QUARK-GLUON TOMOGRAPHY COLLABORATION

QuantOm Collaboration

Argonne NATIONAL LABORATORY

Progress towards a machine learning extraction of GPDs from data

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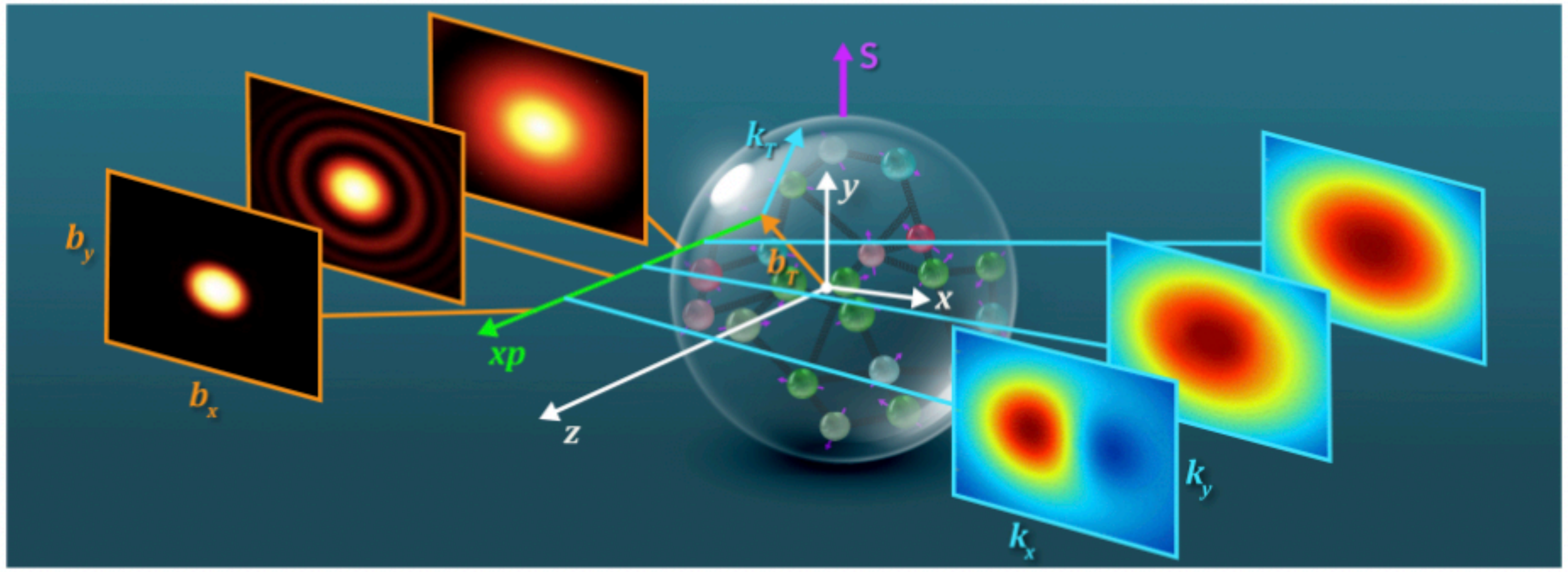
QCD Evolution 5-19-25

Introduction

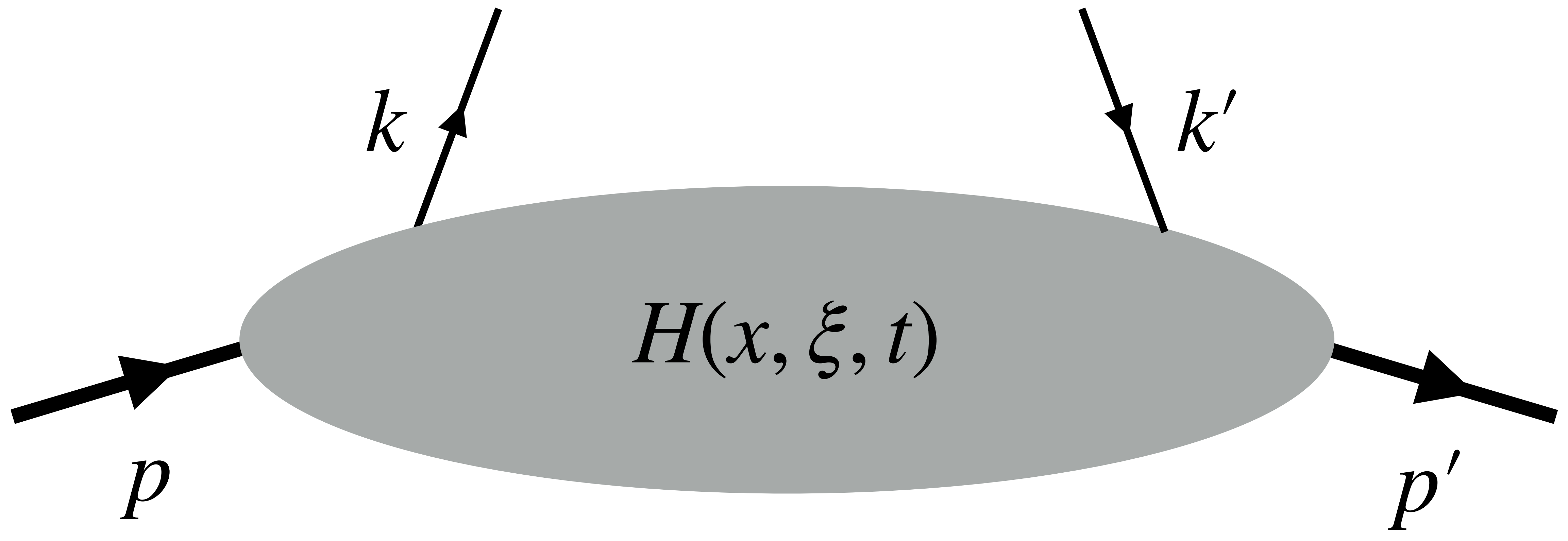
- Generalized Parton Distributions (GPDs) contain information about many hadron properties:
    - 3D structure
    - Spin sum
    - Pressure and shear force distributions
- The goal:
    - Perform a global analysis of GPDs from available data



Properties of GPDs

- Functions of x, ξ, and t:

$$x = \frac{k^+ + k^{'+}}{p^+ + p^{'+}} \quad \xi = \frac{p^{'+} - p^+}{p^+ + p^{'+}} \quad t = (p' - p)^2$$



- Forward limit $(\xi, t \to 0)$:
  - H maps to the unpolarized Parton Distribution Functions (PDFs)
  - $\tilde{H}$ maps to the polarized PDFs
  - Forward limits of E and $\tilde{E}$ do not map to known functions

Properties of GPDs

- Polynomiality:

$$\int_{-1}^{1} dx x^s H^a(x,\xi,t;\mu^2) = \sum_{i=0(\text{even})}^{s} (2\xi)^i A^a_{s+1,i}(t,\mu^2) + \text{mod}(s,2)(2\xi)^{s+1} C^a_{s+1}(t,\mu^2)$$

$$\int_{-1}^{1} dx x^s E^a(x,\xi,t;\mu^2) = \sum_{i=0(\text{even})}^{s} (2\xi)^i B^a_{s+1,i}(t,\mu^2) - \text{mod}(s,2)(2\xi)^{s+1} C^a_{s+1}(t,\mu^2)$$

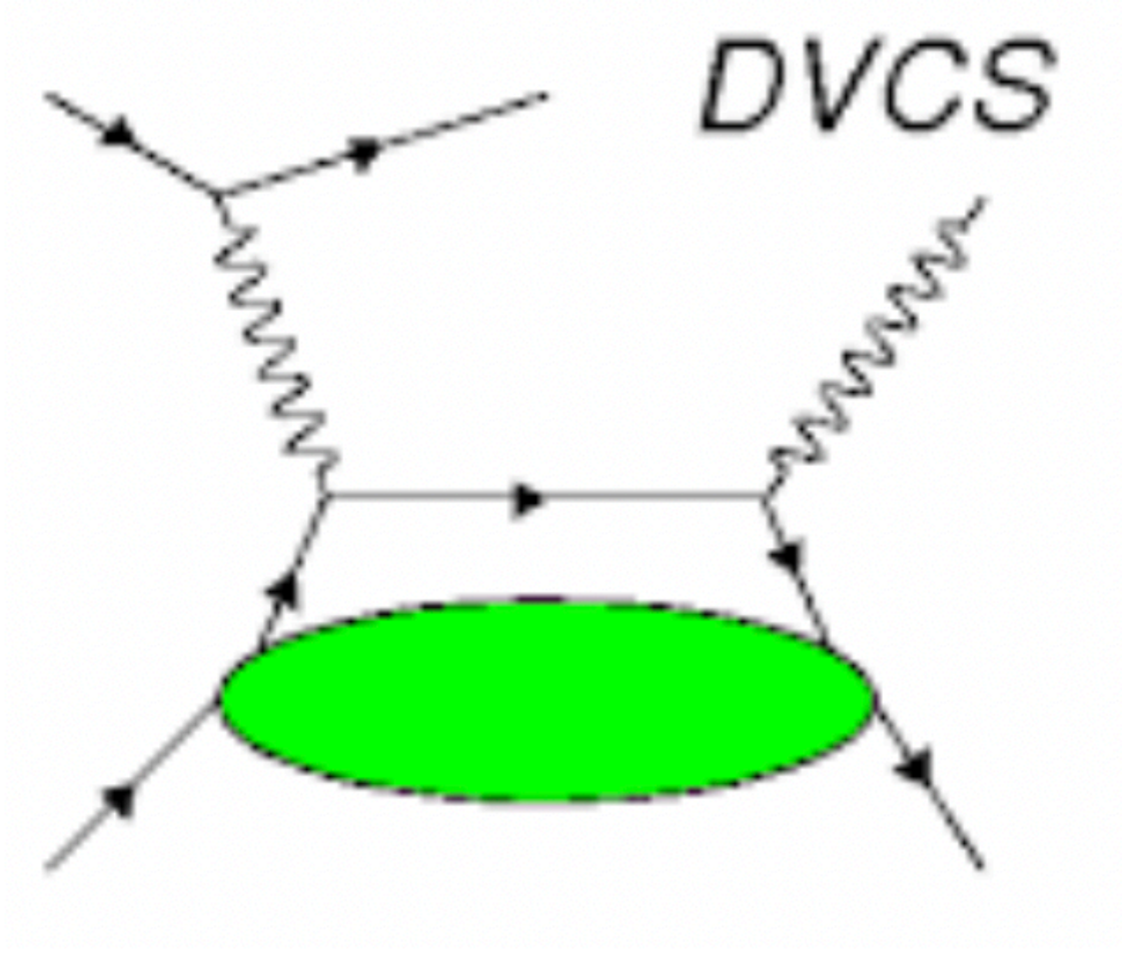
$$\int_{-1}^{1} dx x^s \tilde{H}^a(x,\xi,t;\mu^2) = \sum_{i=0(\text{even})}^{s} (2\xi)^i \tilde{A}^a_{s+1,i}(t,\mu^2)$$

$$\int_{-1}^{1} dx x^s \tilde{E}^a(x,\xi,t;\mu^2) = \sum_{i=0(\text{even})}^{s} (2\xi)^i \tilde{B}^a_{s+1,i}(t,\mu^2)$$

Phenomenological Challenges

- Functions of three variables (x, ξ, and t)
- Inverse Problem:
  - Shadow GPDs (SGPDS) (Bertone, et. al. Phys.Rev.D 103 (2021) 11, 114019):
    - There is an infinite number of functions that can give the same observable.

