

Towards Pixel-Based Imaging of Transverse Momentum Distributions

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QCD Evolution 2025

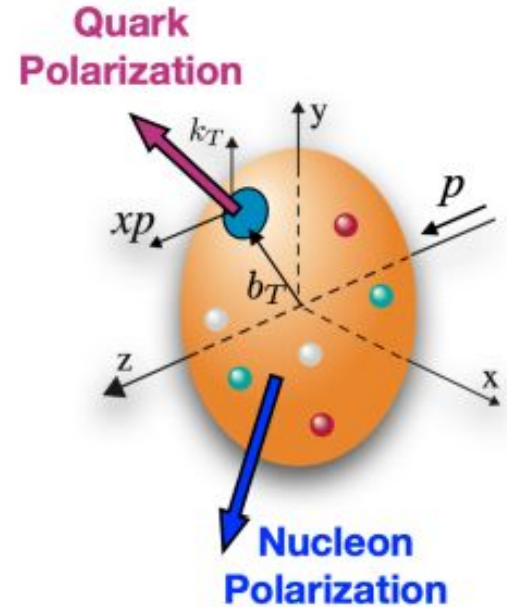


Outline

- **TMDs:** 3D Nucleon Structure, Definition, Importance, Categories
- **Extraction:** From Data, Inverse Problem, UQ
- **Pixel Approach:** Discretization, Advantages, Generative AI
- **Normalizing Flows:** Core Concept & Training
- **Proof of Concept:** Closure Test, Pseudo-Data
- **Results:** Pixel vs. Traditional, Uncertainty Handling
- **Conclusions:** Benefits of Pixel-Based Method

TMD Distributions

- **Transverse Momentum Dependent** Parton Distributions reveal the **3D structure** of protons and neutrons.
- Unlike standard PDFs, TMDs account for a parton's **transverse momentum (k_T)**, not just its collinear motion.
- This k_T information offers a more complete, **spatial view** of parton dynamics within the nucleon.
- **Fragmentation Functions (FFs)** are also crucial, describing how quarks and gluons **hadronize** into observable particles.



TMD Distributions

		Quark Polarization		
		Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	$f_1 = \text{Unpolarized}$		$h_1^\perp = \text{Boer-Mulders}$
	L		$g_1 = \text{Helicity}$	$h_{1L}^\perp = \text{Worm-gear}$
	T	$f_{1T}^\perp = \text{Sivers}$	$g_{1T}^\perp = \text{Worm-gear}$	$h_1^\perp = \text{Transversity}$ $h_{1T}^\perp = \text{Pretzelosity}$

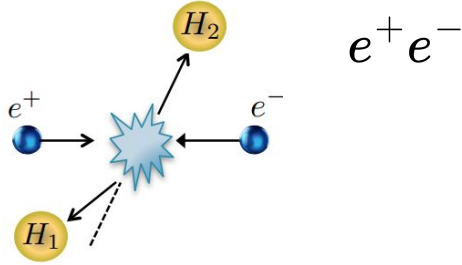
		Quark Polarization		
		Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Unpolarized (or Spin 0) Hadrons		$D_1 = \text{Unpolarized}$		$H_1^\perp = \text{Collins}$
Polarized Hadrons	L		$G_1 = \text{Helicity}$	H_{1L}^\perp
	T	$D_{1T}^\perp = \text{Polarizing FF}$	G_{1T}^\perp	$H_1^\perp = \text{Transversity}$ H_{1T}^\perp

- TMDs are classified into **two categories**, each containing **8 independent functions**.
- They describe the **correlation** between **parton transverse momentum** and the **spin** of both **partons** and the **transverse hadron**.
- Different TMDs give rise to various **hadronic asymmetries** observed in experiments.

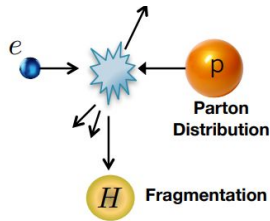
TMDs, as **continuous functions**, inherently carry rich information.

Hadron tomography in transverse momentum space.

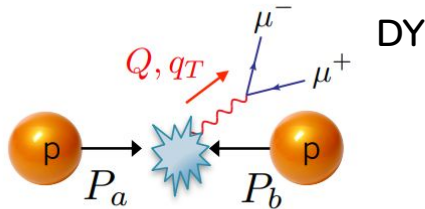
Where do we extract TMDs?



e^+e^-



SIDIS



DY

- TMDs are extracted from diverse data, thanks to **factorization**.
- Reconstructing TMDs from finite data is an **inverse problem**.
- Robust **Uncertainty Quantification (UQ)** is critical for reliable TMD reconstructions.

TMDs extraction

$$\tilde{f}_{i/p_s}^{[\Gamma]0(u)}(x, \mathbf{b}_T, \epsilon, \tau, xP^+) \quad \tilde{\Delta}_{h;i}^{[\Gamma]0(u)}(z, \mathbf{b}_T, \epsilon, \tau, P^+/z)$$

Continuous Function
=
“Infinite Information”

