Andrea Simonelli

Unveiling off lightcone effects in hadronic processes

QCD EVOLUTION

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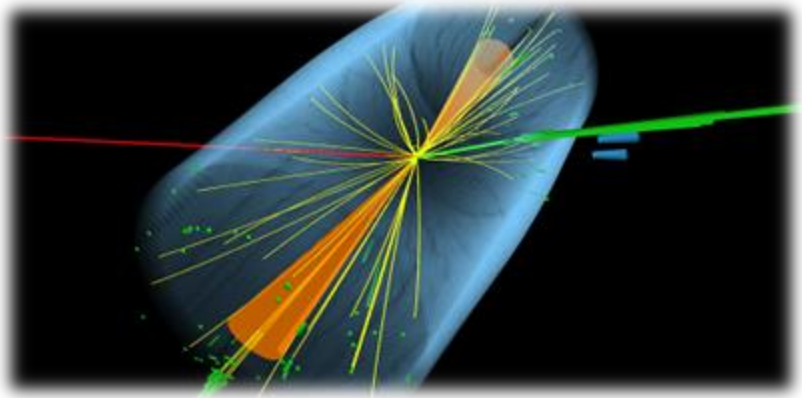
Introduction

What are the off light-cone effects in hadronic processes and how to track them

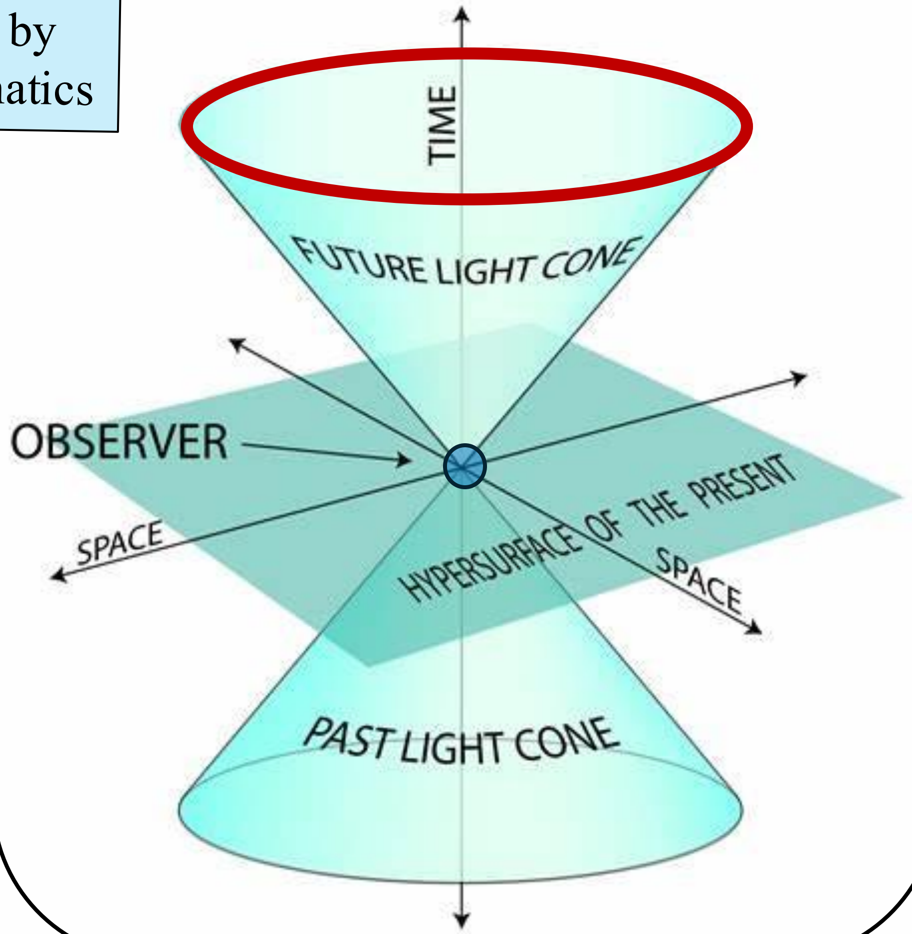
Introduction

Hadronic process



Underlying spacetime structure

Fixed by kinematics



Factorization

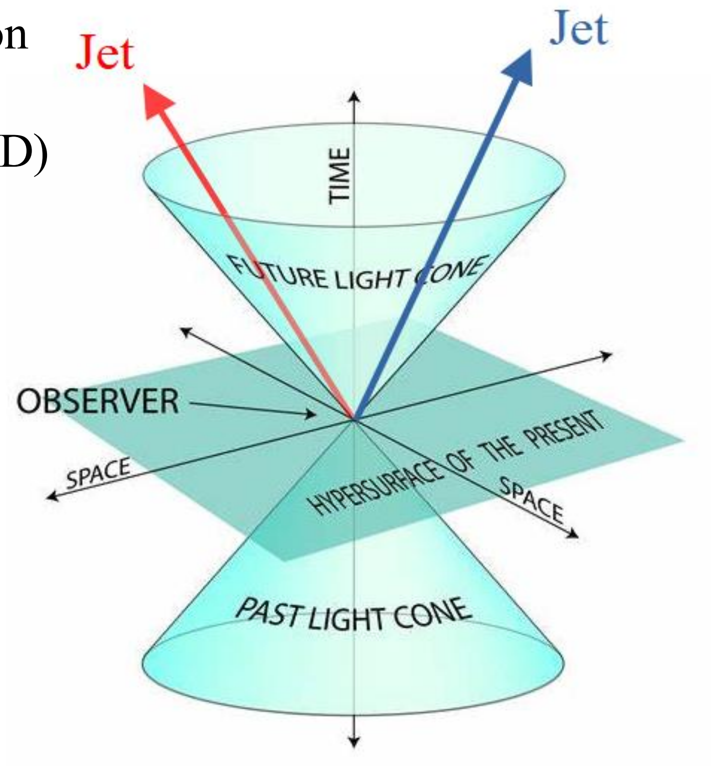
"Untangling" the short-distance contributions (PERTURBATIVE) from the long-distance contributions (NON-PERTURBATIVE)

Introduction

A certain configuration is particularly relevant: **two opposite lightcone directions**

$e^+e^- \to \text{dijet}$ - like

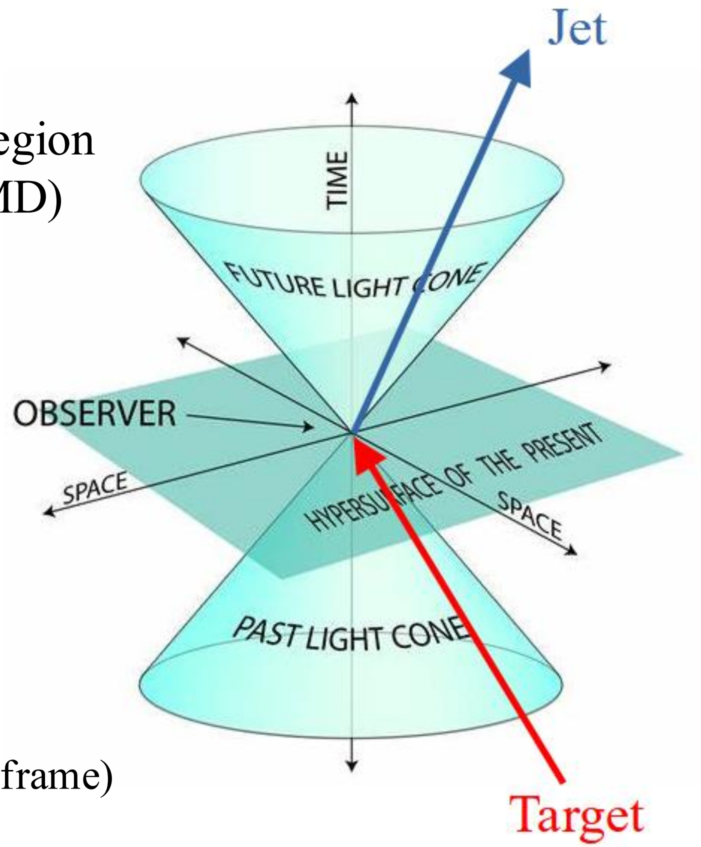
- Thrust distribution
- Double-inclusive annihilation (TMD)
- ...



(center-of-mass frame)

Semi-inclusive DIS - like

- DIS in the endpoint region
- SIDIS (TMD)
- ...

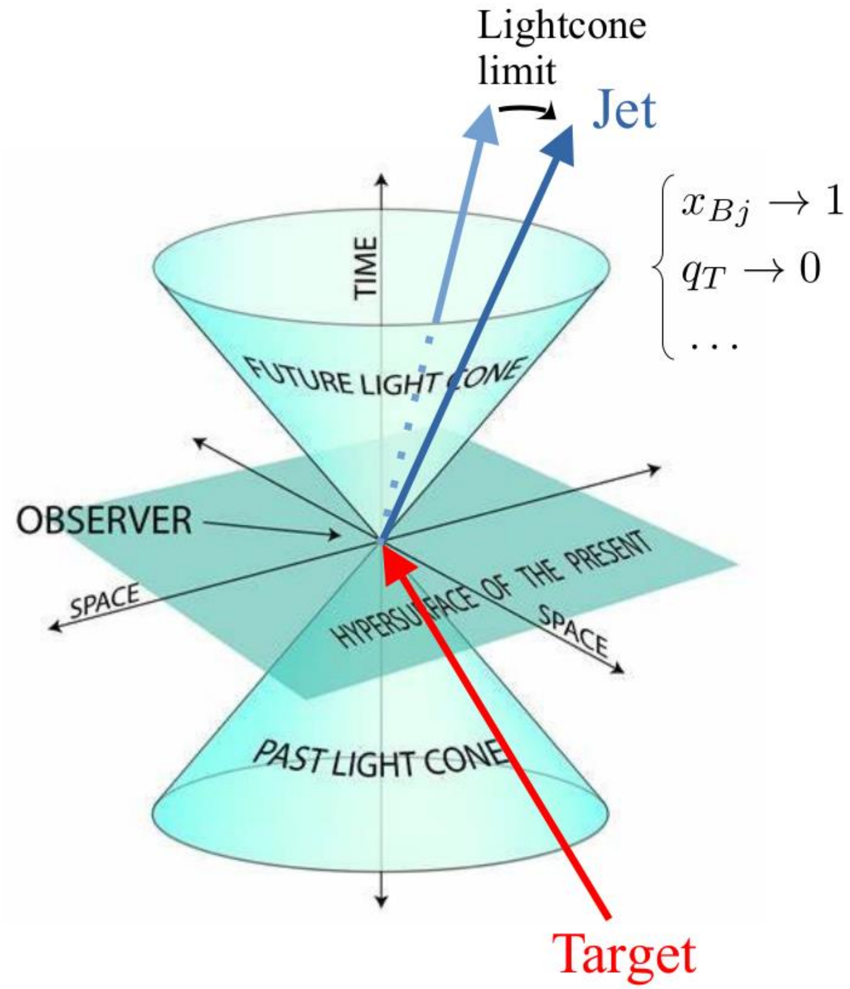


(Breit frame)

