## Partonic structure of hadrons from low to high transverse-momentum



19-23 May 2025 Jefferson Lab

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# Outline

- •Discussion of unpolarized observables in SIDIS @ LP & @ NLP
- •In particular the *NLP* observables (a subtle discussion)
  - $F_{UU,L}$  and  $R_{SIDIS}$  of interest to JLab TMD program 11 GeV, 22 GeV ...
  - $F_{UU}^{\cos\phi_h}$
- Factorization frameworks
  - Collinear  $P_T$
  - TMD
- 1. Challange of Factorization at next to leading power (NLP) in the hard scale 2.  $\implies$  *Matching* "low" (TMD) to "high" (collinear) transverse momentum spectrum  $q_T$  or  $P_{hT}$ 3. Discussion ... power counting

# Importance of NLP Factorization & TMDs

 $\odot$  NLP/SLP TMDs as sizable as leading-power in situations where Q not that large... e.g. the kinematics of fixed-target experiments

•Of interest to probe physics of quark-gluon-quark correlations, only recently explored beyond  $\alpha_{\rm c}$ 

•Experimental info SIDIS on effects related to subleading TMDs available DESY/Zeus, Fermi-LAB, HERMES, COMPASS, JLab •Opportunity for EIC with its large kinematical coverage, for 11 GeV SoLID TMD program & for further groundbreaking progress in this area

•NB: Iff factorization can be established beyond "tree level" @ next to leading order -Global analysis in terms of NLP TMDs

- •Importance of NLP "TMD-like" observables underscored while suppressed by  $(M/Q)^n$  wrt LP observables
- •Their understanding is required for a complete description of "benchmark processes" SIDIS, DY &  $e^+e^-$ ...





# **Challenges of NLPTMDs**

Treatments in the literature are mostly limited to a tree-level formalism until recently

\*First studies beyond tree level : "Matches & Mis-matches" Bacchetta et al. JHEP 2008, Chen et al. PLB 2017

### More recently

A.P. Chen, J.P. Ma, PLB (2017) Bacchetta et al. PLB 2019 MIT group, Gao, Ebert, Stewart JHEP 2022 Gamberg, Kang, Shao, Terry, Zhao arXiv: e-Print:221.13209 Vladimirov, Rodini, Scimemi, Moos, JHEP 2021, 2022, JHEP 2023, PRD 2024 Balitsky 2023 rapidity TMD evolution

• In arXiv: e-Print:221.13209 present a systematic procedure for stress testing TMD factorization for DY & SIDIS at NLP using CSS formalism which addresses disagreements in the literature

NLP TMD observables challenging in comparison to the current state-of-the-art of LP observables



# •**The observable** $F_{UU,L}$ & $R_{SIDIS} = \frac{\sigma_L}{\sigma_T} \sim \frac{F_{UU,L}}{F_{UU,T}}$

Feynman "Photon-Hadron Phys." 1972, Ravndal, PLB 1973

• The observable  $\langle \cos \phi \rangle$  Georgi & Cahn, PRL 1978, PLB 1978 Critique of the perturbative QCD calculation of azimuthal dependence in leptoproduction emphasize importance intrinsic  $k_T$  the early days/birth of TMD physics

# $R_{SIDIS}$ Is there a TMD formalism?

•The best known of these (& relatively not well understood) the ratio of longitudinal and transverse cross section (SIDIS) longitudinal and transverse photon: power supressed  $(M/Q)^2$ 

Feynman 1972 "Photon Hadron Interactions" & Ravndal PLB 1973:

$$R = \frac{\sigma_L}{\sigma_T} = \frac{4\left(m^2 + \langle p_\perp^2 \rangle\right)}{Q^2} \implies \frac{F_{UU,L}}{F_{UU,T}}$$

where  $\langle p_{\perp}^2 \rangle$  is the intrinsic parton transverse momentum ... often assumed that  $F_{UU,L}$  is negligible at low transverse momentum details see Cahn 1989 PRD



