

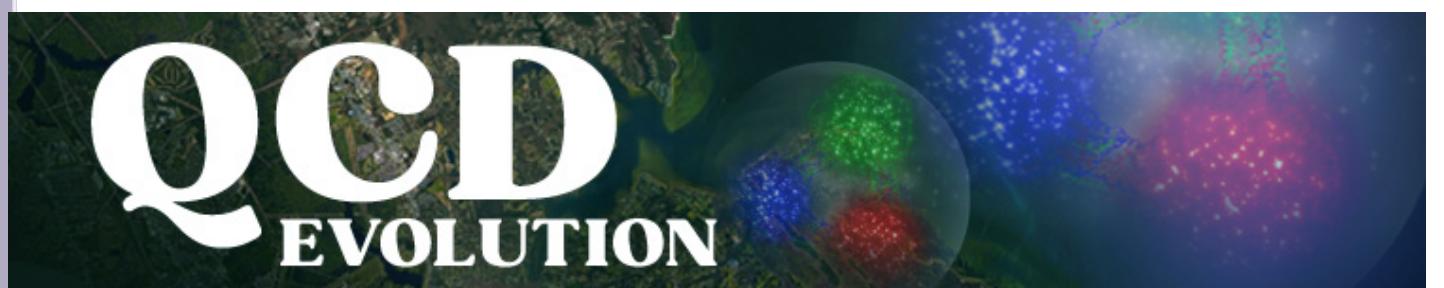
# NEW INSIGHTS INTO LAMBDA FRAGMENTATION FUNCTIONS

Alessia Bongallino

*University of the Basque Country  
UPV/EHU*

**QCD Evolution 2025**

In collaboration with  
V. Bertone, A. Chiefa, M. G. Echevarría,  
E. Nocera, G. Schnell



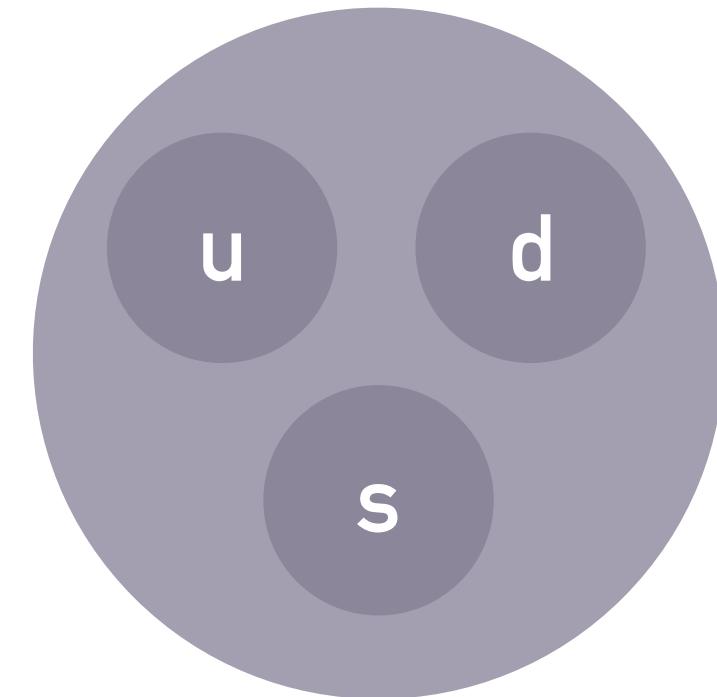
# INTRODUCTION

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Big amount of information collected in the last decades regarding hadron production through **fragmentation functions**. We still need to improve our knowledge on fragmentation functions focusing on specific hadrons.

The  $\Lambda^0$  baryon has a **simple structure**:

- ▶ u and d quarks form a spin-0 isospin singlet state
- ▶ the  $\Lambda^0$  spin is carried almost entirely by the strange quark

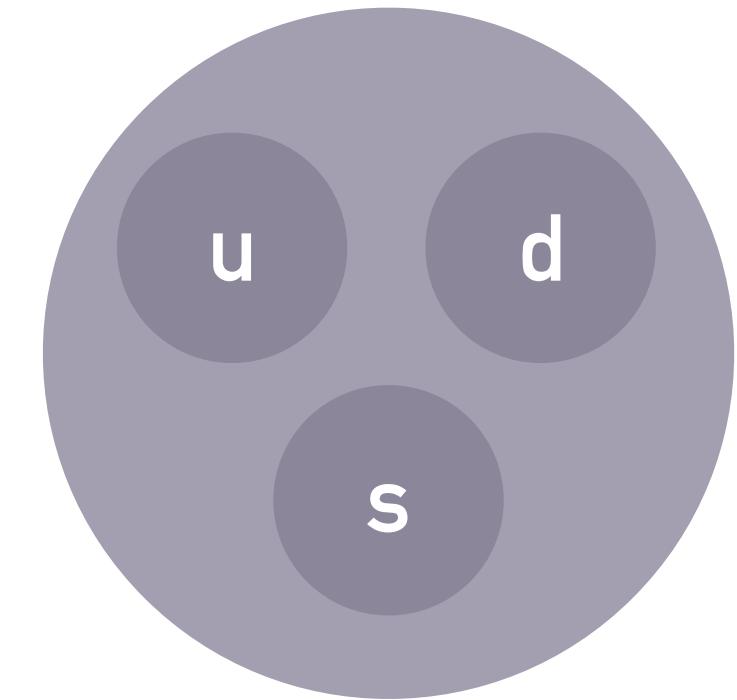


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## State of the art of fits (for unpolarized $\Lambda$ FF)

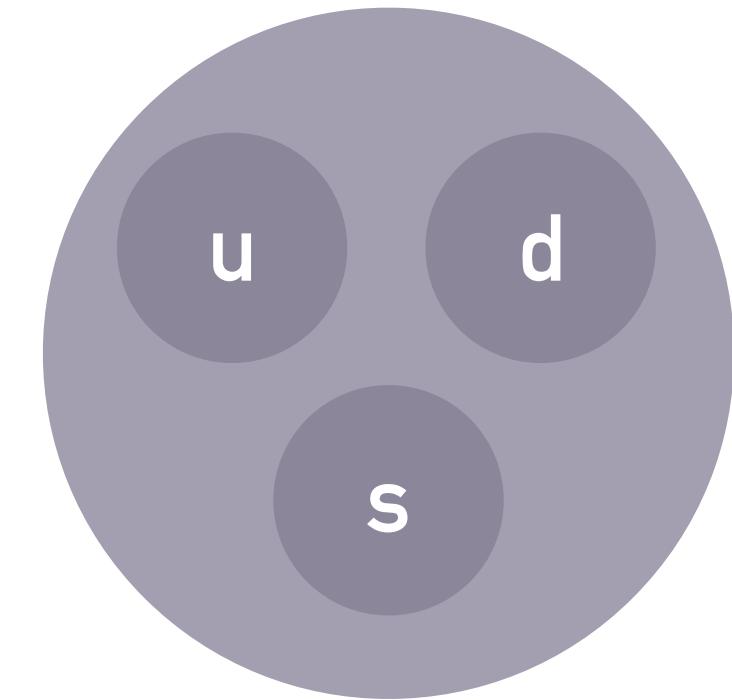
- ▶ D. De Florian, M. Stratmann, W. Vogelsang (1998) , SIA data from LEP (ALEPH, OPAL, DELPHI), TASSO, CELLO, HRS, SLD
- ▶ S. Albino, B. A. Kniehl, G. Kramer (2006) SIA data with  $\sqrt{s} = M_Z$   
(2008) SIA data with  $\sqrt{s} \leq M_Z$  ,  $pp(\bar{p})$  collisions (BRAHMS, PHENIX, STAR)
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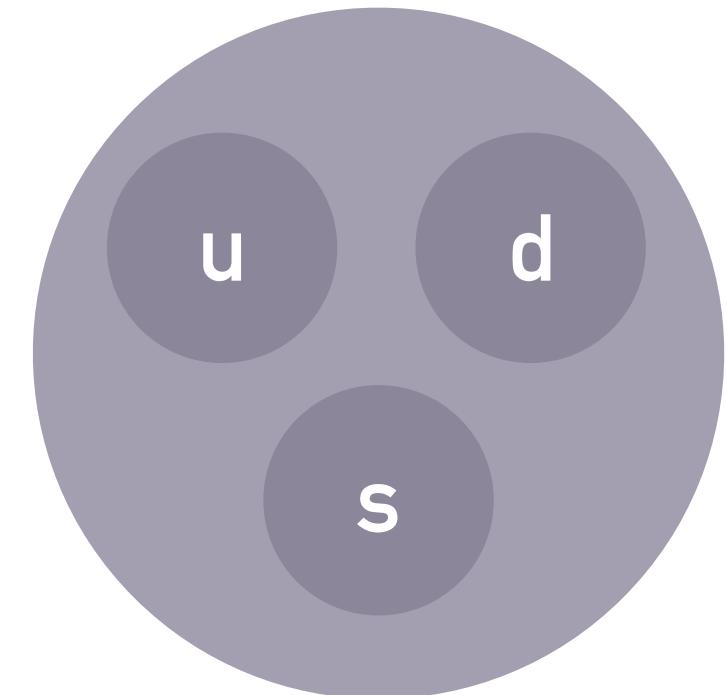
2025 → new global fit including SIA, SIDIS, pp collisions

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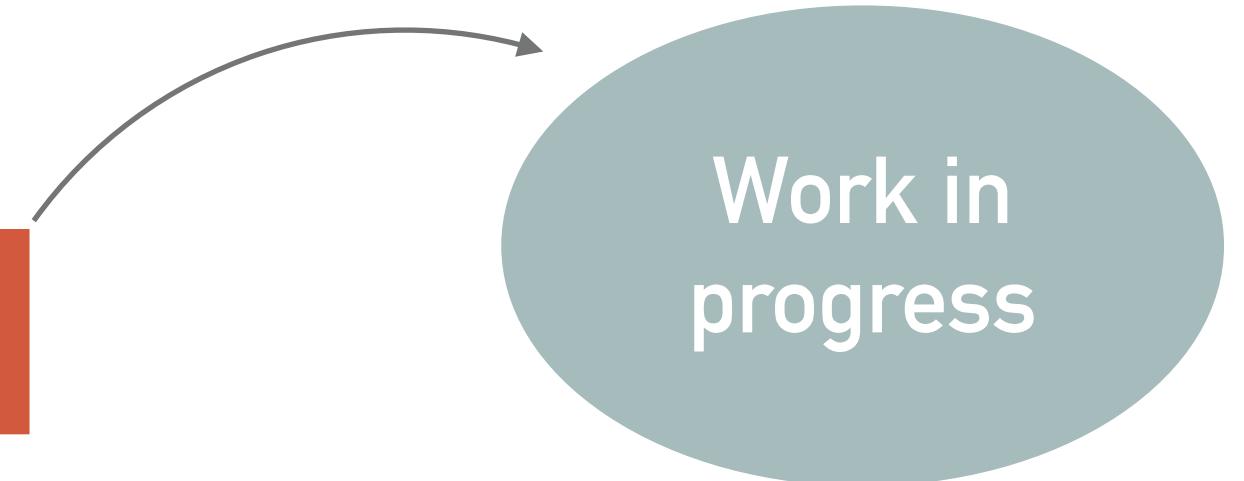
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# EXPERIMENTS

