Kinematic power corrections in DVCS with twist-6 accuracy

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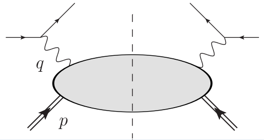
based on

VB, Yao Ji, A. Manashov PRD *111 (2025) 076011*

JLAB, Newport News, 23.05.2025

Planar vs. non-planar kinematics

“Natural” separation of longitudinal and transverse d.o.f. in DIS

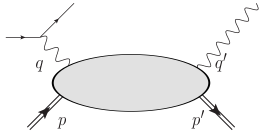


$$p = (p_0, \vec{0}_\perp, p_z)$$
$$q = (q_0, \vec{0}_\perp, q_z)$$

$\Rightarrow$ parton fraction = Bjorken x_B

Planar vs. non-planar kinematics (2)

Many possible choices in DVCS



"DIS frame"

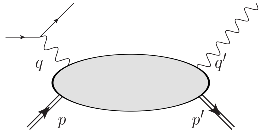
$$p = (p_0, \vec{0}_\perp, p_z)$$
$$q = (q_0, \vec{0}_\perp, q_z)$$

⇒ asymmetry parameter $\xi \simeq x_B/(2 - x_B)$

⇒ momentum transfer $\Delta = p' - p$ (almost) transverse

Planar vs. non-planar kinematics (2)

Many possible choices in DVCS



"Photon frame"

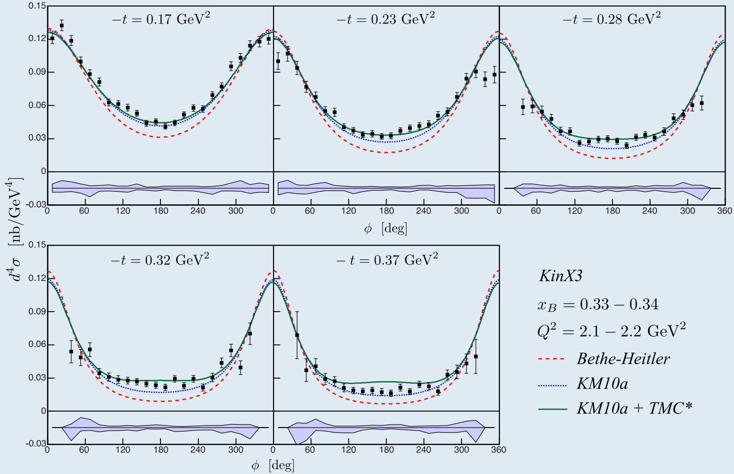
$$\begin{aligned} q' &= (q'_0, \vec{0}_\perp, q'_z) \\ q &= (q_0, \vec{0}_\perp, q_z) \end{aligned}$$

$\Rightarrow$ skewedness parameter $\xi = \frac{x_B(1+t/Q^2)}{2-x_B(1-t/Q^2)}$

$\Rightarrow$ momentum transfer $\Delta = p' - p$ longitudinal

Large effects for the DVCS cross section in certain kinematics

M. Defurne *et al.* [Hall A Collaboration] arXiv:1504.05453



GPD model: KM10a (Kumericki, Mueller, Nucl. Phys. B **841** (2010) 1)

Kinematic power corrections $(\sqrt{-t}/Q)^k$ and $(m/Q)^k$

- Ambiguity in the choice of collinear directions makes "leading-twist" calculations ambiguous. In addition, electromagnetic Ward identities are violated.
  - Repaired by power-suppressed corrections, $(\sqrt{-t}/Q)^k$ and $(m/Q)^k$
  - "Kinematic" — do not involve new nonperturbative input apart from usual GPDs
  - Factorizable

- Twist-four

  VB, A. Manashov, JHEP **01** (2012), 085 ⟵ method
  VB, A. Manashov, D. Müller, B. Pirnay, PRD **89** (2014) 074022

- Twist-six

  VB, Y. Ji, A. Manashov, JHEP **03** (2021), 051 ⟵ method
  VB, Y. Ji, A. Manashov, JHEP **01** (2023), 078 ⟵ scalar target
  VB, Y. Ji, A. Manashov, PRD **111** (2025), 076011