# 20<sup>th</sup> century interpretation from collinear inclusive DIS physics

Recall inclusive C.S. expressed in terms of  $\sigma_T$  and  $\sigma_L$  or structure functions  $F_L(F_2 \& F_1) \& F_T(F_1)$ ,

i.e via absorption of transverse and longitudinal photons

$$R = \frac{\sigma_T}{\sigma_L} = \frac{F_L}{F_T} = \frac{1}{2xF_1} \left\{ F_2 \left( 1 + \frac{4x^2 M^2}{Q^2} \right) - 2xF_1 \right\}$$

• Zero in the scaling limit  $\lim_{Q^2 \to \infty} R \approx \frac{4x^2 M^2}{Q^2} \to 0$ , • However pQCD result  $\{F_2 - 2xF_1\} \sim \frac{\alpha_s(Q)}{2\pi} C_2(F) x$ 

> Comparison of the values of  $R(x, Q^2)$ for hydrogen from the JLab exp. (E99-118) to results of other exps.





### 21<sup>st</sup> century interpretation from DIS to SIDIS)

$$\frac{d\sigma}{dx\,dy\,dz\,d\phi_S\,d\phi_h\,dP_{h\perp}^2} = \frac{\alpha^2}{xQ^2}\,\frac{y}{2\left(1-\varepsilon\right)}\,\left\{F\right\}$$

TMD observable for ??

- $F_{UU,T} = C[f_1 D_1] ,$
- $F_{UU,L} = C[.?.]$

$$\mathcal{C} ig[wfDig] = \sum_a x e_a^2 \int d^2 oldsymbol{p}_T d^2 oldsymbol{k}_T \, \delta^{(2)} ig(oldsymbol{p}_T - oldsymbol{k}_T ol$$

Mulders & Tangerman 1995 Bacchetta et al. JHEP 2007

 $R = \frac{\sigma_L}{\sigma_T} \to \frac{F_{UU,L}}{F_{UU,T}}$ 

 $F_{UU,T} + \varepsilon F_{UU,L} + \ldots$ 

 $\boldsymbol{k}_T + \boldsymbol{q}_T \end{pmatrix} w(\boldsymbol{p}_T, \boldsymbol{k}_T) f^a(x, p_T^2) D^a(z, k_T^2)$ 

## Context TMD Correlator at tree level "twist 4"

K. Goeke, A. Metz, M. Schlegel PLB 2005

**NNLP**:

 $\Phi^{[\gamma^{-}]} = \frac{M^2}{(P^+)^2} \left[ f_3(x, \vec{k}_T^2) + \frac{\varepsilon_T^{ij} k_{Ti} S_{Tj}}{M} f_{3T}^{\perp}(x, \vec{k}_T^2) \right]$ Correlator at tree level @ "twist" 4 previously of academic interest factorization is at best unexplored

*LP & NLP* :  $\Phi^{[\gamma^{+}]} = f_1(x, \vec{k}_T^2) - \frac{\varepsilon_T^{ij} k_{Ti} S_{Tj}}{M} f_{1T}^{\perp}(x, \vec{k}_T^2), \leq M$  $\Phi^{[\gamma^i]} = \frac{M}{P^+} \left[ \frac{k_T^i}{M} \left( f^\perp(x, \vec{k}_T^2) - \frac{\varepsilon_T^{jk} k_{Tj} S_{Tk}}{M} f_T^{\perp\prime}(x, \vec{k}_T^2) \right)^+ \cdots \right]$ 



#### Subleading Quark TMDPDFs

		Quark Chirality		
		Chiral Even	Chiral Odd	
Nucleon Polarization	U	$f^{\perp}\!, g^{\perp}$	$e \;,\; h$	
	L	$f_L^{\perp}, \ g_L^{\perp}$	$e_L,\ h_L$	
	т	$f_T^{},\;f_T^{\perp}\!\!,\;g_T^{},\;g_T^{\perp}$	$e_T^{},\;e_T^{\perp}\!,h_T^{}$	





# Context TMD Correlator at tree level "twist 4"

NNLP: some discussion in Bacchetta et al. Matches and mismatches JHEP 2008 & recent discussion w/ M. Cerruti



**NNLP**:



#### Subleading Quark TMDPDFs

Quark		Quark	Chirality	
		Chiral Even	Chiral Odd	
zation	U	$f^{\perp}\!\!,g^{\perp}$	$e \;,\; h$	
on Polari	L	$f_L^{\perp}, \ g_L^{\perp}$	$e_L,\ h_L$	
Nucle	т	$f_T^{},\;f_T^{\perp}\!\!,\;g_T^{},\;g_T^{}$	$e_T^{},\;e_T^{\perp}\!,h_T^{}$	

+ ...







