Stony Brook University　　Center for Nuclear Theory　　FWF Austrian Science Fund

Probing the Nucleon's Spin Structure: A String-Based Approach to Generalized Parton Distributions

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in collaboration with
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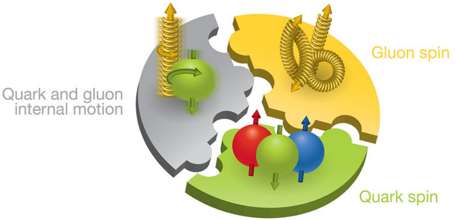
Center for Nuclear Theory, Stony Brook University

QCD Evolution
May 19, 2025

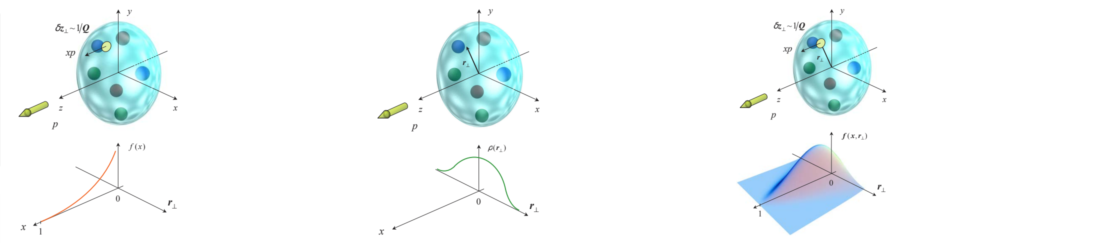
The proton as physics laboratory

- Easily accessible
- High-precision measurements
- But still a lot of discovery potential



Source: EIC Homepage

- Generalized Parton Distributions (GPDs) and their moments contain full information (x, η, Δ) on flavor, spin and mass composition of a given hadron:
  - Parton Distribution Functions (PDFs) $\Rightarrow$ momentum distribution
  - Form Factors $\Rightarrow$ charge, shear, pressure distributions
  - spin, mass



Source: Phys.Rept. 418 (2005) 1-387

Outline

Outline

Prerequisites

GPDs are coefficients in the decomposition of off-forward matrix elements

$$F_{q,g}^{V,A,T}(x,\eta,t,\mu) = \int \frac{\mathrm{d}z^-}{2\pi} e^{ixz^-P} \langle p_2 | \mathcal{O}_{q,g}^{V,A,T} | p_1 \rangle$$

where, e.g.,

$$\mathcal{O}_q^V = \overline{\psi}_q(z_1^-)[z_1^-, z_2^-]\gamma^+\psi_q(z_2^-), \quad \mathcal{O}_g^V = F_a^{+\mu}(z_1^-)[z_1^-, z_2^-]^{ab} g_{\mu\nu} F_b^{\nu+}$$

and thus

$$F_{q,g}^V(x,\eta,t,\mu) = \overline{u}(p_2)\gamma^+ u_1(p_1)\, H_{q,g}(x,\eta,t,\mu) + \frac{i\Delta_\nu}{2m_N}\overline{u}(p_2)\sigma^{+\nu}u(p_1)\, E_{q,g}(x,\eta,t,\mu)$$

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Note: Dependence on three kinematical parameters (tricky), experimental extraction from convolution with scattering kernels → Deconvolution problem (See Kiminad's talk)

Relation to Form Factors

Charge distribution

$$F_1^q(t) = \int_{-1}^{1} \mathrm{d}x\, H_q(x,0,t) = \mathbb{H}_q(n=1,\eta,t),$$

$$F_2^q(t) = \int_{-1}^{1} \mathrm{d}x\, E_q(x,0,t) = \mathbb{E}_q(n=1,\eta,t),$$

Mass distribution

$$A_q(t) + \eta^2 D_q(t) = \int \mathrm{d}x\, x\, H_q(x,\eta,t) = \mathbb{H}_q(n=2,\eta,t),$$

$$A_g(t) + \eta^2 D_g(t) = \int \mathrm{d}x\, H_g(x,\eta,t) = \mathbb{H}_g(n=2,\eta,t).$$