OPE formulation

schematically

$$\mathsf{T}\{j(x)j(0)\} = \sum_N \Big\{ A_N^{\mu_1\ldots\mu_N} \underbrace{\mathcal{O}^N_{\mu_1\ldots\mu_N}}_{\text{twist-2 operators}} + B_N^{\mu_1\ldots\mu_N} \underbrace{\partial^\mu \mathcal{O}^N_{\mu,\mu_1\ldots\mu_N}}_{\text{descendants of twist 2}}$$

$$+ C_N^{\mu_1\ldots\mu_N} \underbrace{\partial^2 \mathcal{O}^N_{\mu_1\ldots\mu_N}}_{\text{descendants}} + D_N^{\mu_1\ldots\mu_N} \underbrace{\partial^\mu\partial^\nu \mathcal{O}^N_{\mu,\nu,\mu_1\ldots\mu_N}}_{\text{descendants}} + \ldots \Big\}$$

$$+ \text{ quark-gluon operators}$$

“kinematic” corrections that repair the frame dependence and Ward identities come from

(1) corrections m/Q and $\sqrt{-t}/Q$ to the ME of twist-two operators (Nachtmann)

(2) higher-twist operators that are obtained from twist-two by adding total derivatives

Problem: matrix elements of some descendant operators over free quarks vanish

Ferrara, Grillo, Parisi, Gatto, '71-'73

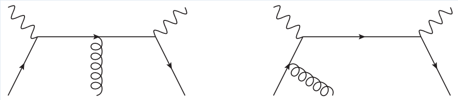
Example

$$\partial^\mu O_{\mu\nu} = 2i\bar{q} g F_{\nu\mu}\gamma^\mu q\,, \qquad O_{\mu\nu} = (1/2)[\bar{q}\gamma_\mu \overset{\leftrightarrow}{D}_\nu\, q + (\mu \leftrightarrow \nu)]$$

- Usual procedure to calculate the coefficient functions does not work

VB, A. Manashov, D. Müller, B. Pirnay '11-'14

- Consider quark-gluon matrix elements



- Use hermiticity of evolution equations for twist-4 operators to separate "kinematic" terms from "genuine" (quark-gluon) contributions

CFT formulation: all twists

$$\mathsf{T}\{j(x)j(0)\} = \sum_N \Big\{ A_N^{\mu_1\cdots\mu_N} \underbrace{\mathcal{O}^N_{\mu_1\cdots\mu_N}}_{\text{twist-2 operators}} + B_N^{\mu_1\cdots\mu_N} \underbrace{\partial^\mu \mathcal{O}^N_{\mu,\mu_1\cdots\mu_N}}_{\text{descendants of twist 2}}$$

$$+ C_N^{\mu_1\cdots\mu_N} \underbrace{\partial^2 \mathcal{O}^N_{\mu_1\cdots\mu_N}}_{\text{descendants}} + D_N^{\mu_1\cdots\mu_N} \underbrace{\partial^\mu\partial^\nu \mathcal{O}^N_{\mu,\nu,\mu_1\cdots\mu_N}}_{\text{descendants}} + \ldots \Big\} + \ldots$$

$$\equiv \sum_N C_N^{\mu_1\cdots\mu_N}(x,\partial)\, \mathcal{O}^N_{\mu_1\cdots\mu_N} + \text{ quark-gluon operators}$$

S. Ferrara, A. F. Grillo and R. Gatto, 1971-1973: "Conformally covariant OPE"

In conformal field theories, the CFs of descendants are related to the CFs of twist-2 operators by symmetry and do not need to be calculated directly

$$A_N^{\mu_1\cdots\mu_N} \overset{O(4,2)}{\mapsto} C_N^{\mu_1\cdots\mu_N}(x,\partial)$$

