNR for large t

— No fitting needed

Simple and robust estimators for matrix elements



Hackett, MW, arXiv:2407.21777 $t = 2m - 1$

Residual bounds

- Lanczos approximation error after finite number of iterations directly computable:

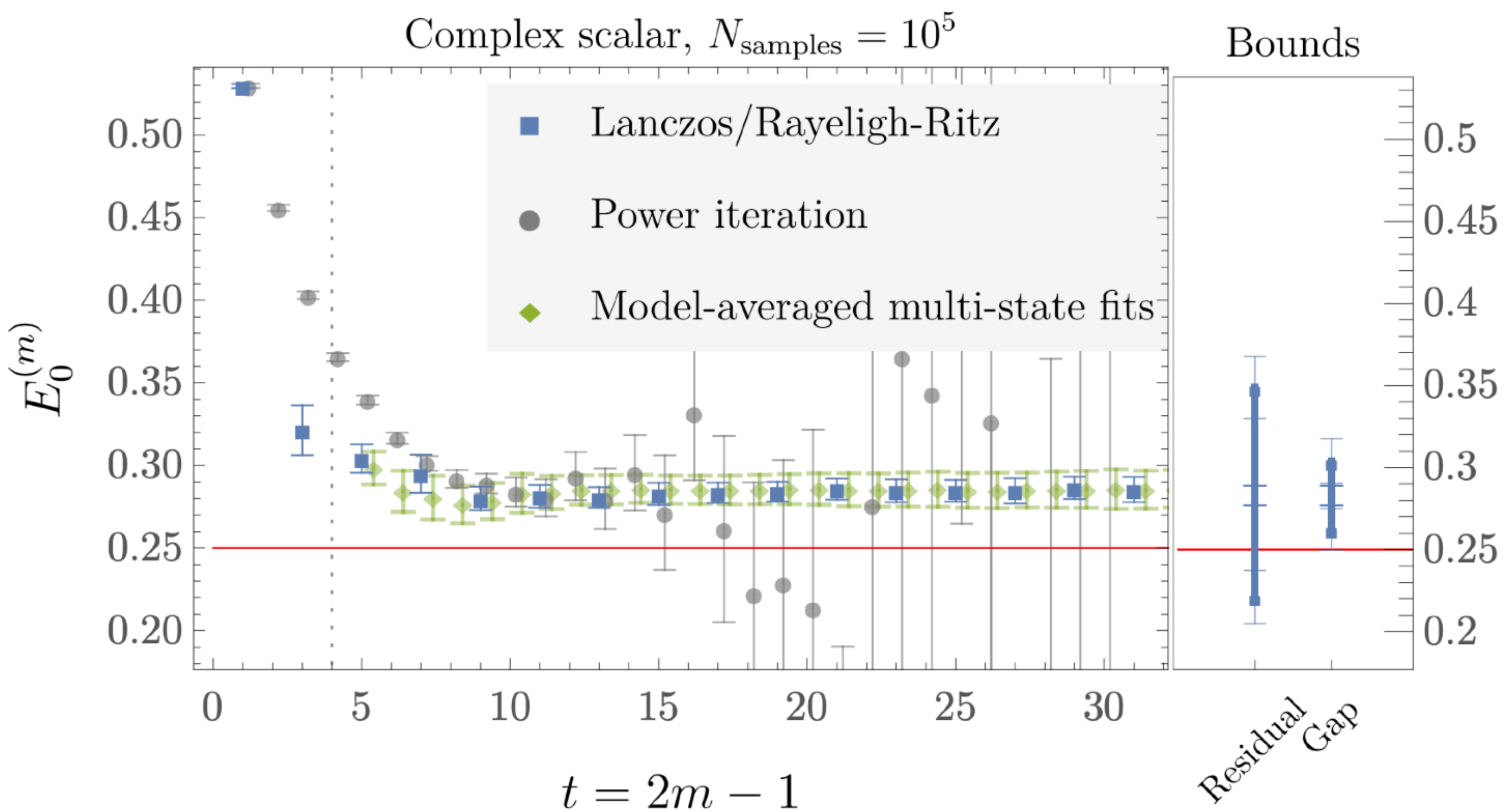
$$\min_{\lambda \in \{\lambda_n\}} |\lambda_0^{(m)} - \lambda| \leq |\beta_{m+1} \omega_{m0}^{(m)}|$$

Eigenvectors of $T^{(m)}$

Matrix element $T^{(m)}_{m(m+1)}$

See Parlett, The Symmetric Eigenvalue Problem (1980)

Rigorous quantification of excited-state effects!



