Upgrading the knowledge of TMD distributions in the proton

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(MAP Collaboration)

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May 20, 2025

QCD Evolution 2025

Transverse-Momentum Distributions (TMDs)

See talks of the morning

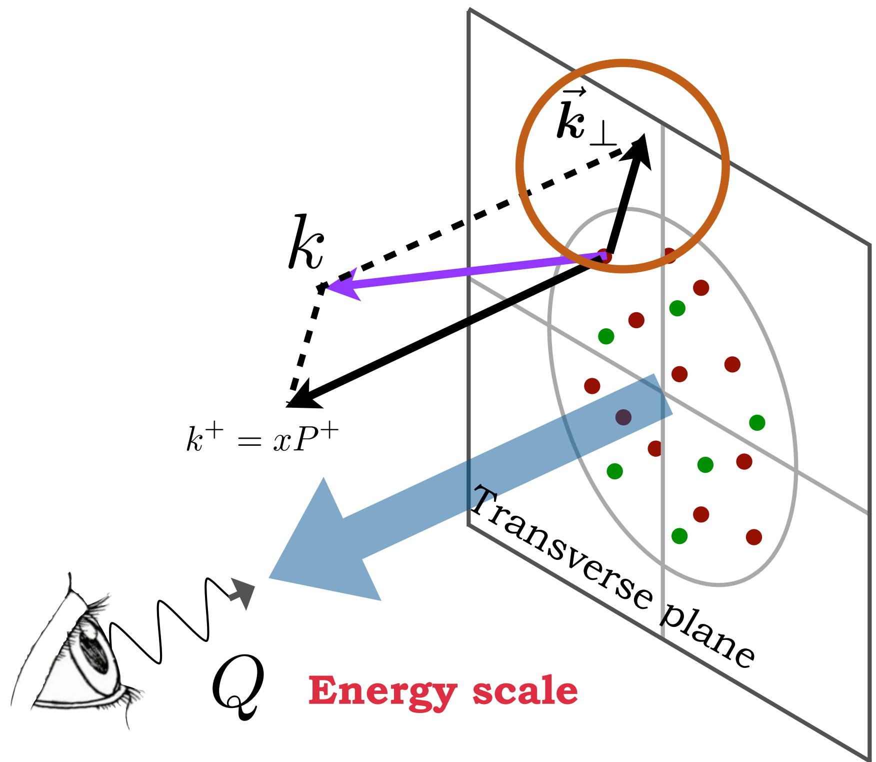
3-***dimensional*** ***map*** of the internal structure of the nucleon

Non-collinear framework

Quark Polarization

Nucleon Pol.	U	L	T
U	f_1		$h_1^\perp$
L		g_1	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1 \; h_{1T}^\perp$

Time-reversal odd   Time-reversal even



TMD PDFs

$$F(x, \boldsymbol{k}_\perp^2, \mu, \zeta)$$

Transverse-Momentum Distributions (TMDs)

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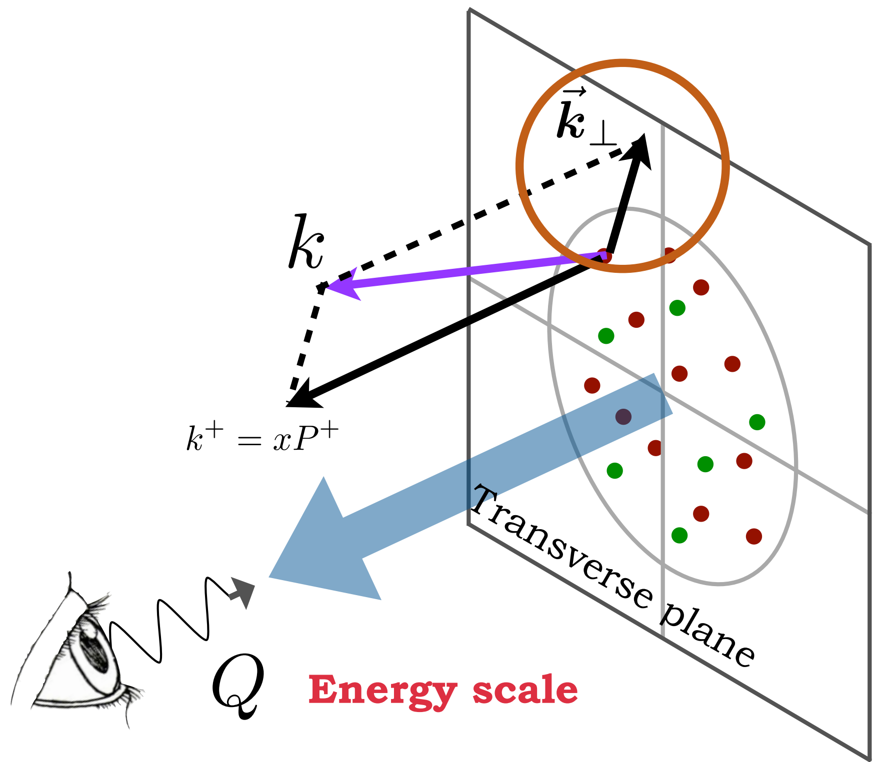

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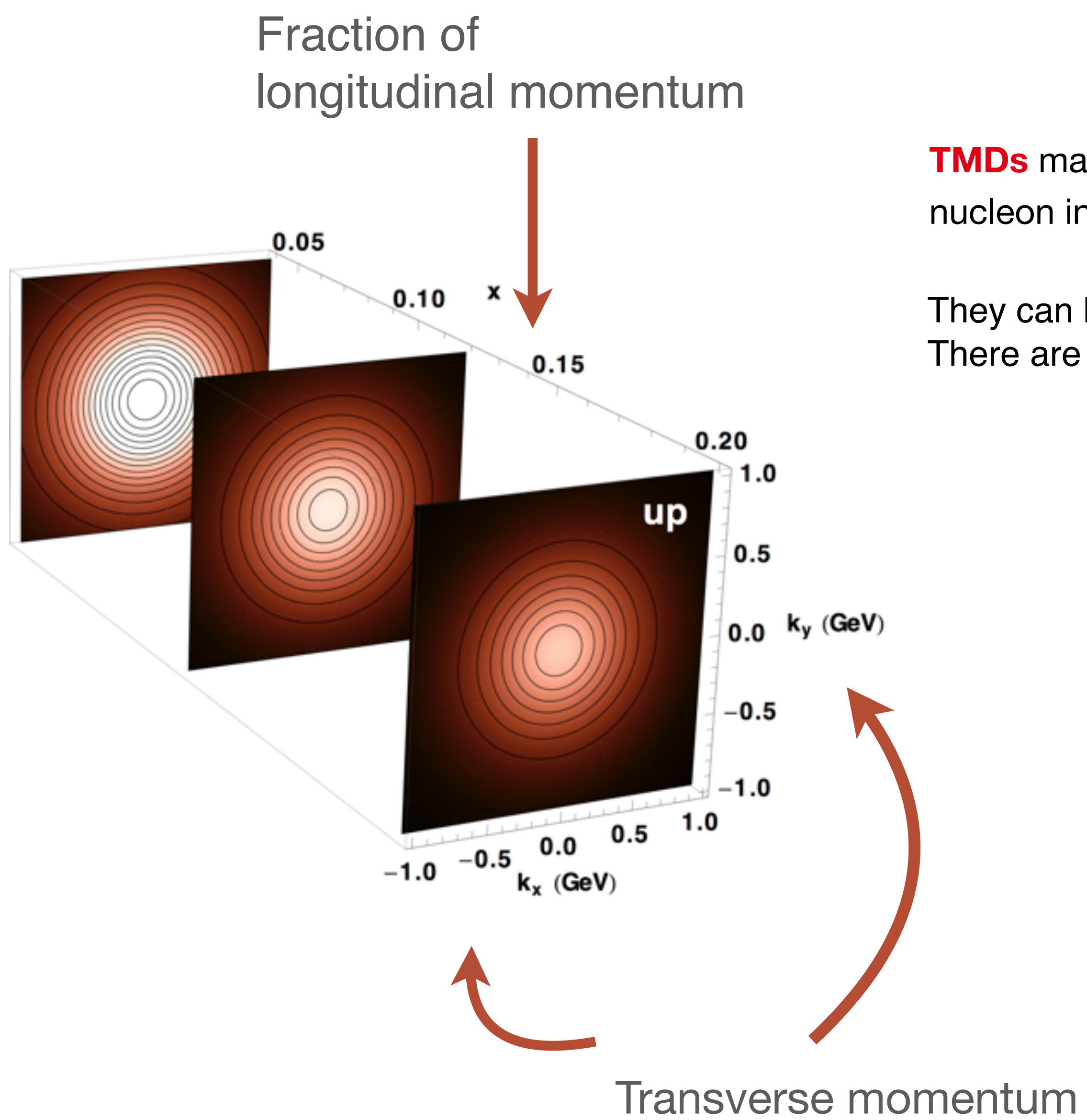
Transverse-Momentum Distributions (TMDs)

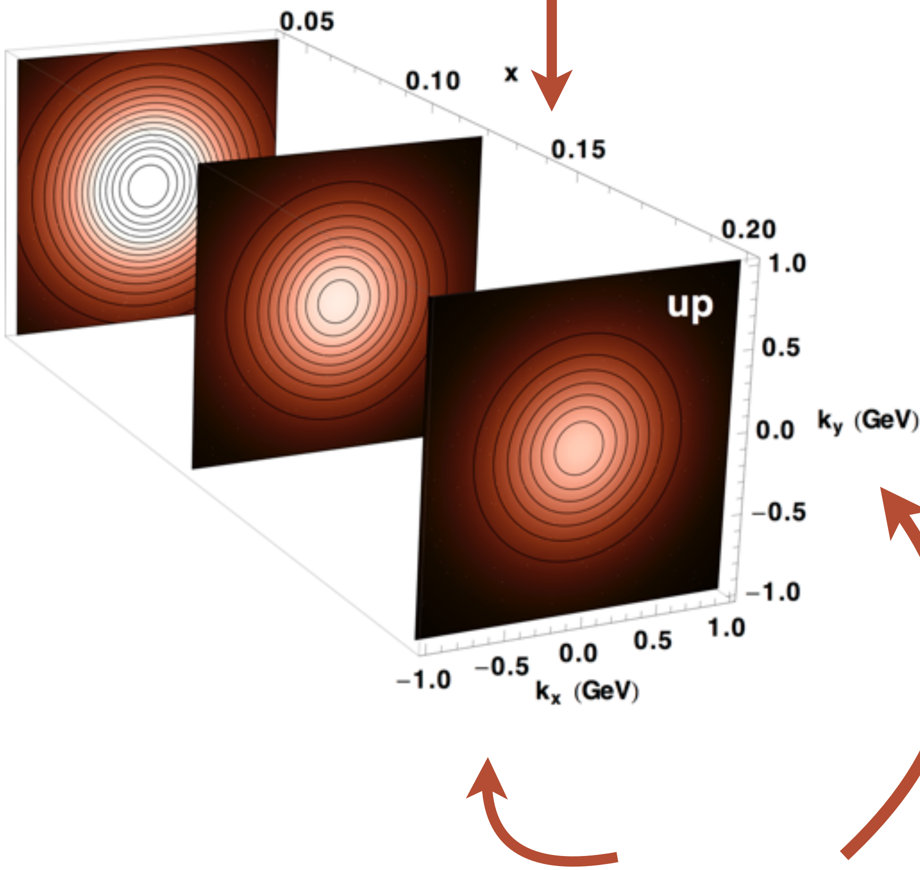


TMDs map the distribution of partons inside the nucleon in 3D in momentum space.

They can be extracted through ***global fits***
There are attempts to calculate them in lattice QCD

Transverse-Momentum Distributions (TMDs)

Fraction of
longitudinal momentum



Transverse momentum

TMDs map the distribution of partons inside the nucleon in 3D in momentum space.

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Are TMDs universal?

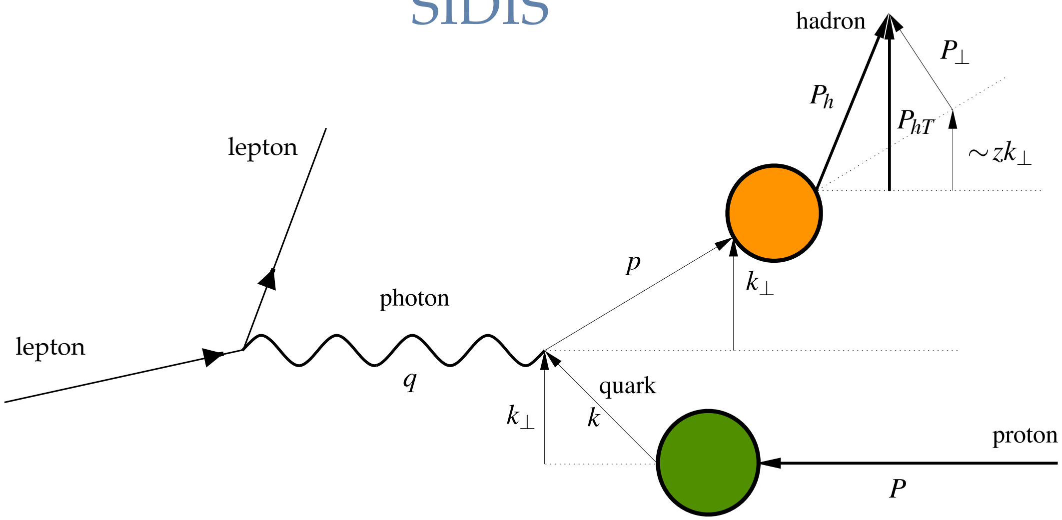
Do they depend on x?

Do they depend on the quark flavor?

TMD factorization: Universality

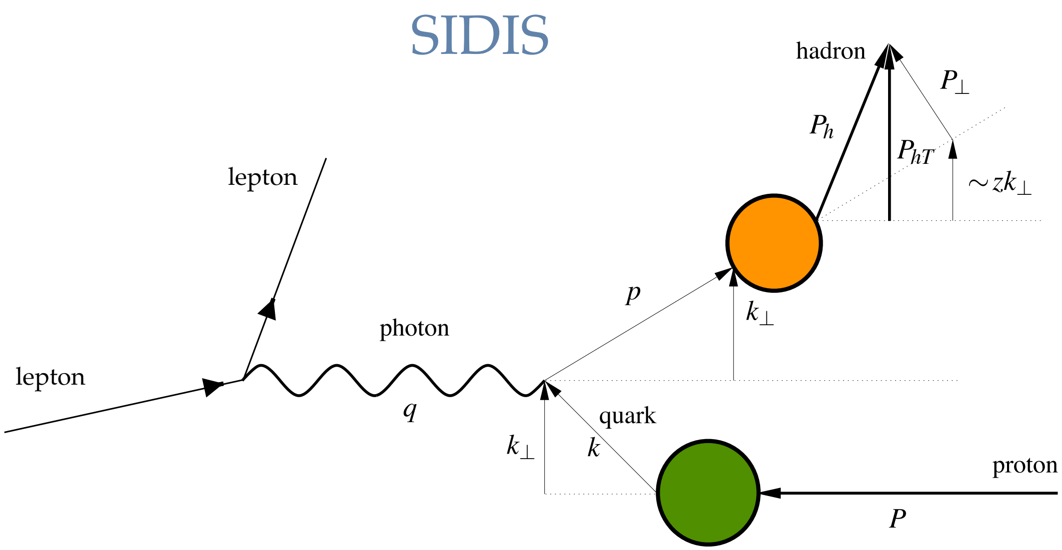
TMD factorization: Universality

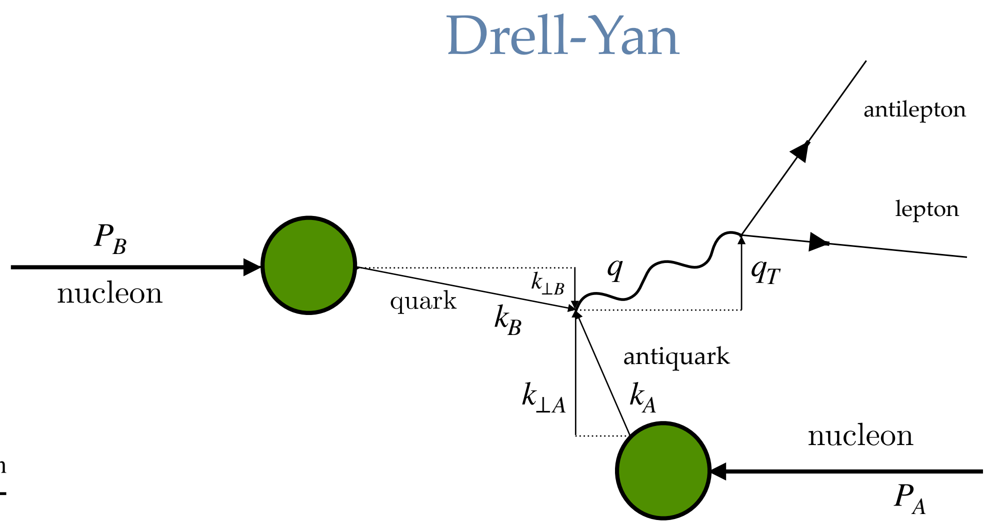
SIDIS



$$F_{UU,T}\big(x, z, |\boldsymbol{q}_T|, Q\big) \sim \int_0^{+\infty} d|\boldsymbol{b}_T||\boldsymbol{b}_T| J_0\big(|\boldsymbol{b}_T||\boldsymbol{q}_T|\big)\,\hat{f}_1^a(x, b_T^2; \mu, \zeta_A)\,\hat{D}_1^{a\to h}(z, b_T^2; \mu, \zeta_B)$$

TMD factorization: Universality





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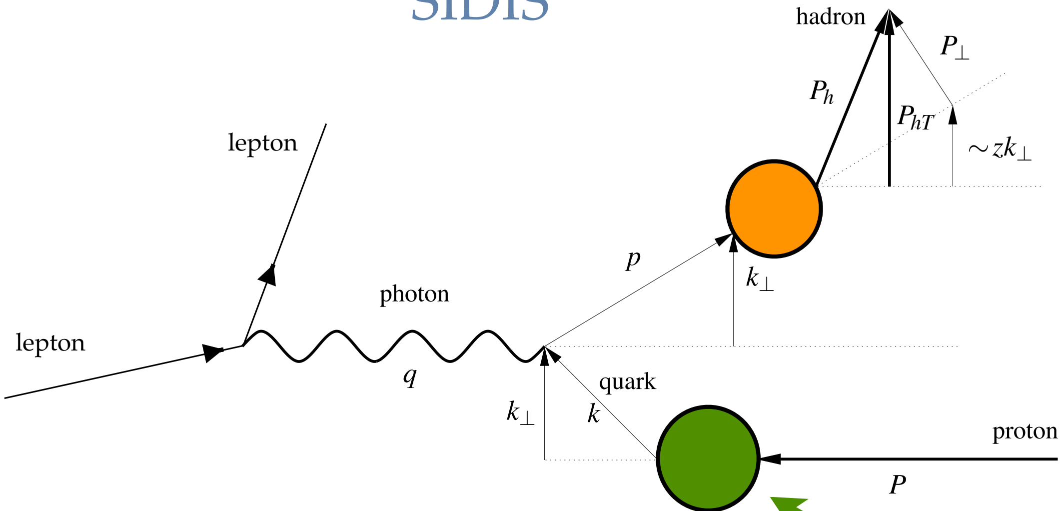
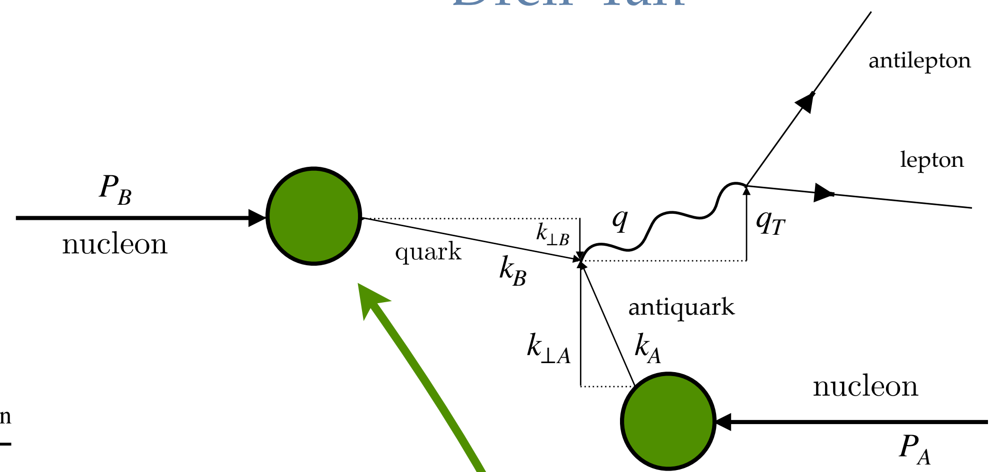
$$F_{UU}^1\left(x_A,x_B,|\boldsymbol{q}_T|,Q\right) \sim \int_0^{+\infty} d|\boldsymbol{b}_T||\boldsymbol{b}_T|J_0\left(|\boldsymbol{b}_T||\boldsymbol{q}_T|\right)\hat{f}_1^a(x_A,b_T^2;\mu,\zeta_A)\hat{f}_1^{\bar{a}}(x_B,b_T^2;\mu,\zeta_B)$$