### $R_{sidis}$ esitmate sizable contribution up to 20%

$$\frac{d\sigma}{dx\,dy\,dz\,d\phi_S\,d\phi_h\,dP_{h\perp}^2} = \frac{\alpha^2}{xQ^2}\,\frac{y}{2\,(1-\varepsilon)} \quad \left\{F_{UU,T} + \varepsilon\,F_{UU,L}\right\}$$

 $\cdot \epsilon$  ratio of longitudinal and transverse photon flux ...

$$\varepsilon = \frac{1 - y - \frac{1}{4}\gamma^2 y^2}{1 - y + \frac{1}{2}y^2 + \frac{1}{4}\gamma^2 y^2}, \qquad \gamma^2 \equiv \frac{4M^2 x^2}{Q^2}$$

• Findings demonstrate  $F_{UU,L}$  can't be ignored substantial & essential for an accurate interpretation of  $F_{UU,T}$ which is associated with leading twist TMDs.

## •What is the physics here?!

**Fig. 18** Estimate of  $R_{\text{SIDIS}} = F_{UU,L}/F_{UU,T}$  versus the hadron transverse momentum  $P_T(P_{hT})$  at fixed values of x and z and for different values of  $Q^2$ , compatible with JLab22 kinematics, using MAP22 TMD analysis [134]



## •What is the physics here?

## Power behavior $F_{UU,L}$ & $F_{UU,T}$



# $F_{UU,L} = rac{4M^2}{Q^2} \mathcal{C} \left[ rac{p_T^2}{M^2} f_1 D_1 ight]$ Bacchetta & Cerruti MAP

## SIDIS "benchmark processes" TMD Factorization & $P_{\perp}$ Collinear Factorization

- <u>TMD</u>: applicable  $\Lambda_{QCD} \sim P_{h\perp} \ll Q$  <u>Collinear</u>: applicable  $P_{h\perp} \sim Q \gg \Lambda_{QCD}$
- $P_{h\perp} \sim \mathbf{k}_T$  or  $\mathbf{p}_T$  intrinsic transverse momentum partons CS described via TMDs
- $P_{h\perp} \gg \mathbf{k}_T$  or  $\mathbf{p}_T$  generated transverse momentum in the final state as perturbative radiation & non-perturbative structure is given by collinear pdfs & FFs



# Can we learn about power corrections $F_{UU,L}$ of $R_{SIDIS}$ @ large $q_T \approx P_{hT}/z$

- Factorization & Matching unpolairzed Colins/Soper/Sterman NPB 1985, Collins Collins, Gamberg, Prokudin, Rogers, Sato, Wang 2016 PRD
- Bacchetta, Boer, Diehl, Mulders JHEP (2008) Ji, Qiu, Vogelsang Yuan PRL. 97, (2006); Phys. Rev. D 73 (2006)



**Fixed Order** Collinear Factorization

#### • Cross section in terms of different "regions"

- W valid for  $q_T \sim k_T \ll Q$  TMD factorization
- *FO* valid for  $k_T \ll p_T \sim Q$  Collinear factorization
- AY subtracts d.c. & in principle,

 $AY \rightarrow W, p_T \rightarrow \infty \text{ and } AY \rightarrow FO, p_T \rightarrow 0$ 

 $\frac{d\sigma(m \leq q_T \leq Q, Q)}{dydq^2 dp_T^2} = W(p_T, Q) + FO(p_T, Q) - AY(p_T, Q) + O\left(\frac{m}{Q}\right)^c$ 







# "Matching" is much more nuanced @ NLP & NNLP Opportunity to learn about low $q_T \sim M$ from large $q_T \sim Q$ Power counting/behavior $F_{UU,L}$ & $F_{UU,T}$ $F_{UU,T} = \mathcal{C}[f_1 D_1]$ $q_T \ll Q$ $F_{UU,L} = \mathcal{O}\left(\frac{M^2}{Q^2}, \frac{q_T^2}{Q^2}\right)$ $F_{UU,T} \sim \frac{1}{q_T^2} \alpha_s \mathcal{F}[f_1 D_1] ,$