Tractable inference via prior

TMD model parametrization

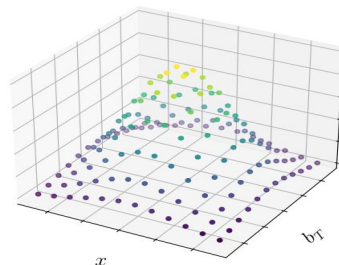
Classical models

See M. Cerutti, P. Barry
Talks

NN representations

See C. Bissolotti's Talk

Pixel-Based
approach

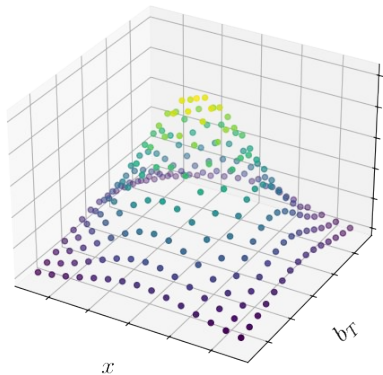


Pixel
Optimization
&
UQ

Reducing Continuum Information

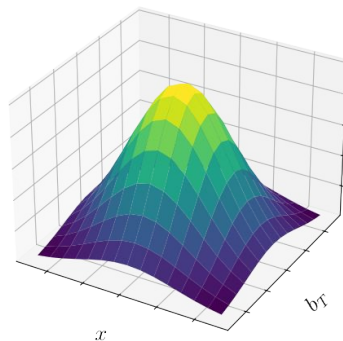
Pixel based Reconstruction of TMDs

Discretized TMD in x and b_T space



Linear Interpolation

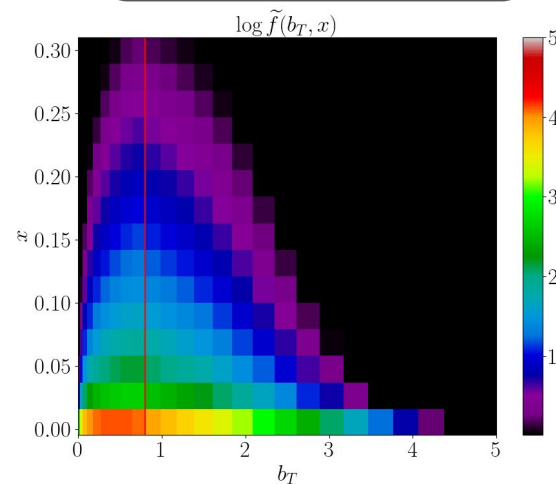
Continuous Function



- Explicit control of *expressivity* and TMD structure via discretization (i.e. # pixels).
- Quantifying *local resolution* from data.
- Improved *Uncertainty Quantification*.
- Model Independent Ansatz

Image Reconstruction using
Generative AI

Pixel-based:
because results can be
represented as an
images!



Generative AI

Understanding, Generating, Reconstructing Images

Generative AI Examples:

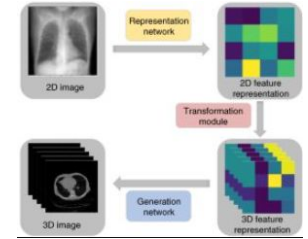
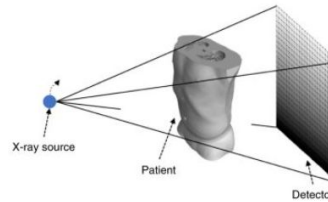
Generation of highly realistic human faces



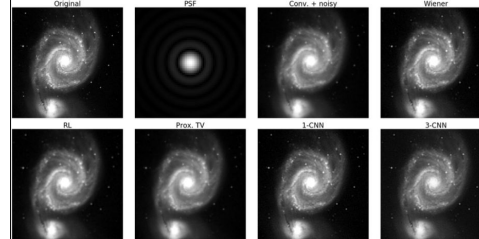
Architectural visualizations and design



Used in Scientific Fields for Reconstruction: Medical Imaging



Astrophysics



Art Restoration




Understanding, Generating, Reconstructing Images

Reconstruction:

Towards Pixel-Based Imaging of Transverse Momentum Distributions

Mentor: Zeph Grunschlag, Jefferson Lab
GCD Fellowship 2023




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- TMDs: 2D Nuclear Structure, Definition, Importance, Categories
- Extensions: From Data, Inverse Problems, UQ
- Pixel Approach: Discretization, Advantages, Generative AI
- Normalizing Flow: Core Concept & Training
- Proof of Concept: Closure Test, Pseudo-Data
- Results: Pixel vs. Traditional, Uncertainty Handling
- Conclusions: Benefits of Pixel-Based Method

TMD Distributions

- Transverse Momentum Dependent** Parton Distribution Functions (TMD PDFs) describe the 2D distribution of partons and are crucial for understanding the spin structure of nucleons.
- Unlike standard PDFs, TMDs account for a parton's transverse momentum (PT), relative to the collision axis.
- They are characterized by a complex, coupled system of partial integro-differential equations.
- Phenomenological Prediction (PPF)** are also crucial, describing how quarks and gluons behave within the observable partons.



TMD Distributions

PDFs are classified into two categories, depending on whether the parton is a quark or a gluon:


- Quark PDFs:** These describe the probability for a quark to carry a certain fraction of the nucleon's momentum and transverse momentum.
- Gluon PDFs:** These describe the probability for a gluon to carry a certain fraction of the nucleon's momentum and transverse momentum.

TMDs are continuous functions, naturally only used in calculations.

Machine learning is becoming more and more important in the field of TMDs.

Where do we extract TMDs?

- TMDs are extracted from observables, thanks to the factorization theorem.
- Reconstructing TMDs from their data is an inverse problem.
- Reduce Uncertainty: Quantification (UQ) is critical to reduce TMD reconstruction errors.



TMDs extraction

Factorization theorem structure

TMD model parametrization

Global models TMD reconstruction

Pixel Reconstruction

Pixel Optimization (UQ)