TMD factorization: Universality

SIDIS



Drell-Yan



Same functions

$$F_{UU,T}(x, z, |\boldsymbol{q}_T|, Q) \sim \int_0^{+\infty} d|\boldsymbol{b}_T||\boldsymbol{b}_T| J_0(|\boldsymbol{b}_T||\boldsymbol{q}_T|)\, \hat{f}_1^a(x, b_T^2; \mu, \zeta_A)\, \hat{D}_1^{a\to h}(z, b_T^2; \mu, \zeta_B)$$

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TMD factorization: Universality

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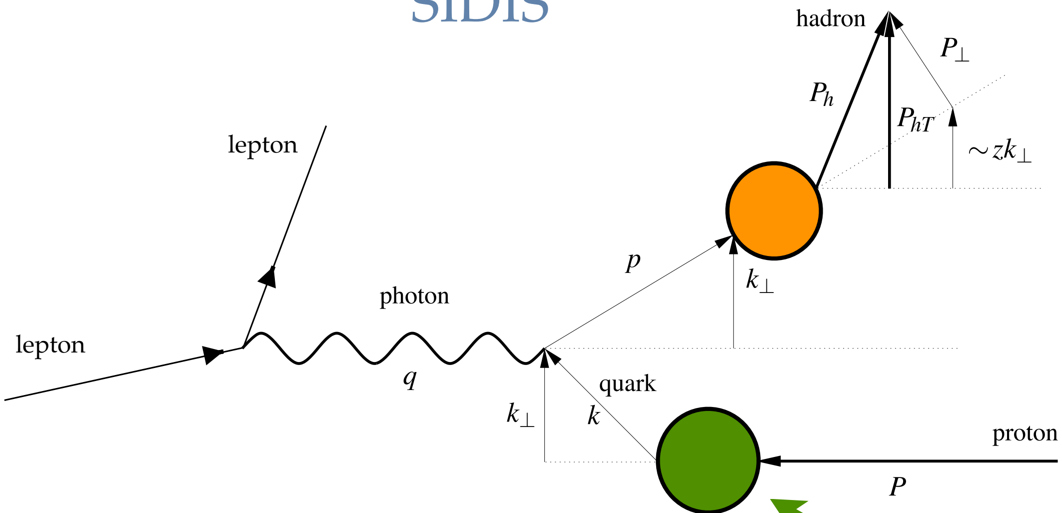

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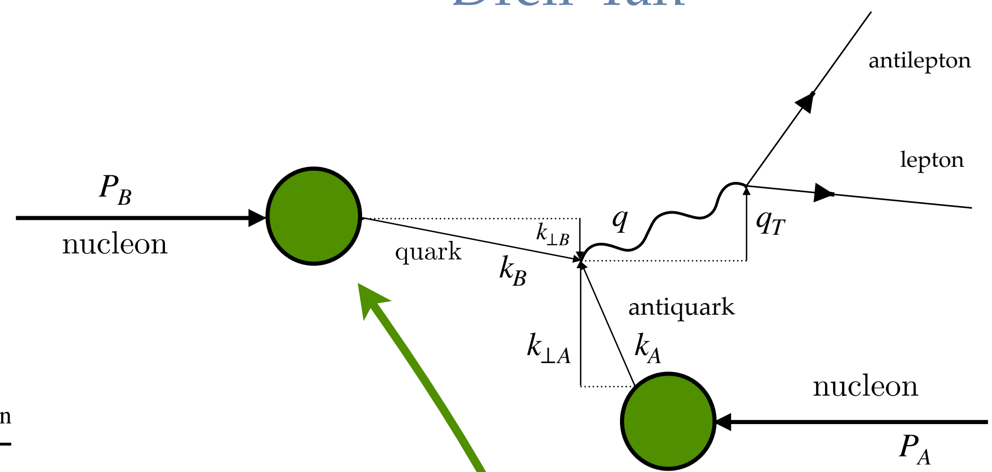

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GLOBAL FITs

Structure of a TMD

TMD in Fourier space

$$\hat{F}(x, b_T^2; \mu, \zeta) = \int \frac{d^2 \boldsymbol{k}_\perp}{(2\pi)^2} e^{i \boldsymbol{b}_T \cdot \boldsymbol{k}_\perp} F(x, k_\perp^2; \mu, \zeta)$$

See talks of the morning

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See talks of the morning

$$\hat{f}_1^q(x, b_T^2; \mu, \zeta) = \sum_j C_{q/j}(x, b_*; \mu_{b_*}, \mu_{b_*}^2) \otimes f_1^j(x, \mu_{b_*})$$

J. Collins, "Foundation of Perturbative QCD"

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b_*-prescription

$$\hat{f}_1^q(x, b_T^2; \mu, \zeta) = \sum_j C_{q/j}(x, b_*; \mu_{b_*}, \mu_{b_*}^2) \otimes f_1^j(x, \mu_{b_*}) \qquad : \mathrm{A}$$

Perturbative TMD at the initial scale

Structure of a TMD

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Perturbative TMD at the initial scale

$$\times \exp\left\{ K(b_*; \mu_{b_*}) \ln \frac{\sqrt{\zeta}}{\mu_{b_*}} + \int_{\mu_{b_*}}^{\mu} \frac{d\mu'}{\mu'} \left[\gamma_F - \gamma_K \ln \frac{\sqrt{\zeta}}{\mu'} \right] \right\} \qquad : \mathrm{B}$$

Evolution to final scale (of the process)

Structure of a TMD

TMD in Fourier space

$$\hat{F}(x, b_T^2; \mu, \zeta) = \int \frac{d^2\boldsymbol{k}_\perp}{(2\pi)^2} e^{i\boldsymbol{b}_T \cdot \boldsymbol{k}_\perp} F(x, k_\perp^2; \mu, \zeta)$$

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Evolution to final scale (of the process)

$$\times f_{NP}(x, b_T^2) \exp\left\{ g_K(b_T^2) \ln \frac{\sqrt{\zeta}}{\sqrt{\zeta_0}} \right\} \quad : \mathrm{C}$$

Non-perturbative part of the TMD

J. Collins, "Foundation of Perturbative QCD"

Structure of a TMD

TMD in Fourier space

$$\hat{F}(x, b_T^2; \mu, \zeta) = \int \frac{d^2\boldsymbol{k}_\perp}{(2\pi)^2} e^{i\boldsymbol{b}_T\cdot\boldsymbol{k}_\perp} F(x, k_\perp^2; \mu, \zeta)$$

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Perturbative TMD at the initial scale

Perturbative

$$\times \exp\left\{K(b_*; \mu_{b_*}) \ln\frac{\sqrt{\zeta}}{\mu_{b_*}} + \int_{\mu_{b_*}}^{\mu} \frac{d\mu'}{\mu'}\left[\gamma_F - \gamma_K \ln\frac{\sqrt{\zeta}}{\mu'}\right]\right\} \qquad : \mathrm{B}$$

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Collinear extractions

See talks of the morning

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Perturbative TMD at the initial scale

Perturbative

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Non-perturbative part of the TMD

Parameterization

J. Collins, “Foundation of Perturbative QCD”

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Evolution to final scale (of the process)

$$\times f_{NP}(x, b_T^2) \exp\left\{ g_K(b_T^2) \ln \frac{\sqrt{\zeta}}{\sqrt{\zeta_0}} \right\} \quad : C$$

Non-perturbative part of the TMD

Parameterization

GLOBAL FITs

Available Global Fits

	Accuracy	*SIDIS*	*DY*	*N of points*	*χ^2/N_{data}*	*Flavor Dependence*
Pavia 2017 Bacchetta, Delcarro, et al., JHEP 06 (2017)	NLL	✔	✔	8059	1.55	✘
SV 2019 Scimemi, Vladimirov, JHEP 06 (2020)	N^3LL^-	✔	✔	1039	1.06	✘
MAPTMD22 Bacchetta, Bertone, et al., JHEP 10 (2022)	N^3LL^-	✔	✔	2031	1.06	✘
MAPTMD24 Bacchetta, Bertone, et al., JHEP 08 (2024)	**N^3LL**	✔	✔	**2031**	**1.08**	✔
ART25 Moos, Scimemi, et al., arXiv:2503.11201	N^3LL^+	✔	✔	1209	1.05	✔

MAPTMD24:

3D structure of the proton with **flavor dependence**

TMD fitting framework

NangaParbat

https://github.com/MapCollaboration/NangaParbat

README.md

Nanga Parbat is a fitting framework aimed at the determination of the non-perturbative component of TMD distributions.

Download

You can obtain NangaParbat directly from the github repository:

https://github.com/MapCollaboration/NangaParbat

For the last development branch you can clone the master code:

```
git clone git@github.com:MapCollaboration/NangaParbat.git
```

MAPTMD24: included data sets

Drell-Yan data 484

Fixed-target:

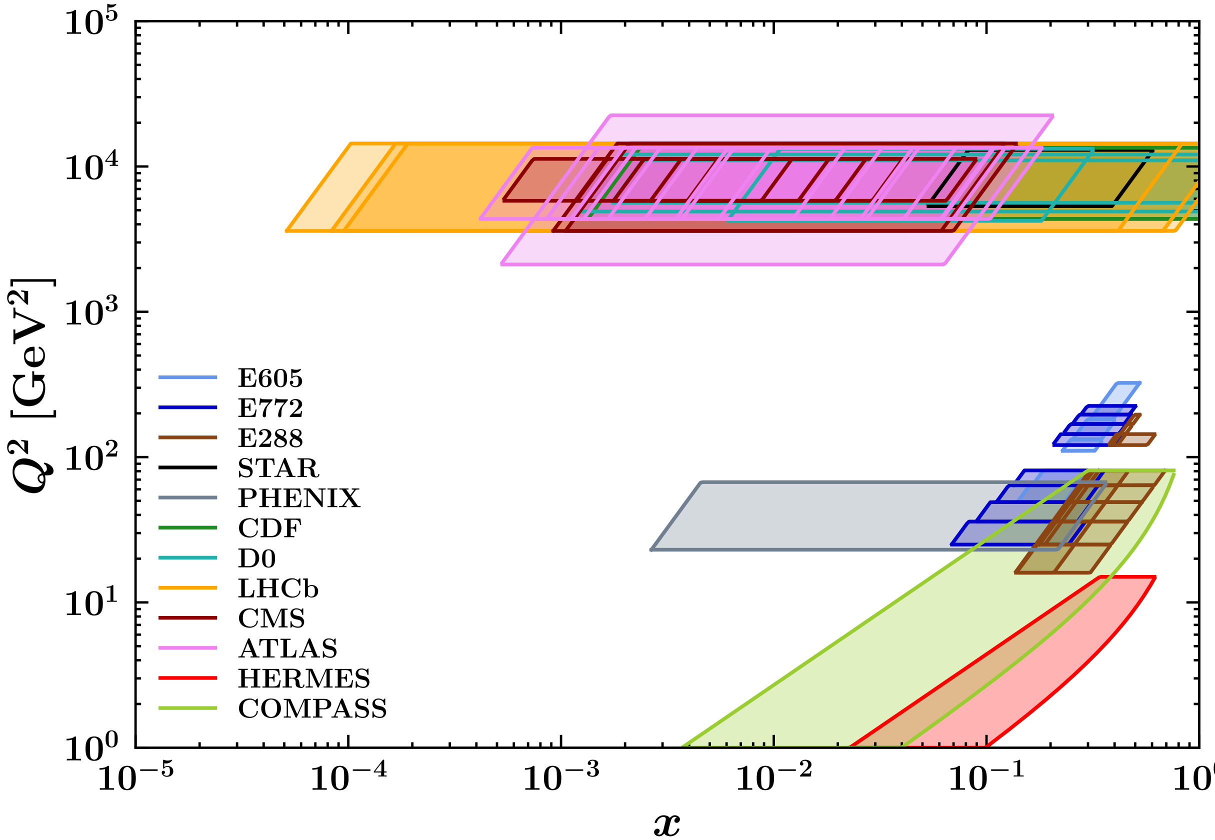
E288, E605, E772

Collider mode:

RHIC, Tevatron, LHC

SIDIS data 1547

HERMES, COMPASS



Total number of data: 2031

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Perturbative TMD at the initial scale

$$\times \exp\left\{ K(b_*; \mu_{b_*}) \ln \frac{\sqrt{\zeta}}{\mu_{b_*}} + \int_{\mu_{b_*}}^{\mu} \frac{d\mu'}{\mu'} \left[\gamma_F - \gamma_K \ln \frac{\sqrt{\zeta}}{\mu'} \right] \right\} \qquad : \mathrm{B}$$

Evolution to final scale (of the process)

$$\times f_{NP}(x, b_T^2) \exp\left\{ g_K(b_T^2) \ln \frac{\sqrt{\zeta}}{\sqrt{\zeta_0}} \right\} \qquad : \mathrm{C}$$

Non-perturbative part of the TMD

J. Collins, "Foundation of Perturbative QCD"

MAP22: Collinear input

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$$\hat{f}_1^q(x, b_T^2; \mu, \zeta) = \sum_j C_{q/j}(x, b_*; \mu_{b_*}, \mu_{b_*}^2) \otimes f_1^j(x, \mu_{b_*}) \qquad : \mathrm{A}$$

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Collinear distributions

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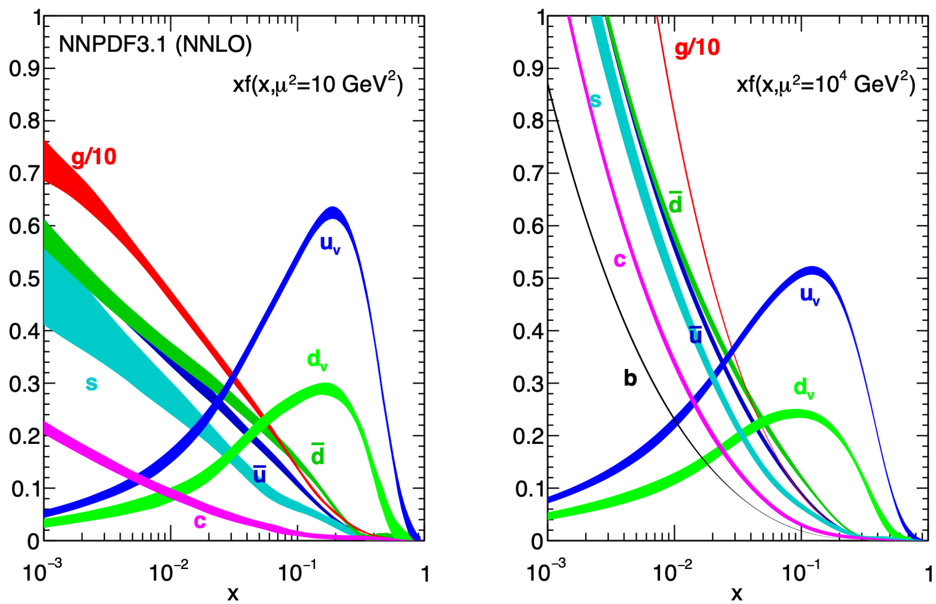
Collinear distributions

Input for PDFs: NNPDF3.1



NNPDF Collaboration, Ball, et al., EPJ C 77 (2017)

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Collinear distributions

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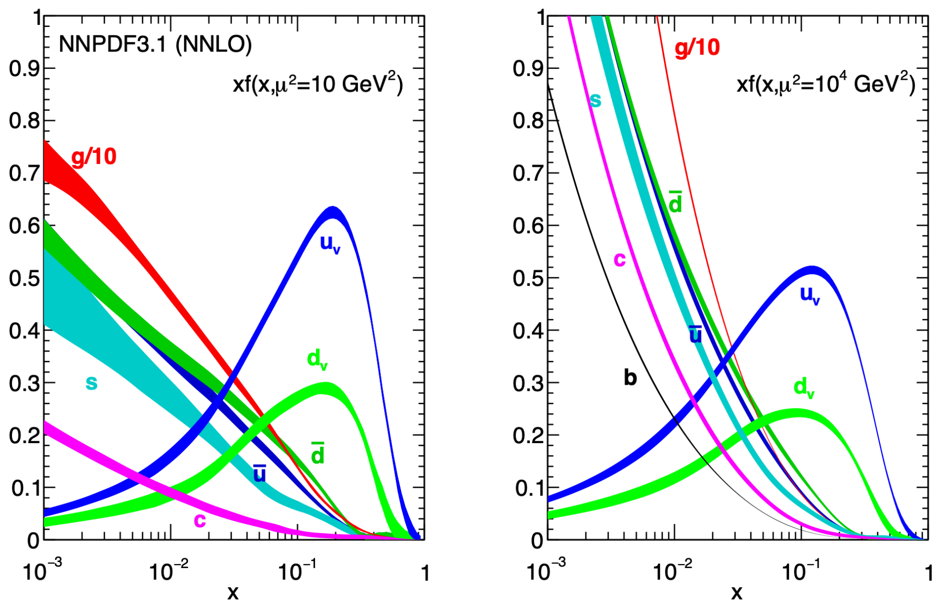
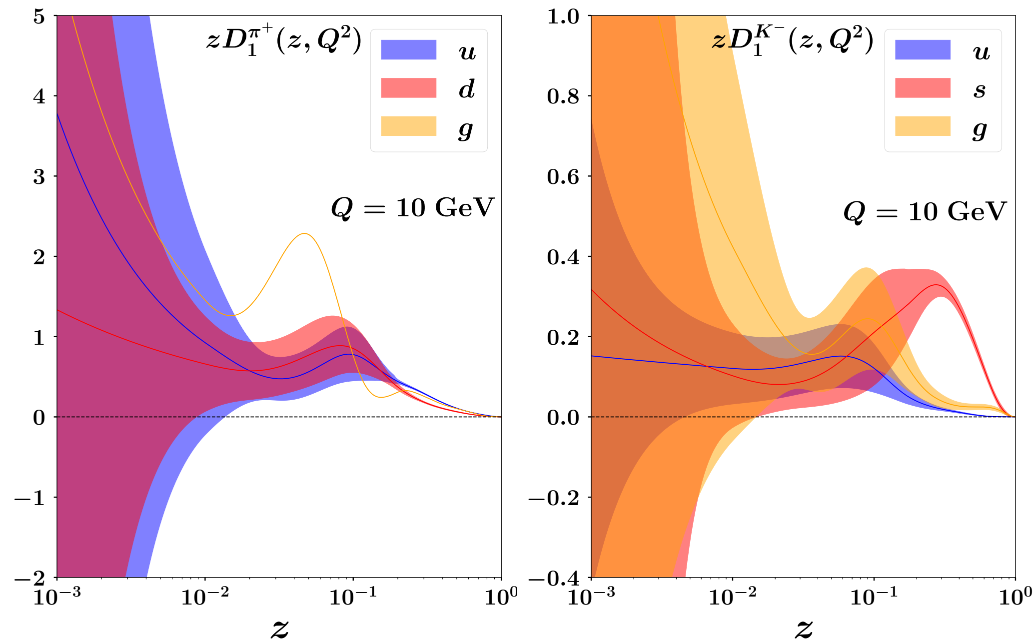


