D-term and dispersion relations beyond kinematic twist-4

VÍCTOR MARTÍNEZ-FERNÁNDEZ

POSTDOC AT IRFU/CEA (PARIS-SACLAY, FRANCE) AND CFNS (USA)

Work in collaboration with:

Cédric Mezrag (IRFU/CEA)

Outline

- Introduction: GPDs, Compton and gravitational form factors.
- Dispersion relation beyond Born and Bjorken approx.
- Subtraction constant and double distributions.
- *D*-term extraction at LO and NLO.
- Take aways.

Generalized Parton Distributions

GPD

Generalized Parton Distribution ≈ "3D version of a PDF (Parton Distribution Function)." With x the average fraction of the hadron's longitudinal momentum carried by a quark:

$$H_f(x,\xi,t) = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ix\bar{p}^+z^-} \langle p'|\bar{\mathrm{q}}_f(-z/2)\gamma^+\mathcal{W}[-z/2,z/2]\mathrm{q}_f(z/2)|p\rangle\Big|_{z_\perp=z^+=0}$$

$$t=\Delta^2=(p'-p)^2\,,\quad \xi=-\frac{\Delta n}{2\bar{p}n}\,,\quad \bar{p}=\frac{p+p'}{2}$$

Importance

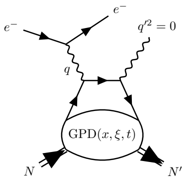
- Connected to **QCD energy-momentum tensor**. GPDs are a way to study "mechanical" properties and to address the hadron's spin puzzle (*X. Ji's sum rule**).
- **Tomography:**$ distribution of quarks in terms of the longitudinal momentum and in the transverse plane.

$$f(x,\vec{b}_\perp) = \int \frac{d^2\vec{\Delta}_\perp}{4\pi^2}\, e^{-i\vec{b}_\perp\cdot\vec{\Delta}_\perp} H_f(x,0,\vec{\Delta}_\perp^2)$$

*PRD 55 (1997) 7114-7125; $Burkardt, Int. J. Mod. Phys. A 21 (2006) 926-929.

Accesing GPDs: DVCS

- In the 1990s, Müller et al.,† Ji* and Radyushkin# introduced GPDs and deeply virtual Compton scattering (DVCS):



- At LO ($O(\alpha_s^0)$) and LT ($\Lambda/Q^2 \to 0$, $\Lambda \in \{|t|, M^2\}$):

$$\mathcal{H}_{\mathrm{DVCS}}(\xi,t) = -\mathrm{PV}\left(\int_{-1}^{1} dx \, \frac{1}{x-\xi} H^{(+)}(x,\xi,t)\right) + \int_{-1}^{1} dx \;\, i\pi\delta(x-\xi) H^{(+)}(x,\xi,t) \,,$$

$$H^{(+)}(x,\xi,t) = H(x,\xi,t) - H(-x,\xi,t) \,.$$

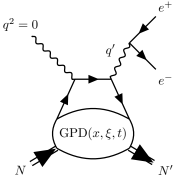
- $\xi = \frac{-n\Delta}{2\bar{p}n} \,, \quad \bar{p} = \frac{p+p'}{2} \,, \quad \Delta = p' - p \,, \quad t = \Delta^2 \,.$

† Fortsch. Phys. 42 (1994) 101-141. * PRD 55 (1997) 7114-7125. # PLB 449 (1999) 81-88.

Higher-twist corrections to DVCS off proton: Braun et al., PRD 111 (2025) 7, 076011... and earlier works.

Other processes

- Timelike Compton scattering (TCS)

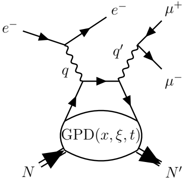


Like DVCS but $\xi \to -\xi$.

LT: Berger, Diehl and Pire, EPJC 23, 675–689 (2002).

Higher twists: VMF, Pire, Sznajder & Wagner, PRD 111 (2025) 7, 074034.

- **Double** deeply virtual Compton scattering (**D**DVCS)



LT: Belitsky & Müller, PRL 90, 022001 and PRD 68, 116005 (2003); Guidal & Vanderhaeghen, PRL 90, 012001 (2003); Deja, VMF, Pire, Sznajder & Wagner, PRD 107, 094035 (2023); Alvarado, Hoballah & Voutier, arXiv:2502.02346 (2025).

Higher twists: VMF, Pire, Sznajder & Wagner, PRD 111 (2025) 7, 074034.

$$\mathrm{Im}\left(\mathcal{H}_{\mathrm{DDVCS}}(\rho, \xi, t)\right) \propto H^{(+)}(\rho, \xi, t)\,, \qquad \rho \overset{\mathrm{LT}}{=} \xi \frac{Q^2 - Q'^2}{Q^2 + Q'^2}$$

QCD energy-momentum tensor (EMT), $\Theta^{\mu\nu}$

$\Theta^{\mu\nu}$ parameterization $\rightarrow$ gravitational form factors (GFFs):

$$\langle p', s' | \Theta_a^{\mu\nu}(0) | p, s \rangle =$$

$$= \bar{u}(p', s') \left\{ \frac{\bar{p}^\mu \bar{p}^\nu}{M} A_a(t) + \frac{\Delta^\mu \Delta^\nu - \eta^{\mu\nu} \Delta^2}{M} C_a(t) + M \eta^{\mu\nu} \bar{C}_a(t) \right.$$

$$\left. + \frac{\bar{p}^{\{\mu} i \sigma^{\nu\}\rho} \Delta_\rho}{4M} [A_a(t) + B_a(t)] + \frac{\bar{p}^{[\mu} i \sigma^{\nu]\rho} \Delta_\rho}{4M} D_a(t) \right\} u(p, s) \, .$$

$a =$ quarks and gluons.

GPDs and GFFs:

$$\int dx \, x^{\mathscr{P}_{f,\,g}} \begin{Bmatrix} H_f(x, \xi, t) \\ E_f(x, \xi, t) \end{Bmatrix} = \begin{Bmatrix} A_f(t) + 4\xi^2 C_f(t) \\ B_f(t) - 4\xi^2 C_f(t) \end{Bmatrix} ,$$

$$J_{f,\,g} = \frac{A_{f,\,g}(0) + B_{f,\,g}(0)}{2} = \frac{1}{2} \int dx \; x^{\mathscr{P}_{f,\,g}} \left[H_{f,\,g}(x, \xi, 0) + E_{f,\,g}(x, \xi, 0) \right] ,$$

and other relations.