Large

Low

 $q_T \gg M$   $F_{UU,L} \sim \frac{1}{Q^2} \alpha_s \mathcal{F}[f_1 D_1]$ 

# For large $q_T \sim P_{hT}/z$ well established for Leading POWER

• large  $P_T$ ,  $F_{UU,L} = 2 F_{UU}^{\cos 2\phi_h}$  — Bacchetta et al. JHEP 2008 "Matches & Mis-matches": in principle hard gluon raditation – "collinear  $P_T$  factorization applies CSS 1985 Catani et al. 1997–2015, Nadolsky, Vogelsang Koike NPB 2005... many others

$$\begin{split} F_{UU,T} &= \frac{1}{Q^2} \frac{\alpha_s}{(2\pi z)^2} \sum_a x e_a^2 \int_x^1 \frac{d\hat{x}}{\hat{x}} \int_z^1 \frac{d\hat{z}}{\hat{z}} \,\delta\!\left(\frac{q_T^2}{Q^2} - \frac{(1-\hat{x})(1-\hat{z})}{\hat{x}\hat{z}}\right) \\ & \times \left[f_1^a\!\left(\frac{x}{\hat{x}}\right) D_1^a\!\left(\frac{z}{\hat{z}}\right) C_{UU,T}^{(\gamma^* q \to qg)} + f_1^a\!\left(\frac{x}{\hat{x}}\right) D_1^g\!\left(\frac{z}{\hat{z}}\right) C_{UU,T}^{(\gamma^* q \to gq)} + f_1^g\!\left(\frac{x}{\hat{x}}\right) D_1^a\!\left(\frac{z}{\hat{z}}\right) C_{UU,T}^{(\gamma^* g \to q\bar{q})} \right] \end{split}$$







(b)

$$-\frac{(1-\hat{x})(1-\hat{z})}{\hat{x}\hat{z}}
ight)$$





$$p_b$$
 (b')



# $\frac{d\sigma}{dx\,dy\,dz\,d\phi_S\,d\phi_h\,dP_{h\perp}^2} = \frac{\alpha^2}{xQ^2}\,\frac{y}{2\left(1-\varepsilon\right)} \left\{F_{UU,T} + \varepsilon F_{UU,L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h} + \varepsilon\cos(2\phi_h)\,F_{UU}^{\cos\,2\phi_h}\right\}$















Opportunity  $R_{SIDIS}$  & large  $P_T$ 

$$F_{UU,T} = \frac{1}{Q^2} \frac{\alpha_s}{(2\pi z)^2} \sum_a x e_a^2 \int_x^1 \frac{d\hat{x}}{\hat{x}} \int_z^1 \frac{d\hat{z}}{\hat{z}} \,\delta\left(\frac{q_T^2}{Q^2} - \frac{Q^2}{2}\right) \\ \times \left[f_1^a\left(\frac{x}{\hat{x}}\right) D_1^a\left(\frac{z}{\hat{z}}\right) C_{UU,T}^{(\gamma^* q \to qg)}\right]$$

Asymptotic region



$$F_{UU,L} = \left(\frac{2}{Q^2}\right) \frac{\alpha_s}{2\pi^2 z^2} \sum_a x e_a^2 \left[ f_1^a(x) D_1^a(z) L\left(\frac{Q^2}{q_T^2}\right) \right] \left[ f_1^a(x) D_1^a(z) L\left(\frac{Q^2}{q_T^2}\right) \right] \left[ f_1^a(x) D_1^a(z) + \frac{Q^2}{q_T^2} \right] \left[ f_1^a(x) D_1^a(z) + \frac$$

$$L\left(\frac{Q^2}{q_T^2}\right) \equiv C_F \left[2\ln\left(\frac{Q^2}{q_T^2}\right)\right]$$

Power behavior  $F_{UU,L}$  &  $F_{UU,T}$  at  $M \ll q_T \ll Q$ 

 $\left(\frac{(1-\hat{x})(1-\hat{z})}{\hat{x}\,\hat{z}}\right)$ 

 $+ f_1^a \left(\frac{x}{\hat{x}}\right) D_1^g \left(\frac{z}{\hat{z}}\right) C_{UU,T}^{(\gamma^* q \to gq)} + f_1^g \left(\frac{x}{\hat{x}}\right) D_1^a \left(\frac{z}{\hat{z}}\right) C_{UU,T}^{(\gamma^* g \to q\bar{q})} \right]$ 



