Kinematic power corrections in TMD factorization

Based on **S.Piloñeta** and **A.Vladimirov** **JHEP12(2024)059** and work in progress

QCD Evolution 2025

Sara Piloñeta. May 20th 2025

PID2022-136510NB-C31

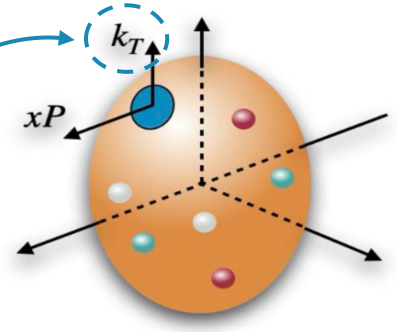
- What do we do? We are interested in studying the structure of nucleons

TMD factorization theorem

→ Transverse Momentum Dependent Distributions (TMDs)

→ Very powerful tool but has some **limitations**

Parton's transverse momentum



Introduction

The necessity for power corrections

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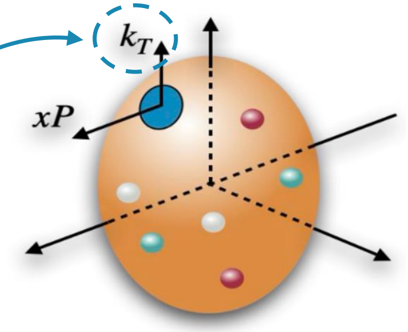
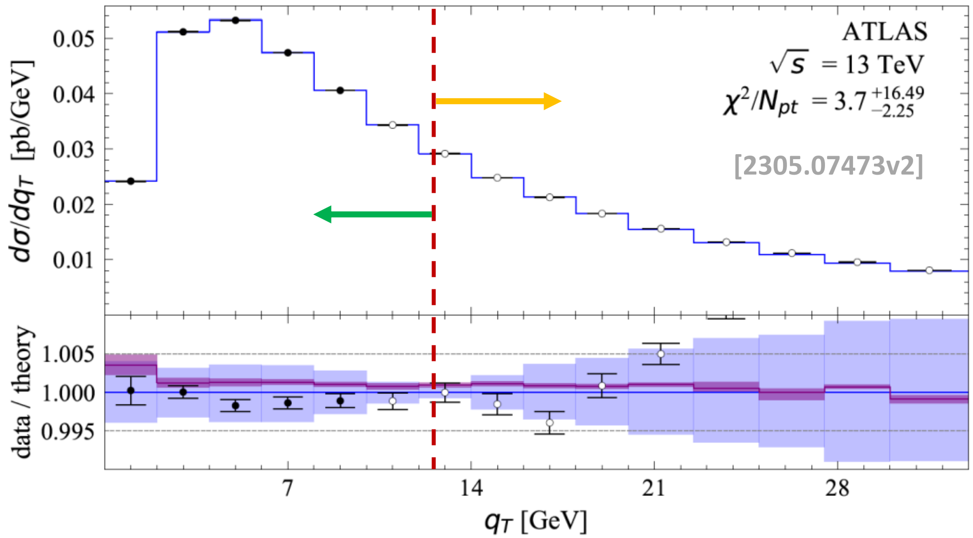


Z-boson production ATLAS measurement comparison



→ Very **precise description** at **low q_T**

→ **Prediction** systematically **lower** than the data at **larger q_T**

Signal of the **necessity** for **power corrections**

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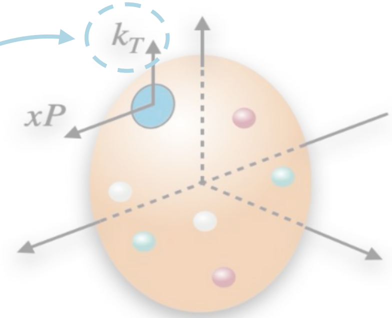
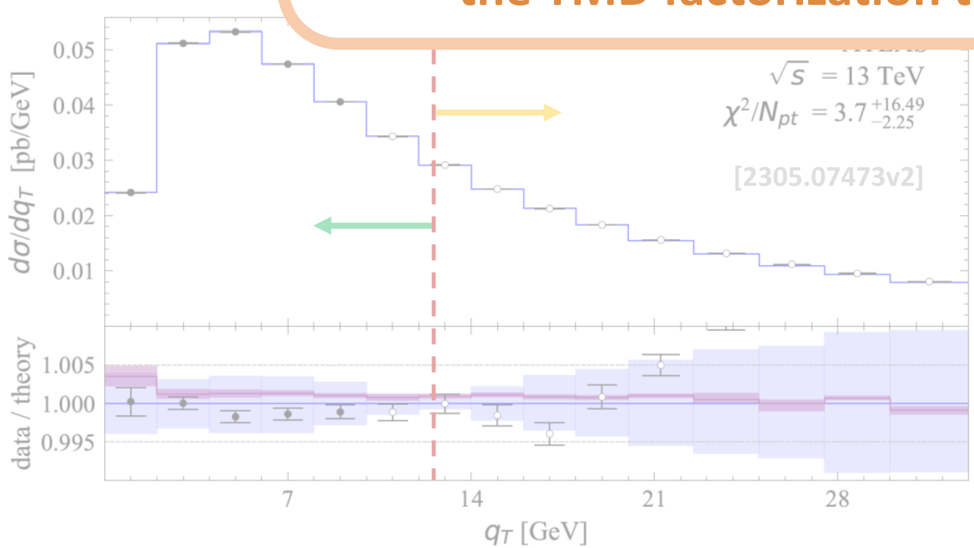


Z-boson pr

This has motivated us to explore the structure of the TMD factorization theorem deeper



- Very precise description at low q_T
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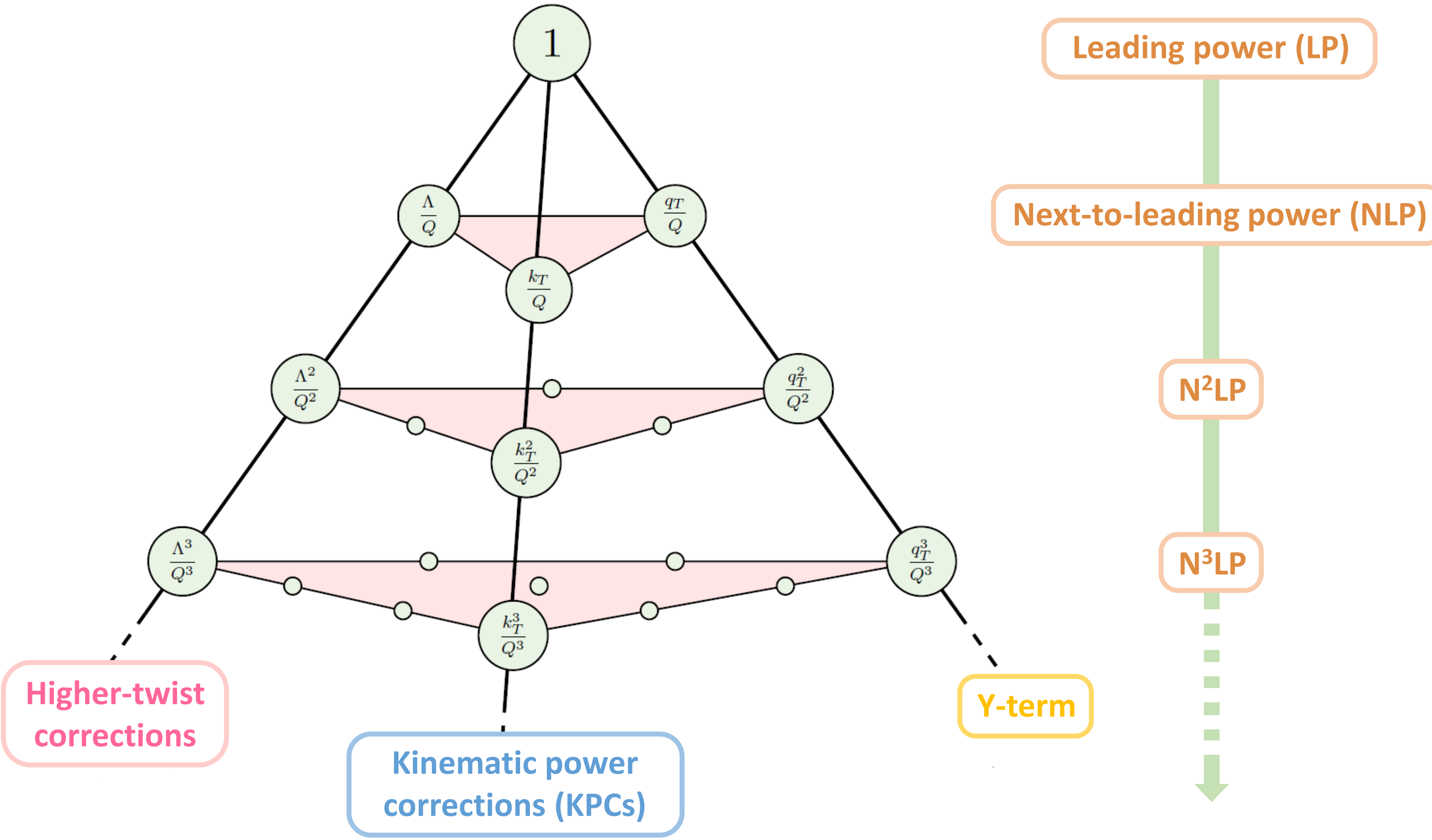
Signal of the necessity for power corrections