The Inverse Problem



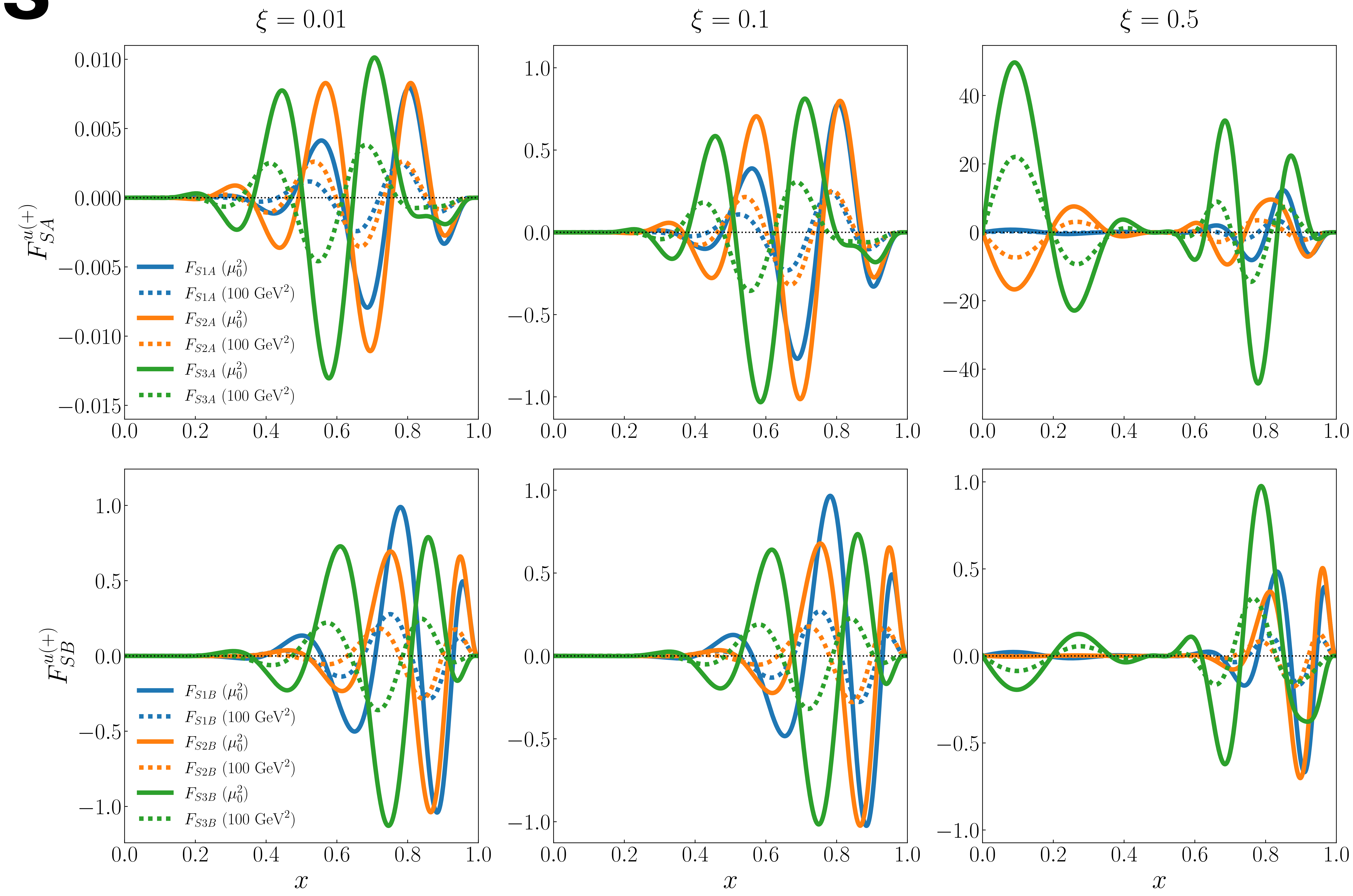
- Deeply virtual Compton scattering:
    - Compton Form Factors ($\mathscr{H}, \mathscr{E}, \tilde{\mathscr{H}}, \tilde{\mathscr{E}}$):

$$\mathscr{H}(\xi, t, Q^2) = \int_{-1}^{1} dx \sum_a C^a(x, \xi, Q^2, \mu^2)\, H^a(x, \xi, t; \mu^2) \qquad \tilde{\mathscr{H}}(\xi, t, Q^2) = \int_{-1}^{1} dx \sum_a \tilde{C}^a(x, \xi, Q^2, \mu^2)\, H^a(x, \xi, t; \mu^2)$$

$$\mathscr{E}(\xi, t, Q^2) = \int_{-1}^{1} dx \sum_a C^a(x, \xi, Q^2, \mu^2)\, E^a(x, \xi, t; \mu^2) \qquad \tilde{\mathscr{E}}(\xi, t, Q^2) = \int_{-1}^{1} dx \sum_a \tilde{C}^a(x, \xi, Q^2, \mu^2)\, E^a(x, \xi, t; \mu^2)$$

- x-dependence is lost in the integration
- SGPDS are functions that give zero contribution to the CFFs:
    - While a fit could obtain a GPD: Does the x-dependence represent the true GPD?

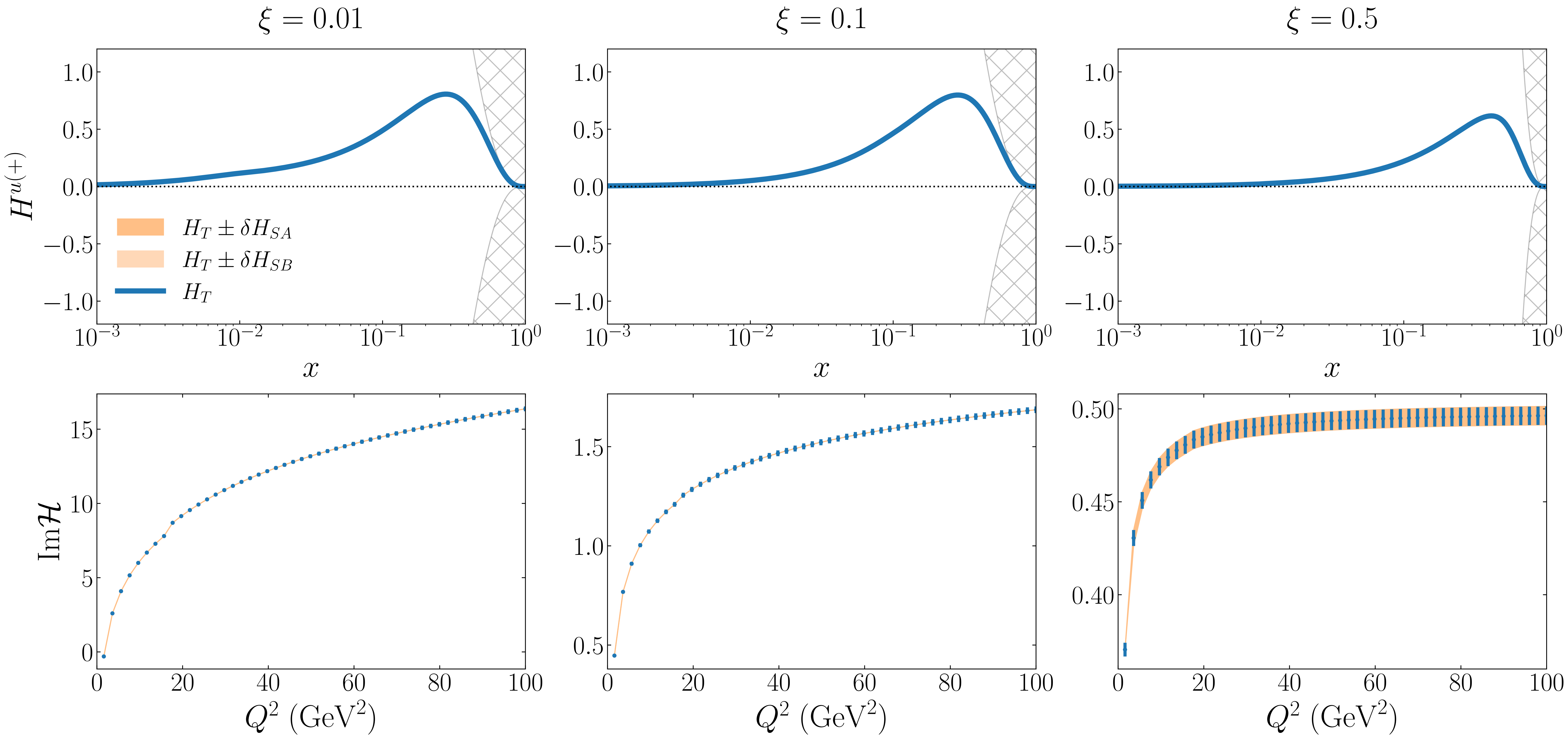
SGPDs



E. M. et al, Phys.Rev.D 108 (2023) 3, 036027

SGPDs

- Evidence that evolution can help constrain at least some SGPDs but unknown if this is true in general.



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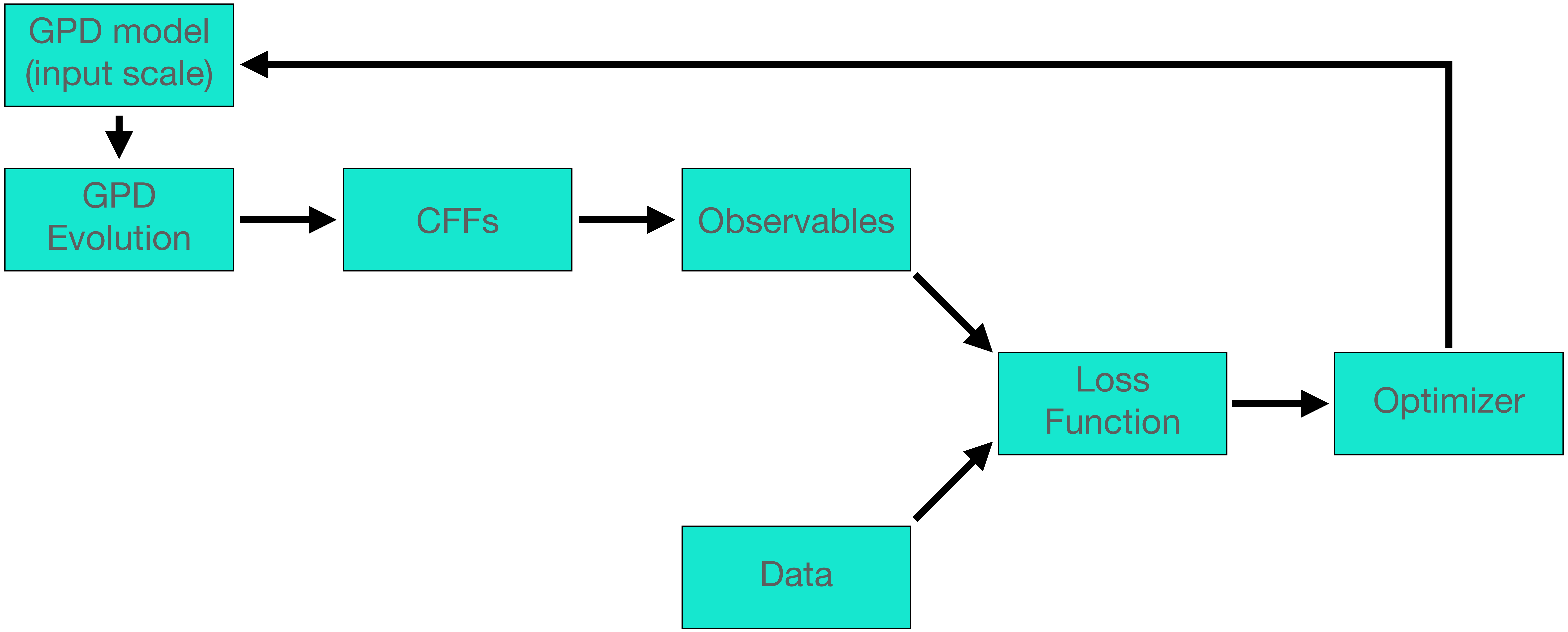
SGPDs