NNPDF Collaboration, Ball, et al., EPJ C 77 (2017)

Input for FFs: MAPFF1.0



MAP Collaboration, Khalek, Bertone, et al., PLB 834 (2022)

Perturbative accuracy

$$\exp\left\{K(b_*;\mu_{b_*})\ln\frac{\sqrt{\zeta}}{\mu_{b_*}}+\int_{\mu_{b_*}}^{\mu}\frac{d\mu'}{\mu'}\left[\gamma_F-\gamma_K\ln\frac{\sqrt{\zeta}}{\mu'}\right]\right\}$$: B

Perturbative accuracy

$$\exp\left\{K(b_*;\mu_{b_*})\ln\frac{\sqrt{\zeta}}{\mu_{b_*}}+\int_{\mu_{b_*}}^{\mu}\frac{d\mu'}{\mu'}\left[\gamma_F-\gamma_K\ln\frac{\sqrt{\zeta}}{\mu'}\right]\right\} \quad : \mathrm{B}$$

Resummation of large logs $S_{\text{pert}}(\mu_b,\mu)=1+\sum_{k=0}^{\infty}\boxed{R_{\mathrm{N}^k\mathrm{LL}}}$

$$S_{\text{pert}}(\mu_b,\mu)=1+\sum_{k=0}^{\infty}\sum_{n=1+[k/2]}^{\infty}\left(\frac{\alpha_S(\mu)}{4\pi}\right)^n\sum_{k=1}^{2n}L^{2n-k}R^{(n,2n-k)}$$

Perturbative accuracy

$$\exp\left\{K(b_*;\mu_{b_*})\ln\frac{\sqrt{\zeta}}{\mu_{b_*}}+\int_{\mu_{b_*}}^{\mu}\frac{d\mu'}{\mu'}\left[\gamma_F-\gamma_K\ln\frac{\sqrt{\zeta}}{\mu'}\right]\right\}$$: B

Resummation of large logs $$S_{\text{pert}}(\mu_b,\mu)=1+\sum_{k=0}^{\infty}R_{\text{N}^k\text{LL}}$$

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$$L=\ln\left(\frac{\mu^2}{\mu_b^2}\right)$$

Perturbative accuracy

$$\exp\left\{K(b_*;\mu_{b_*})\ln\frac{\sqrt{\zeta}}{\mu_{b_*}}+\int_{\mu_{b_*}}^{\mu}\frac{d\mu'}{\mu'}\left[\gamma_F-\gamma_K\ln\frac{\sqrt{\zeta}}{\mu'}\right]\right\} \quad : \mathrm{B}$$

Resummation of large logs $\quad S_{\text{pert}}(\mu_b,\mu)=1+\sum_{k=0}^{\infty}R_{\mathrm{N}^k\mathrm{LL}}$

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Accuracy	H and C	K and γ_F	γ_K	PDF/FF and α_S evol.
LL	0	-	1	-
NLL	0	1	2	LO
NLL’	1	1	2	NLO
NNLL	1	2	3	NLO
NNLL’	2	2	3	NNLO
$\mathrm{N^3LL^-}$	2	3	4	NNLO/NLO
$\mathrm{N^3LL}$	2	3	4	NNLO
$\mathrm{N^3LL'}$	3	3	4	$\mathrm{N^3LO}$

Bacchetta, Bertone, Bissolotti, et al., JHEP 07 (2020)
TMD handbook, Boussarie, et al., 2023

NP parametrization

$$f_{NP}(x, b_T^2) \exp\left\{ g_K(b_T^2) \ln \frac{\sqrt{\zeta}}{\sqrt{\zeta_0}} \right\} \qquad : \mathrm{C}$$

Bacchetta, Gamberg, Goldstein, et al., PLB 659 (2008)
Bacchetta, Conti, Radici, PRD 78 (2008)
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MAPTMD24

	χ^2/N_{data}		
Configuration	DY	SIDIS	Total
MMHT+DSS (MAP22)	1.66	0.87	**1.06**
NNPDF+MAPFF (MAP24 FI)	1.58	1.34	**1.39**

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Data set	N_{dat}	χ_0^2/N_{dat}
DY collider total	251	2.06
Dy fixed target total	233	1.24
HERMES total	344	0.71
COMPASS total	1203	0.92
SIDIS total	1547	0.87
Total	2031	1.06

Data set	N_{dat}	χ_0^2/N_{dat}
DY collider total	251	2.14
Dy fixed target total	233	0.68
HERMES total	344	2.72
COMPASS total	1203	0.99
SIDIS total	1547	1.38
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TU QUOQUE,

~~BRUTE~~

HERMES

MAPTMD24: new approach

Solution: we need **flavor dependence** to obtain a good agreement between theory and experiments

MAPTMD24: new approach

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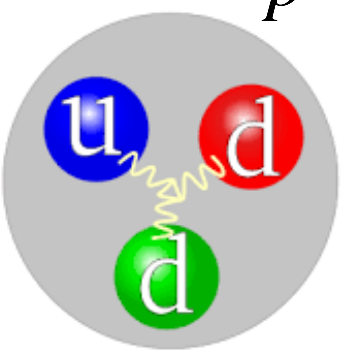

u, d

$\bar{u}, \bar{d}$

$s \ (sea)$

MAPTMD24: new approach

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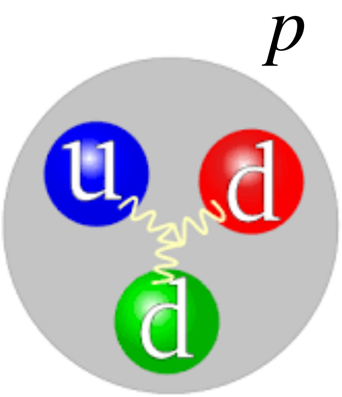

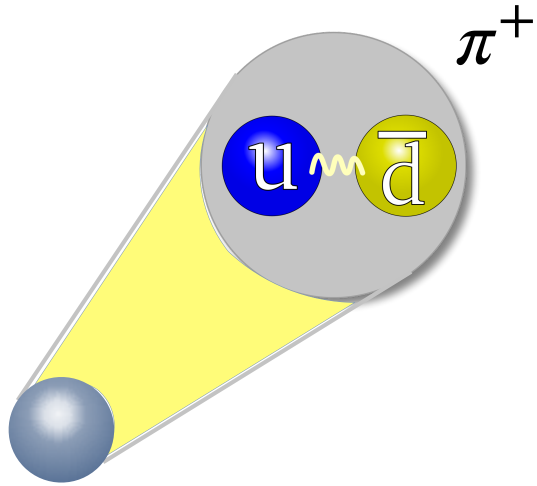

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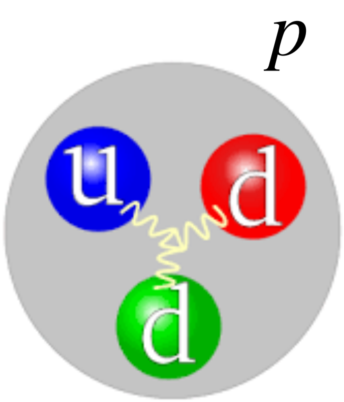

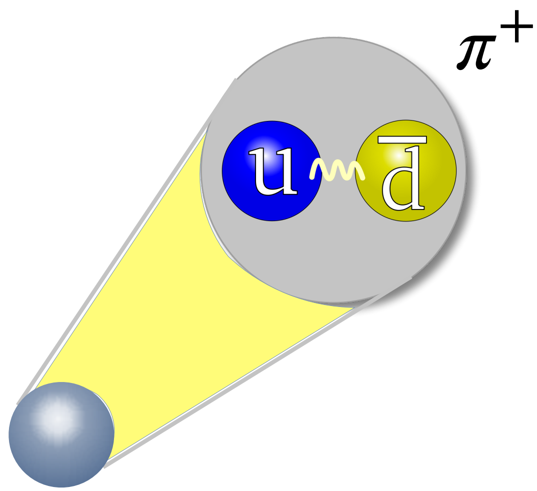

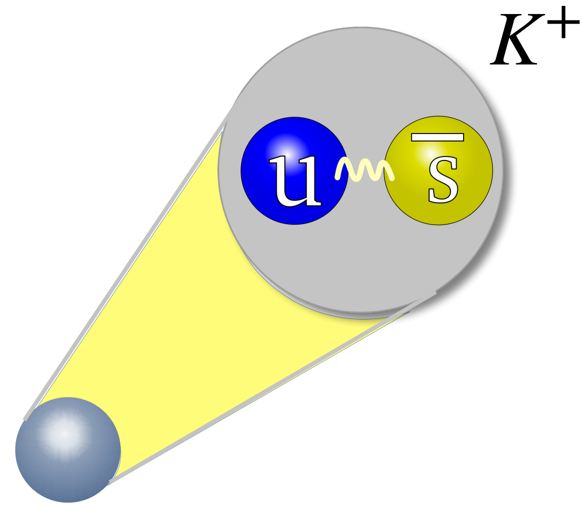

u, d	$u \to \pi^+, \ldots$	$u \to K^+, \ldots$
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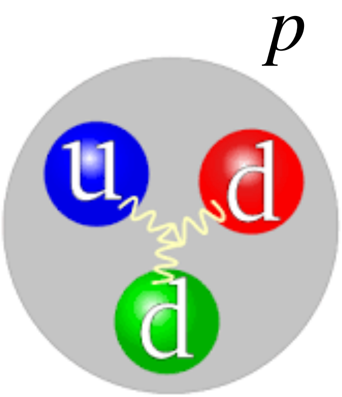

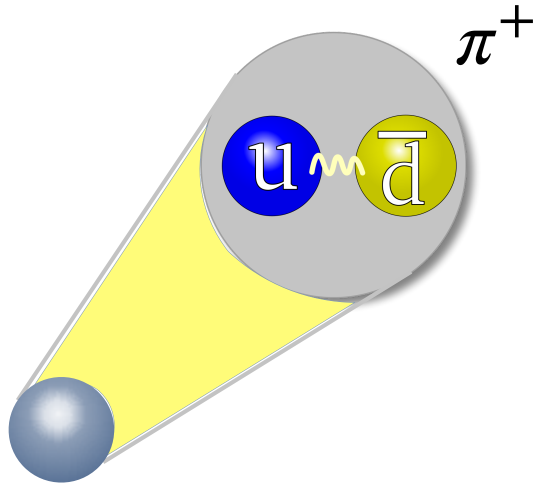

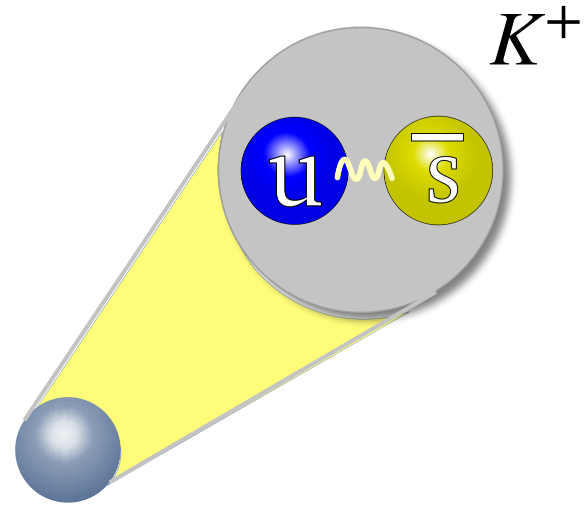

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charge conjugation

MAPTMD24: new approach

high sensitivity to flavor dependence

low sensitivity to flavor dependence

MAPTMD24: new approach

high sensitivity to flavor dependence

$$e + p \rightarrow e' + \pi^+ + X$$
$$e + p \rightarrow e' + \pi^- + X$$
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+ deuteron target

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$$p + p \to \gamma^*/Z \to l^+ + l^-$$

$q\bar{q}$ in the initial state

low sensitivity to flavor dependence

MAPTMD24: results

	$\mathrm{N^3LL}$			
Data set	N_{dat}	χ^2_D	χ^2_λ	χ^2_0
DY collider total	251	1.37	0.28	1.65
DY fixed-target total	233	0.63	0.31	0.94
HERMES total	344	0.81	0.24	1.05
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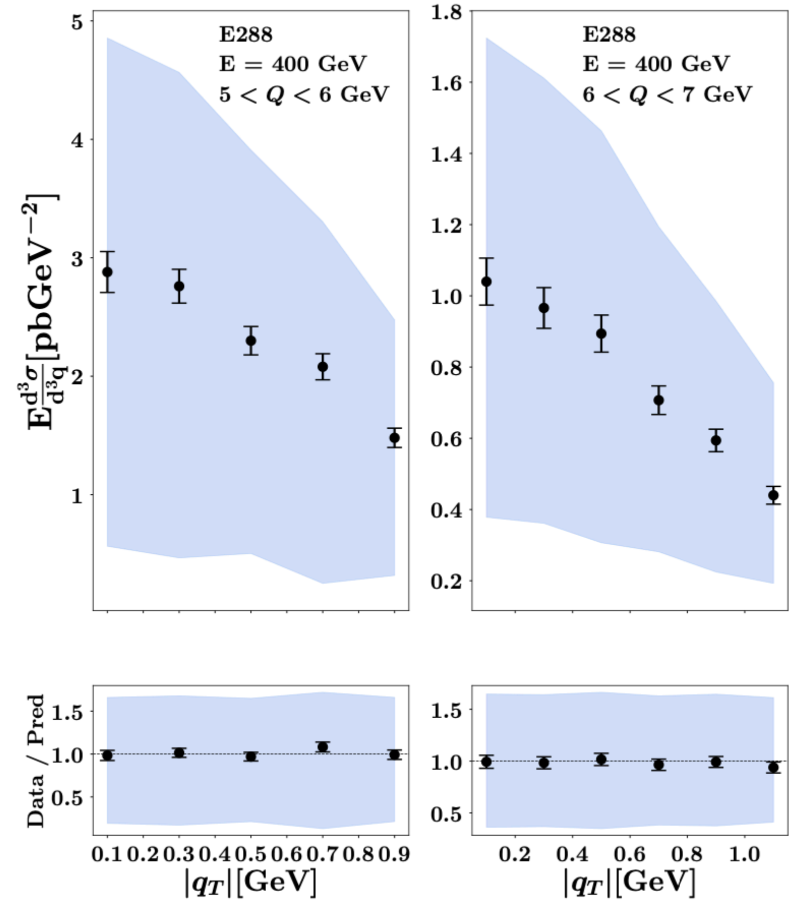

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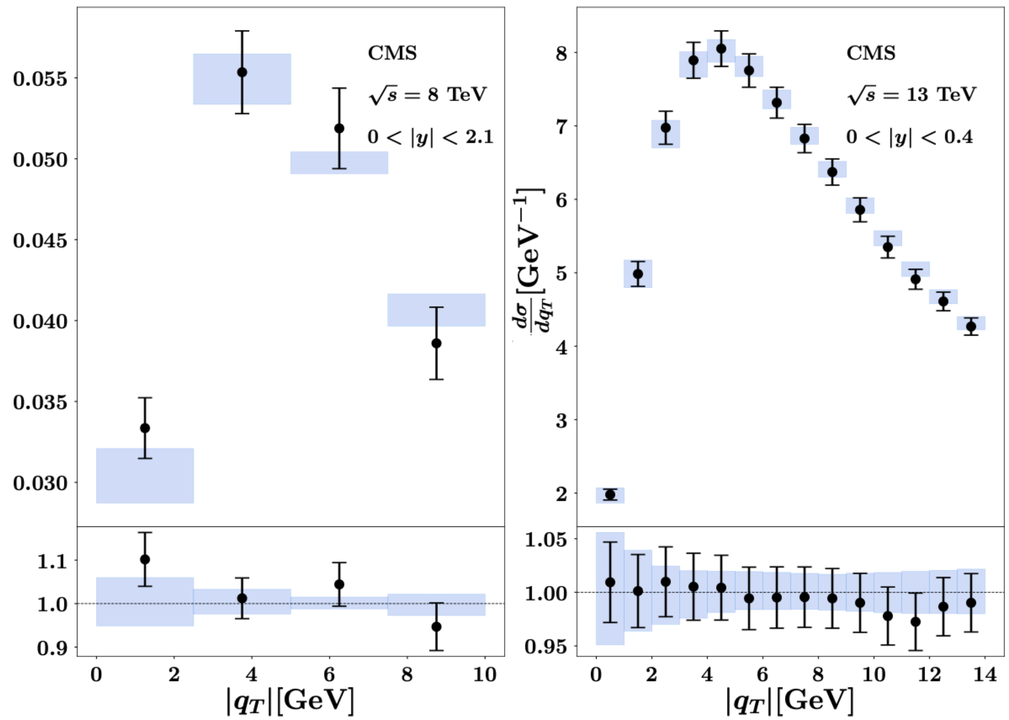

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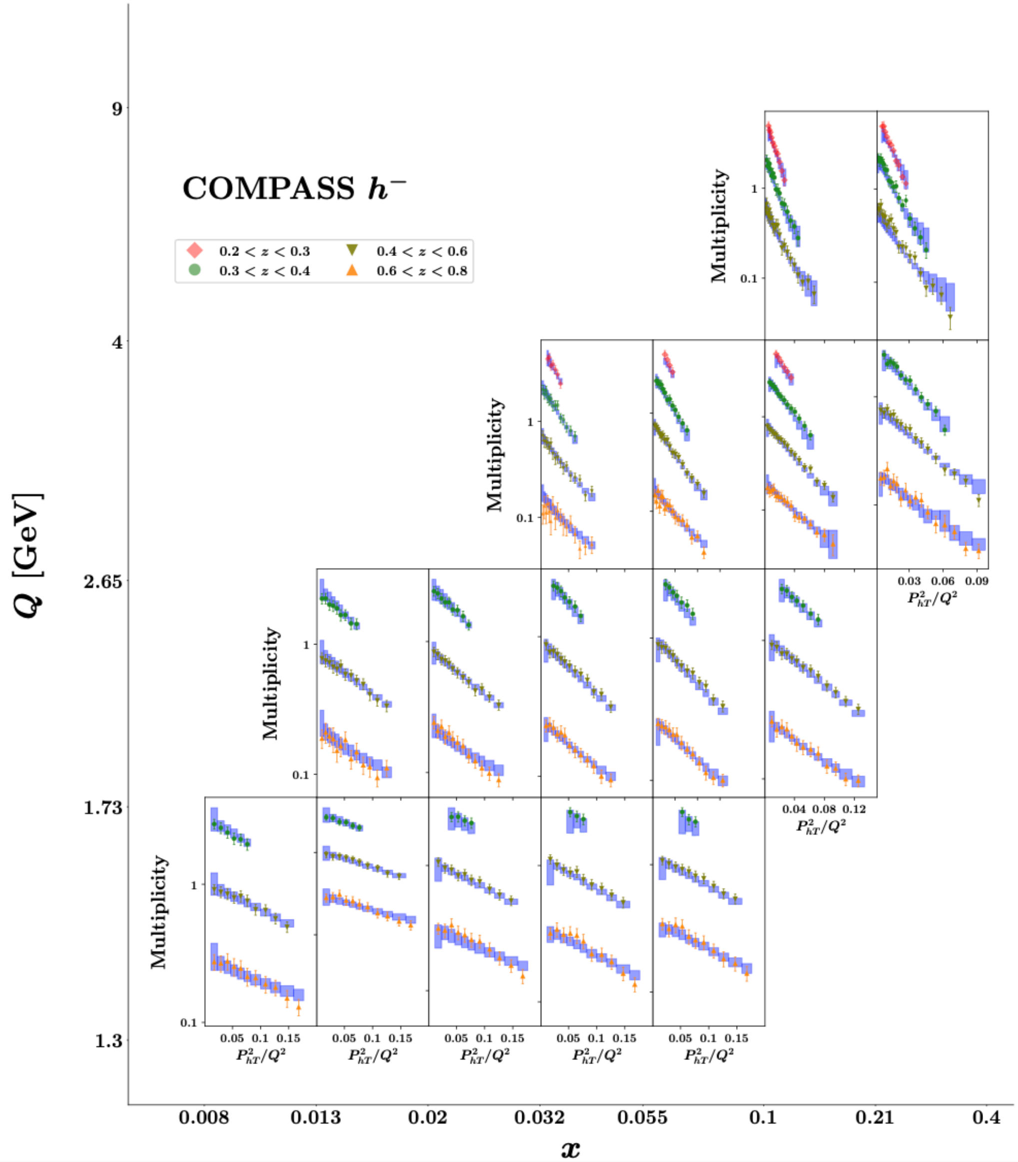