The hierarchy of the TMD factorization theorem

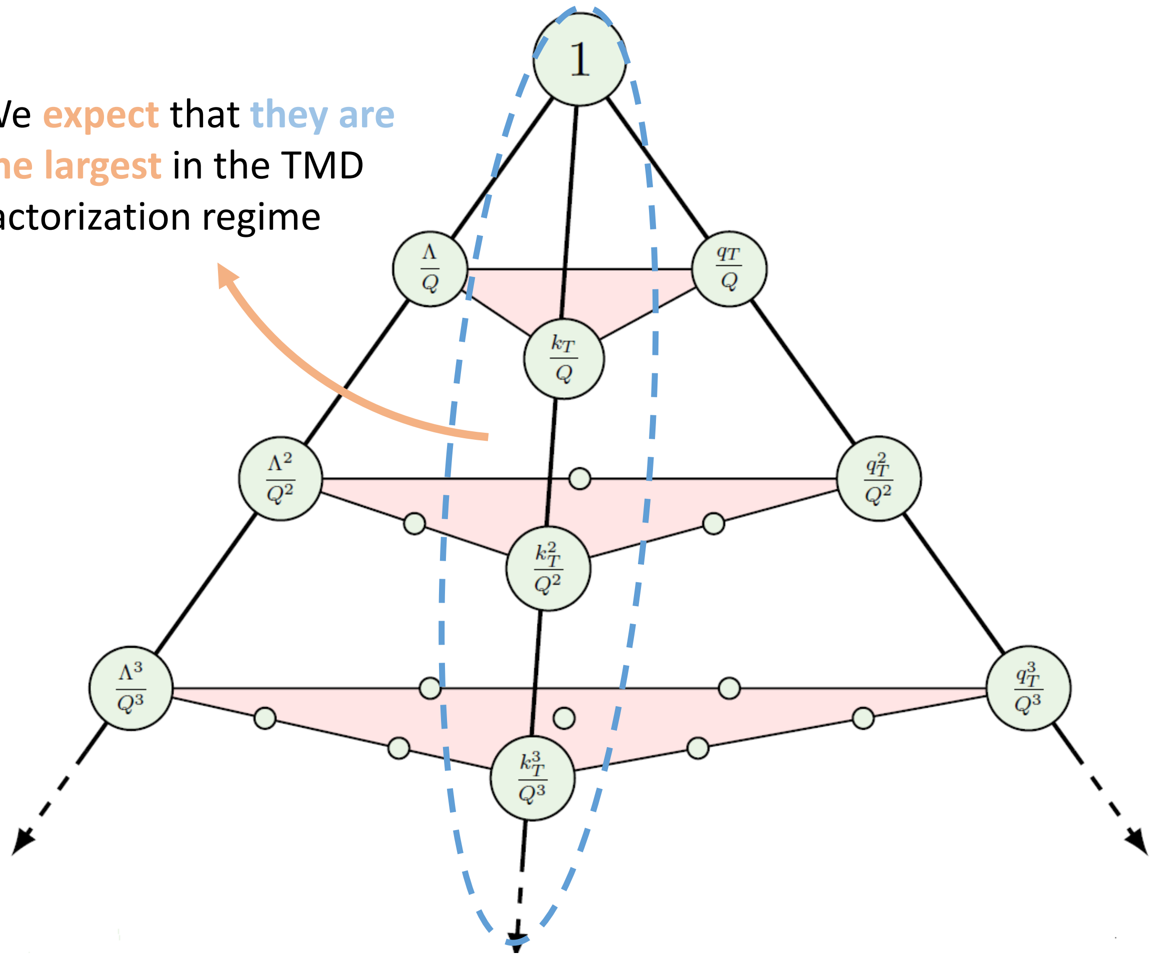
- There is a whole "pyramid" of power corrections



The TMD-with-KPCs factorization theorem

- We focus on the Kinematic power corrections (KPCs) that follow the LP term

We expect that they are the largest in the TMD factorization regime



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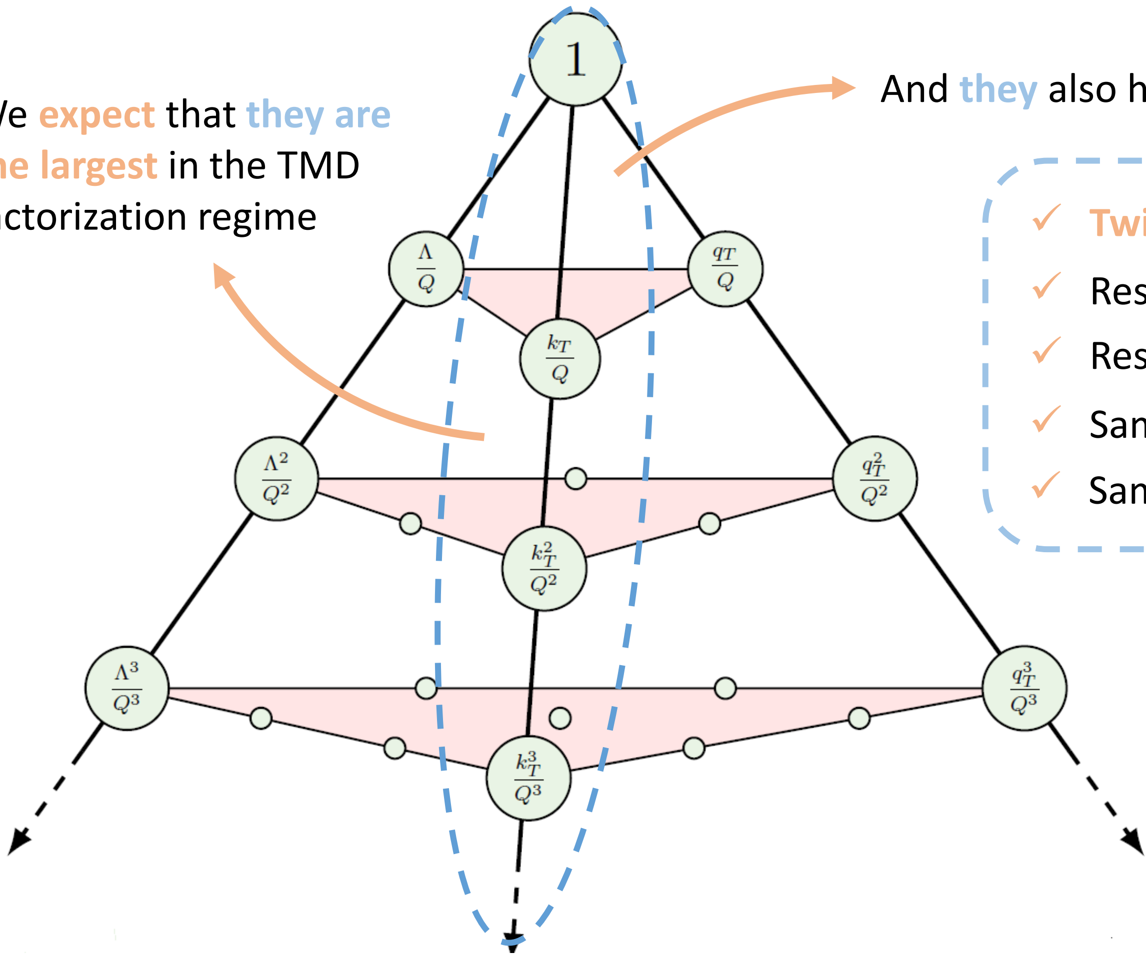

And they also have nice properties!