- Most parametric models lack the flexibility to thoroughly sample SGPDs
  - Different parametric models can fit the data equally well but could yield significantly different results
- Need a highly flexible model to accurately account for uncertainties while minimizing bias:
  - Use Neural Networks (NNs)
- Developed a machine learning framework for GPD extraction
  - Utilizing Bayesian Monte Carlo approach consistent with methods used by the Jefferson Lab Angular Momentum (JAM) Collaboration

The machinery



- All pieces are backward differentiable to facilitate machine learning

The machinery

- Loss function:
  - Typical chi squared function

$$\sum \left(\frac{\text{data} - \text{theory}}{\text{uncertainty}} \right)^2$$

- Optimizer:
  - Use PyTorch Adam algorithm
    - Stochastic Gradient Descent

Closure test

- Tested the machinery with a simple parametric model
- GPD model:
  - Utilizes double distributions to ensure the GPDs satisfy polynomiality:

$$H^f(x,\xi,t;\mu_0^2) = \int_\Omega d\beta d\alpha\, \delta(x-\beta-\xi\alpha)\left[H^f_{DD}(\beta,\alpha,t;\mu_0^2) + \xi\delta(\beta)D^f(\alpha,t;\mu_0^2)\right] \qquad \tilde{H}^f(x,\xi,t;\mu_0^2) = \int_\Omega d\beta d\alpha\, \delta(x-\beta-\xi\alpha)\left[\tilde{H}^f_{DD}(\beta,\alpha,t;\mu_0^2)\right]$$

$$E^f(x,\xi,t;\mu_0^2) = \int_\Omega d\beta d\alpha\, \delta(x-\beta-\xi\alpha)\left[E^f_{DD}(\beta,\alpha,t;\mu_0^2) - \xi\delta(\beta)D^f(\alpha,t;\mu_0^2)\right] \qquad \tilde{E}^f(x,\xi,t;\mu_0^2) = \int_\Omega d\beta d\alpha\, \delta(x-\beta-\xi\alpha)\left[\tilde{E}^f_{DD}(\beta,\alpha,t;\mu_0^2)\right]$$

  - Double distributions:
    - Use GK model (Kroll, Moutarde, Sabatie, Eur. Phys. J. C (2013) 73:2278):

$$H^{uv}_{DD}(\alpha,\beta,t) = \Theta(\beta)N\beta^{-a}(1-\beta)^b(c_0 + c_1\beta^{1/2} + c_2\beta + c_3\beta^{3/2})\frac{\Gamma(2n+2)}{2^{2n+1}\Gamma^2(n+1)}\frac{\left[(1-\beta)^2-\alpha^2\right]^n}{(1-\beta)^{2n+1}}\,\beta^{-\alpha_p t}e^{\beta_p t}$$

  - D term:
    - Use first three terms of a Gegenbauer series (Goeke, Polyakov, Vanderhaeghan, Prog. Part. Nucl. Phys. 47, 401 (2001)

$$D^q(\alpha,t;\theta_D) = \frac{1-\alpha^2}{N_f}\left[d_1C_1^{3/2} + d_3C_3^{3/2} + d_5C_5^{3/2}\right](1-t/\lambda)^{-\alpha_D}$$

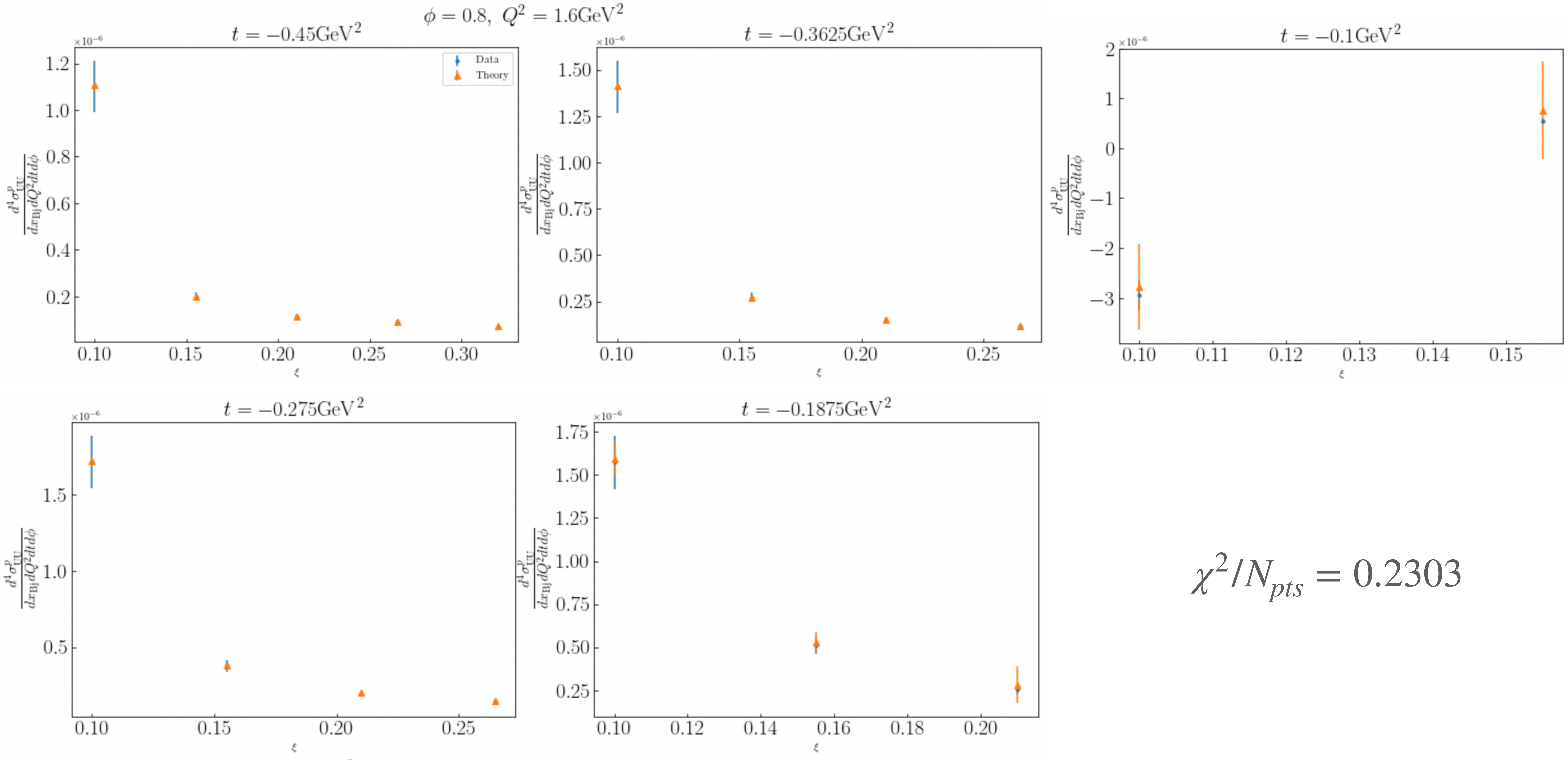
Closure test

- Generated pseudodata for various DVCS observables from model GPDs:
  - Assume 10% uncertainty for all data points
- Fitted parameters (31 in total):
  - Fit uv and dv double distribution parameters:
    - For H and $\tilde{H}$, keep pdf parameters fixed and only fit profile function parameters
  - Fit the coefficients and the t dependence parameters in the D term (same for all quark flavors)

Closure test

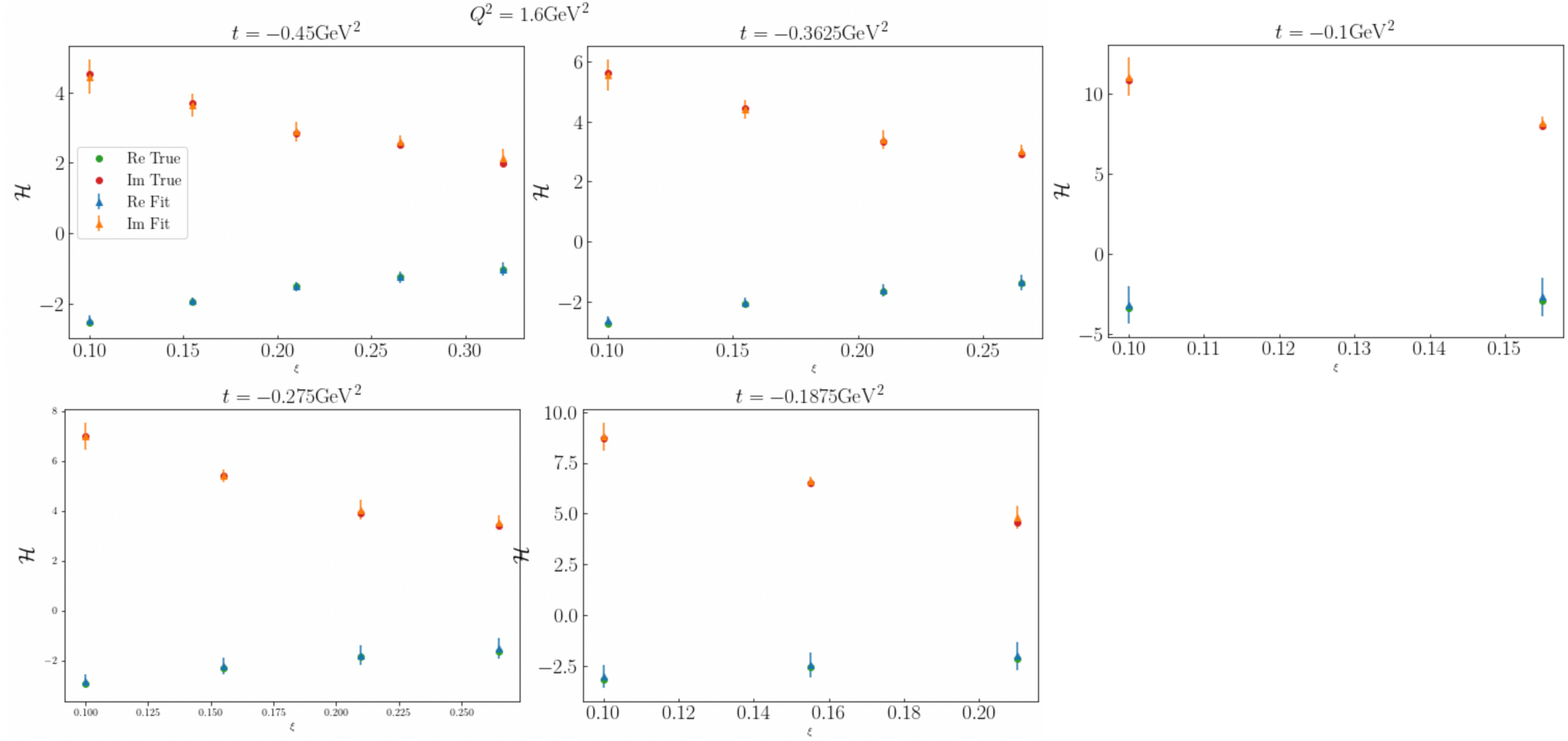
- Monte Carlo fit:
  - Conduct multiple fits (called replicas):
    - For each replica:
      - Starting parameters are randomly sampled
      - Data values sampled from Gaussian distribution
  - Calculate average and standard deviation of all replicas