 $+\left(P_{qq}\otimes f_1^a+P_{qg}\otimes f_1^g\right)(x)D_1^a(z)\right]$  $\left(\frac{2}{2}\right) + f_1^a(x) \left(D_1^a \otimes P_{qq}'' + D_1^g \otimes P_{gq}''\right)(z)$  $+ \left(P_{qq}'' \otimes f_1^a + P_{qg}'' \otimes f_1^g\right)(x) D_1^a(z)$ 

### Large to intermidate to low $q_T$ regions Colliner SIDIS vs. truncated moments

$$\int_{P_{hT}^{2} \min/z^{2}}^{q_{T}^{2} \max} F(x, z, Q^{2}, P_{hT})$$

 $F_{UU,L}$  truncated moment converges " $P_T$  integrable"

Small power corrections ?
Small <u>TMD contribution</u> ?

 $F_{UU,T}$  truncated moment as expected diverges



### Intermidate region to low $q_T$ region

### • $F_{UU,T}(x, z, q_T, Q)$ & $F_{UU,L}(x, z, q_T, Q)$

![](_page_19_Figure_2.jpeg)

![](_page_19_Picture_3.jpeg)

$$R_{SIDIS} & \& \sigma_{L} \sim F_{UU,L} \text{ at large } p_{T}$$

$$\frac{d\sigma}{dx \, dy \, dz \, dq_{T}^{2} \, d\varphi} = \frac{\pi \alpha^{2} yz}{4Q^{2}} \left[ \underbrace{\sinh^{2} \vartheta F_{UU,L}}_{\sigma_{1}} - \frac{1}{2} (2 + \sinh^{2} \vartheta) F_{UU,T}}_{\sigma_{0}} - \underbrace{\sinh 2 \vartheta F_{UU}^{\cos \varphi}}_{\sigma_{1}} \cos \varphi + \frac{1}{2} \sinh^{2} \vartheta F_{UU}^{\cos 2\varphi}}_{\sigma_{2}} \cos \varphi \right]$$
e.g.
$$F_{UU,T} = \frac{1}{Q^{2}} \frac{\alpha_{s}}{(2\pi z)^{2}} \sum_{a} x e_{a}^{2} \int_{x}^{1} \frac{d\hat{x}}{\hat{x}} \int_{z}^{1} \frac{d\hat{z}}{\hat{z}} \, \delta\left(\frac{q_{T}^{2}}{Q^{2}} - \frac{(1 - \hat{x})(1 - \hat{z})}{\hat{x}\hat{z}}\right) \times \left[f_{1}^{a}\left(\frac{x}{\hat{x}}\right) D_{1}^{a}\left(\frac{z}{\hat{z}}\right) C_{UU,T}^{(\gamma^{*}q \to qq)} + f_{1}^{g}\left(\frac{x}{\hat{x}}\right) D_{1}^{a}\left(\frac{z}{\hat{z}}\right) C_{UU,T}^{(\gamma^{*}g \to q\bar{q})} + \int_{1}^{g} \left(\frac{x}{\hat{x}}\right) D_{1}^{a}\left(\frac{z}{\hat{z}}\right) C_{UU,T}^{(\gamma^{*}g \to q\bar{q})} = \frac{F_{UU,L}}{F_{UU,T}}$$

### Nb:SIDIS truncated moments

 $\int_{-\infty}^{q_T^2 \max} F(x, z, Q^2, P_{hT})$  $\mathbf{J}_{P_{hT\min}^2/z^2}$ 

Nb: Bands are generated by computing the observable on subset of JAM replicas (from recent W+charm analysis) & taking the mean  $\pm$  standard deviation

![](_page_20_Figure_5.jpeg)

