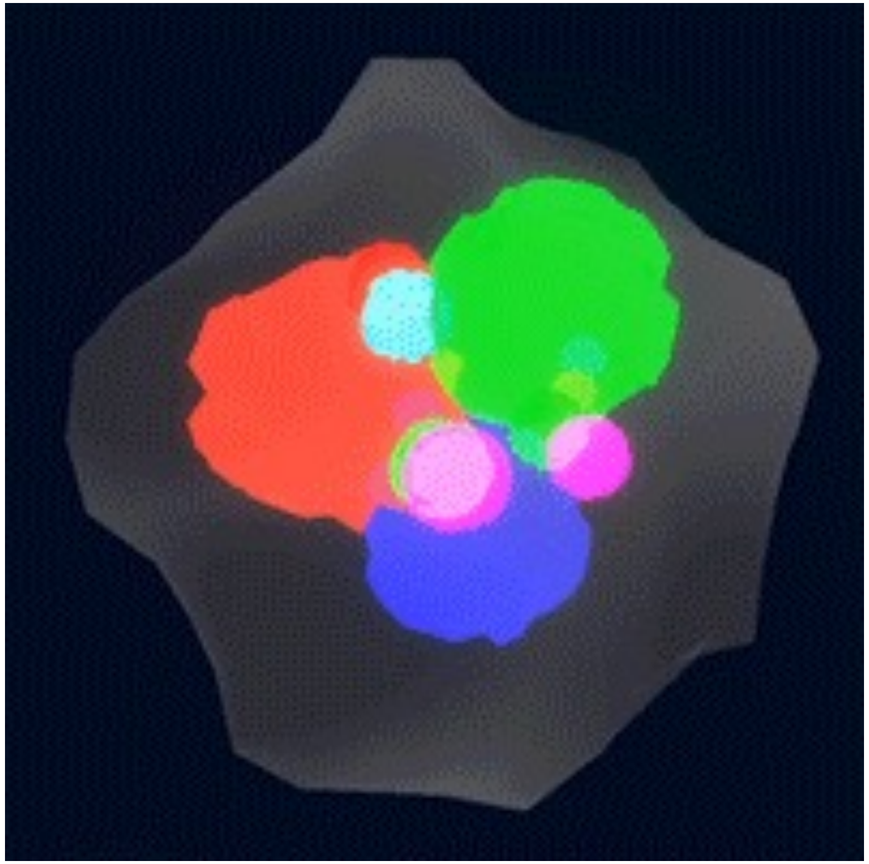
Probing the proton's correlated spatial structure through exclusive processes

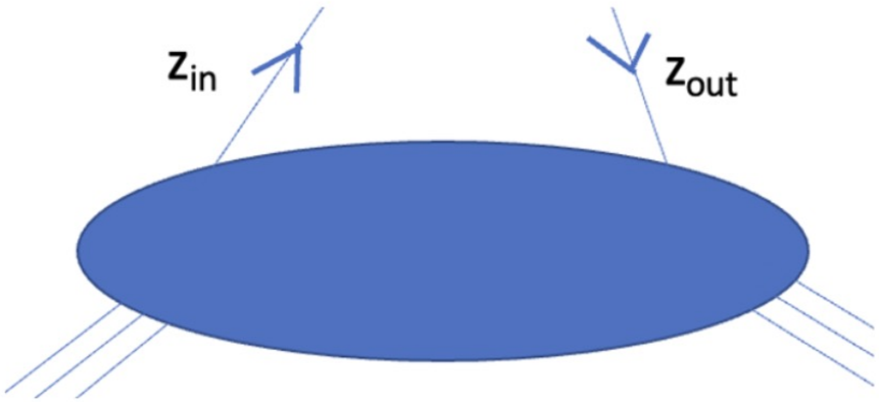


Zaki Panjsheeri and Simonetta Liuti
University of Virginia
QCD Evolution Workshop
19 May 2025

Outline

1. Review the connection between generalized parton distributions (GPDs) and the impact parameter-dependent parton distribution functions (IPPDFs)
2. Introduce the formalism of two-body densities and double GPDs
3. Identify the observables for two-body densities in ultraperipheral collisions (UPCs)

One-body densities



proton + quark → proton + quark
correlation function

$$W^{\Gamma}_{\Lambda,\Lambda'} = \int \frac{dz^-_{in} d^2 z_{in,T}}{(2\pi)^3} \frac{dz^-_{out} d^2 z_{out,T}}{(2\pi)^3} e^{i(k^+_{in} z^-_{in} - \mathbf{k}_{in,T}\cdot\mathbf{z}_{in,T})} e^{-i(k^+_{out} z^-_{out} - \mathbf{k}_{out,T}\cdot\mathbf{z}_{out,T})} \times \langle P', \Lambda' | \bar{\psi}(0, z^-_{out}, \mathbf{z}_{out,T}) \Gamma \psi(0, z^-_{in}, \mathbf{z}_{in,T}) | P, \Lambda\rangle$$

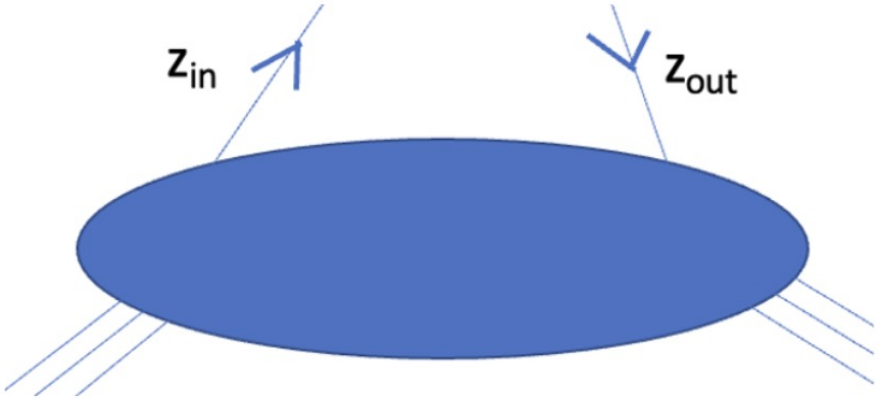
We can see immediately that the correlation function that defines GPDs exhibits in the longitudinal component, for zero skewness $\Delta^+ = k^+_{in} - k^+_{out} = 0$, the form of a momentum distribution:

$$W^{\Gamma}_{\Lambda,\Lambda'} = \int \frac{dz^-_{in} d^2 z_{in,T}}{(2\pi)^3} \frac{dz^-_{out} d^2 z_{out,T}}{(2\pi)^3} e^{ik^+_{in}(z^-_{in} - z^-_{out})} e^{i(-\mathbf{k}_{in,T}\cdot\mathbf{z}_{in,T})} e^{-i(-\mathbf{k}_{out,T}\cdot\mathbf{z}_{out,T})} \times \langle P', \Lambda' | \bar{\psi}(0, z^-_{out}, \mathbf{z}_{out,T}) \Gamma \psi(0, z^-_{in}, \mathbf{z}_{in,T}) | P, \Lambda\rangle$$

- Focusing here on the collinear limit in which the gauge link $U(z_{in}, z_{out})$ is unity
- Renormalization for GPDs is similar to that of PDFs due to collinearity

One-body densities

$$W^{\Gamma}_{\Lambda,\Lambda'} = \int \frac{dz^-_{in} d^2 z_{in,T}}{(2\pi)^3} \frac{dz^-_{out} d^2 z_{out,T}}{(2\pi)^3} e^{i(k^+_{in} z^-_{in} - \mathbf{k}_{in,T}\cdot \mathbf{z}_{in,T})} e^{-i(k^+_{out} z^-_{out} - \mathbf{k}_{out,T}\cdot \mathbf{z}_{out,T})}$$
$$\times \langle P', \Lambda' | \bar{\psi}(0, z^-_{out}, \mathbf{z}_{out,T}) \Gamma \psi(0, z^-_{in}, \mathbf{z}_{in,T}) | P, \Lambda \rangle$$



proton + quark → proton + quark correlation function

To get this into the form of a GPD, start by performing a change of variables:

$$z \equiv z_{in} - z_{out},\ b \equiv \frac{1}{2}(z_{in} + z_{out}) \implies z_{in} = b + \frac{z}{2}, z_{out} = b - \frac{z}{2}.$$

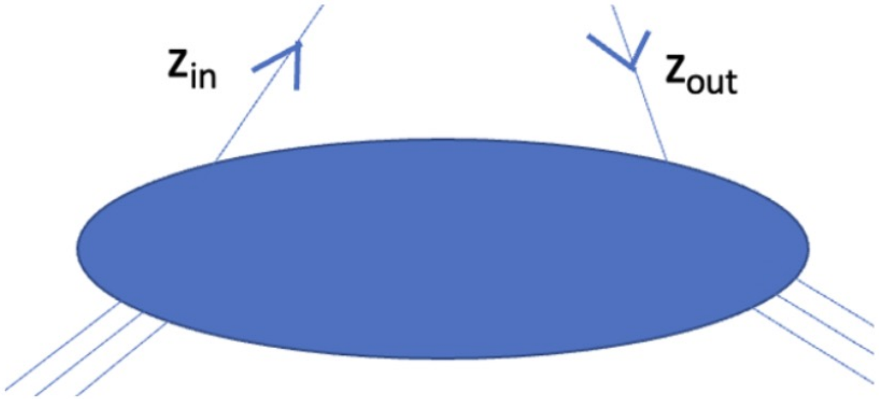
$$\Delta \equiv k_{in} - k_{out};\ k \equiv \frac{1}{2}(k_{in} + k_{out}),$$

$$W^{\Gamma}_{\Lambda,\Lambda'} = \int \frac{db^- d^2 b_T}{(2\pi)^3} \frac{dz^- d^2 z_T}{(2\pi)^3} e^{i(b^-\Delta^+ + z^- k^+ - \mathbf{b}_T\cdot\Delta_T - \mathbf{z}_T\cdot\mathbf{k}_T)}$$
$$\times \langle P', \Lambda' | \bar{\psi}(0, b^- - \frac{z^-}{2}, \mathbf{b}_T - \frac{\mathbf{z}_T}{2}) \Gamma \psi(0, b^- + \frac{z^-}{2}, \mathbf{b}_T + \frac{\mathbf{z}_T}{2}) | P, \Lambda \rangle$$

We can thus conclude that Δ is conjugate to b and k is conjugate to z

One-body densities

$$W^{\Gamma}_{\Lambda,\Lambda'} = \int \frac{dz^-_{in} d^2 z_{in,T}}{(2\pi)^3} \frac{dz^-_{out} d^2 z_{out,T}}{(2\pi)^3} e^{i(k^+_{in} z^-_{in} - \mathbf{k}_{in,T}\cdot \mathbf{z}_{in,T})} e^{-i(k^+_{out} z^-_{out} - \mathbf{k}_{out,T}\cdot \mathbf{z}_{out,T})} \\ \times \langle P', \Lambda' | \bar{\psi}(0, z^-_{out}, \mathbf{z}_{out,T}) \Gamma \psi(0, z^-_{in}, \mathbf{z}_{in,T}) | P, \Lambda \rangle$$



proton + quark → proton + quark correlation function

Applying translational invariance of the correlation function and integrating over k_T gives

$$\begin{aligned} W^{\Gamma}_{\Lambda,\Lambda'} &= \int \frac{dz^-}{2\pi} e^{iz^-(k^+ + \Delta^+/2)} \langle P', \Lambda' | \bar{\psi}(0,0,0) \Gamma \psi(0, z^-, 0) | P, \Lambda \rangle \\ &= \int \frac{dz^-}{2\pi} e^{iXp^+z^-} \langle P', \Lambda' | \bar{\psi}(0,0,0) \Gamma \psi(0, z^-, 0) | P, \Lambda \rangle \,, \end{aligned}$$

Defining skewness ζ and parton momentum fraction X

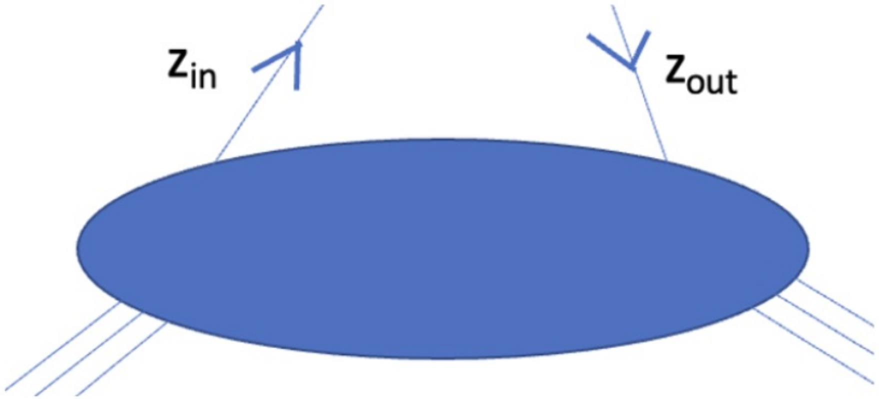
$$\zeta = \frac{\Delta^+}{p^+} \text{ and } X - \frac{\zeta}{2} = \frac{k^+}{p^+}$$