...as well as Drell-Yan like (both directions in the past)

Introduction

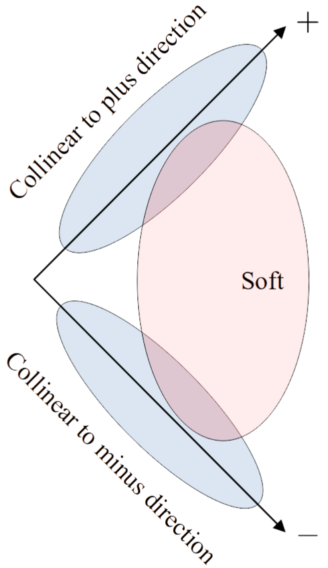
A certain configuration is particularly relevant: **two opposite lightcone directions**



In the real world, the exact lightcone is an idealized scenario:

- Mass effects (true if $Q \to \infty$)
- Kinematic limit

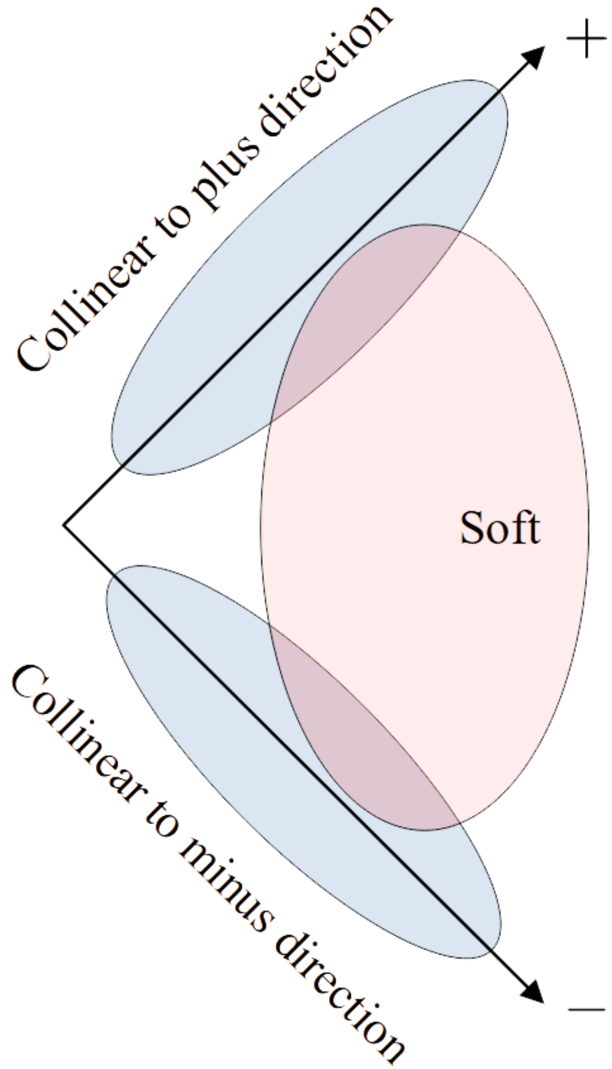
However, the behavior in the *lightcone limit* (i.e. the limit in which the two directions lie exactly on two opposite lightcone directions) has *universal properties due to soft correlations*



The real elephant in the room!

Introduction

A certain configuration is particularly relevant: **two opposite lightcone directions**



Entangling Soft Radiation

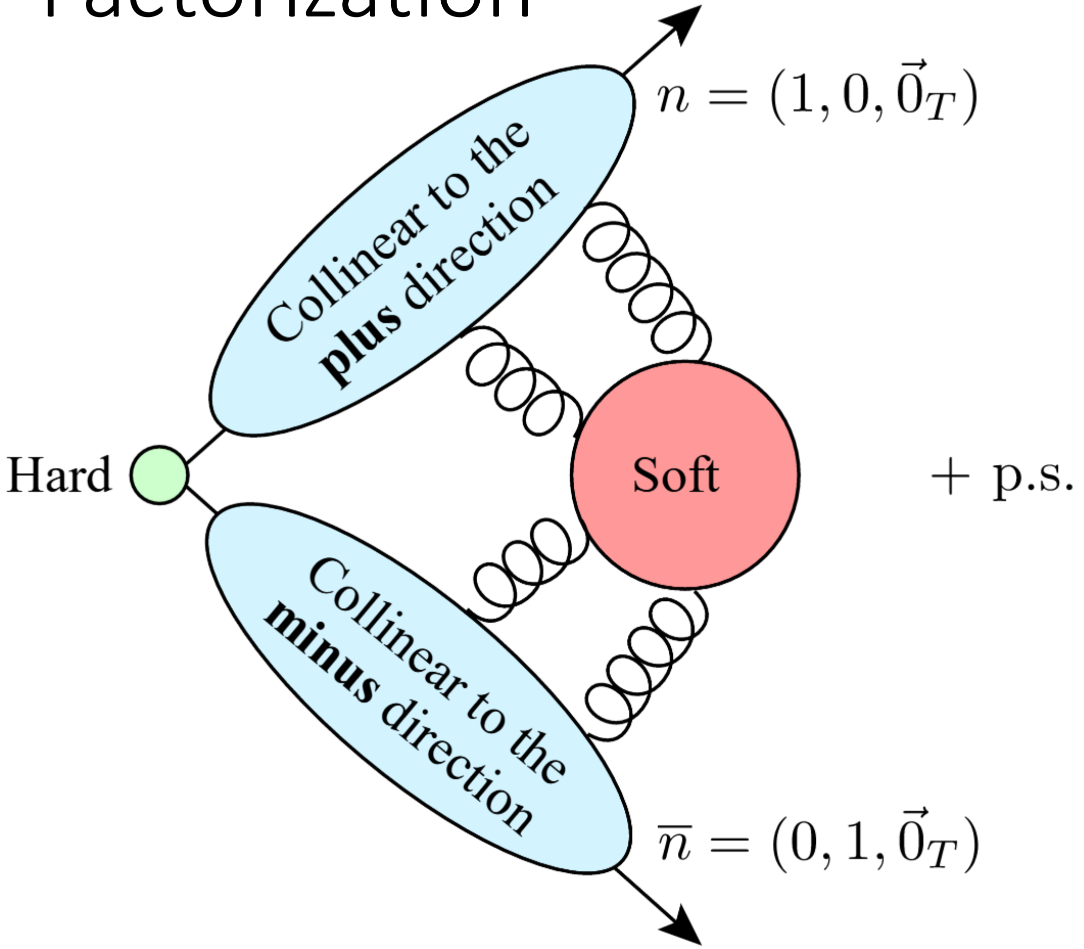
Subtractions

Rapidity Divergences

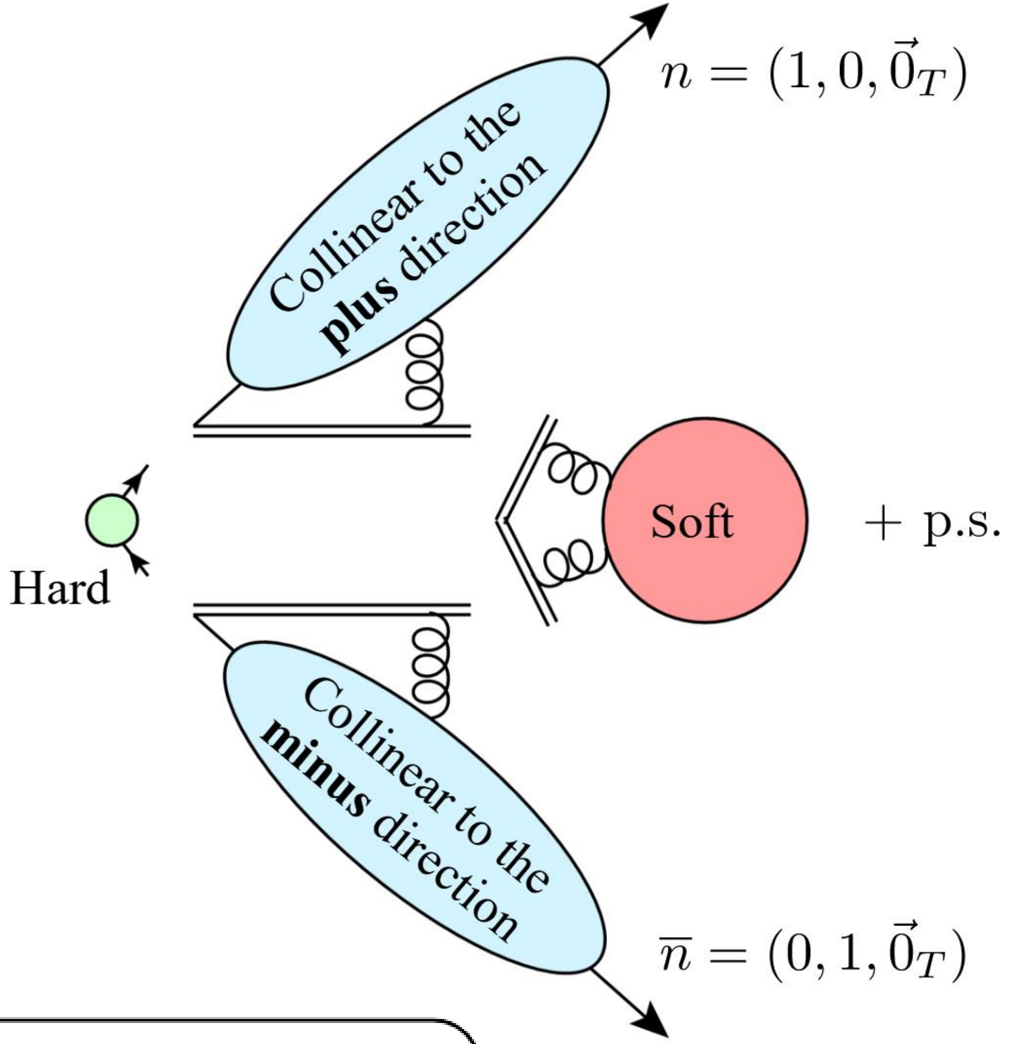
An old and familiar problem in TMD physics.