Image Reconstructing using Generative AI

Pixel based Reconstruction of TMDs

Discretized TMD in a pixelated space

Generative Function

Pixel Based Reconstruction

Image Reconstruction using Generative AI

Generative AI

Understanding, Generating, Reconstructing Images

Generative AI Examples

Image Reconstruction using Generative AI

Generative AI

Understanding, Generating, Reconstructing Images

Pixelated AI-Generated Presentation

Generative AI Examples

Image Reconstruction using Generative AI

Training Loop Overview

Generative Function

Pixel Based Reconstruction

Image Reconstruction using Generative AI

Normalizing Flow: Core concept

Generative Function

Pixel Based Reconstruction

Image Reconstruction using Generative AI

Normalizing Flow: Core concept

Generative Function

Pixel Based Reconstruction

Image Reconstruction using Generative AI

Proof of Concept: Closure Test for TMD Extraction

Generative Function

Pixel Based Reconstruction

Image Reconstruction using Generative AI

SIDIS Multiplicities and Cross Section

Generative Function

Pixel Based Reconstruction

Image Reconstruction using Generative AI

Compass Data

Generative Function

Pixel Based Reconstruction

Image Reconstruction using Generative AI

Reference Models: Non-perturbative Terms

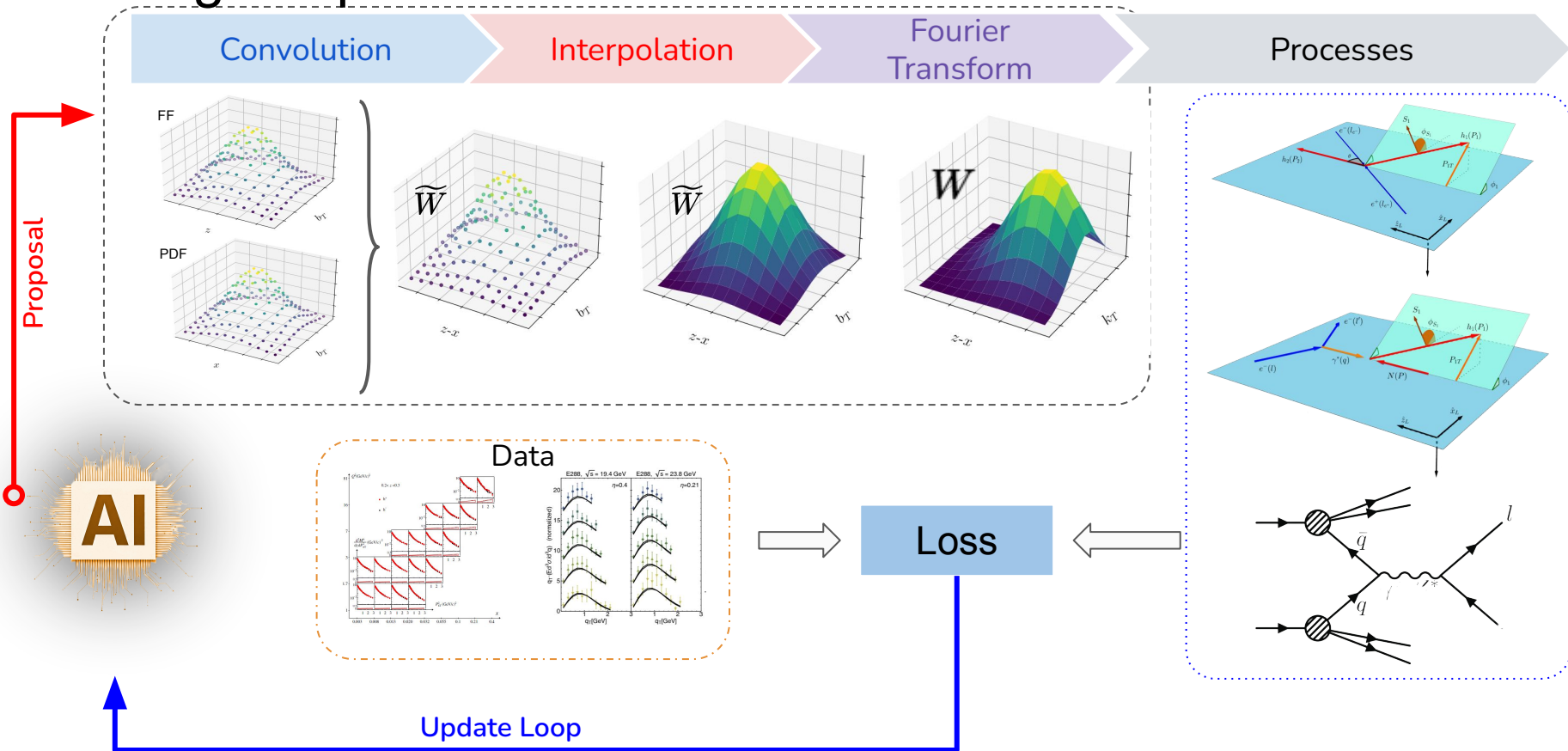
Generative Function

Pixel Based Reconstruction

Image Reconstruction using Generative AI



Training Loop Overview



Normalizing Flow: Core concept

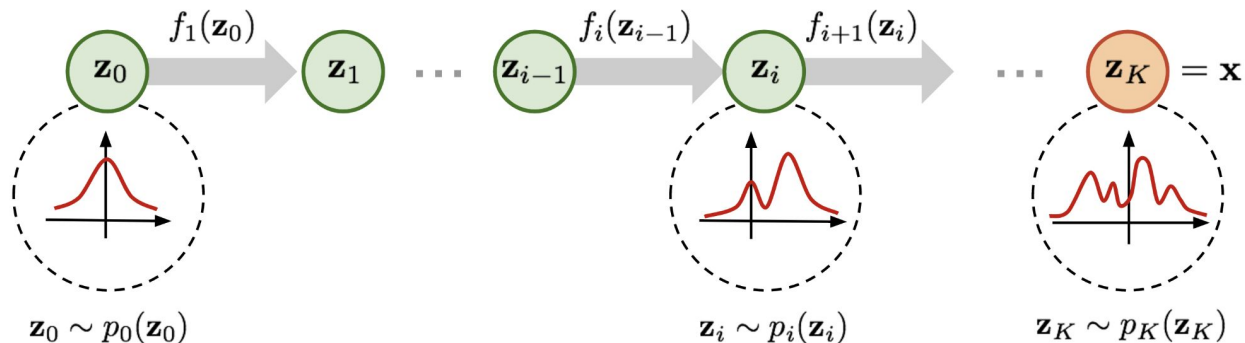
$$\overset{\text{Posterior}}{-2 \ln P(TMD|data)} = \overset{\text{Chi-square}}{\chi^2(data|TMD)} + \overset{\text{Gaussian prior}}{\sum_i (TMD/\delta)^2}$$

The AI generates pixels and, through training, learns their distribution.