Identifying the non-local, one-body operator:

$$\mathcal{O}_1(z) = \overline{\psi}\,(0)\;\Gamma\,\psi\,(z)\,,$$

One-body densities

$$W^{\Gamma}_{\Lambda,\Lambda'} = \int \frac{dz^-_{in} d^2 z_{in,T}}{(2\pi)^3} \frac{dz^-_{out} d^2 z_{out,T}}{(2\pi)^3} e^{i(k^+_{in} z^-_{in} - \mathbf{k}_{in,T}\cdot \mathbf{z}_{in,T})} e^{-i(k^+_{out} z^-_{out} - \mathbf{k}_{out,T}\cdot \mathbf{z}_{out,T})}$$
$$\times \langle P', \Lambda' | \bar{\psi}(0, z^-_{out}, \mathbf{z}_{out,T}) \Gamma \psi(0, z^-_{in}, \mathbf{z}_{in,T}) | P, \Lambda \rangle$$



proton + quark → proton + quark
correlation function

We thus obtain a correlation function that can be parameterized in terms of GPDs, where we take $\Gamma = \gamma^+$ for the leading twist (twist-2) contribution:

$$\boxed{\begin{aligned} W^{\gamma^+}_{\Lambda,\Lambda'} &= \int \frac{dz^-}{2\pi} e^{iXp^+z^-} \langle P', \Lambda' | \bar{\psi}(0,0,0)\gamma^+ \psi(0, z^-, 0) | P, \Lambda \rangle \\ &= \frac{1}{2P^+}\Big[H^q(X,\zeta,t)\bar{u}(p',\Lambda')\gamma^+ u(p,\Lambda) + E^q(X,\zeta,t)\bar{u}(p',\Lambda')\frac{\sigma^{i+}\Delta_i}{2M} u(p,\Lambda)\Big] \end{aligned}}$$

One-body densities

- Connection to the coordinate space
- Project out the good components of the Dirac fields

$$H_q(X,0,t) = \int \frac{dz^-}{2\pi}\, e^{iXp^+z^-} \langle p-\Delta \mid \bar{\psi}_+(0)\, \psi_+(z^-) \mid p \rangle$$

- Insert the complete set of states

$$\begin{aligned} H_q(X,0,t) &= \int \frac{dz^-}{2\pi}\, e^{iXp^+z^-} \sum_{\mathcal{X}} \langle p-\Delta \mid \bar{\psi}_+(0) \mid \mathcal{X}\rangle\langle \mathcal{X} \mid \psi_+(z^-) \mid p\rangle\Big|_{\substack{z^+=0\\ \mathbf{z}_T=0}} \\ &= \int d^2\mathbf{k}_T^{\mathcal{X}}\, dk_{\mathcal{X}}^+\, \delta(k_{\mathcal{X}}^+ - (1-X)p^+)\, \langle p-\Delta \mid \bar{\psi}_+(0) \mid \mathcal{X}\rangle\, \langle \mathcal{X} \mid \psi_+(0) \mid p\rangle \end{aligned}$$

Momentum conservation at the proton-quark-X vertex: $\mathbf{k}_{T,in} = -\mathbf{k}_T^{\mathcal{X}}$

One-body densities

- We can now define the vertex functions,

$$\phi(k^+_{\mathcal{X}}, \mathbf{k}_{T,\mathcal{X}}) \;\to\; \phi(X, \mathbf{k}_{T,in}) = \langle \mathcal{X} \mid \psi_+(0) \mid p \rangle, \qquad \phi^*(X, \mathbf{k}_{T,out} = \mathbf{k}_{T,in} - \Delta) = \langle p' \mid \bar{\psi}_+(0) \mid \mathcal{X} \rangle$$

and see that the GPDs are non-diagonal in the transverse component:

$$H_q(X, 0, t) = \int d^2\mathbf{k}_{T,in}\, \phi^*(X, \mathbf{k}_{T,in} - \mathbf{\Delta})\phi(X, \mathbf{k}_{T,in}).$$

We only see this at this stage because the transverse component of the momentum transfer is an external variable

Upon Fourier transformation in k_{in} and k_{out}, we obtain

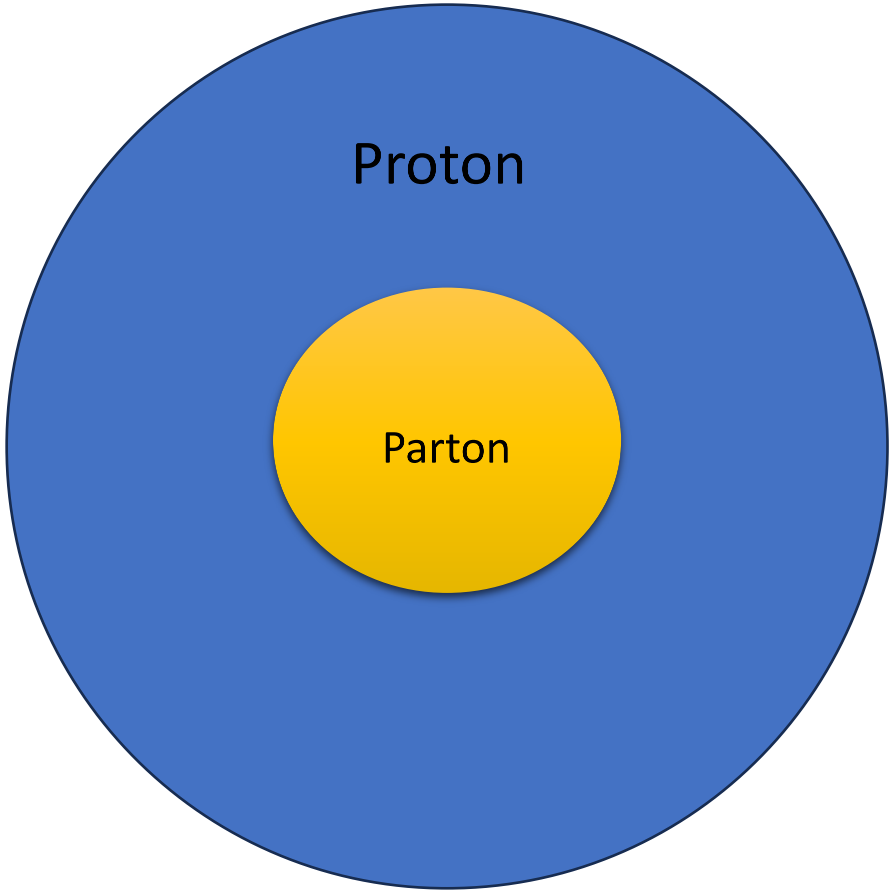
$$H_q(X, 0, t) = \int d^2\mathbf{b}\, e^{i\mathbf{b}\cdot\mathbf{\Delta}}\, \tilde{\phi}^*(X, \mathbf{b})\, \tilde{\phi}(X, \mathbf{b})$$

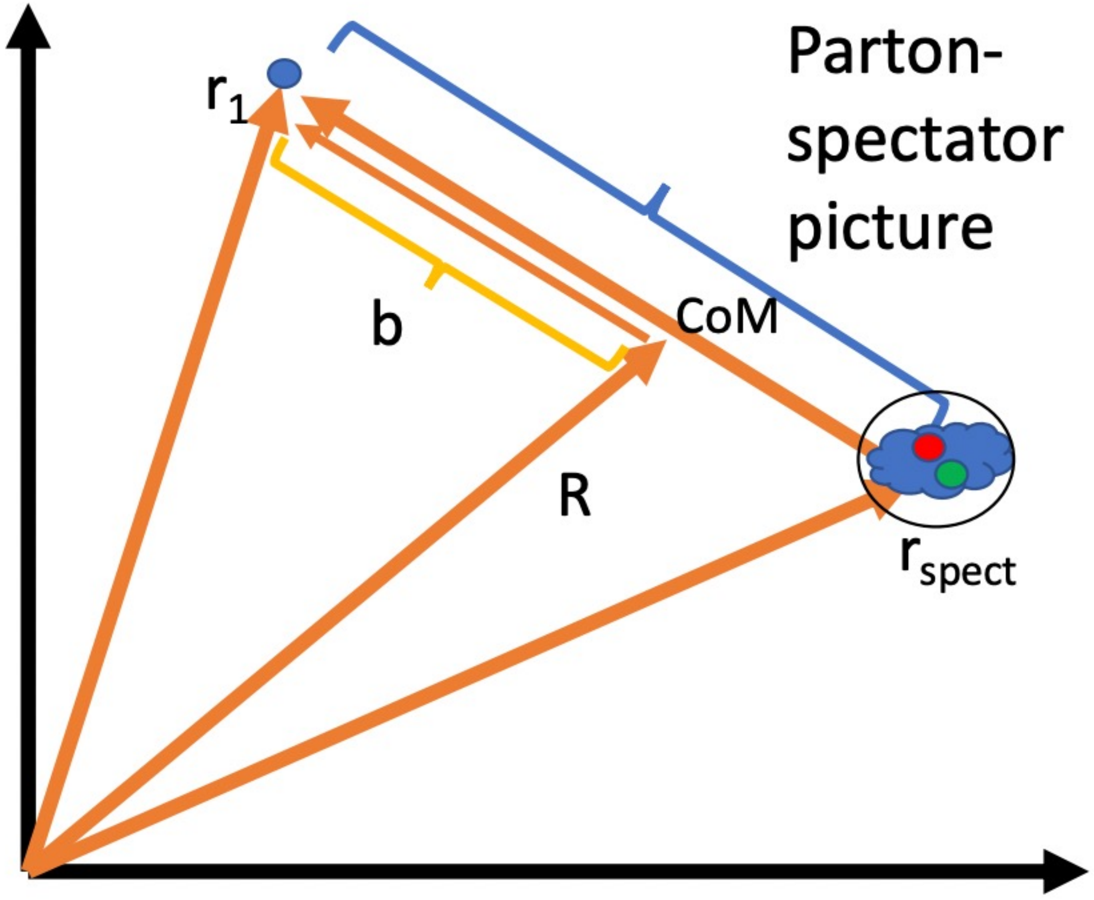
$$\rho_q(X, \mathbf{b}) = \tilde{\phi}^*(X, \mathbf{b})\, \tilde{\phi}(X, \mathbf{b})\,.$$

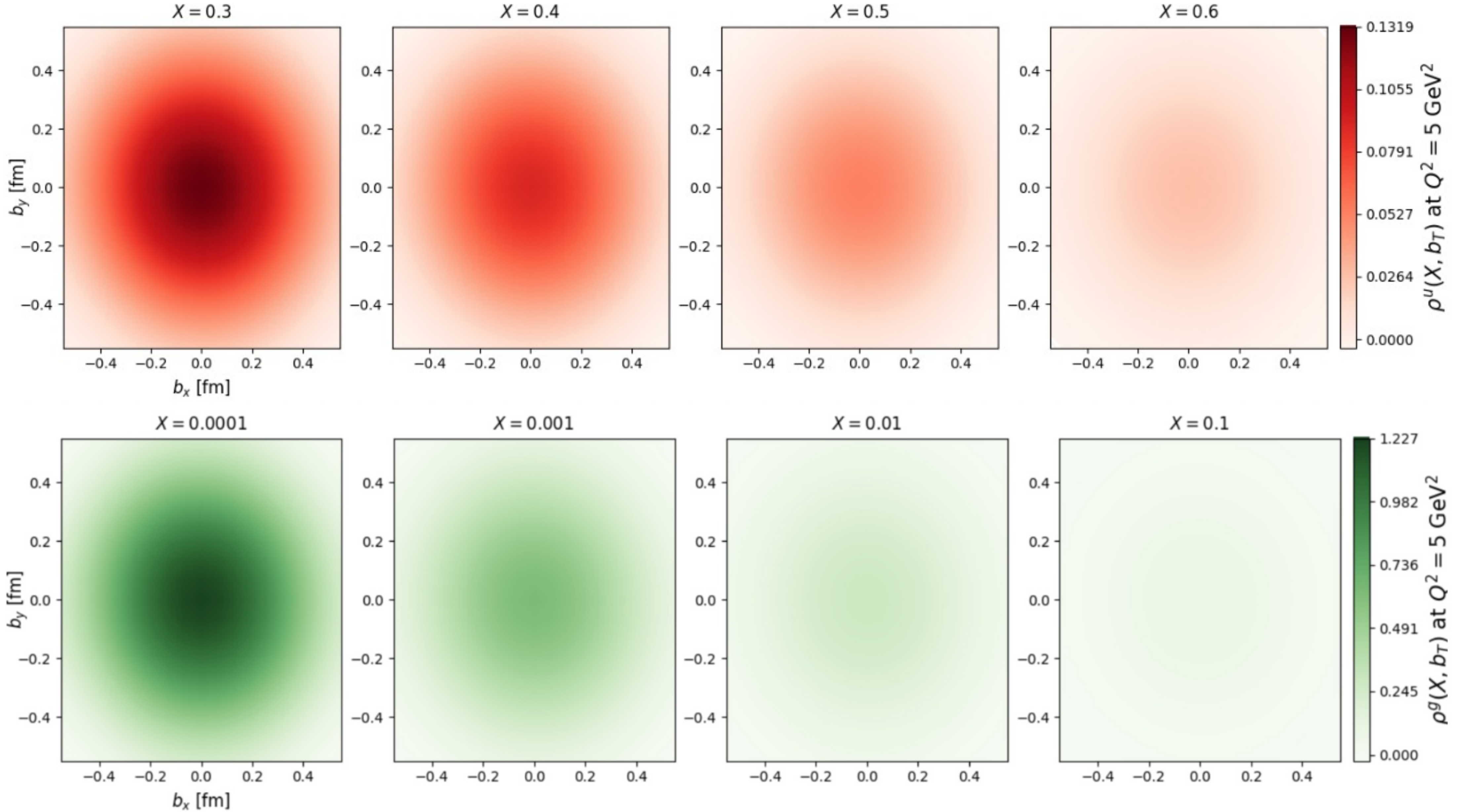
One-body diagonal density distribution

One-body densities

What do we see about the proton so far?