Closure test results



$$\chi^2/N_{pts} = 0.2303$$

Closure test results



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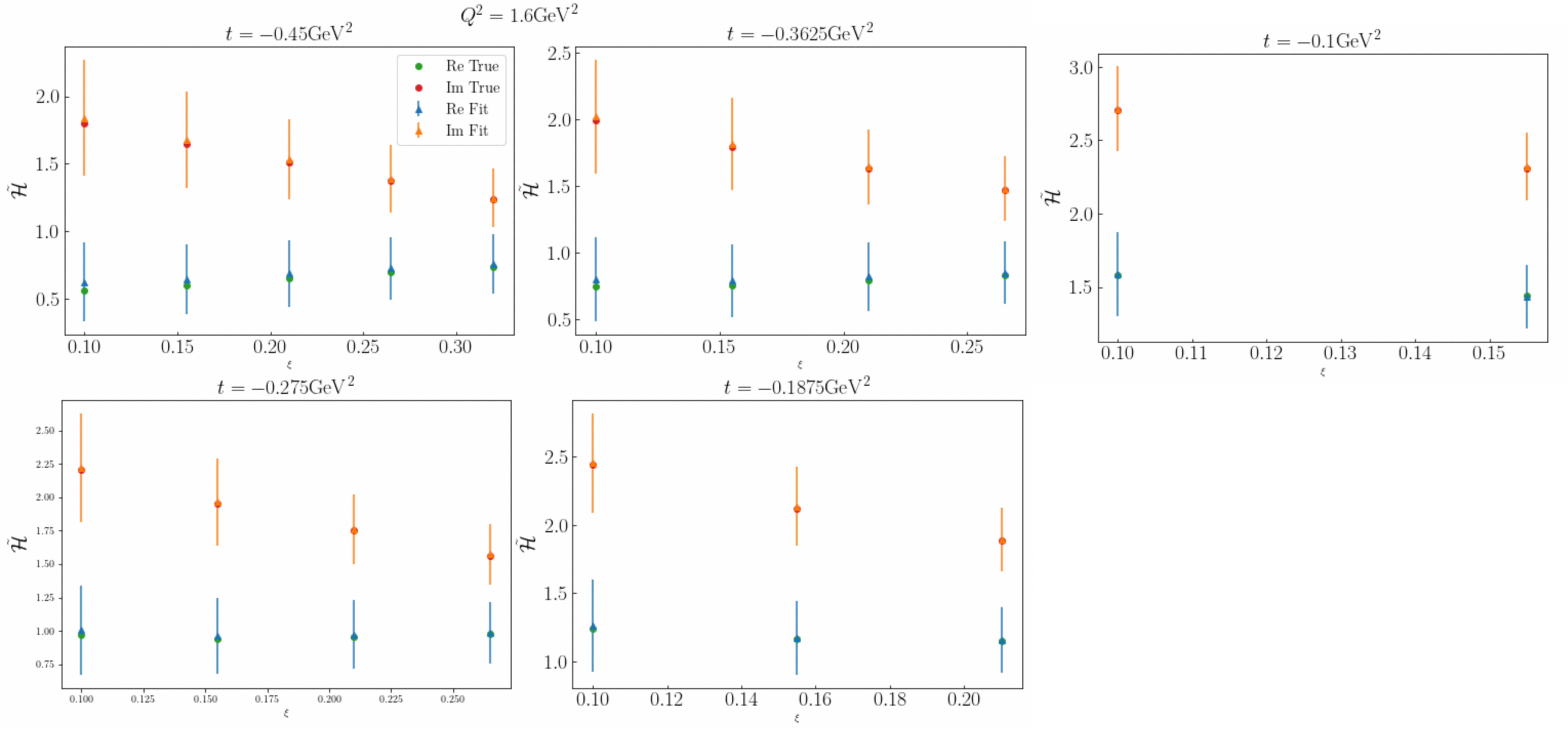

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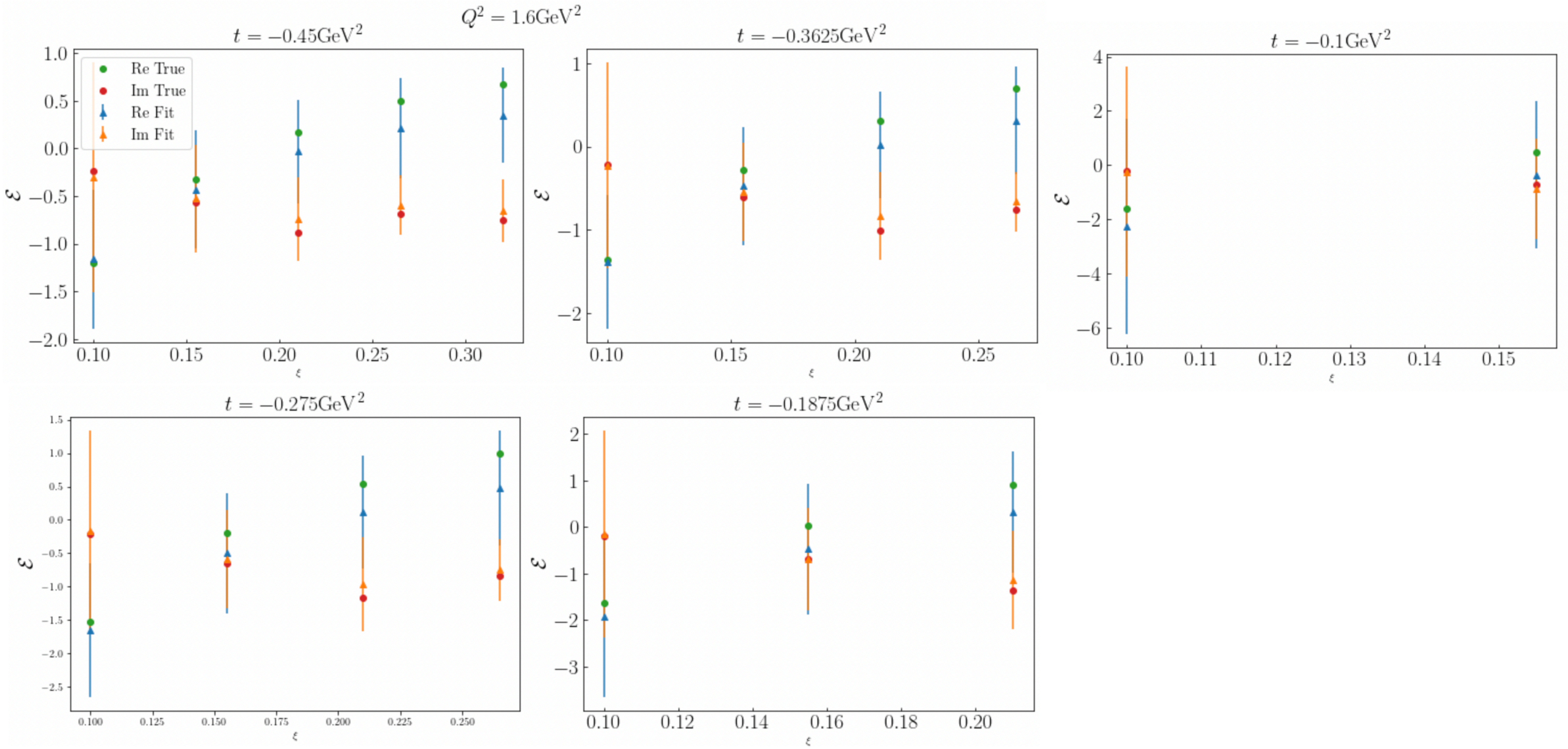

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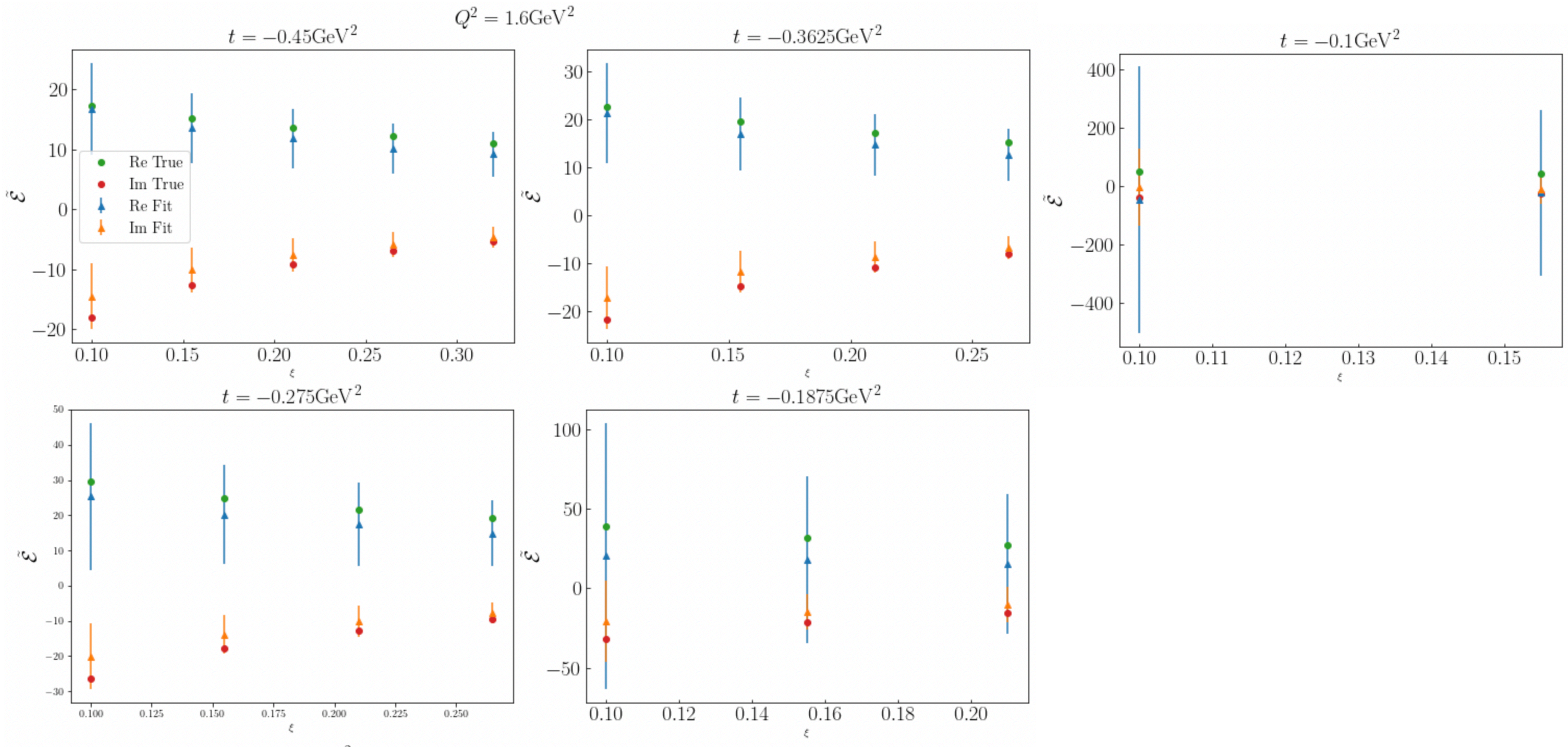