$\mathscr{P}_{f,\,g} = 1$, quarks flavor f; 0, gluons.

The D-term

Double distribution parameterization of GPDs:

$$H_a(x,\xi,t) = \iint_{\mathbb{D}} d\beta d\alpha\ \delta(x-\beta-\alpha\xi)\left[\beta^{\mathscr{P}_a} F_a(\beta,\alpha,t) + \xi^{1+\mathscr{P}_a} D_a(\alpha,t)\delta(\beta)\right],$$

and the 1st moments of GPDs:

$$C_a(t) = \frac{1}{4}\int_{-1}^{1} d\alpha\ \alpha^{1-\mathscr{P}_a} D_a(\alpha,t)\,.$$

C_a is related to the pressure inside the hadron:[‡, ‡‡]

$$p_a(r) = M\int \frac{d^3\boldsymbol{\Delta}}{(2\pi)^3}\, e^{-i\boldsymbol{\Delta}\cdot r}\left\{-\bar{C}_a(t) + \frac{2}{3}\frac{t}{M^2}\, C_a(t)\right\}.$$

D_a-term $\mapsto$ GFF **C_a** $\mapsto$ **pressure** p_a .

Our goal: Extraction of the D-term beyond LO and LT.

[‡]Polyakov, Schweitzer, Int. J. Mod. Phys. A33(26), 1830025 (2018).

[‡‡]Dutrieux, Lorcé, Moutarde, Sznajder, Trawiński, Wagner, EPJC 81 (2021) 4, 300.

$\sum_a \bar{C}_a = 0$. $\bar{C}_a$ is related to higher-twist distributions: Leader, Lorcé, Phys. Rept. 541(3), 163 (2014);

Leader, PLB 720, 120 (2013) [Erratum: PLB 726, 927–927 (2013)]; Tanaka, PRD 98(3), 034009 (2018).

D-term extraction

Dispersion relation: inversion problem for the D-term $(\alpha, t) \to t$:

$$\mathcal{H}(\xi, t) = \int \frac{dx}{\xi} \; C(x/\xi) H(x, \xi, t)\,,$$

$$\mathrm{Re}\,(\mathcal{H}) + \int \frac{\mathrm{Im}\,(\mathcal{H})}{\xi - \xi'} \underset{\mathrm{LT}}{\sim} \int d\alpha \; C(\alpha) D(\alpha, t)$$

What do NLO and twist corrections do to this expression?

Kinematic higher-twist corrections

- LT: $Q^2 \to \infty$, Bjorken limit.
- Kinematic power corrections = kinematic higher-twist corrections:

$$\text{kin. power corrections} = O\left(\left(\frac{|t|}{Q^2}\right)^P, \left(\frac{M^2}{Q^2}\right)^P\right),$$

$$P = \frac{\tau_{\text{kin}} - 2}{2}.$$

Braun, Ji & Manashov, JHEP 03 (2021) 051; JHEP 01 (2023) 078.

Why to go beyond leading twist?

1. Nucleon tomography:

$$f(x, \vec{b}_\perp) = \frac{1}{4\pi^2} \int d^2\vec{\Delta}_\perp \; e^{-i\vec{b}_\perp \cdot \vec{\Delta}_\perp} H_f(x, 0, t = -\vec{\Delta}_\perp^2).$$

2. Increase the range of useful experimental data.
3. Universality tests $\to \mathcal{H}^{++}_{\text{DVCS}} \overset{\text{LO, HT}}{\neq} \left(\mathcal{H}^{++}_{\text{TCS}}\right)^*$.

Dispersion relation

Analyticity of the scattering amplitude

Following procedure in Dutrieux, Meisgny, Mezrag & Moutarde, EPJC 85 (2025) 1, 105:

DVCS: fixed t + momentum conservation: amplitude $= \mathcal{F}(s)$.

Causality $\Rightarrow$ extension to the upper-half of the complex plane: $\mathcal{F}(s) \to \mathcal{F}(s + i0)$.

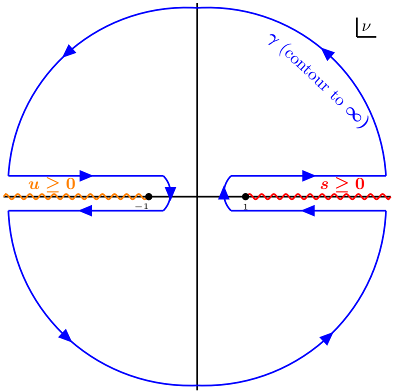
$$s + u = -\mathbb{Q}^2 \left[1 - \frac{2M^2}{\mathbb{Q}^2} \right] .$$

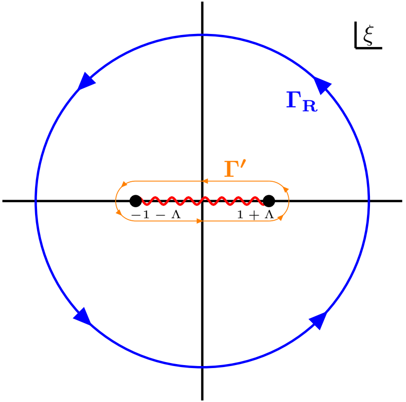
1. $s + u < 0$ if $M^2/\mathbb{Q}^2 < 1/2 \Rightarrow$
   2. $\exists$ region for both $s, u < 0 \Rightarrow$
      3. no particle production & $\mathrm{Im}(\mathcal{F}) = 0$ **(optical theorem)** $\Rightarrow$
         4. analytic continuation to the lower-half of the complex plane:

            Schwartz's reflexion principle: $\mathcal{F}(s - i0) = \mathcal{F}^*(s + i0)$

Domain of (granted) analyticity in ν and ξ

$$s \to \nu \simeq 1/\xi + \Lambda\,, \qquad \Lambda = 2M^2/\mathbb{Q}^2$$





Path γ in ν's complex plane $\to$ Diehl & Ivanov, EPJC 52 (2007) 919-932.