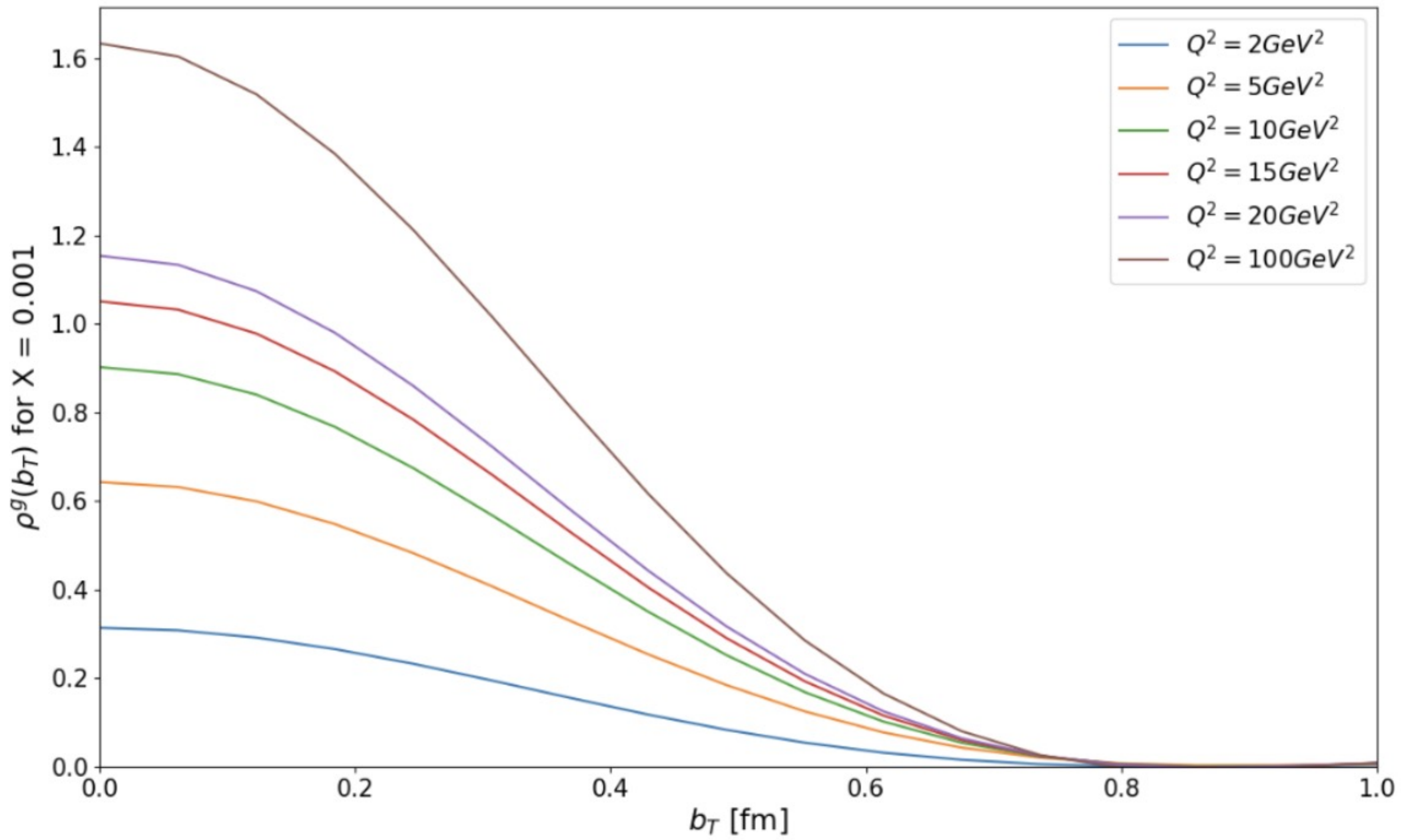




IPPDF calculated from the u quark and gluon GPDs in:

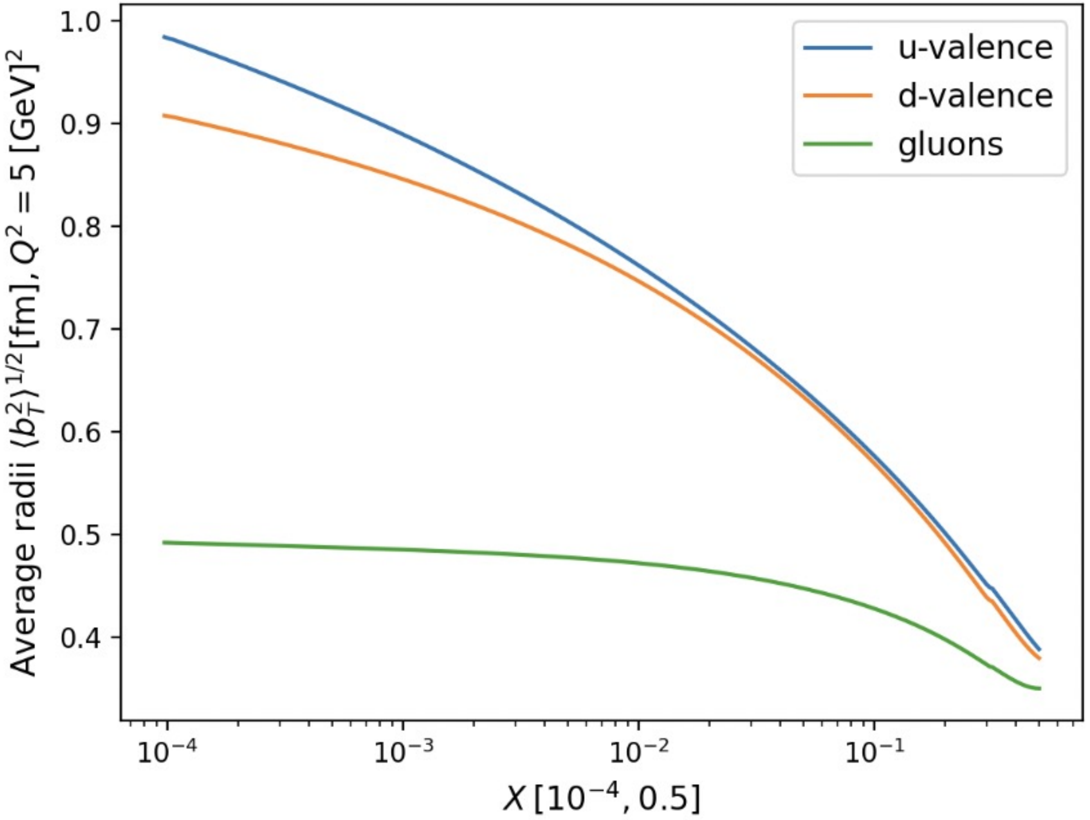
B. Kriesten. P. Velie, E. Yeats, F. Y. Lopez, & S. Liuti, *Phys.Rev.D* 105 (2022) 5, 056022

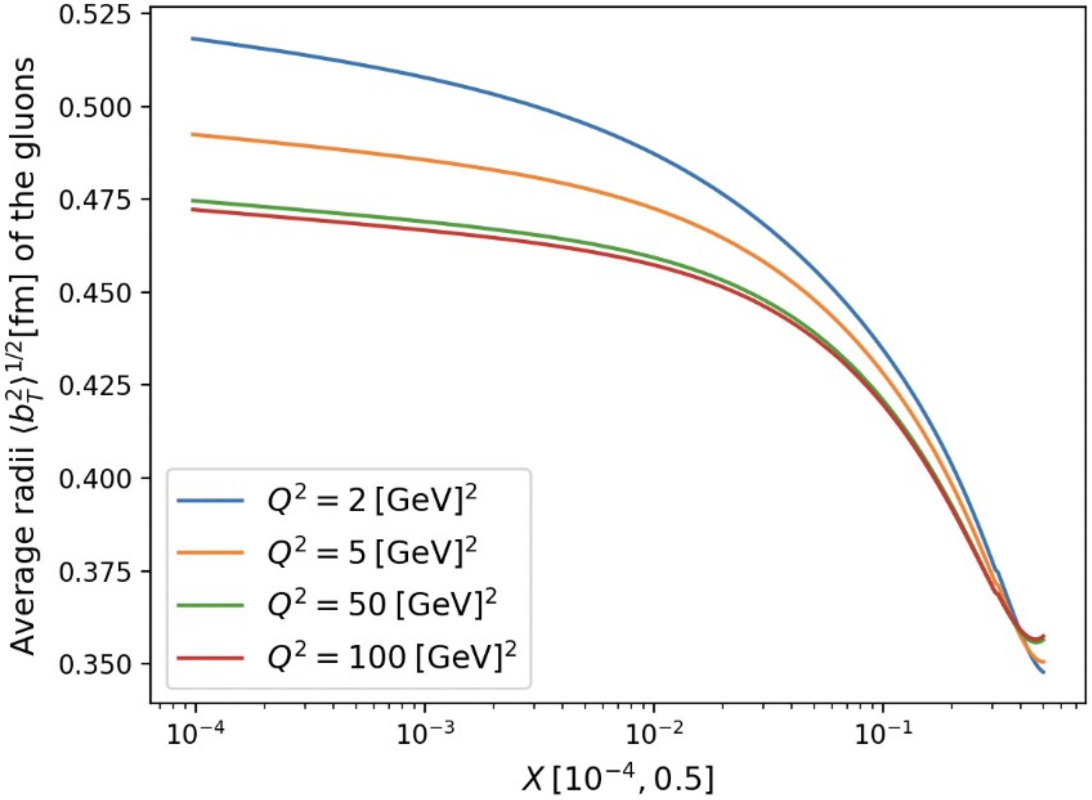
Q^2 Evolution



Evolving the distributions and studying the Q^2-dependence of the gluon IPPDFs

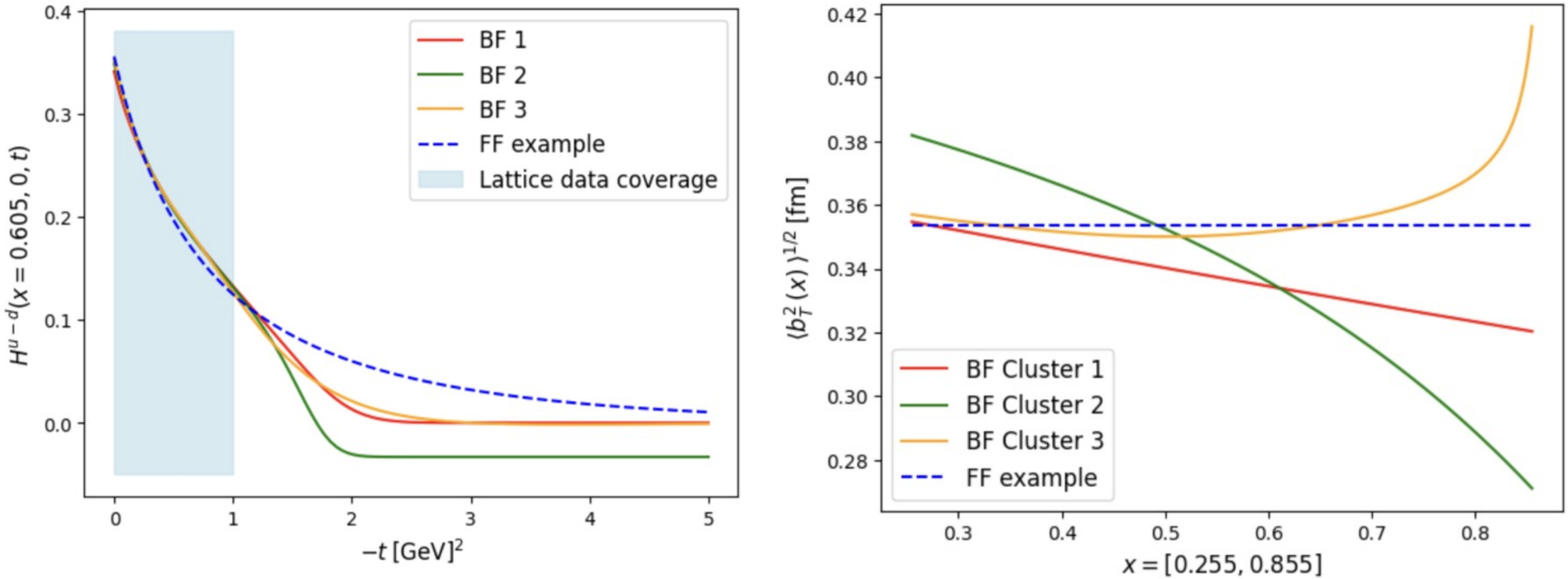
One-body densities





Average radii calculations

$$\langle b_T^2(X) \rangle^{1/2} = \frac{\int d^2 b_T b_T^2 \rho^{q,g}(X, b_T)}{\int d^2 b_T \rho^{q,g}(X, b_T)}$$