To achieve this, we need:

- **χ^2 or log-likelihood distribution:** This depends on the TMD pixel values.
- **Gaussian prior for pixels:** Addresses the ill-posed nature of the problem, allowing pixel reconstruction from data, and limits pixel size.
- **Model learns the log-posterior distribution:** This represents the pixel distribution allowed by the data.
- **Result:** We can generate pixels and their distribution.

Normalizing Flow: Core concept



Posterior

$$P(TMD|data)$$

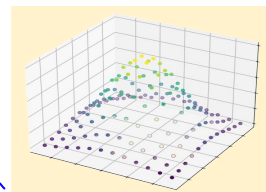
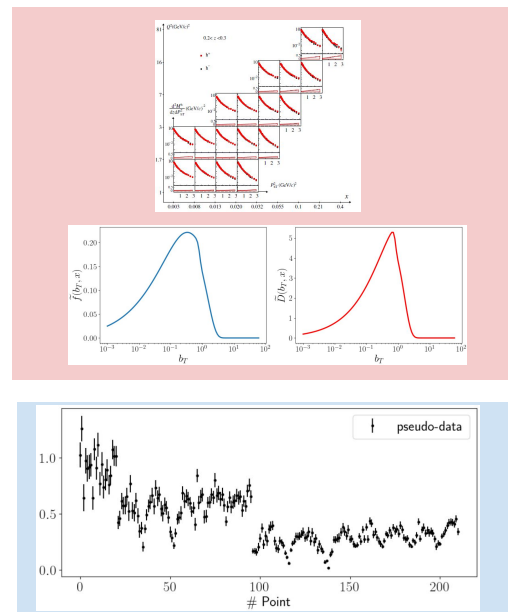
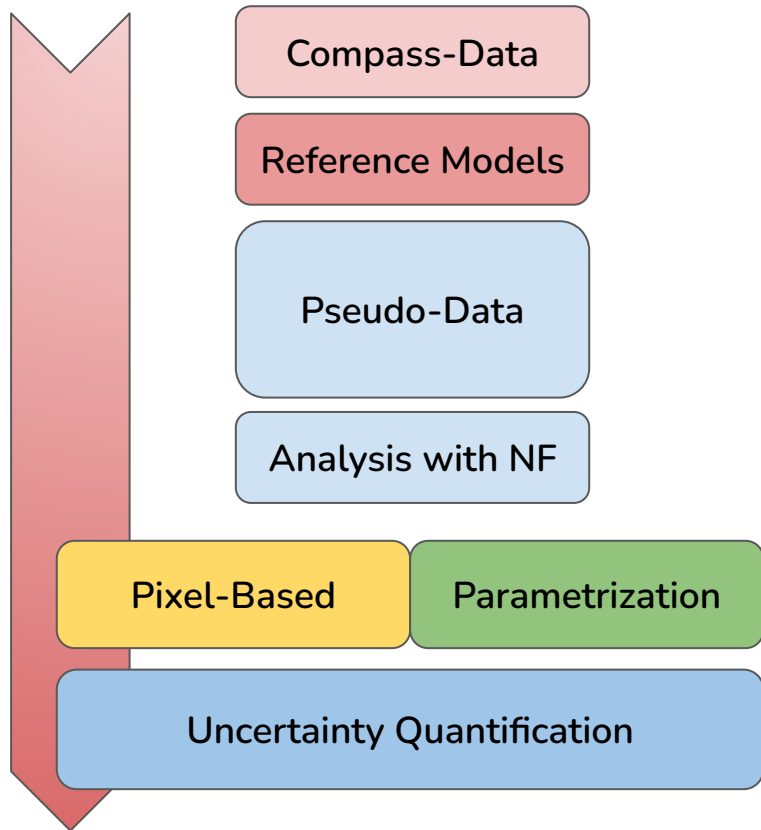
KLD loss

$$D_{KL}(P||Q) = \int P(x) \log \frac{P(x)}{Q(x)} dx$$

- **Approximating Complex Distributions** : NF learn to transform a simple distribution (e.g., a Gaussian) into a target distribution (e.g. TMD's log-posterior).
- **Objective** : Match the log-posterior distribution using a "reverse KLD loss".
No explicit calculation of the normalization factor
- **Invertible Transformations** : A **Flow** is a series of invertible transformations.

The target distribution is the log-posterior, which combines information from experimental data and prior expectations within a **Bayesian inference** framework.

Proof of Concept: Closure Test for TMD Extraction



$$M_f(b_T, x; b_{max}) = \exp(-\alpha_f b_T^2 R(b_T; b_{max})/4)$$

$$g_K(b_T; b_{max}) = -g_2 b_T^2 R(b_T; b_{max})/2$$

SIDIS Multiplicities and Cross Section

SIDIS Multiplicities:

$$\frac{d^2 M^h(x, z, P_{h\perp}^2, Q^2)}{dz dP_{h\perp}^2} = \left(\frac{d^4 \sigma^h}{dx dQ^2 dz dP_{h\perp}^2} \right) / \left(\frac{d^2 \sigma^{DIS}}{dx dQ^2} \right)$$