Dispersion relation

- $\mathcal{F} \to \mathcal{H}^{AB}$, A: in-photon polariz., B: out-photon polariz..
- Schwartz + homotopy of Γ_{R} and Γ' = **dispersion relation:**

$$\sum_{j=0}^{n} h_j^{AB} \frac{1}{\xi^j} = \frac{1}{\pi} \mathrm{PV} \int_{-(1+\boldsymbol{\Lambda})}^{1+\boldsymbol{\Lambda}} d\xi' \; \frac{\mathrm{Im}(\mathcal{H}^{AB}(\xi'))}{\xi' - \xi} \left(\frac{\xi'}{\xi}\right)^n + \mathrm{Re}(\mathcal{H}^{AB}(\xi))$$

- $|\xi| > 1 \Rightarrow$ GPDs on $x \in (-\xi, \xi) \Rightarrow \underbrace{\mathrm{Im}(\mathcal{H}^{++}(\xi))}_{\text{NLO+tw-4} \,\in\, \text{DGLAP region } |x| > |\xi|} = 0$:

$$\sum_{j=0}^{n} h_j^{AB} \frac{1}{\xi^j} = \frac{1}{\pi} \mathrm{PV} \int_{\mathbf{-1}}^{\mathbf{1}} d\xi' \; \frac{\mathrm{Im}(\mathcal{H}^{AB}(\xi'))}{\xi' - \xi} \left(\frac{\xi'}{\xi}\right)^n + \mathrm{Re}(\mathcal{H}^{AB}(\xi))$$

Subtraction constant and DDs

$$\sum_{j=0}^{n} h_j^{AB} \frac{1}{\xi^j} \Rightarrow \boldsymbol{h_0^{++}} = \textbf{subtraction constant of the DR}\,.$$

With quark GPD

$$H(x,\xi,t) = \iint_{\mathbb{D}} d\beta d\alpha \; \delta(x-\beta-\alpha\xi)\left[F(\beta,\alpha,t) + \xi D(\alpha,t)\delta(\beta)\right]\,,$$

we get:

Subtraction constant beyond Born and Bjorken approximations (PRELIMINARY)

$$h_0^{++} = \int_{-1}^{1} d\alpha \; \left[\mathrm{Re}\left(T_0^{++}(\alpha)\right) + \frac{t}{\mathbb{Q}^2}\mathrm{Re}\left(T_1^{++}(\alpha)\right)\right] \boldsymbol{D(\alpha)}$$

$$- 4\frac{M^2 - t/4}{\mathbb{Q}^2} \iint_{\mathbb{D}} d\beta d\alpha \; \boldsymbol{F(\beta,\alpha)}\beta\,\mathrm{Re}\left(T_1^{++\,(1)}(\alpha)\right)\,,$$

where $f^{(n)}(\alpha) = \partial_\alpha^n f(\alpha)$ and

$T_0^{++} \sim$ LT(LO + NLO) + tw-4,

$\boldsymbol{T_1^{++}} \sim$ **LT(LO) + tw-4** $\Rightarrow$ $\boldsymbol{F(\beta,\alpha)}$ **contribution** $\rightarrow$ **novel result**

$F(\beta,\alpha)$-term by means of CFFs

Recap, DR's subtraction constant:

$$h_0^{++} = \left(\int D\right) - 4\frac{M^2 - t/4}{\mathbb{Q}^2}\iint_{\mathbb{D}} d\beta d\alpha\ \boldsymbol{F(\beta,\alpha)}\beta\,\mathrm{Re}\left(T_1^{++\,(1)}(\alpha)\right).$$

From the DR, we extract the coefficient h_2^{++} which leads us to:

$$\frac{2}{\pi}\mathrm{PV}\int_0^1 d\xi'\ \xi'\mathrm{Im}\left(\mathcal{H}^{++}_{\mathrm{tw}-4}(\xi',t) - \mathcal{H}^{++}_{\mathrm{tw}-4}(\xi',0)\right) =$$

$$= \iint_{\mathbb{D}} d\beta d\alpha\ \beta\left[\frac{t}{\mathbb{Q}^2}F(\beta,\alpha,t)\left\{-3\,T_1^{++\,(1)}(\alpha) + \mathbf{R}^{(1)}(\alpha) - \beta^2\,T_1^{++\,(3)}(\alpha)\right\}\right.$$

$$\left. - \beta^2\frac{4M^2}{\mathbb{Q}^2}\,T_1^{++\,(3)}(\alpha)\left(F(\beta,\alpha,t) - F(\beta,\alpha,0)\right)\right]$$

$$\underbrace{\simeq}_{(?)}(-3+\kappa)\frac{t}{\mathbb{Q}^2}\iint_{\mathbb{D}} d\beta d\alpha\ F(\beta,\alpha)\beta\ T_1^{++\,(1)}(\alpha).$$

PRELIMINARY,
we are working on this

$f^{(n)} = \partial^n_\alpha f\,,\qquad \mathbf{R}^{(1)}(\alpha)\mapsto \kappa T_1^{++\,(1)}(\alpha) + \epsilon_\kappa(\alpha)\,,\ \ |\kappa| = O(1).$

$F(\beta,\alpha)$-term in h_0^{++} might be obtained from data on CFFs $\longrightarrow$ avoid modelling $F(\beta,\alpha)$.

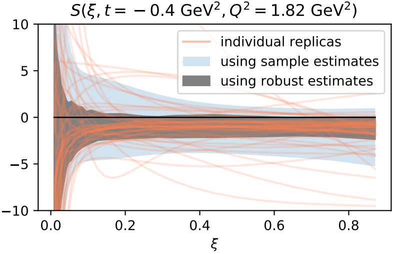
D-term extraction

Subtraction constant from CFFs

DR choosing $n = 0$ to extract h_0^{++} :

$$h_0^{++} = \text{Re}\left(\mathcal{H}^{++}(\xi)\right) - \frac{2}{\pi}\text{PV}\int_0^1 d\xi' \; \frac{\xi' \,\text{Im}\left(\mathcal{H}^{++}(\xi')\right)}{\xi^2 - \xi'^2}$$

With NLO coefficient functions → 100 sets of CFFs[&] → neural network extraction of 100 samples/**replicas** of $h_0^{++} = S$ at NLO and LT:



Example of 50 replicas for $\boldsymbol{S = h_0^{++}}$ from Dutrieux, Meisgny, Mezrag & Moutarde, EPJC 85 (2025) 85:105.

[&] Dutrieux, Lorcé, Moutarde, Sznajder, Trawiński & Wagner, EPJC 81(4), 300 (2021).