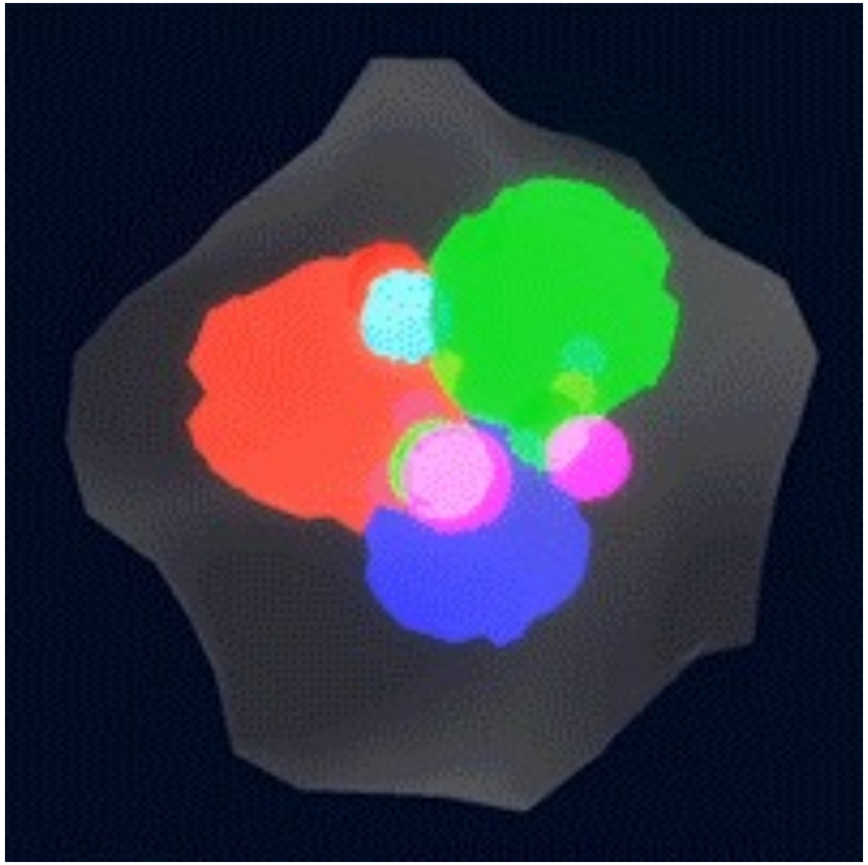
One-body densities from lattice data + symbolic regression

Generalized Parton Distributions from Symbolic Regression

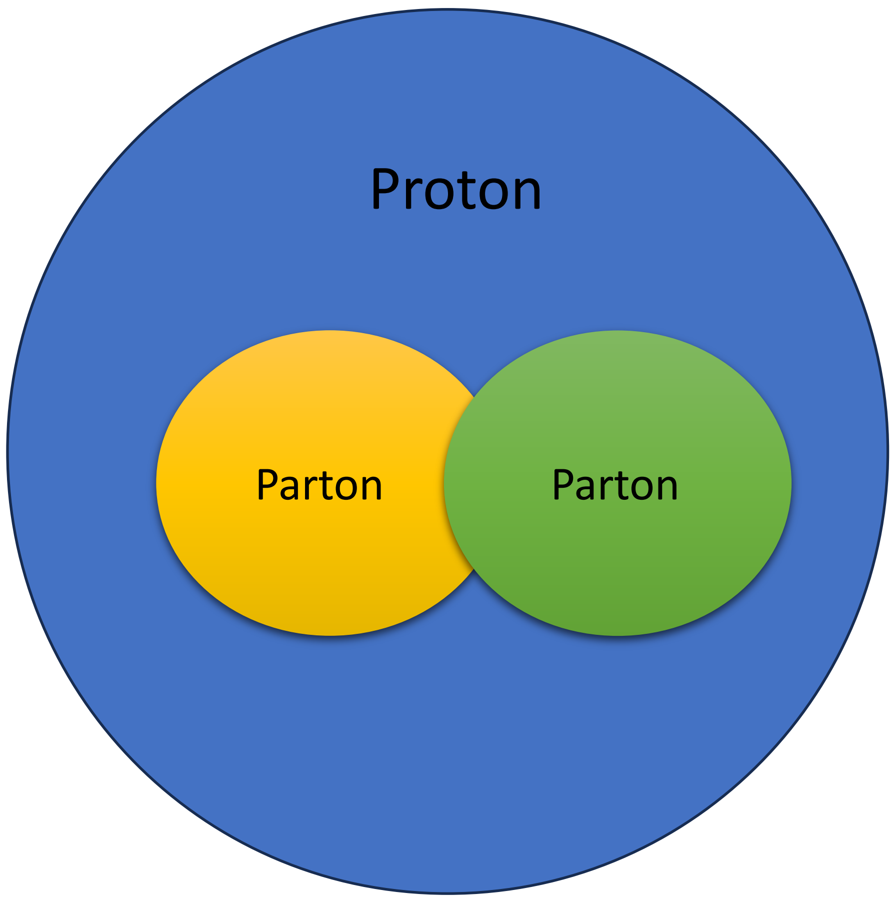
arXiv:2504.13289

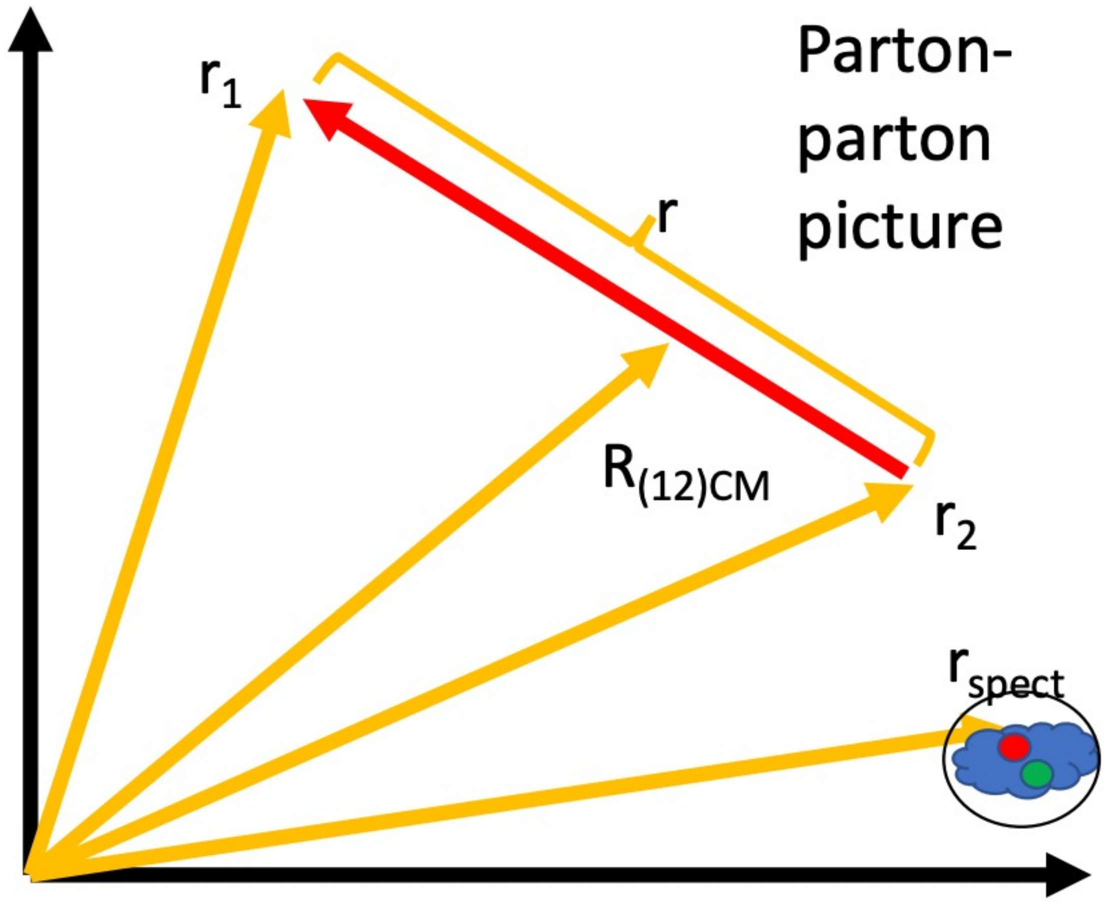
Andrew Dotson,[1,*] Zaki Panjsheeri,[1,†] Anusha Reddy Singireddy,[1] Douglas Q. Adams,[1,‡] Emmanuel Ortiz-Pacheco,[2] Marija Čuić,[1] Yaohang Li,[1] Huey-Wen Lin,[2,§] Simonetta Liuti,[1,¶] Matthew D. Sievert,[3,**] Marie Boër,[4] Gia-Wei Chern,[1] Michael Engelhardt,[3] and Gary R. Goldstein[5]
EXCLAIM Collaboration

How can we move *beyond* this one-body picture and get closer to →



Two-body densities





Two-body densities

The most general two-body density is given by

$$\rho_2^{q_1,q_2} = \rho_2^{q_1,q_2}(x_1, \mathbf{b_1}, x_2, \mathbf{b_2})$$

The joint probability of finding parton q_1 with momentum fraction x_1 at b_1 and parton q_2 with momentum fraction x_2 at b_2 (dynamically correlated particle motion)

A special case:

$$\rho_2^{q_1,q_2} = \rho_1^{q_1}(x_1, \mathbf{b_1})\rho_1^{q_2}(x_2, \mathbf{b_2})$$

The probability of independently finding parton q_1 with momentum fraction x_1 at b_1 and parton q_2 with momentum fraction x_2 at b_2 (dynamically uncorrelated particle motion)

Two-body densities

- Once we have the two-body density, we can define average relative distances between partons as well as the average overlap of two partons

$$
\begin{aligned}
\mathbf{r} &= \mathbf{b}_1 - \mathbf{b}_2 \\
\mathbf{R}_{12} &= \frac{\mathbf{b}_1 + \mathbf{b}_2}{2}
\end{aligned}
$$

$$
\begin{aligned}
\langle \mathbf{r}^2 \rangle (X_1, X_2) &= \frac{1}{\mathcal{N}} \int\int d^2\mathbf{r}\, d^2\mathbf{R}_{12}\, \left|\mathbf{r}^2\right|\; \rho_2 \left(X_1, \mathbf{R}_{12} + \frac{\mathbf{r}}{2}; X_2, \mathbf{R}_{12} - \frac{\mathbf{r}}{2} \right) \\
\langle \mathbf{R}_{12}^2 \rangle (X_1, X_2) &= \frac{1}{\mathcal{N}} \int\int d^2\mathbf{r}\, r^2\mathbf{R}_{12}\, \left|\mathbf{R}_{12}^2\right|\; \rho_2 \left(X_1, \mathbf{R}_{12} + \frac{\mathbf{r}}{2}; X_2, \mathbf{R}_{12} - \frac{\mathbf{r}}{2} \right) \\
\mathcal{N} &= \int\int d^2\mathbf{r}\, r^2\mathbf{R}_{12}\; \rho_2 \left(X_1, \mathbf{R}_{12} + \frac{\mathbf{r}}{2}; X_2, \mathbf{R}_{12} - \frac{\mathbf{r}}{2} \right) \\
\langle A(X_1, X_2) \rangle &= \frac{1}{\mathcal{N}} \int d^2\mathbf{b}_1 d^2\mathbf{b}_2\, \rho_2^{ij}(X_1, \mathbf{b}_1; X_2, \mathbf{b}_2) A(\mid \mathbf{b}_1 - \mathbf{b}_2 \mid),
\end{aligned}
$$