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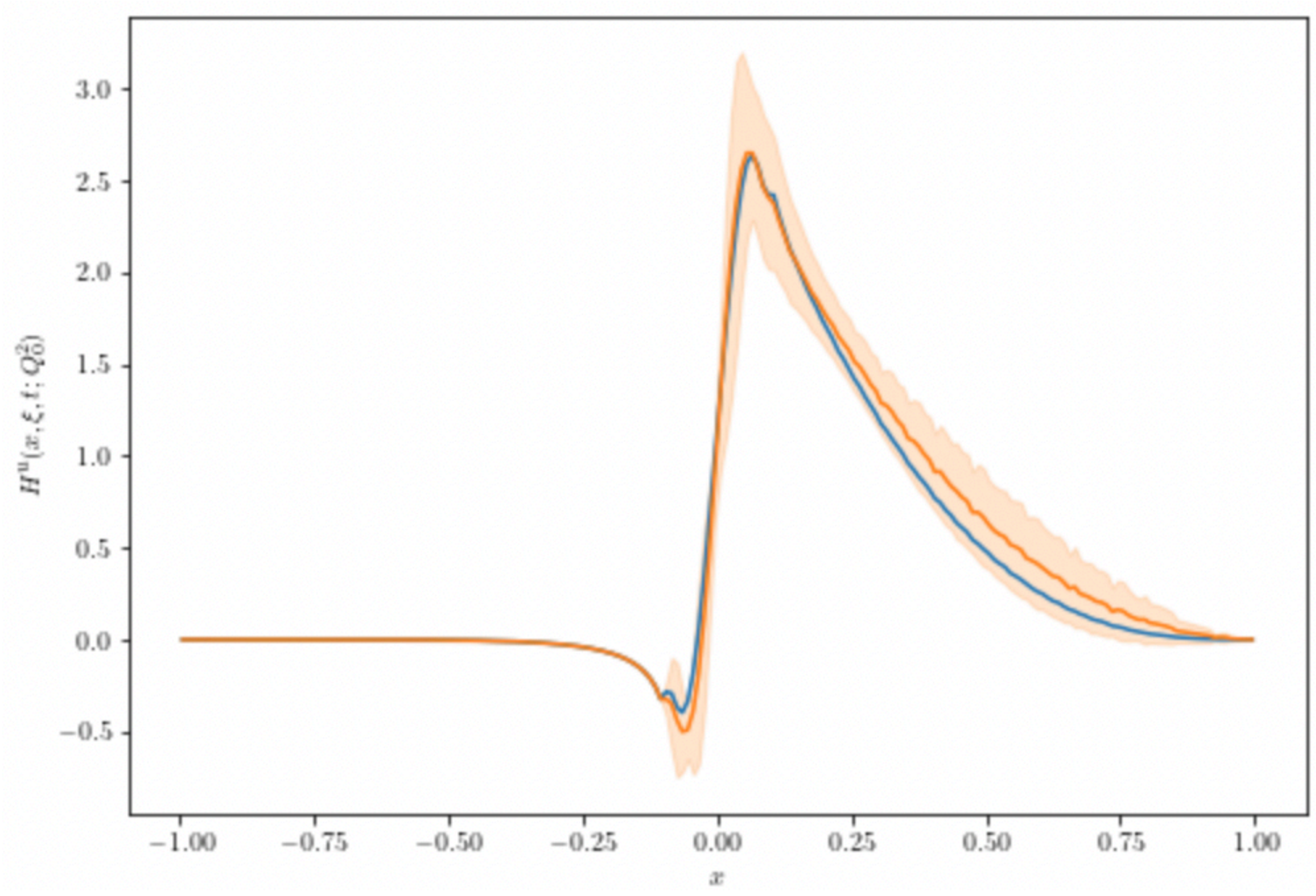

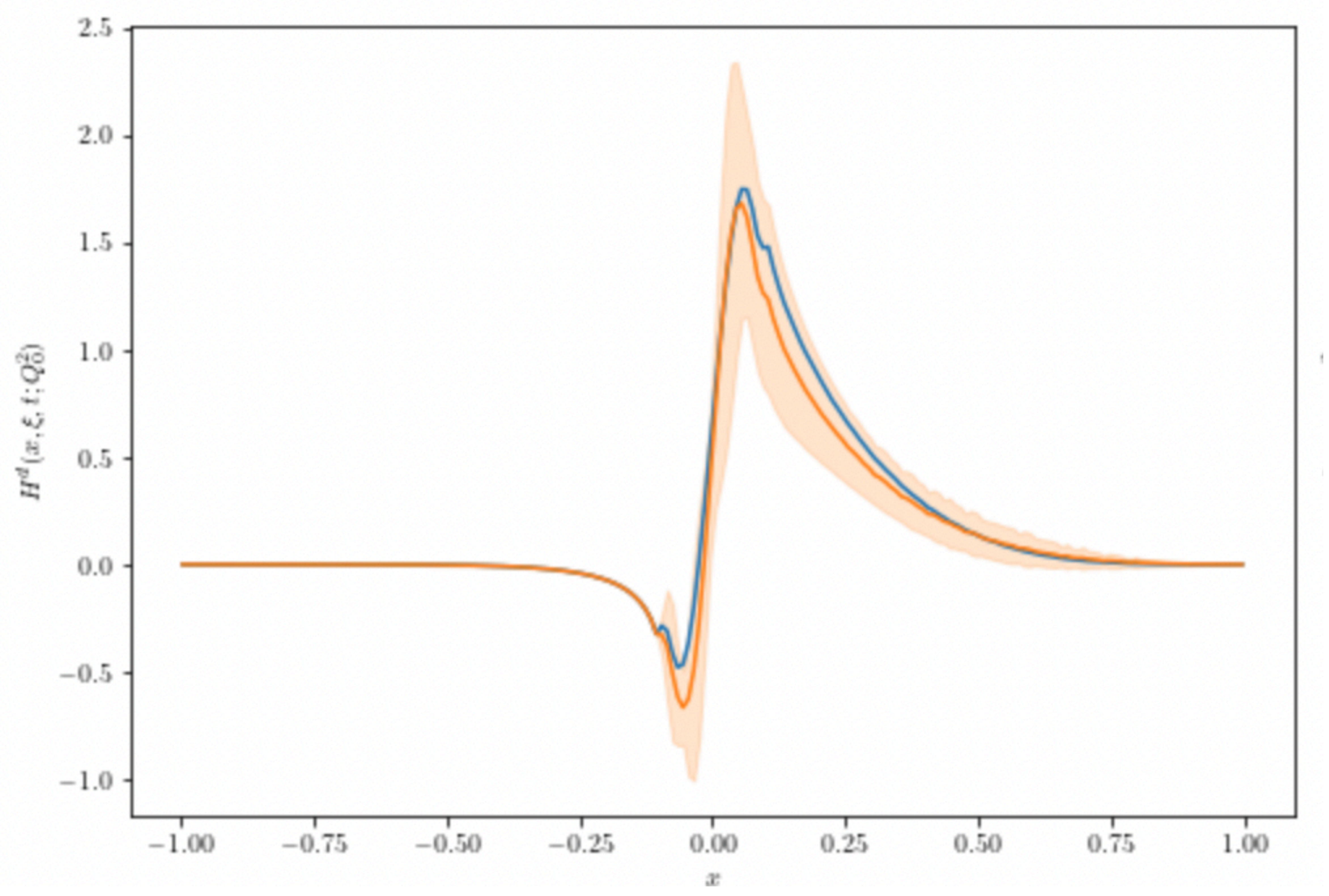

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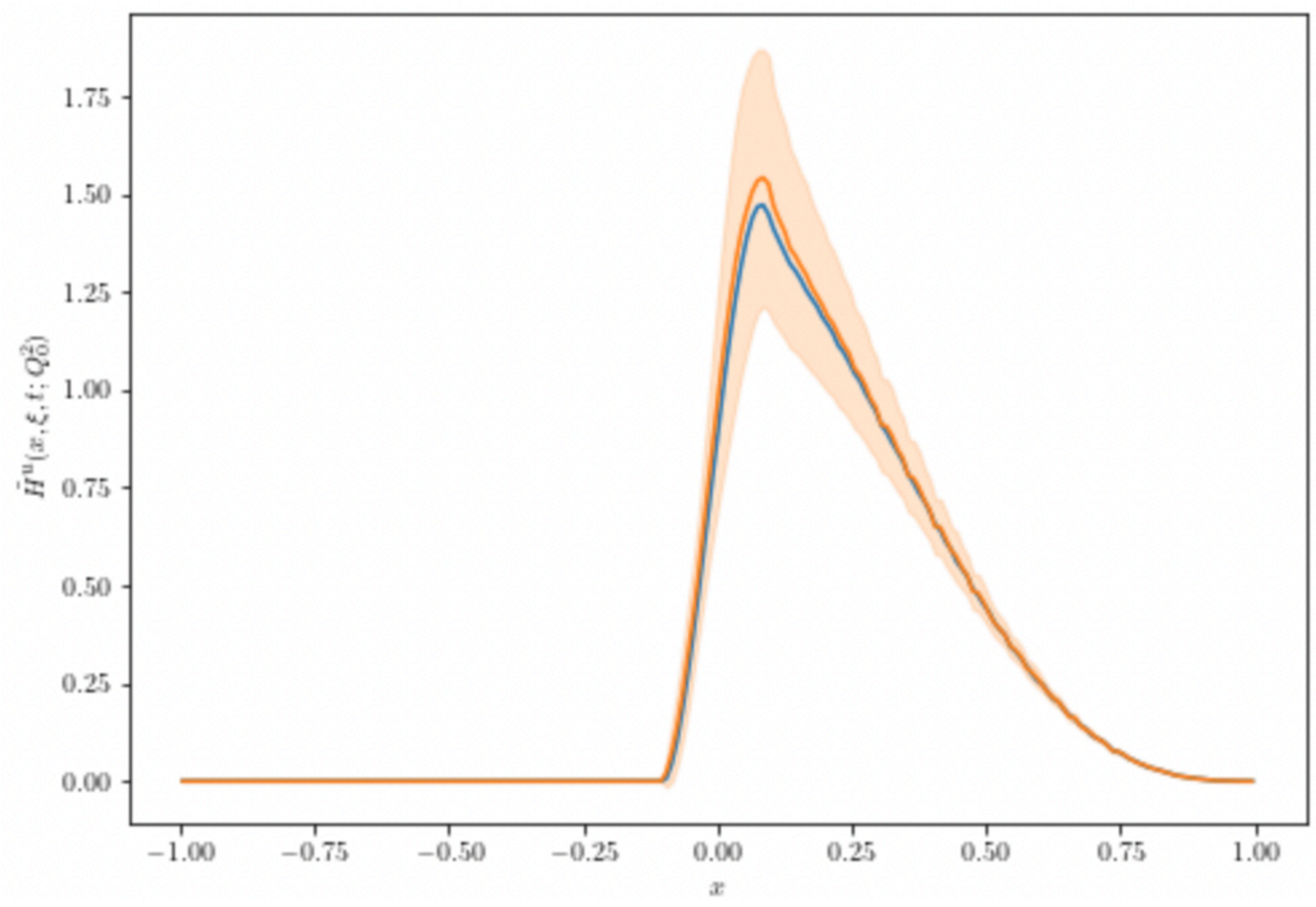