- ✓ Twist-2 terms at $q_T \ll Q$
- ✓ Restore EM-gauge invariance
- ✓ Restore frame-invariance
- ✓ Same TMD distributions as LP
- ✓ Same coefficient function as LP

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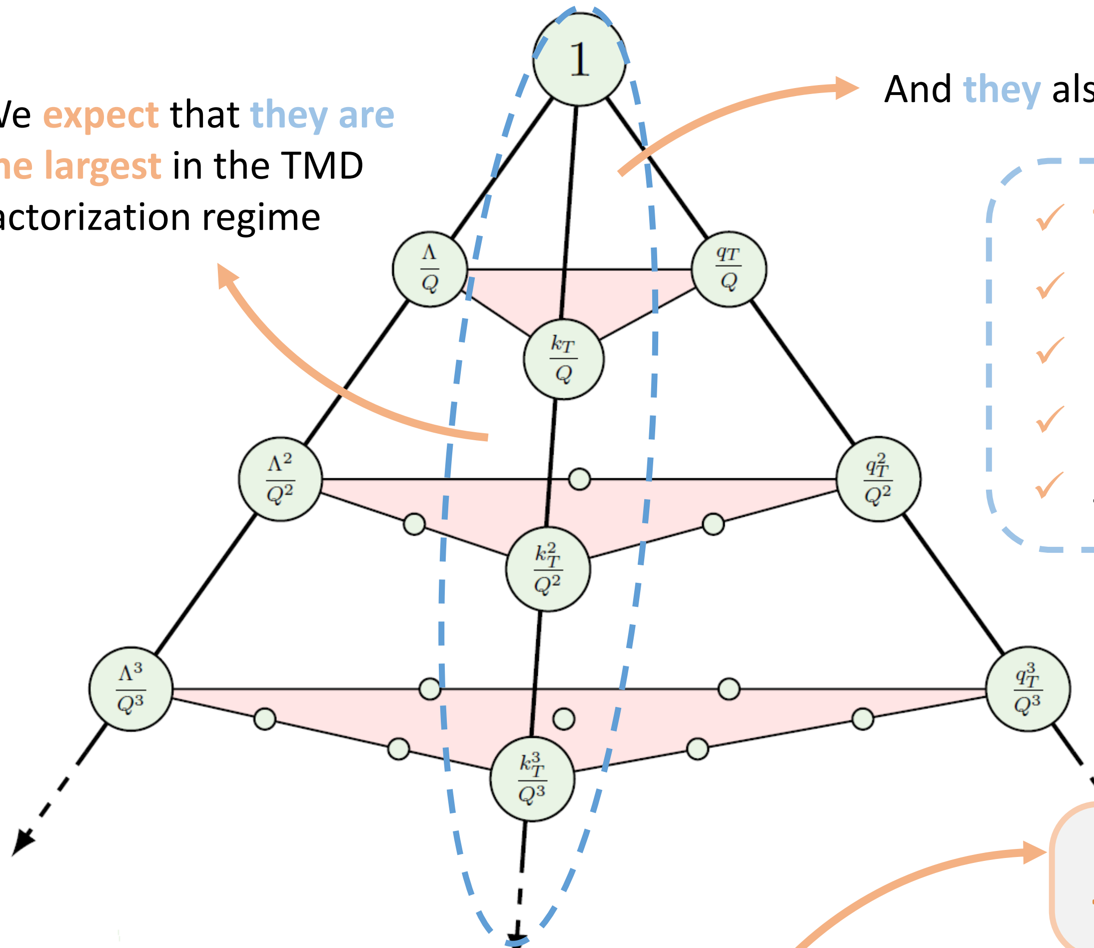

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Theoretical framework including them

TMD-with-KPCs factorization theorem

For details, see [AV, 2307.13054v2]

Angular distributions of Drell-Yan leptons

- **Feasibility** of using the **TMD-with-KPCs factorization** → unpolarized Drell-Yan as a starting point

→ ***Angular distributions** of Drell-Yan leptons in the TMD factorization approach*

→ First practical application of this framework [S. Piloñeta, AV, 2407.06277]

Angular decomposition of the cross-section

$$\frac{d\sigma}{dp_T dy dQ d\Omega} = \frac{3}{16\pi}\boxed{\frac{d\sigma^{U+L}}{dp_T dy dQ}}\left[(1+\cos^2\theta) + \frac{1-3\cos^2\theta}{2}\boxed{A_0} + \sin 2\theta\cos\phi\boxed{A_1} + \frac{\sin^2\theta\cos 2\phi}{2}\boxed{A_2} + \sin\theta\cos\phi\boxed{A_3} + \cos\theta\boxed{A_4} + \sin^2\theta\sin 2\phi\boxed{A_5} + \sin 2\theta\sin\phi\boxed{A_6} + \sin\theta\sin\phi\boxed{A_7}\right]$$

A_U → **Unpolarized angle-integrated** cross-section

$\boxed{A_{0,\dots,7}}$

→ Only **four** at **LP** $\{A_U,\ A_2,\ A_4,\ A_5\}$

→ The **rest** are **power-suppressed**

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- **Feasibility** of using the **TMD-with-KPCs factorization** ⟹ unpolarized Drell-Yan as a starting point

  ⟹ ***Angular distributions*** *of Drell-Yan leptons in the TMD factorization approach*

  ⟹ First practical application of this framework [S. Piloñeta, AV, 2407.06277]

Angular decomposition of the cross-section

$$\frac{d\sigma}{dp_T dy dQ d\Omega} =$$

$\boxed{A_U}$ ⟹ **Unpolarized angle-integrated** cross-section

When we incorporate the resummed KPCs we can describe them all!

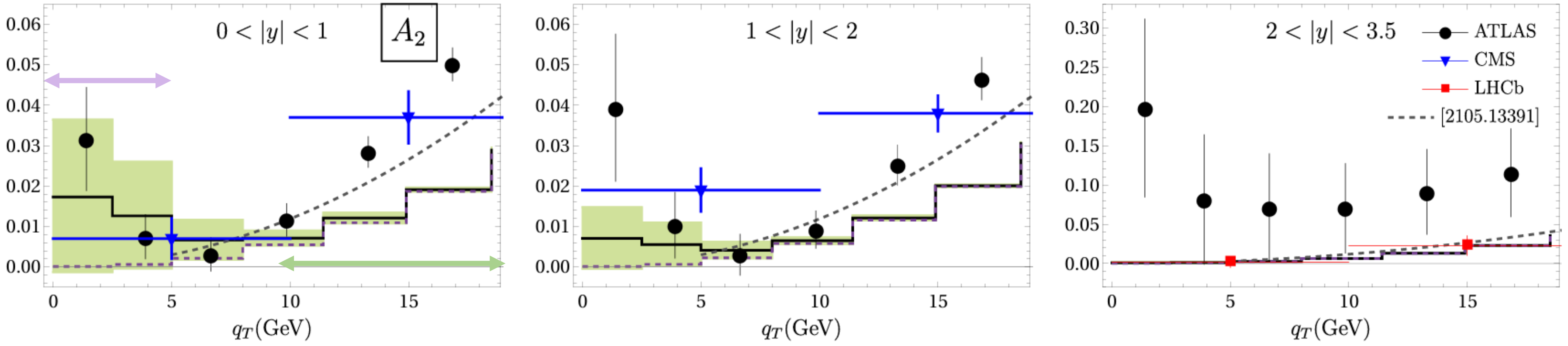
$$\frac{\sin^2\theta\cos 2\phi}{2}\boxed{A_2} + \sin\theta\cos\phi\boxed{A_3} + \cos\theta\boxed{A_4}$$

$$\sin^2\theta\sin 2\phi\boxed{A_5} + \sin 2\theta\sin\phi\boxed{A_6} + \sin\theta\sin\phi\boxed{A_7}\Big]$$

$\boxed{A_{0,\ldots,7}}$ ⟹ Only **four** at **LP** $\{A_U,\ A_2,\ A_4,\ A_5\}$

⟹ The **rest** are **power-suppressed**

Angular distribution A_2



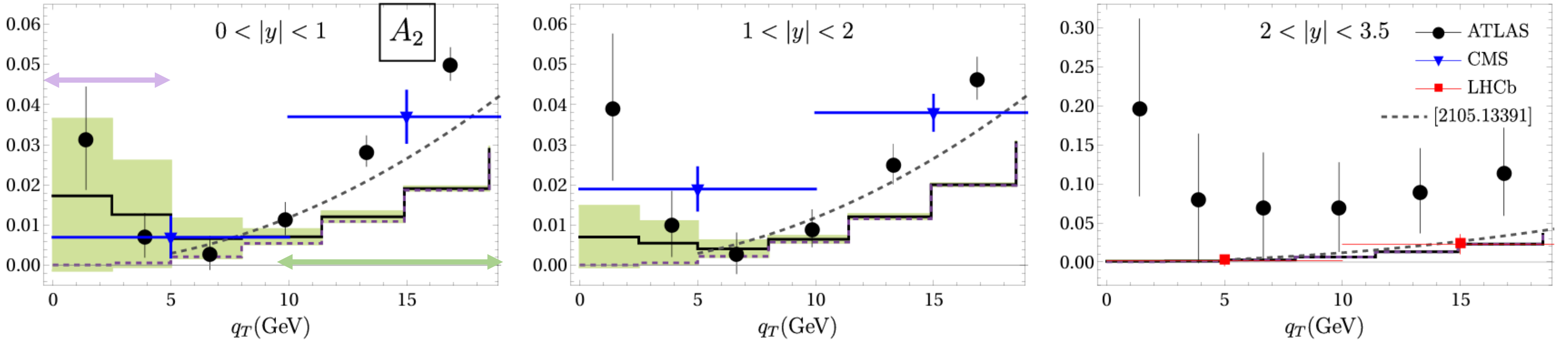
$$A_2 \sim \left[(v^2 - a^2)_q \frac{h_1^\perp h_1^\perp}{M^2} + (v^2 + a^2)_q \frac{f_1 f_1}{Q^2} \right]$$