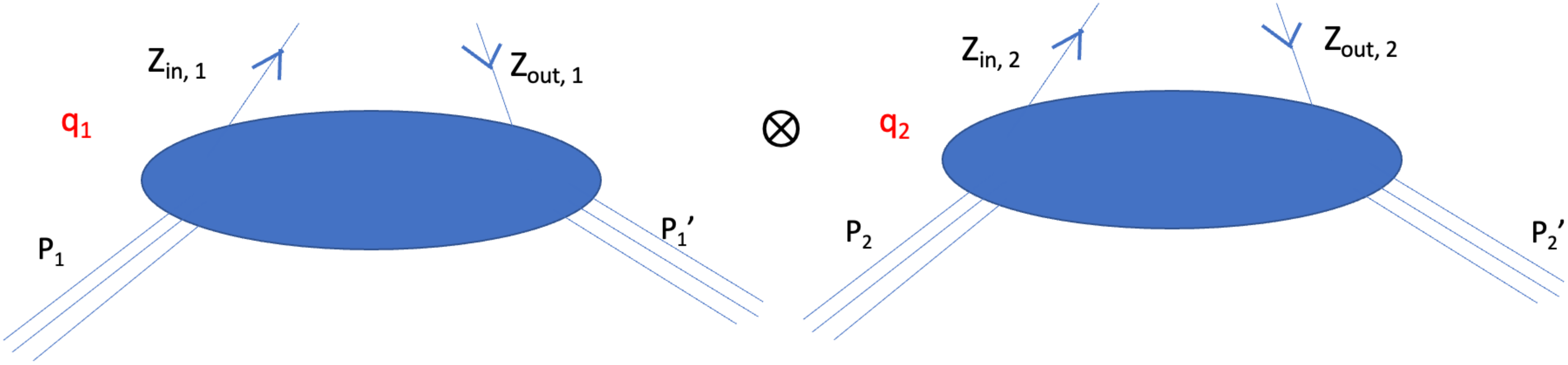
$$
A(d) = 1 - \frac{2}{\pi}\delta\sqrt{1-\delta^2} - \frac{2}{\pi}\arctan\frac{\delta}{\sqrt{1-\delta^2}} \qquad \delta = d/(2a)
$$

Model the average overlap as the geometric overlap of two circles weighted by the two-body density

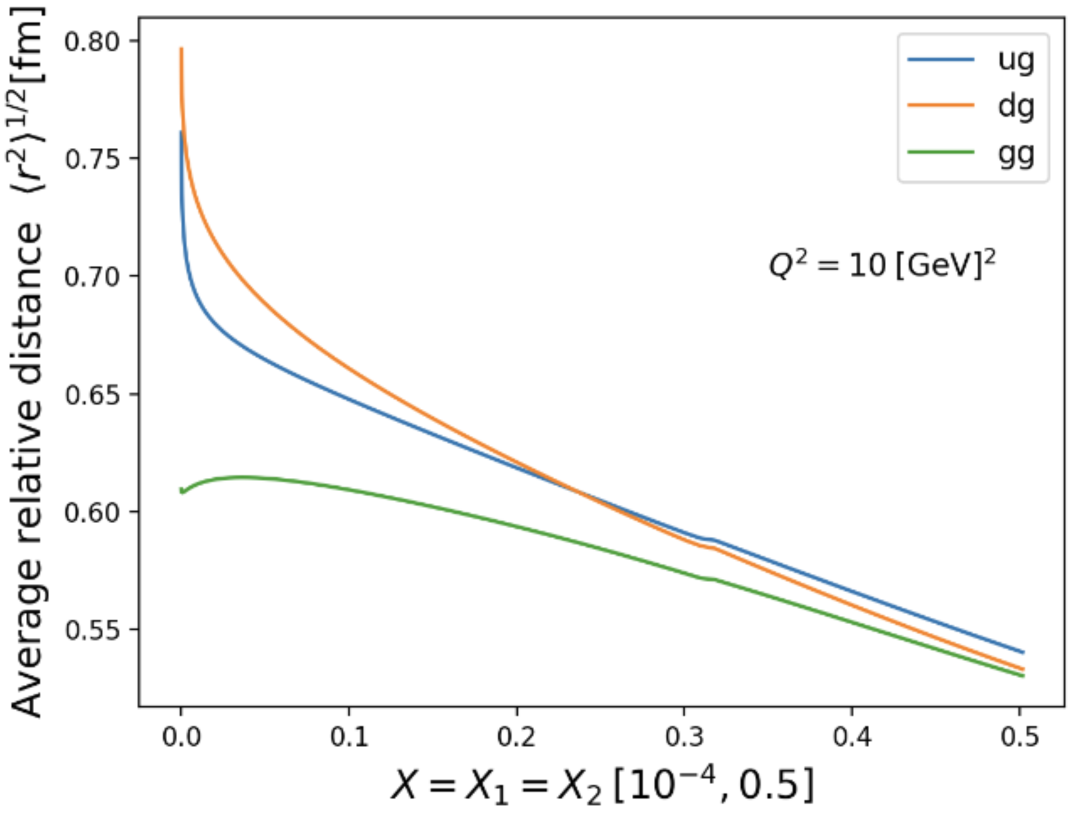
Two-body densities

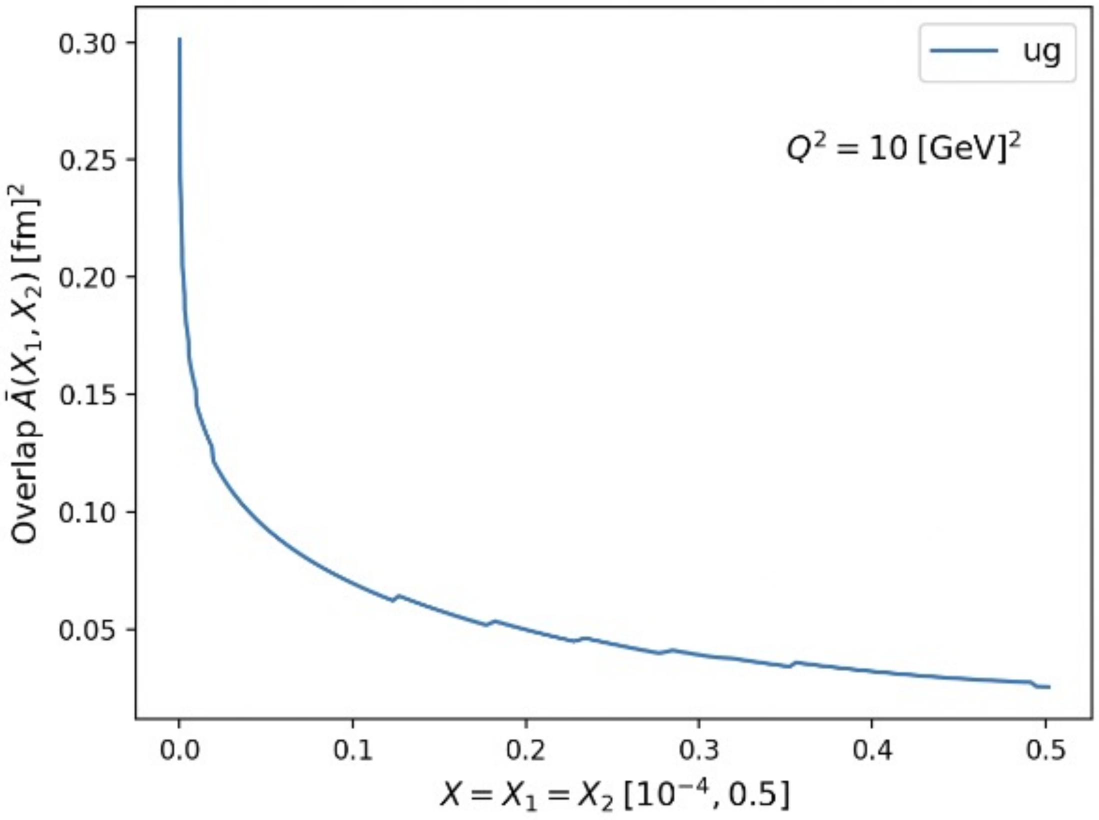
- The uncorrelated case arises from the convolution of two one-body correlation functions

$$\rho_2^{q_1,q_2} = \rho_1^{q_1}(x_1, \mathbf{b_1})\rho_1^{q_2}(x_2, \mathbf{b_2})$$



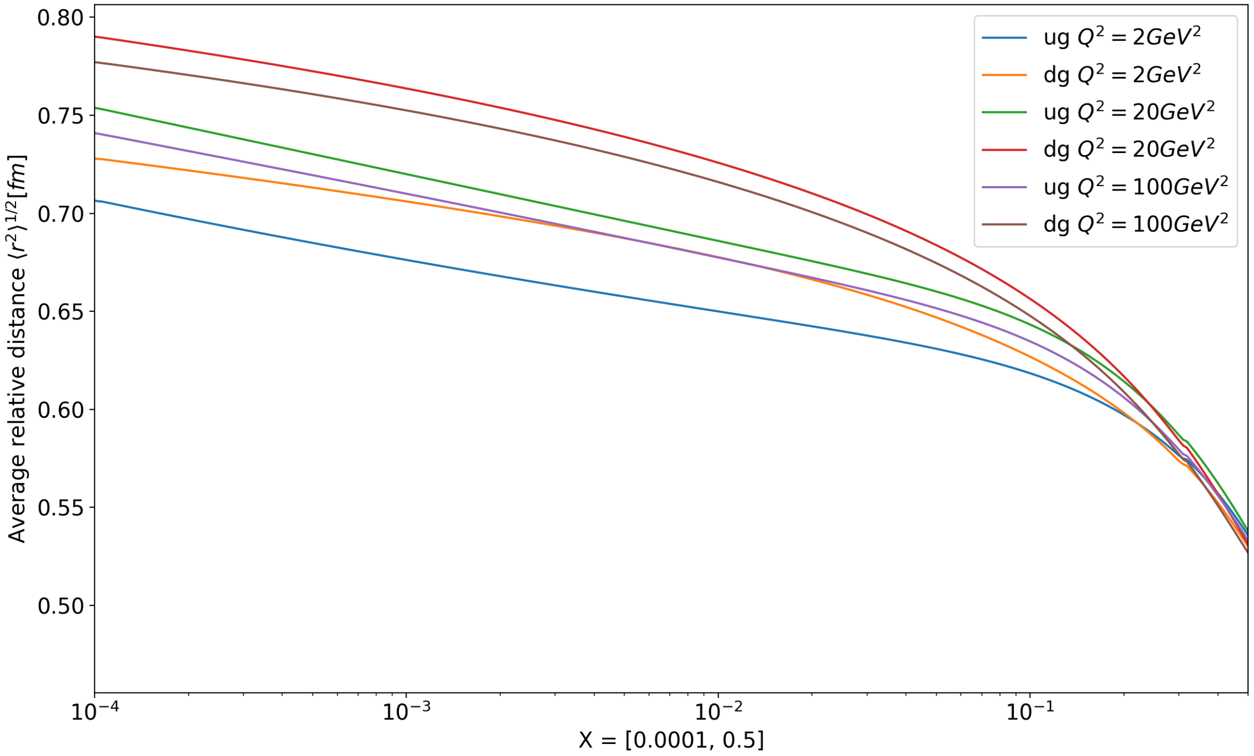
Relative distances and overlap

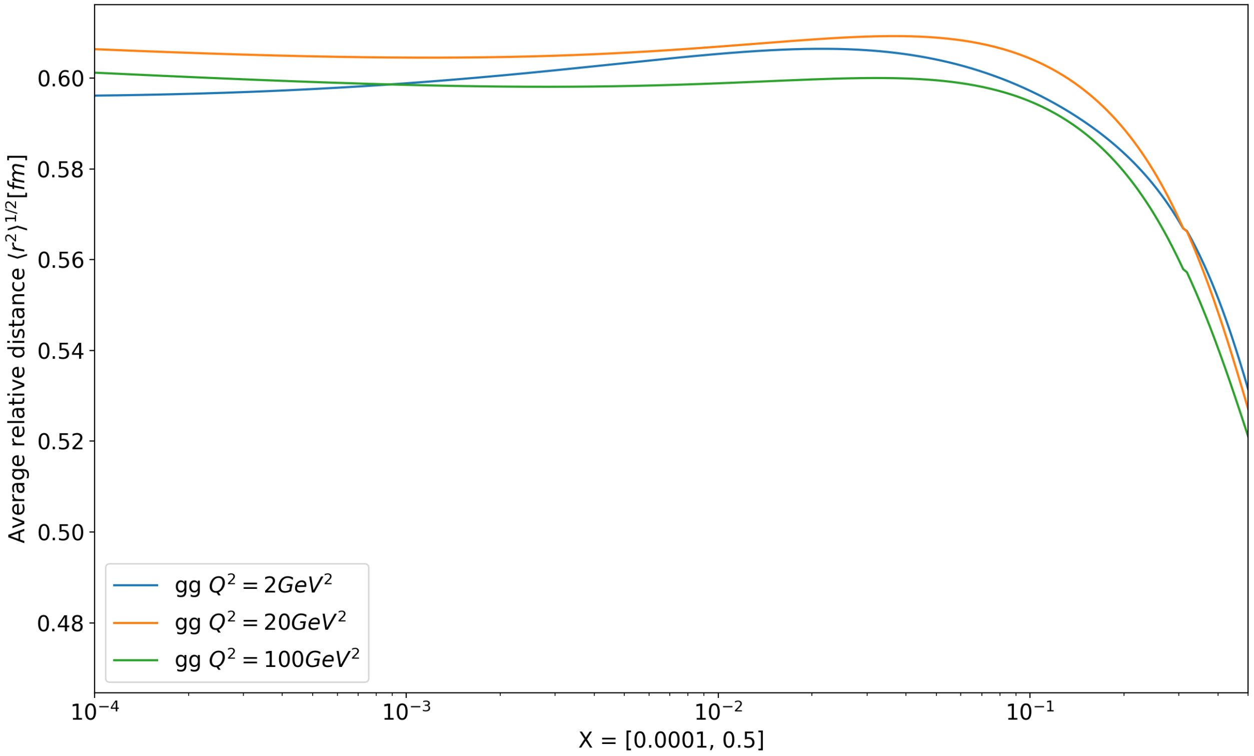




Q^2 Evolution

- Evolving the distributions and studying the Q^2 dependence of the average distance and overlap