Unpolarized Cross section

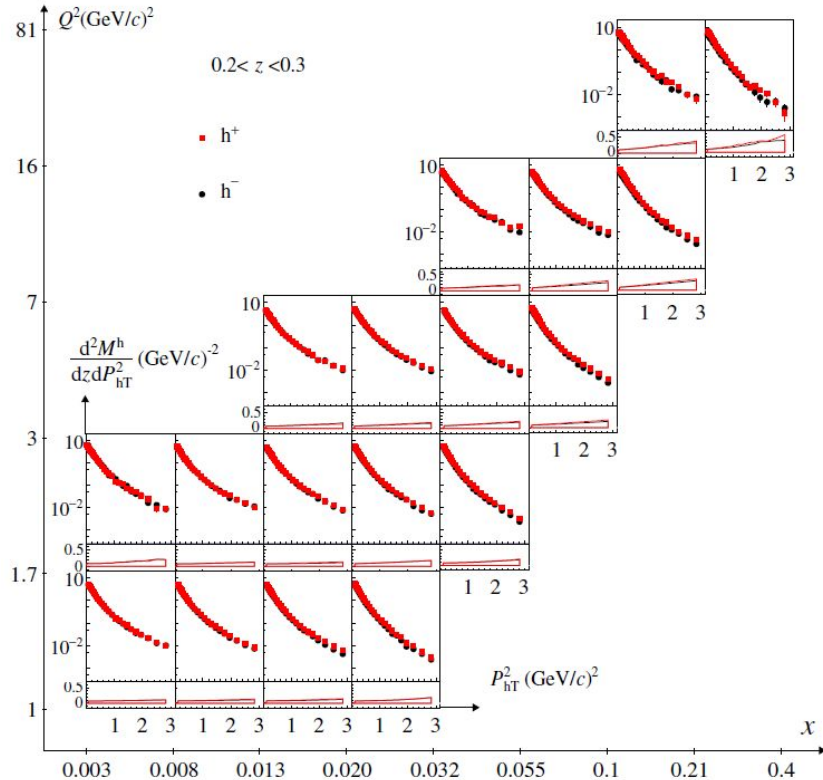
$$\frac{d\sigma}{dx dy dz d\phi_S d\phi_h dP_{h\perp}^2} = \frac{\alpha_{em}^2}{xy Q^2} \frac{y^2}{2(1-\epsilon)} \left(1 + \frac{\gamma^2}{2x} \right) F_{UU,T}$$

$$\frac{d\sigma^{DIS}}{dx dQ^2} = \frac{4\pi\alpha_{em}^2}{x Q^2} \mathcal{Y}_+ F_2(x, Q^2)$$

Form Factor

$$\begin{aligned} F_{UU}(x, z, q_T^2, Q^2) &= \mathcal{C}[f_1 D_1] \\ &= x H(Q^2) \sum_a e_a^2 \int d^2 p_T d^2 k_T \delta^{(2)}(\mathbf{p}_T - \mathbf{k}_T - \mathbf{q}_T) f_1^a(x, p_T^2) D_1^a(z, k_T^2) \\ &= x H(Q^2) \sum_a e_a^2 \frac{1}{2\pi} \int db_T b_T J_0(q_T b_T) \tilde{f}_1^a(x, b_T) \tilde{D}_1^a(z, b_T) \end{aligned}$$

Compass Data



PHYS. REV. D 97, 032006 (2018)

Data selection and kinematic cuts:

$$Q^2 > 5 \text{ GeV}^2$$

$$q_T/Q < 0.5$$

$$z < 0.6$$

$$had = h^+$$

211 points

Extract Reference Models

Pseudo-data:

Same statistics and kinematic cuts

Reference Models: Non-perturbative Terms

TMD PDF

$$\tilde{f}_{j/P}(x, \mathbf{b}_T; \mu, \zeta_F) = \sum_j \int_x^1 \frac{d\hat{x}}{\hat{x}} C_{f/j}(x/\hat{x}, \mathbf{b}_*; \mu_b^2, \mu_b, g(\mu_b)) f_{j/P}(\hat{x}, \mu_b) \\ \times \exp \left\{ \ln \frac{\sqrt{\zeta_F}}{\mu_b} \tilde{K}(b_*; \mu_b) + \int_{\mu_b}^{\mu} \frac{d\mu'}{\mu'} \left[\gamma_F(g(\mu'); 1) - \ln \frac{\sqrt{\zeta_F}}{\mu'} \gamma_K(g(\mu')) \right] \right\} \exp \left\{ g_{j/P}(x, b_T) + g_K(b_T) \ln \frac{\sqrt{\zeta_F}}{\sqrt{\zeta_{F,0}}} \right\} \\ M_f(b_T, x; b_{max})$$

Non-perturbative Models:

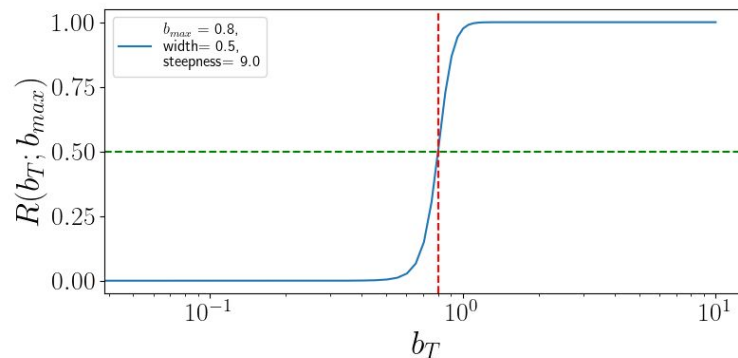
$$M_f(b_T, x; b_{max}) = \exp(-\alpha_f b_T^2 R(b_T; b_{max})/4) \quad \alpha_f = 0.84$$

$$M_D(b_T, z; b_{max}) = \exp(-\alpha_D b_T^2 R(b_T; b_{max})/4z^2) \quad \alpha_D = 0.24$$