ψ_+ nucleons

Generic nucleon operator:

$$N_\Gamma = \epsilon_{abc}(d_a^T C\Gamma u_b)(1+\gamma_4)u_c$$

Leading-twist spin:

$$\Gamma \in \{\gamma_+, \gamma_5\gamma_+\}$$

Burkardt, Ji, Yuan, Phys. Lett. B 545 (2002)

Xi, Ma, Yuan, Eur. Phys. J. C 33 (2004)

Braun et al, Null. Phys. B 589 (2000)

Zanotti et al [CSSM], PRD 68 (2003)

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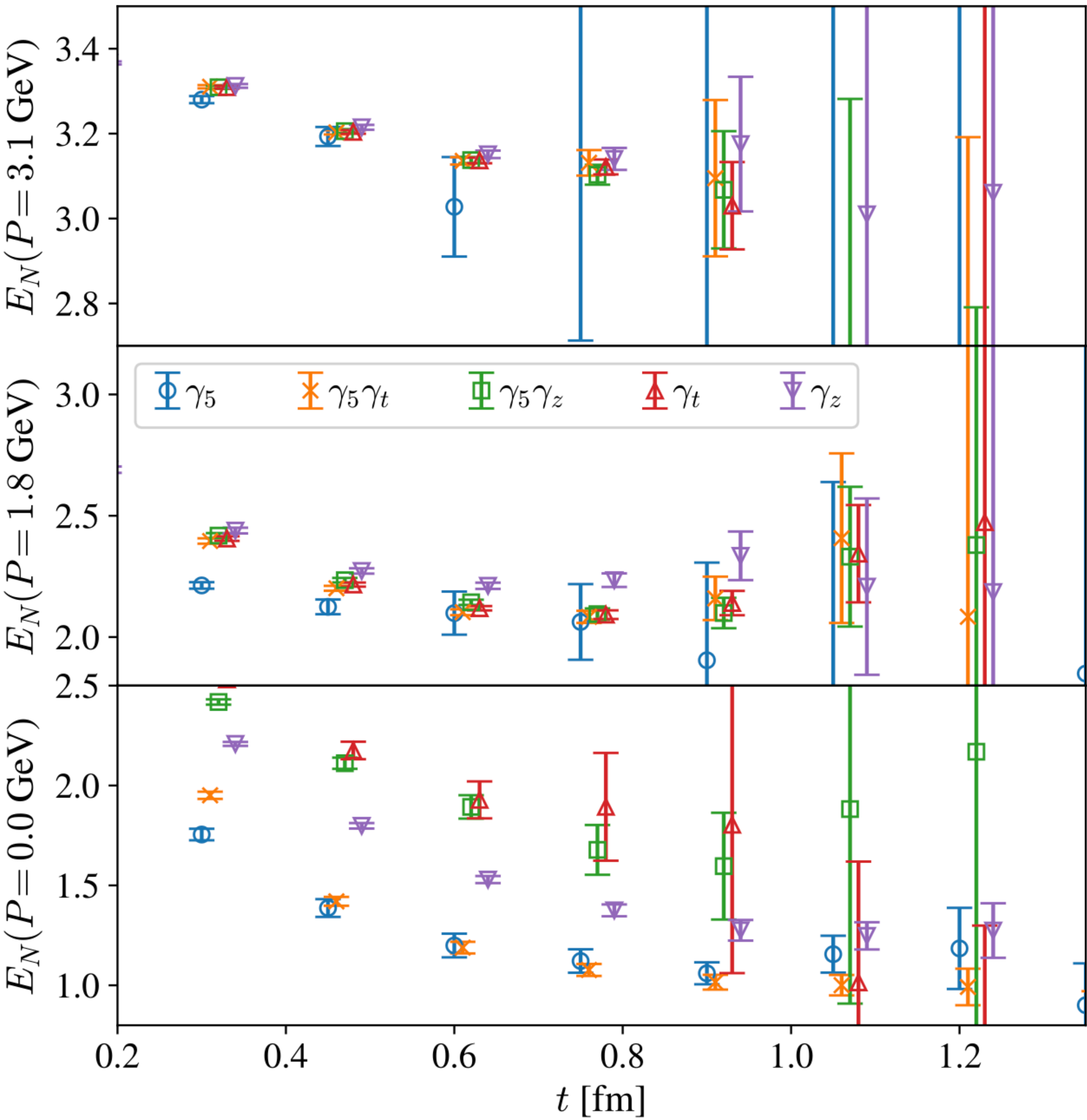
Braun et al, Null. Phys. B 589 (2000)