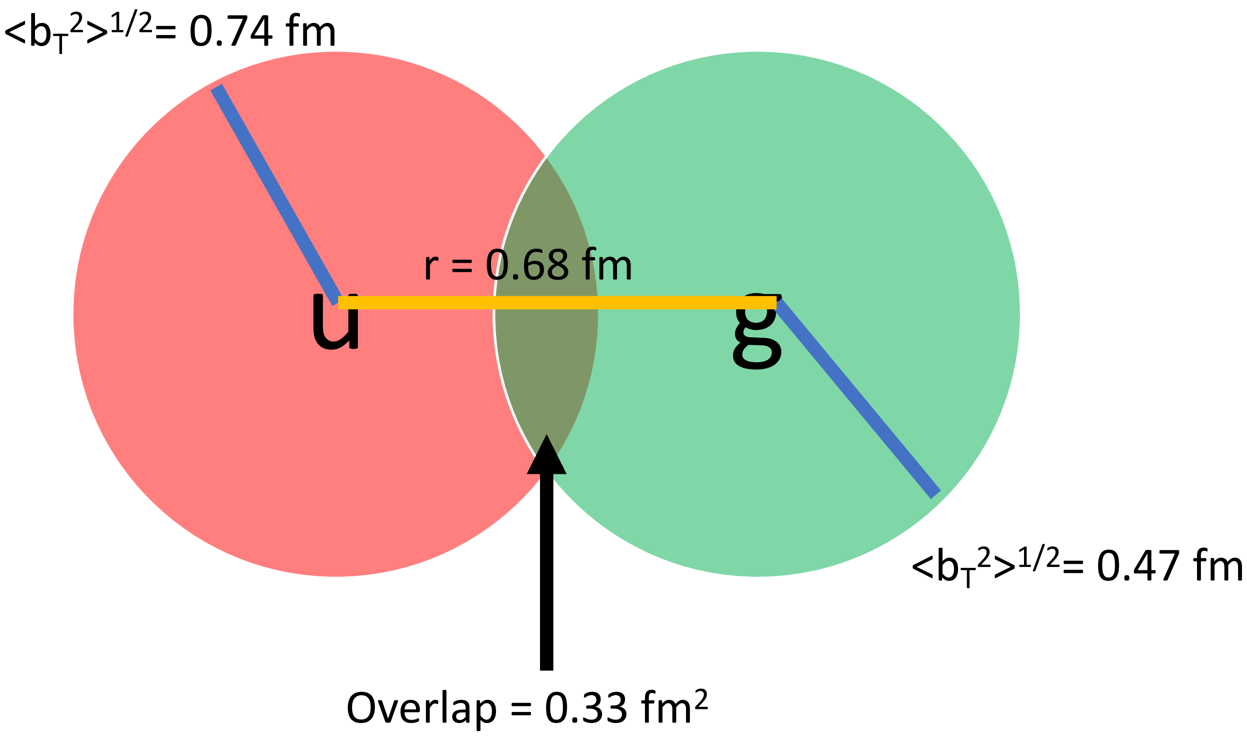


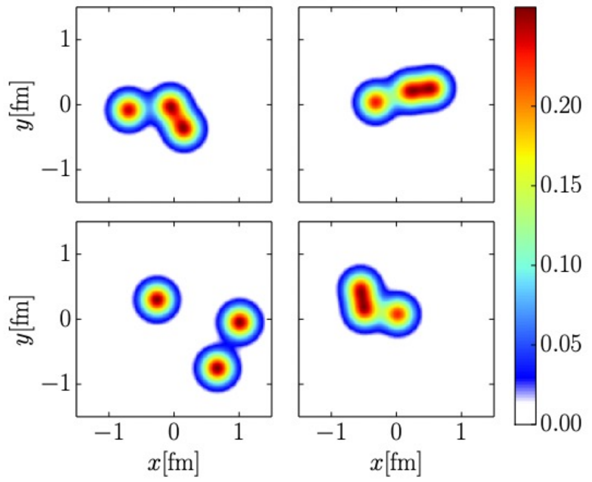


	This work	Mantysaari and Schenke
Observable	Double GPDs through UPCs	Incoherent and coherent J/ψ cross section
Picture	u, d, g have varying overlaps, rel. dist.	u, d valence surrounded by gluon cloud
Determining size	$\langle b_T^2 \rangle^{1/2}$ average radius for u, d, g	B_q gluon hotspot size
Determining separation	$\langle r^2 \rangle^{1/2}$ average rel. dist.	MC sampling with Gaussian of width B_{qc}

To study the correlated case, we need to introduce a new observable.

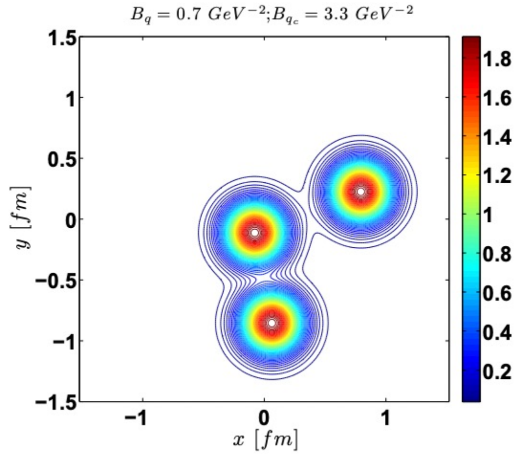
X = 0.01, Q^2 = 20 GeV^2





(a) $B_{qc} = 3.3$ GeV^{-2}, $B_q = 0.7$ GeV^{-2}

Heikki Mäntysaari and Björn Schenke Phys. Rev. D **94**, 034042



Traini, M.C., Blaizot, JP. *Eur. Phys. J. C* **79**, 327 (2019)

Short introduction to ultraperipheral collisions

- UPCs are peripheral collisions in which the impact parameter b is larger than twice the nuclear radius R, making them “ultra” peripheral.

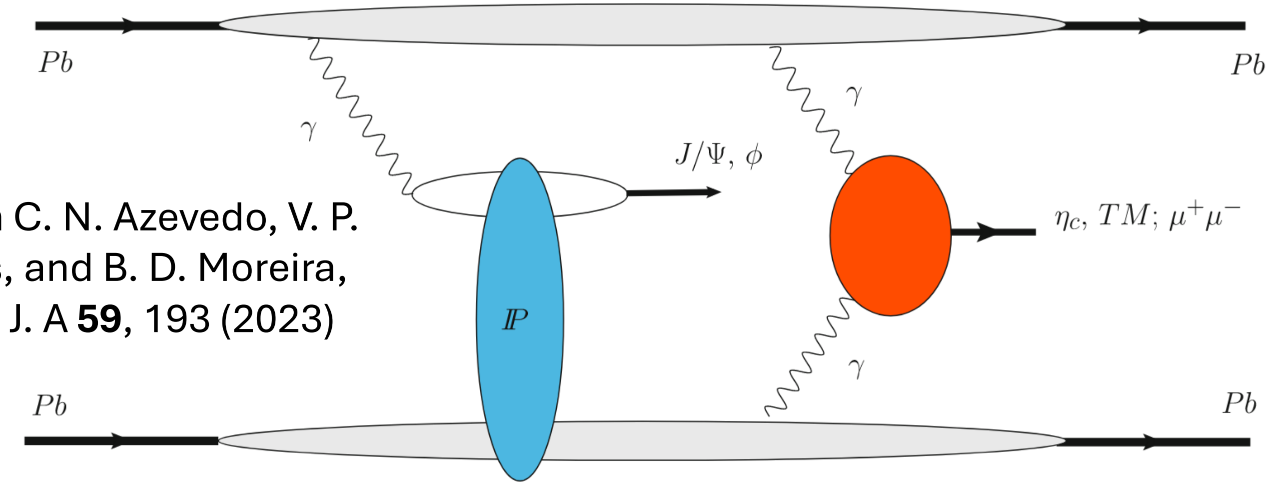
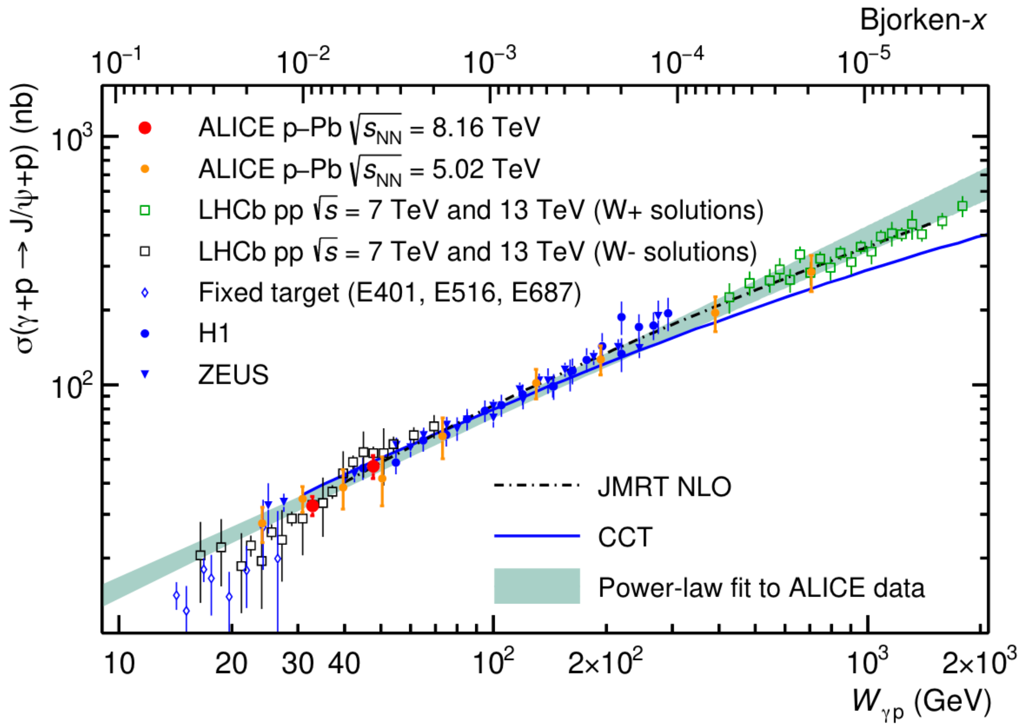


Figure from C. N. Azevedo, V. P. Gonçalves, and B. D. Moreira, Eur. Phys. J. A **59**, 193 (2023)

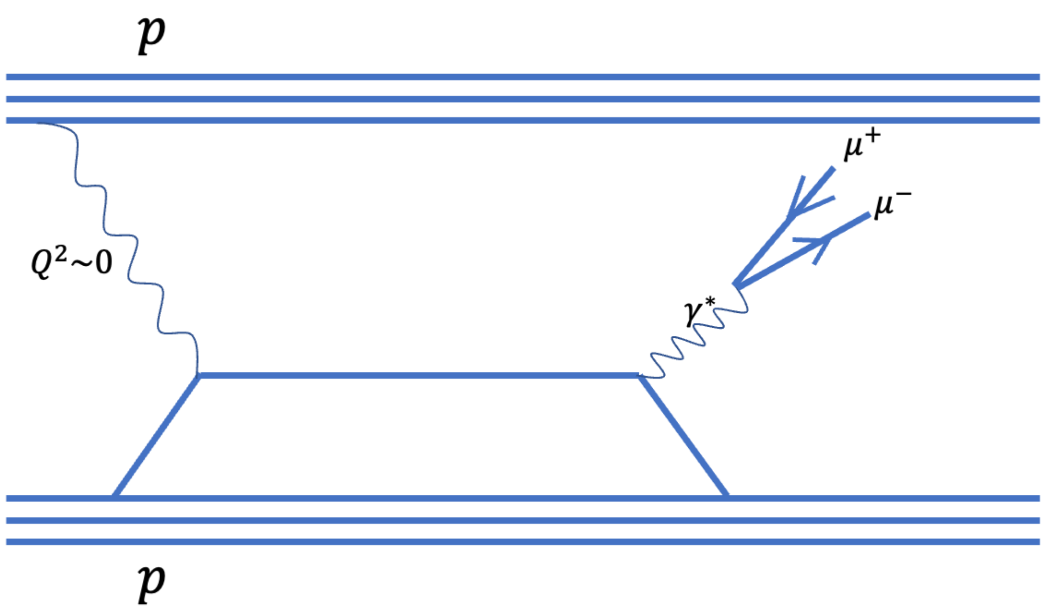


- Recent results from the ALICE Collaboration on exclusive J/psi photon-production

ALICE Collaboration et al., Phys. Rev. D **108**, 112004 (2023).