Dominant at larger q_T

Dominant at $q_T \to 0$

- Contains both Boer-Mulders and unpolarized distributions

Angular distribution A_2



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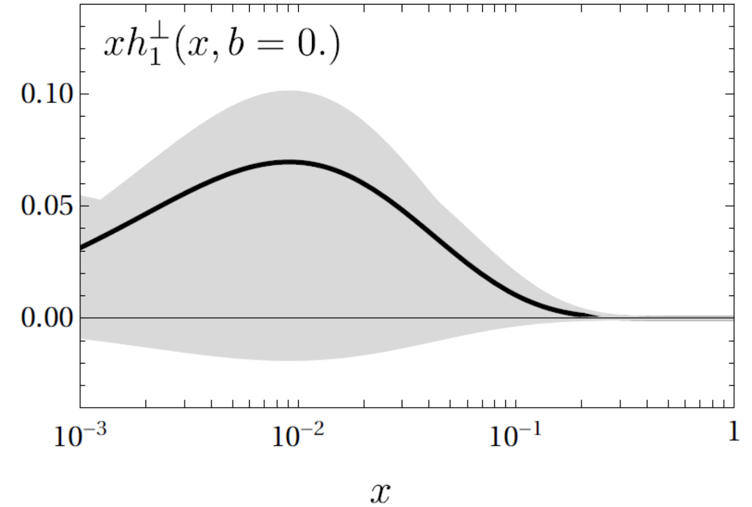
- Boer-Mulders distribution extraction from A_2
  - Very primitive model
    - New fit is required
  - First evidence (?) of twist-3 effects at LHC

$$x|E(-x,0,x)|$$



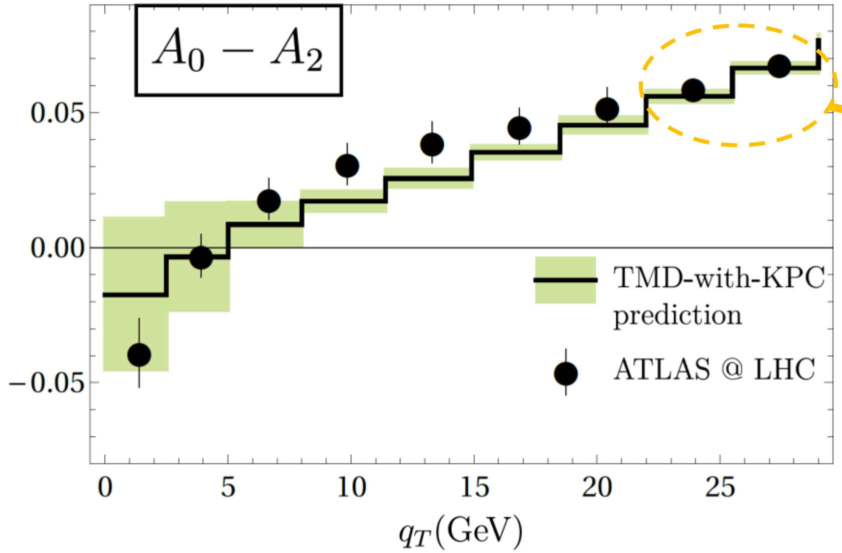
Lam-Tung relation ($A_0 - A_2$) description

- If we use the TMD factorization theorem the relation does not hold at LP

$$\Sigma_{LT} \sim \boldsymbol{k}^2/M^2\, h_1^\perp h_1^\perp$$

The double Boer-Mulders is negligible!

- Using the A_2 and A_0 computed including KPCs the theoretical expression also contains f_1



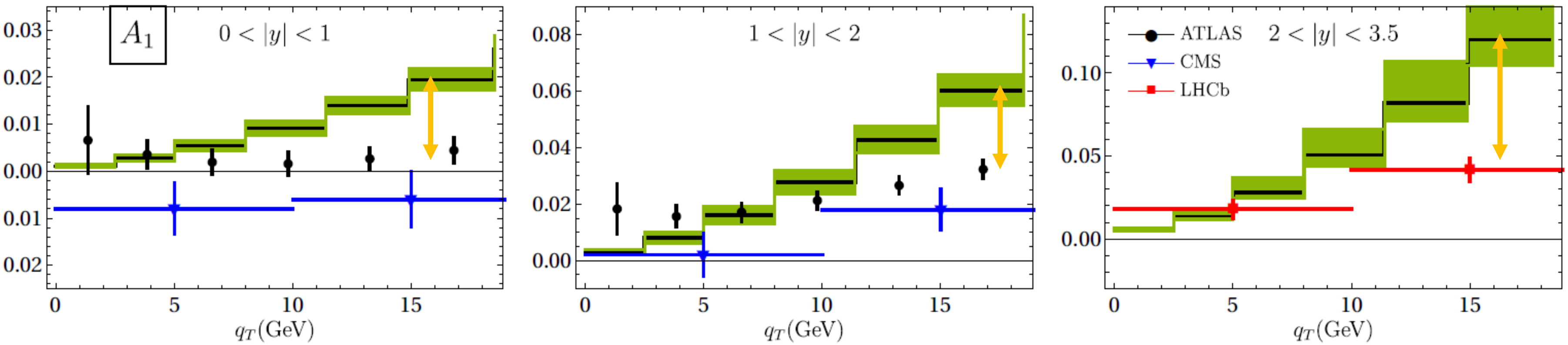
Y-term suppressed by an extra factor

Smaller q_T/Q corrections

Good agreement in contrast to A_0 and A_2 independently

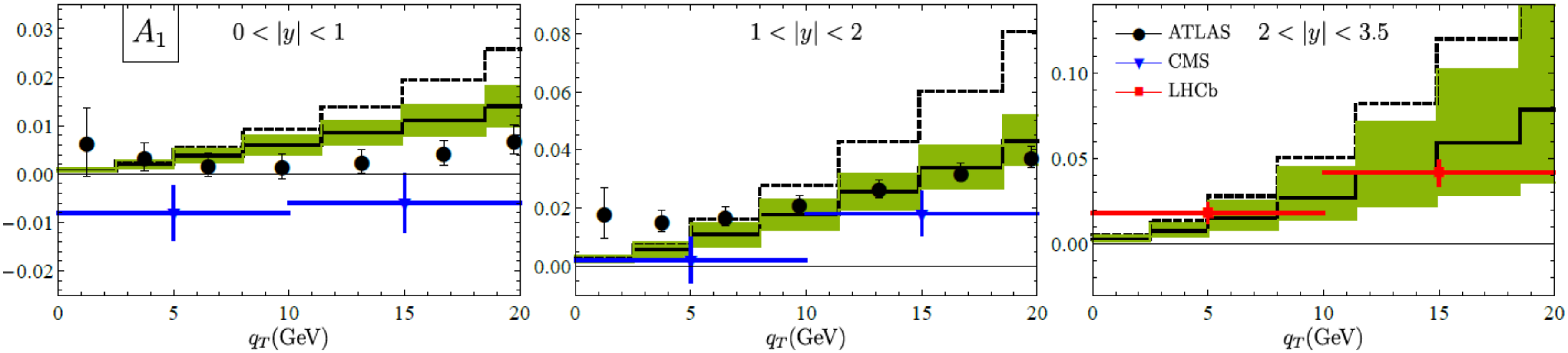
Good description only possible due to KPCs inclusion!