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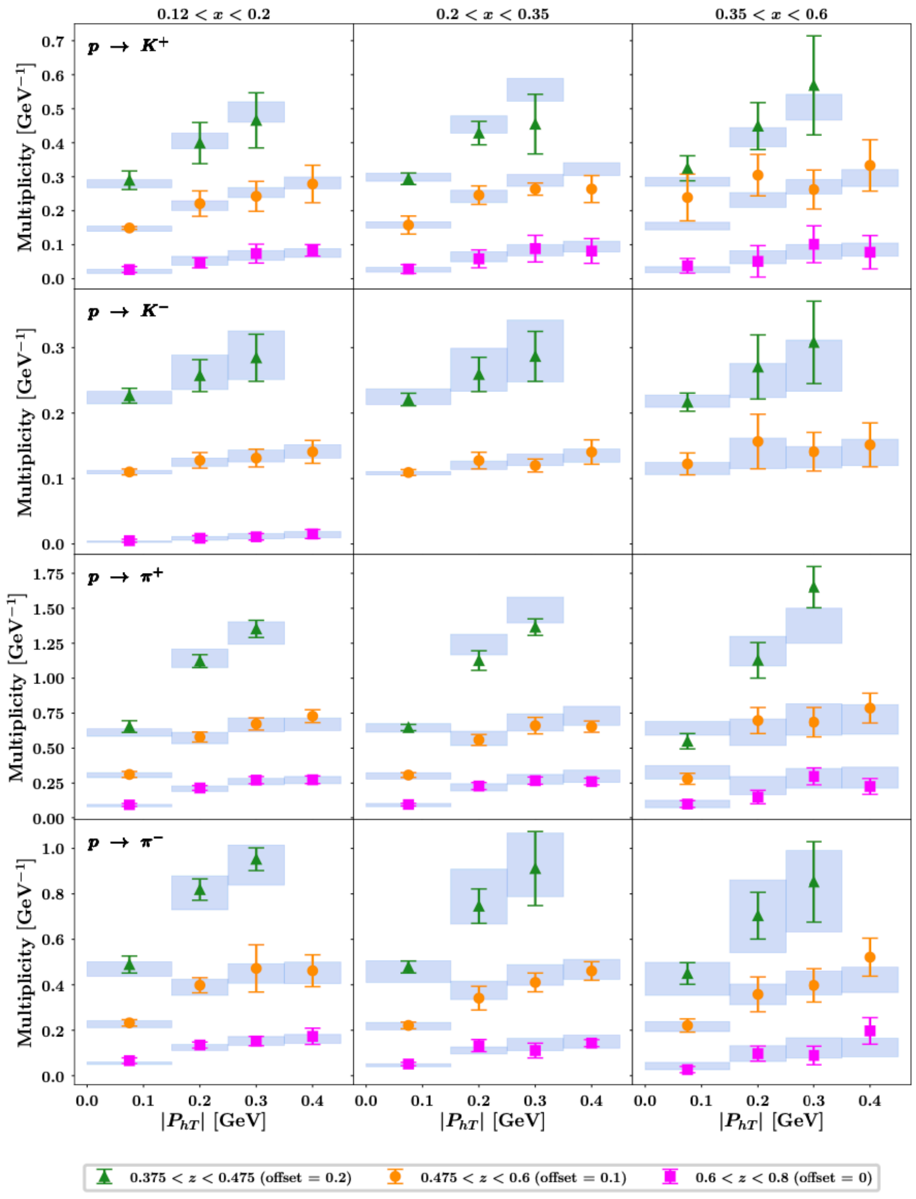

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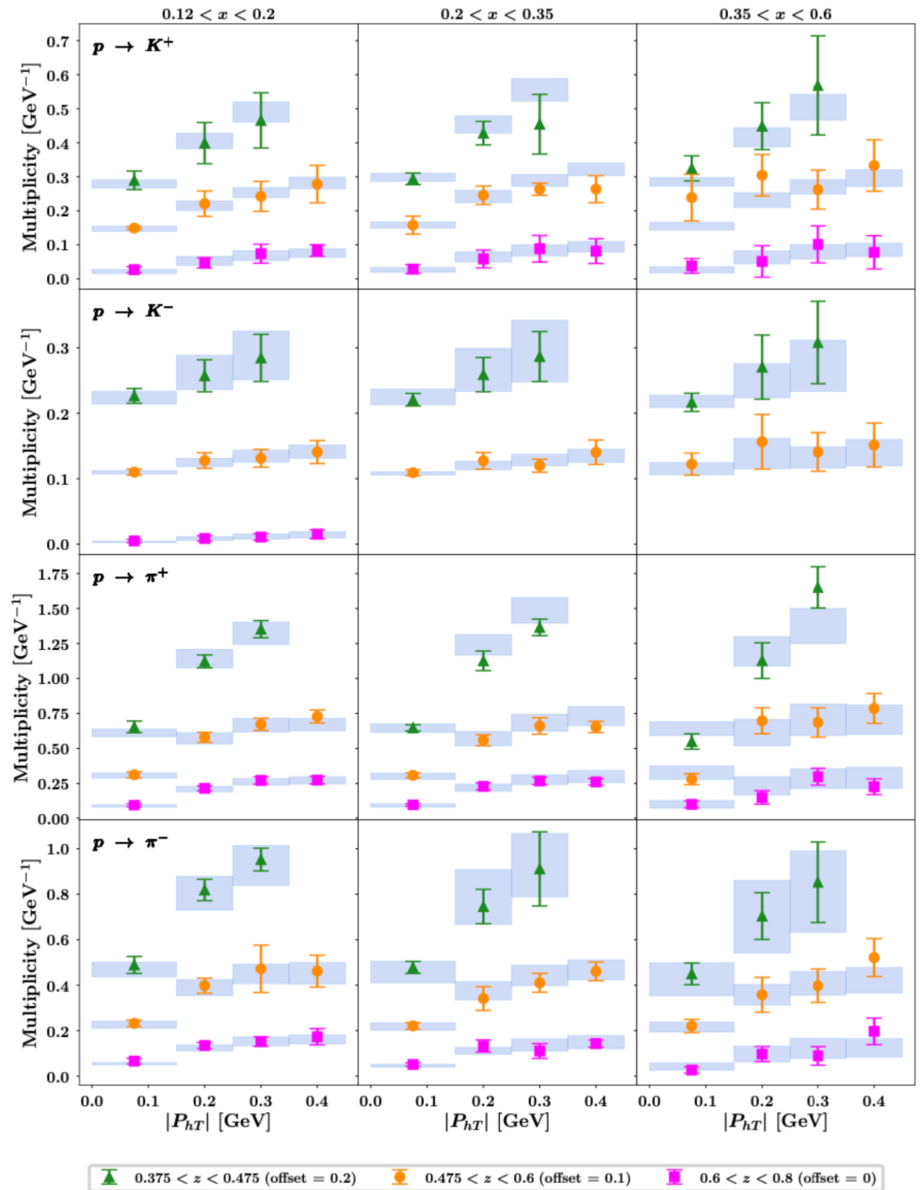
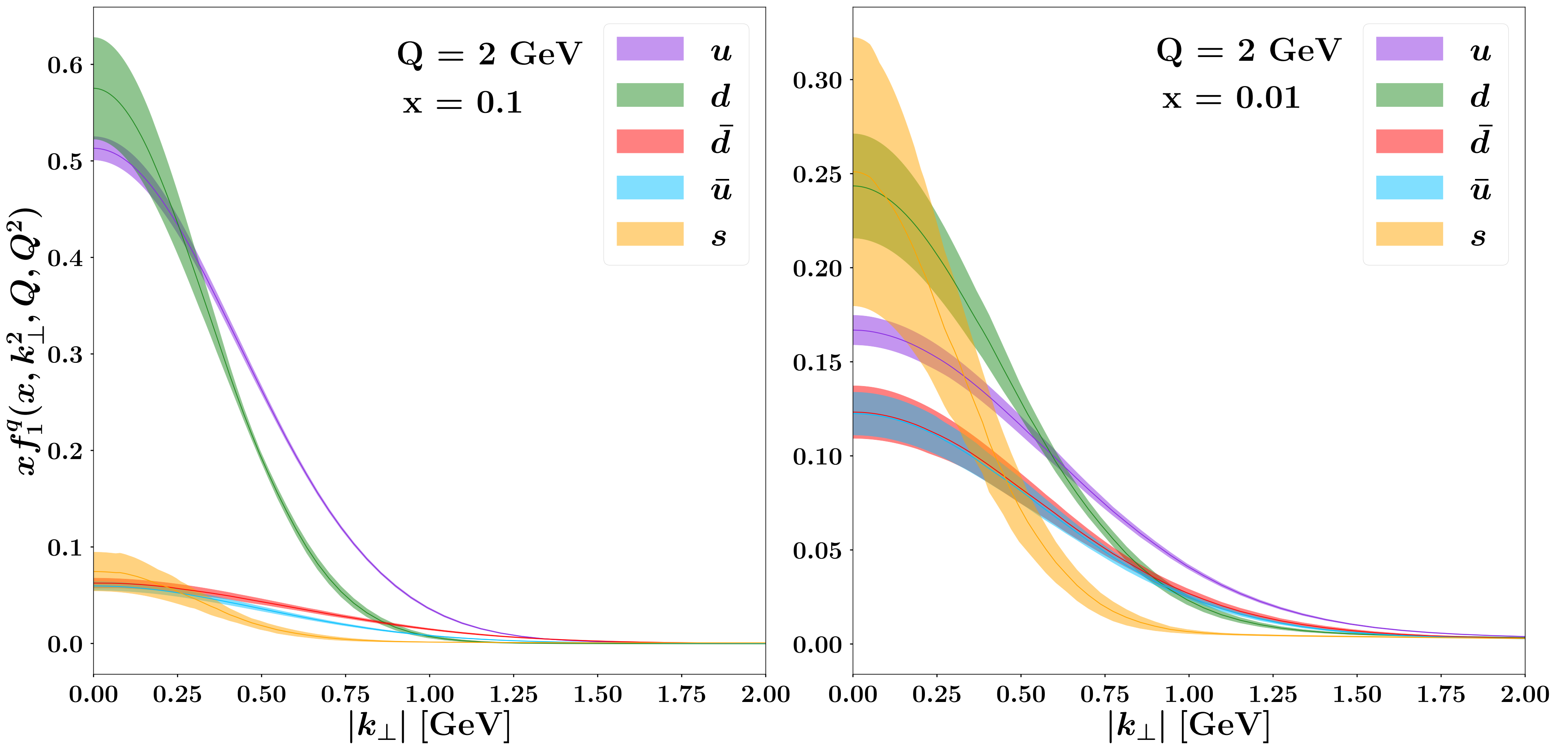


The agreement between theory and HERMES data has increased a lot!

MAPTMD24: results

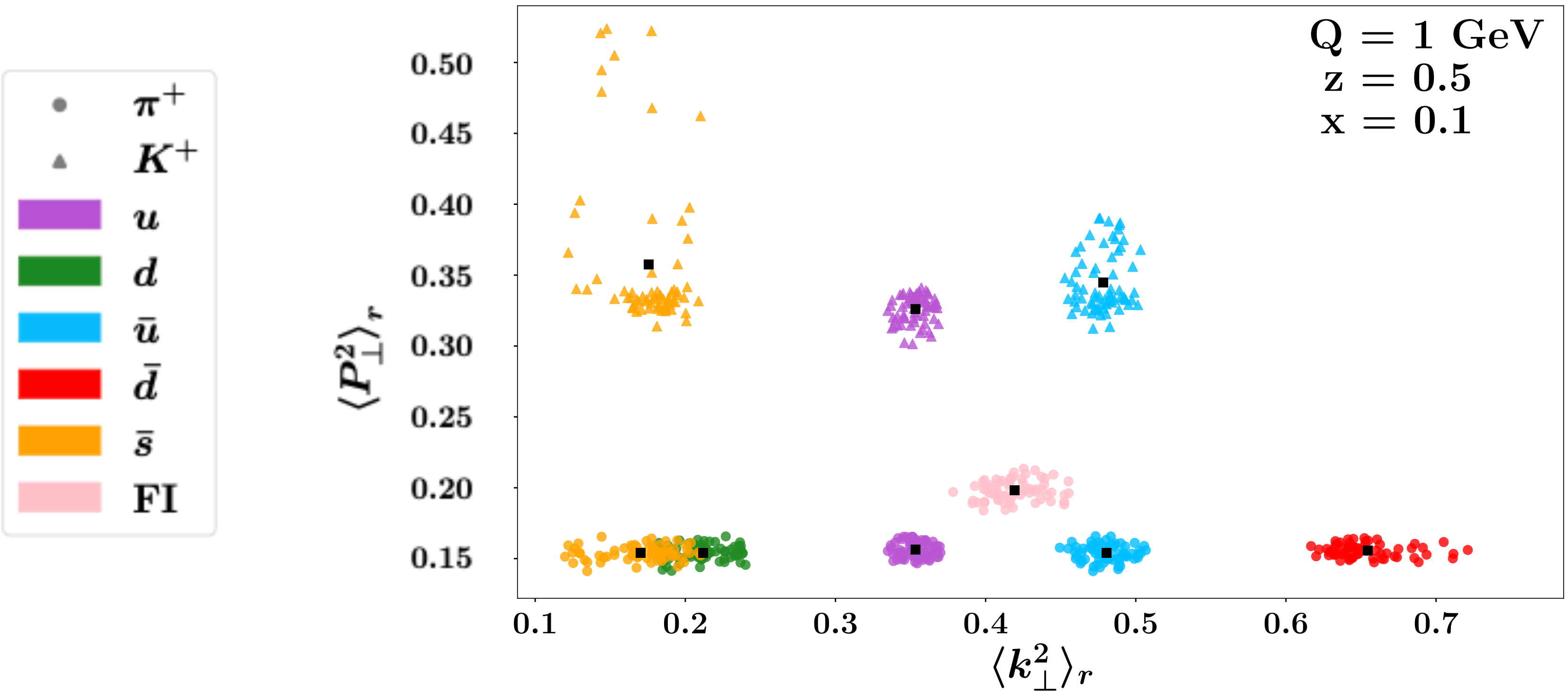
Flavor-dependent TMD PDFs



Evidence of different behaviors for different flavors

MAPTMD24: results

TMD's "effective width"



Evidence of different behaviors for different flavors

Evidence of different behaviors for different measured hadrons

Longitudinal polarization vs transverse momentum

Transverse-Momentum Distributions (TMDs)

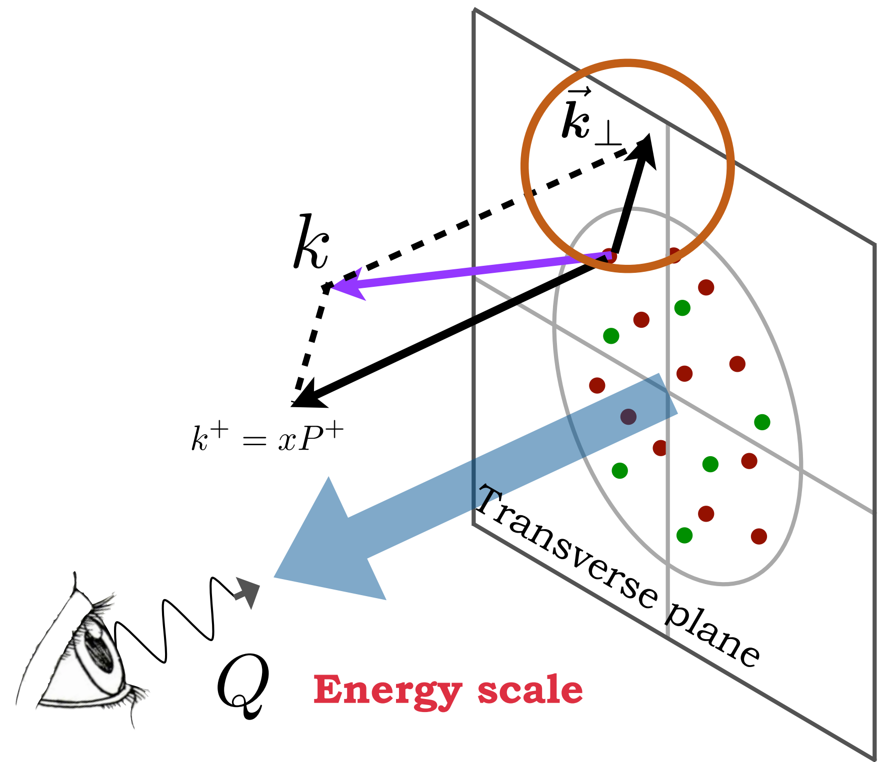
3-***dimensional*** ***map*** of the internal structure of the nucleon

Non-collinear framework

Quark Polarization

Nucleon Pol.	U	L	T
U	f_1		$h_1^\perp$
L		g_1	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1 \, h_{1T}^\perp$

Time-reversal odd   Time-reversal even



TMD PDFs

$$F(x, \boldsymbol{k}_\perp^2, \mu, \zeta)$$

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3-***dimensional*** ***map*** of the
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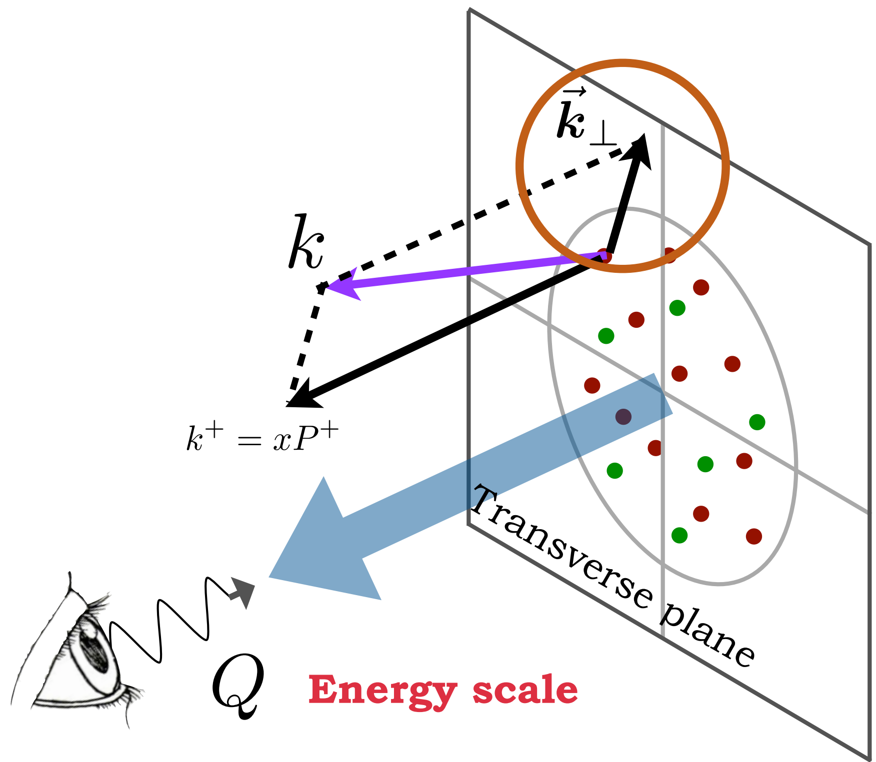

Quark Polarization

Nucleon Pol.