Higher moments contain information on **higher-spin** probes with additional skewness dependence fixed by polynomiality:

$$\mathbb{H}_g(n,\eta,t) = \frac{1}{2}\int_{-1}^{1} \mathrm{d}x\, x^{n-2} H_q(x,\eta,t) = \sum_{k=0}^{n-2} \eta^k A_{n,k}^g(t) + \eta^n D_n^g(t)$$

D. Müller, A. Schäfer, Nucl.Phys.B 739 (2006)

Conformal moment expansion for arbitrary GPD $G_{q,g} = H_{q,g}, \widetilde{H}_{q,g}, E_{q,g}, \ldots$

$$G_{q,g}(x,t,\eta) = \sum_{n=1,2}^{\infty} (-1)^{n\pm 1} p_n^{q,g}(x,\eta) \mathbb{G}_{q,g}(n,t,\eta)$$

with $p_n^{q,g}(x,\eta) \sim C_{n-1,n-2}^{3/2,5/2}(-x/\eta)$ forming an ONB $\Rightarrow$ project out conformal moments

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Divergent as a sum of polynomials $\Rightarrow$ resum via extension to complex conformal spin-j

$$n \to j, \qquad \sum_n (-1)^{n\pm 1} \to \frac{1}{2i} \oint_{\mathcal{C}} \frac{\mathrm{d}j}{\sin \pi j}, \qquad p_n^{q,g} \to p_j^{q,g}$$

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Analytic continuation uniquely fixed by polynomiality, crossing symmetry...
$\Rightarrow$ Mellin-Barnes Integral:

$$G_q(x,t,\eta) \sim \frac{1}{2i} \int_{c-i\infty}^{c+i\infty} \frac{\mathrm{d}j}{\sin \pi j} p_j^{q,g}(x,\eta) \mathbb{G}_q(j,t,\eta)$$

Example: Quark GPD

$$H_q(x,\eta,t,\mu) = \sum_{n=1}^{\infty} (-1)^{n-1} p_n(x,\eta) \mathbb{H}_q(n,\eta,t,\mu)$$

$$p_n(x,\eta) = \frac{1}{\eta^n} p_n\left(\frac{x}{\eta}\right), \qquad p_n(x) = \theta(1-|x|) \frac{2^{n-1}\Gamma(3/2+n)}{\Gamma(3/2)\Gamma(2+n)} (1-x^2) C_{n-1}^{3/2}(-x)$$

Gegenbauer polynomials $C_n^{3/2}(x)$ diagonalize leading order evolution equations and form an orthonormal basis.

The non-singlet isovector and isoscalar GPDs are obtained by utilizing reflection symmetry ($p_n(-x,\eta) = (-1)^n p_n(x,\eta)$) of the conformal partial waves

$$H^{(-)}_{u\pm d}(x,\eta,t,\mu) = \sum_{n=1}^{\infty}(-1)^{n-1}(p_n(x,\eta) - p_n(-x,\eta)\mathbb{H}^{(-)}_{u\pm d}(n,\eta,\Delta,\mu)$$

which corresponds to the exchange of a Regge trajectory with spin 1,3,5... cf. Vector Meson Dominance (VMD) of electromagnetic form factors

Input Form Factors

Utilizing the MSTW (AAC) PDFs for the unpolarized (polarized) case

$$\mathbb{H}^{(-)}_{u+d}(j,\eta,t;\mu_0) = \int_0^1 dx\, \frac{u_v(x;\mu_0)+d_v(x;\mu_0)}{x^{1-j+\alpha'_{u+d}t}} \overset{j=1}{=} 3(F_1^p(t)-F_1^n(t))\,,$$

we can extract the Regge slopes from know form factors. And evolve to leading-order via

$$\mathbb{H}^{(-)}_{u\pm d}(j,\eta,t,\mu) = \mathbb{H}^{(-)}_{u\pm d}(j,\eta,t,\mu_0)\left(\frac{\alpha_s(\mu_0^2)}{\alpha_s(\mu^2)}\right)^{\frac{\gamma_{j-1}^{qq;\mathrm{NS}}}{\beta_0}}\,.$$