Angular distribution A_1



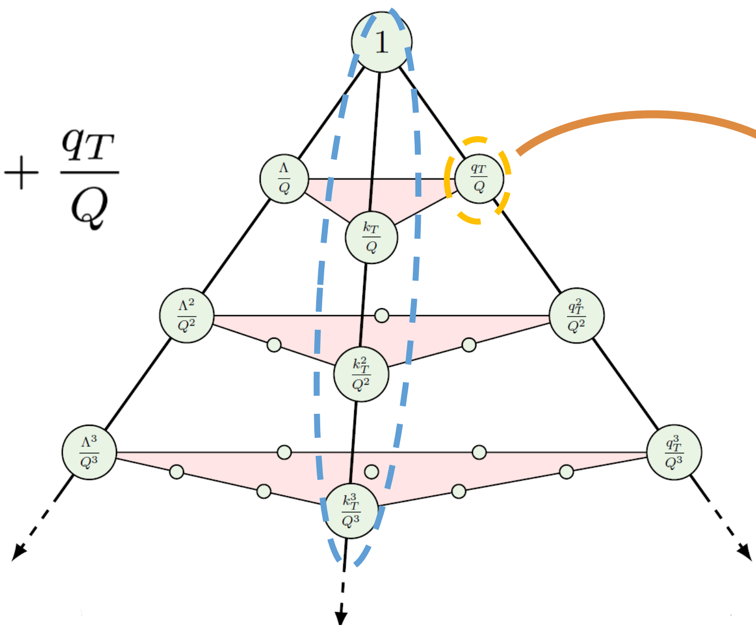
- Problems with the A_1 data description at larger values of q_T

Angular distribution A_1



- **Problems** with the A_1 data description at **larger values of q_T**

$$\text{KPCs} + \frac{q_T}{Q}$$



Including the computation of the **leading q_T/Q correction** fixes the jump!

[A.Arroyo, I.Scimemi, AV, 2503.24336]

Moving to SIDIS: structure functions computation using KPCs

- Now, let's shift our focus to SIDIS → essential process for probing hadron structure

Cross-section decomposition in terms of structure functions

$$\frac{d\sigma}{dx\,dy\,d\psi\,dz\,d\phi_h\,d\boldsymbol{p}_\perp^2} = \frac{\alpha_{\rm em}^2}{xyQ^2}\frac{y^2}{2(1-\varepsilon)}\Big\{F_{UU,T} + \varepsilon F_{UU,L} + \sqrt{2\varepsilon(1+\varepsilon)}\cos\phi_h F_{UU}^{\cos\phi_h} + \varepsilon\cos(2\phi_h) F_{UU}^{\cos 2\phi_h} + \lambda_e\sqrt{2\varepsilon(1-\varepsilon)}\sin\phi_h F_{LU}^{\sin\phi_h} + \ldots$$

We want to compute them including KPCs

$$\Big\{F_{UU,T} + \ldots\Big\} = \frac{x}{4z}\frac{1-\varepsilon}{Q^2}L_{\mu\nu}W^{\mu\nu}$$

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$$\Big\{F_{UU,T} + \ldots\Big\} = \frac{x}{4z}\frac{1-\varepsilon}{Q^2}\underbrace{L_{\mu\nu}}_{1}W^{\mu\nu}$$

1 Lepton tensor conveniently decomposed via the tensors $P^\mu,\ q^\mu,\ p_\perp^\mu$

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2 Hadron tensor computed using the TMD-with-KPCs factorization theorem

- Same coefficient function as LP $\quad q_\mu W^{\mu\nu} = 0$
- Main difference with LP → Convolution integral

A closer look at the convolution integral

- The convolution integral is more complicated now

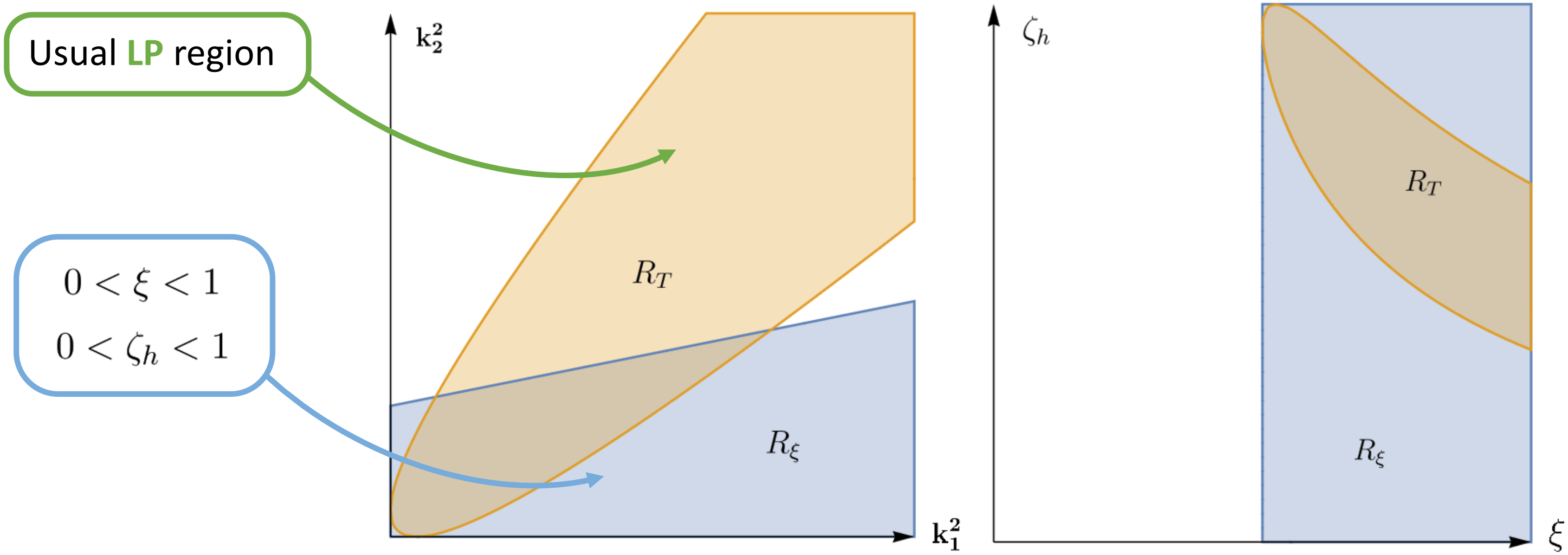
LP case

$$\mathcal{C}_{LP}[A, f_1, D_1] \sim \int d^2\boldsymbol{k}_1 d^2\boldsymbol{k}_2\, \delta(\boldsymbol{q_T} + \boldsymbol{k_1} - \boldsymbol{k_2})\, f_1(x_1, \boldsymbol{k_1^2}) D_1(z_1, \boldsymbol{k_2^2})$$

Additional dependance
coming from extra δ-functions

KPCs case

$$\mathcal{C}_{KPC}[A, f_1, D_1] \sim \int d^2\boldsymbol{k}_1 d^2\boldsymbol{k}_2\, \delta(\boldsymbol{q_T} + \boldsymbol{k_1} - \boldsymbol{k_2})\, f_1(\xi(x_1, \boldsymbol{k_{1,2}^2}), \boldsymbol{k_1^2}) D_1(\zeta_h(z_1, \boldsymbol{k_{1,2}^2}), \boldsymbol{k_2^2})$$



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Additional dependance
coming from extra δ-functions

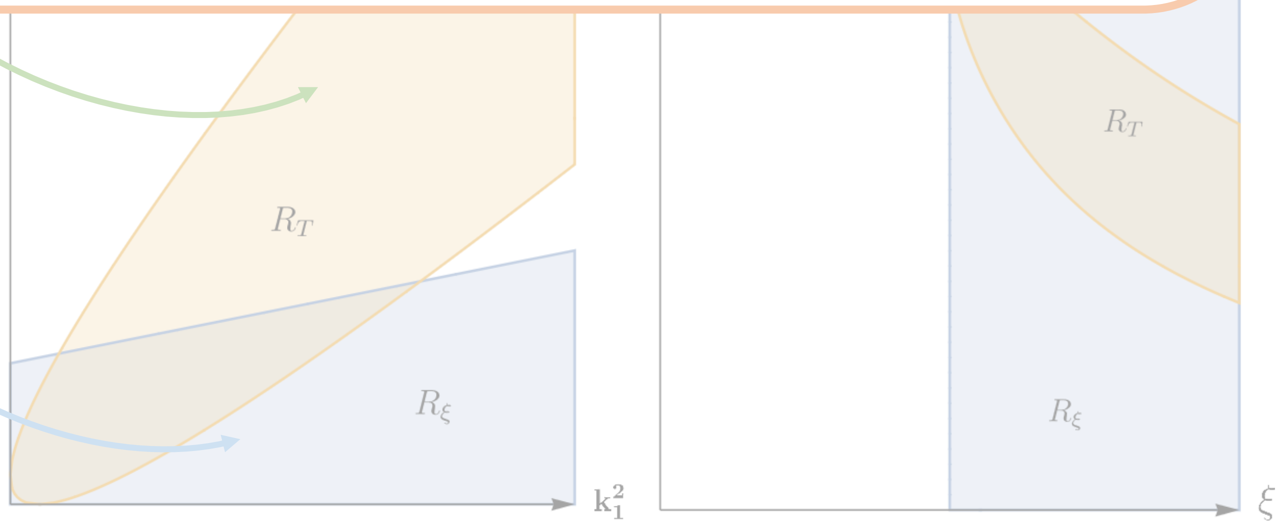
KPCs c… $\ldots), \boldsymbol{k}_2^2)$

The convolution integral and the KPCs have been implemented in arTeMiDe, although a few improvements are still needed

github.com/VladimirovAlexey /artemide-development

Usual …

$0 < \xi < 1$
$0 < \zeta_h < 1$



Focusing on $F_{UU,T}$ and $F_{UU,L}$

- I have obtained the **theoretical expressions** for all of them using the **TMD-with-KPCs factorization**