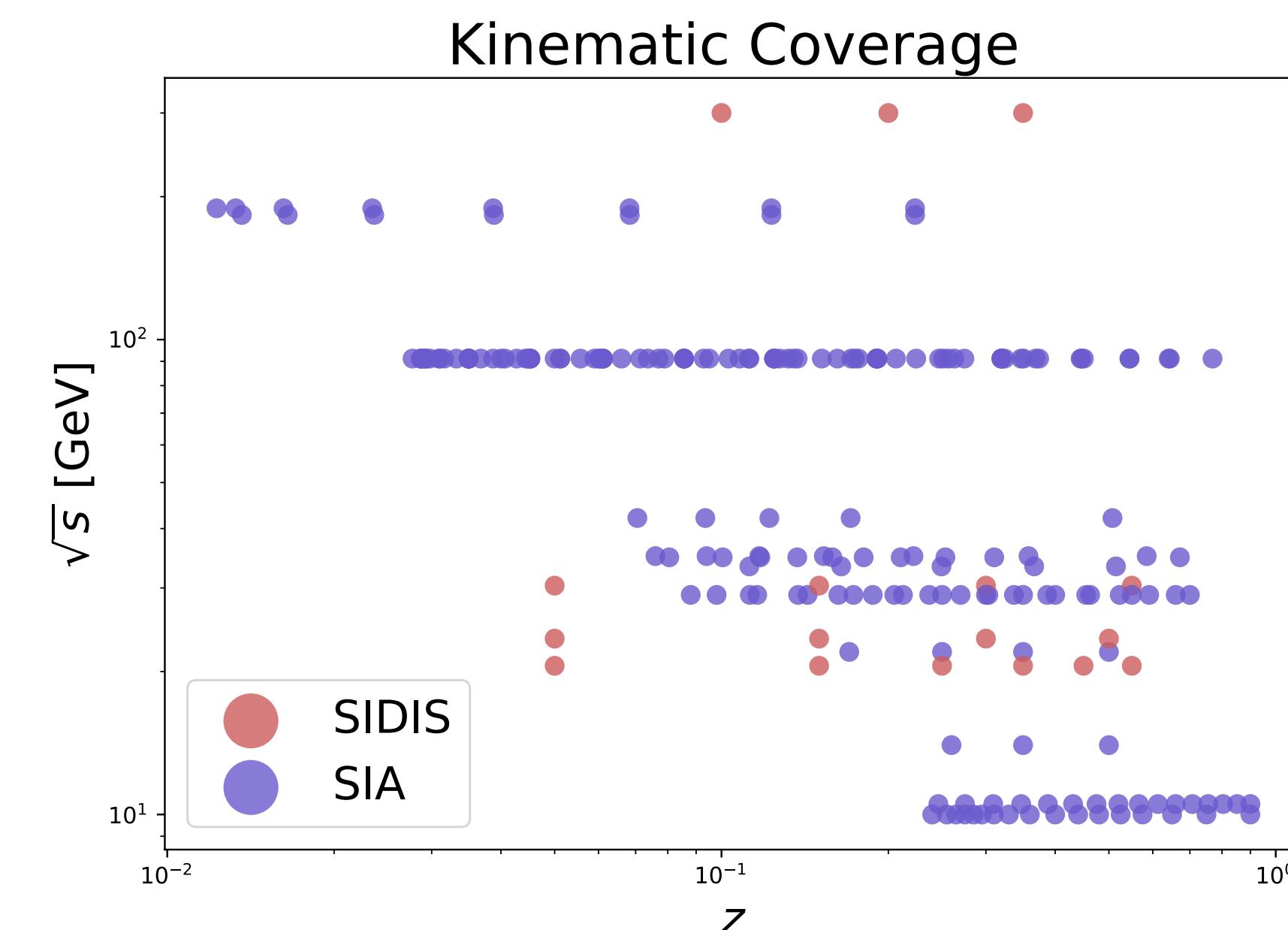
## SIA

Experiment	Observable	$\sqrt{s}$ (GeV)
ARGUS	Multiplicity	10
BELLE	Cross section	10.52
TASSO @ 14,22,34	Cross section	14,22,34
TASSO @ 33.3	Cross section	33.3
TASSO @ 34.8,42.1	Cross section	34.8, 42.1
CELLO	Multiplicity	35
HRS	Cross section	29
MARK II	Cross section	29
ALEPH	Multiplicity	91.2
DELPHI @ 91 GeV	Multiplicity	91.2
OPAL	Multiplicity	91.2
SLD unt, uds-tag, c-tag, b-tag	Multiplicity	91.2
DELPHI @ 183,189	Multiplicity	183,189

# SIDIS

Experiment	Observable	$\sqrt{s}$ (GeV)
Chicago-Harvard -Illinois-Oxford @ Fermilab	Multiplicity	20
EMC	Multiplicity	23.5
E665	Multiplicity	30
H1	Multiplicity	300

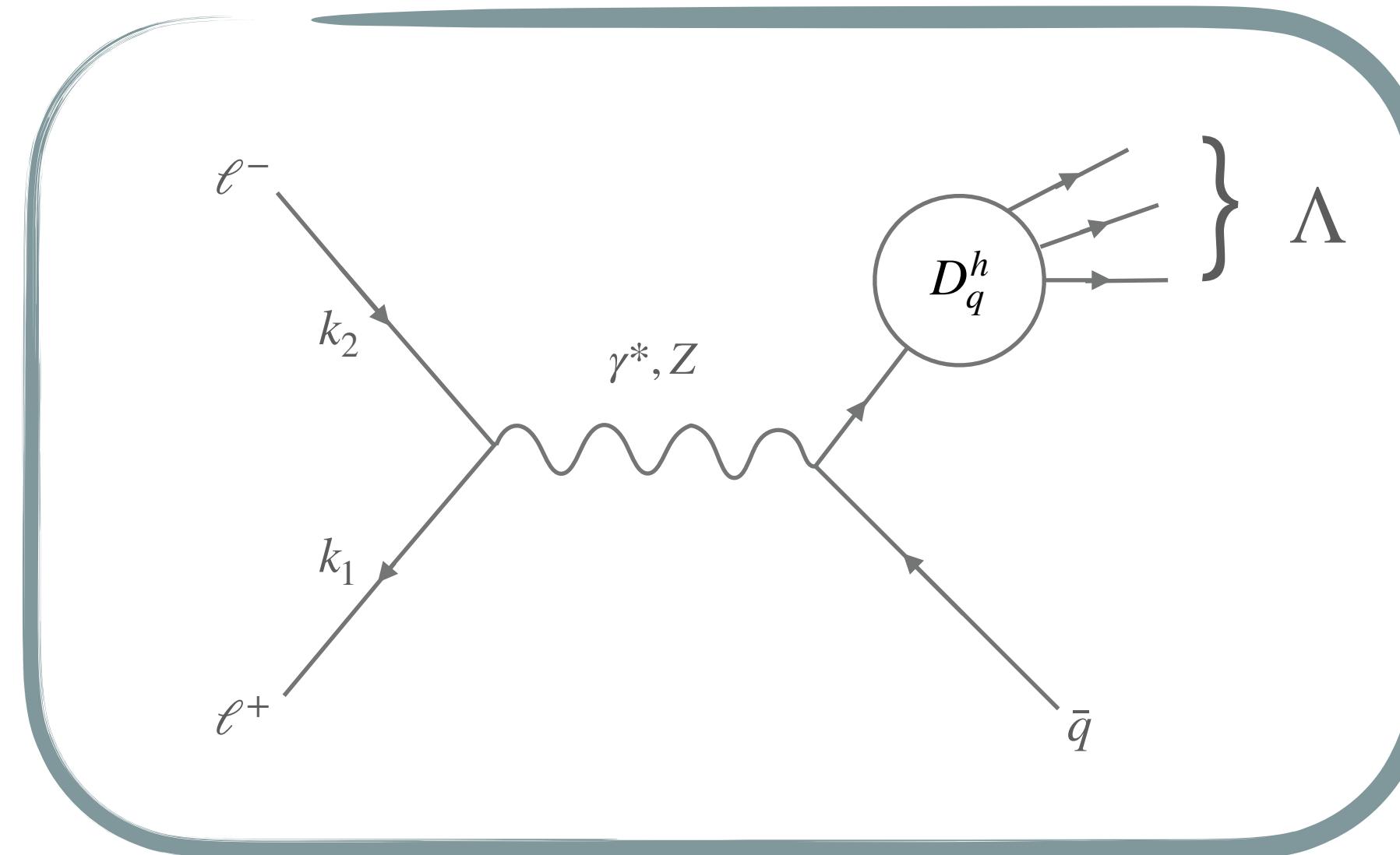
NC



# THEORETICAL BASIS

SIA  
kinematics

$$Q^2 = q^2$$
$$z = \frac{2P_h \cdot q}{Q^2}$$
$$\sqrt{s} = Q$$



## SINGLE INCLUSIVE ANNIHILATION

$$l^+(k_1) + l^-(k_2) \rightarrow \Lambda(P_\Lambda) + X$$

### ► Differential cross section

$$\frac{d\sigma^h}{dz}(z, Q) = \frac{4\pi\alpha^2}{Q^2} F_2^h(z, Q)$$

NNPDF Collaboration, V. Bertone et al.,  
Eur.Phys.J.C 77 (2017) 8, 516

### ► Fragmentation structure function

$$F_2^h(z, Q) = \frac{1}{n_f} \sum_q^{n_f} e_q^2(Q) \left[ C_{2,q}^S(z, \alpha_s(Q)) \otimes D_\Sigma^h(z, Q) + C_{2,q}^{NS}(z, \alpha_s(Q)) \otimes D_{NS}^h(z, Q) + C_{2,g}^S(z, \alpha_s(Q)) \otimes D_g^h(z, Q) \right]$$

# THEORETICAL BASIS

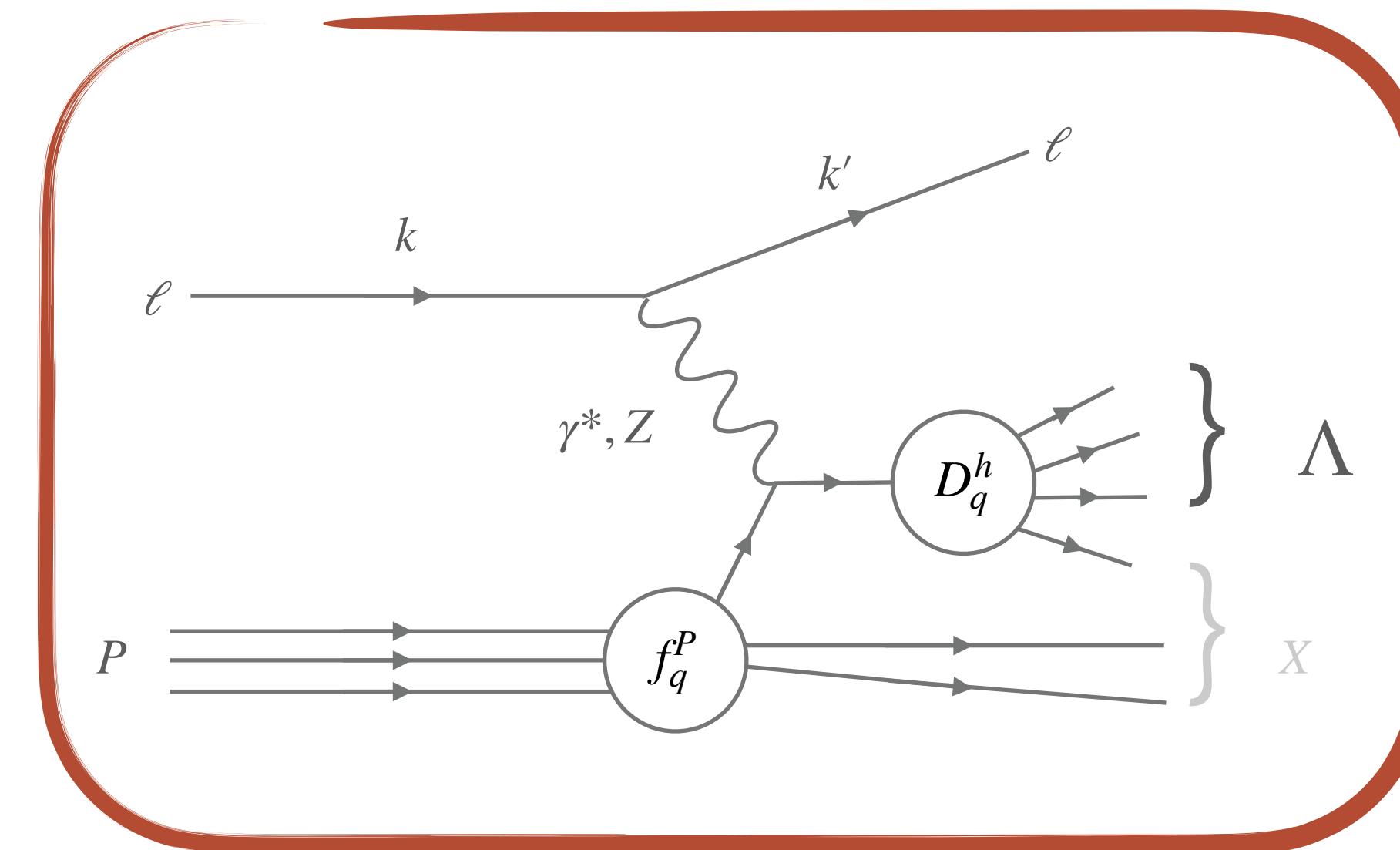
## SIDIS Kinematics

$$Q^2 = -q^2$$

$$x = \frac{Q^2}{2p \cdot q}$$

$$z = \frac{P \cdot P_h}{P \cdot q}$$

$$y = \frac{Q^2}{x_s}$$



## SEMI INCLUSIVE DIS

$$\ell(k) + N(P) \rightarrow \ell(k') + \Lambda(P_\Lambda) + X$$

### Differential cross section ( $Q \ll M_Z$ )

$$\frac{d^3\sigma}{dxdQdz} = \frac{4\pi\alpha^2}{xQ^3} [(1 + (1 - y)^2 F_2(x, z, Q) - y^2 F_L(x, z, Q)]$$