Difficulties at NLO

Moments no longer evolve autonomously but evolution has upper-triangular form

$$F_j^q(\eta, t, \mu) = \mathcal{E}_{j-1}(\mu, \mu_0) F_j(\eta, t, \mu_0) + \sum_{k=1}^{j-2} \eta^{j-k} \mathcal{B}_{j-1,k-1}(\mu, \mu_0) \mathcal{E}_{k-1}(\mu, \mu_0) F_k^q(\eta, t, \mu_0)$$

where $\mathcal{E}$ and $\mathcal{B}$ are evolution operators containing various anomalous dimensions and elements of the special conformal algebra

Analytic continuation of discrete sums

First example of fractional finite sum

$$\sum_{k=1}^{-\frac{1}{2}} \frac{1}{k} = -2 \log 2, \qquad \text{(Euler, 1813)}$$

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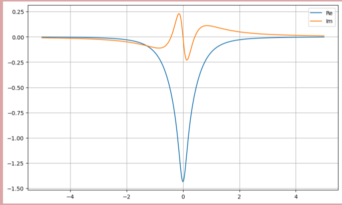
$$\sum_{k=k_0}^{j} f_k \to \sum_{k=k_0}^{\infty} (f_k - f_{k+j}) \to -\frac{1}{2i}\oint \frac{\mathrm{d}k}{\tan(\pi k)}(f(k) - f(k+j-k_0+1)),$$

see Nucl.Phys.B 1010 (2025) 116762 for a recent discussion in the GUMP framework.

$$\oint \to \int_{k_0-\epsilon-i\infty}^{k_0-\epsilon+i\infty}$$



$$= -1.38629\ldots = -2\log 2 \quad \checkmark$$

Singlet Moments

- Spin-averaged gluon GPD

$$H_g^{(+)}(n,\eta,t,\mu) = \sum_{n=2}^{\infty}(-1)^{n+1}(p_n^g(x,\eta)+p_n^g(-x,\eta))\mathbb{H}_g^{(+)}(n,\eta,t,\mu),$$

cf. glueball dominance of gravitational form factors.

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- Additional complications of mixing between quark and gluon singlet GPD through evolution equations resolved by taking the diagonal combination

$$\mathbb{H}_j^{\pm}(j,\eta,t,\mu_0)=\frac{1}{2}\sum_{q=1}^{N_f}\mathbb{H}_q^{(+)}(j,\eta,t,\mu_0)+\frac{1}{2}\left(\frac{\gamma_{j-1}^{qg}}{\gamma_{j-1}^{qq}-\gamma_{j-1}^{\mp}}\right)\mathbb{H}_g^{(+)}(j,\eta,t,\mu_0)$$

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- Quark and gluon singlet contributions poorly constrained by experiment

$\Rightarrow$ Use insights from Holography

Outline

The Holographic Principle

Different forms of the AdS/CFT correspondence

	4d $\mathcal{N}=4$ Super Yang-Mills (SYM)	**IIB String Theory on $AdS_5 \times S_5$**
Strongest form	any N and $\lambda = g^2_{\rm YM} N$	Quantum string theory, $g_s \neq 0$, $\alpha'/L^2 \neq 0$
Strong form	$N \to \infty$, λ fixed but arbitrary	Classical string theory, $g_s \to 0$, $\alpha'/L^2 \neq 0$
Weak form	$N \to \infty$, λ **large**	**Classical supergravity,** $g_s \to 0$, $\alpha'/L^2 \to 0$

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Holographic dictionary

Gauge theory	**Gravity theory**
Gauge theory in flat spacetime	Boundary of gravitational theory
Gauge invariant operators	Fields sourcing these operators
Energy scale	Radial coordinate
Global symmetry	Gauge symmetry