  ↳ But in **this talk** I am going to **focus on $F_{UU,T}$** and $F_{UU,L}$

$$F_{UU,T} = \frac{x}{4z} F_0 (\mathcal{S}_1^{\mu\nu} - \underbrace{\mathcal{S}_0^{\mu\nu}}) W_U^{\mu\nu} \qquad F_{UU,L} = \frac{x}{4z} F_0 (2\underbrace{\mathcal{S}_1^{\mu\nu}}) W_U^{\mu\nu}$$

$$\mathcal{S}_0^{\mu\nu} = g^{\mu\nu} - \frac{q^\mu q^\nu}{Q^2} \qquad \mathcal{S}_1^{\mu\nu} = \frac{(2xP^\mu + q^\mu)(2xP^\nu + q^\nu)}{(1+\gamma^2)Q^2}$$

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The theoretical expression for $F_{UU,T}$ including KPCs is

$$F_{UU,T} = \frac{x}{2z} Q^2 \, \mathcal{C}_{KPC}[1, f_1 D_1] + \frac{1}{2} F_{UU,L}$$

$$\mathcal{C}_{LP}[1, f_1 D_1] + \varepsilon \, F_{UU,L}$$

Cross-section gets modified

$$\mathcal{C}_{KPC}[1, f_1 D_1] + \left(\frac{1}{2} + \varepsilon\right) F_{UU,L}$$

The inclusion of the KPCs implies that $F_{UU,T}$ is not exactly 1 as it was in the LP case!

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$$\mathcal{C}_{LP}[1, f_1 D_1] + \varepsilon F_{UU,L} \quad \xrightarrow{\text{Cross-section gets modified}} \quad \mathcal{C}_{KPC}[1, f_1 D_1] + \left(\frac{1}{2} + \varepsilon\right) F_{UU,L}$$

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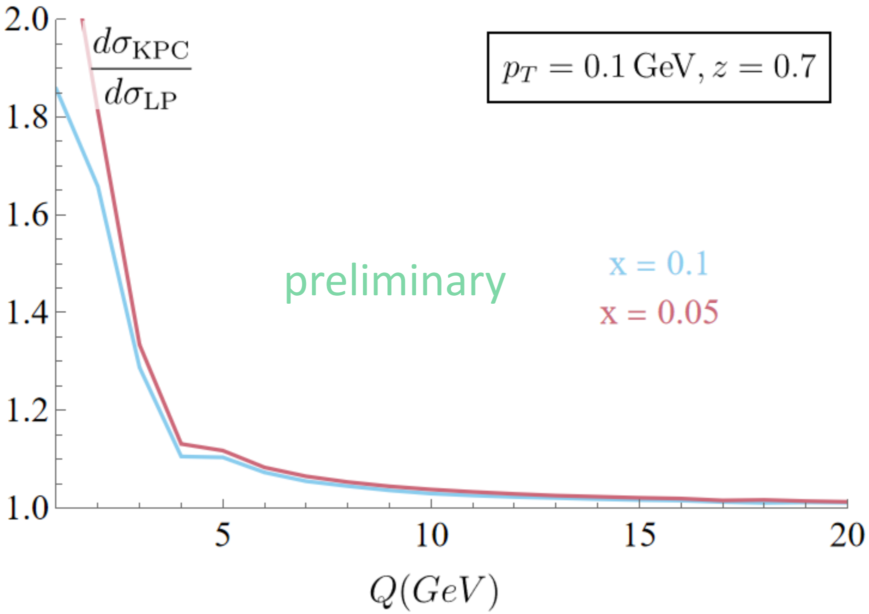
So far, I have studied the KPCs impact on

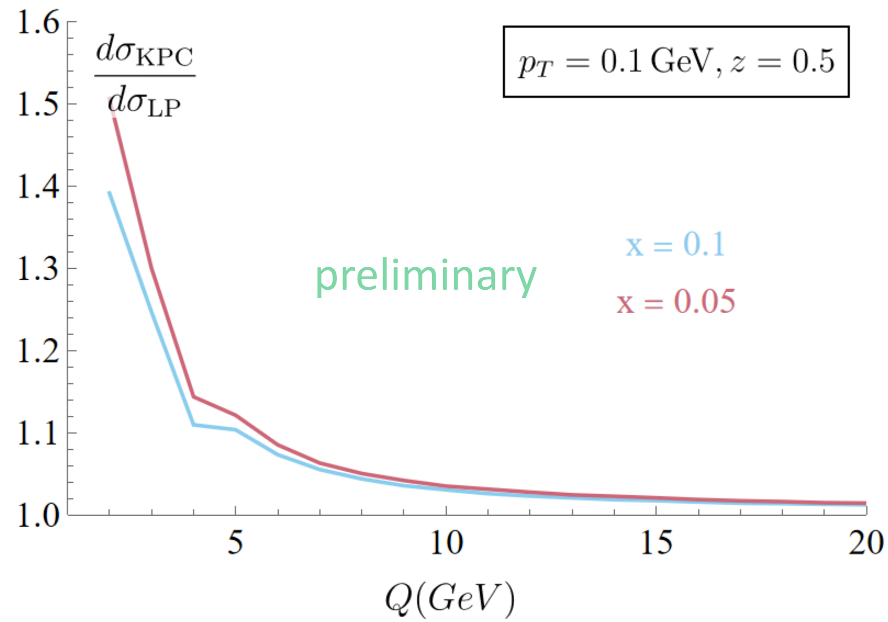
- The SIDIS cross-section
- The structure functions $F_{UU,T}$ and $F_{UU,L}$

Work is still in progress!

Ratio of KPC-summed to LP cross-sections

- **Studying** the **size** of $\mathcal{C}_{KPC}[1, f_1 D_1] + \frac{1}{2} F_{UU,L}$ in **comparison with** $\mathcal{C}_{LP}[1, f_1 D_1]$