$$g_K(b_T; b_{max}) = -g_2 b_T^2 R(b_T; b_{max})/2 \quad g_2 = 0.29$$

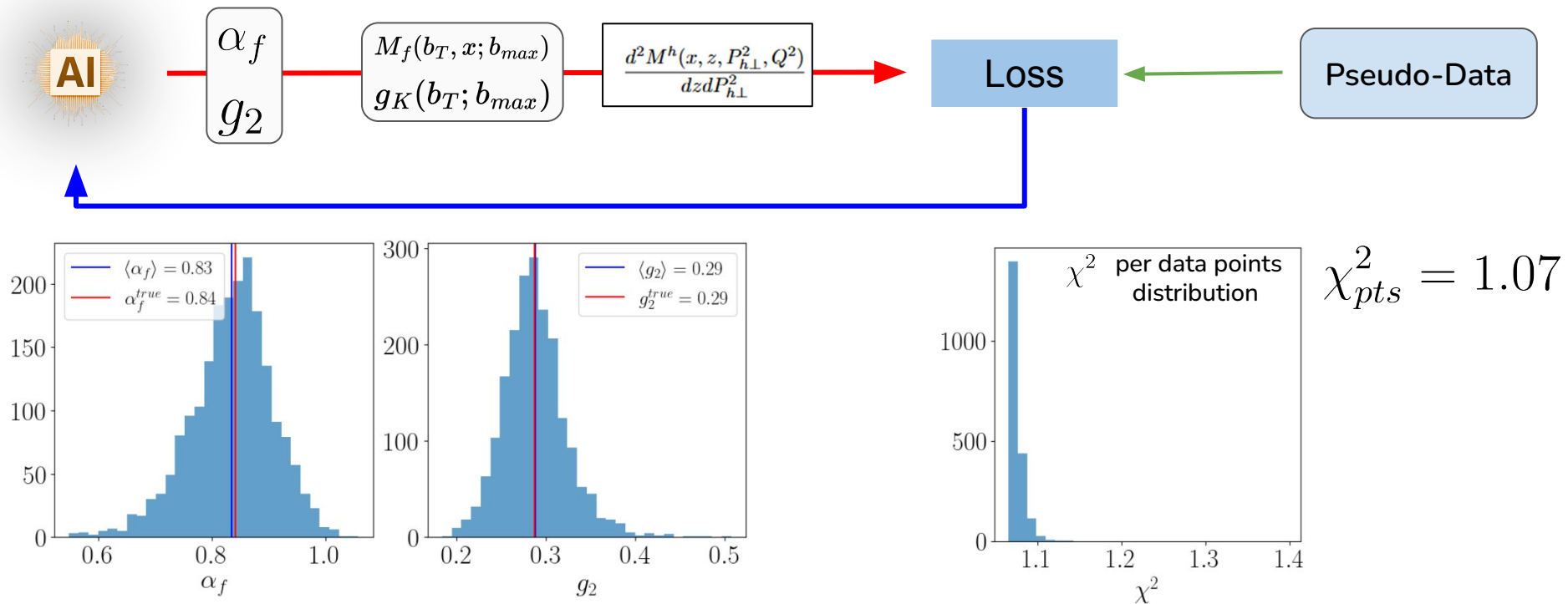
Transition Function

$$R(b_T; b_{max}) = 1 - 1/(1 + \exp(stps \cdot (b_T - b_{max})/wdt))$$



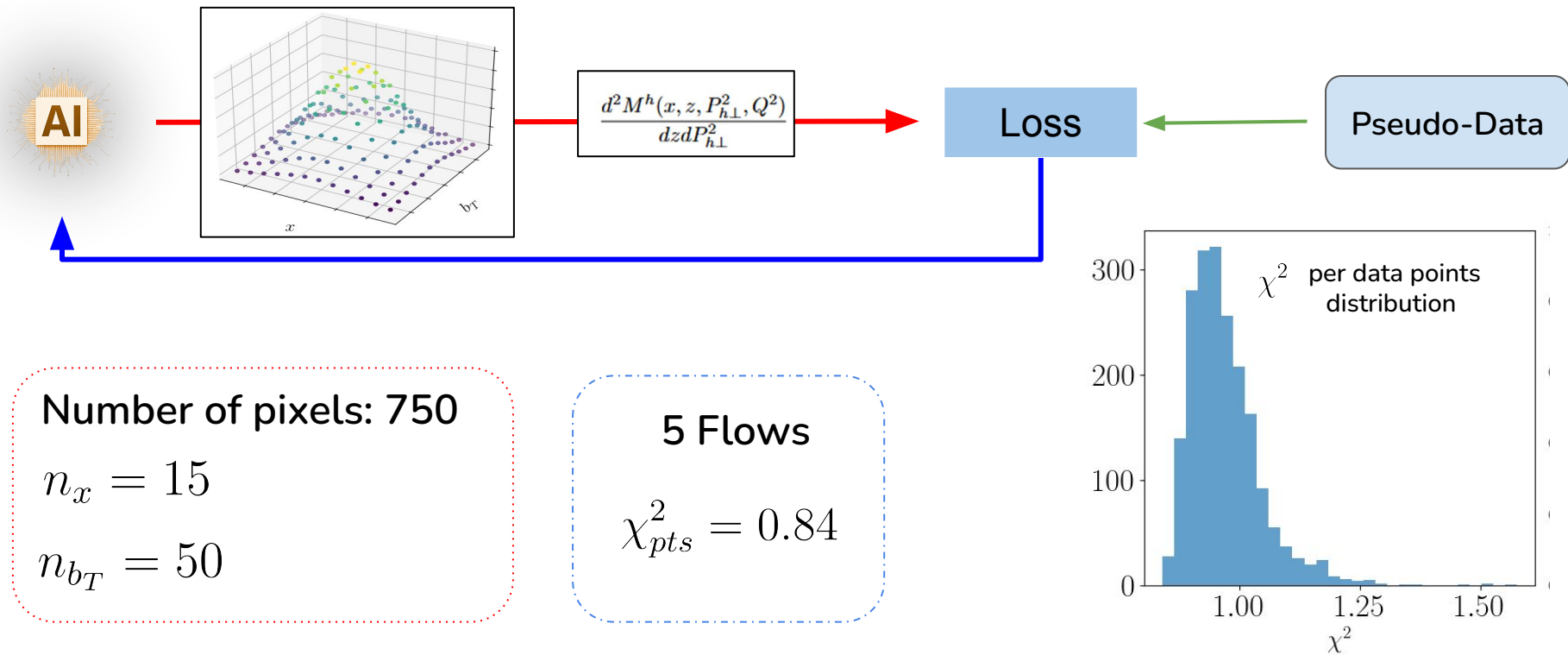