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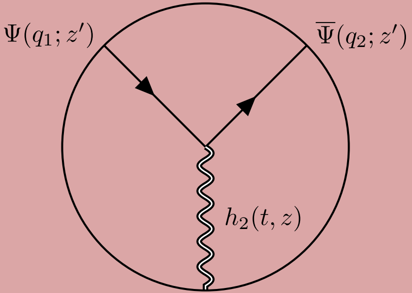
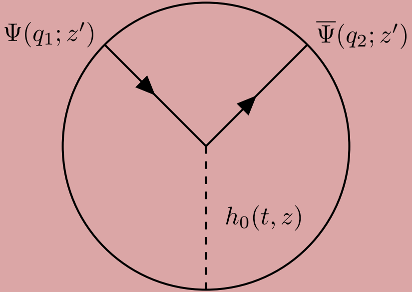
Holographic QCD

Generalization to non-conformal and non-supersymmetric case

Gravitational A and D form factors

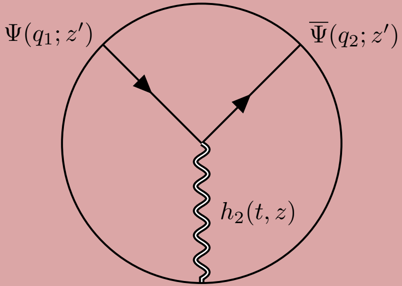




$$S_{\rm grav} \sim \int \mathrm{d}^d x \mathrm{d}z \, h_{MN} \, T^{MN}_{\rm QCD}$$

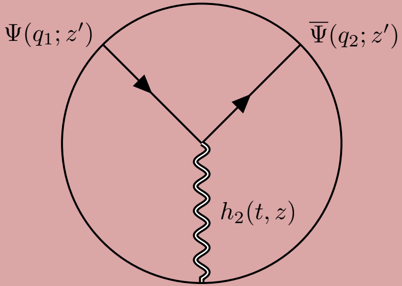
$$\langle q_2 | T^{\mu\nu} | q_1 \rangle = \overline{u}(q_2) \left(A(k) \gamma^{(\mu} p^{\nu)} + B(k) \frac{i p^{(\mu} \sigma^{\nu)\alpha} k_\alpha}{2 m_N} + D(k) \frac{k^\mu k^\nu - \eta^{\mu\nu} k^2}{m_N} \right) u(q_1)$$

K. A. Mamo, I. Zahed, PRD 101 (2020) 8, 086003



- Form factors defined by overlap integrals of holographic wave functions

K. A. Mamo, I. Zahed, PRD 101 (2020) 8, 086003



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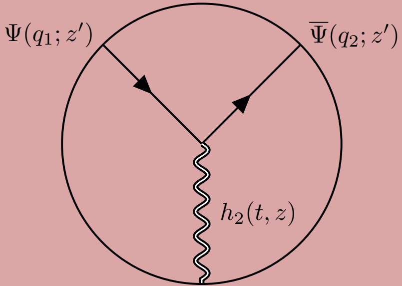

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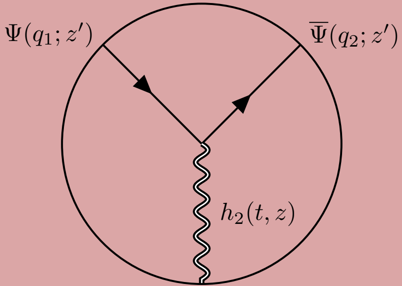

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- Normalizable nucleon modes $\psi_{L,R}(z)$ obtained from chiral boundary spinors with Dirac action
- Kaluza-Klein decomposition $\Phi(x^\mu, z) = \sum_n \varphi(x^\mu)\phi_n(z)$ yields mode equations for $\phi_n(z)$ with $m_n^2 \sim \kappa^2 n$ for $\phi(z) = \kappa^2 z^2$

$$A(t) = \frac{1}{2g_5^2} \int \mathrm{d}z \sqrt{g} e^{-\phi} z \left(\psi_R^2(z) + \psi_L^2(z)\right) \mathcal{H}(k, z), \quad t = -k^2$$