	U	L	T
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TMD PDFs

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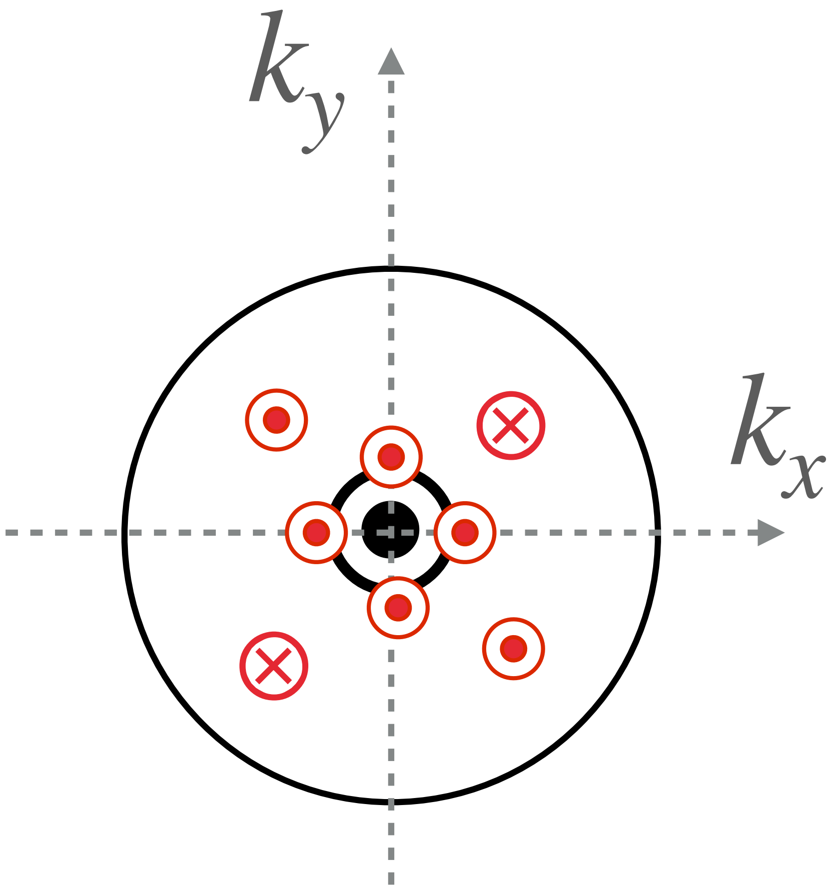
Time-reversal odd

Time-reversal even

Transverse-Momentum Distributions (TMDs)

$$g_1^q(x, k_\perp) = q^+ - q^-$$



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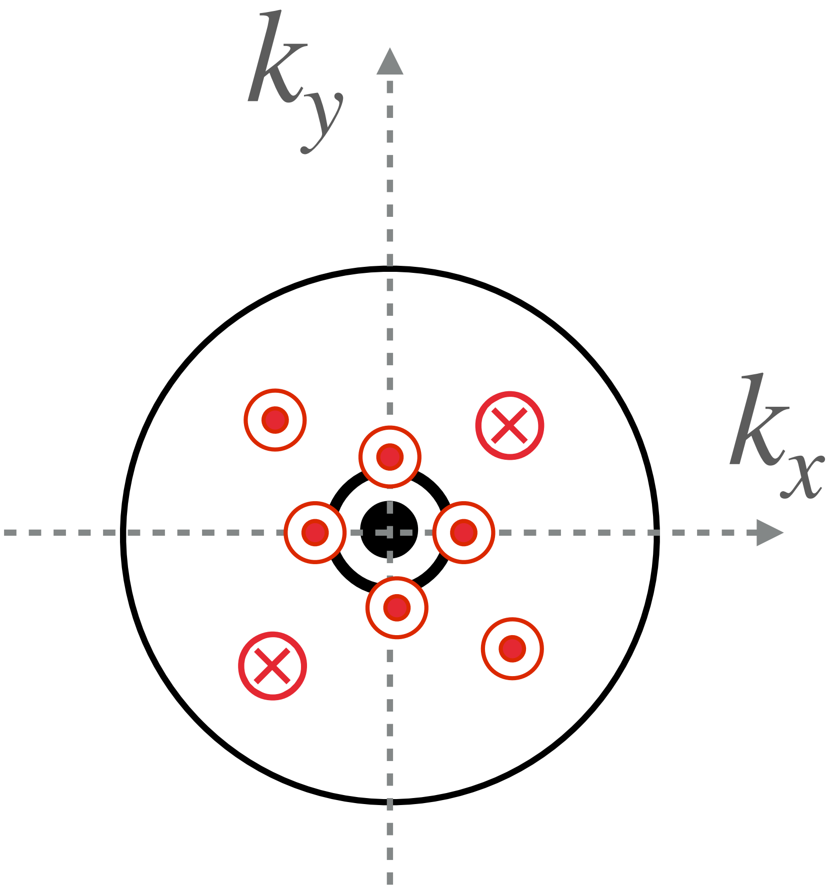

How the proton polarization reflects on its 3D structure?

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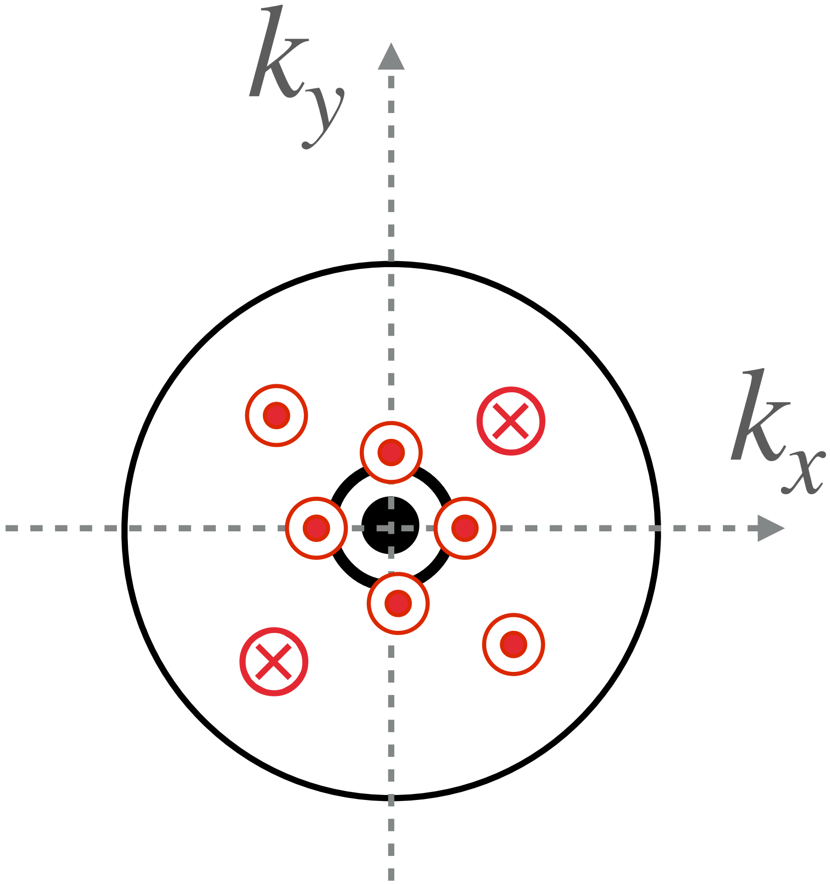

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How the quark polarization distorts their transverse momentum?

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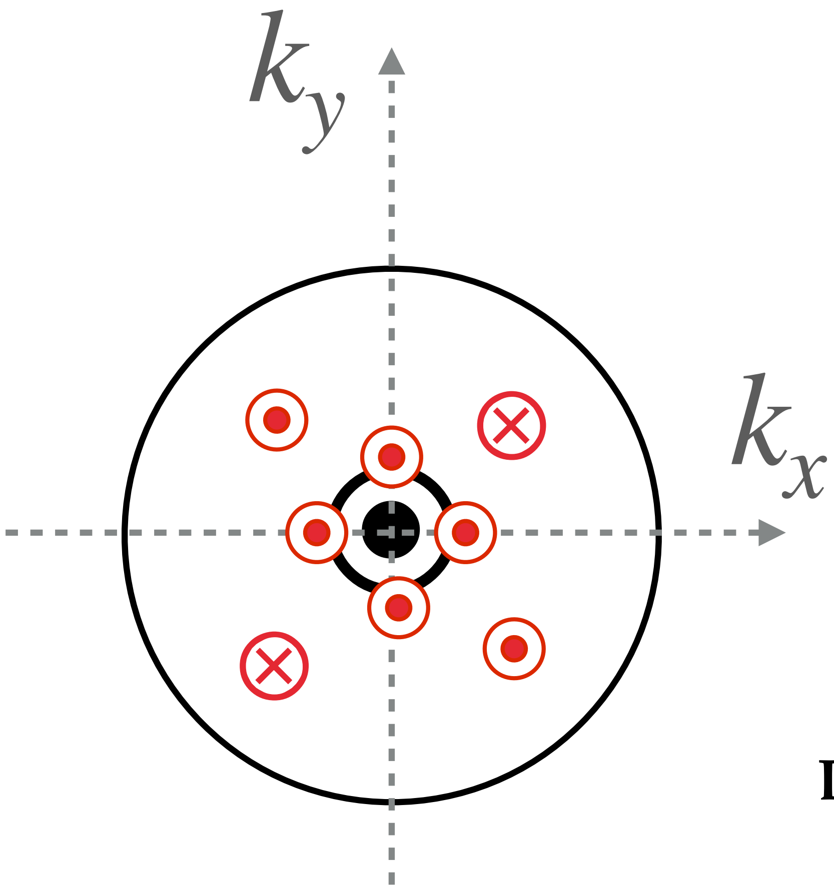


How the proton polarization reflects on its 3D structure?

How the quark polarization distorts their transverse momentum?

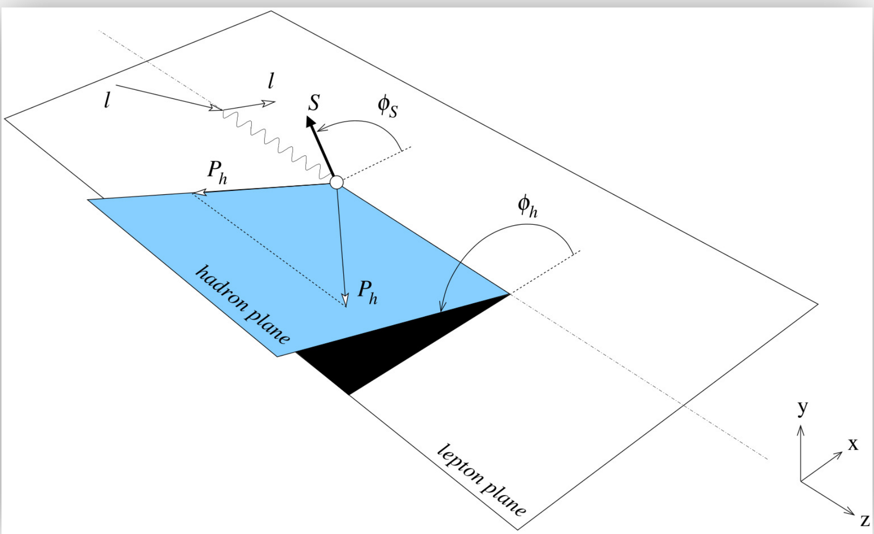
Do quarks with spin parallel to the proton have smaller transverse momentum?

Helicity distribution

Analysis of longitudinally polarized process

SIDIS

$$\ell^{\rightleftarrows}(l) + N^{\leftrightarrows}(P) \to \ell(l') + h(P_h) + X$$



A. Bacchetta et al., Phys.Rev.D 70 (2004), 117504

DOUBLE SPIN ASYMMETRY

$$A_1 = \frac{d\sigma^{\rightarrow\leftarrow} - d\sigma^{\rightarrow\rightarrow} + d\sigma^{\leftarrow\rightarrow} - d\sigma^{\leftarrow\leftarrow}}{d\sigma^{\rightarrow\leftarrow} + d\sigma^{\rightarrow\rightarrow} + d\sigma^{\leftarrow\rightarrow} + d\sigma^{\leftarrow\leftarrow}}$$

M. Diehl and S. Sapeta, Eur. Phys. J. C 41, 515 (2005)

Available Fits of the Helicity TMD

	Accuracy	*SIDIS*	*DY*	*N of points*	χ^2/N_{data}	*Flavor Dependence*
YLSZM fit Transverse Nucleon Tomography Collaboration PRL 134 (2025)	NNLL	✔	❌	253	0.74	❌
MAP22pol MAP Collaboration PRL 134 (2025)	NNLL	✔	❌	291	1.09	❌

TMD factorization: Universality

Longitudinal-Spin Asymmetry - TMD factorization

$$A_1(x, z, Q, |\boldsymbol{P}_{hT}|) = \frac{\sum_{a=q,\bar{q}} e_a^2 \int_0^{+\infty} d|\boldsymbol{b}_T|^2 J_0\left(\frac{|\boldsymbol{b}_T||\boldsymbol{P}_{hT}|}{z}\right) \hat{g}_1^a(x, |\boldsymbol{b}_T|^2, Q)\, \hat{D}_1^{a\to h}(z, |\boldsymbol{b}_T|^2, Q)}{\sum_{a=q,\bar{q}} e_a^2 \int_0^{+\infty} d|\boldsymbol{b}_T|^2 J_0\left(\frac{|\boldsymbol{b}_T||\boldsymbol{P}_{hT}|}{z}\right) \hat{f}_1^a(x, |\boldsymbol{b}_T|^2, Q)\, \hat{D}_1^{a\to h}(z, |\boldsymbol{b}_T|^2, Q)}$$

- Large energy scale $Q^2 \gg M^2$
- Small transverse momentum $q_T^2 \ll Q^2$

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MAP Collaboration, Bacchetta et al., JHEP 10 (2022)

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$\Longrightarrow$ Experimental observables in terms of universal objects

MAP22pol: fit setup

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Collinear distributions

NNPDFpol1.1

Unpolarized from MAP22

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Unpolarized from MAP22

Perturbative Accuracy NNLL Highest possible

since C^g known up to NLO

Gutiérrez-Reyes et al., Phys. Lett. B (2017)

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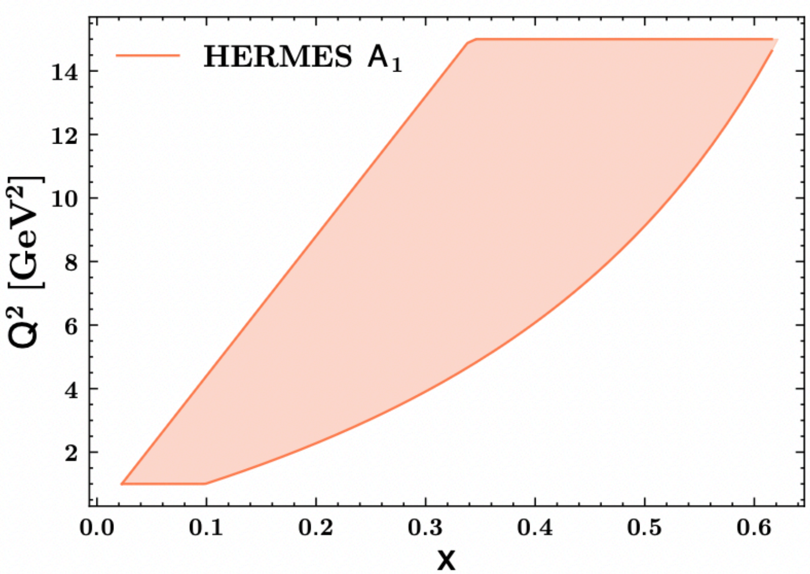
Experimental data



Airapetian et al. (HERMES), Phys. Rev. D (2019)

Consistent MAP22 kinematic cuts are applied

CLAS6 and COMPASS datasets excluded due to the cut

Total number of data: 291

NP parametrization

NP parametrization

$$g_{NP}(x, \boldsymbol{k}_\perp^2, Q_0) = f_{NP}^{MAP22}(x, \boldsymbol{k}_\perp^2, Q_0)\ \frac{e^{-\frac{k_\perp^2}{\omega_1(x)}}}{k_{norm}(x)}$$