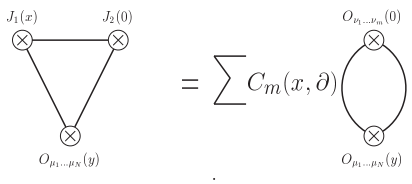
Conformal triangles

A.M. Polyakov, 1970:

$$\langle O_1(x_1)\, O_2(x_2)\rangle = \frac{\text{const}}{|x_1-x_2|^{2\Delta_1}}\,\delta_{\Delta_1\Delta_2}$$

$$\langle O_1(x_1)\, O_2(x_2)\, O_3(x_3)\rangle = \frac{\text{const}}{|x_1-x_2|^{\Delta_1+\Delta_2-\Delta_3}\,|x_1-x_3|^{\Delta_1+\Delta_3-\Delta_2}\,|x_2-x_3|^{\Delta_3+\Delta_2-\Delta_1}}$$

- ← Δ_k is a scaling dimension (canonical + anomalous)



- ← exact to all orders of perturbation theory

S. Ferrara, A. F. Grillo and R. Gatto, Lett.Nuovo.Cim. 2(1971) 1363

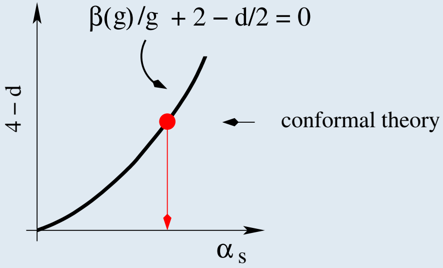
Conformal symmetry in QCD?

QCD is not a conformal theory, but

$$\mathcal{A}_{\rm QCD} = \mathcal{A}^{\rm conf}_{\rm QCD} + O(\beta(\alpha_s))$$

"Conformal QCD": QCD in $d - 2\epsilon$ at Wilson-Fischer critical point $\beta(\alpha_S) = 0$

V.B., A. Manashov, Eur.Phys.J.C 73 (2013) 2544



- Done (to LO):

$$\mathcal{A}^{(\pm\pm)} \sim 1 + \frac{1}{Q^2} + \frac{1}{Q^4} + \dots \qquad \checkmark$$

$$\mathcal{A}^{(\pm 0)} \sim \frac{1}{Q} + \frac{1}{Q^3} + \dots \qquad \checkmark$$

$$\mathcal{A}^{(\pm\mp)} \sim \frac{1}{Q^2} + \frac{1}{Q^4} + \dots \qquad \checkmark$$

- further powers can be calculated if necessary

- Observe:
  - — factorization valid at twist 6 (IR divergences cancel)
  - — target mass corrections absorbed in the dependence on $t_{min} = -\frac{\xi^2 m^2}{1-\xi^2}$

Compare DIS, Nachtmann variable

$$\xi_N = \frac{2x_B}{1 + \sqrt{1 + \frac{4x_B^2 m^2}{Q^2}}} = x_B \left(1 - \frac{x_B^2 m^2}{Q^2} + \dots\right)$$

- On a nucleus $m \mapsto Am$, $x_B \mapsto x_B/A$, $\xi \mapsto \xi/A$, hence TMCs are the same
  $\to$ factorization not in danger

Kinematics and notations

- BMP frame

$$P_\mu = \frac{1}{2}(p_\mu + p'_\mu) = \alpha q_\mu + \beta q'_\mu + P^\perp_\mu$$

- Photon polarization vectors

$$\varepsilon^0_\mu = -\Big(q_\mu - q'_\mu q^2/(q\cdot q')\Big)\,/\sqrt{-q^2}\,,$$
$$\varepsilon^\pm_\mu = (P^\perp_\mu \pm i\bar{P}^\perp_\mu)/(\sqrt{2}|P_\perp|)\,,$$

- Helicity amplitudes

$$\mathcal{A}_{\mu\nu} = \varepsilon^+_\mu \varepsilon^-_\nu \mathcal{A}^{++} + \varepsilon^-_\mu \varepsilon^+_\nu \mathcal{A}^{--} + \varepsilon^0_\mu \varepsilon^-_\nu \mathcal{A}^{0+} + \varepsilon^0_\mu \varepsilon^+_\nu \mathcal{A}^{0-} + \varepsilon^+_\mu \varepsilon^+_\nu \mathcal{A}^{+-} + \varepsilon^-_\mu \varepsilon^-_\nu \mathcal{A}^{-+}$$

- Kinematic invariants

$$\xi = -\frac{\Delta\cdot q'}{2P\cdot q'} = \frac{x_B(1+t/Q^2)}{2 - x_B(1-t/Q^2)} \qquad |P_\perp|^2 = \frac{1-\xi^2}{4\xi^2}(t_{\min} - t)$$