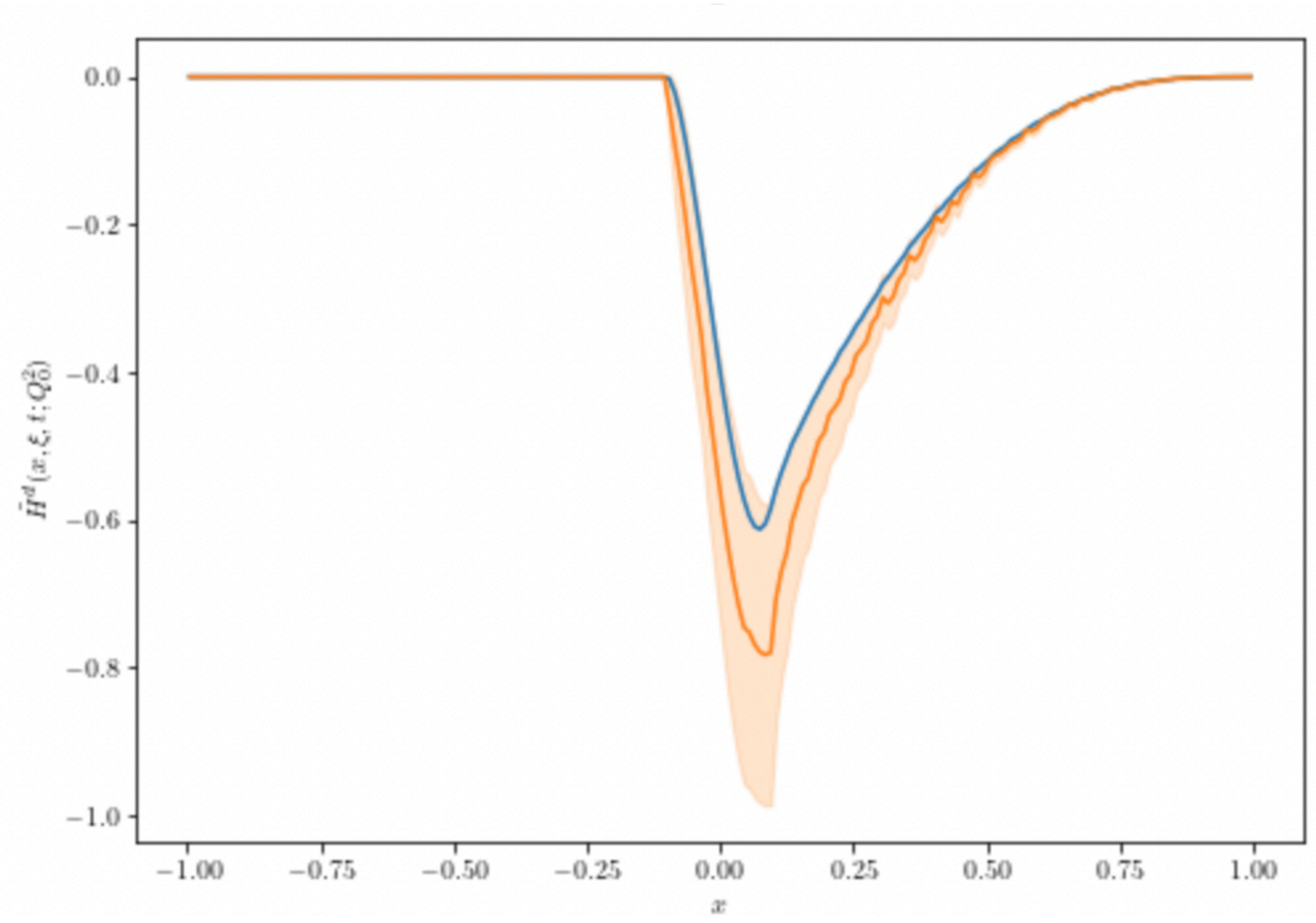

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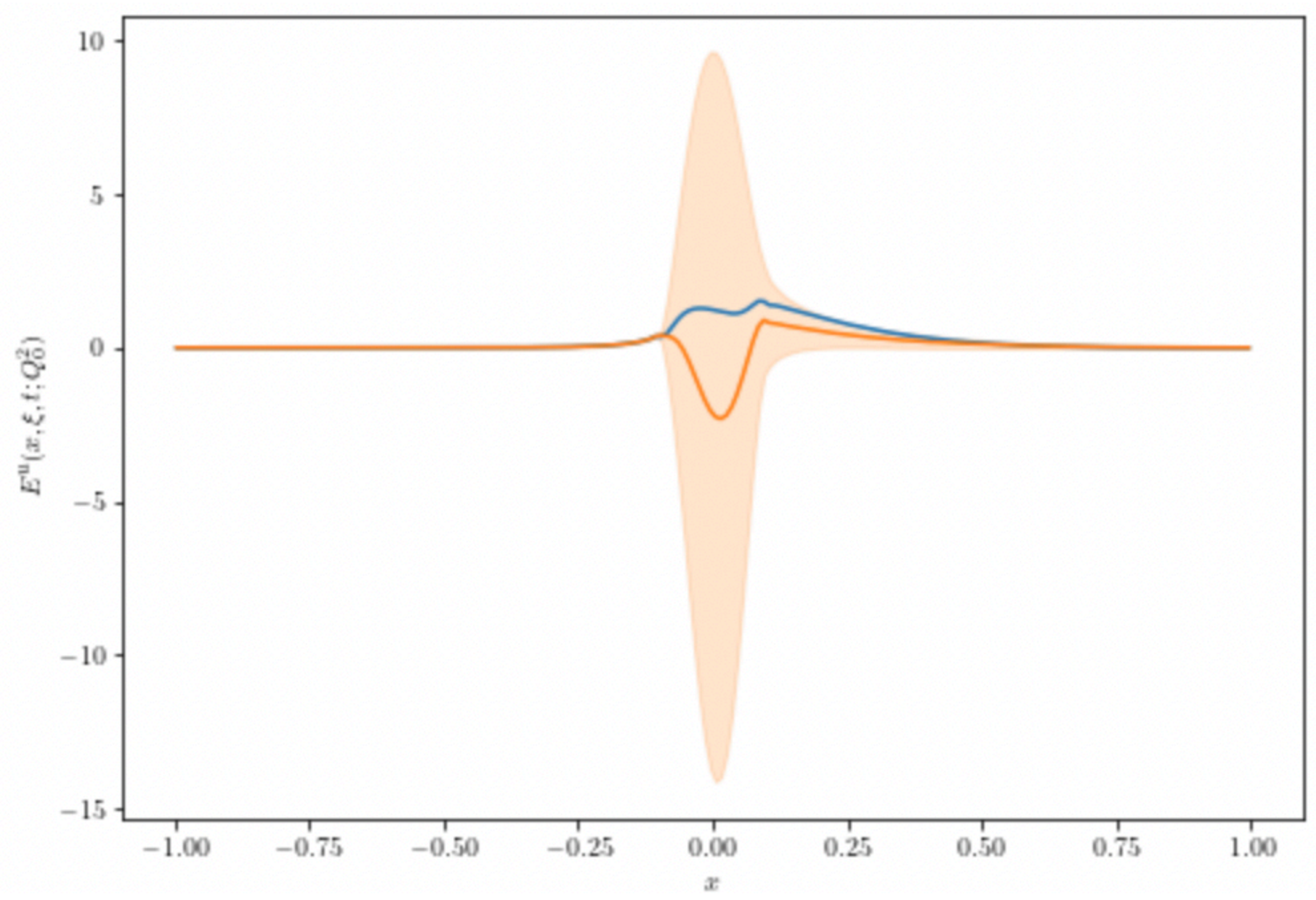