**MAPFF1.0**

R. A. Khalek et al.,  
Phys.Rev.D 104 (2021) 3, 034007

### Fragmentation structure function within collinear factorisation ( $Q \gg \Lambda_{QCD}$ )

$$F_i(x, z, Q) = x \sum_{q\bar{q}} e_q^2 \left[ (C_{i,qq}(x, z, Q) \otimes f_q(x, Q) + C_{i,qg}(x, z, Q) \otimes f_g(x, Q)) \otimes D_q^\Lambda(z, Q) \right. \\ \left. + (C_{i,gq}(x, z, Q) \otimes f_q(x, Q)) \otimes D_g^\Lambda(z, Q) \right]$$

$$i = 2, L$$

# THEORETICAL BASIS

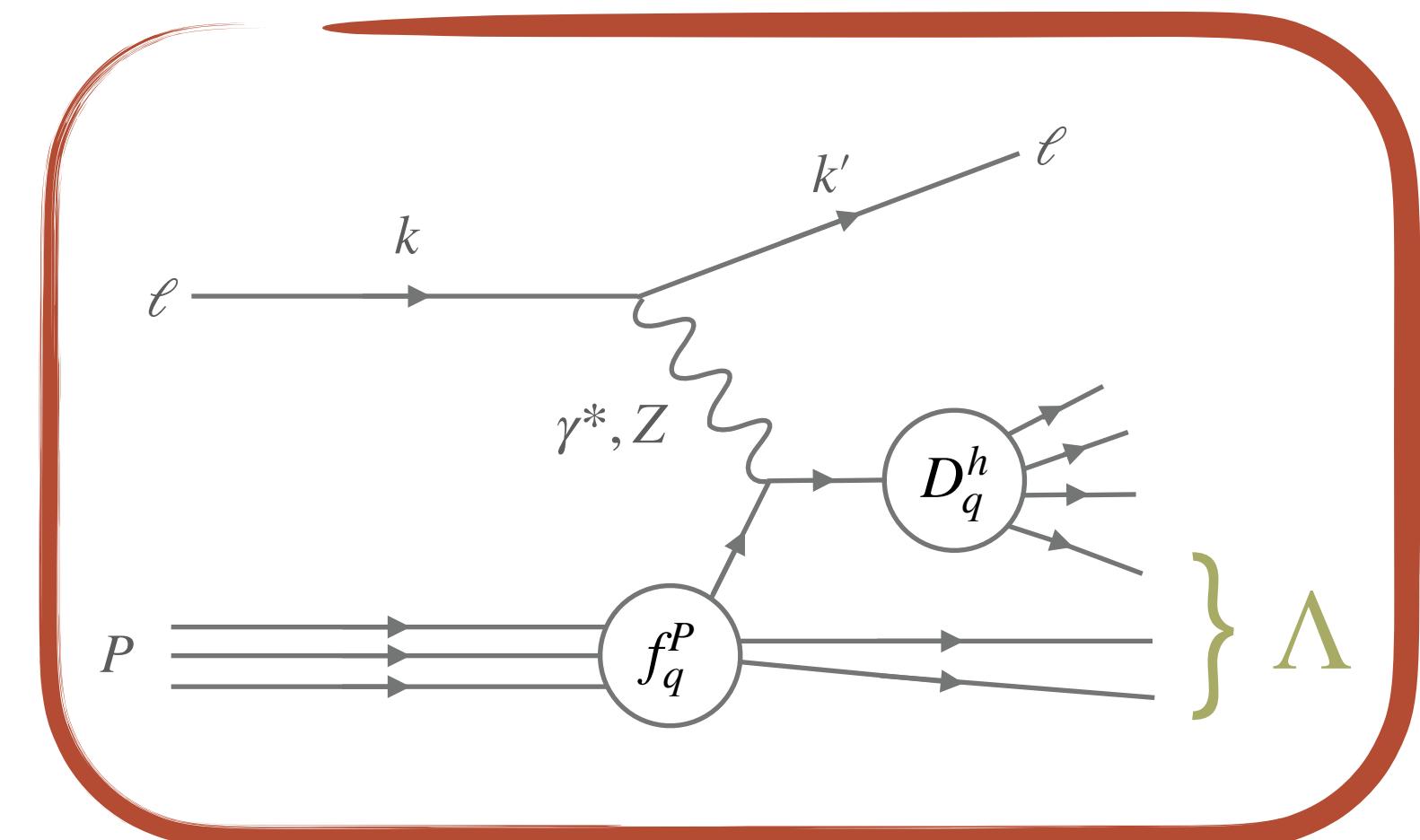
All datasets provide  $\frac{1}{\sigma_T} \frac{d\sigma^h}{dx_F}$   $\rightarrow$

$x_F < 0$	target fragmentation region
$x_F > 0$	current fragmentation region

# THEORETICAL BASIS

All datasets provide

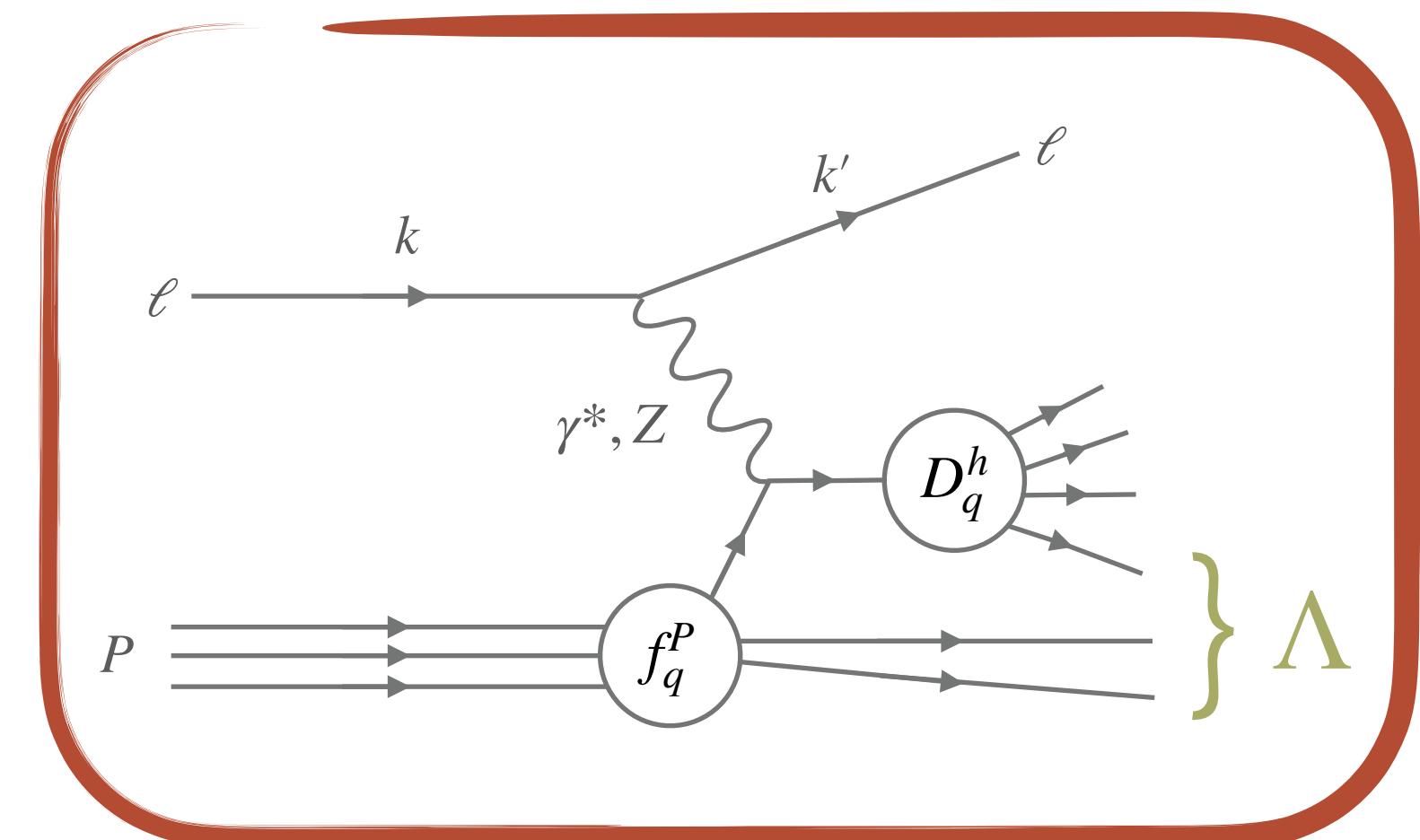
$$\frac{1}{\sigma_T} \frac{d\sigma^h}{dx_F} \rightarrow \begin{cases} x_F < 0 & \text{target fragmentation region} \\ x_F > 0 & \text{current fragmentation region} \end{cases}$$



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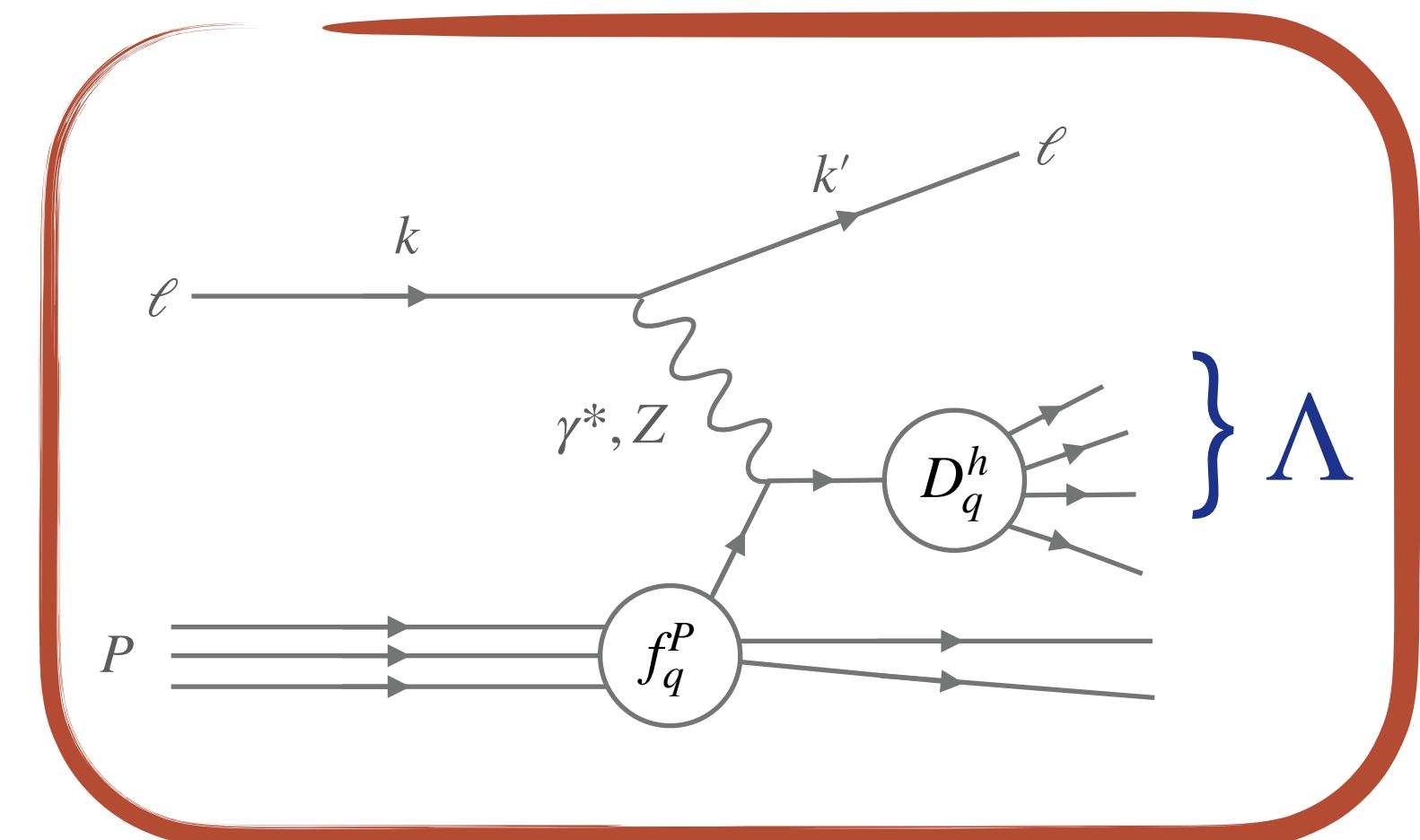
Handled with **fracture functions**