Can we adapt TMD factorization to a general 2-opposite lightcone direction situation?

Factorization



- Kinematic approximations
- Ward identities
- Subtractions

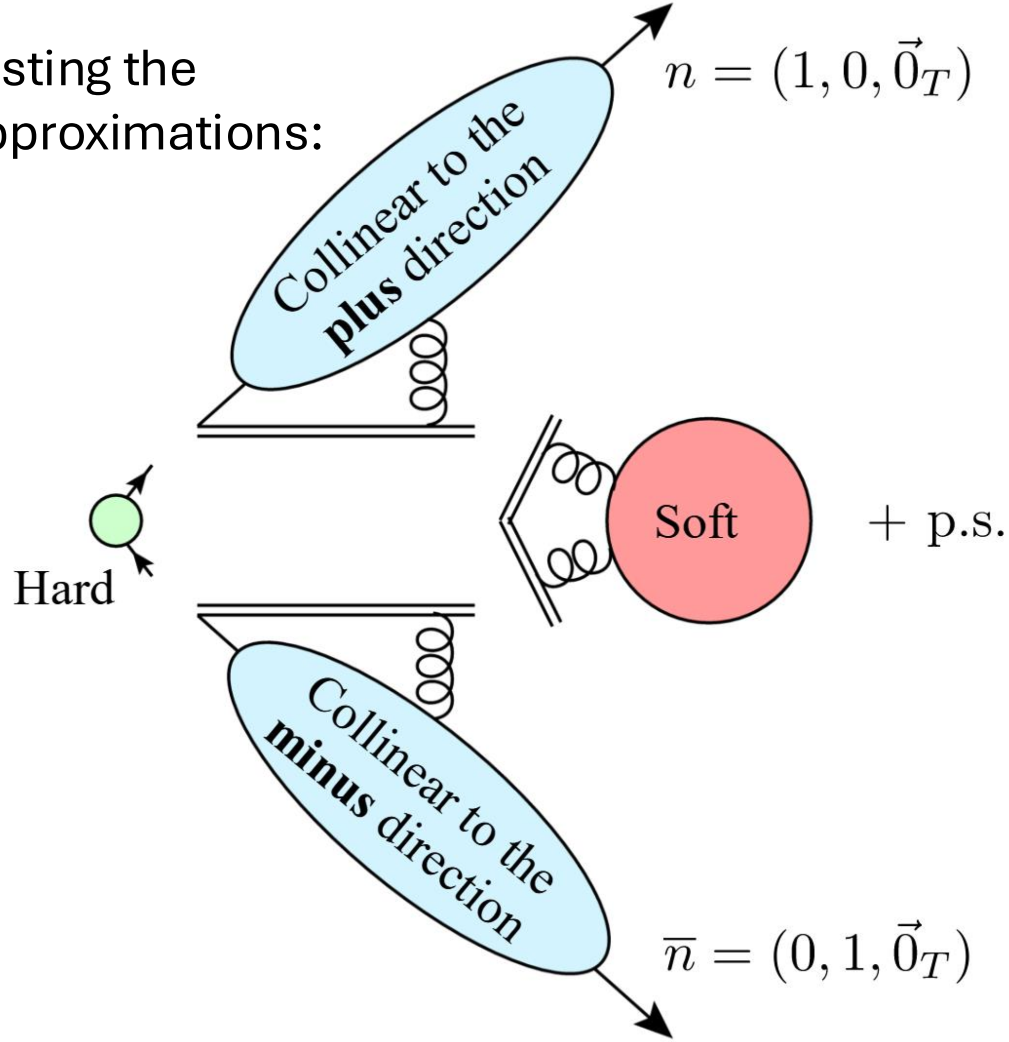


Typical **factorization theorem** for processes characterized by 2 opposite LC directions

$$H \cdot C_+ \cdot S \cdot C_-$$

Factorization

Testing the approximations:



The Wilson lines lie *exactly* on the light-cone

RAPIDITY DIVERGENCES

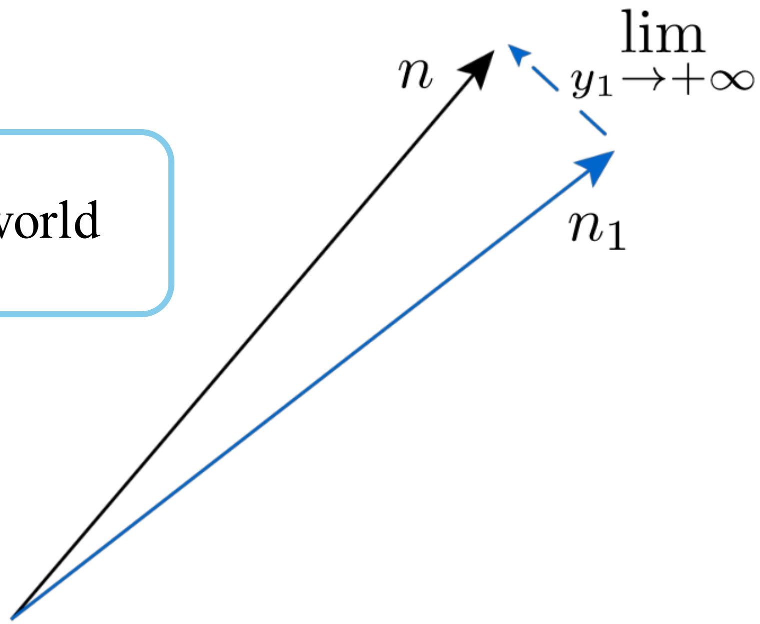
$$\int_0^\infty \frac{dk^+}{k^+}$$

$$H \cdot C_+ \cdot S \cdot C_-$$

Long-distance operators are ill-defined when considered singularly

This is a symptom that we are missing something: too strong approximations?

Off lightcone effects



Deviation from the lightcone ⟷ Deviation from the idealistic world

How can we parametrize such deviation?

A simple (and realistic) choice is the introduction of tilts:

$$n = (1, 0, \vec{0}_T) \to n_1 = (1, \pm e^{-2y_1}, \vec{0}_T),$$
$$\overline{n} = (0, 1, \vec{0}_T) \to n_2 = (\pm e^{2y_2}, 1, \vec{0}_T)$$

The lightcone limit corresponds to $y_{1,2} \to \pm\infty$

The choice of the sign of the tilts and the orientation (future vs past) is crucial for the validity of factorization

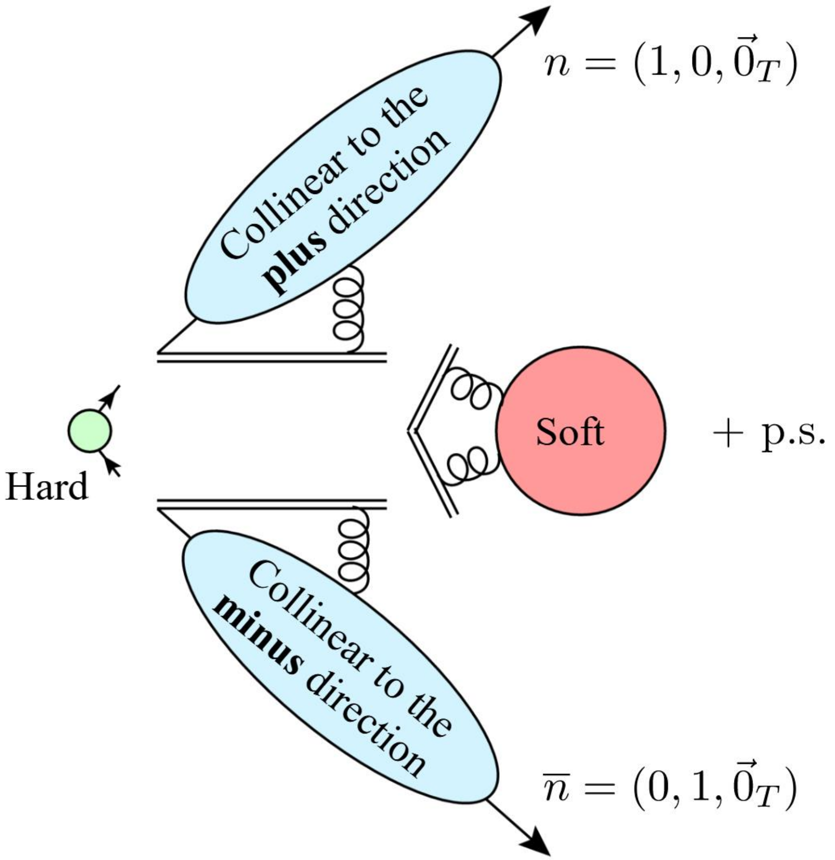
We can now track the effects of going off the lightcone: do they impact leading-power (LP) factorization?