Short introduction to ultraperipheral collisions

- Time-like Compton scattering (TCS) in UPCs



$$\frac{d\sigma_{Np\to N(l^+l^-)p}}{dy dQ^2 d\cos\theta d\phi} = n_Z(\omega) \frac{d\sigma_{\gamma p\to(l^+l^-)p}}{dy dQ^2 d\cos\theta d\phi}$$

Connect the photon-proton scattering amplitude for TCS to UPCs using the Weissacker-Williams "equivalent photon factor"

$$n_Z(\omega) = \frac{2Z^2\alpha_{EM}}{2\pi}\left(XK_0(X)K_1(X) - \frac{X^2}{2}[K_1^2(X) - K_0^2(X)]\right)$$

$$X = 2\omega R_A/\gamma_L$$

Photon energy: $\omega \propto \exp(y)$

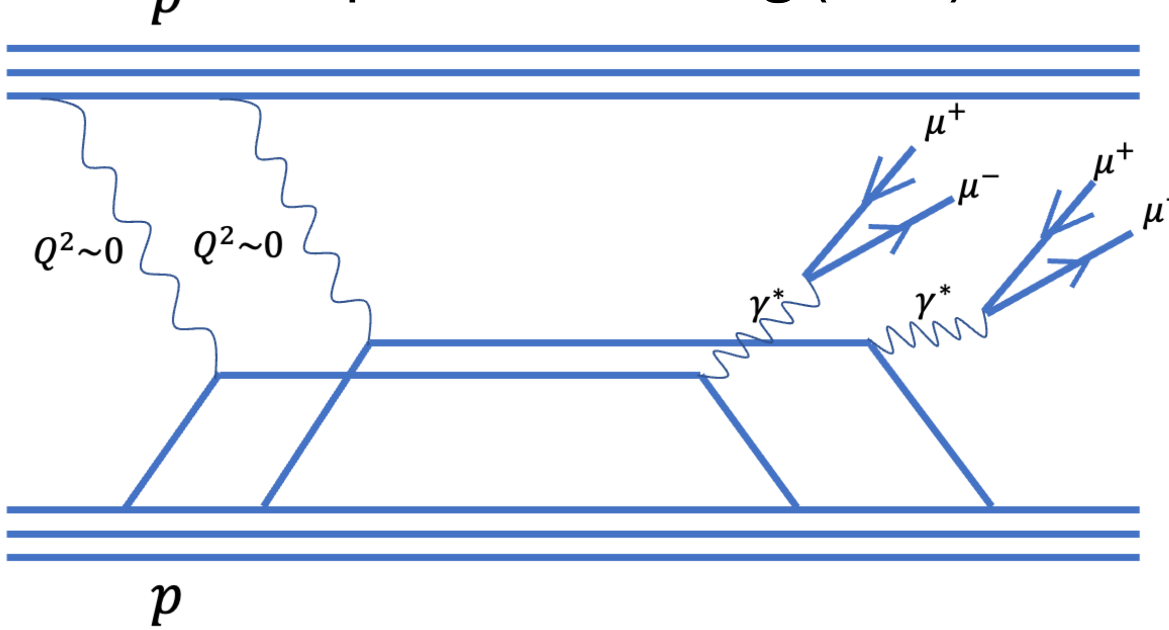
Formalism in Y.-P. Xie and V. P. Gonçalves, Physics Letters B **839**, 137762 (2023);
B. Pire, L. Szymanowski, and J. Wagner, Phys. Rev. D **79**, 014010 (2009).

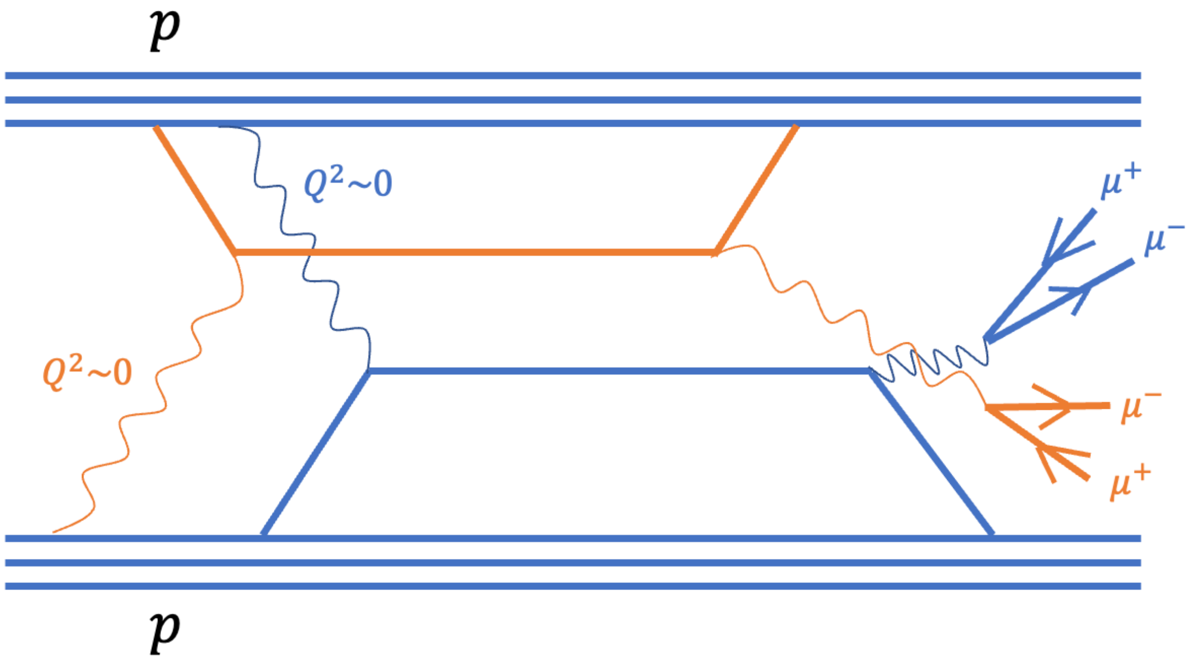
- UPCs at the LHC can get to small skewness, complementing Jefferson Lab's large skewness capabilities

Uncorrelated double TCS in UPCs

Predictions forthcoming → In collaboration with members of ALICE

- We envision ultraperipheral collisions (UPCs) as the experimental probe of the two-body densities → uncorrelated double time-like Compton scattering (TCS) in UPCs

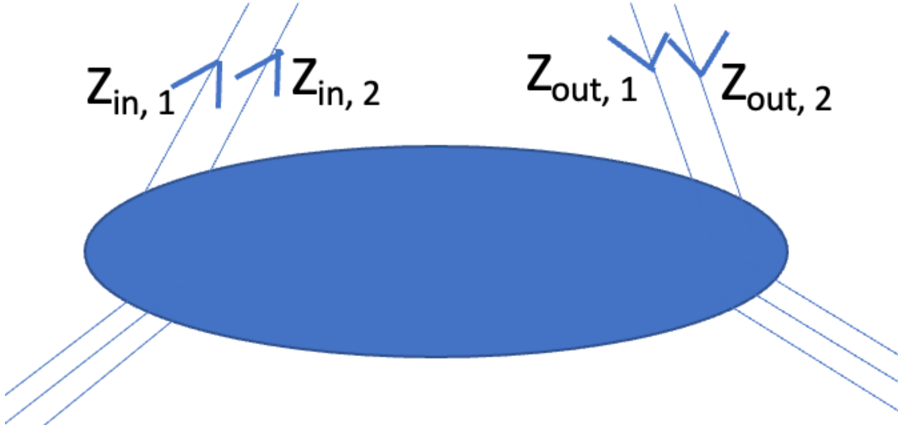




$$d\sigma_{h_1h_2\to h_1(l^+l^-)_a(l^+l^-)_bh_2} = n^a_{h_1}(\omega_1)\,\sigma_{\gamma_a h_2\to(l^+l^-)_a h_2} \times n^b_{h_1}(\omega_2)\,\sigma_{\gamma_b h_2\to(l^+l^-)_b h_2}$$

$$d\sigma_{h_1h_2\to h_1(l^+l^-)_a(l^+l^-)_bh_2} = n^a_{h_1}(\omega_1)\,\sigma_{\gamma_a h_1\to(l^+l^-)_a h_1} \times n^b_{h_2}(\omega_2)\,\sigma_{\gamma_b h_2\to(l^+l^-)_b h_2}$$

Correlated two-body densities



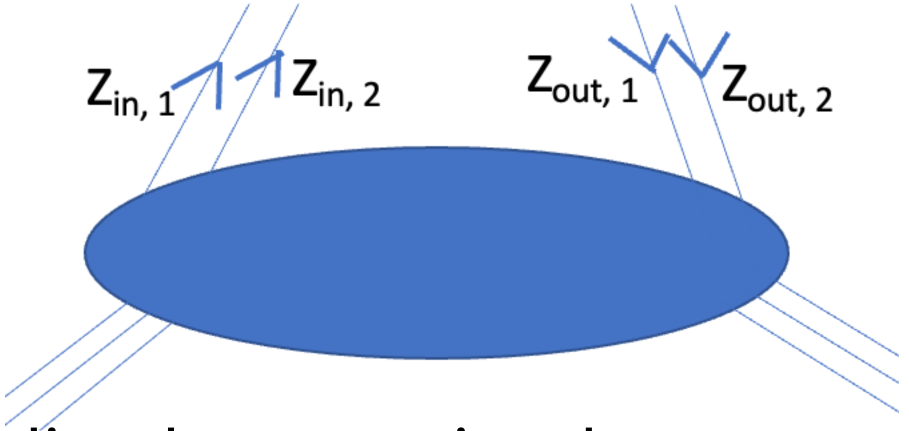
- We define the double-parton correlation function for exclusive processes as

$$W^{\Gamma}_{\Lambda,\Lambda'} = \int \frac{dz^-_{1,in} d\mathbf{z}_{1,T,in}}{(2\pi)^3} \frac{dz^-_{2,in} d\mathbf{z}_{2,T,in}}{(2\pi)^3} \int \frac{dz^-_{1,out} d\mathbf{z}_{1,T,out}}{(2\pi)^3} \frac{dz^-_{2out} d\mathbf{z}_{2,T,out}}{(2\pi)^3}$$
$$\times\, e^{i(k_{1,in} z_{1,in} + k_{2,in} z_{2,in})}\, e^{-i(k_{1,out} z_{1,out} + k_{2,out} z_{2,out})} \langle p', \Lambda' | \overline{\psi}(z_{1,out})\, \Gamma \psi(z_{1,in})\, \overline{\psi}(z_{2,out})\, \Gamma \psi(z_{2,in}) | p, \Lambda \rangle \Big|_{z_1^+ = z_2^+ = 0}$$

- Gives us access to the proton's correlated spatial structure
- Renormalization scale dependence here is non-trivial but this object is still collinear

Compare to the inclusive sector, e.g., Diehl, M., Ostermeier, D. & Schäfer, A., *J. High Energ. Phys.* **2012**, 89 (2012); A. V. Manohar and W. J. Waalewijn, Phys. Rev. D **85**, 114009 (2012); Kasemets & Scopetta, *Adv. Ser. Direct. High Energy Phys.* 29 (2018)

Correlated two-body densities



- Just as in the one-body case, one can immediately recognize that this is a momentum distribution in the longitudinal component in the zero skewness case,

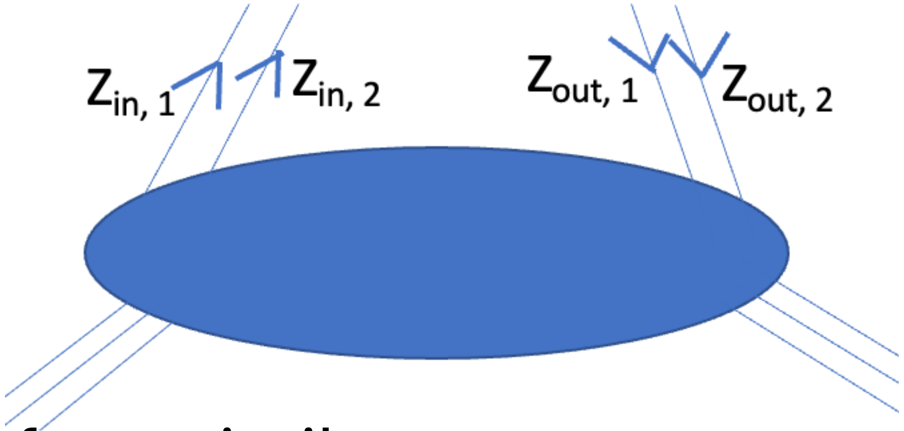
$$
\begin{aligned}
W^{\Gamma}_{\Lambda,\Lambda'} \;=\; & \int \frac{dz^-_{1,in} d^2 z_{1,in,T}}{(2\pi)^3} \frac{dz^-_{1,out} d^2 z_{1,out,T}}{(2\pi)^3} \int \frac{dz^-_{2,in} d^2 z_{2,in,T}}{(2\pi)^3} \frac{dz^-_{2,out} d^2 z_{2,out,T}}{(2\pi)^3} \\
& \times e^{ik^+_{1,in}(z^-_{1,in}-z^-_{1,out})} e^{ik^+_{2,in}(z^-_{2,in}-z^-_{2,out})} e^{i(-k_{T_1,in}\cdot z_{T_1,in}-k_{T_2,in}\cdot z_{T_2,in})} e^{i(-k_{T_1,out}\cdot z_{T_1,out}-k_{T_2,out}\cdot z_{T_2,out})} \\
& \times \langle P',\Lambda'|\,\bar\psi(0,z^-_{1,out},z_{T_1,out})\Gamma\psi(0,z^-_{1,in},z_{T_1,in})\bar\psi(0,z^-_{2,out},z_{T_2,out})\Gamma\psi(0,z^-_{2,in},z_{T_2,in})\,|P,\Lambda\rangle
\end{aligned}
$$

$$
\boxed{\Delta_i^+ \;=\; k^+_{i,\text{in}} - k^+_{i,\text{out}} = 0}
$$

$$
\boxed{
\begin{aligned}
k_1^{in} &\equiv \left(X_1 p^+, k_1^{in,-}, \mathbf{k}_{1T}^{in}\right) \\
k_2^{in} &\equiv \left(X_2 p^+, k_2^{in,-}, \mathbf{k}_{2T}^{in}\right) \\
k_1^{out} &\equiv \left((X_1-\zeta_1)p^+, k_1^{out,-}, \mathbf{k}_{1T}^{in} - \mathbf{\Delta}_{1T}\right) \\
k_2^{out} &\equiv \left((X_2-\zeta_2)p^+, k_2^{out,-}, \mathbf{k}_{2T}^{in} - \mathbf{\Delta}_{2T}\right)
\end{aligned}
}
$$

For a similar scenario in nuclei, see P. Papakonstantinou, E. Mavrommatis, and T. S. Kosmas, The Two-Body Momentum Distribution in Finite Nuclei, Nuclear Physics A **713**, 81 (2003).