Zanotti et al [CSSM], PRD 68 (2003)

Euclidean analogs

$$\Gamma \in \{\gamma_t, \gamma_5\gamma_t, \gamma_z, \gamma_5\gamma_z\}$$

Large SNR improvement for $P_z \gtrsim 2$ GeV

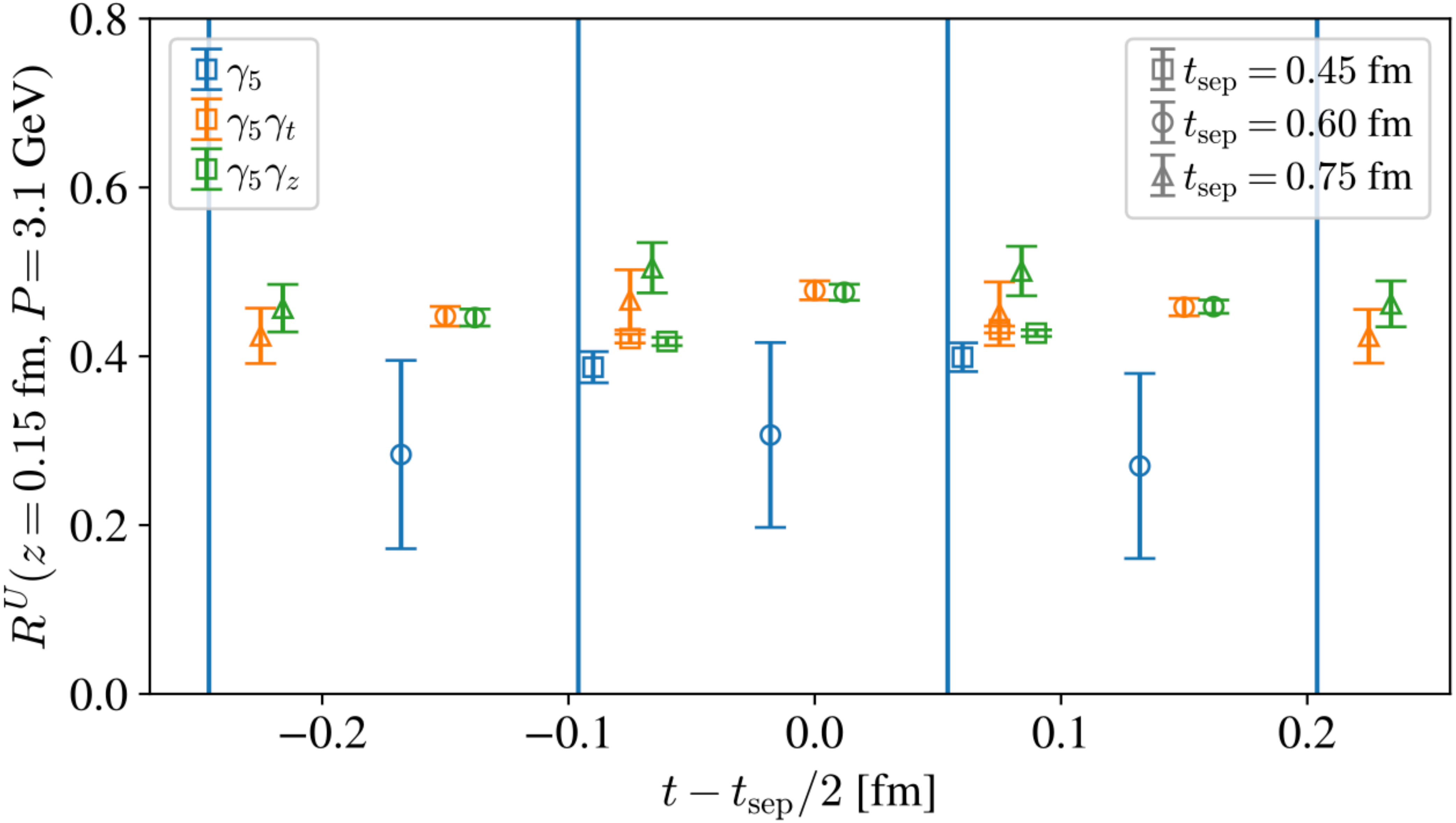


Zhang, Grebe, Hackett, MW, Zhao, arXiv:2501.00729

ψ_+ nucleon matrix elements

$$\Gamma \in \{\gamma_t, \gamma_5\gamma_t, \gamma_z, \gamma_5\gamma_z\}$$

Quasi-PDF matrix elements exhibit similar SNR enhancements

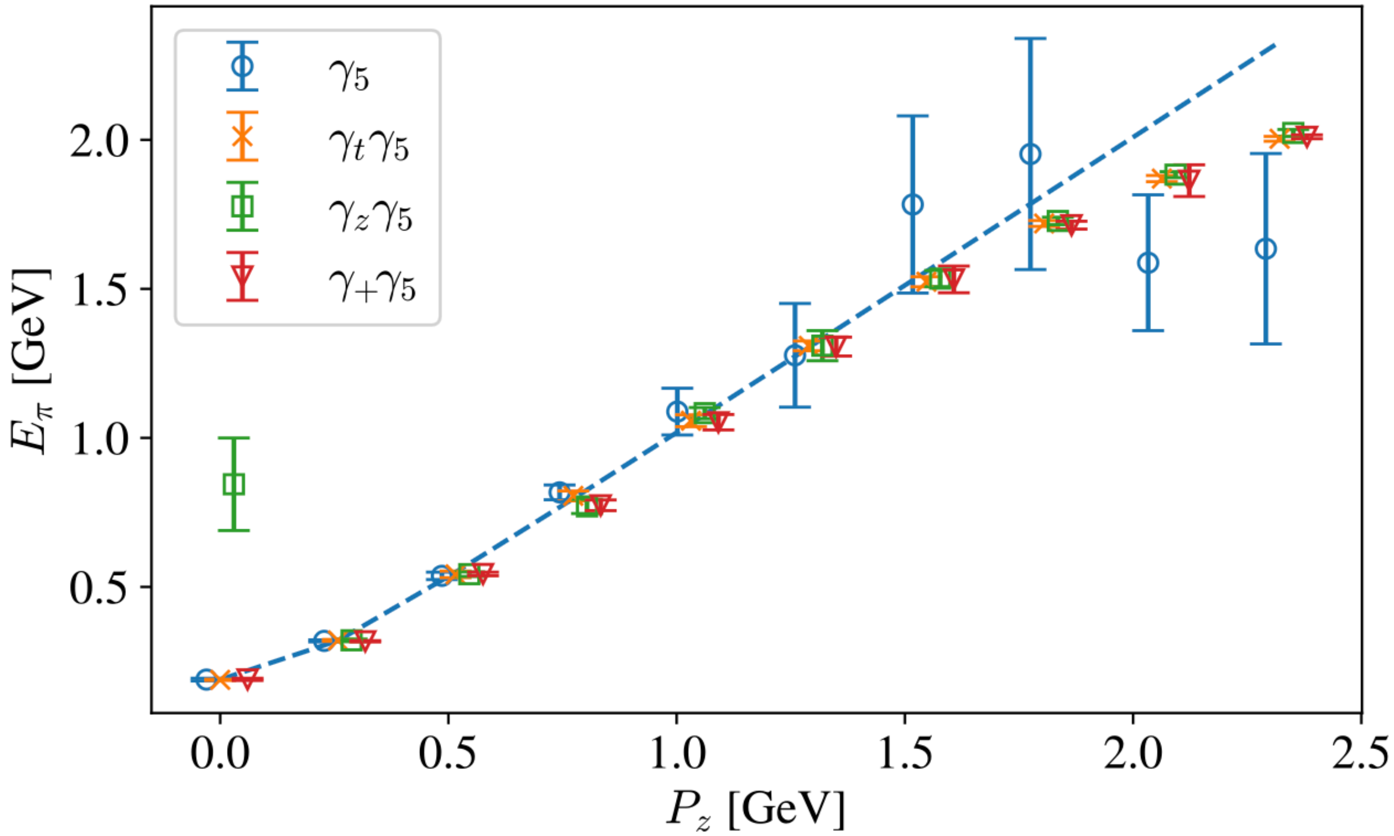


Zhang, Grebe, Hackett, MW, Zhao, arXiv:2501.00729

Kinematically-enhanced interpolating operators for boosted hadrons

Zhang, Grebe, Hackett, MW, Zhao, arXiv:2501.00729

Boosted hadron states in LQCD are easier to study using operators built from leading-twist ψ_+ spinors



- SNR enhanced by $O(P^2/M^2)$
- Excited-state effects reduced for sufficiently large P
- Theoretical enhancement seen in real LQCD data: 2000x / 100x improvement in effects statistics for pions / nucleons with ~2 GeV momenta
- Easy to implement — just add a γ_t or γ_z to your pion and nucleon operators!