- Proportional to f_{NP}^{MAP22}
- x-dependent

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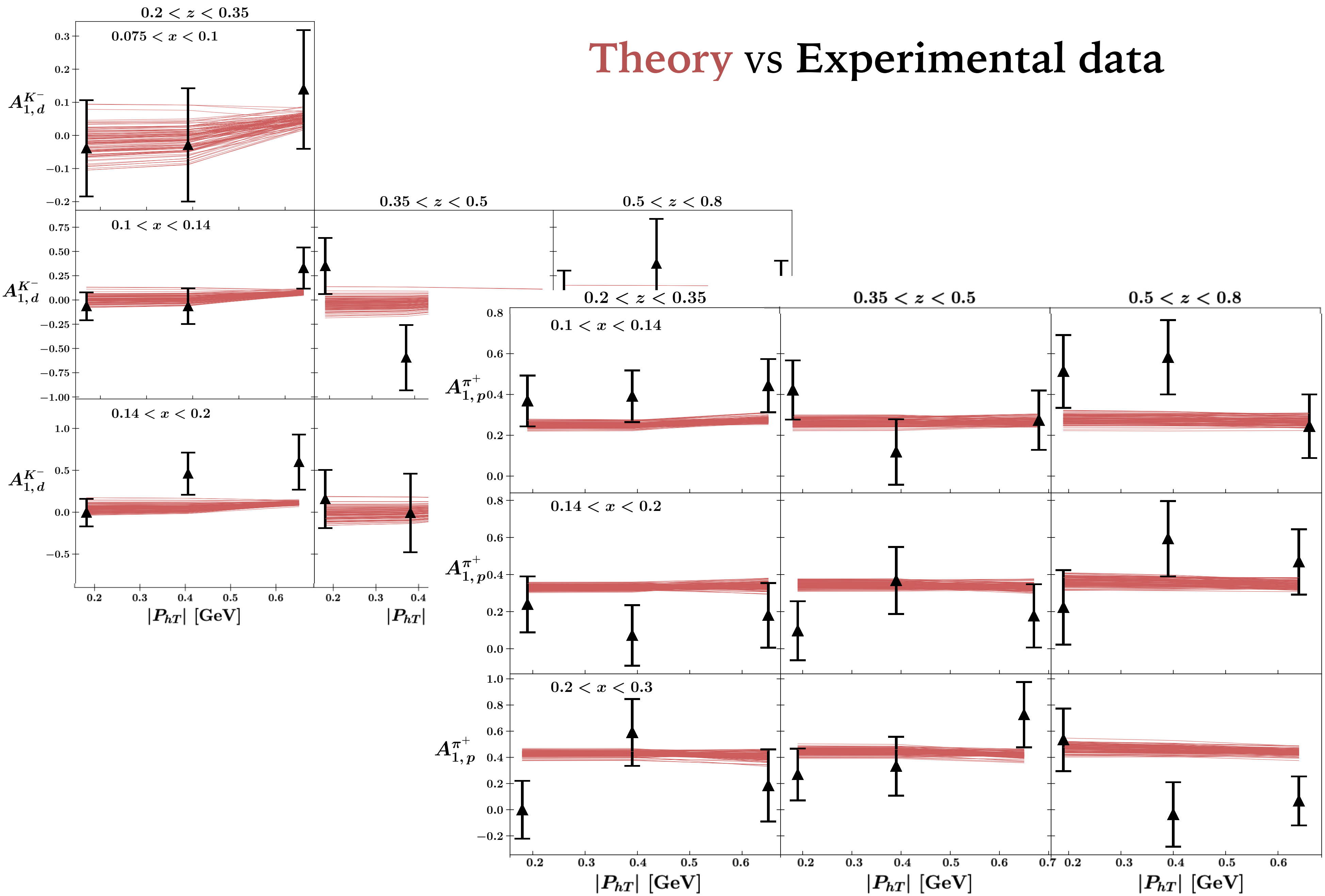
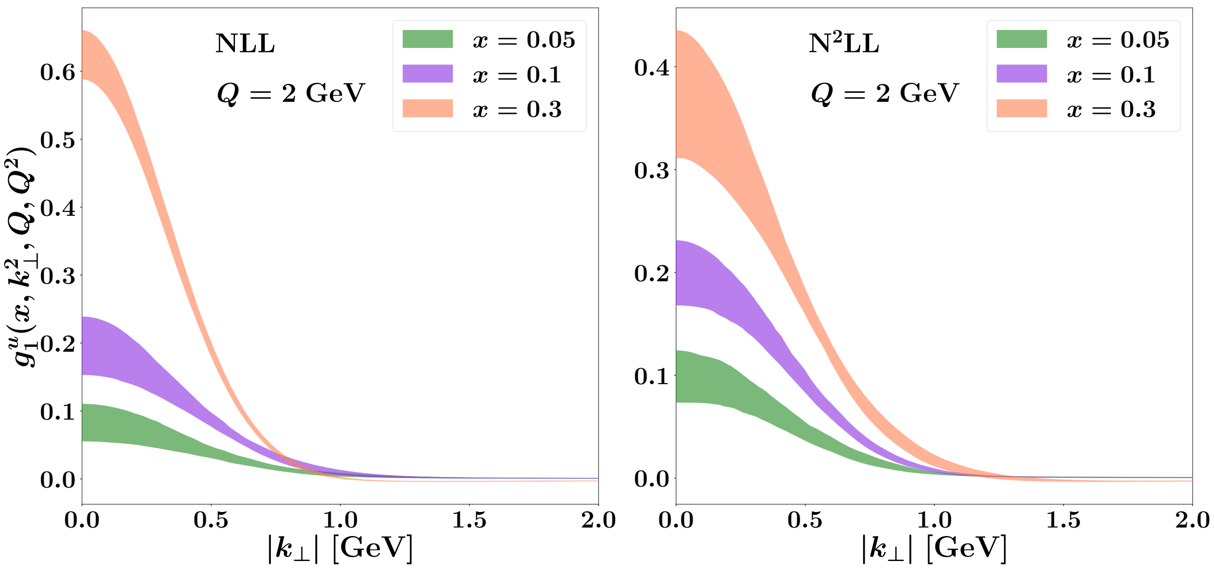
MAP22pol: results

Theory vs Experimental data

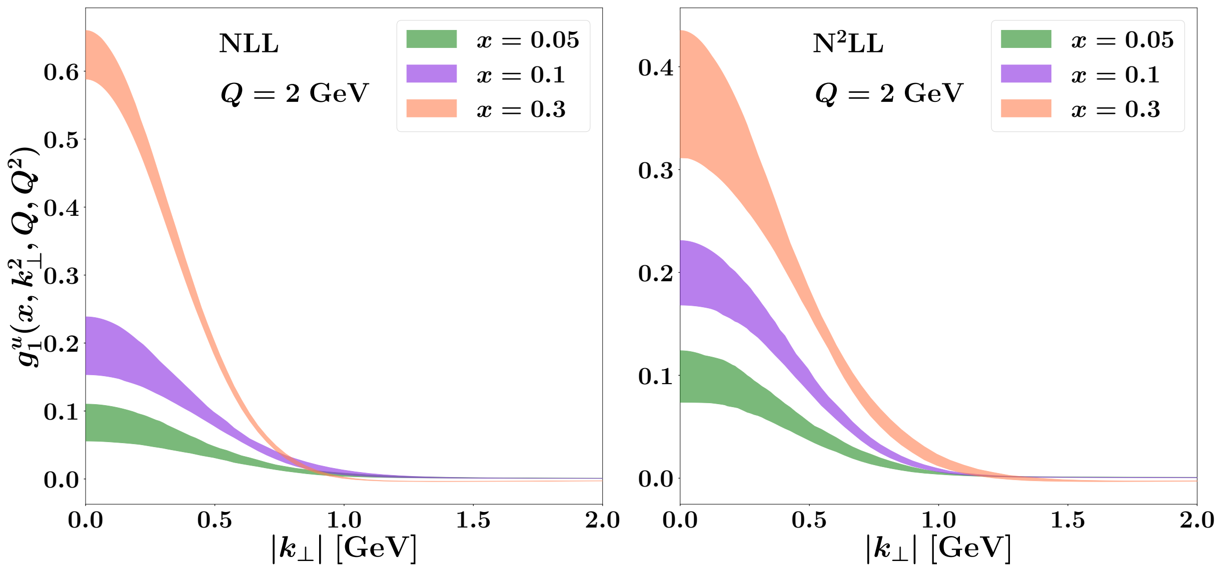


MAP22pol: results



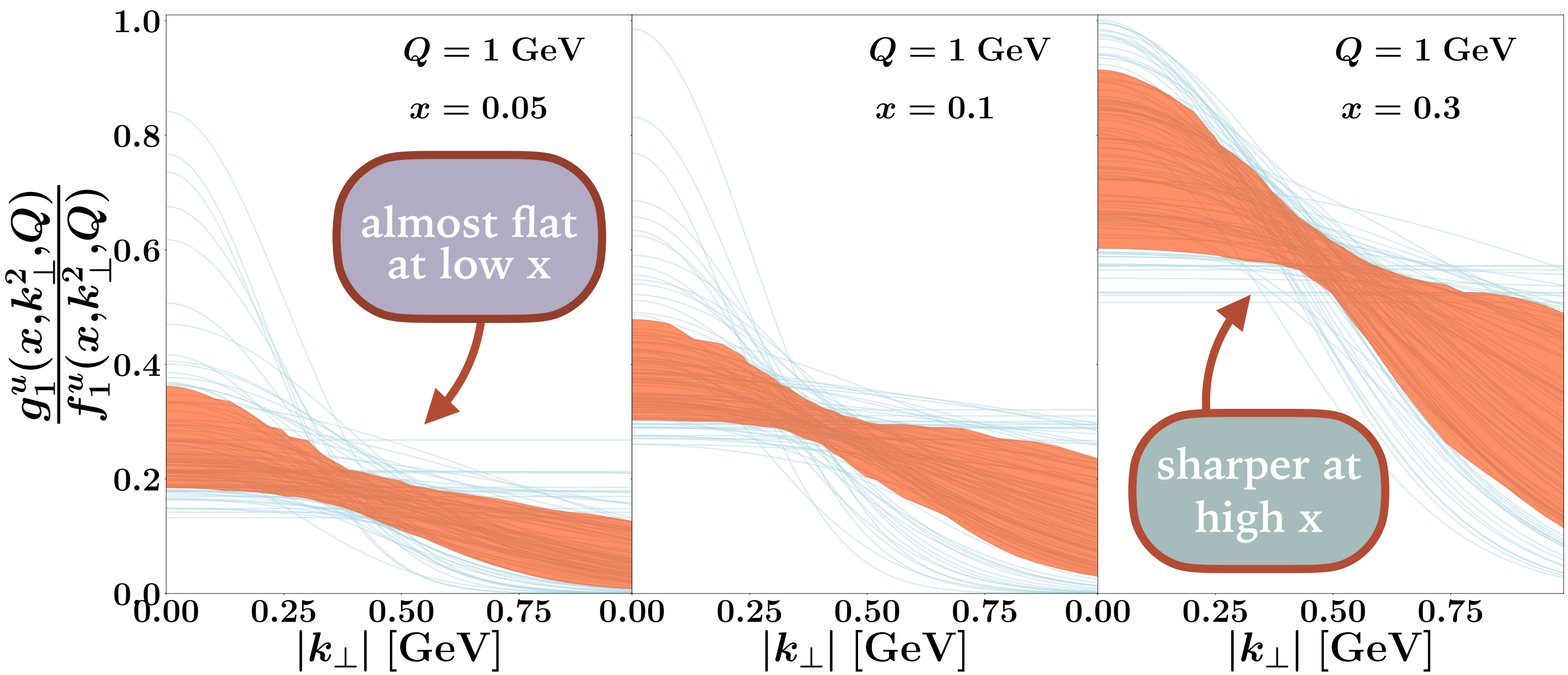
Extracted helicity TMDs

MAP22pol: results

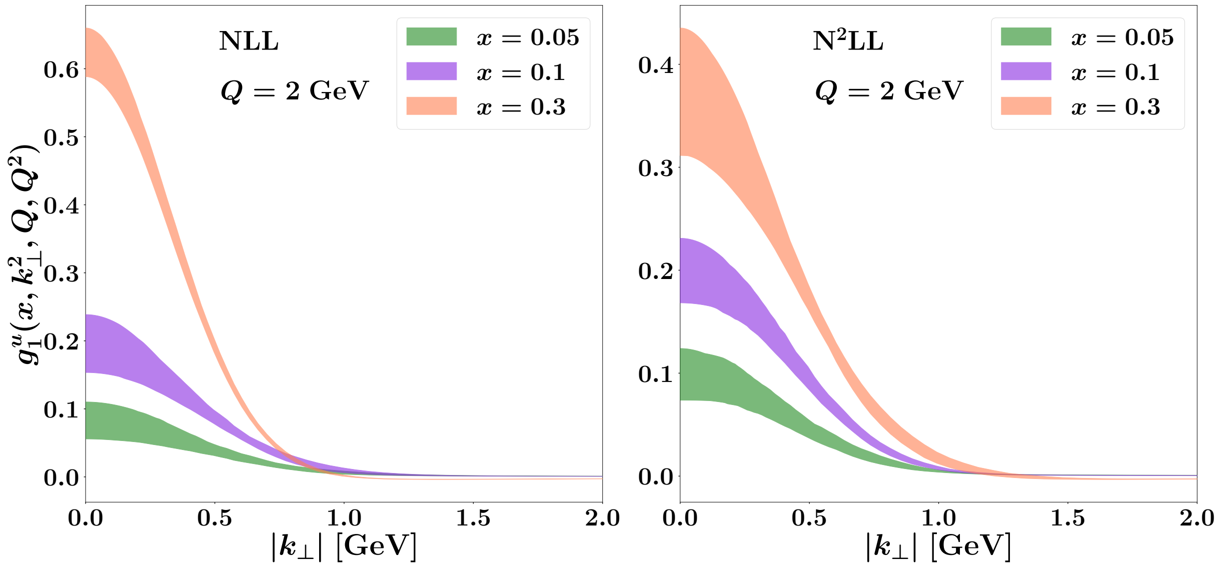


Extracted helicity TMDs

Helicity ratio



MAP22pol: results



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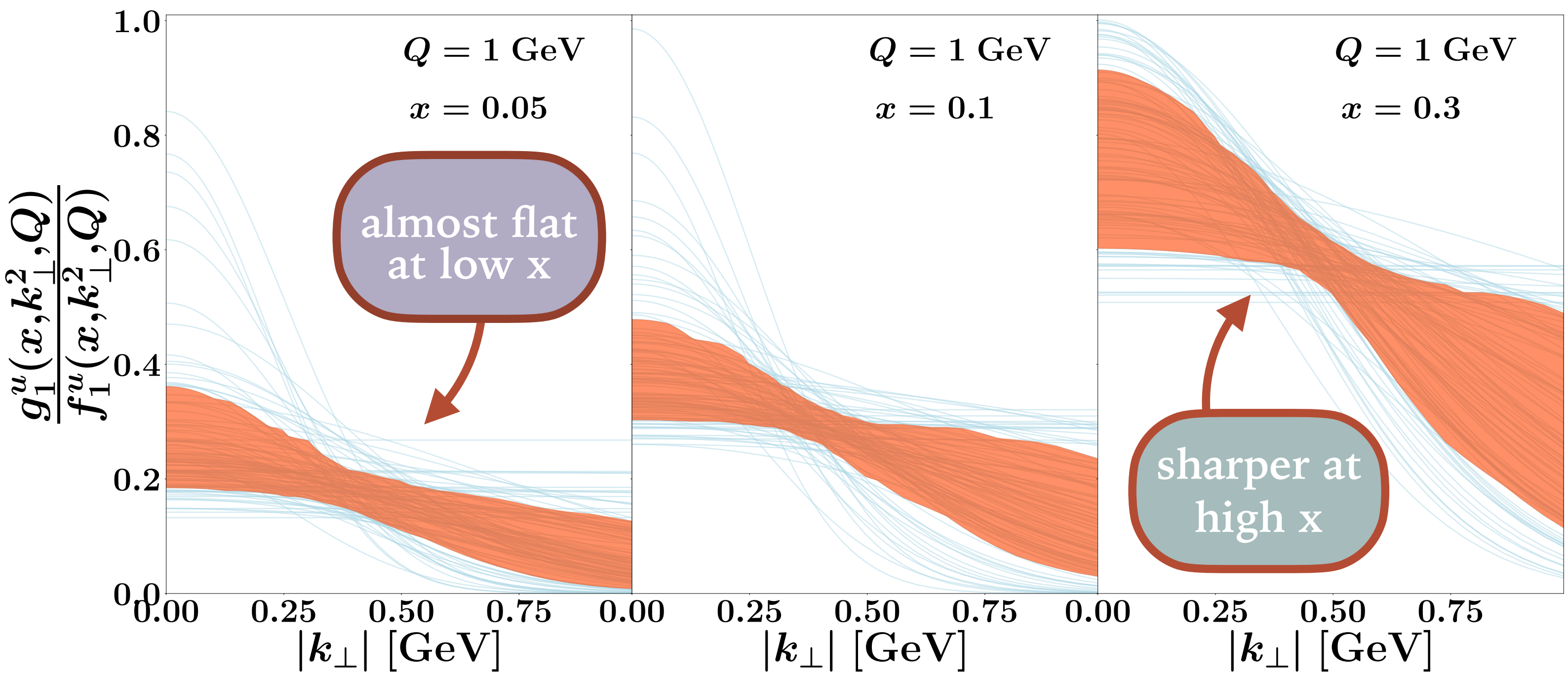

Conclusions

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New advanced computational framework for QCD global fits

with MAP Collaboration

Detailed study on extracted TMD FFs from global fits

with MAP Collaboration

Stress test of CSS formalism in TMD extractions

with A. Simonelli

Backup

Structure of a TMD: NP content

$$f_{NP}(x, b_T^2) \exp\left\{ g_K(b_T^2) \ln \frac{\sqrt{\zeta}}{\sqrt{\zeta_0}} \right\} \qquad : \mathrm{C}$$

$$\boxed{\mu_b > \mu} \qquad \infty \xleftarrow{b_T \ll 1} \mu_b = \frac{2e^{-\gamma_E}}{|\boldsymbol{b}_T|} \xrightarrow{b_T \gg 1} 0 \qquad \boxed{\alpha_S(\mu_b) \to +\infty}$$

b_*-prescription

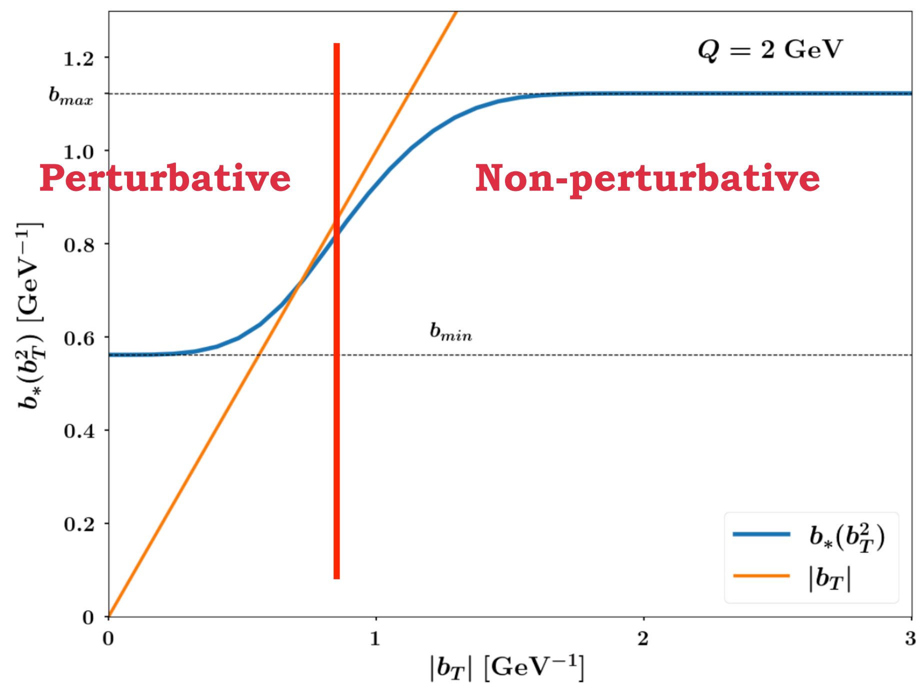
$$b_{\max} = 2e^{-\gamma_E}$$

$$b_{\min} = \frac{2e^{-\gamma_E}}{\mu}$$

Collins, Soper, Sterman, Nucl. Phys. B250 (1985)

Collins, Gamberg, et al., PRD (2016)

Bacchetta, Echevarria, Mulders, et al., JHEP 11 (2015)

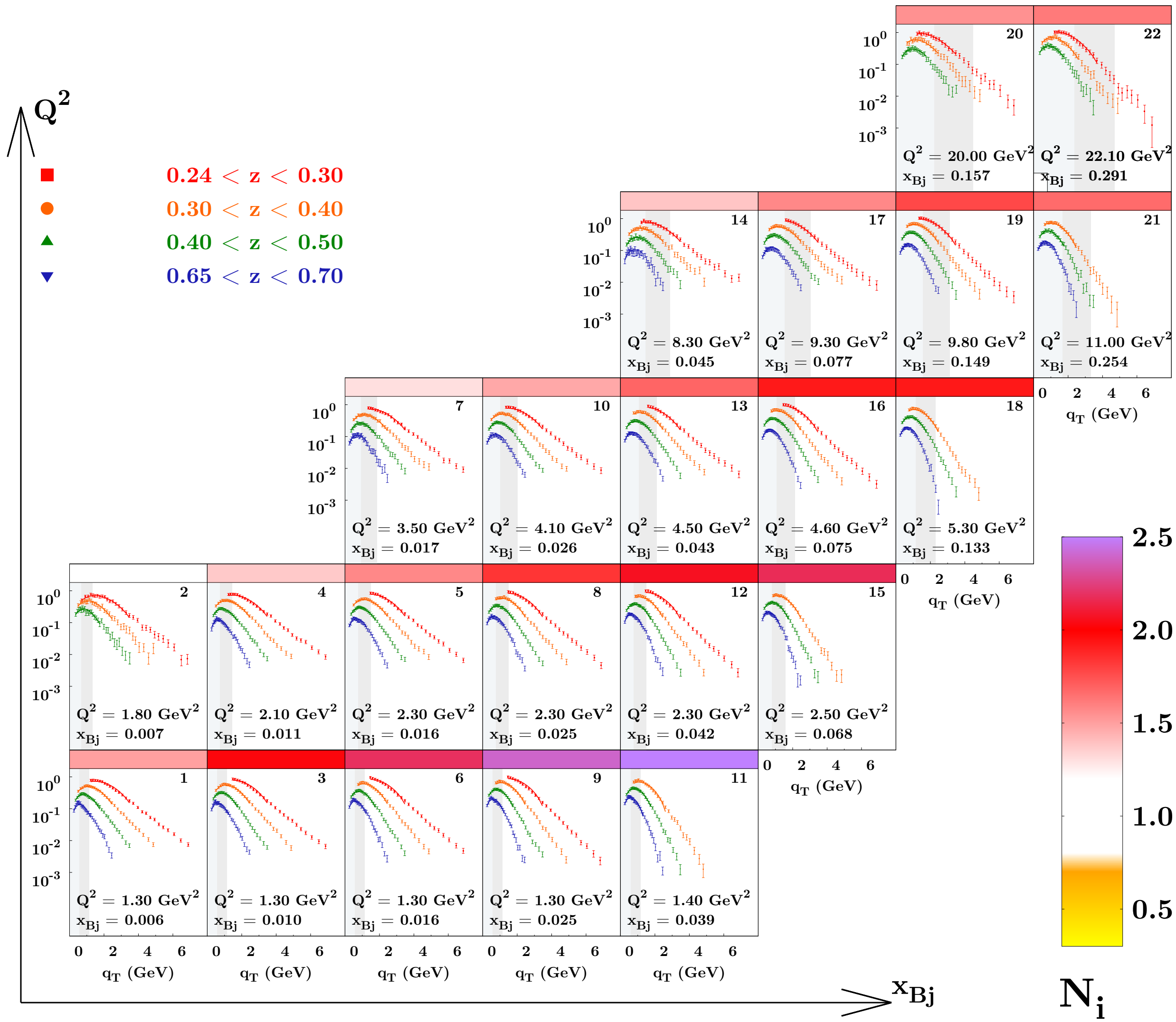


$$\hat{f}_1(x, b_T^2; \mu, \zeta) = \left[\frac{\hat{f}_1(x, b_T^2; \mu, \zeta)}{\hat{f}_1(x, b_*(b_T^2); \mu, \zeta)} \right] \hat{f}_1(x, b_*(b_T^2); \mu, \zeta) \equiv \boxed{f_{\mathrm{NP}}(x, b_T^2; \zeta)} \hat{f}_1(x, b_*(b_T^2); \mu, \zeta)$$