D-term and subtraction constant

- Parameterization of the D-term:

$$D_f(\alpha, t, \mu^2) = (1-\alpha^2) \sum_{\text{odd } n} d_{n,\,f}(t,\mu^2) \underbrace{C_n^{(3/2)}(\alpha)}_{\text{Gegenbauer poly.}} \,,$$

$$D_g(\alpha, t, \mu^2) = \frac{3}{2}(1-\alpha^2)^2 \sum_{\text{odd } n} d_{n,g}(t,\mu^2) C_{n-1}^{(5/2)}(\alpha) \,,$$

$$d_{n,\,a}(t,\mu^2) = d_{n,\,a}(0,\mu^2) \cdot (1 - t[\mathrm{GeV}^2]/0.8^2)^{-3} \,, \quad a \in \{f, g\} \,.$$

Extract $d_{n,\,a}(0,\mu^2)$ from h_0^{++}

- At LO and LT (no $F(\beta,\alpha)$ at LT):

$$h_0^{++} = \frac{4}{(1 - t[\mathrm{GeV}^2]/0.8^2)^3} \sum_f e_f^2 \sum_{\text{odd } n} d_{n,\,f}(0,\mu^2) \,.$$

Match the formula above (and NLO equivalent) to the replicas for h_0^{++}.

Shadow D-term

Definition (Shadow D-term)

$D \to D + D_{\text{shadow}} \Longrightarrow h_0^{++}$ **UNCHANGED,** *at a given energy scale* μ_0^2 .

- **At LO and LT (no $F(\beta, \alpha)$):**

$$h_0^{++} = \frac{4}{(1 - t[\mathrm{GeV}^2]/0.8^2)^3} \sum_f e_f^2 \sum_{\mathrm{odd}\ n} d_{n,\,f}(0, \mu^2)\,.$$

Truncating for $n \in \{1, 3\}$, **LO D_{shadow} at μ_0^2:**

$$\boldsymbol{d_{1,f}(0, \mu_0^2) = -d_{3,f}(0, \mu_0^2) = \lambda\,.} \tag{1}$$

- **Evolution $\mu_0^2 \to \mu^2$ at LT:**

$h_{0,\,S}^{++} \to$ contribution to h_0^{++} from D_{shadow} :

$$d_{n,f}(0, \mu^2) = \left(\frac{\alpha_s(\mu^2)}{\alpha_s(\mu_0^2)}\right)^{\frac{2\gamma_n}{\beta_0}} d_{n,f}(0, \mu_0^2) \Rightarrow h_{0,\,S}^{++} \overset{\text{lin.}}{\simeq} \lambda \left[1 - \frac{\alpha_s(\mu^2)}{\alpha_s(\mu_0^2)}\right]\,.$$

- **Uncertainty estimator:**

$$\text{Experimental uncertainty: } \Delta h_0^{++} \Rightarrow \sigma_{S,\,d1f} \approx \sigma_{S,\,d3f} \approx \frac{\Delta h_0^{++}}{1 - \frac{\alpha_s(\mu^2)}{\alpha_s(\mu_0^2)}}\,. \tag{2}$$

LO h_0^{++} fit: $n=1$ vs $n=3$, radiative gluons

$$d_{1,\,uds} := d_{1,\,u} = d_{1,\,d} = d_{1,\,s}\,.$$

LO h_0^{++}, $n=1$, radiative gluons
$d_{1,\,uds}(0,\mu_0^2)$ free (only)

$d_{1,\,uds}(0, 2\ \mathrm{GeV}^2) =$	-0.6 ± 1.1
$d_{1,\,g}(0, 2\ \mathrm{GeV}^2) =$	-0.8 ± 1.5
$d_{1,\,c}(0, 2\ \mathrm{GeV}^2) =$	-0.003 ± 0.005

LO h_0^{++}, $n=3$, radiative gluons
$d_{1,\,uds}(0,\mu_0^2)$, $d_{3,\,uds}(0,\mu_0^2)$ free

$d_{1,\,uds}(0, 2\ \mathrm{GeV}^2) =$	-2.1 ± 26.6
$d_{3,\,uds}(0, 2\ \mathrm{GeV}^2) =$	1.5 ± 26.5
$d_{1,\,g}(0, 2\ \mathrm{GeV}^2) =$	-2.9 ± 37
$d_{3,\,g}(0, 2\ \mathrm{GeV}^2) =$	0.2 ± 4.1

$n=1$ vs $n=3$:

$$d_{1,\,uds} \approx -d_{3,\,uds} \overset{(1)}{\Rightarrow} \text{shadow } D\text{-term} \overset{\substack{(2),\\ \mu_0^2=1.4,\\ \mu^2=2.5}}{\Longrightarrow} \sigma_{d1f} \approx \sigma_{d3f} \approx 25.5\,.$$

Conclusion: most uncertainty is contamination by LO D_{shadow}.

Fits hereafter from Dutrieux, Meisgny, Mezrag & Moutarde, EPJC 85 (2025) 85:105.

LO vs NLO h_0^{++} fits: $n = 1$, radiative gluons

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NLO h_0^{++}, $n = 1$, radiative gluons
$d_{1,\,uds}(0, \mu_0^2)$ free (only)

$d_{1,\,uds}(0, 2\ \mathrm{GeV}^2) =$	-0.7 ± 1.3
$d_{1,\,g}(0, 2\ \mathrm{GeV}^2) =$	-0.9 ± 1.8
$d_{1,\,c}(0, 2\ \mathrm{GeV}^2) =$	-0.003 ± 0.006

LO vs NLO:

$$\mu_g^2 = 0.09\,(\text{gluon radiation threshold}) \Rightarrow d_{n,\,g}(0,2) = \frac{\Gamma_n^{gq}(2,1)}{\Gamma_n^{qq}(2,1)} d_{n,\,uds}(0,2)\,.$$

$$h_0^{++} \overset{\mathrm{NLO}}{\approx} -0.3 d_{1,\,g} + 2.65 d_1\big|_{\mathrm{quarks}}$$

Conclusion: gluons account for a 10% effect in h_0^{++}, hence the LO-NLO similarity.