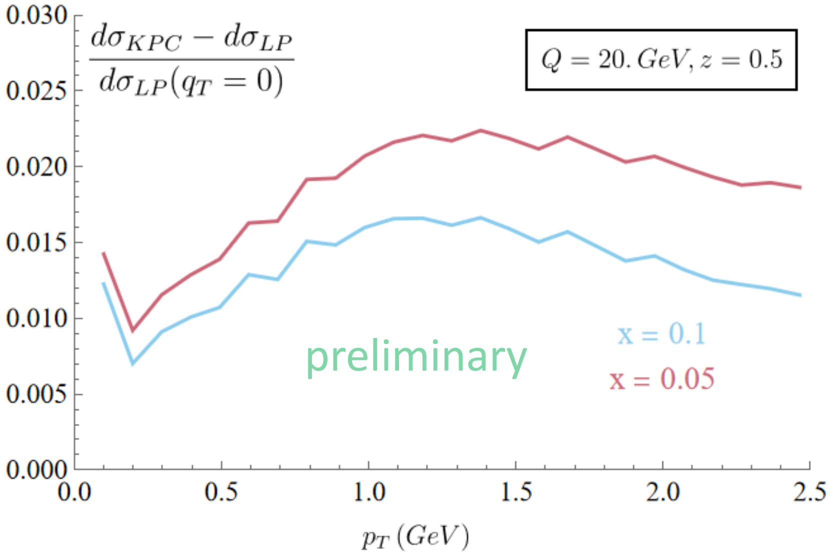


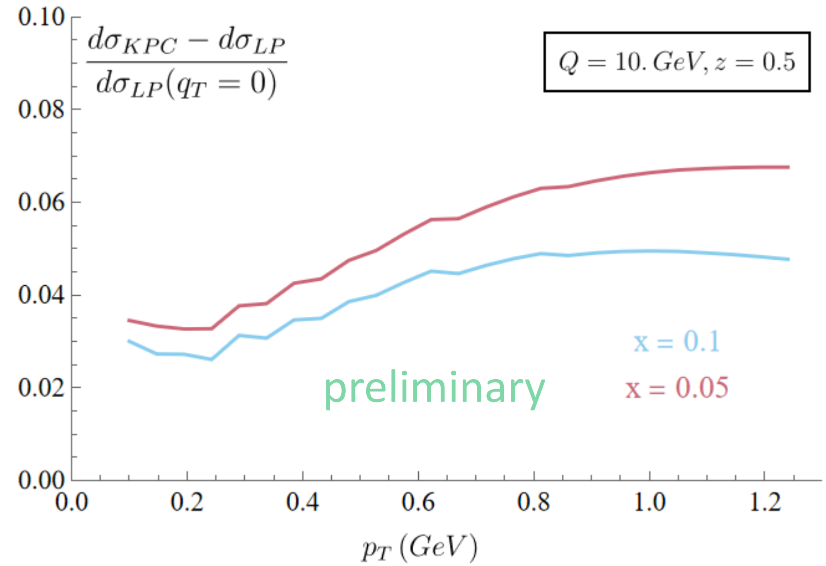


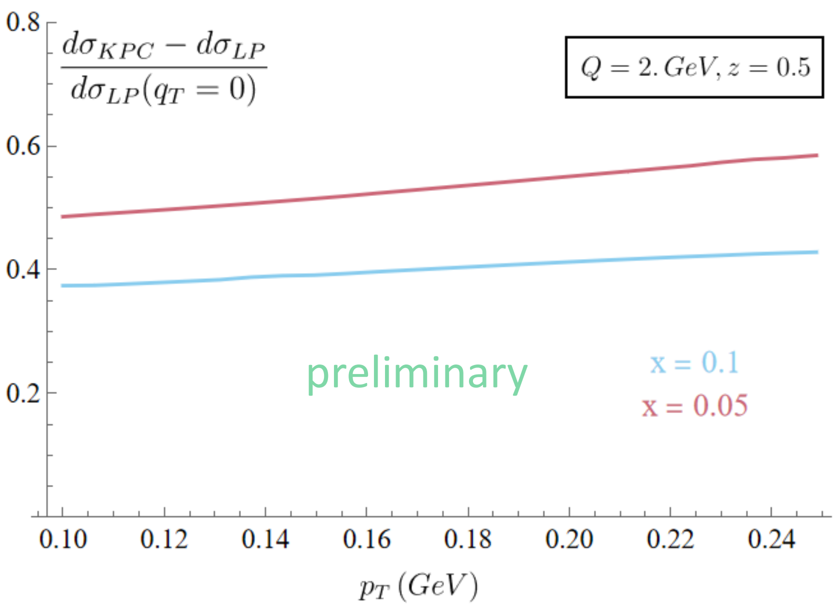
✓ **Cross-section** with **KPCs** is **bigger**

- **Specially** at **low energies**
- Satisfies that $\lim_{Q \to \infty} d\sigma_{\mathrm{KPC}} = d\sigma_{\mathrm{LP}}$

Cross-sections difference relative to the $q_T = 0$ LP cross-section







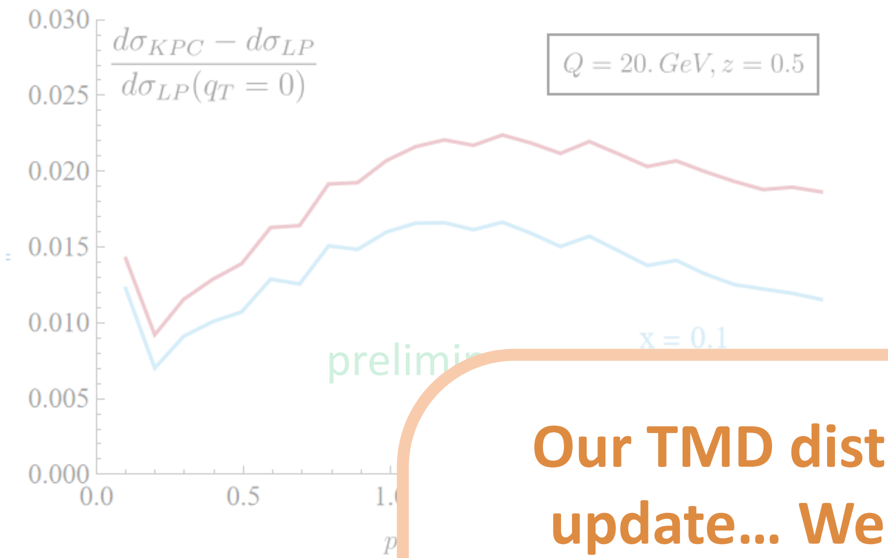
- ✓ Almost flat increase of the cross-section

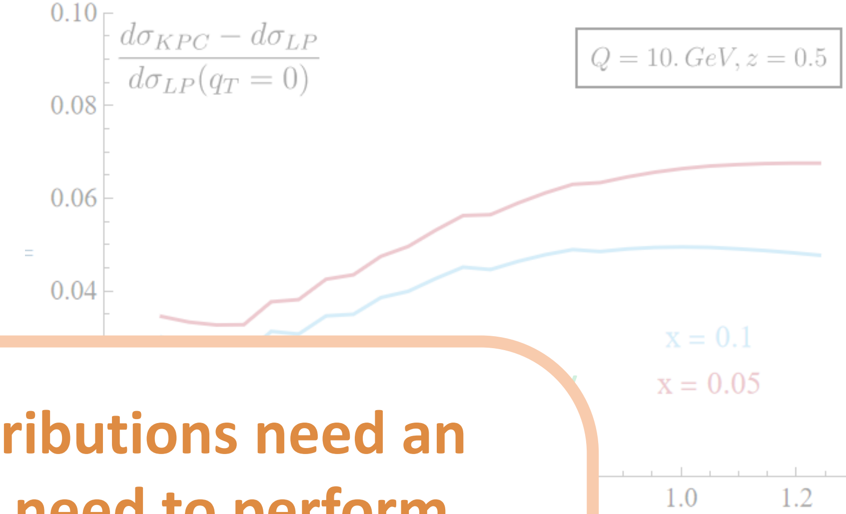
The inclusion of KPCs mostly changes the normalization

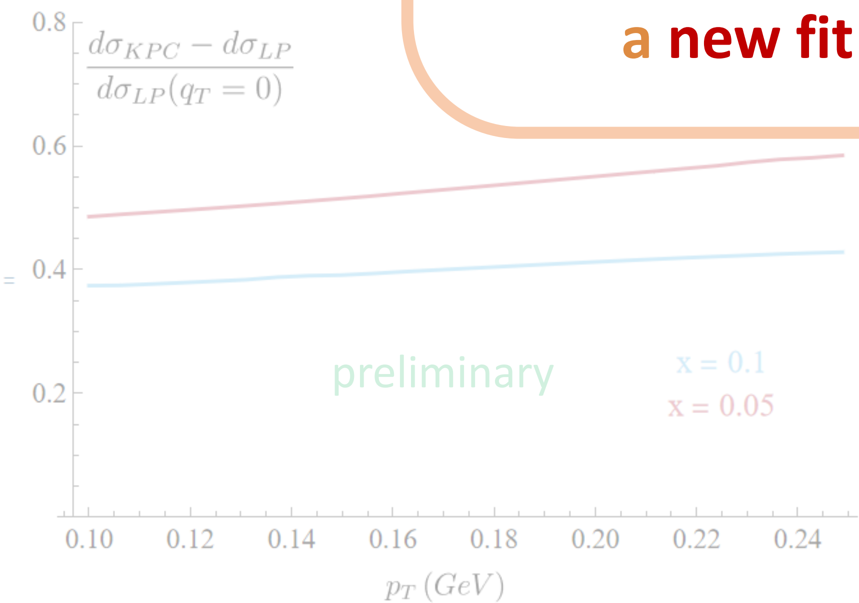
- ✓ KPCs very important at low energies

40-60% at 2 GeV!