Normalization of SIDIS calculation

Normalization issue confirmed also in other analyses from different collaborations

Vladimirov, JHEP 12 (2023)



Gonzalez-Hernandez, PoS DIS2019 (2019)

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Sun, Isaacson, Yuan, Yuan, IJNP A (2014)

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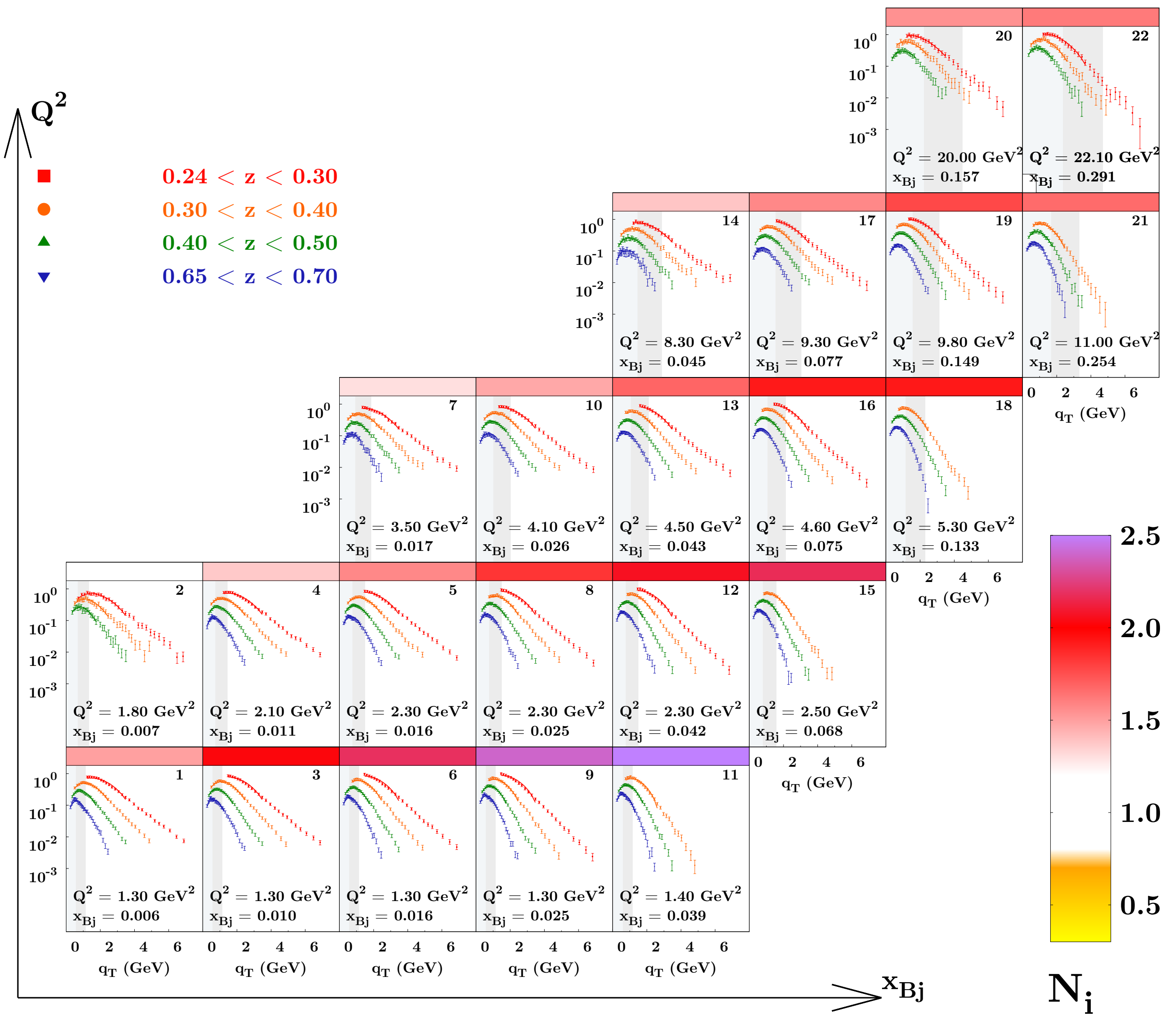

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MAP22 work solution

Good agreement for almost all bins

MAP Collaboration, JHEP 10 (2022)

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SIDIS multiplicity

$$M(x, z, P_{hT}, Q) = \frac{d\sigma}{dx dQ dz dP_{hT}} \bigg/ \frac{d\sigma}{dx dQ}$$

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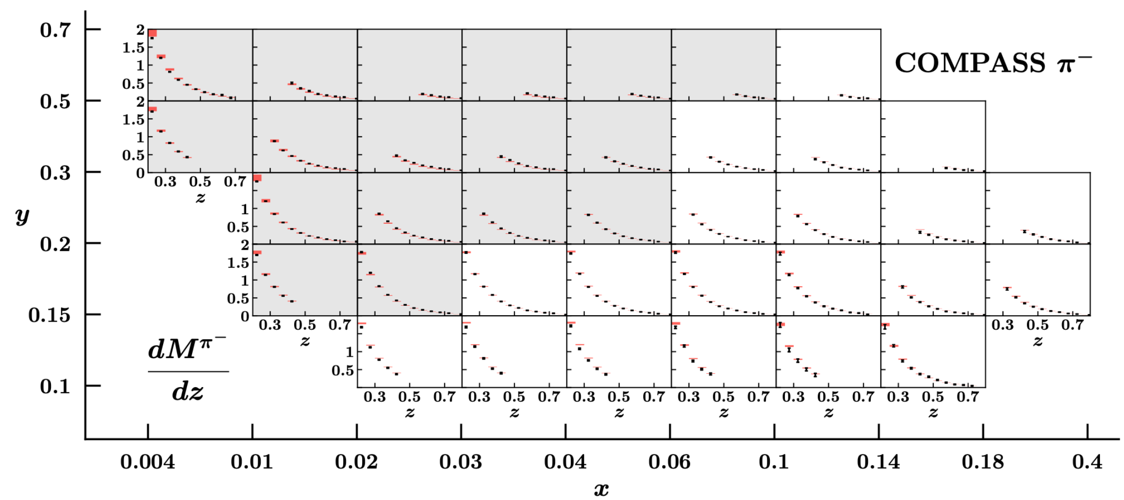

Khalek, Bertone, Nocera, et al., PRD 104 (2021)

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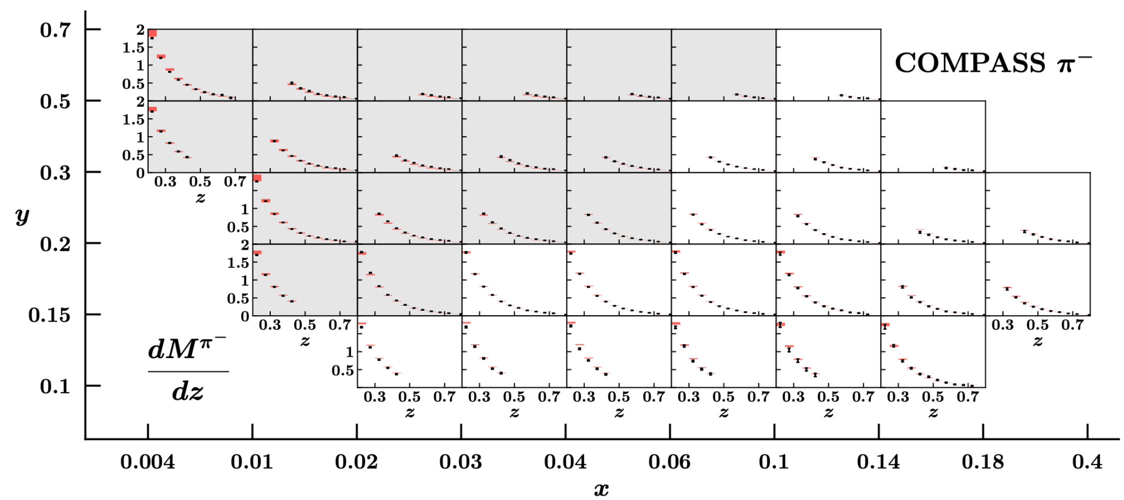
Normalization of prediction such that

$$\int dP_{hT} W(x,z,Q,P_{hT}) = \frac{d\sigma}{dxdQdz}$$

Piacenza, PhD thesis (2020)

Good agreement theory/data



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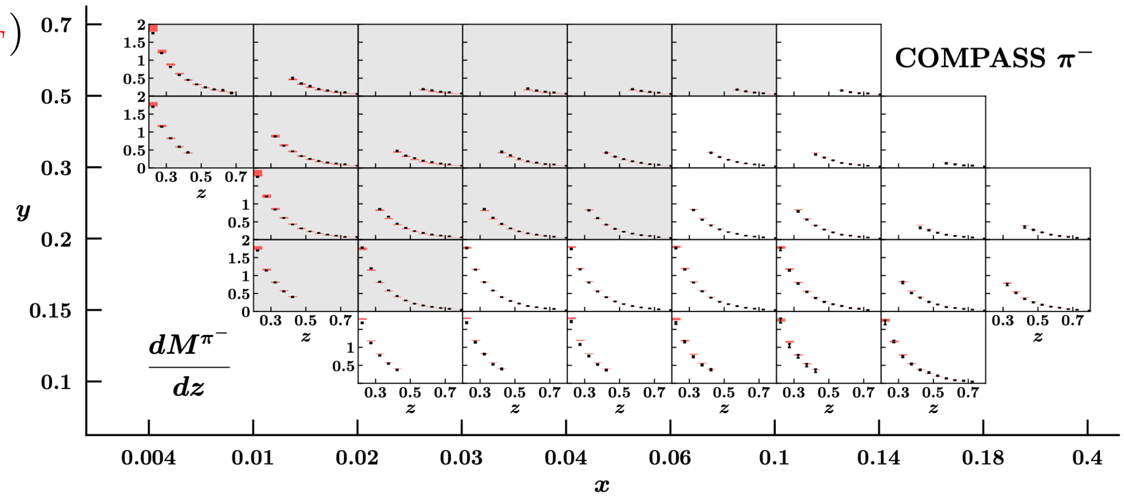

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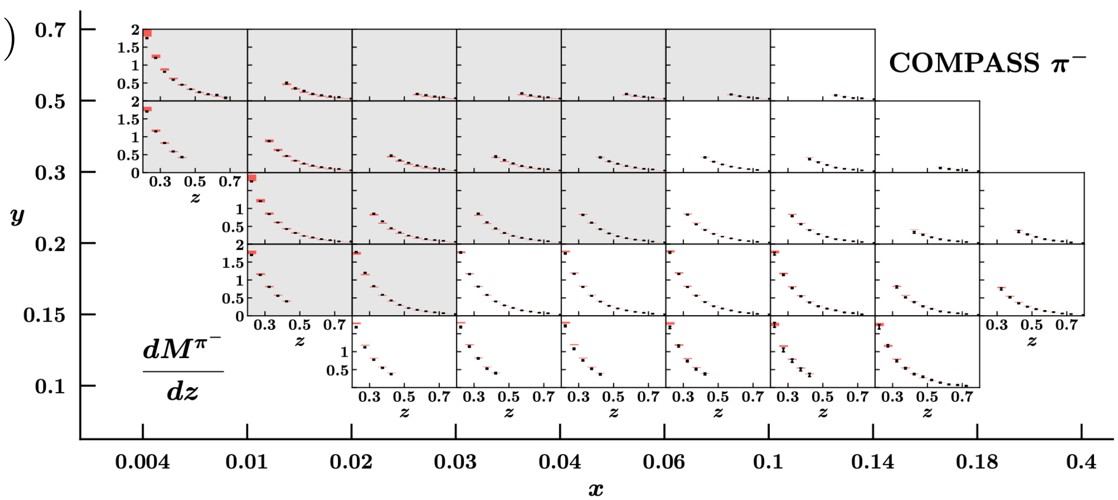

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Calculable before the fit

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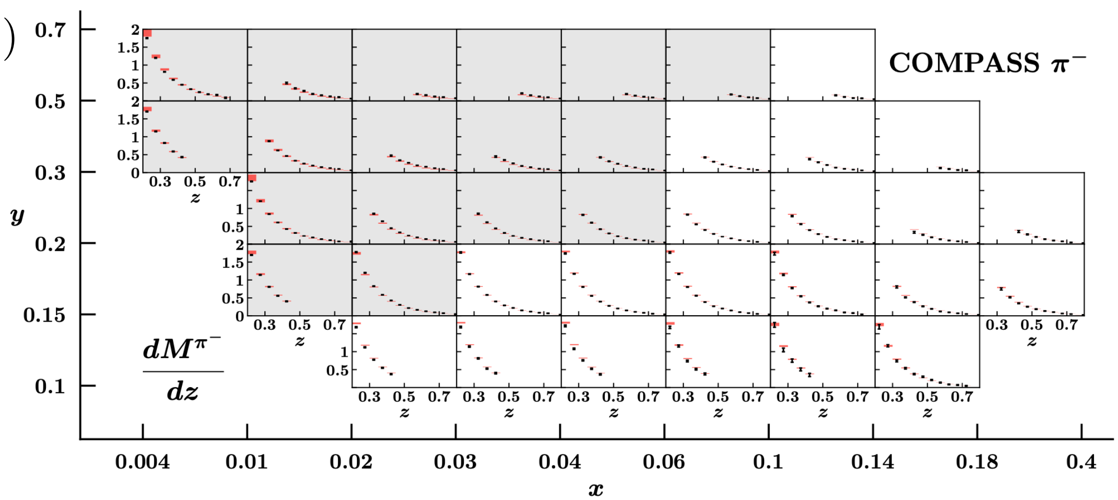


Khalek, Bertone, Nocera, et al., PRD 104 (2021)

MAP Collaboration, JHEP 10 (2022)

MAPTMD24 — Error analysis

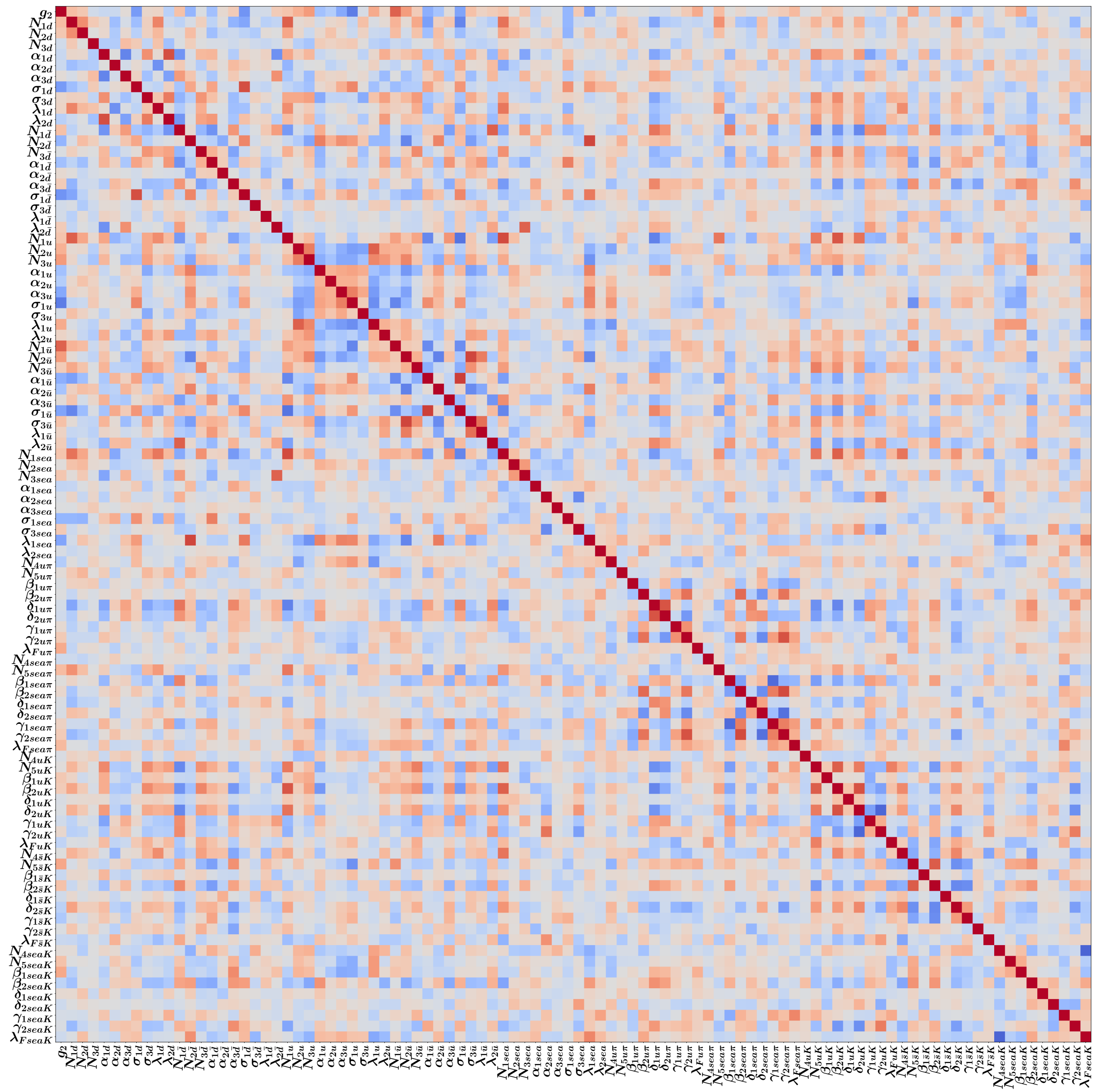
Error propagation

↓

100 Monte Carlo replicas of data

100 Monte Carlo replicas of PDFs

100 Monte Carlo replicas of FFs



MAPTMD24

	χ^2/N_{data}		
Configuration	DY	SIDIS	Total
MMHT+**DSS** (MAP22)	1.66	0.87	**1.06**
NNPDF+**MAPFF** (MAP24 FI)	1.58	1.34	**1.40**