- Holographic computations suggest that skewness dependence is only contained in D and arises through non-degeneracy of tensor and scalar glueball spectrum

$$\mathbb{F}_g^{(+)}(j,\eta,t,\mu_0) = \mathcal{A}_g(j,t;\mu_0) + \mathcal{D}_{g\eta}(j,\eta,\Delta;\mu_0), \qquad \mathcal{A}_g(j,t;\mu_0) = \int_0^1 \mathrm{d}x\, \frac{xg(x;\mu_0)}{x^{2-j+\alpha'_T t}},$$

$$\mathcal{D}_{g\eta}(j,\eta,\Delta,\mu_0) = \left(\hat{d}_j(\eta,t) - 1\right) \times \left[\mathcal{A}_g(j,t,\mu_0) - \mathcal{A}_{gS}(j,t,\mu_0)\right],$$

$$A(t,\mu_0) = \mathcal{A}_g(j=2,t,\mu_0), \quad \eta^2 D(t,\mu_0) = \mathcal{D}_{g\eta}(j=2,t,\mu_0)\,.$$

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R. Nishio, T. Watari, PRD 90 (2014) 12, 125001

- Skewness dependence $\hat{d}_j(\eta,t)$ determined from 2-to-2 open (quark) and closed (gluon) string scattering amplitude in cubic string field theory

$$\hat{d}_j(\eta,t) = {}_2F_1\left(-\frac{j}{2}, -\frac{j-1}{2}; \frac{1}{2}-j; \frac{4m_N^2}{-t}\eta^2\right)$$

The 10-dimensional type-II supergravity actions contain various form fields

$$\text{IIA} \qquad\qquad\qquad \text{IIB}$$
$$C_1, \quad B_2, \quad C_3 \qquad\qquad C_2, \quad B_2, \quad C_4$$

whose dynamics are governed by (twisted) field strengths.

Helicity from Holography

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IIA	IIB
$C_1, \quad B_2, \quad C_3$	$C_2, \quad B_2, \quad C_4$

whose dynamics are governed by (twisted) field strengths.
(Broken) Supersymmetry requires the inclusion of a Chern-Simons term

$$S_{\rm CS}^{\rm Dp} = T_p \sum_q \int_{\rm Dp} \sqrt{\hat{\mathcal{A}}(\mathcal{R})} {\rm Tr} \exp\left(2\pi\alpha' F + B\right) \wedge C_q$$

Form fields contain $1^{\pm -}$ glueballs whose interactions with the proton are governed by the Chern-Simons term

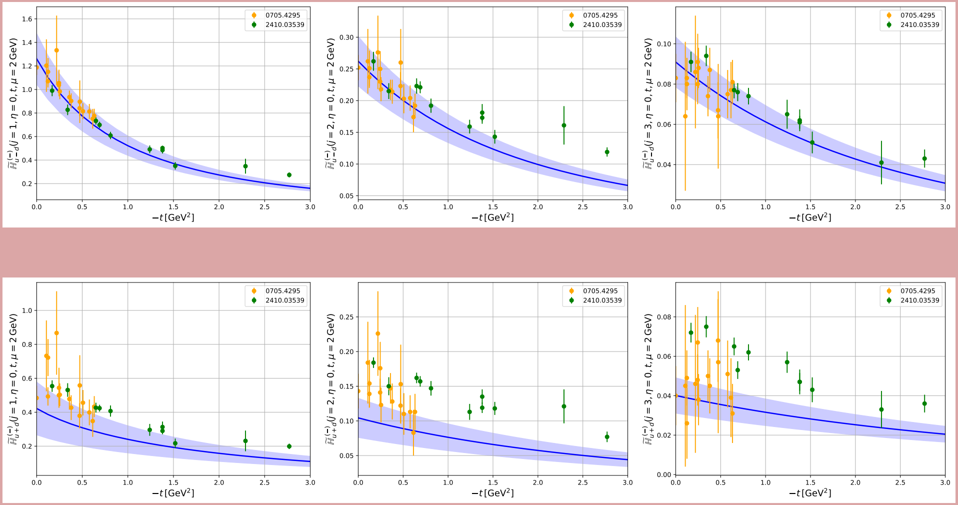
Parametrization of Moments

- Reggeize (N)LO MSTW and AAC PDFs to parametrize input moments
- Non-singlet moments fixed by (axial) Pauli and Dirac form factors
- Singlet moments fixed by η' meson trajectory (helicity) or lattice (unpolarized)
- Gluon moments fixed using gravitational A and D form factors from holography