Correlated two-body densities



- To define the correlated double GPD, we perform similar transformations as before:

$$\begin{aligned} z_i &= z_{i,in} - z_{i,out} \\ b_i &= \frac{1}{2}(z_{i,in} + z_{i,out}) \end{aligned} \qquad \begin{aligned} \Delta_i &= k_{i,\mathrm{in}} - k_{i,\mathrm{out}} \\ k_i &= \frac{1}{2}(k_{i,\mathrm{in}} + k_{i,\mathrm{out}}) \end{aligned} \qquad \Delta_i^+ = k_{i,\mathrm{in}}^+ - k_{i,\mathrm{out}}^+ = 0$$

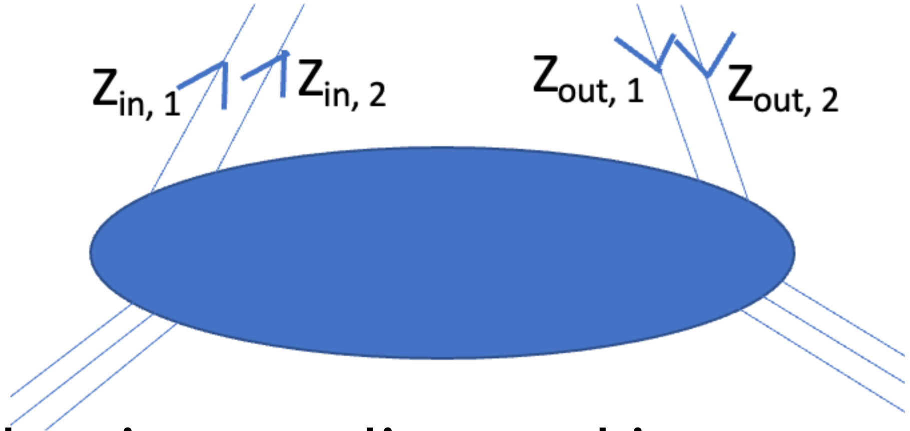
$$W^{\Gamma}_{\Lambda,\Lambda'} = \int \frac{d\mathbf{y}_T}{(2\pi)^2} e^{i\Delta_{2T}\cdot y_T} \int \frac{dz_1^-}{2\pi} \frac{dz_2^-}{2\pi} e^{ik_1^+ z_1^-} e^{ik_2^+ z_2^-} \langle p', \Lambda' | \overline{\psi}(0) \Gamma \psi(z_1) \overline{\psi}\left(y - \frac{z_2 - z_1}{2}\right) \Gamma \psi\left(y + \frac{z_2 + z_1}{2}\right) | p, \Lambda \rangle$$

Translated the fields, integrate over external variables, and integrated over the transverse components of z_1 and z_2

Identify a relative distance between the two partons:

$$y = b_2 - b_1$$

Correlated two-body densities



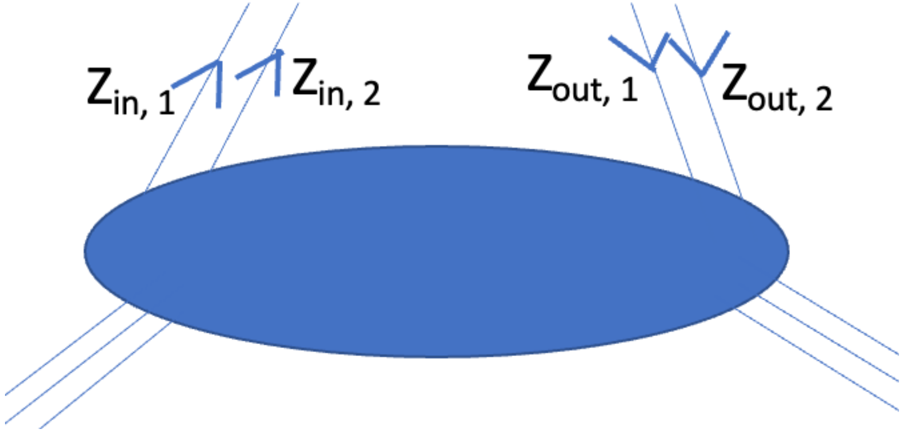
- We next look for a momentum distribution that is non-diagonal in the transverse momentum and therefore diagonal in coordinate space. Insert the complete set of states:

$$W^{\Gamma}_{\Lambda,\Lambda'} = \int \frac{d\mathbf{y}_T}{(2\pi)^2}\, e^{i\Delta_{2T}\cdot y_T} \int \frac{dz_1^-}{2\pi}\frac{dz_2^-}{2\pi}$$
$$\times\; e^{i(k_1 z_1)}\; e^{i(k_2 z_2)} \sum_{\mathcal{X}} \langle p',\Lambda'|\overline{\psi}_+(0)\,\overline{\psi}_+(y+z_1/2-z_2/2)\,|\mathcal{X}\rangle\langle\mathcal{X}|\psi_+(z_1)\,\psi_+(y+z_1/2+z_2/2)\,|p,\Lambda\rangle$$

- We now define a two-body quark-quark GPD (double GPD):

$$H_{qq}(X_1,X_2,0,t_1,t_2) = \int d^2\mathbf{k}_T \int \frac{dy^- d\mathbf{y}_T}{(2\pi)^3}\, e^{i(\Delta_2-k_2)y}\langle p',\Lambda'|\overline{\psi}_+(0)\,\overline{\psi}_+(y)\,|\mathcal{X}\rangle\langle\mathcal{X}|\psi_+(0)\,\psi_+(y)\,|p,\Lambda\rangle$$

Correlated two-body densities



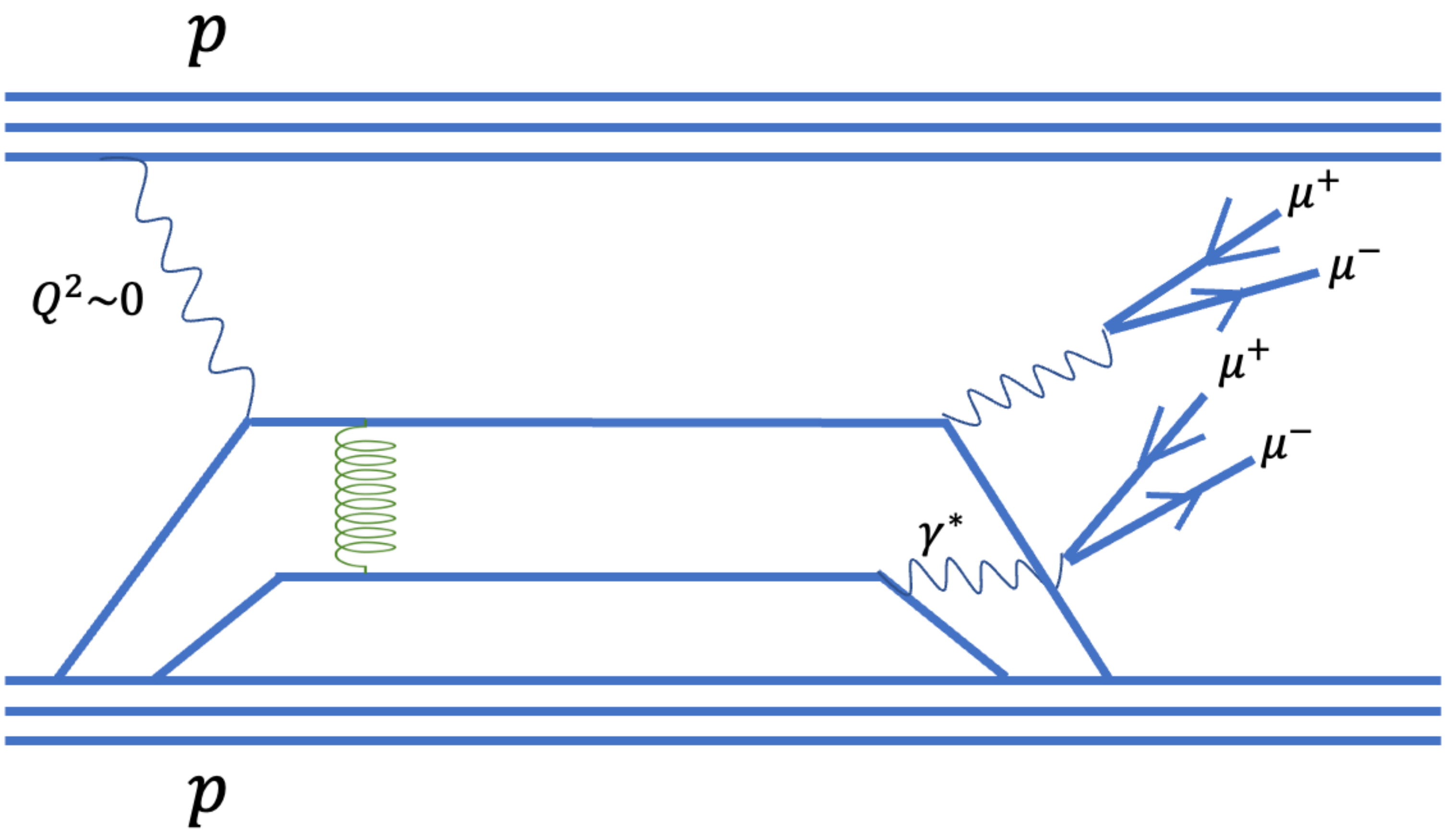
- Define the vertex functions, similarly to how we did in the one-body case:

$$\Phi(X_1, X_2, \mathbf{k}_{1,T}, \mathbf{k}_{2,T}; y) = \langle \mathcal{X} \mid \psi_+(0)\psi_+(y) \mid p\rangle$$
$$\Phi^*(X_1, X_2, \mathbf{k}'_{1,T}, \mathbf{k}'_{2,T}; y) = \langle p - \Delta \mid \overline{\psi}_+(0)\overline{\psi}_+(y) \mid \mathcal{X}\rangle$$

$$\mathbf{k}'_{1(2),T} = \mathbf{k}_{1(2),T} - \Delta_{1(2)}$$

$$H_{qq}(X_1, X_2, 0, t_1, t_2) = \int \frac{dy^- d\mathbf{y}_T}{(2\pi)^3}\, e^{i(\Delta_2 - k_2)y} \int d^2\mathbf{k}_{1,T} d^2\mathbf{k}_{2,T} \Phi^*(X_1, X_2, \mathbf{k}'_{1,T}, \mathbf{k}'_{2,T}; y)\Phi(X_1, X_2, \mathbf{k}_{1,T}, \mathbf{k}_{2,T}; y)$$

Observable for correlated two-body densities



Correlated double TCS

$$d\sigma_{h_1 h_2 \to h_1 (l^+ l^-)_a (l^+ l^-)_b h_2} = n_{h_1}(\omega_1)\, \sigma_{\gamma h_2 \to (l^+ l^-)_a (l^+ l^-)_b h_2}$$

Conclusions

- The one-body density on its own already reveals a great deal about proton structure
- Moving from a one-body density picture to a two-body density picture gives us access to the proton's internal many-body structure
  - We introduced double GPDs to study the correlations between the partons inside the proton with two-body densities
  - UPCs are an avenue for extracting such two-body densities from experiment

Soon to be posted on arXiv