F. A. Ceccopieri, D. Mancusi (2013)

# THEORETICAL BASIS

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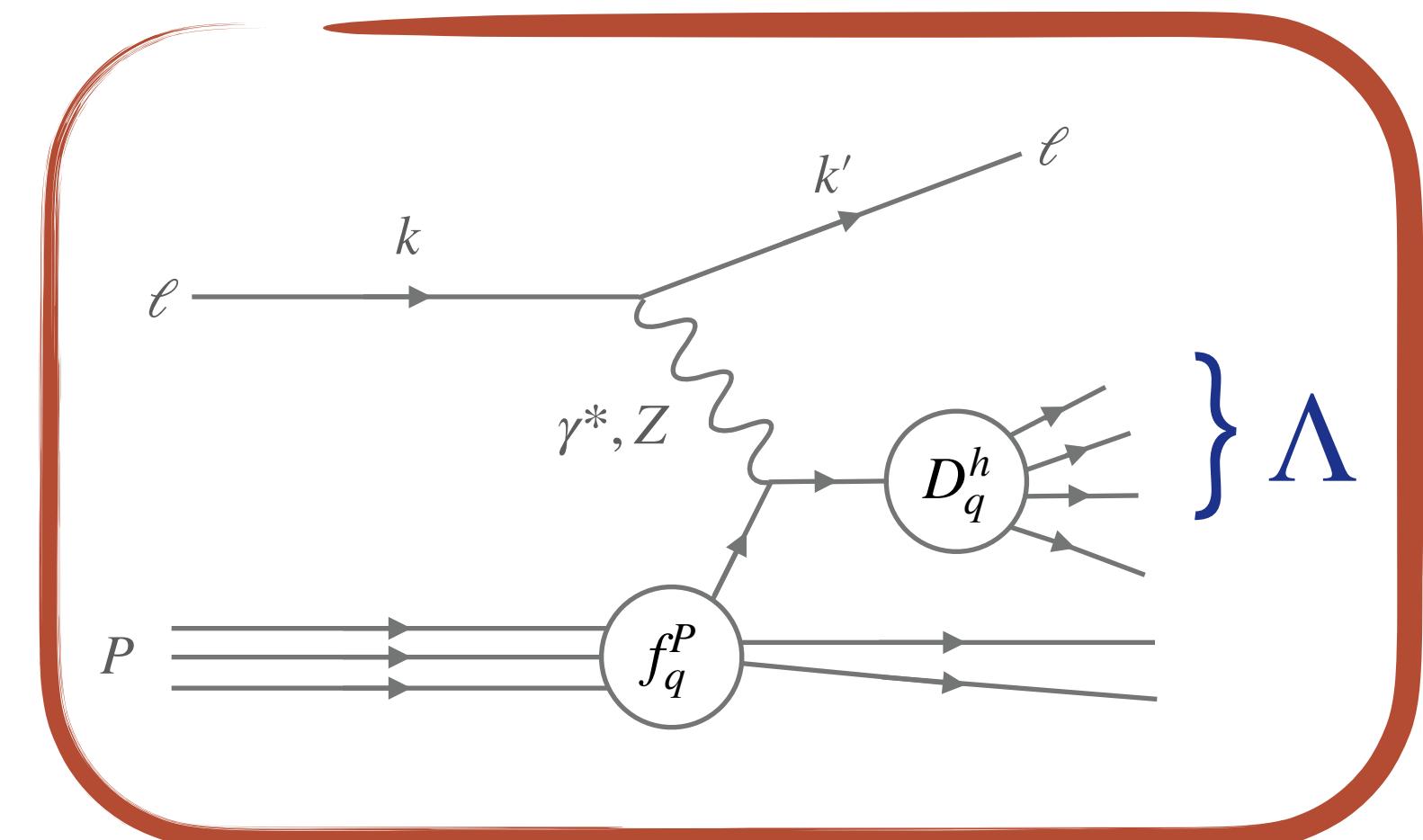


# THEORETICAL BASIS

All datasets provide

$$\frac{1}{\sigma_T} \frac{d\sigma^h}{dx_F}$$

$x_F < 0$  target fragmentation region  
 $x_F > 0$  current fragmentation region



We restrict to  $x_F > 0$  for proper SIDIS analysis, with  $x_F = z$

$$\frac{dM}{dz} = \left[ \int_{Q_{min}}^{Q_{max}} dQ \int_{x_{min}}^{x_{max}} dx \int_{z_{min}}^{z_{max}} dz \frac{d^3\sigma}{dx dQ dz} \right] \Bigg/ \left[ \Delta z \int_{Q_{min}}^{Q_{max}} dQ \int_{x_{min}}^{x_{max}} dx \frac{d^2\sigma}{dx dQ} \right]$$

**MAPFF1.0**  
R. A. Khalek et al.,  
Phys.Rev.D 104 (2021) 3, 034007  
Bin by bin integration

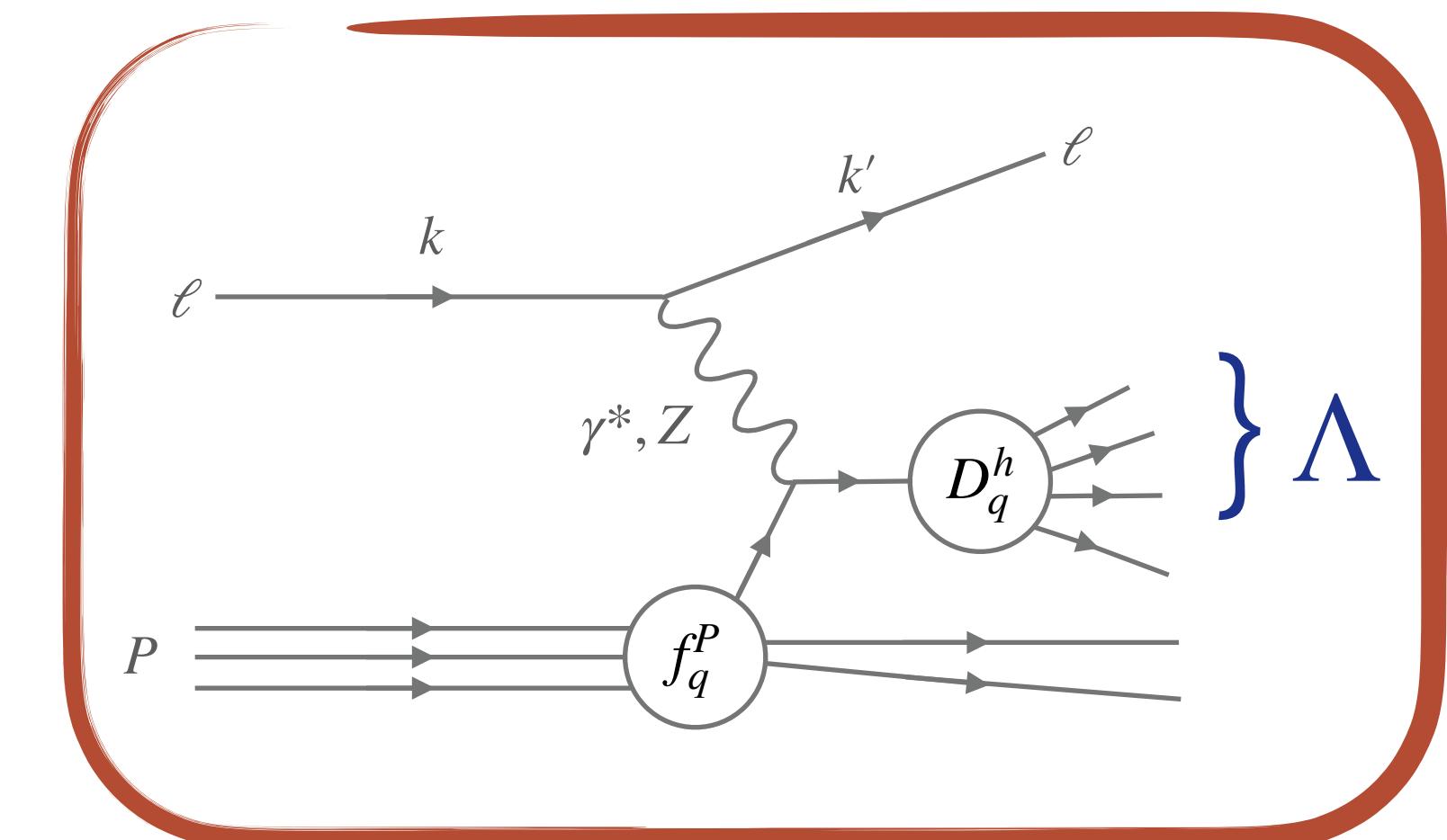
# THEORETICAL BASIS

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$$\frac{dM}{dz} = \left[ \int_{Q_{min}}^{Q_{max}} dQ \int_{x_{min}}^{x_{max}} dx \int_{z_{min}}^{z_{max}} dz \frac{d^3\sigma}{dx dQ dz} \right] / \left[ \Delta z \int_{Q_{min}}^{Q_{max}} dQ \int_{x_{min}}^{x_{max}} dx \frac{d^2\sigma}{dx dQ} \right]$$

**MAPFF1.0**

R. A. Khalek et al.,  
Phys.Rev.D 104 (2021) 3, 034007

Bin by bin integration

Experiment	$Q^2$ range	x range
CHIO @ Fermilab	$4 < Q^2 < 50$	$0.001 < x < 1$
EMC	$Q^2 > 4$	$0.02 < x < 1$
E665	$Q^2 > 1$	$0.003 < x < 1$
H1	$10 < Q^2 < 70$	$0.0001 < x < 0.01$

Whole range of integration in x and Q  
(as specified by the collaboration)

Phase space reduction

$$Q_{max} < \sqrt{s}$$

$$x_{min} = \max \left[ x_{min}^{collab}, \frac{Q^2}{sy_{max}} \right]$$

$$x_{max} = \min \left[ x_{max}^{collab}, \frac{Q^2}{sy_{min}}, \frac{Q^2}{Q^2 + W^2} \right]$$

# METHODOLOGY

- Nanga Parbat



**Nanga Parbat: a TMD fitting framework**

Nanga Parbat is a fitting framework aimed at the determination of the non-perturbative component of TMD distributions.

- APFEL++



PASSED codefactor A CodeQL passing DOI 10.5281/zenodo.10522420

A PDF evolution library in C++

- NNAD & ceres-solver

## NNAD

NNAD stands for Neural Network Analytic Derivatives and is a C++ implementation of the analytic derivatives of a feed-forward neural network with arbitrary architecture with respect to its free parameters. We implemented the back-propagation method using three strategies: analytic, automatic (when interfaced with ceres-solver) and numeric differentiation.

# METHODOLOGY

Data is assumed to be sampled from a multivariate Gaussian distribution:

$$\mathcal{G}(\mathbf{x}^{(k)}) \propto \exp \left[ (\mathbf{x}^{(k)} - \boldsymbol{\mu})^T C^{-1} (\mathbf{x}^{(k)} - \boldsymbol{\mu}) \right]$$

- Nanga Parbat

Chi square,  
Replica generation

Statistical framework  
relies on Monte Carlo  
sampling method

With  $\mathbf{x}^{(k)} = \{x_1^{(k)}, x_2^{(k)}, \dots, x_{N_{dat}}^{(k)}\}$ ,  $k = 1, \dots, N_{rep}$  replicas of a set of  $N_{dat}$  values

$\boldsymbol{\mu} = \{\mu_1, \mu_2, \dots, \mu_{N_{dat}}\}$  expectation values = measured data

$$C_{ij} = \delta_{ij} \sigma_{i,\text{unc}}^2 + \sum_{\beta} \sigma_{i,\text{corr}}^{(\beta)} \sigma_{j,\text{corr}}^{(\beta)}$$

Elements of the covariance matrix

Generation of replicas using the Cholesky decomposition  $\mathbf{L}$ , ( $\mathbf{C} = \mathbf{L} \cdot \mathbf{L}^T$ )

$$\mathbf{x}^{(k)} = \boldsymbol{\mu} + \mathbf{L} \cdot \mathbf{r}^{(k)}$$

With  $\mathbf{r}^{(k)}$  such that

$$\frac{1}{N_{rep}} \sum_k^{N_{rep}} x_i^{(k)} \simeq \mu_i, \quad \frac{1}{N_{rep}} \sum_k^{N_{rep}} x_i^{(k)} x_j^{(k)} \simeq \mu_i \mu_j + C_{ij}$$

# METHODOLOGY

- ▶ Computation of SIA and SIDIS cross sections

- APFEL++

$$\frac{d\sigma^h}{dz}(z, Q) = \frac{4\pi\alpha^2}{Q^2} F_2^h(z, Q)$$

$$\frac{d^3\sigma}{xdQdz} = \frac{4\pi\alpha^2}{xQ^3} [(1 + (1 - y)^2 F_2(x, z, Q^2) - y^2 F_L(x, z, Q^2)]$$