Naively, one might say no. After all, tilts are ultimately mass effects.
However, this conclusion is non-trivial and, most importantly, not guaranteed *a priori*.

Rapidity scale separation

All the operators are now defined off the light-cone:

$$H \cdot C_+(y_1) \cdot S(y_1, y_2) \cdot C_-(y_2)$$



$$y \gtrsim y_1$$

$$y_1 \gtrsim y \gtrsim y_2$$

$$y_2 \gtrsim y$$

There is a clear and transparent separation in rapidity

Regularization of rapidity divergences

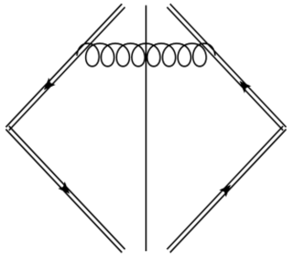
The tilts provide a natural regularization (at operator level) for the rapidity divergences:

$$\mathcal{P}\mathrm{exp}\left\{ -ig_0 \int_0^\infty d\tau n^\mu A_\mu^{(0)}(a+\tau n) \right\}$$

$$\Rightarrow \quad \mathcal{P}\mathrm{exp}\left\{ -ig_0 \int_0^\infty d\tau n_{1,2}^\mu A_\mu^{(0)}(a+\tau n_{1,2}) \right\}$$

Resulting in the off lightcone eikonals propagators: $\frac{1}{k^+} \mapsto \frac{1}{k^+ \pm e^{2y_2} k^-}, \quad \frac{1}{k^-} \mapsto \frac{1}{k^- \pm e^{-2y_1} k^+}$

1. Gauge invariance ✓
2. Soft exponentiation ✓
3. Genuine off lightcone terms ✓



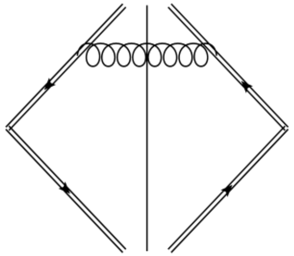
Naively zero, but actually ill-defined on the lightcone!

Regularization of rapidity divergences

What if the rapidity regulator is defined on the lightcone?
Very popular choices are:

- Delta regulator $\int d\tau n^\mu \rightarrow \int d\tau e^{-\delta\,\tau} n^\mu$
- Rapidity Renormalization Group $\frac{1}{k^\pm} \rightarrow \frac{\nu^\eta}{(k^\pm)^{1+\eta}}$

1. Gauge invariance ⚠ (Only for covariant gauges and after cancellation of regulators)
2. Soft exponentiation ✔ (although difficult in RRG)
3. Genuine off lightcone terms ✘



Non trivial in non-covariant gauges, e.g.
$A^3 = 0$

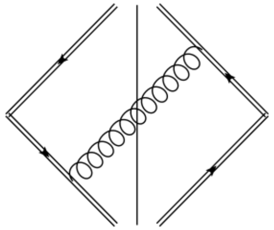
Universal K-P Decomposition

Each operator in off lightcone factorization:

$$\mathcal{O}(\ldots, y_1, y_2) \propto \exp\{\mathbf{K}\left[K(\ldots), y_1, y_2\right] + \mathbf{P}\left[P(\ldots), y_1, y_2\right] + \mathcal{O}(e^{-2y_1}, e^{2y_2})\}$$

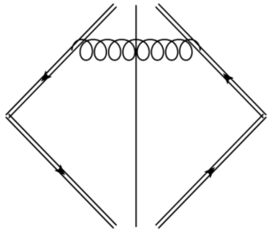
Leading asymptotic behavior in the light-cone limit.
If the tilts are removed, this is the leading rapidity divergent term.

It is a functional of the **Collins-Soper kernel K** typical of TMD observables.
In covariant gauges, it is associated to gluon exchanges between *opposite* directions.



Sub-Leading asymptotic behavior in the light-cone limit.
If the tilts are removed, this term (might) introduce a sub-leading rapidity divergence.

It is a functional of **P**, a (perhaps never mentioned before) universal function.
In covariant gauges, it is associated to gluon exchanges between *the same* directions.



Light-cone suppressed terms in the light-cone limit.
If the tilts are removed, this terms do not contribute.

Universal K-P Decomposition

Each operator in off lightcone factorization:

$$\mathcal{O}(\ldots, y_1, y_2) \propto \exp\{\mathbf{K}\,[K(\ldots), y_1, y_2] + \mathbf{P}\,[P(\ldots), y_1, y_2] + \mathcal{O}(e^{-2y_1}, e^{2y_2})\}$$

Two sources of off lightcone effects:

1. The dependence on the tilts
2. The dependence on the P-terms

Three possible scenarios:

Lightcone factorization theorem
Off-lightcone effects cancel in both the factorized operators and the cross section.
E.g. DIS at threshold

Operators sensitive to off lightcone effects
Soft and collinear operators are defined off the lightcone, yet the cross section remains independent of off lightcone effects.
E.g. TMD factorized cross sections.

Factorization sensitive to off lightcone effects
Tilts are intimately connected to kinematic variables and do not cancel.
E.g. single inclusive thrust and transverse momentum distribution of e+e- annihiliation (BELLE)

Unveiling the Collins-Soper kernel in inclusive DIS at threshold

Andrea Simonelli,[1,*] Alberto Accardi,[2,3,†] Matteo Cerutti,[2,3,‡] Caroline S. R. Costa,[4,§] and Andrea Signori[5,6,¶]

2502.15033 [hep-ph]