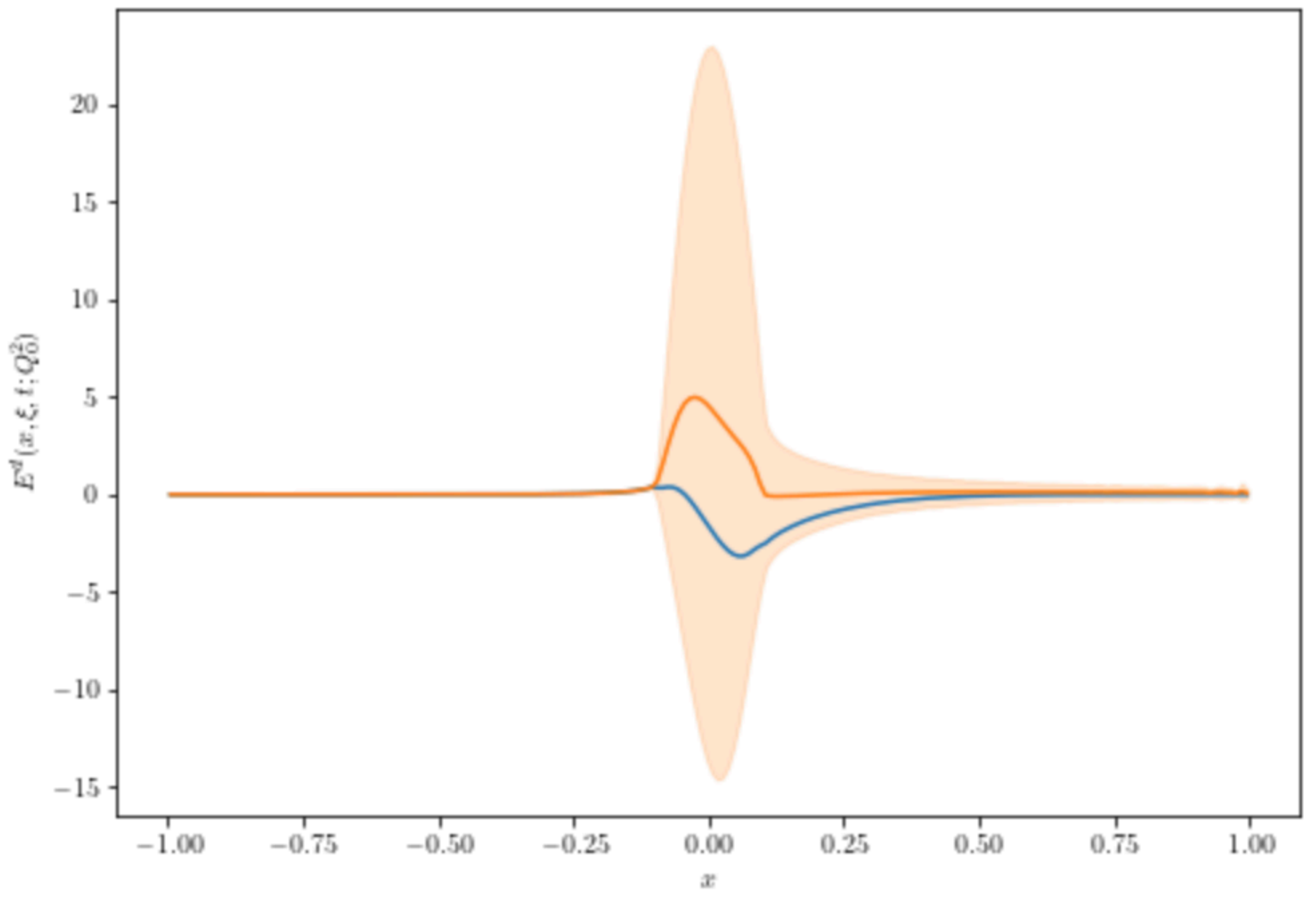

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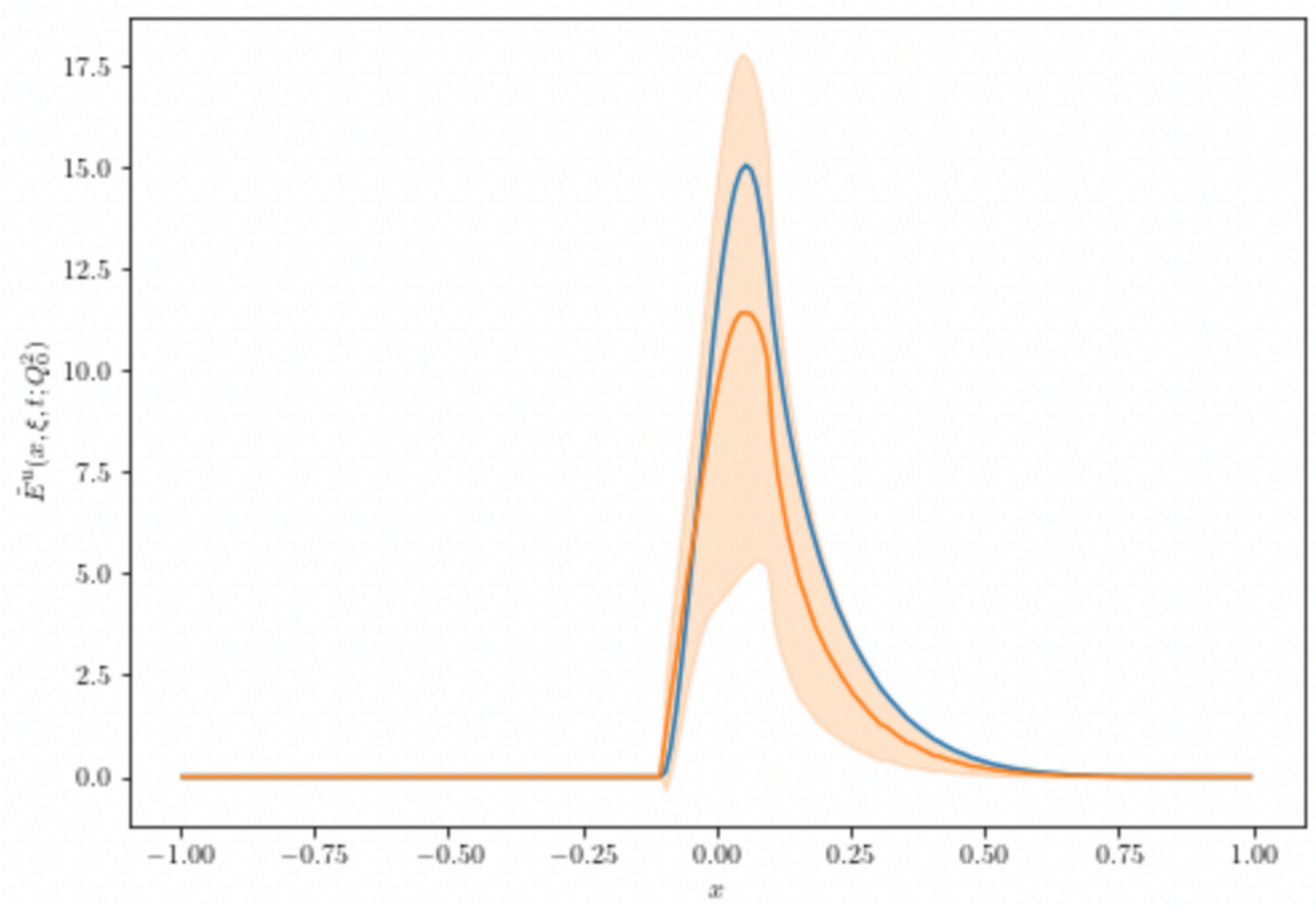
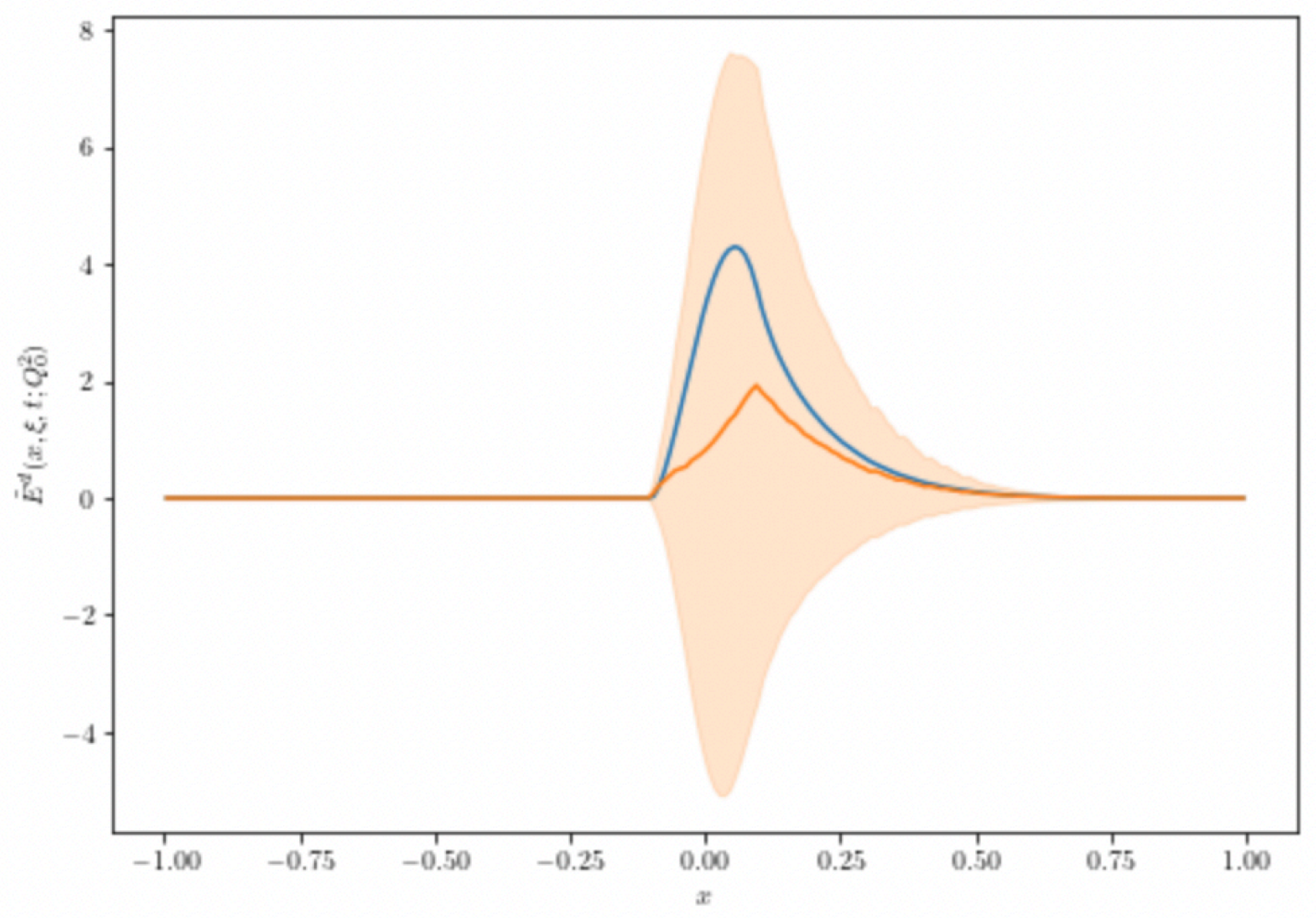
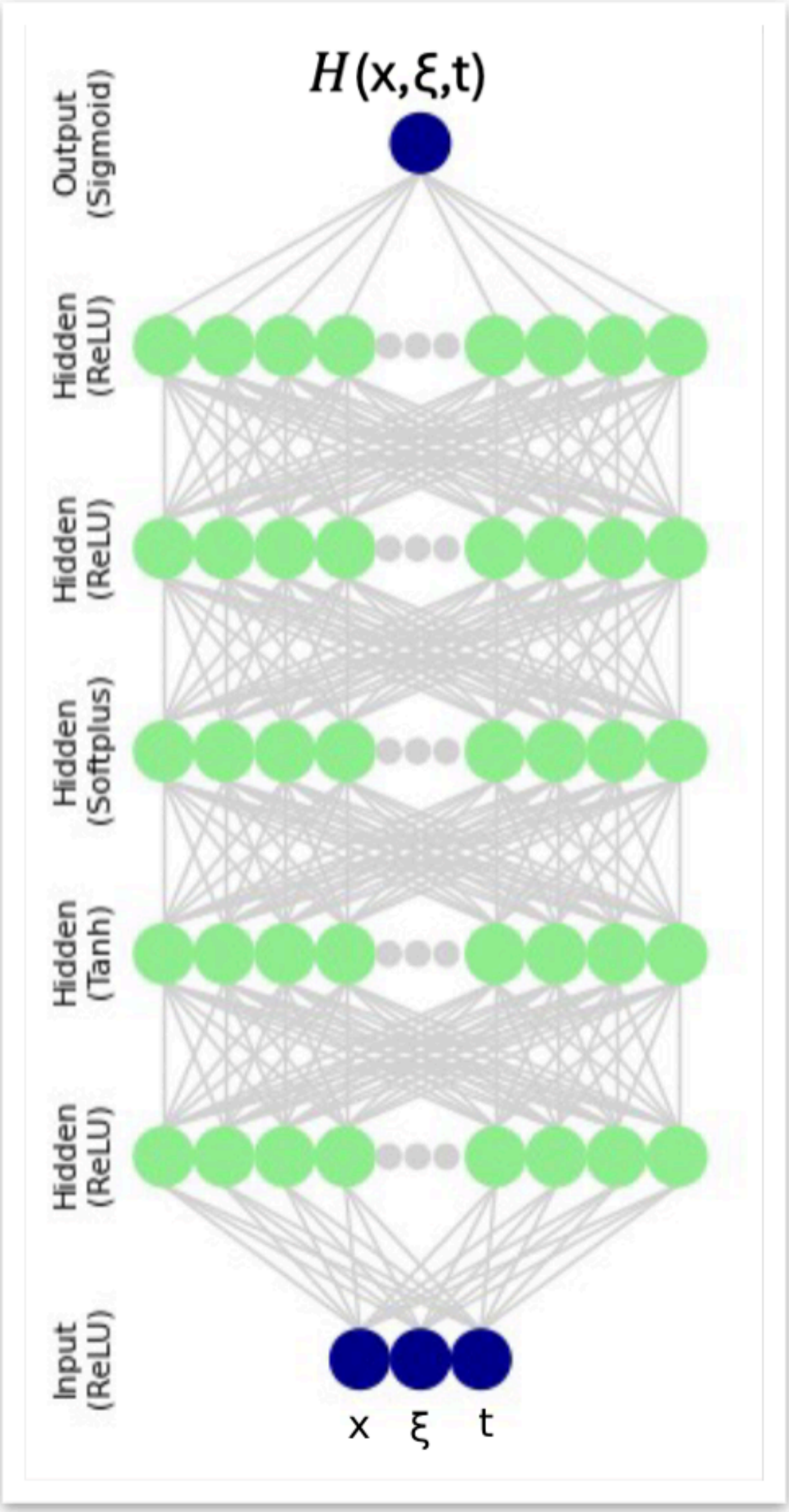