- ▶ Perturbative expansion of the coefficient functions (NLO and NNLO)

$$C(x, z, Q) = \sum_n \left( \frac{\alpha_s(Q)}{4\pi} \right)^n C^{(n)}(x, z)$$

- ▶ DGLAP Evolution of FFs
- ▶ PDF for SIDIS

# METHODOLOGY

- NNAD & ceres-solver

Parameterisation choice:

single one-layered feed-forward Neural Network

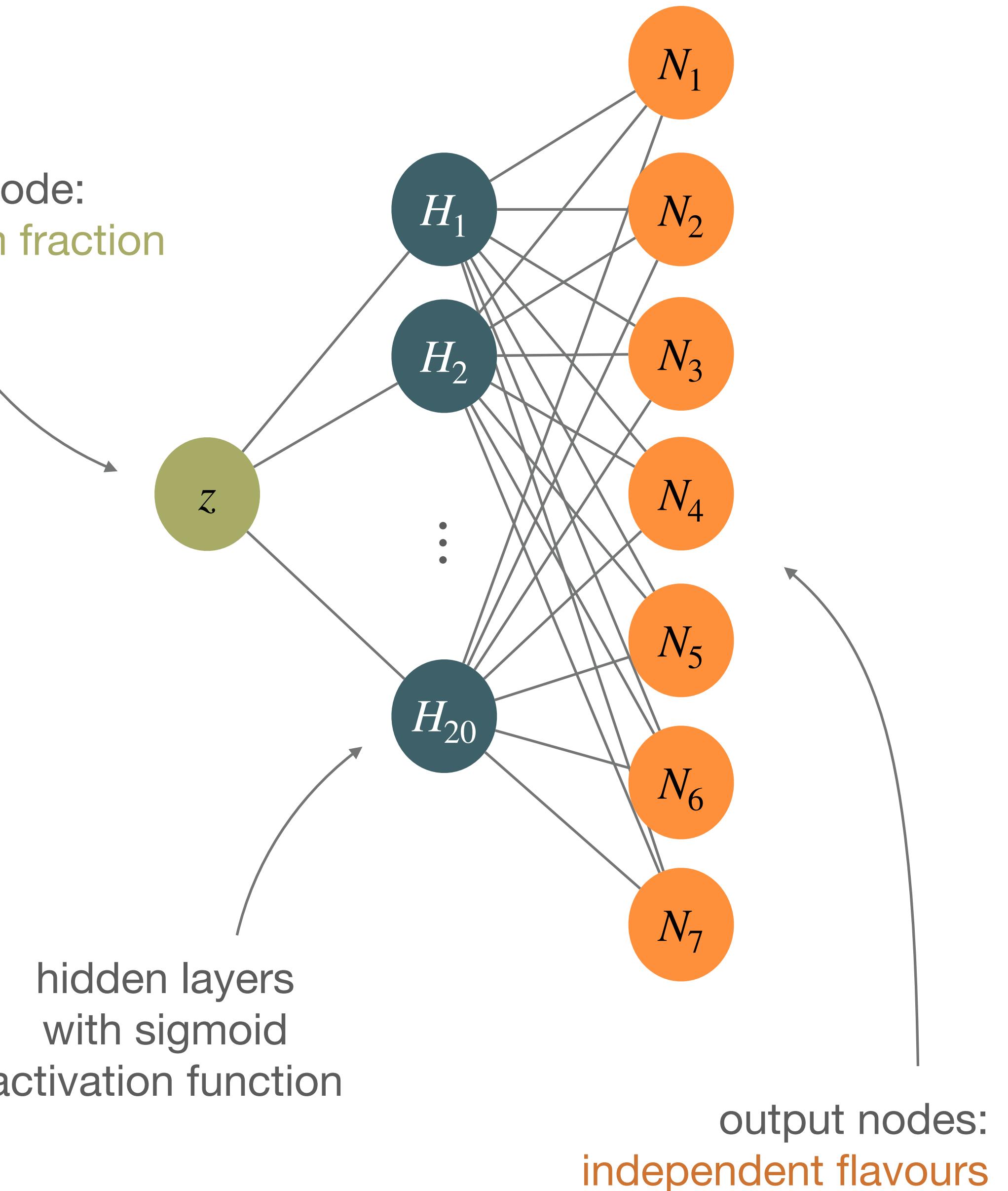
$$\mathcal{N}_i(z, \theta)$$

$$z D_i^\Lambda(z, \mu_0 = 5 \text{ GeV}) = (\mathcal{N}_i(z, \theta) - \mathcal{N}_i(1, \theta))^2$$

7 independent flavours

$$D_u^\Lambda, \quad D_d^\Lambda, \quad D_s^\Lambda, \quad D_{\bar{u}}^\Lambda = D_{\bar{d}}^\Lambda = D_{\bar{s}}^\Lambda, \quad D_{c^+}^\Lambda, \quad D_{b^+}^\Lambda, \quad D_g^\Lambda$$

input node:  
momentum fraction



# METHODOLOGY

Parameterisation

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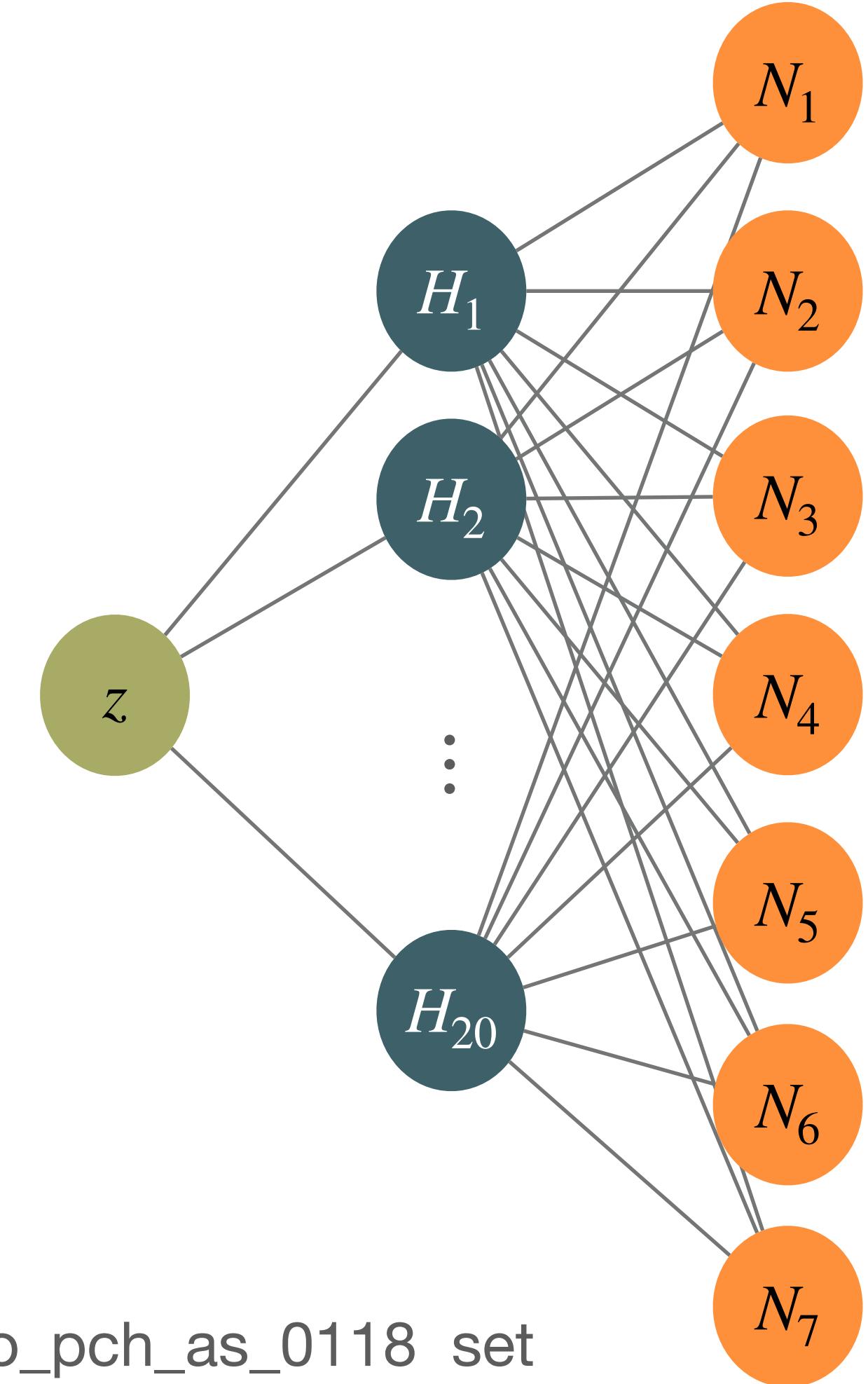
$$D_u^\Lambda, \quad D_d^\Lambda, \quad D_s^\Lambda, \quad D_{\bar{u}}^\Lambda = D_{\bar{d}}^\Lambda = D_{\bar{s}}^\Lambda, \quad D_{c^+}^\Lambda, \quad D_{b^+}^\Lambda, \quad D_g^\Lambda$$

$$\chi^{2(k)} \equiv (T(\theta^{(k)}) - x^{(k)})^T C^{-1} (T(\theta^{(k)}) - x^{(k)})$$

Propagation of PDF correlated uncertainties for SIDIS using the NNPDF31\_nlo\_pch\_as\_0118 set

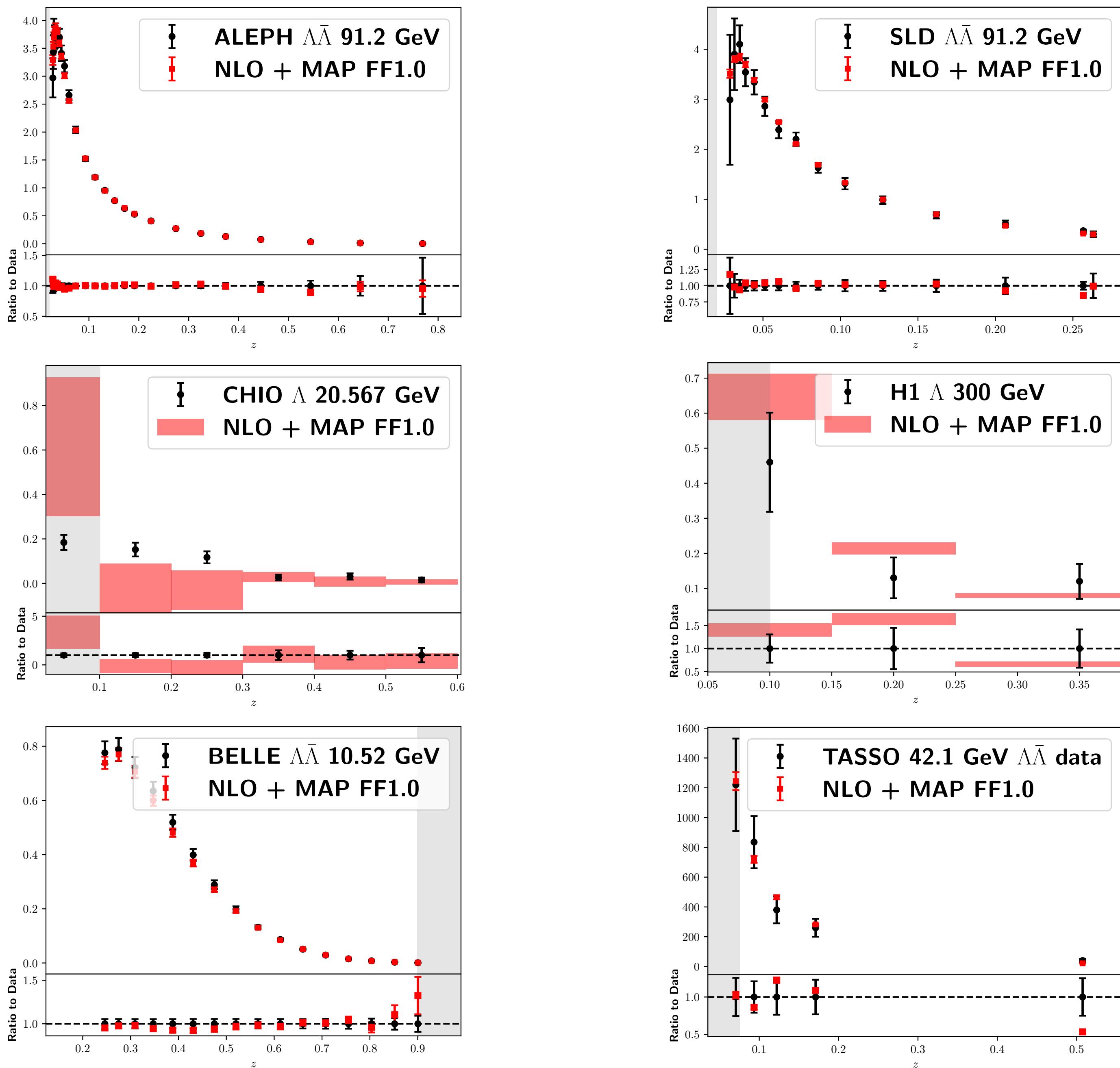
Minimisation algorithm: **Levenberg-Marquardt** implemented in **Ceres-solver**