![](_page_20_Figure_6.jpeg)

$$R_{SIDIS} & \& \sigma_{L} \sim F_{UU,L} \text{ at large } p_{T}$$

$$\frac{d\sigma}{dx \, dy \, dz \, dq_{T}^{2} \, d\varphi} = \frac{\pi \alpha^{2} yz}{4Q^{2}} \left[ \underbrace{\sinh^{2} \vartheta F_{UU,L}}_{\text{on}} - \frac{1}{2} (2 + \sinh^{2} \vartheta) F_{UU,T}}_{\sigma_{0}} - \underbrace{\sinh 2 \vartheta F_{UU}^{\cos \varphi}}_{\sigma_{1}} \cos \varphi + \frac{1}{2} \sinh^{2} \vartheta F_{UU}^{\cos 2\varphi}}_{\sigma_{2}} \cos \varphi \right]$$
e.g.
$$F_{UU,T} = \frac{1}{Q^{2}} \frac{\alpha_{s}}{(2\pi z)^{2}} \sum_{a} x e_{a}^{2} \int_{x}^{1} \frac{d\hat{x}}{\hat{x}} \int_{z}^{1} \frac{d\hat{z}}{\hat{z}} \, \delta\left(\frac{q_{T}^{2}}{Q^{2}} - \frac{(1 - \hat{x})(1 - \hat{z})}{\hat{x}\hat{z}}\right) \times \left[f_{1}^{a}\left(\frac{x}{\hat{x}}\right) D_{1}^{a}\left(\frac{z}{\hat{z}}\right) C_{UU,T}^{(\gamma^{*}q \rightarrow qq)} + f_{1}^{a}\left(\frac{x}{\hat{x}}\right) D_{1}^{a}\left(\frac{z}{\hat{z}}\right) C_{UU,T}^{(\gamma^{*}g \rightarrow qq)} + f_{1}^{g}\left(\frac{x}{\hat{x}}\right) D_{1}^{a}\left(\frac{z}{\hat{z}}\right) C_{UU,T}^{(\gamma^{*}g \rightarrow qq)} = \frac{F_{UU,L}}{F_{UU,T}}$$

$$Ilab \text{ II GeV} \quad x = 0.$$

Comments:

 $\bullet {\rm Truncated}$  moment is sig. larger than  $P_T$ integrated SIDIS—indication of •power corrections ? • TMD contribution ?

![](_page_21_Figure_4.jpeg)

![](_page_21_Figure_5.jpeg)

 $R_{SIDIS} \& \sigma_L \sim F_{UU,L}$  at large  $p_T$ 

#### Jlab 11 GeV x = 0.3 & z = 0.5

![](_page_22_Figure_2.jpeg)

#### Jlab 22 GeV x = 0.3 & z = 0.5

![](_page_22_Figure_4.jpeg)

![](_page_22_Picture_5.jpeg)

- gluon contribute large uncertainty @ hi-x (see delta function)
- $g \rightarrow 0$  ie gluon PDF set to zero
- Residis can be useful to pin down the g @ lg x

![](_page_23_Figure_3.jpeg)

 $R_{SIDIS} \& \sigma_L \sim F_{UUL}$  at large  $p_T$ 

 $\mathbf{FO} = \sum_{q} e_q^2 \int_{\frac{q_T^2}{\alpha^2} \frac{xz}{1-x} + x}^{1} \frac{d\xi}{\xi - x} H(\xi) \ \mathbf{f}_q(\xi, \mu) \ \mathbf{d}_q(\zeta(\xi), \mu) + O(\alpha_S^2) + O(m^2/q^2)$ 

![](_page_23_Figure_6.jpeg)

# The observable $\langle \cos \phi \rangle$

$$\frac{d\sigma}{dx\,dy\,dz\,d\phi_S\,d\phi_h\,dP_{h\perp}^2} = \frac{\alpha^2}{xQ^2}\,\frac{y}{2\,(1-\varepsilon)} \quad \left\{F_{UU,T} + \varepsilon\,F_{UU,L} + \sqrt{\frac{1}{2}}\right\}$$

 $\int d\sigma^{(1)} \cos \phi = \int d^2 P_T \cos \phi \frac{d\sigma}{dx_H \, dy \, dz_H \, d^2 P_T}$ 

No assumption of mechanism

![](_page_24_Picture_5.jpeg)