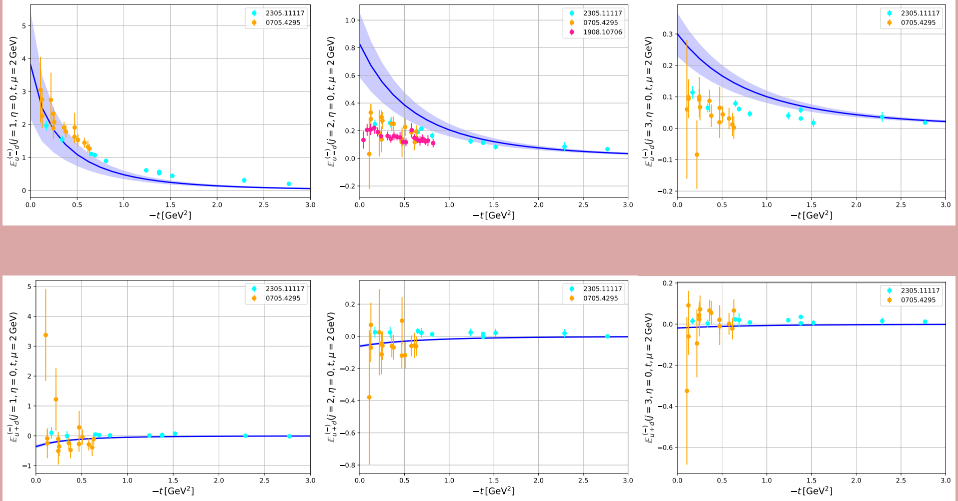
Outline

Comparison to Lattice QCD

Non-singlet $\widetilde{H}$ Moments

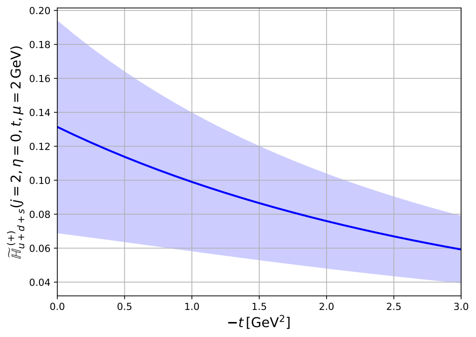


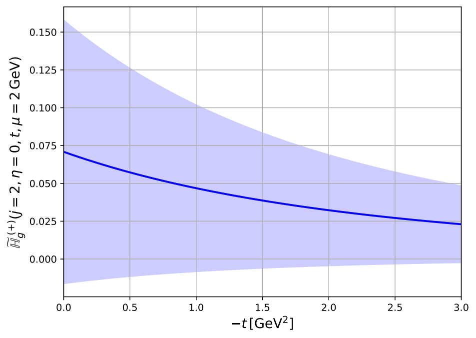
Non-singlet E Moments



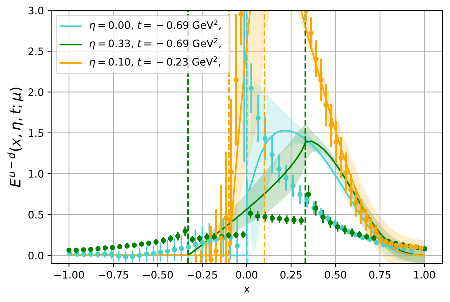
Singlet moments

- Unpolarized H moments $\to$ Kiminad's talk (almost unchanged at NLO)
- In AdS_n with point-like baryons $B(t) = 0 \Rightarrow$ No singlet E GPDs.