Example: Helicity-conserving amplitude

$$\mathcal{A}_V^{\pm,\pm} = \frac{(vq')}{(qq')} V_0^{(1)} + \frac{(vP)}{m^2} V_0^{(2)}\,, \qquad v^\mu = \bar{u}(p')\gamma^\mu u(p)$$

$$\begin{aligned}
V_0^{(1)} &= -\Big(1+\frac{\widehat{t}}{4}\Big)\Big(M\otimes T_0\Big) - \frac{\widehat{t}}{2}\Big(M\otimes T_{10}\Big) + \frac{1}{4}\widehat{t}^2\Big(M\otimes T_{11}\Big) - \frac{1}{2}D_\xi^2|\widehat{P}_\perp|^2\Big(M\otimes\Big(T_2+2\widehat{t}\,T_V\Big)\Big) \\
&\quad + \frac{1}{8}D_\xi^3|\widehat{P}_\perp|^4 D_\xi\Big(M\otimes T_3\Big)\,, \\
V_0^{(2)} &= -\Big(1+\frac{\widehat{t}}{4}\Big)\Big(E\odot T_0\Big) - \frac{\widehat{t}}{2}\Big(E\odot T_{10}\Big) + \frac{\widehat{t}^2}{4}\Big(E\odot T_{11}\Big) - \frac{1}{2}D_\xi|\widehat{P}_\perp|^2 D_\xi\Big(E\odot\Big(T_2+2\widehat{t}\,T_V\Big)\Big) \\
&\quad + \frac{1}{8}D_\xi^2|\widehat{P}_\perp|^4 D_\xi^2\Big(E\odot T_3\Big) - \widehat{m}^2\Big\{D_\xi\Big(M\otimes\Big(T_2+2\widehat{t}T_V\Big)\Big) - \frac{1}{2}D_\xi^2|\widehat{P}_\perp|^2\Big(M\otimes T_3\Big)\Big\}
\end{aligned}$$

$$\widehat{t} = \frac{t}{(qq')}\,, \qquad \widehat{m}^2 = \frac{m^2}{(qq')}\,, \qquad |\widehat{P}_\perp|^2 = \frac{|P_\perp|^2}{(qq')}, \qquad (qq') = -\frac{1}{2}(Q^2+t)$$

$$M(x,\xi,t) = H(x,\xi,t) + E(x,\xi,t) \qquad \boxed{D_\xi = (-2\xi^2\partial_\xi)}$$

Coefficient functions (complete set)

$$T_0(z) = \frac{1}{\bar{z}}\,,$$

$$T_{00}(z) = \frac{\bar{z}}{z}\ln\bar{z}\,,$$

$$T_{11}(z) = \left(2 - \frac{1}{z}\right)\ln\bar{z}\,,$$

$$T_A(z) = -\frac{1}{\bar{z}}\Big(\mathrm{Li}_2(z) - \zeta_2\Big) + \frac{1}{z}\ln\bar{z}\,,$$

$$T_3(z) = \frac{2z+1}{\bar{z}}\Big(\mathrm{Li}_2(z) - \zeta_2\Big) - \frac{1}{2}\left(7 - \frac{1}{z}\right)\ln\bar{z}\,.$$

$$T_1(z) = \ln\bar{z}\,,$$

$$T_{10}(z) = \frac{1}{z}\ln\bar{z}\,,$$

$$T_V(z) = \frac{1}{\bar{z}}\Big(\mathrm{Li}_2(z) - \zeta_2\Big) - \ln\bar{z}\,,$$

$$T_2(z) = \frac{1}{\bar{z}}\Big(\mathrm{Li}_2(z) - \zeta_2\Big) - \frac{1}{2z}\ln\bar{z}\,,$$

- They are analytic functions of z with a cut from 1 to ∞, apart from T_0 which has a pole singularity.

Convolution integrals involve the CFs on the upper side of the cut: $T(z) \mapsto T(z+i\epsilon)$ for $x > \xi > 0$.