Closure Test: Parametrization Based

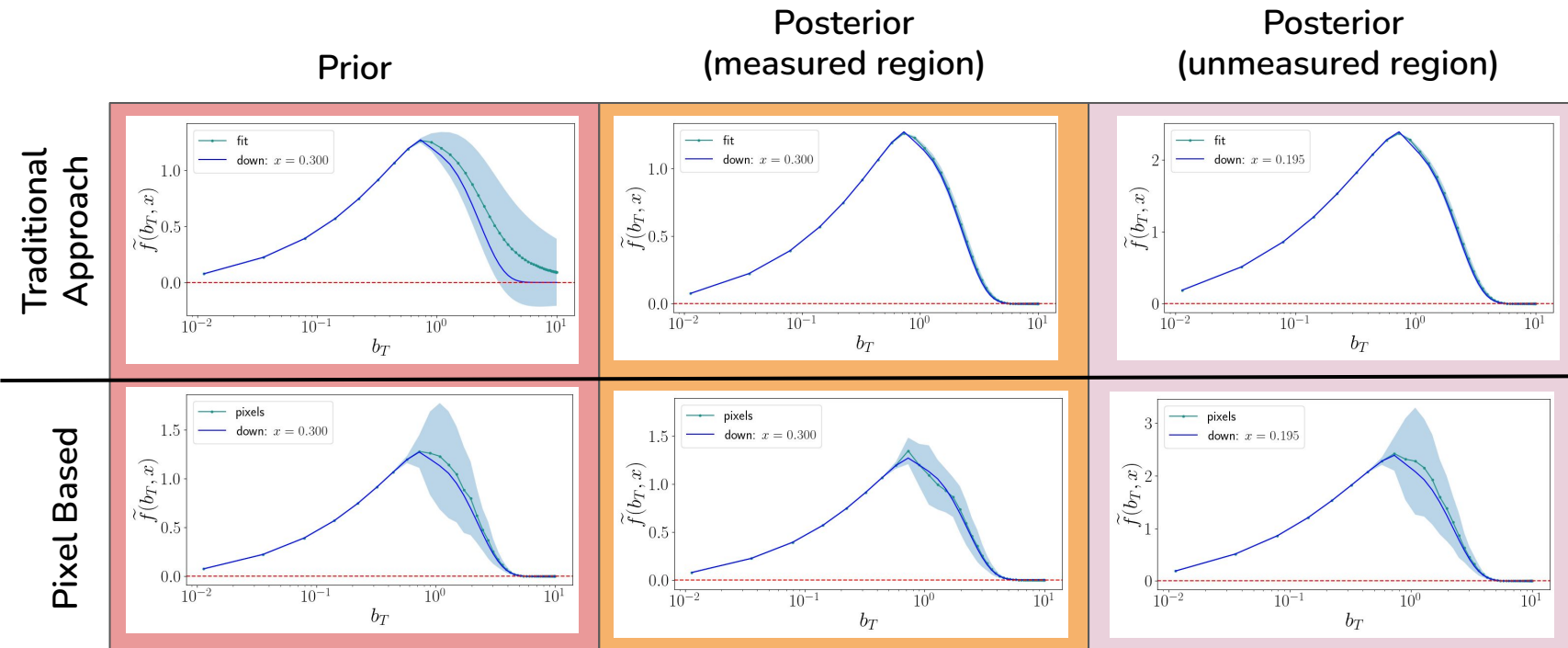
PDF parameters fitted. FF is fixed.



Closure Test: Pixel-based

PDF pixels reconstructed. FF is fixed.





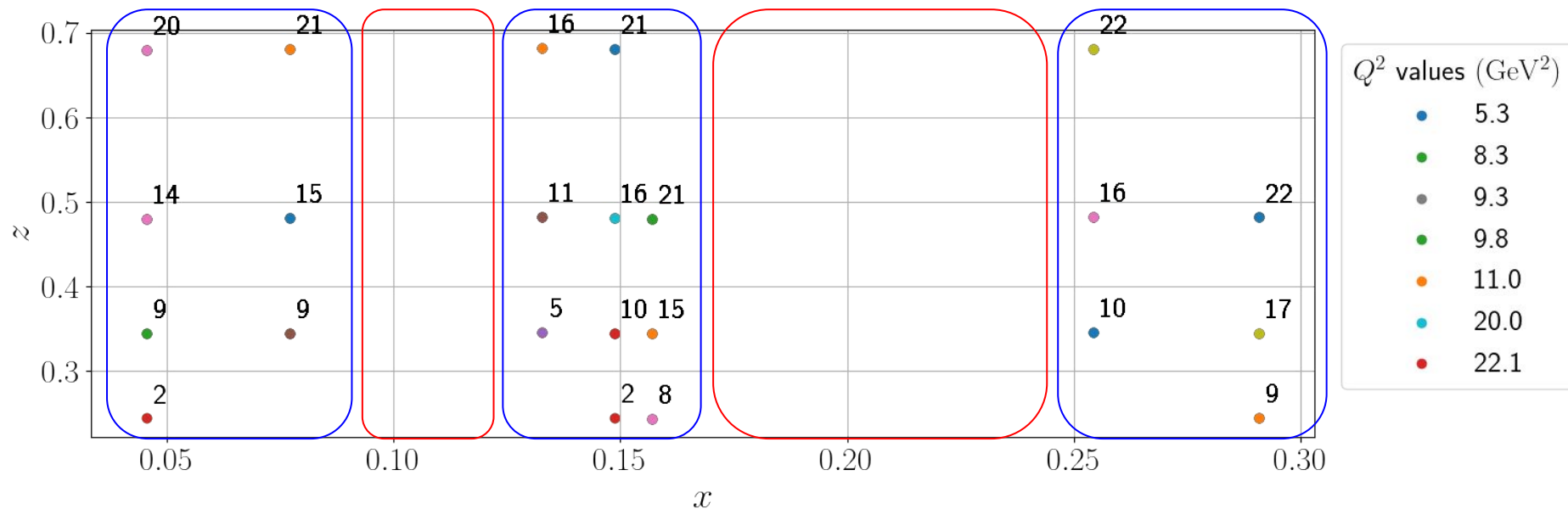
Traditional Approach:

- Smaller uncertainties
- Artificial constraints on extrapolated region

Pixel Based:

- Error band shrinks in measured region
- Band remains constant in unmeasured region (like Prior)

Pseudo Data



Measured regions

Unmeasured regions

Conclusions

- **Transverse Momentum Distributions (TMDs)** provide a **3D view** of nucleon structure.
- A **novel pixel-based approach** reconstructs TMDs using **Generative AI**, specifically **Normalizing Flows**.
- This pixel method allows **explicit control of expressivity**, improved **uncertainty quantification**, and **model independence**.
- Normalizing Flows learn the **log-posterior distribution of TMD pixels**, matching experimental data with prior expectations.
- **Closure tests** with pseudo-data demonstrate that the pixel-based approach offers more **realistic uncertainty handling** compared to traditional methods.

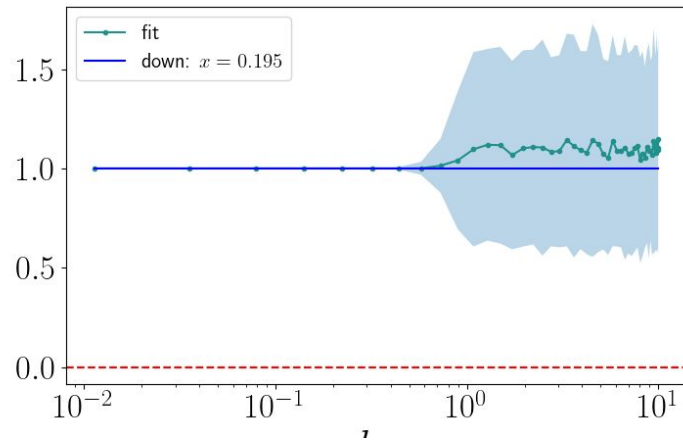
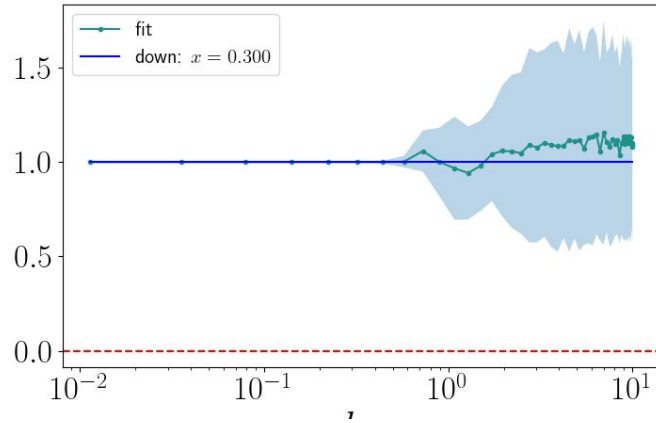
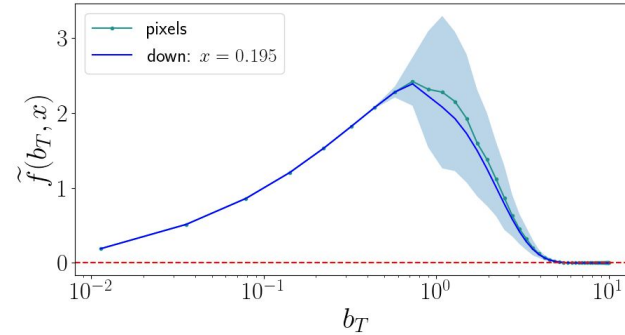
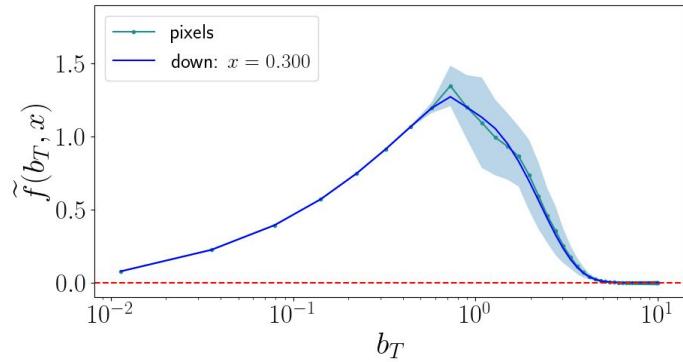
Outlooks:

- **Simultaneous PDF and FF Reconstruction**
- **Compass Real Data Analysis**

Thank you
for your
attention



$$\tilde{f} = OPE \cdot SDK \cdot \overbrace{e^{-\alpha_f b_T^2 R/4 - g_2 b_T^2 R/2 \ln(Q/Q_0)}}^{\text{reference model}} \overbrace{e^{-\epsilon_j R - \epsilon_K R \ln(Q/Q_0)}}^{\text{pixels}}$$



$$\tilde{f}_{i/p_s}^{[\Gamma]0(u)}(x, \mathbf{b}_T, \epsilon, \tau, xP^+) = \int \frac{db^-}{2\pi} e^{-ib^-(xP^+)} \langle p(P, S) | [\bar{\psi}^i(b^\mu) W(b^\mu, 0) \frac{\Gamma}{2} \psi^i(0)]_\tau | p(P, S) \rangle$$

Tomography in transverse momentum space

$$\tilde{\Delta}_{h;i}^{[\Gamma]0(u)}(z, \mathbf{b}_T, \epsilon, \tau, P^+/z) = \frac{1}{4N_c z} \text{Tr} \int \frac{db^-}{2\pi} \sum_X e^{ib^-(P^+/z)} \Gamma_{\alpha\alpha'}^+ \langle 0 | [W\psi_i^{0\alpha}(b)]_\tau | h(P, S), X \rangle \langle h(P, S), X | [W\bar{\psi}_i^{0\alpha'}(0)]_\tau | 0 \rangle$$