**SIDIS Kinematics dictionary**  $Q^2 = -q^2, \quad \mathbf{P}_T = \mathbf{P}_{2T}, \quad \phi,$ 

$$x_H = rac{Q^2}{2P_1 \cdot q}, \quad y = rac{P_1 \cdot q}{P_1 \cdot k_1}, \quad z_H =$$

and the parton variables

$$x = \frac{x_H}{\xi} = \frac{Q^2}{2p_1 \cdot q}, \quad z = \frac{z_H}{\xi'} = \frac{p_1 \cdot p_2}{p_1 \cdot q}.$$

![](_page_24_Picture_11.jpeg)

![](_page_24_Picture_12.jpeg)

## TMDs (a) "twist-3" NLP

## The beginning of TMD physics? $\langle \cos \phi \rangle$

### • <u>Georgi Politzer, PRL 1978</u> "Measurement $\langle \cos \phi \rangle$ provides clean test of predictions of PQCD

~12-15% ... clean test of QCD "... since such effects would not arise as a result of limited transverse momentum associated with confined quarks..."

 Cahn, PLB 1978, (& earlier paper by Ravndal, PLB 1972) Critique QCD calculation of azimuthal dependence emphasize importance intrinsic  $k_T$  ...

"...Results cast doubt on the utility of such azimuthal asymmetry as a clean test of quantum chromodynamics " (i.e. of G&P 78)

![](_page_25_Figure_8.jpeg)

# **Clean tests of QCD?**

# PHYSICAL REVIEW LETTERS

Volume 40

#### 2 JANUARY 1978

#### Clean Tests of Quantum Chromodynamics in $\mu p$ Scattering

Howard Georgi

Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138

and

H. David Politzer California Institute of Technology, Pasadena, California 91125 (Received 25 October 1977)

Hard gluon bremsstrahlung in  $\mu p$  scattering produces final-state hadrons with a large component of momentum transverse to the virtual-photon direction. Quantum chromodynamics can be used to predict not only the absolute value of the transverse momentum, but also its angular distribution relative to the muon scattering plane. The angular correlations should be insensitive to nonperturbative effects.

![](_page_26_Figure_10.jpeg)

![](_page_26_Figure_12.jpeg)

FIG. 1. Diagrams contributing to semi-inclusive  $\mu$ -parton scattering to first order in  $\alpha_s$ . k (p) denotes muon (parton) momentum. The wavy line is a virtual photon. The curly line is a gluon.

Pert. QCD  $\alpha_s = g^2/4\pi$ 

![](_page_26_Picture_15.jpeg)

![](_page_26_Figure_16.jpeg)

![](_page_26_Picture_17.jpeg)

![](_page_27_Picture_0.jpeg)

Volume 78B, number 2,3

PHYSICS LETTERS

#### **AZIMUTHAL DEPENDENCE IN LEPTOPRODUCTION: A SIMPLE PARTON MODEL CALCULATION**<sup>☆</sup>

#### Robert N. CAHN

Department of Physics, University of Michigan, Ann Arbor, MI 48109, USA

Received 5 June 1978

Semi-inclusive leptoproduction,  $\varrho + p \rightarrow \varrho' + h + X$ , is considered in the naive parton model. The scattered parton shows an azimuthal asymmetry about the momentum transfer direction. Simple derivations for the effects in ep, vp and  $\overline{v}p$  scattering are given. Reduction of the asymmetry due to fragmentation of partons into hadrons is estimated. The results cast doubt on the utility of such azimuthal asymmetry as a clean test of quantum chromodynamics.

![](_page_27_Figure_8.jpeg)

![](_page_27_Picture_9.jpeg)

25 September 1978

### Simple parton model argument allowing for transverse momentum in Mandelstam variables...

$$\sigma_{ep} \propto \hat{s}^{2} + \hat{u}^{2} \propto \left[1 - \frac{2p_{\perp}}{Q}\sqrt{1-y}\cos\phi\right]^{2} + (1-y)^{2}\left[1 - \frac{2p_{\perp}}{Q\sqrt{1-y}}\cos\phi\right]^{2}$$

$$NLP! \quad \frac{p_{\perp}}{Q}$$

$$\langle\cos\phi\rangle_{ep} = -\left[\frac{2p_{\perp}}{Q}\right]\frac{(2-y)\sqrt{1-y}}{1+(1-y)^{2}}$$