Compton Form factor $\mathcal{H}^{++}(x_B, t, Q^2)$

VB, Y. Ji, A. Manashov, PRD **111** (2025), 076011

- All numerical results are for the GK12 GPD model

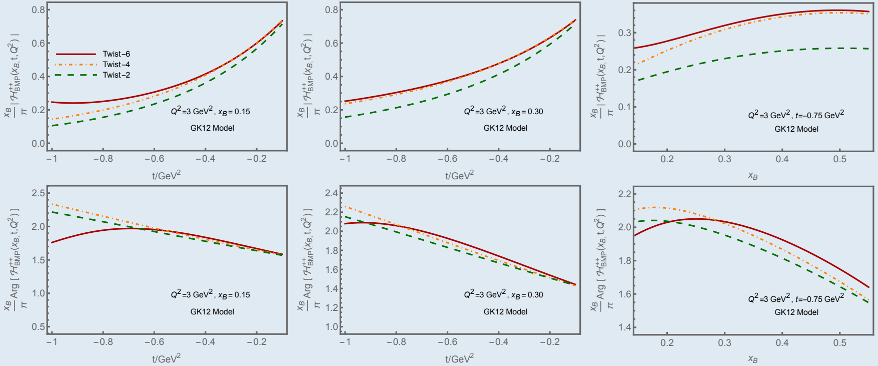


Figure: Kinematic power corrections to the absolute value and phase of the BMP Compton Form Factor $\mathcal{H}^{++}_{\rm BMP}(x_B, t, Q^2)$

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VB, Y. Ji, A. Manashov, PRD **111** (2025), 076011

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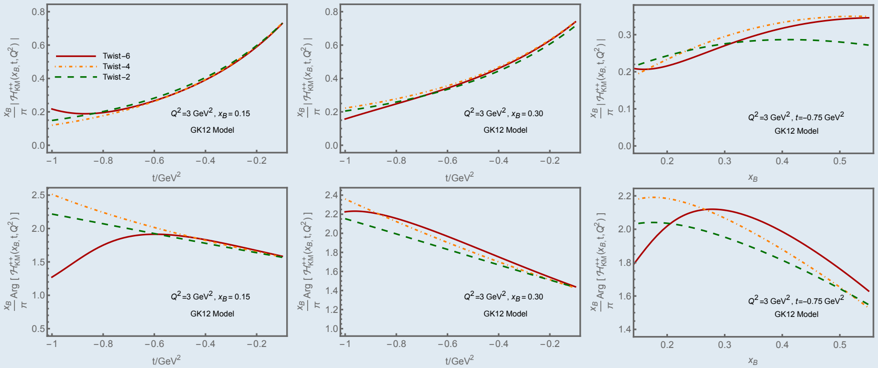


Figure: Kinematic power corrections to the absolute value and phase of the KM Compton Form Factor $\mathcal{H}^{++}_{\rm KM}(x_B, t, Q^2)$.

Helicity-flip Compton Form factor $\mathcal{H}^{0+}(x_B, t, Q^2)$

VB, Y. Ji, A. Manashov, PRD **111** (2025), 076011

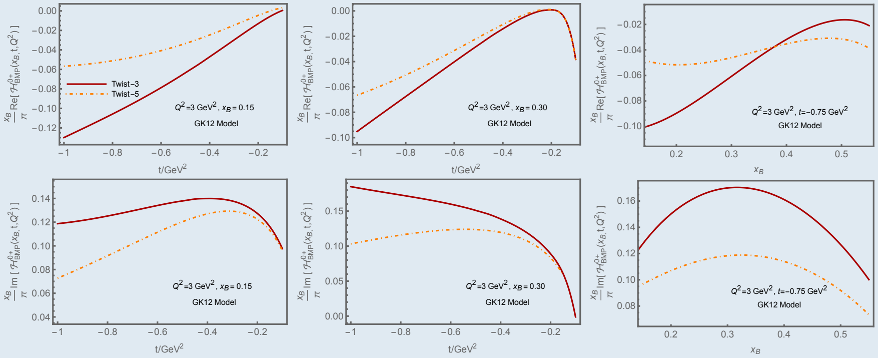
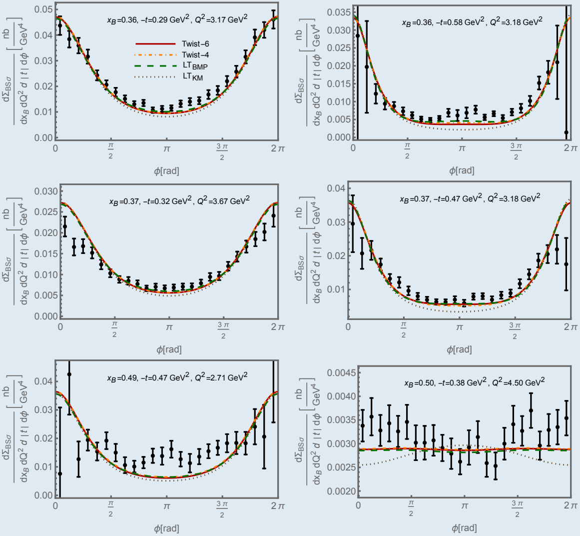