NN and analytical derivatives provided by **NNAD**



# RESULTS

Experiment	$\chi^2$ per point	$N_{dat}$ after cuts
<hr/>		
SIDIS		
E665 $\Lambda$	1.65	2
E665 $\bar{\Lambda}$	1.87	2
EMC $\Lambda$	3.55	3
H1 $\Lambda + \bar{\Lambda}$	0.91	2
CHIO Fermilab $\Lambda$	1.17	4
CHIO Fermilab $\bar{\Lambda}$	1.56	4
<hr/>		
SIA $\Lambda + \bar{\Lambda}$		
ALEPH	0.45	25
ARGUS	1.43	16
BELLE	0.70	15
CELLO	0.60	7
DELPHI 91.2 GeV	4.75	11
DELPHI 183 GeV	3.37	5
DELPHI 189 GeV	2.58	5
HRS	0.96	12
MARK II	3.09	15
OPAL	0.88	15
SLD B	0.52	8
SLD C	1.61	8
SLD UDS	1.99	8
SLD	0.65	16
TASSO 14 GeV	0.25	3
TASSO 22 GeV	0.51	4
TASSO 33 GeV	0.77	5
TASSO 34.8 GeV	2.77	10
TASSO 42.1 GeV	1.04	4
<hr/>		
Total	1.54	209



# RESULTS

Experiment	$\chi^2/N_{dat}$	
	NLO	NNLO
<hr/>		
SIDIS		
E665 $\Lambda$	1.65	1.66
E665 $\bar{\Lambda}$	1.87	1.83
EMC $\Lambda$	3.55	3.79
H1 $\Lambda + \bar{\Lambda}$	0.91	0.95
CHIO Fermilab $\Lambda$	1.17	1.06
CHIO Fermilab $\bar{\Lambda}$	1.56	1.43
<hr/>		
SIA $\Lambda + \bar{\Lambda}$		
ALEPH	0.45	0.45
ARGUS	1.43	1.39
BELLE	0.70	0.61
CELLO	0.60	0.59
DELPHI 91.2 GeV	4.75	4.71
DELPHI 183 GeV	3.37	3.30
DELPHI 189 GeV	2.58	2.50
HRS	0.96	0.95
MARK II	3.09	3.01
OPAL	0.88	0.86
SLD B	0.52	0.49
SLD C	1.61	1.63
SLD UDS	1.99	2.08
SLD	0.65	0.66
TASSO 14 GeV	0.25	0.25
TASSO 22 GeV	0.51	0.51
TASSO 33 GeV	0.77	0.78
TASSO 34.8 GeV	2.77	2.75
TASSO 42.1 GeV	1.04	1.06
<hr/>		
Total	1.54	1.51

► Extension of calculation at N2LO  $\rightarrow \chi^2 = 1.51$

# RESULTS

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Experiment	$\chi^2/N_{dat}$	
	NLO	NNLO
SIDIS		
E665 $\Lambda$	1.65	1.66
E665 $\bar{\Lambda}$	1.87	1.83
EMC $\Lambda$	3.55	3.79
H1 $\Lambda + \bar{\Lambda}$	0.91	0.95
CHIO Fermilab $\Lambda$	1.17	1.06
CHIO Fermilab $\bar{\Lambda}$	1.56	1.43
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ALEPH	0.45	0.45
ARGUS	1.43	1.39
BELLE	0.70	0.61
CELLO	0.60	0.59
DELPHI 91.2 GeV	4.75	4.71
DELPHI 183 GeV	3.37	3.30
DELPHI 189 GeV	2.58	2.50
HRS	0.96	0.95
MARK II	3.09	3.01
OPAL	0.88	0.86
SLD B	0.52	0.49
SLD C	1.61	1.63
SLD UDS	1.99	2.08
SLD	0.65	0.66
TASSO 14 GeV	0.25	0.25
TASSO 22 GeV	0.51	0.51
TASSO 33 GeV	0.77	0.78
TASSO 34.8 GeV	2.77	2.75
TASSO 42.1 GeV	1.04	1.06
Total	1.54	1.51

► Extension of calculation at N2LO  $\rightarrow \chi^2 = 1.51$

► SIA only, symmetric parameterisation  $\rightarrow \chi^2 = 1.42$

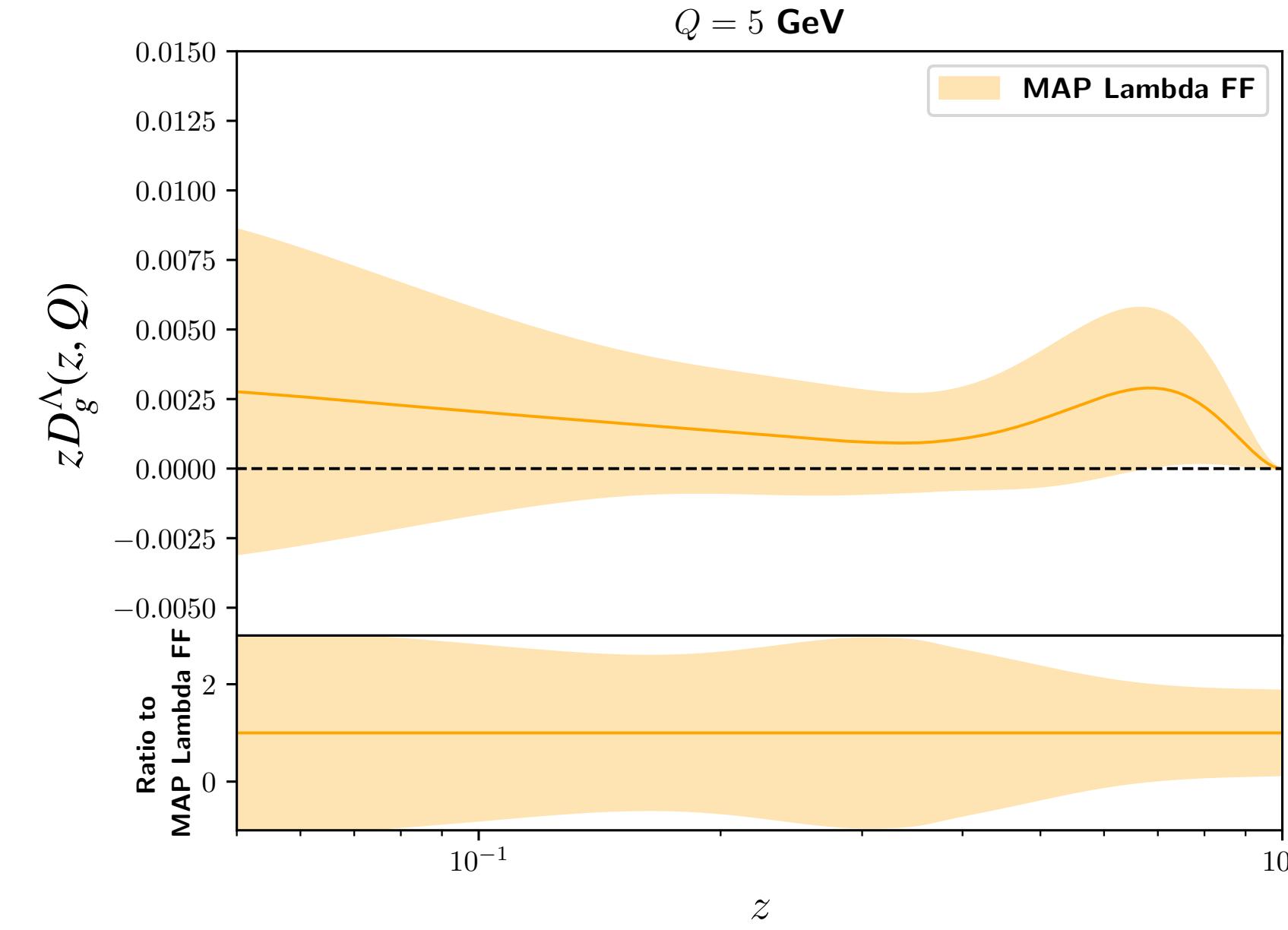
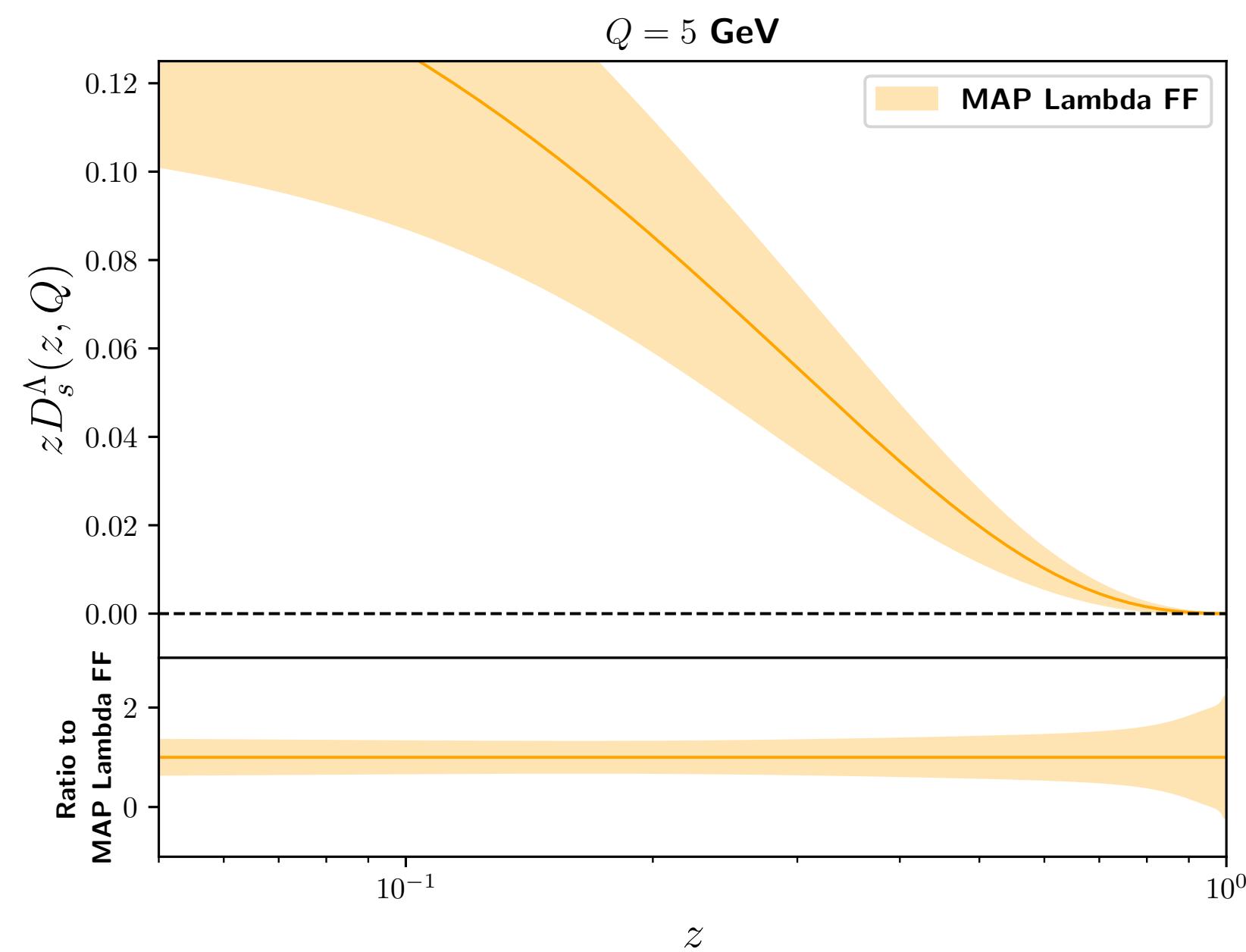
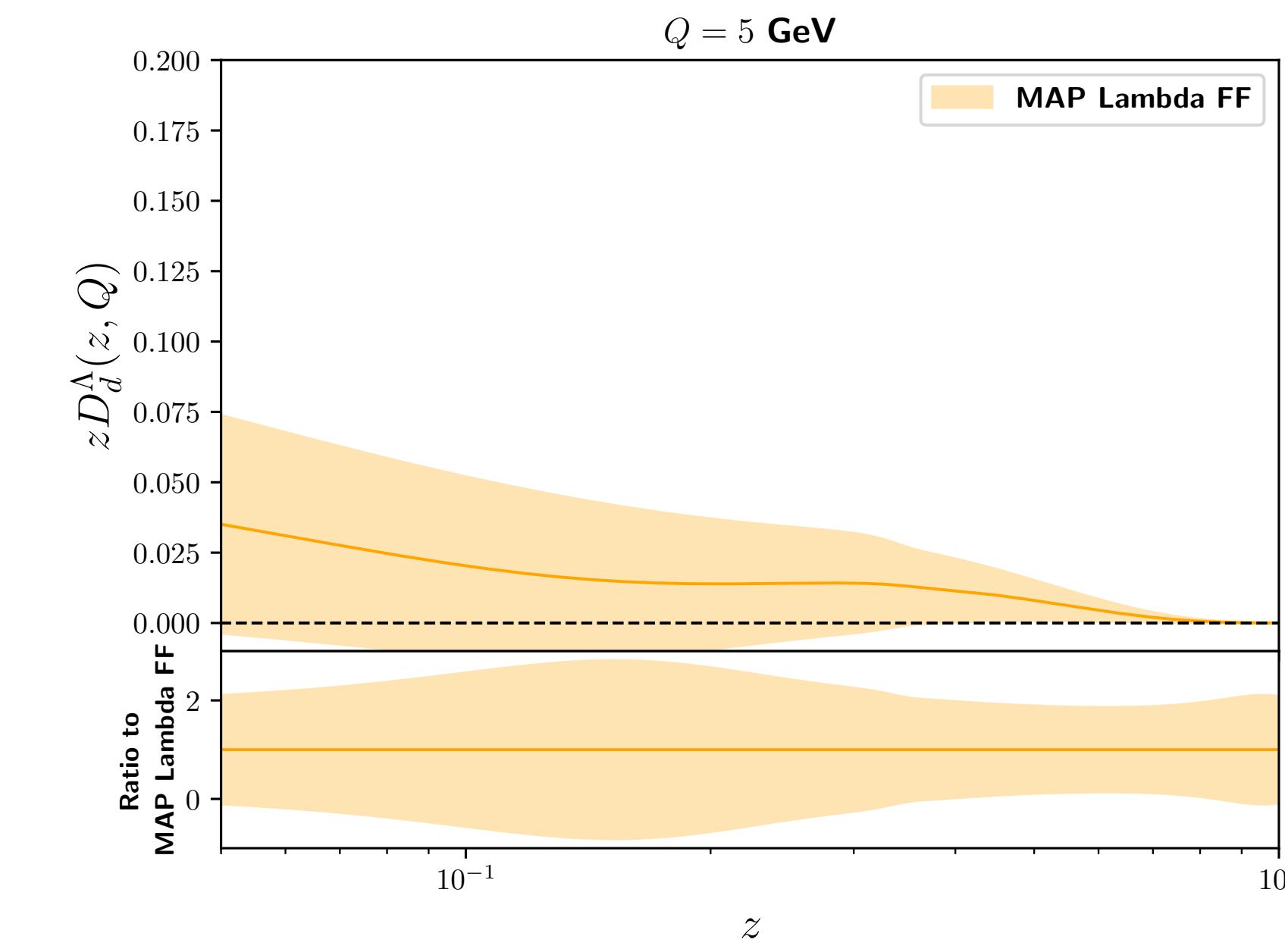
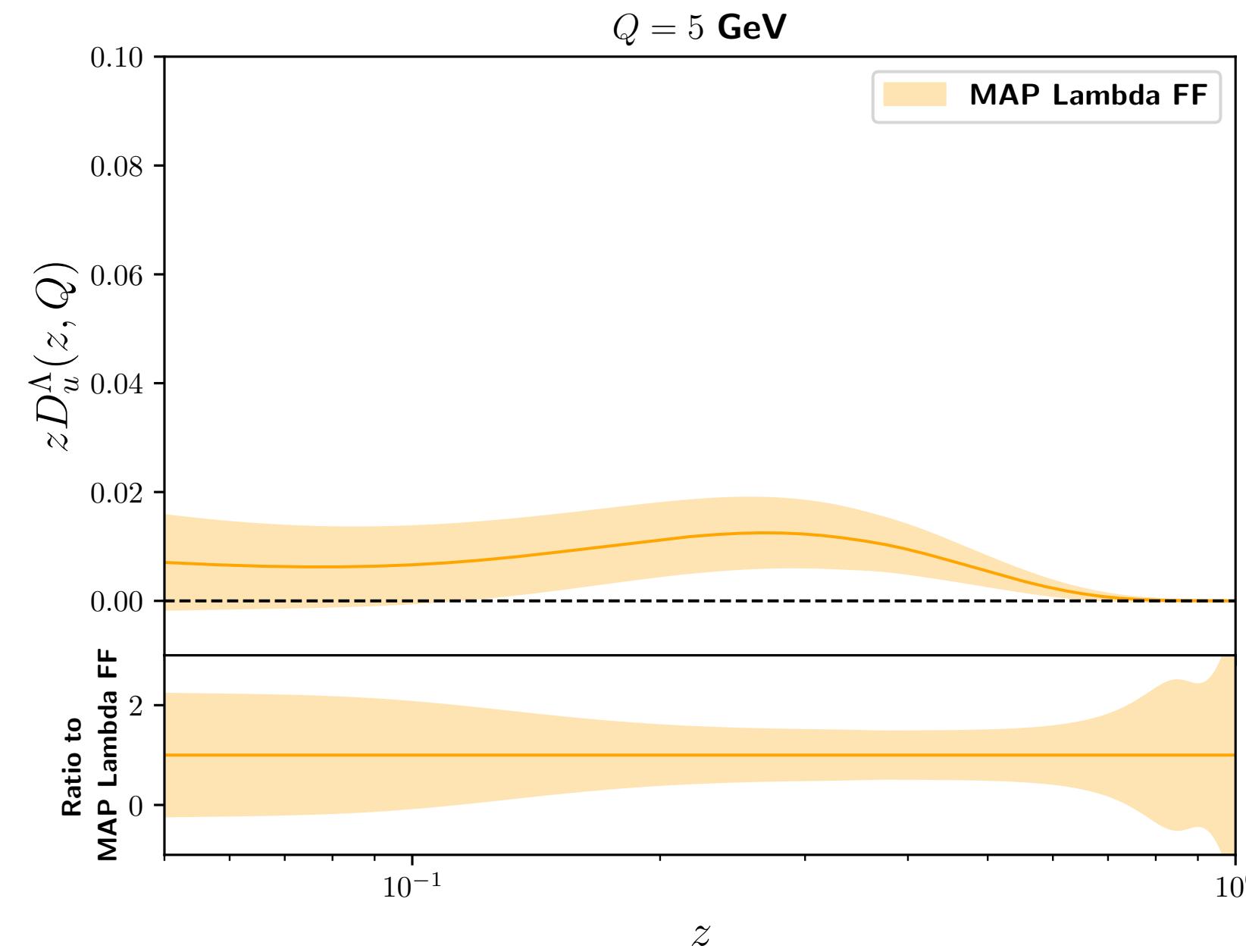
$$D_{u^+}^\Lambda, D_{d^+}^\Lambda, D_{s^+}^\Lambda, D_{c^+}^\Lambda, D_{b^+}^\Lambda, D_g^\Lambda$$