NLO h_0^{++} fit: $n = 1$ vs $n = 3$, radiative gluons

NLO h_0^{++}, $n = 1$, radiative gluons
$d_{1,\,uds}(0, \mu_0^2)$ free (only)

$d_{1,\,uds}(0, 2\ \mathrm{GeV}^2) =$	-0.7 ± 1.3
$d_{1,\,g}(0, 2\ \mathrm{GeV}^2) =$	-0.9 ± 1.8
$d_{1,\,c}(0, 2\ \mathrm{GeV}^2) =$	-0.003 ± 0.006

NLO h_0^{++}, $n = 3$, radiative gluons
$d_{1,\,uds}(0, \mu_0^2)$, $d_{3,\,uds}(0, \mu_0^2)$ free

$d_{1,\,uds}(0, 2\ \mathrm{GeV}^2) =$	-1.7 ± 21
$d_{3,\,uds}(0, 2\ \mathrm{GeV}^2) =$	0.7 ± 15
$d_{1,\,g}(0, 2\ \mathrm{GeV}^2) =$	-2 ± 30
$d_{3,\,g}(0, 2\ \mathrm{GeV}^2) =$	0.1 ± 2.3

$n = 1$ vs $n = 3$:

$$\mu_g^2 = 0.09 \text{ (gluon radiation threshold)} \Rightarrow d_{n,\,g}(0,2) = \frac{\Gamma_n^{gq}(2, 0.09)}{\Gamma_n^{qq}(2, 0.09)} d_{n,\,uds}(0,2)\,.$$

$$h_0^{++} \overset{\mathrm{NLO}}{=} (2.65-0.36) d_{1,\,uds}(0,2) + (3.36+0.05) d_{3,\,uds}(0,2) \Rightarrow d_{1,\,uds}^{\mathrm{shadow}}(0,2) = -1.5 d_{3,\,uds}^{\mathrm{shadow}}(0,2)\,.$$

$$\sigma_{d1f} \approx 1.5\sigma_{d3f} \approx 1.5 \times 15 \approx 22.5\,.$$

Conclusion: uncertainties are due to NLO D_{shadow}.

NLO h_0^{++} fit: $d_{1,a}$, unconstrained gluons

LO h_0^{++}, $n = 1$, radiative gluons
$d_{1,uds}(0, \mu_0^2)$ free (only)

$d_{1,uds}(0, 2\ \mathrm{GeV}^2) =$	-0.6 ± 1.1
$d_{1,g}(0, 2\ \mathrm{GeV}^2) =$	-0.8 ± 1.5
$d_{1,c}(0, 2\ \mathrm{GeV}^2) =$	-0.003 ± 0.005

NLO h_0^{++}, $n = 1$, free gluons
$d_{1,uds}(0, \mu_0^2)$, $d_{1,g}(0, \mu_0^2)$ free

$d_{1,uds}(0, 2\ \mathrm{GeV}^2) =$	-1.1 ± 7.7
$d_{1,g}(0, 2\ \mathrm{GeV}^2) =$	-6 ± 78
$d_{1,c}(0, 2\ \mathrm{GeV}^2) =$	-0.02 ± 0.27

Radiative vs free gluons:

Larger quark uncertainty for free gluons:

$$h_{0,\mathbf{g}}^{++} \propto \alpha_s(\mu^2)\Gamma^{\mathbf{gg}}(\mu^2, \mu_0^2) d_{1,\mathbf{g}}(0, \mu_0^2) \approx \alpha_s(\mu^2)\Gamma^{\mathbf{qq}}(\mu^2, \mu_0^2) d_{1,\mathbf{g}}(0, \mu_0^2)\,.$$

Conclusion: gluon and quark distributions are strongly correlated.

Take aways

- Analyticity of scattering amplitude $\leftrightarrow$ dispersion relation at all orders $\leftrightarrow$ extraction of D-term.
- Shadow D-term $\leftrightarrow$ inverse problem $\leftrightarrow$ uncertainties.
- Kin. twist-4: **subtraction const** $\sim \int \boldsymbol{D} + \int \boldsymbol{F}$ $\leftrightarrow$ **new result.**

- Our scientific program:
  1. $\int F \sim \int \mathrm{Im}\,(\mathcal{H}_{\mathrm{tw\text{-}4}})$?
  2. twist corrections = less uncertainty in d_is ?

Thank you!

Complementary slides

Conformal group (CG)

Set of transformations $z \to z'$ such that the metric is re-scaled as:

$$g_{\mu\nu}(z) \to g'_{\mu\nu}(z') = \Omega^2(z) g_{\mu\nu}(z)\,.$$

$$\text{CG} = \text{Poincaré} + \text{dilations} + \underbrace{\text{special conformal transformations}}_{\text{inversion + translation + inversion}}.$$

Main tool: shadow-operator formalism

- Use of conformal symmetry to constrain the coefficients of the expansion around the light-cone:

$$\mathscr{P}_{\ell_n,n} = \int d^D z\; \widetilde{\mathcal{O}}_{\alpha_1\cdots\alpha_n}(z)|0\rangle\langle 0|\mathcal{O}^{\alpha_1\cdots\alpha_n}(z)\,,$$

$\widetilde{\mathcal{O}}$ is the *shadow operator* of $\mathcal{O}$ with scaling dimension $\widetilde{\ell}_n = D - \ell_n$ for a D-dimensional spacetime,

$$\mathcal{O}_1(z_1)\mathcal{O}_2(z_2) = \sum_{\ell_n,n}\int d^D z\;\left[\langle\mathcal{O}_1(z_1)\mathcal{O}_2(z_2)\widetilde{\mathcal{O}}_{\beta_1\cdots\beta_n}(z)\rangle e^{izr}\right]\,\mathcal{O}^{\beta_1\cdots\beta_n}(y)\Big|_{y=0}\,,$$

$$r^\mu = -i\partial_y^\mu\,,$$

$\langle\mathcal{O}_1(z_1)\mathcal{O}_2(z_2)\widetilde{\mathcal{O}}_{\beta_1\cdots\beta_n}(z)\rangle \to$ **up to a constant,** it is given by the constraints to impose conformal covariance by means of:

$$\Lambda^\mu(z_1,z_2,z_3) = \frac{1}{2}\partial_3^\mu \ln\frac{(z_2-z_3)^2}{(z_1-z_3)^2}\,,\quad \partial_3^\mu = \frac{\partial}{\partial z_{3,\,\mu}}\,,$$

$$Z(z_1,z_2,z_3) = \left(\frac{(z_1-z_2)^2}{(z_1-z_3)^2(z_2-z_3)^2}\right)^{T/2}\,,\quad T\in\mathbb{R}\,,$$

$$\mathcal{I}^\mu_\nu(z) = \delta^\mu_\nu - 2\frac{z^\mu z_\nu}{z^2}\,.$$

The conformal basis

- At LO, the expansion of $j^\mu j^\nu$ is given in a basis of 3 operators: $\mathcal{O}_N^{(k)}$ for $k \in \{0, 1, 2\}$ with

$$\mathcal{O}_N^{(k)}(y) = \partial_y^{\alpha_1} \cdots \partial_y^{\alpha_k} \mathcal{O}_{\alpha_1 \cdots \alpha_k \alpha_{k+1} \cdots \alpha_N}(y) z_{12}^{\alpha_{k+1}} \cdots z_{12}^{\alpha_N} \,.$$