Figure: Real (upper panels) and imaginary (lower panels) parts of the helicity-flip BMP Compton Form Factor $\mathcal{H}^{0+}_{\rm BMP}(x_B, t, Q^2)$

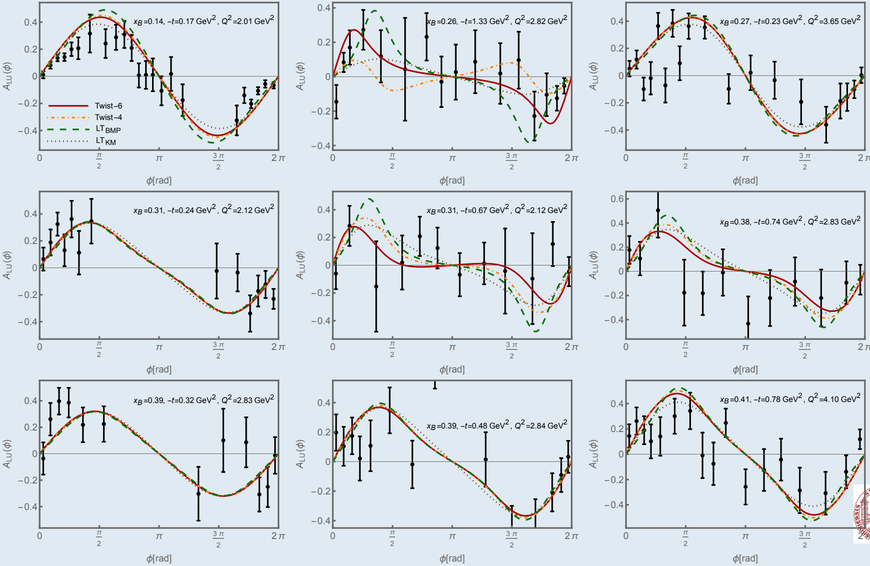
Spin-averaged cross sections

F. Georges et al. (JLAB Hall A), PRL 128, 252002 (2022)



Beam spin asymmetries

G. Christiaens et al. (CLAS), PRL 130, 211902 (2023)



Summary and Outlook

1. **Towards NNLO accuracy**
   - Two-loop coefficient functions for DVCS
     — sizeable corrections, completed for light quarks
     — new: DDVCS, flavor-nonsinglet only
   - Three-loop evolution equations for GPDS
     — flavor-nonsiglet in position space, singlet for the first few moments
     — pressing issue: numerical implementation, also in NLO
   - Threshold resummations at $x \to \xi$
     — completed to NNLL; new: DDVCS,

2. **Kinematic power corrections**
   - new: Twist-six accuracy, $(\sqrt{-t}/Q)^3$, $(m/Q)^3$
     — complete results available, numerical code (B.Pirnay + ...)
     — good convergence if expansion organized in $1/(Q^2+t)$
     — large effects for parts of phase space and in collider kinematics
     — coherent DVCS from nuclei: Target mass corrections do not spoil factorization

3. **Further issues**
   — establishing NLO accuracy (at minimum) as standard of the field
   — GPDs from Compton form factors; Neural networks or ansätze?
   — t-dependence of “genuine” higher-twist contributions; models