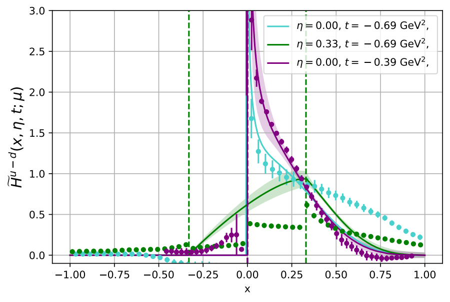




Note: Large errors due to simple Gaussian error propagation (vs. Hessian)



(a)



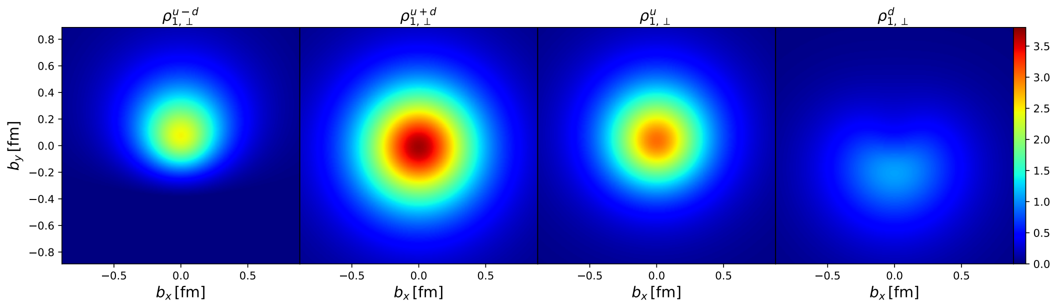
(b)

(a) Non-singlet isovector GPD E compared to ETMC 2020 (green, turquoise), PRL 127 (2021) 18, 182001 (purple), PRD 110 (2024) 3, 3 (orange) at $\mu = (2,2,3)$ GeV and (b) evolved non-singlet isovector axial GPD compared to ETMC 2020 (green) and PLB 824 (2022) 136821 (purple) at a resolution of $\mu = (2,3)$ GeV.

Impact Parameter Space

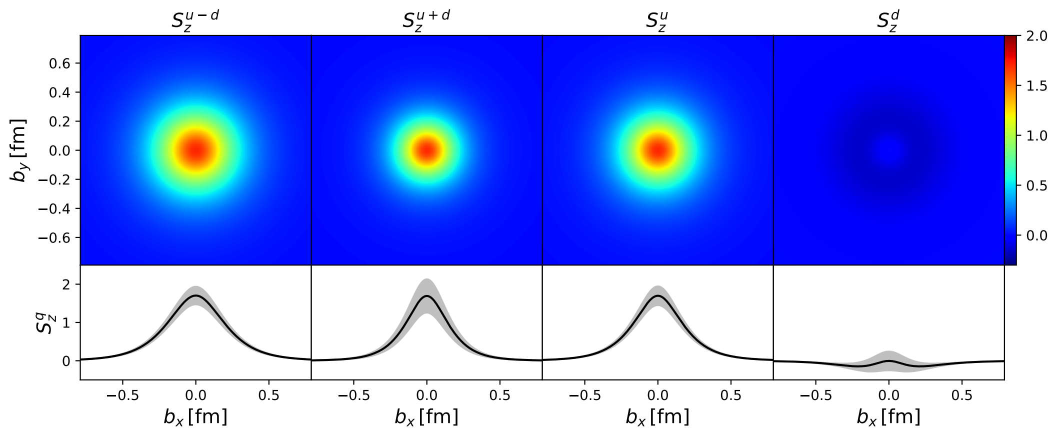
Vector-type quark moments for transversely polarized proton

$$\rho^q_{n,\perp} = \int \frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2} \left(\mathbb{H}^{(-)}_q(n, \eta = 0, -\Delta^2_\perp) + i\frac{\Delta_y}{2m_N}\mathbb{E}^{(-)}_q(n, \eta = 0, -\Delta^2_\perp) \right) e^{-i\vec{b}_\perp \cdot \Delta_\perp}$$