- Not all traces have been removed, so write this operators by means of their geometric LT components. In general, for a scalar operator ($\partial^\mu = \partial / \partial y_\mu$):

$$\mathcal{O}(y) = [\mathcal{O}(y)]_{\mathrm{LT}} - \sum_{k=1}^{\infty} \int_0^1 dt \, \left(\frac{-y^2}{4} \right)^k \frac{(\partial^2)^k}{k!(k-1)!} \frac{(1-t)^{k-1}}{t^k} \mathcal{O}(ty)$$

$$= [\mathcal{O}(y)]_{\mathrm{LT}} + \frac{y^2}{4} \int_0^1 \frac{dt}{t} \left[\partial^2 \mathcal{O}(ty) \right]_{\mathrm{LT}} + \frac{y^4}{32} \int_0^1 dt \, \frac{1-t}{t^3} \left[\partial^4 \mathcal{O}(ty) \right]_{\mathrm{LT}} + O(y^6) \,.$$

Connection to light-ray operators

- Light-ray operator definition:

$$\mathscr{O}(\lambda_1,\lambda_2)=\sum_f\left(\frac{e_f}{e}\right)^2\frac{1}{2}[\bar{\mathrm{q}}_f(\lambda_1 z)\not{z}\mathrm{q}_f(\lambda_2 z)-(\lambda_1\leftrightarrow\lambda_2)]_{\mathrm{LT}}\,,\quad \boldsymbol{z^2\neq 0}\,.$$

- Braun et al. proved them to be related to the conformal basis by:

$$\mathscr{O}(\lambda_1,\lambda_2)=\sum_{\substack{N>0,\\ \text{even}}}\rho_N\lambda_{12}^{N-1}\int_0^1 du\,(u\bar{u})^N\left[\mathcal{O}_N^{(0)}(\lambda_{21}^u z)\right]_{\mathrm{LT}}\,.$$

- $\mathcal{O}_N^{(k)}\to\mathscr{O}$ by the above integral and similar relations for $k\neq 0$.

Braun-Ji-Manashov conformal OPE

$$T^{\mu\nu} = i \int d^4z\, e^{iq'z} \langle p'|\mathbb{T}\{j^\nu(z) j^\mu(0)\}|p\rangle =$$

$$\frac{1}{i\pi^2} i \int d^4z\, e^{iq'z} \Big\{ \frac{1}{(-z^2+i0)^2} \left[g^{\nu\mu}\mathscr{O}(1,0) - z^\nu \partial^\mu \int_0^1 du\ \mathscr{O}(\bar{u},0) - z^\mu(\partial^\nu - i\Delta^\nu) \int_0^1 dv\ \mathscr{O}(1,v) \right]$$

$$- \frac{1}{-z^2+i0} \left[\frac{i}{2}(\Delta^\mu \partial^\nu - (\nu \leftrightarrow \mu)) \int_0^1 du \int_0^{\bar{u}} dv\ \mathscr{O}(\bar{u},v) - \frac{t}{4} z^\nu \partial^\mu \int_0^1 du\ u \int_0^{\bar{u}} dv\ \mathscr{O}(\bar{u},v) \right]$$

$$+ \cdots$$

Operators $\mathscr{O}$ above are understood as matrix elements, that is:

$$\langle p'|\mathscr{O}(\lambda_1,\lambda_2)|p\rangle = \frac{2i}{\lambda_{12}} \iint_{\mathbb{D}} d\beta d\alpha\ \underbrace{\left[e^{-i\ell_{\lambda_1,\lambda_2} z} + O(z^2) \right]}_{\text{LT}} \Phi^{(+)}(\beta,\alpha,t)\,,$$

where

$$\ell_{\lambda_1,\lambda_2} = -\lambda_1 \Delta - \lambda_{12}\left[\beta \bar{p} - \tfrac{1}{2}(\alpha+1)\Delta\right]$$

and

$$\Phi^{(+)}(\beta,\alpha,t) = \partial_\beta F + \partial_\alpha G \quad \leftrightarrow \quad H(x,\xi,t) = \iint_{\mathbb{D}} d\beta d\alpha\ \delta(x-\beta-\alpha\xi)[F+\xi G]\,.$$

More details on: Braun, Ji & Manashov, JHEP 03 (2021) 051; JHEP 01 (2023) 078.

Compton tensor for spin-0 target & helicity amplitudes

- Spin-0 target $\Rightarrow$ vector component of $T^{\mu\nu}$ is enough.
- Parameterization of $T^{\mu\nu} \to$ **helicity amplitudes, $\mathcal{A}^{AB}$.**
- Spin-0 $\Rightarrow$ total of **5 independent** $\mathcal{A}^{AB}$s thanks to parity conservation.

$$T^{\mu\nu} = \mathcal{A}^{00} \frac{-i}{QQ'R^2} \left[(qq')(Q'^2 q^\mu q^\nu - Q^2 q'^\mu q'^\nu) + Q^2 Q'^2 q^\mu q'^\nu - (qq')^2 q'^\mu q^\nu \right]$$
$$+ \mathcal{A}^{+0} \frac{i\sqrt{2}}{R|\bar{p}_\perp|} \left[Q' q^\mu - \frac{qq'}{Q'} q'^\mu \right] \bar{p}_\perp^\nu - \mathcal{A}^{0+} \frac{\sqrt{2}}{R|\bar{p}_\perp|} \bar{p}_\perp^\mu \left[\frac{qq'}{Q} q^\nu + Q q'^\nu \right]$$
$$+ \mathcal{A}^{+-} \frac{1}{|\bar{p}_\perp|^2} \left[\bar{p}_\perp^\mu \bar{p}_\perp^\nu - \widetilde{\bar{p}}_\perp^\mu \widetilde{\bar{p}}_\perp^\nu \right] - \mathcal{A}^{++} g_\perp^{\mu\nu} \,,$$

- Read out projectors $\to \mathcal{A}^{AB} = \boldsymbol{\Pi}_{\mu\nu}^{(AB)} T^{\mu\nu}$.
- Scale of DDVCS: $\mathbb{Q}^2 = Q^2 + Q'^2 + t$.