# RESULTS

FFS

N2LO

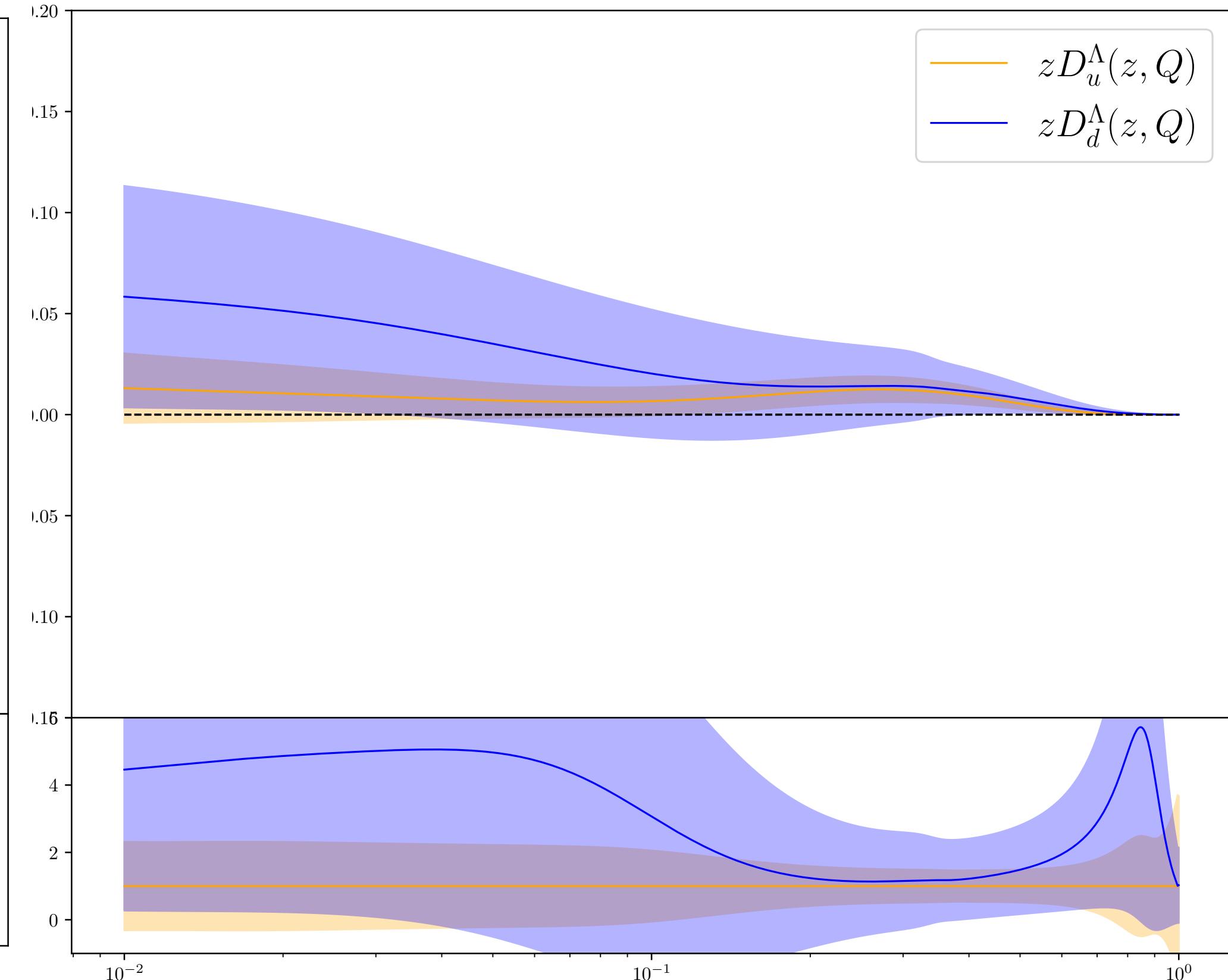
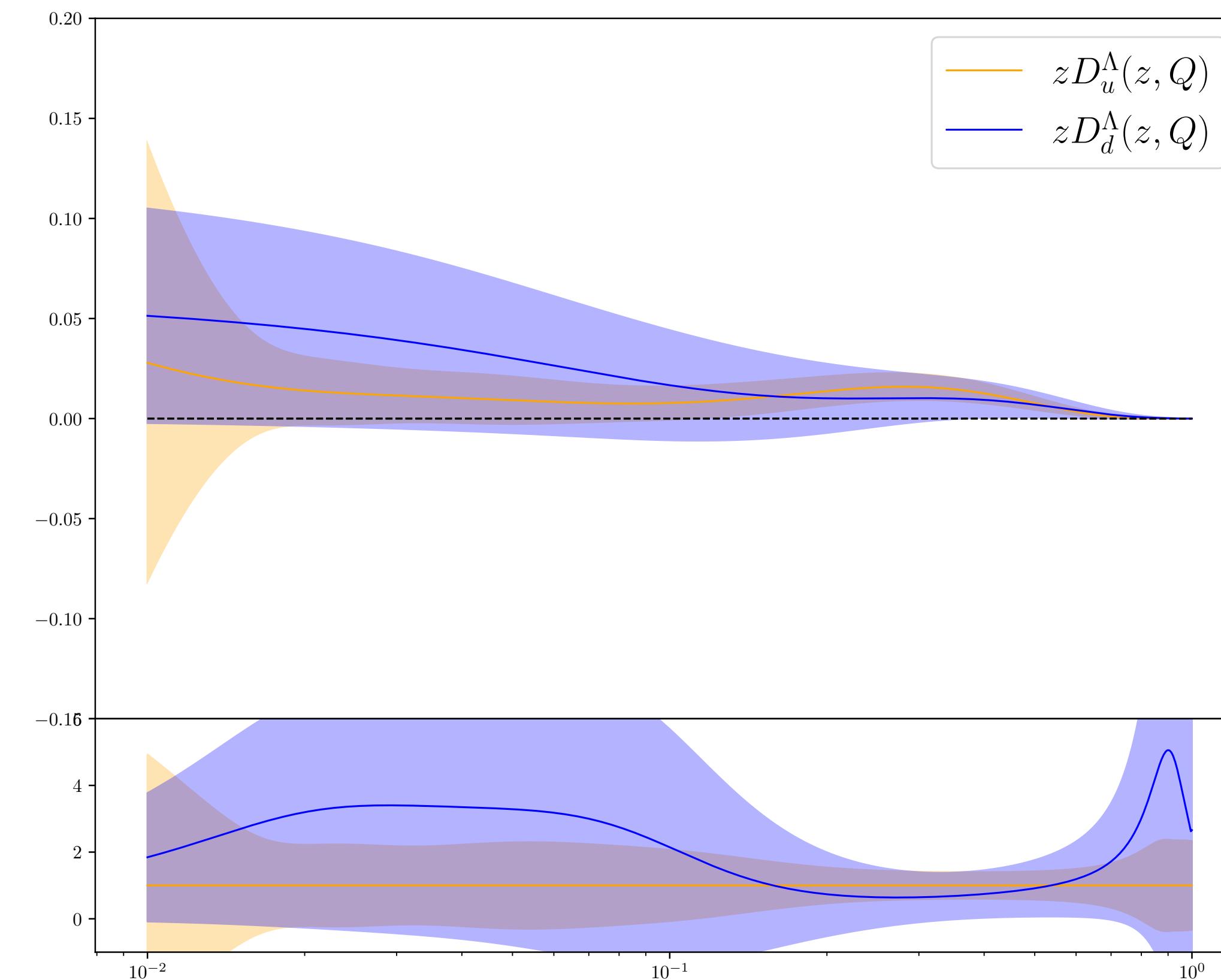
$D_s^\Lambda$  dominates!