![](_page_27_Picture_13.jpeg)

$$\frac{d\sigma}{dx\,dy\,dz\,d\phi_S\,d\phi_h\,dP_{h\perp}^2} = \frac{\alpha^2}{xQ^2}\,\frac{y}{2\,(1-\varepsilon)} \quad \left\{F_{UU,T} + \varepsilon\,F_{UU,L} + \frac{\omega^2}{2\,(1-\varepsilon)}\right\}$$

![](_page_28_Picture_2.jpeg)

Preprints: JLAB-THY-23-3780, LA-UR-21-20798, MIT-CTP/5386

#### TMD Handbook

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TMD Handbook

#### **10 - Subleading TMDs**

L. Gamberg, A. Metz, I. Stewart

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# Cahn intrinsic $k_T$

![](_page_28_Picture_11.jpeg)

### Subleading Quark TMDPDFs

		Quark Chirality		
		Chiral Even	Chiral Odd	
zation	U	$f^{\perp}\!\!,g^{\perp}$	$e \;,\; h$	
on Polari	L	$f_L^\perp,\ g_L^\perp$	$e_L,\ h_L$	
Nucle	т	$f_T^{},\ f_T^{},\ g_T^{},\ g_T^{},\ g_T^{}$	$e_T^{},\;e_T^{\perp}\!\!,h_T^{},h_T^{\perp}$	

![](_page_28_Figure_14.jpeg)

# However again large $q_T$ angular modulation $\cos \phi_h$ effect & $\cos 2\phi_h$

 $F_{UU,T} = \frac{1}{Q^2} \frac{\alpha_s}{(2\pi z)^2} \sum_a x e_a^2 \int_x^1 \frac{d\hat{x}}{\hat{x}} \int_z^1 \frac{d\hat{z}}{\hat{z}} \,\delta\bigg(\frac{q_T^2}{Q^2} - \frac{1}{2} \frac{d\hat{z}}{\hat{z}}\bigg) \,\delta\bigg(\frac{q_T^2}{Q^2}$ 

![](_page_29_Picture_2.jpeg)

![](_page_29_Picture_3.jpeg)

$$rac{(1-\hat{x})(1-\hat{z})}{\hat{x}\hat{z}}
ight
angle$$

 $\times \left[ f_1^a \Big( \frac{x}{\hat{x}} \Big) D_1^a \Big( \frac{z}{\hat{z}} \Big) C_{UU,T}^{(\gamma^* q \to qg)} + f_1^a \Big( \frac{x}{\hat{x}} \Big) D_1^g \Big( \frac{z}{\hat{z}} \Big) C_{UU,T}^{(\gamma^* q \to gq)} + f_1^g \Big( \frac{x}{\hat{x}} \Big) D_1^a \Big( \frac{z}{\hat{z}} \Big) C_{UU,T}^{(\gamma^* g \to q\bar{q})} \right] \right]$ 

![](_page_30_Picture_0.jpeg)

![](_page_30_Figure_1.jpeg)

![](_page_30_Figure_2.jpeg)

# factorization theorems

• NLP factorization based on "TMD formalism"

—extend the tree level Amsterdam formalism and beyond leading order CSS, Ji Ma Yuan, Abyat Rogers, framework vs. SCET and Background Field Methods

- ・ "Intrinsic" NLP TMDs related thru EOM in terms "kinematic" & "dynamical"
- - Bacchetta, Boer, Diehl, Mulders JHEP 2008, Bacchetta et al. PLB 2019
- Report progress in this *necessary condition* NLP factorization (not yet sufficient)
- observables used to study intrinsic 3-D momentum structure of the nucleon—Opportunity for Jlab, EIC, COMPASS study of transverse momentum nucleon structure

# Thank You

# Summary

We explore NLP  $(M/Q)^n$  contributions in large  $q_T$  and TMD regions via power counting and

• Considier  $R_{SIDIS}$  & revisit "Cahn effect" & matching related to early importance intrinsic  $k_T$ 

• Consider RG consistency of matching to collinear factorization & issues of resummation

• In doing so, we provide the basis for performing global analysis & phenomenology of one the earliest

![](_page_31_Picture_17.jpeg)

![](_page_31_Picture_18.jpeg)