Double DVCS' $\mathcal{A}^{++}$ = LT + tw-4 + O(tw-6), at LO

$$
\begin{aligned}
\mathcal{H}^{++} = \mathcal{A}^{++} = \int_{-1}^{1} dx \Bigg\{ &- \left(1 - \frac{t}{2\mathbb{Q}^2} + \frac{t(\xi-\rho)}{\mathbb{Q}^2}\partial_\xi\right) \frac{H^{(+)}}{x-\rho+i0} \\
&+ \frac{t}{\xi\mathbb{Q}^2}\Bigg[\mathbb{P}_{(\mathrm{i})} + \mathbb{P}_{(\mathrm{ii})} - \frac{\xi L}{2} + \frac{\widetilde{\mathbb{P}}_{(\mathrm{iii})} - \widetilde{\mathbb{P}}_{(\mathrm{i})}}{2} \\
&\qquad - \frac{\xi}{x+\xi}\left(\ln\left(\frac{x-\rho+i0}{\xi-\rho+i0}\right) - \frac{\xi+\rho}{2\xi}\ln\left(\frac{-\xi-\rho+i0}{\xi-\rho+i0}\right) - \widetilde{\mathbb{P}}_{(\mathrm{i})}\right)\Bigg] H^{(+)} \\
&- \frac{t}{\mathbb{Q}^2}\partial_\xi\Bigg[\Bigg(\mathbb{P}_{(\mathrm{i})} + \mathbb{P}_{(\mathrm{ii})} - \frac{\xi L}{2} \\
&\qquad - \frac{\xi}{x+\xi}\left(\ln\left(\frac{x-\rho+i0}{\xi-\rho+i0}\right) - \frac{\xi+\rho}{2\xi}\ln\left(\frac{-\xi-\rho+i0}{\xi-\rho+i0}\right) - \widetilde{\mathbb{P}}_{(\mathrm{i})}\right)\Bigg) H^{(+)}\Bigg] \\
&+ \frac{\xi^2\bar{p}_\perp^2}{\mathbb{Q}^2} 2\xi\partial_\xi^2\left[\left(\mathbb{P}_{(\mathrm{i})} + \mathbb{P}_{(\mathrm{ii})} - \frac{\xi L}{2} + \frac{\widetilde{\mathbb{P}}_{(\mathrm{iii})} + \widetilde{\mathbb{P}}_{(\mathrm{i})}}{2}\right) H^{(+)}\right]\Bigg\} \\
&+ O(\mathrm{tw\text{-}6})\,.
\end{aligned}
$$

- $\xi^2 \bar{p}_\perp^2 = \xi^2 M^2 - t\left(\xi^2 - 1\right)/4\,.$
- **All amplitudes in VMF, Pire, Sznajder & Wagner, PRD 111 (2025) 7, 074034.**

- Coefficient functions of $\mathcal{A}^{++}$:

$$\mathbb{P}_{(\mathrm{i})}(x/\xi,\rho/\xi) = \frac{\xi-\rho}{x-\xi}\mathrm{Li}_2\left(-\frac{x-\xi}{\xi-\rho+i0}\right),$$

$$\widetilde{\mathbb{P}}_{(\mathrm{i})}(x/\xi,\rho/\xi) = -\frac{\xi-\rho}{x-\xi}\ln\left(\frac{x-\rho+i0}{\xi-\rho+i0}\right),$$

$$\mathbb{P}_{(\mathrm{ii})}(x/\xi,\rho/\xi) = \frac{\xi-\rho}{x+\xi}\left[\mathrm{Li}_2\left(-\frac{x-\xi}{\xi-\rho+i0}\right)-(x\to-\xi)\right],$$

$$\widetilde{\mathbb{P}}_{(\mathrm{iii})}(x/\xi,\rho/\xi) = -\frac{\xi+\rho}{x+\xi}\ln\left(\frac{x-\rho+i0}{-\xi-\rho+i0}\right),$$

$$L = \int_0^1 dw\ \frac{-4}{x-\xi-w(x+\xi)}\int_0^1 du\ \ln\left(1+\frac{\bar{u}[x-\xi-w(x+\xi)]}{\xi-\rho+i0}\right)C_{\bar{u},\bar{u}w},$$

$$C_{\bar{u},v} = \ln\left(\frac{\bar{u}-v}{1-v}\right)+\frac{1}{1-v}.$$

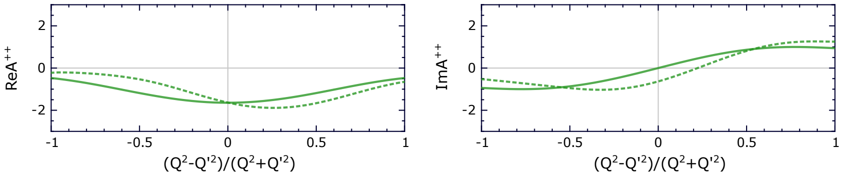
- From the DDVCS result:

$$\begin{cases}\rho\to\xi\Rightarrow \text{DVCS}^{\#}, \\ \rho\to-\xi(1-2t/Q'^2)\Rightarrow \text{TCS to twist-4 accuracy.}\end{cases}$$

DVCS for spin-0 target was already computed in: Braun, Ji & Manashov, JHEP 01 (2023) 078.

Phenomenology for pion target

π-GPD model: J. M. Morgado-Chávez et al., PRD 105, 094012 (2022).



$\xi = 0.2$, $\mathbb{Q}^2 = 1.9\ \mathrm{GeV}^2$, $t = -0.6\ \mathrm{GeV}^2$.
Solid line: LT. Dashed line: LT + tw-4.