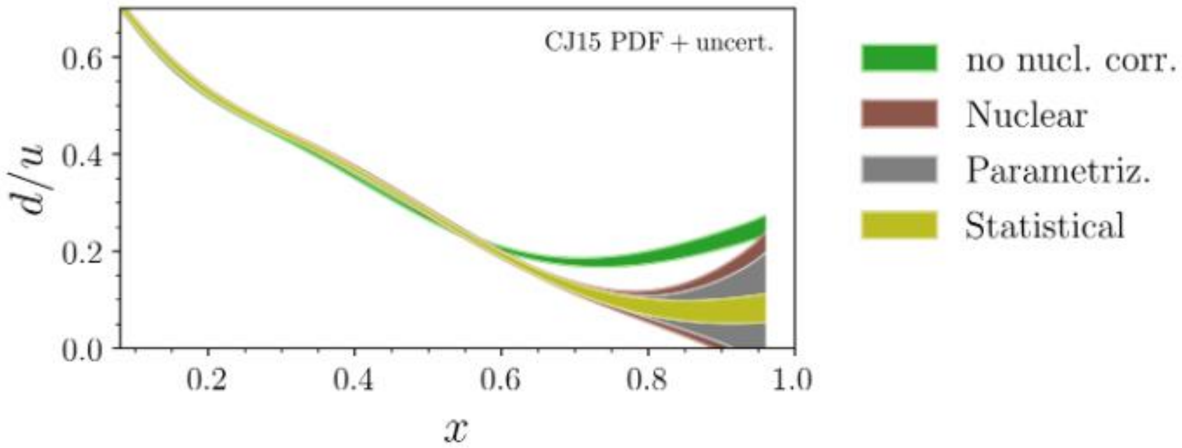
An interesting case

Inclusive DIS in the endpoint region

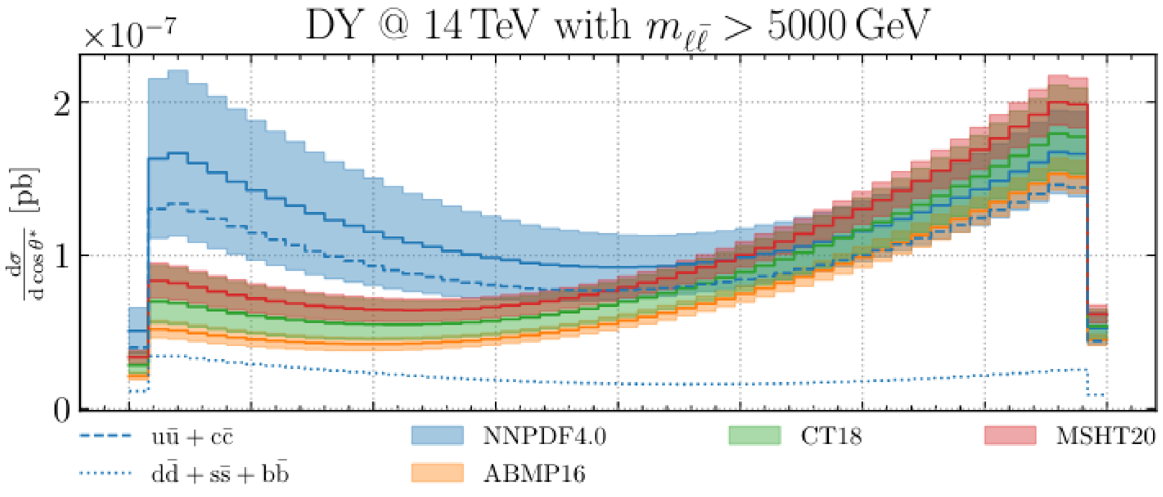
PDFs in the threshold limit

- d/u ratio inside Proton

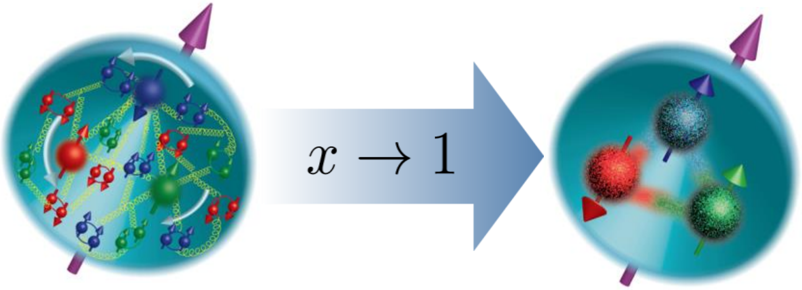
Phys.Rev.D 109 (2024) 7, 074036



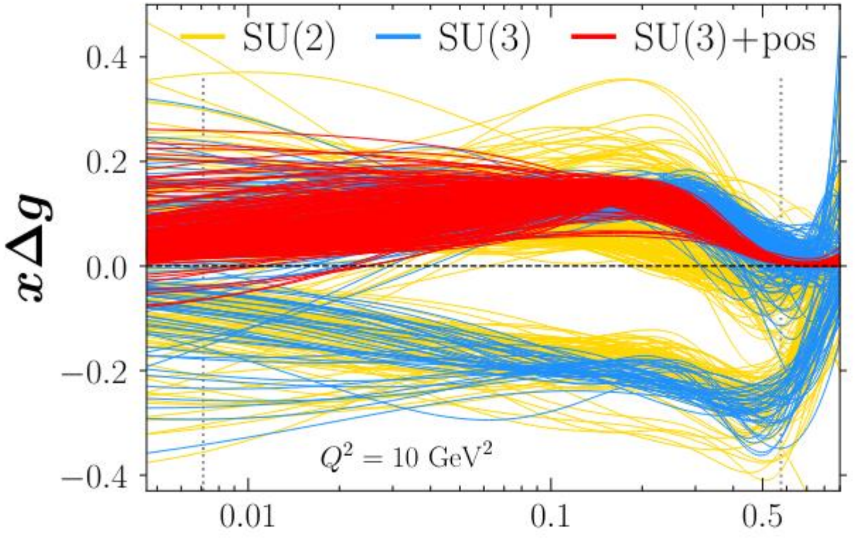
- Search for new physics BSM



Eur.Phys.J.C 82 (2022) 12, 1160



- Theory constraints (positivity bounds)



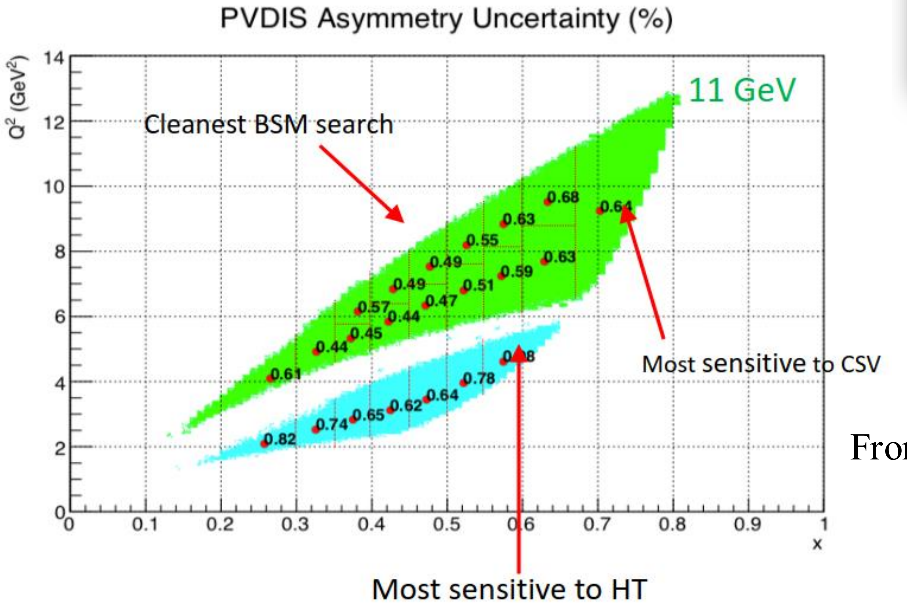
(JAM) Phys.Rev.D 105 (2022) 7, 074022

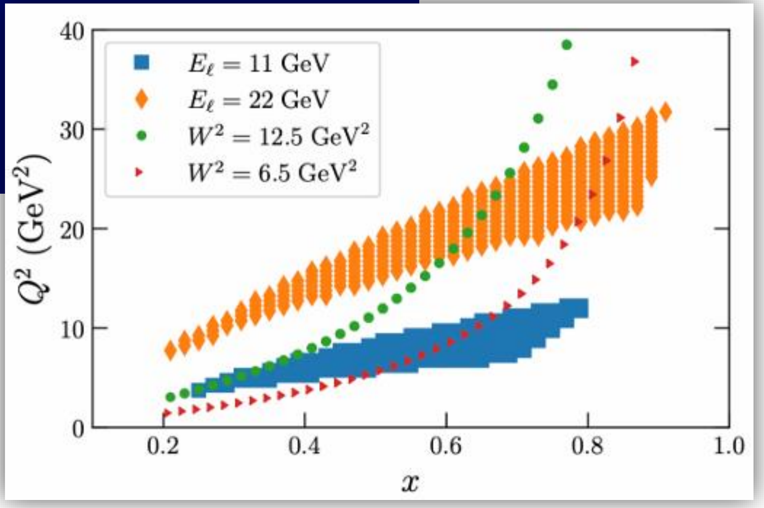
PDFs in the threshold limit

Strong interaction physics at the luminosity frontier with 22 GeV electrons at Jefferson Lab

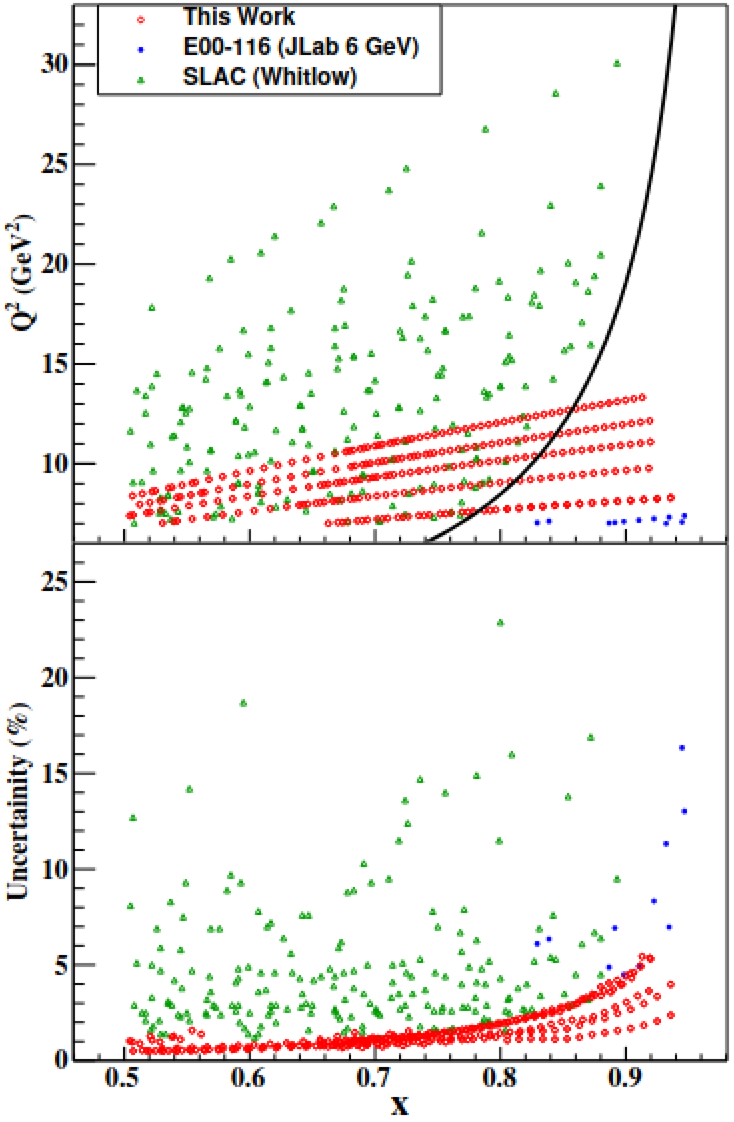
Review | Published: 04 September 2024

Volume 60, article number 173, (2024) Cite this article





From Zein-Eddine Meziani's talk



(JLAB Hall-C) 2409.15236 [hep-ex]

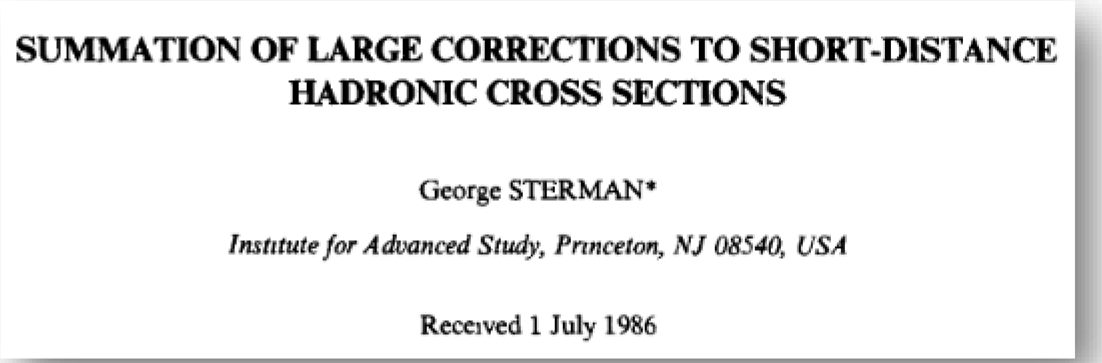
Past literature

QCD

- Sterman (1986)
- Catani, Trentadue (1989)

SCET

- Becher, Neubert, Pecjak (2007)
- Fleming, Labun (2012)
- Chay, Kim (2013)



SUMMATION OF LARGE CORRECTIONS TO SHORT-DISTANCE HADRONIC CROSS SECTIONS

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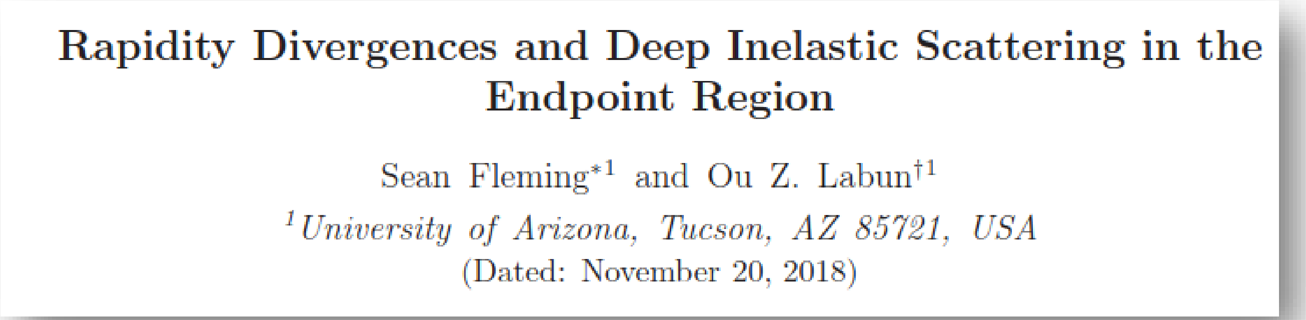
Received 1 July 1986



Rapidity Divergences and Deep Inelastic Scattering in the Endpoint Region

Sean Fleming*[1] and Ou Z. Labun†[1]

[1] *University of Arizona, Tucson, AZ 85721, USA*

(Dated: November 20, 2018)

Is there still more to uncover?