Neural Network Model

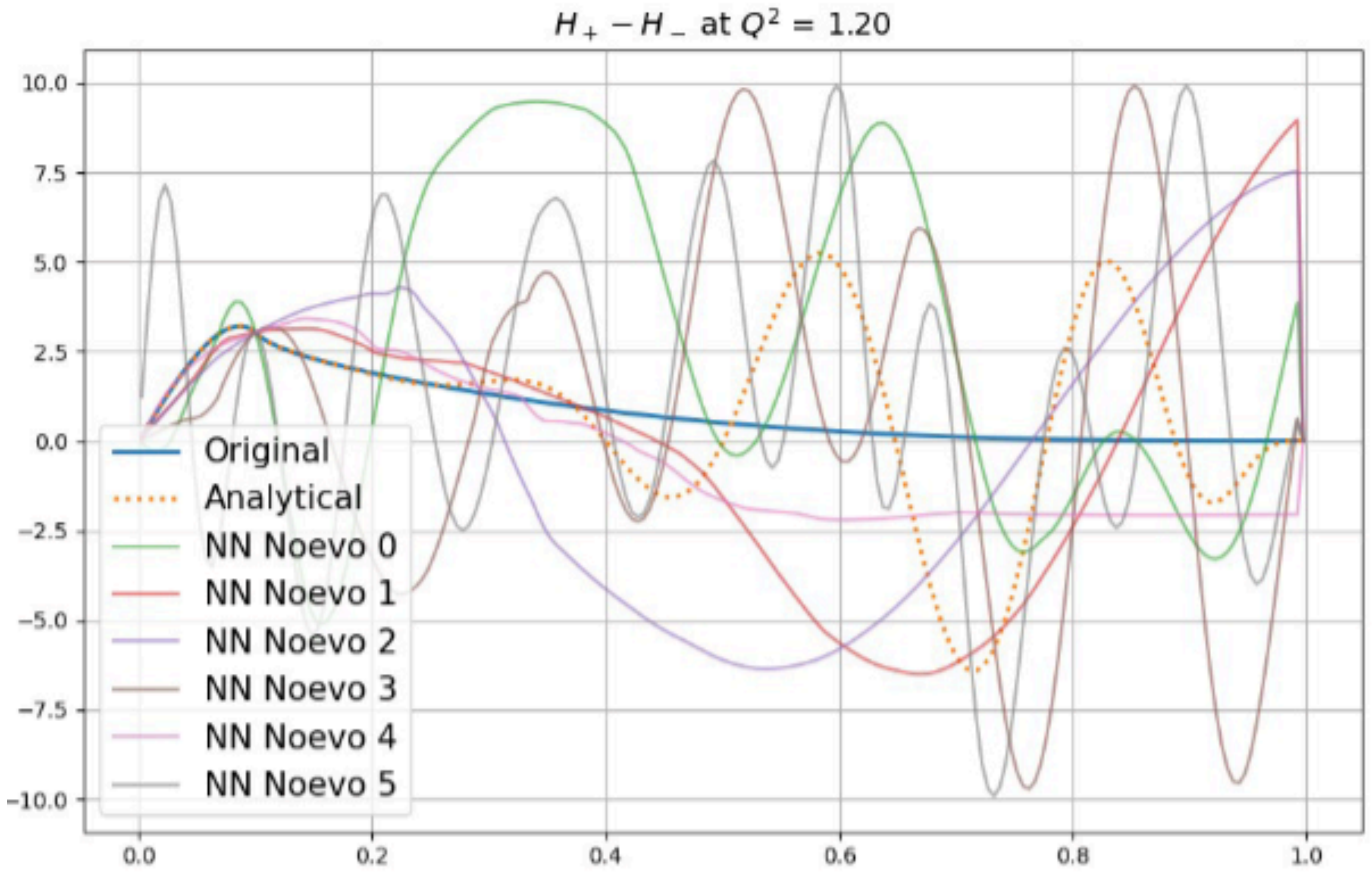
- Setup a NN to model the GPD H directly:
  - Currently not imposing polynomiality or forward limit
- Generated CFF data using the parametric model
- Train the NN to the CFF data

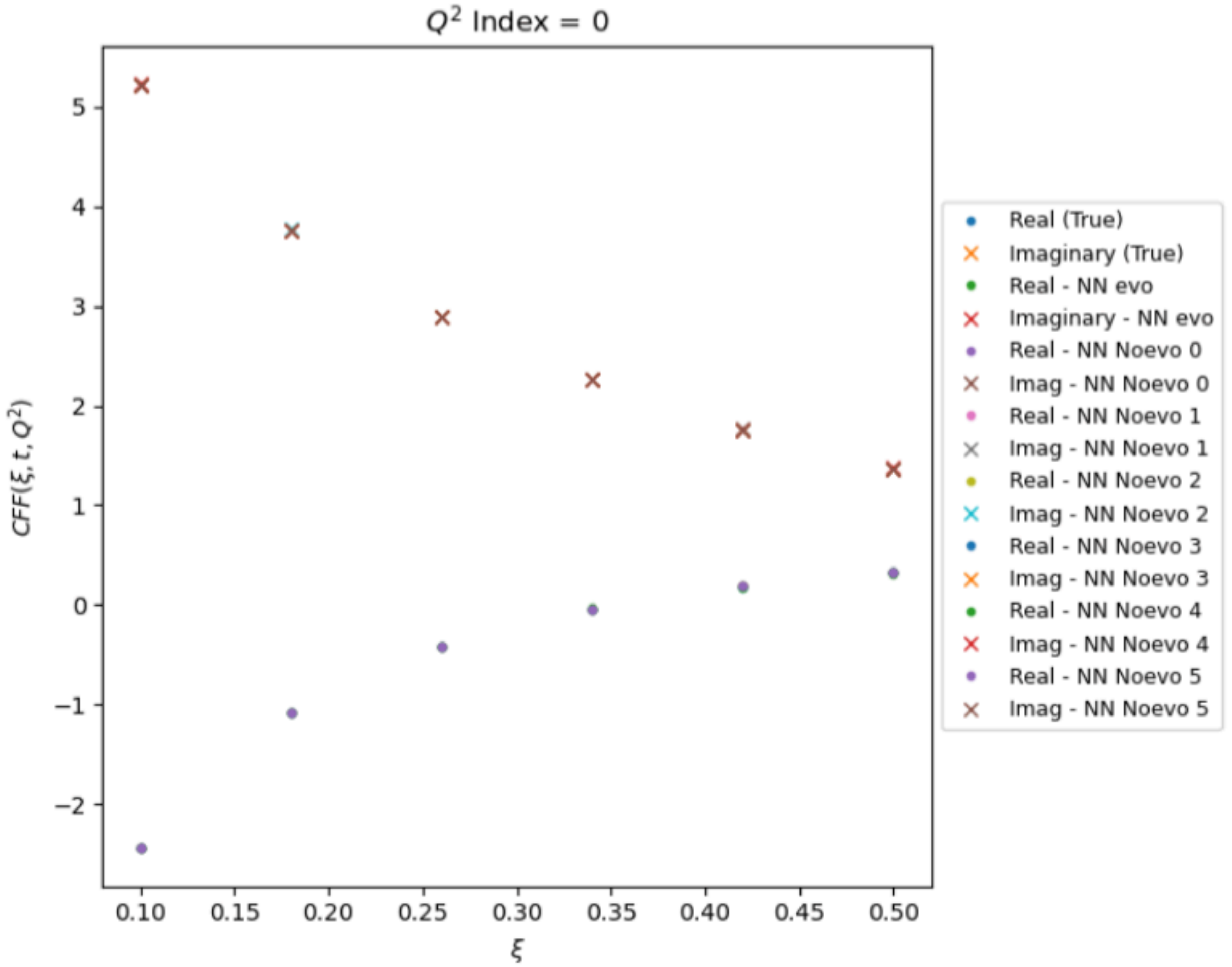


Neural Network Model

- NN is able to find significantly different functions that all give the same CFFs when trained to data at only one Q^2

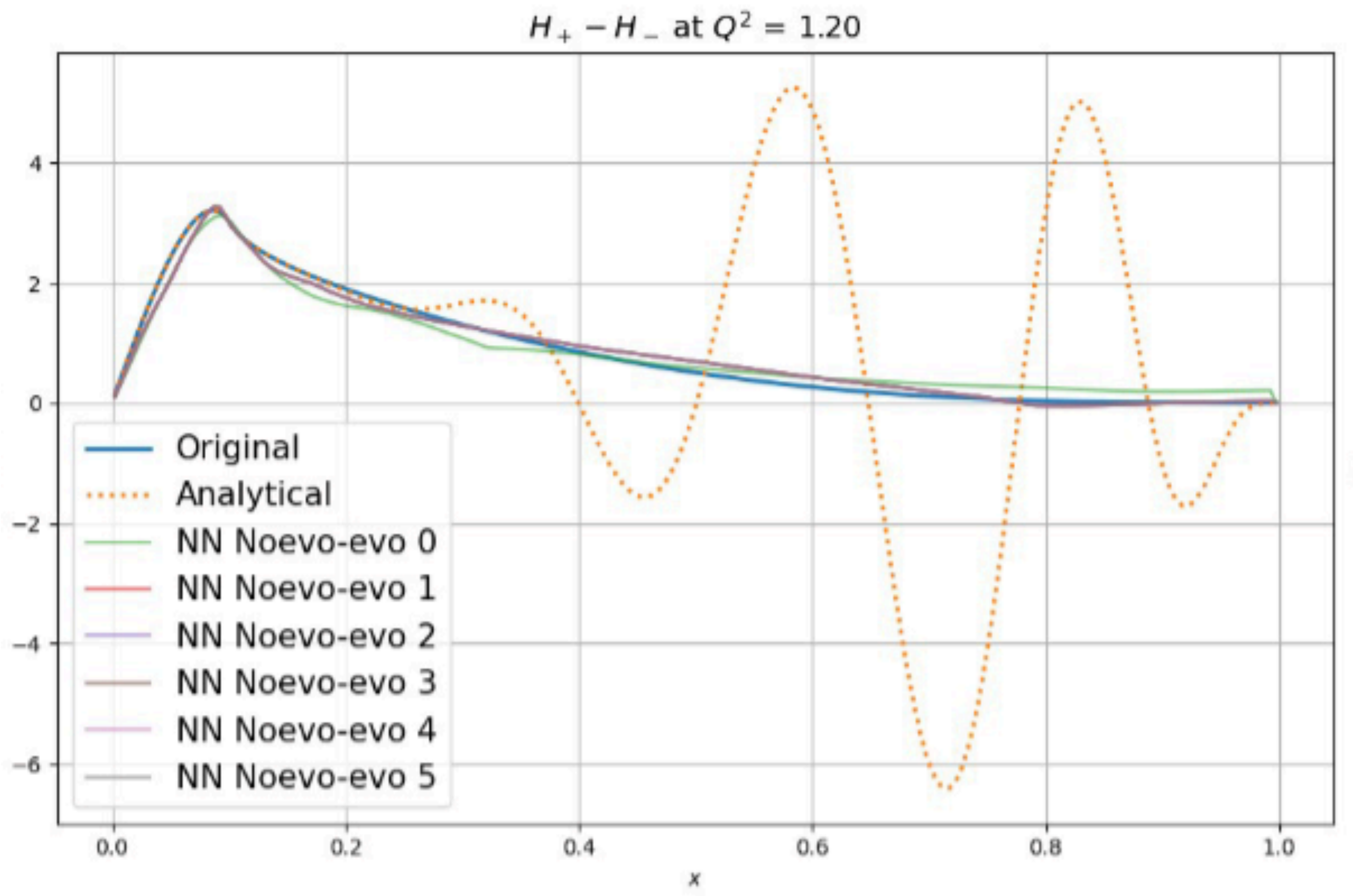
- NN is able to sample SGPDs





Effect of Evolution

- Can use the NN setup to explore the impact of evolution on SGPDs
- Take the NN initially trained at fixed energy and train to data at multiple energy scales
- NN model is able to match the truth when evolution is included.



Conclusion and Next Steps

- Summary:
  - Successful closure tests of fitting machinery with parametric model of the GPDs
  - Developed a NN model capable of capturing SGPDs
    - Thus far, NN has been able to find the true GPD once data at multiple Q^2 is used in the training
- Next Steps:
  - Parametric model:
    - Conduct an analysis with real data
  - NN model:
    - Implement means of enforcing polynomiality and forward limits on the NN at the GPD level
    - Expand to modeling all four GPDs with multiple flavors
    - Conduct an analysis with real data