- The higher-twist corrections **break** the simple LO LT relation: $\mathcal{A}^{++}_{\mathrm{DVCS}} \overset{\mathrm{LO,\ LT}}{=} \left(\mathcal{A}^{++}_{\mathrm{TCS}}\right)^*$.

LT NLO coefficient functions

$$T_{\rm LT}^{++}(x/\xi) = \left[C_0(x/\xi) + C_1(x/\xi) + \ln\left(\frac{Q^2}{\mu_{\rm F}^2}\right) C_{\rm coll}(x/\xi) \right] - (x/\xi \to -x/\xi)\,,$$

$$\begin{aligned} C_0(x/\xi) &= -\frac{\xi}{x+\xi-i0}\,, \\ C_1(x/\xi) &= \frac{\alpha_S C_F}{4\pi}\frac{\xi}{x+\xi-i0}\left[9 - 3\frac{x+\xi}{x-\xi}\ln\left(\frac{x+\xi}{2\xi}-i0\right) - \ln^2\left(\frac{x+\xi}{2\xi}-i0\right)\right], \\ C_{\rm coll}(x/\xi) &= \frac{\alpha_S C_F}{4\pi}\frac{\xi}{x+\xi-i0}\left[-3 - 2\ln\left(\frac{x+\xi}{2\xi}-i0\right)\right], \end{aligned}$$

$+$ gluon terms.

NLO and higher-twist coefficient functions:
$\boldsymbol{C_i \to \mathrm{Im}(C_i) \propto \theta(x-\xi),\ \delta(x-\xi)}$**, accessing therefore:**

$$\begin{cases} \text{DGLAP:} & |x| > |\xi| \ \checkmark \\ \text{ERBL:} & |x| < |\xi| \ \times \end{cases}$$

Belitsky, Müller, PLB 417 (1998) 129; Ji, Osborne, PRD 58 (1998) 094018; Ji, Osborne, PRD 57 (1998) 1337; Mankiewicz, Piller, Stein, Vänttinen, Weigl, PLB 425 (1998) 186; Pire, Szymanowski & Wagner, PRD 83 (2011) 034009.

Towards the dispersion relation (DR)

Incoming-photon helicity: A
Outgoing-photon helicity: B

Analyticity $\Rightarrow \mathcal{H}^{AB}(\xi) = \sum_{j=0}^{\infty} h_j^{AB} \frac{1}{\xi^j}$,

$\boldsymbol{h_j^{AB}} \in \mathbb{R}$ (Schwartz) **.**

Integration over $\boldsymbol{\Gamma_{\mathrm{R}}}$ and $\boldsymbol{\Gamma'}$:

1) $$\oint_{\Gamma_{\mathrm{R}}} d\xi' \, \frac{\mathcal{H}^{AB}(\xi')}{\xi' - \xi} \left(\frac{\xi'}{\xi}\right)^n = 2\pi i \sum_{j=0}^{n} h_j^{AB} \frac{1}{\xi^j}$$

2) $$\oint_{\Gamma'} d\xi' \, \frac{\mathcal{H}^{AB}(\xi')}{\xi' - \xi} \left(\frac{\xi'}{\xi}\right)^n =$$

$$= \mathrm{PV} \int_{-(1+\Lambda)}^{1+\Lambda} d\xi' \, \frac{\mathcal{H}^{AB}(\xi' - i0) - \mathcal{H}^{AB}(\xi' + i0)}{\xi' - \xi} \left(\frac{\xi'}{\xi}\right)^n$$

$$+ i\pi \left[\mathcal{H}^{AB}(\xi - i0) + \mathcal{H}^{AB}(\xi + i0)\right]$$

Schwartz + homotopy of Γ_{R} and Γ' = **dispersion relation (DR):**

$$\sum_{j=0}^{n} h_j^{AB} \frac{1}{\xi^j} = \frac{1}{\pi} \mathrm{PV} \int_{-(1+\boldsymbol{\Lambda})}^{1+\boldsymbol{\Lambda}} d\xi' \, \frac{\mathrm{Im}(\mathcal{H}^{AB}(\xi'))}{\xi' - \xi} \left(\frac{\xi'}{\xi}\right)^n + \mathrm{Re}(\mathcal{H}^{AB}(\xi))$$

LO h_0^{++} fit: $d_{1,a}$, unconstrained gluons

LO h_0^{++}, $n=1$, radiative gluons
$d_{1,uds}(0,\mu_0^2)$ free (only)

$d_{1,uds}(0, 2\ \mathrm{GeV}^2) =$	-0.6 ± 1.1
$d_{1,g}(0, 2\ \mathrm{GeV}^2) =$	-0.8 ± 1.5
$d_{1,c}(0, 2\ \mathrm{GeV}^2) =$	-0.003 ± 0.005

LO h_0^{++}, $n=1$, free gluons
$d_{1,uds}(0,\mu_0^2)$, $d_{1,g}(0,\mu_0^2)$ free

$d_{1,uds}(0, 2\ \mathrm{GeV}^2) =$	-0.6 ± 1.1
$d_{1,g}(0, 2\ \mathrm{GeV}^2) =$	-11 ± 132
$d_{1,c}(0, 2\ \mathrm{GeV}^2) =$	-0.04 ± 0.47

Radiative (generation at $\mu_g^2 = 0.09$) vs free gluons:

$$d_{1,uds}(\mu^2) = \Gamma_1^{qq}(\mu^2,\mu_0^2)\left[1 + \frac{\Gamma_1^{qg}(\mu^2,\mu_0^2)\Gamma_1^{gq}(\mu_0^2,\mu_g^2)}{\Gamma_1^{qq}(\mu^2,\mu_0^2)\Gamma_1^{qq}(\mu_0^2,\mu_g^2)}\right] \times d_{1,uds}(\mu_0^2)\,.$$

$$\mu_0^2 = 1\,,\ \mu^2 = 2.5 \Rightarrow \frac{\Gamma_1^{qg}(\mu^2,\mu_0^2)}{\Gamma_1^{qq}(\mu^2,\mu_0^2)} \approx \frac{1}{60}\,.$$

Gluon distribution needs to be 60 times larger than the quark distribution to contribute similarly.

Conclusion: At LO, DVCS is NOT sensitive to $d_{1,g}$. Evolution does not allow for assessment on $d_{1,g}$.