Past literature...unresolved issues

From Sterman (1986)

$$F(x,Q^2) = |H_{\mathrm{DI}}(Q)|^2 \int_x^1 (\mathrm{d}y/y)\phi(y,p^+,p\cdot n) \times \int_0^{y-x} (\mathrm{d}w/[1-w]) V(wp^+, wp\cdot n) J[l^2, l\cdot n] + O(1-x)^0$$

Explicit soft function

Gauge choice $A^3 = 0$ ⟶ No rapidity divergences?

From Becher, Neubert, Pecjak (2007)

$$F_2^{\mathrm{ns}}(x,Q^2) = \sum_q e_q^2 \, |C_V(Q^2,\mu)|^2 \, Q^2 \int_x^1 d\xi \, J\Big(Q^2 \frac{\xi - x}{x}, \mu\Big) \, \phi_q^{\mathrm{ns}}(\xi,\mu) \, .$$

Where is the soft function? What about rapidity divergences?

From Flemin, Labun (2012)

" [Rapidity anomalous dimensions] reveal sensitivity to IR scales, which may signal a *breakdown* of rapidity factorization in $\mathrm{SCET_{II}}$ "

What is the role of soft function?
What happens to rapidity divergences?
How PDFs at threshold are defined?

Off lightcone factorization answers all this questions!

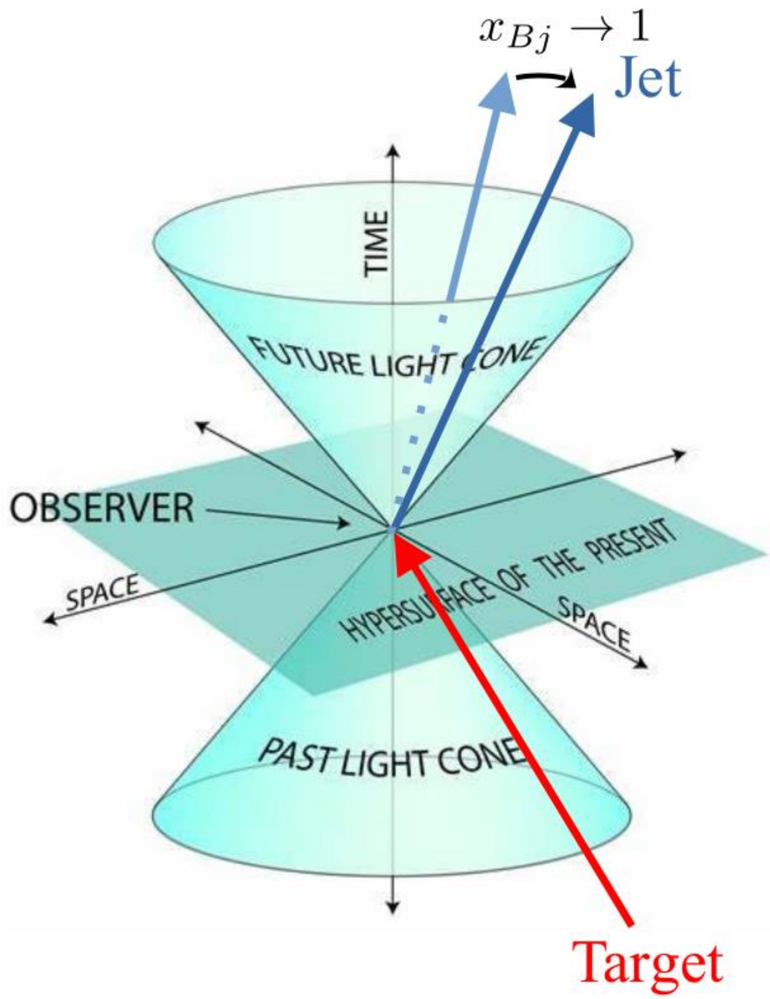
DIS in the endpoint region

Invariant mass is limited by kinematics

$$W^2 = \frac{1 - x_{Bj}}{x_{Bj}} Q^2$$

Strong analogies with TMD SIDIS

The final state becomes more and more jet-like as Bjorken x increases and the spread of transverse momentum is limited



Two (nearly) opposite lightcone directions

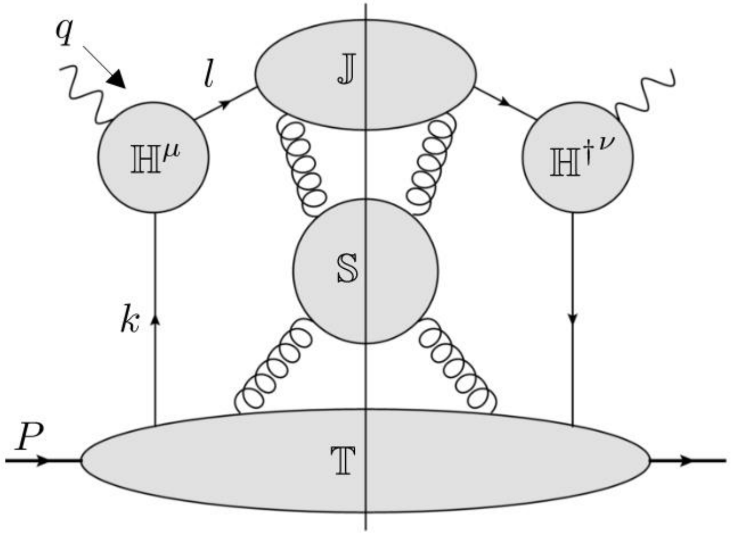
Future pointing Wilson line, with a **time-like tilt** for the jet direction (minus).
Past pointing Wilson line, with **space-like tilt** for the target direction (plus)

$$n = (1, 0, \vec{0}_T) \quad \mapsto \quad n_1 = (1, -e^{-2y_1}, \vec{0}_T),$$
$$\overline{n} = (0, 1, \vec{0}_T) \quad \mapsto \quad n_2 = (e^{2y_2}, 1, \vec{0}_T).$$

(Physical choice)

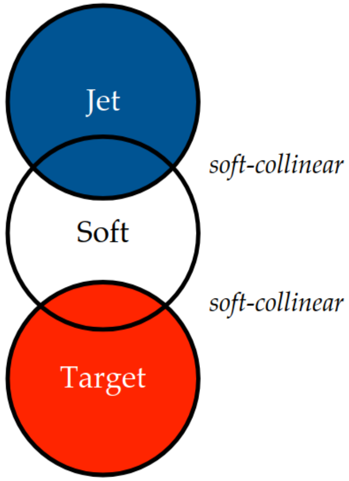
This choice is consistent with Glauber gluons treatment.

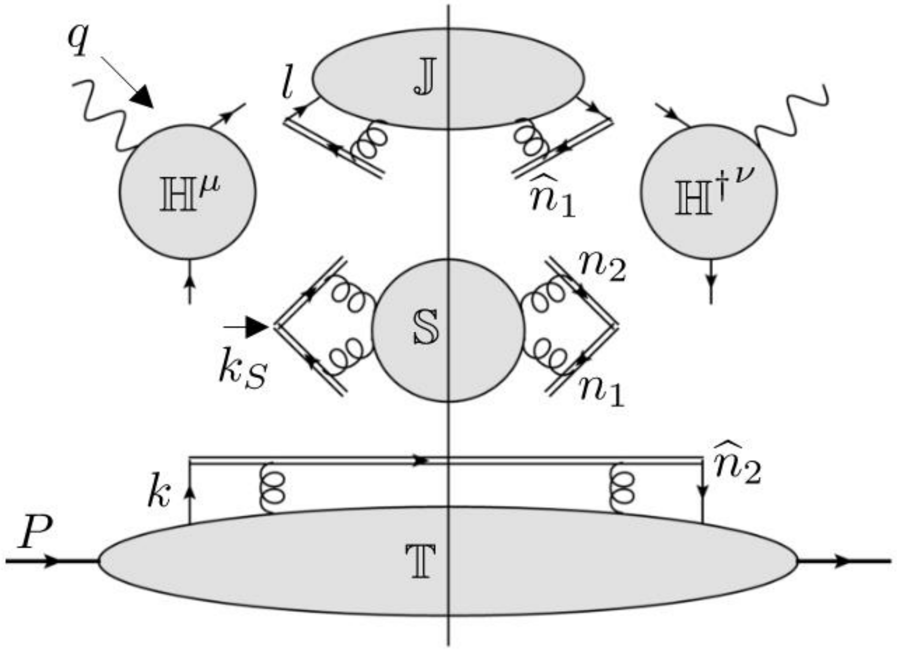
DIS in the endpoint region



Off lightcone factorization

Soft and collinear operators are defined off the lightcone and properly subtracted