# RESULTS

NLO

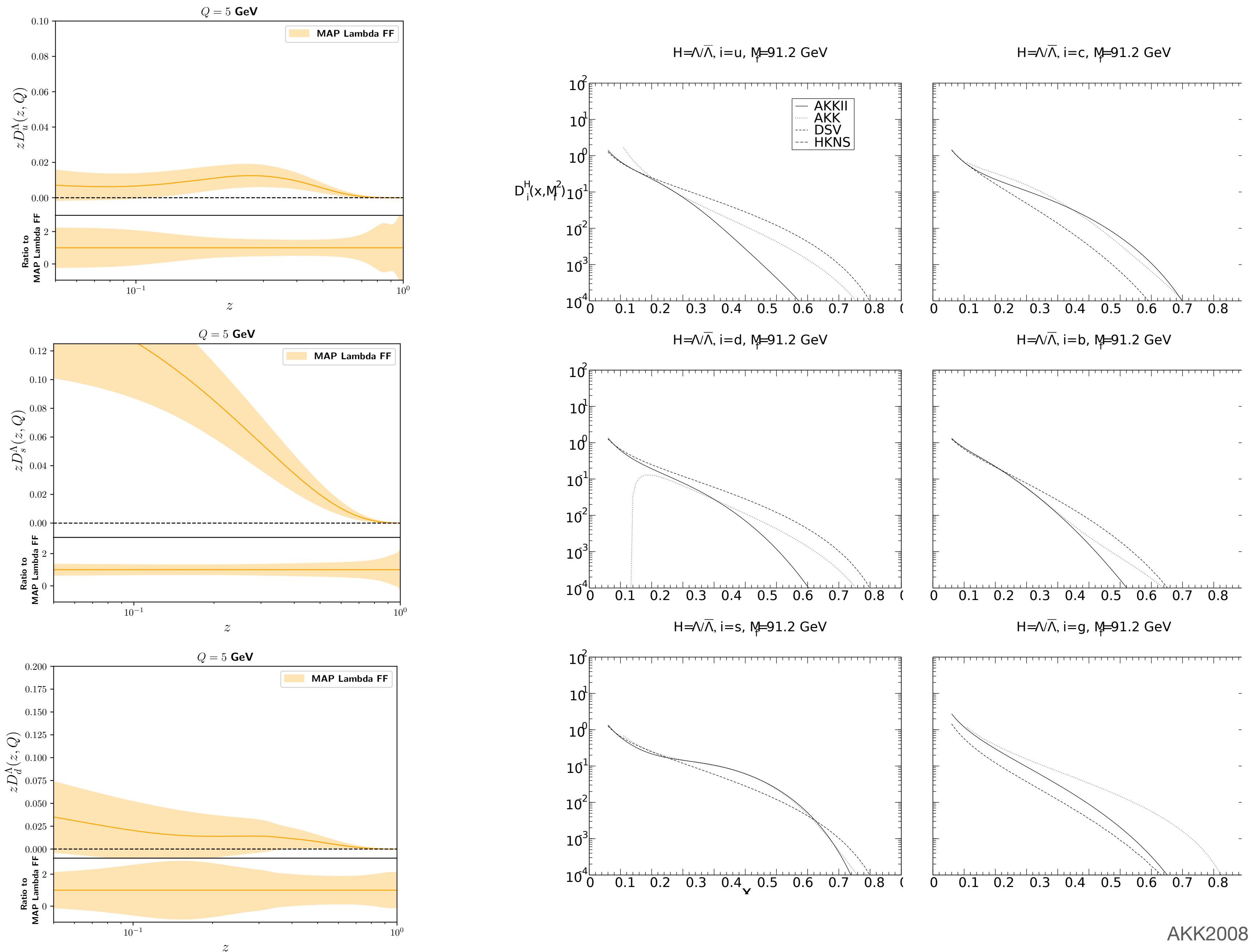
$D_u^\Lambda$  vs  $D_d^\Lambda$



Compatible!

# RESULTS

## Comparison with DSV & AKK



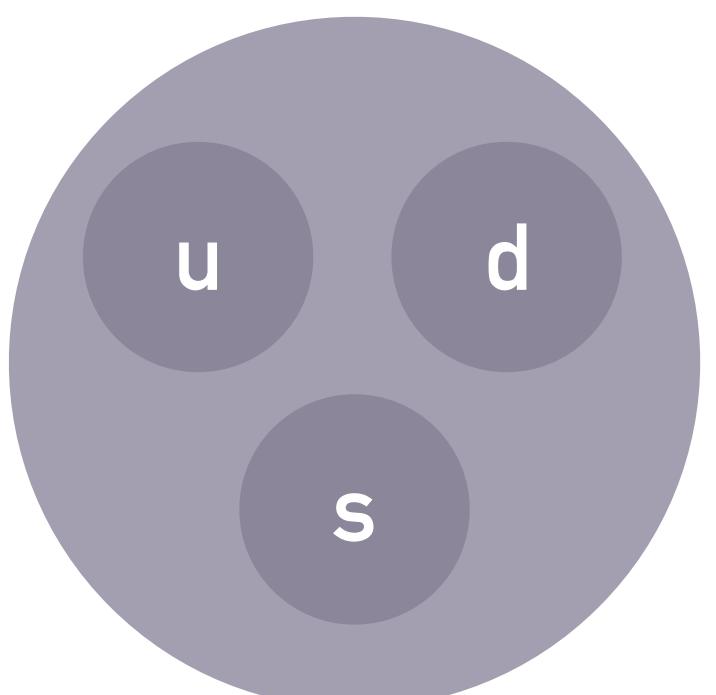
# CONCLUSIONS

**RECAP:** New steps towards a better comprehension of the  $\Lambda$  baryon production in fragmentation

- ▶ Determination of  $\Lambda$  unpolarized collinear fragmentation function from SIA and SIDIS NC data by means of Neural Network and Monte Carlo sampling method as statistical framework at NLO and N2LO
- ▶  $\chi^2=1.51$  at N2LO obtained requiring 7 independent flavours

Next steps:

- ▶ Extend dataset to CC SIDIS and pp collisions
- ▶ Include target and hadron mass corrections for SIDIS
- ▶ Try new flavour combinations (such as requiring isospin symmetry)
- ▶ Improve kinematic constraints in the analysis



# IWHSS & QCD-N 2025

01-05th Sep, 2025  
San Sebastián, Spain



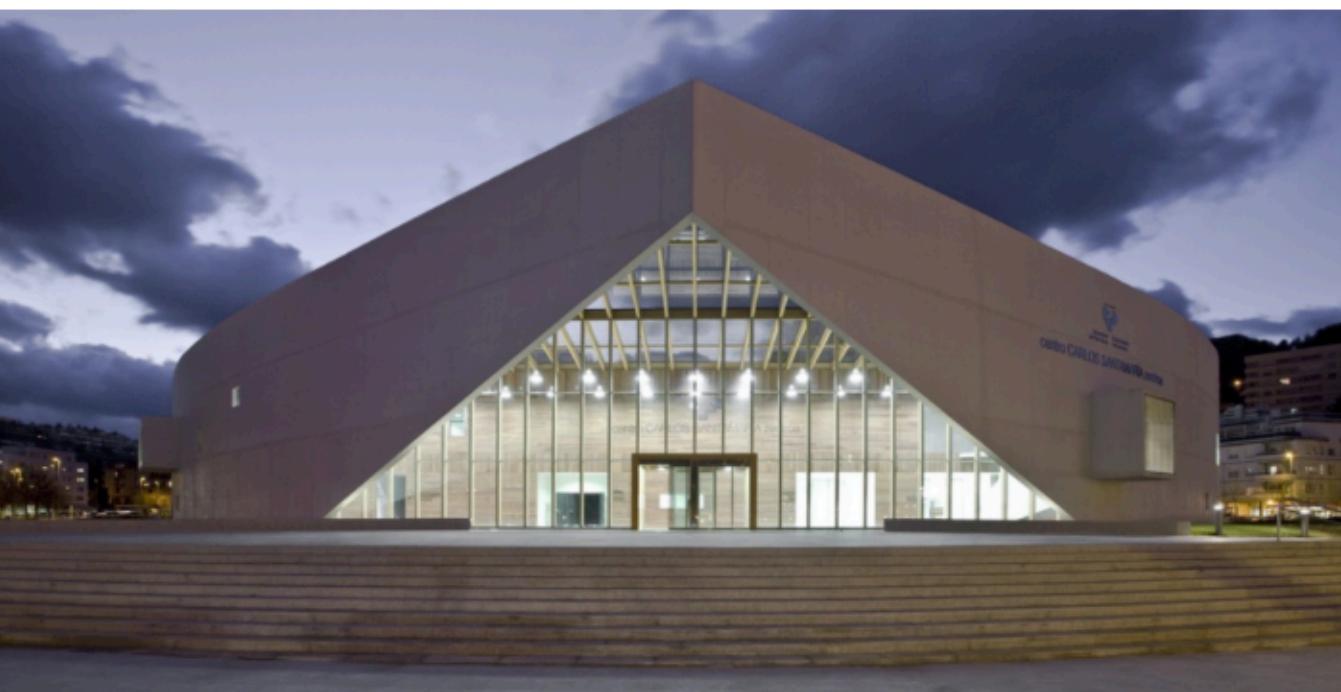
Joint International Workshop on Hadron Structure and Spectroscopy (IWHSS 2025) and the QCD Structure of the Nucleon (QCD-N'25)

01-05 set 2025  
Centro Carlos Santamaría  
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Inserisci il termine di ricerca

Sintesi  
Call for abstract  
Registration Fee  
Pre-registration  
Venue, Travel & Accommodation  
Committees

Contact  
[iwhss-qcdn-2025@cern.ch](mailto:iwhss-qcdn-2025@cern.ch)



Harut Avakian  
Miguel Echevarria  
Bakur Parsamyan  
Gunar Schnell

● Inizio 1 set 2025, 08:30  
● Finisce 5 set 2025, 17:00  
Europe/Zurich

● Centro Carlos Santamaría  
Plaza Elhuyar, 2  
20018 San Sebastián  
Spain  
[Vai alla mappa](#)

**i** The five day long joint "21st International Workshop on Hadron Structure and Spectroscopy" and 6th workshop on the "QCD Structure of the Nucleon" (IWHSS-QCD-N'25) will be held in San Sebastián, Spain, from September 1 to 5, 2025.

The joint workshop is a collaborative effort between the IWHSS and QCD-N workshop series, with the organizational oversight being handled by the HERMES, COMPASS, and JLab communities.

The IWHSS series comprises 21 editions of annual workshops organized by COMPASS Collaboration on Hadron Structure and Spectroscopy, with most recent editions being the [IWHSS-CPHI-2024](#) (Yerevan, Armenia), [IWHSS-2023](#) (Prague, Czechia) and [IWHSS-2022](#) (CERN, Switzerland).

The prior editions of the QCD-N meetings, organized by HERMES Collaboration, were convened in the following locations: [Alcalá de Henares](#) (2021), [Getxo](#) (2016), [Bilbao](#)(2012), [Frascati](#) (2006) and [Ferrara](#) (2002).

A designated session of the workshop will be allocated for the celebration of the **30th anniversary of HERMES data taking**.

The joint workshop will be preceded by the COMPASS Collaboration meeting on September 1 (morning).

<https://indico.cern.ch/event/1527657/>