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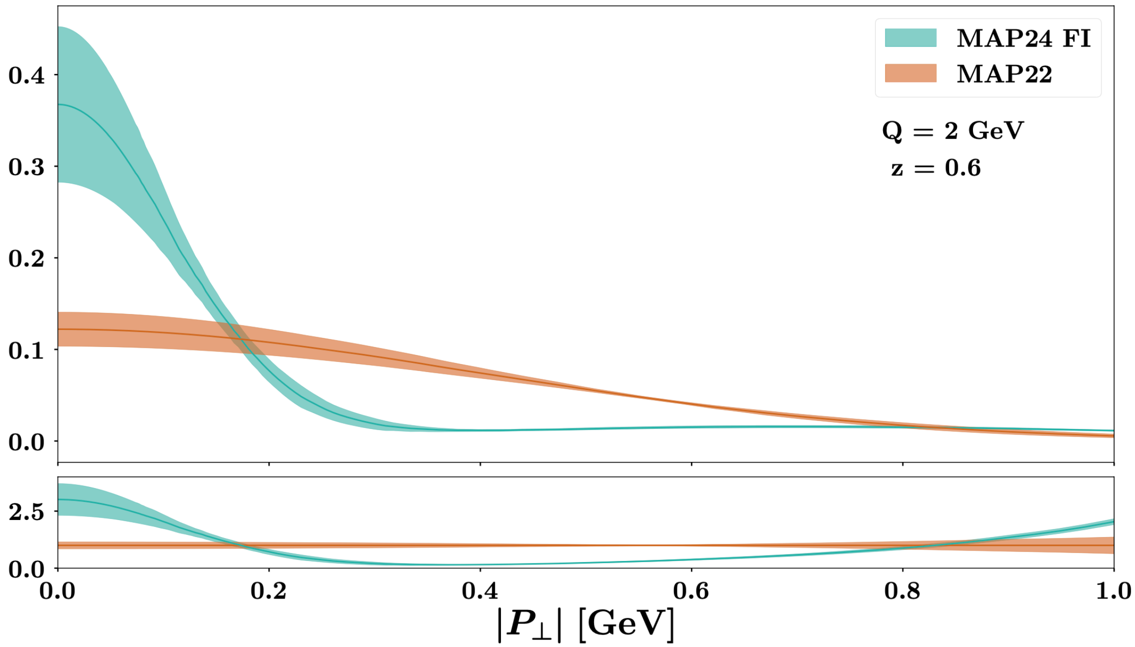

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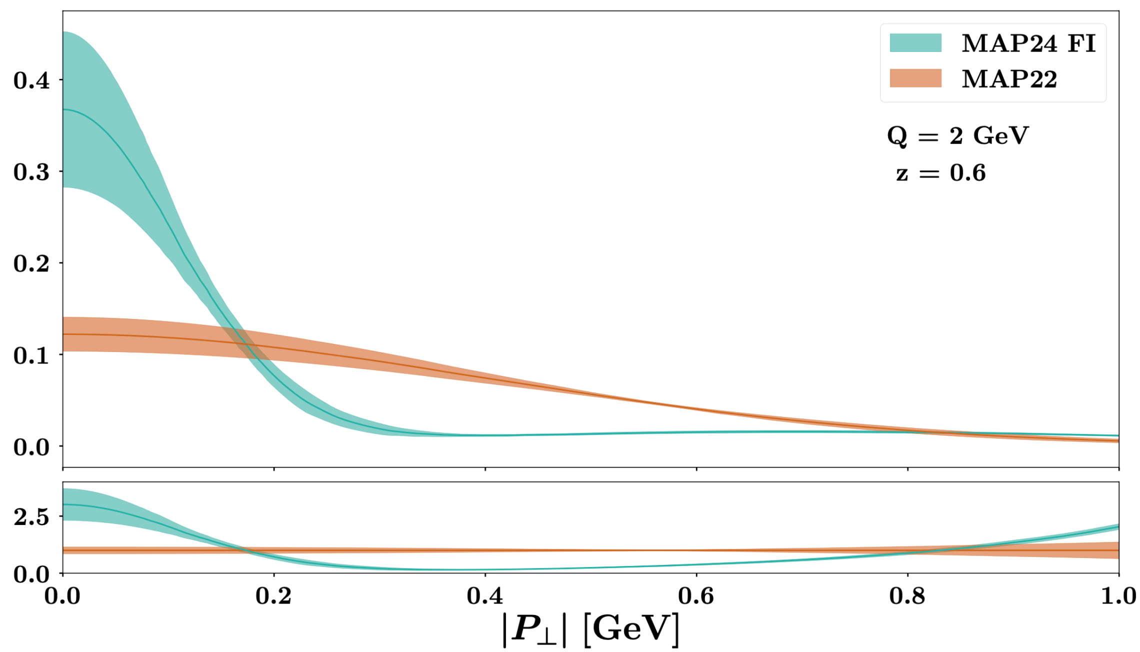
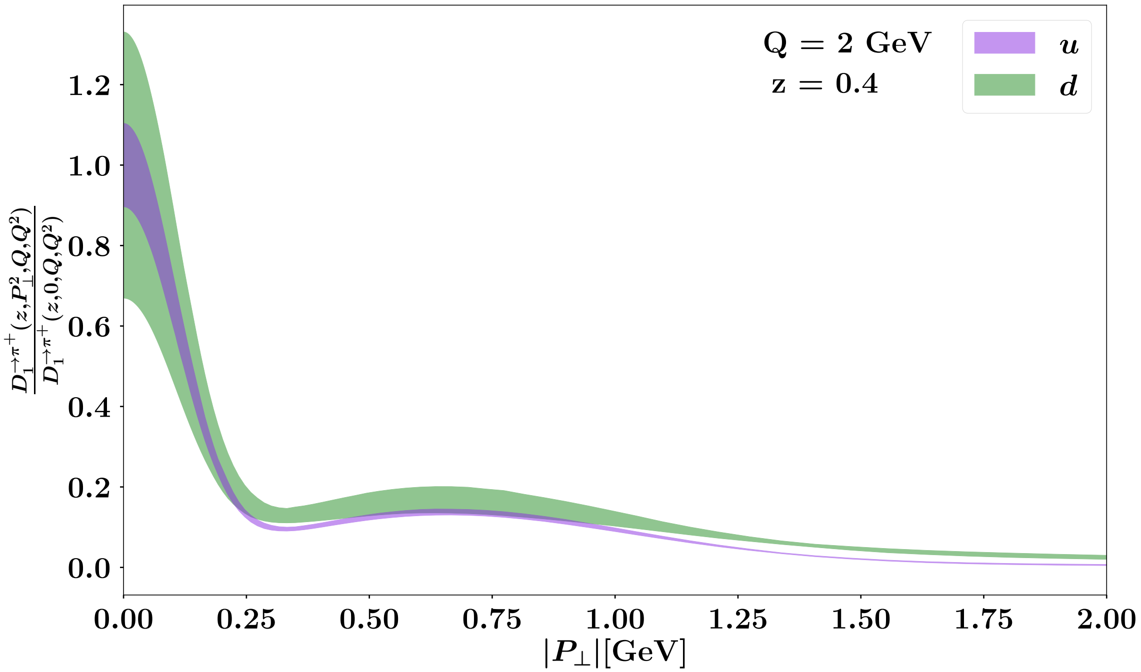


MAPFF1.0nnlo

- approx NNLO
- NN approach
- New behaviors
- Smaller uncertainties

MAPTMD24: results

Flavor-dependent TMD FFs



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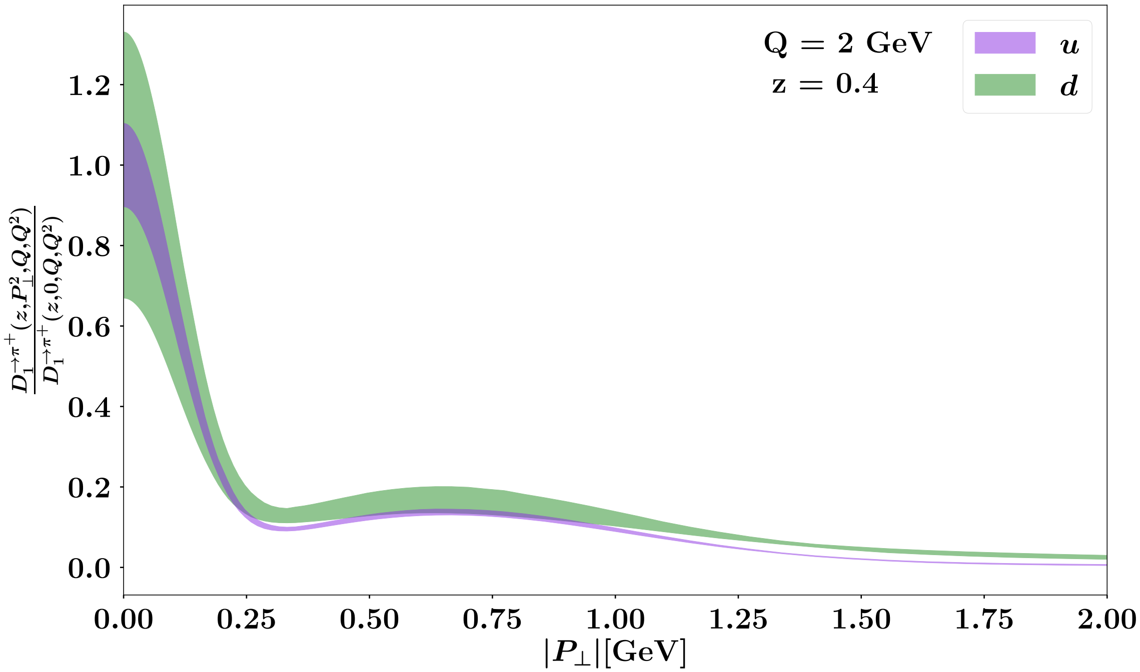

Small evidence of different behaviors for different flavors

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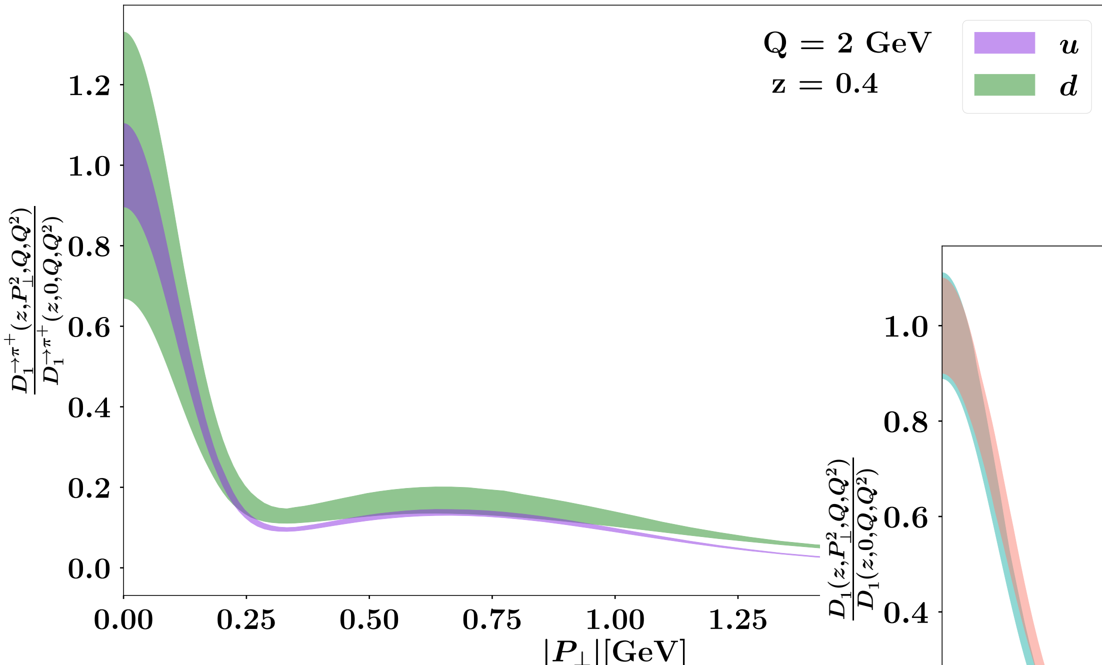

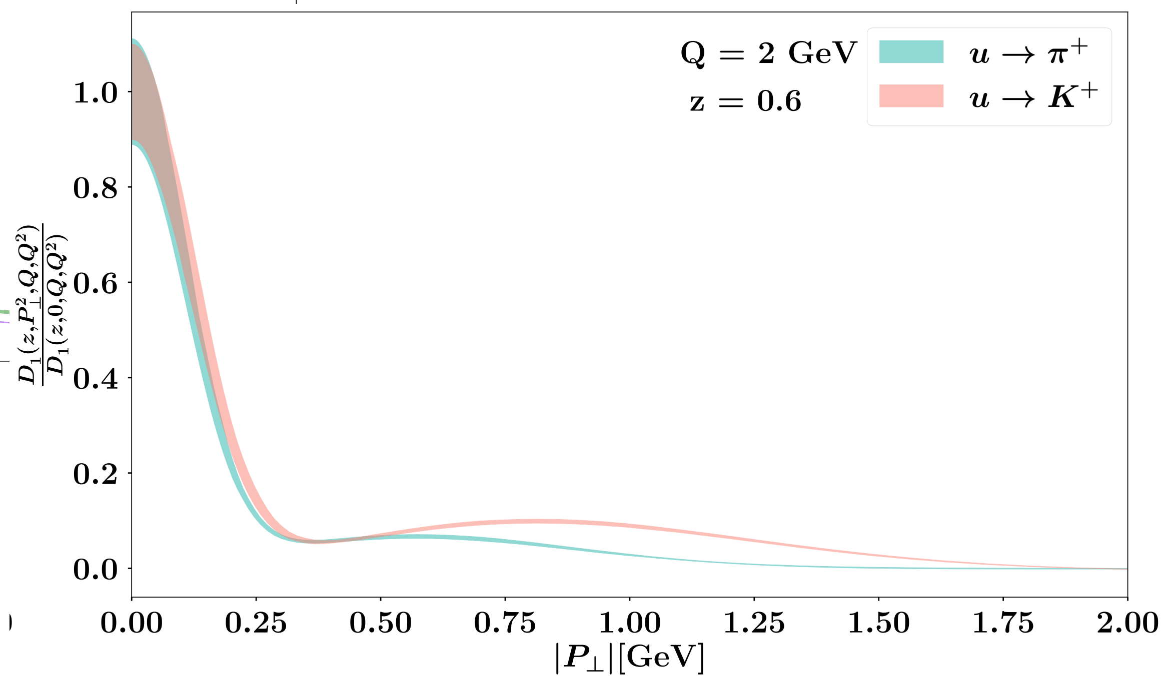

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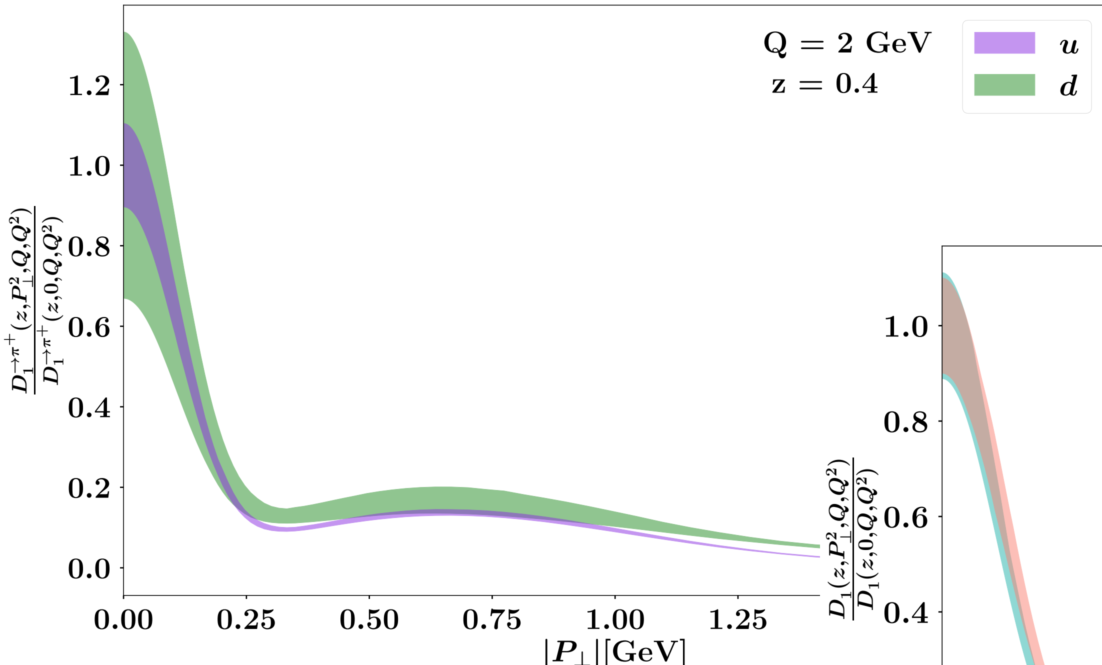

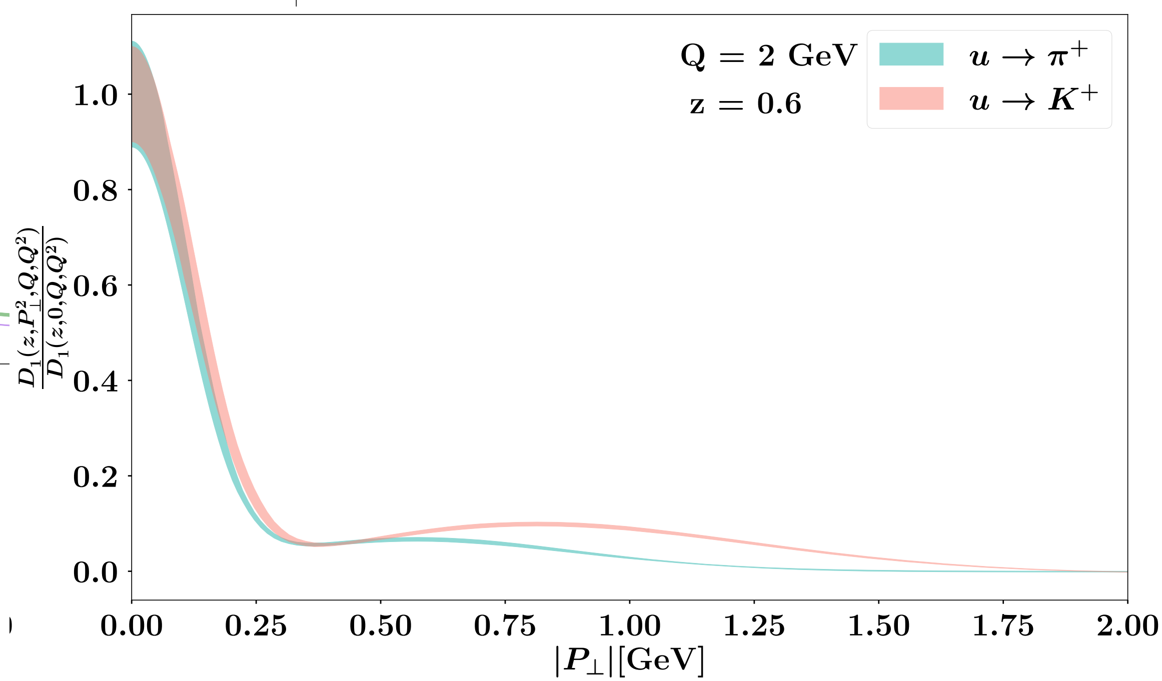

Small evidence of different behaviors for different flavors

Some evidence of different behaviors for different measured hadrons