Gauge invariance is guaranteed, and all rapidity divergences are regularized by the tilts

DIS in the endpoint region

From our last paper 2502.15033 [hep-ph]

$$x\, W^{\mu\nu} = \mathbf{u}^{\mu\nu}\, H(Q^2) \int_x^1 \frac{d\xi}{\xi} \int_0^{\xi - x} d\rho \times \mathcal{F}_j^{\rm sub}(\xi; y_1)\, S\Big(\frac{\rho}{\xi}; y_1, y_2\Big)\, \mathcal{J}_j^{\rm sub}\Big(\frac{\xi - x - \rho}{x}; y_2\Big), \qquad (4)$$

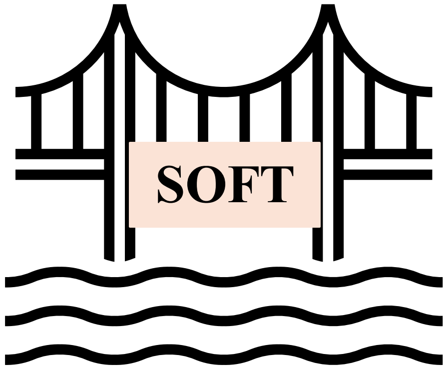
Subtractions, handling of rapidity divergences, universality of lightcone asymptotic behavior...all clearer in **Mellin space**

It is equivalent to Sterman's result, with the difference that soft and collinear operators are defined off the lightcone.
Remember:

y_1: large and positive y_2: large and negative

The soft function bridges the rapidity gap between the target and the jet

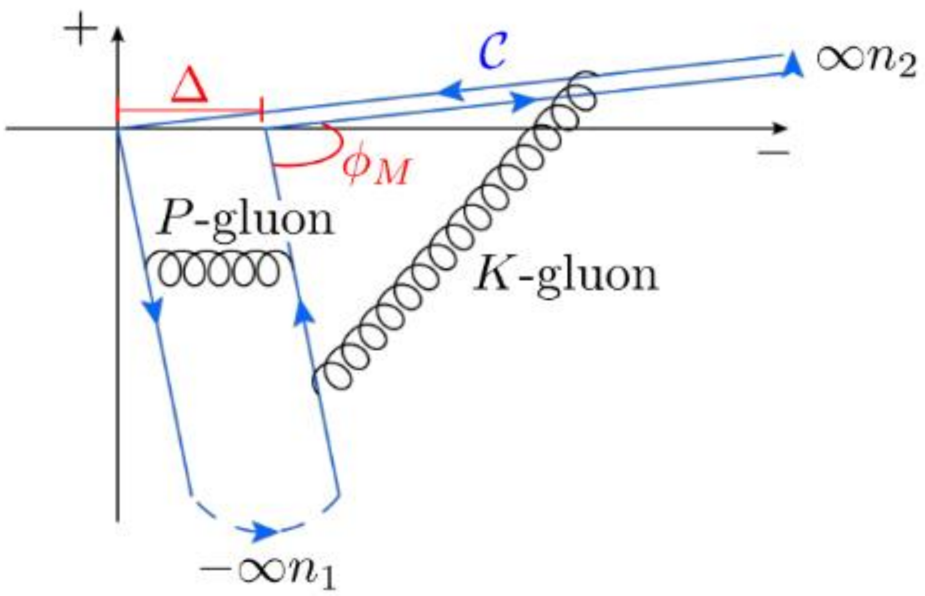
TARGET
large and positive rapidity



JET
large and negative rapidity

Soft function of DIS at threshold

It is the vacuum expectation value of this Wilson loop:



$\phi_M = y_1 - y_2$ $\quad\Delta = -iN/P^+$

$L_N = \log\left(\frac{\mu}{\sqrt{2}P^+} N e^{\gamma_E}\right)$

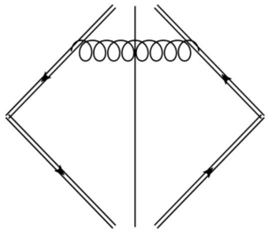
K-P decomposition

$$S_{DIS}(N,\mu;y_1,y_2) = \exp\left\{ \int_{y_2}^{y_1} dy\, K\big(a_S(\mu), L_N + y\big) + \frac{1}{2}\left[P\big(a_S(\mu), L_N + y_1\big) + P\big(a_S(\mu), L_N + y_2\big)\right] \right\}$$

Universal Collins-Soper kernel

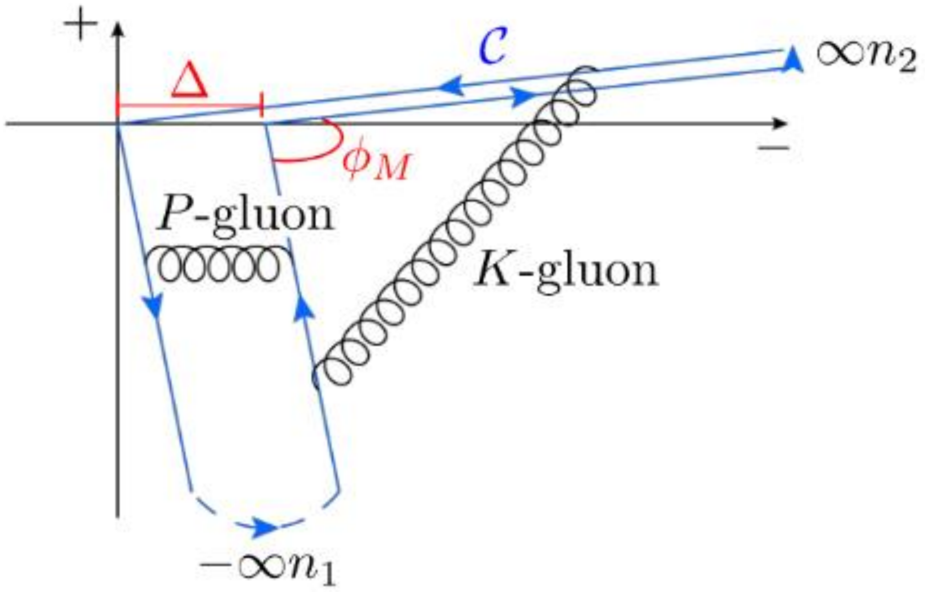
Universal P-function

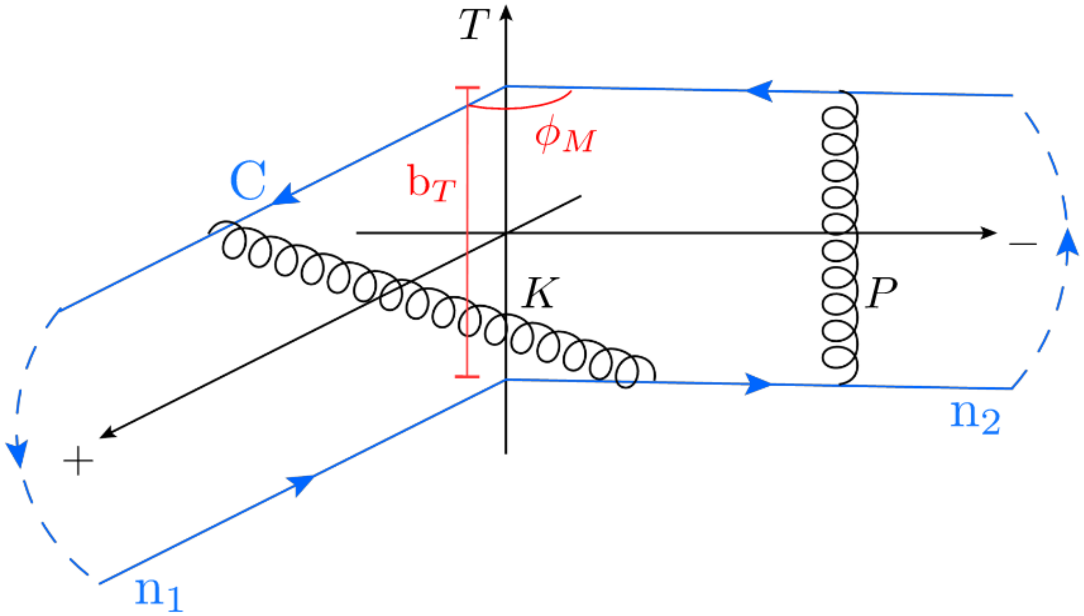
Note that gauge invariance would not hold without the P-terms



Comparison with TMD Soft Function (SIDIS)

It is the vacuum expectation value of this Wilson loop:





$\phi_M = y_1 - y_2 \qquad \Delta = -iN/P^+$

$$L_N = \log\left(\frac{\mu}{\sqrt{2}P^+} N e^{\gamma_E}\right)$$

$\phi_M = y_1 - y_2 \qquad \Delta = b_T$

$$L_b = \log\left(\frac{\mu b_T}{2} e^{\gamma_E}\right)$$

Common geometric features

Comparison with TMD Soft Function (SIDIS)

K-P decomposition

Universal Collins-Soper kernel

$$S_{DIS}(N,\mu;y_1,y_2) = \exp\left\{ \int_{y_2}^{y_1} dy\, K\big(a_S(\mu), L_N + y\big) + \frac{1}{2}\left[P\big(a_S(\mu), L_N + y_1\big) + P\big(a_S(\mu), L_N + y_2\big)\right] \right\}$$

Universal P-function

K-P decomposition

Universal Collins-Soper kernel

$$S_{TMD}(b_T,\mu;y_1,y_2) = \exp\left\{ (y_1 - y_2)\, K\big(a_S(\mu), L_b\big) + P\big(a_S(\mu), L_b\big) \right\}$$

Universal P-function

The TMD case is a particular (and easier!) configuration, where K and P are independent of rapidities