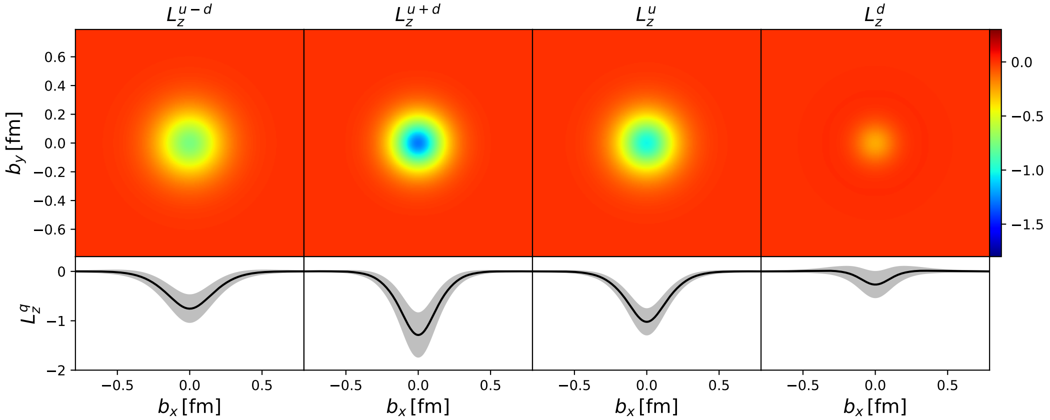
Quark Helicity

$$S_z^q(\vec{b}_\perp, \mu) = \frac{1}{2}\int \frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2} \widetilde{\mathbb{H}}_q(n=1, \eta=0, -\Delta_\perp^2, \mu)\, e^{-i\vec{b}_\perp\cdot\Delta_\perp}$$



Quark Orbital Angular Momentum

$$L_z^q(\vec{b}_\perp) = \frac{1}{2}\int \frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2}\left(\mathbb{H}_q(2,0,-\Delta_\perp^2) + \mathbb{E}_q(2,0,-\Delta_\perp^2) - \widetilde{\mathbb{H}}_q(1,0,-\Delta_\perp^2)\right) e^{-i\vec{b}_\perp\cdot\Delta_\perp}$$



Outline

1. GPDs and the Conformal Moment Expansion
2. Insights from Holography
3. Results
4. Conclusion and Outlook

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- Stay tuned for an open source release of a corresponding Python package

[a] PRL 132 (2024) 25, 251904

Thank you for your attention!

Questions, comments, suggestions: florian.hechenberger@stonybrook.edu

Backup Slides

Summary of extracted quantities

q	$u+d$	$u-d$	u	d
J^q	$+0.171(22)$	$+0.421(86)$	$+0.296(44)$	$-0.125(44)$
S_z^q	$+0.206(56)$	$+0.640(81)$	$+0.423(49)$	$-0.217(49)$
L_z^q	$-0.035(60)$	$-0.219(118)$	$-0.127(66)$	$+0.092(66)$
C_z^q	$-1.463(170)$	$-0.398(179)$	$-0.930(120)$	$-0.532(120)$

(Nested) Harmonic Sums

- NLO anomalous dimensions contain various (nested harmonic sums)

$$S_{\pm m}(n) = \sum_{i=1}^{n} \frac{(\mathrm{sign}\, m)^i}{i^m}, \quad S_{\pm m, j_1, \ldots, j_p}(n) = \sum_{i=1}^{n} \frac{(\mathrm{sign}\, m)^i}{i^m} S_{j_1, \ldots, j_p}(i)$$

  easily implemented in computer algebra software, though numerically costly.
- Non-diagonal part of evolution equation contains elements of special conformal algebra (d_{jk}, g_{jk}...). Defined by overlap integrals of Gegenbauer polynomials $\rightarrow$ Careful analytic continuation

Spin-Orbit Correlation (neglecting $\mathcal{O}(m_q)$ from transversity moments)

$$C_z^q(\vec{b}_\perp) = \frac{1}{2}\int \frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2}\left(\widetilde{\mathbb{H}}_q(2,0,-\Delta_\perp^2) - \mathbb{H}_q(1,0,-\Delta_\perp^2)\right)\, e^{-i\vec{b}_\perp\cdot\Delta_\perp}$$