Cross-sections difference relative to the q_T = 0 LP cross-section







Our TMD distributions need an update… We need to perform a new fit including KPCs!

the cross-section

The inclusion of KPCs mostly changes the normalization

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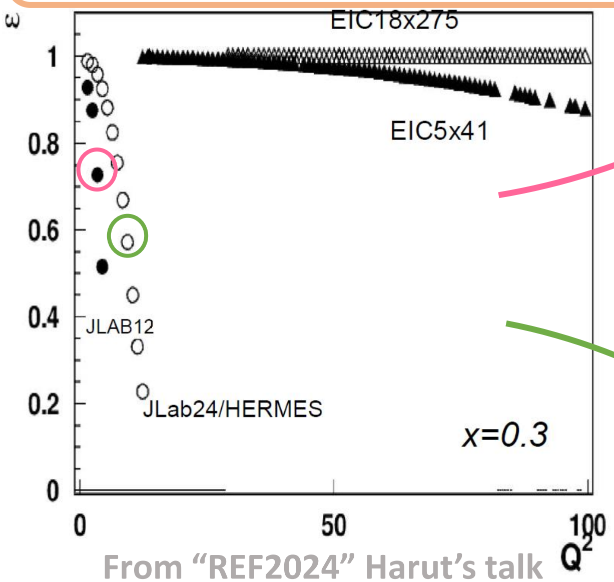
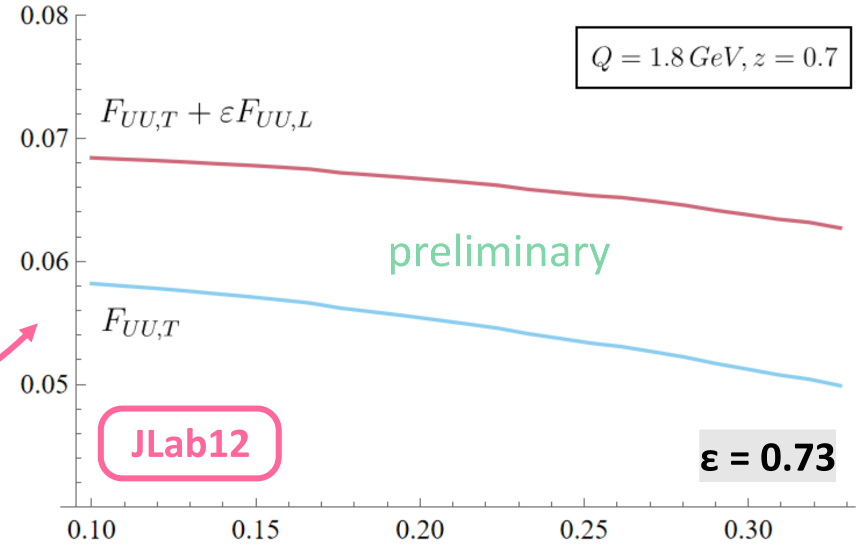
The contribution of longitudinal photons

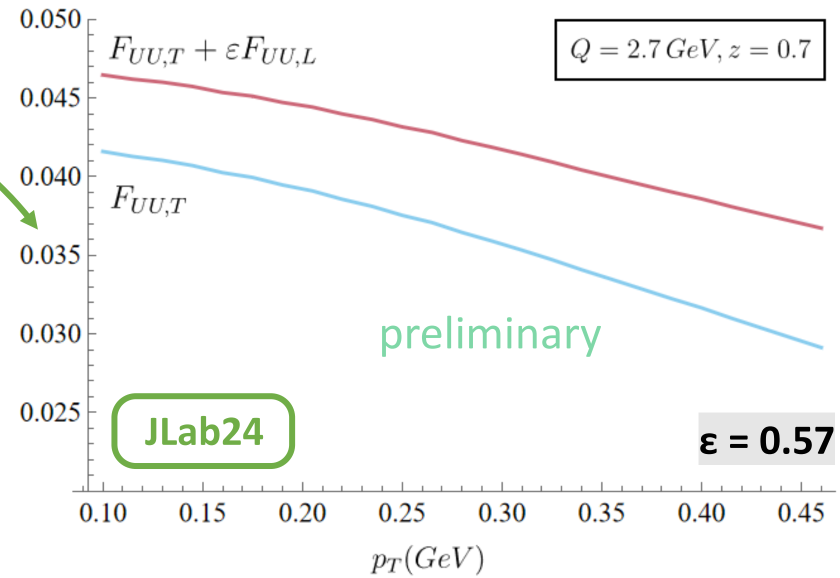
What about the contribution of longitudinal photons?

ε values for different experiments



From "REF2024" Harut's talk



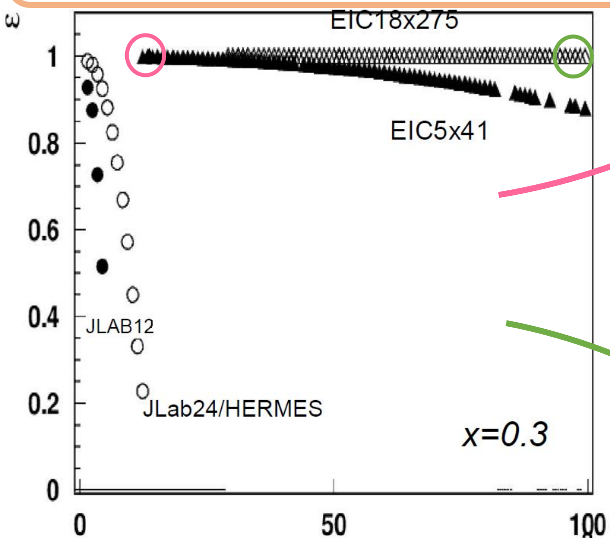


At low energies, the contribution of the $F_{UU,L}$ structure function is not negligible!

The contribution of longitudinal photons

What about the contribution of **longitudinal photons**?

ε values for different experiments



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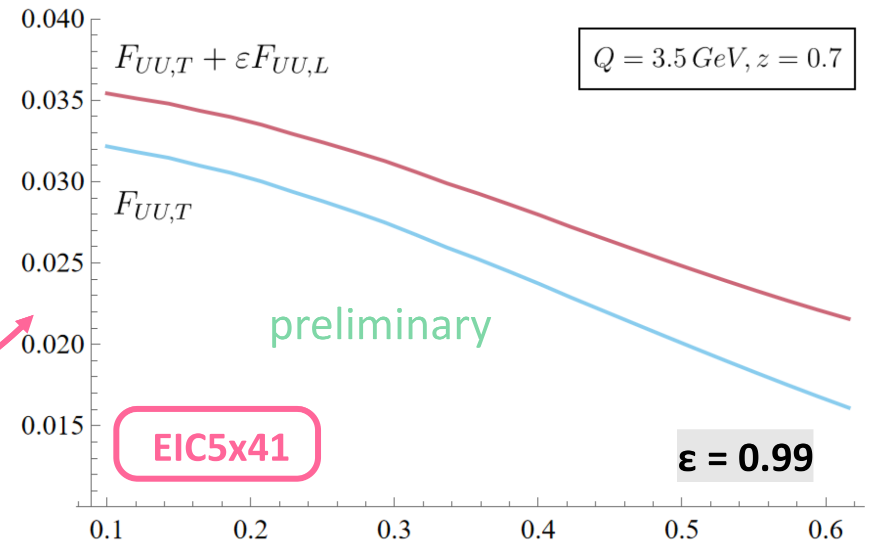

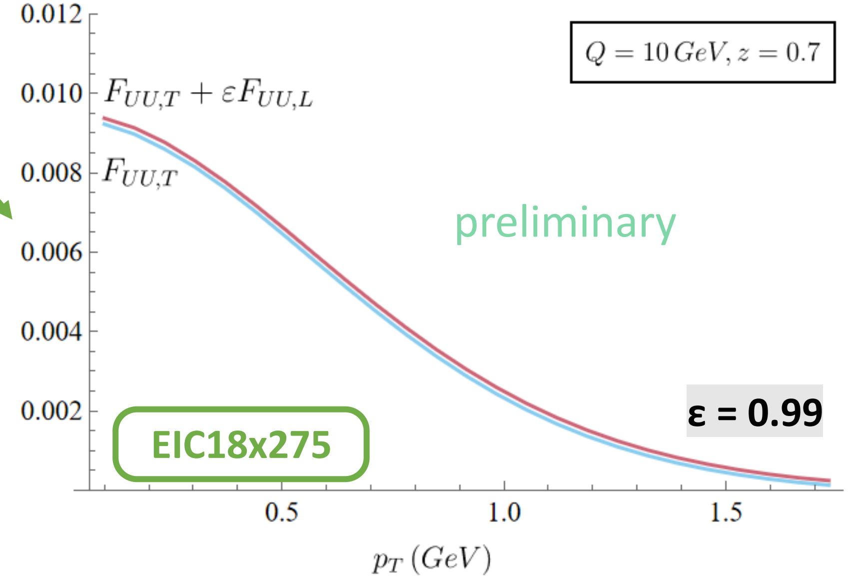

As Q grows, the contribution of $F_{UU,L}$ becomes less important, even if ε is very close to 1

Conclusions

- The TMD-with-KPCs factorization theorem [AV, 2307.13054v2] has been tested
  - The angular distributions of Drell-Yan leptons can be satisfactorily described
  - It gives a nice description of the Lam-Tung relation

- The subleading $F_{UU,T}$ and $F_{UU,L}$ SIDIS structure functions have been computed using it
  - The SIDIS cross-section grows when including KPCs
  - The $F_{UU,L}$ contribution is not negligible at low energies like $2 - 5$ GeV

Our TMD distributions need an update… We need to perform a new fit including KPCs!

Conclusions

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Thank you for your attention! ☺

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