Target and Jet Function

Before subtraction, familiar definitions but tilted off the lightcone:

$$\mathcal{J}_j^{\text{uns}}(l^2/Q^2;\mu,\widehat{y}_1) = Z_J^{\text{uns}}(l^2/Q^2;\mu,\widehat{y}_1)\frac{Q^2}{2\pi l^-} \times \text{Disc}\int\frac{d^4\omega}{(2\pi)^4}e^{-il\cdot\omega}\,\text{Tr}\langle 0|\gamma^-\overline{\psi}_j^{(0)}(\omega)W_1[0,\omega]\psi_j^{(0)}(0)|0\rangle$$

Two past pointing WL joined at infinity

$$W_1[0,\omega] = W_{\widehat{n}_1}^\dagger[\omega,-\infty]W_{-\infty}W_{\widehat{n}_1}[0,-\infty]$$

$$\mathcal{F}_j^{\text{uns}}(\xi;\mu,\widehat{y}_2) = Z_T^{\text{uns}}(\xi;\mu,\widehat{y}_2)\int\frac{d\sigma^-}{2\pi}e^{-i\xi P^+\sigma^-} \times \text{Tr}_{\text{C,D}}\langle P|T\,\overline{\psi}_j^{(0)}(\sigma)W_2[0,\sigma]\frac{\gamma^+}{2}\psi_j^{(0)}(0)|P\rangle$$

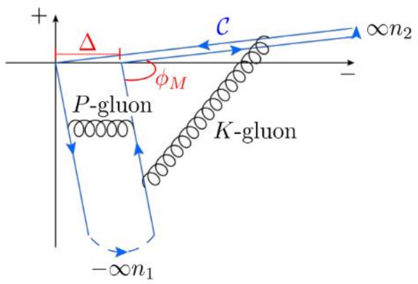
Two future pointing WL joined at infinity

$$W_2[0,\sigma] = W_{\widehat{n}_2}^\dagger[\sigma,\infty]W_{\infty}W_{\widehat{n}_2}[0,\infty]$$

In Mellin space subtractions are transparent:

$$\mathcal{F}_i^{\text{uns}} = \mathcal{F}_i \times S_T + \mathcal{O}(1/N)$$
$$\mathcal{J}_j^{\text{uns}} = \mathcal{J}_j \times S_J + \mathcal{O}(1/N)$$

Soft-collinear operators *coincide* (up to labels) with the DIS Soft function



Target and Jet Function

$$\mathcal{J}_j^{\text{sub}}(N;\mu,y_2) = \lim_{\widehat{y}_1\to+\infty} \frac{\mathcal{J}_j^{\text{uns}}(N;\mu,\widehat{y}_1)}{S_J(N;\mu,\widehat{y}_1,y_2)}$$

$$= \mathcal{J}_j^{\text{sub}}(N;\mu,-L_N)\exp\left\{-\int_{y_2}^{-L_N} K\big(a_S(\mu),L_N+y\big)dy - \frac{1}{2}P\big(a_S(\mu),L_N+y_2\big)\right\} + \mathcal{O}\left(\frac{1}{N}\right)$$

Universal Collins-Soper kernel

Universal P-function

The reference (killing logs) rapidity scale for the Jet function is *large and negative*

K-P decompositions

$$\mathcal{F}_j^{\text{sub}}(N;\mu,y_1) = \lim_{\widehat{y}_2\to-\infty} \frac{\mathcal{F}_j^{\text{uns}}(N;\mu,\widehat{y}_2)}{S_T(N;\mu,y_1,\widehat{y}_2)}$$

$$= \mathcal{F}_j^{\text{sub}}(N;\mu,L_N)\exp\left\{-\int_{L_N}^{y_1} K\big(a_S(\mu),L_N+y\big)dy - \frac{1}{2}P\big(a_S(\mu),L_N+y_1\big)\right\} + \mathcal{O}\left(\frac{1}{N}\right)$$

Universal Collins-Soper kernel

Universal P-function

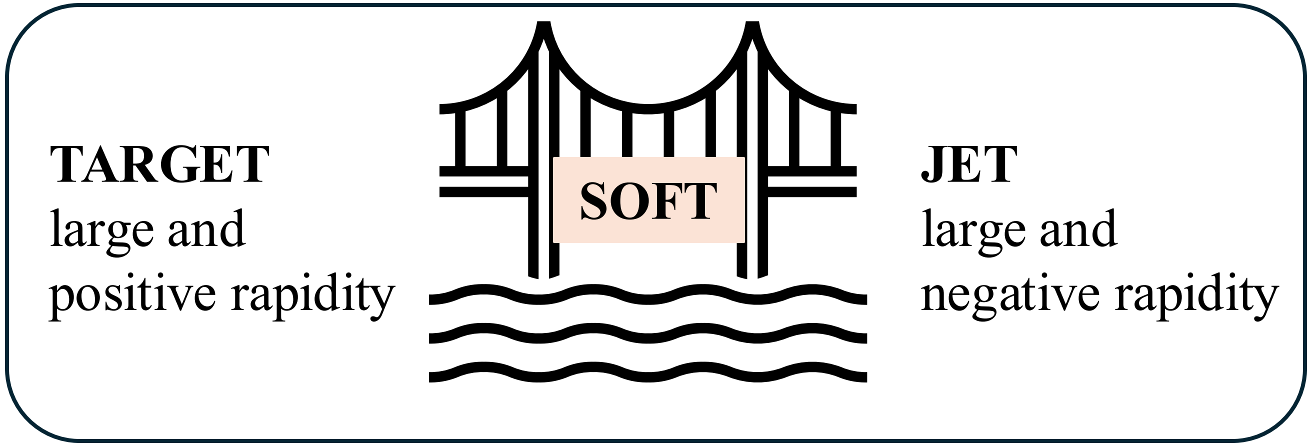
The reference (killing logs) rapidity scale for the Target function is *large and positive*

Cancellation of off lightcone effects

All off lightcone effects (tilts, P-terms) have been cancelled out in the cross section

$$W^{\mu\nu} = \mathbf{u}^{\mu\nu}\, H(\mu, Q) \int \frac{dN}{2\pi i} x^{-N}$$

$$\mathcal{F}_j^{\text{sub}}(N; \mu, L_N) e^{\int_{-L_N}^{L_N} dy\, K(a_S(\mu), L_N + y)} \mathcal{J}_j^{\text{sub}}(N; \mu, -L_N).$$



What these operators have to do with familiar threshold operators?

Square root definition

The P-terms cancels out at the cross-section level.
It is possible to re-define the **Target** and the **Jet function** in such a way that P-terms cancel out already at the level of the operators.

$$\mathcal{J}_j(N;\mu,y_n) = \lim_{\substack{\widehat{y}_1\to+\infty\\ \widehat{y}_2\to-\infty}} \mathcal{J}_j^{\text{uns}}(N;\mu,\widehat{y}_1)\sqrt{\frac{S(N;\mu,y_n,\widehat{y}_2)}{S(N;\mu,\widehat{y}_1,\widehat{y}_2)S(N;\mu,\widehat{y}_1,y_n)}}$$

$$\mathcal{F}_j(N;\mu,y_n) = \lim_{\substack{\widehat{y}_1\to+\infty\\ \widehat{y}_2\to-\infty}} \mathcal{F}_j^{\text{uns}}(N;\mu,\widehat{y}_2)\sqrt{\frac{S(N;\mu,\widehat{y}_1,y_n)}{S(N;\mu,\widehat{y}_1,\widehat{y}_2)S(N;\mu,y_n,\widehat{y}_2)}}.$$

→ Only depend on the universal Collins-Soper kernel

It is the very same rearrangement adopted for common definition of TMDs!
The evolution is in fact CSS-like:

$$\begin{cases} \dfrac{\partial \log \mathcal{J}_j}{dy_n} = K\left(a_S(\mu), L_N + y_n\right) \\ \dfrac{\partial \log \mathcal{J}_j}{d\log\mu} = \gamma_{\mathcal{J}}\left(a_S(\mu), L_J - L_N - y_n\right) \end{cases}$$

$$\begin{cases} \dfrac{\partial \log \mathcal{F}_j}{dy_n} = -K\left(a_S(\mu), L_N + y_n\right) \\ \dfrac{\partial \log \mathcal{F}_j}{d\log\mu} = \gamma_{\mathcal{F}}\left(a_S(\mu), L_T - L_N + y_n\right) \end{cases}$$

Factorization theorem

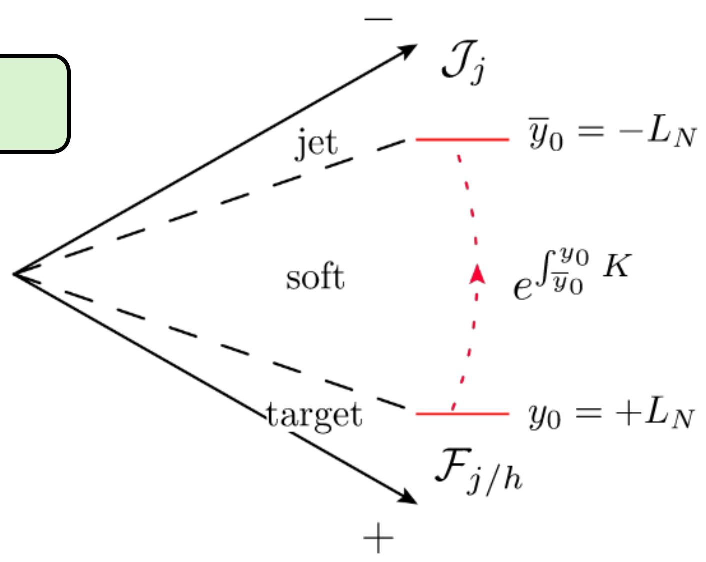
The factorization theorem becomes:

$$W^{\mu\nu} = \mathbf{u}^{\mu\nu} H(\mu, Q) \int \frac{dN}{2\pi i} x^{-N} \mathcal{F}_j(N;\mu,y_n)\mathcal{J}_j(N;\mu,y_n)$$

Where:

$$\mathcal{F}_j(N;\mu,y_n)\mathcal{J}_j(N;\mu,y_n) = \mathcal{F}_j^{\text{sub}}(N;\mu,y_0) e^{\int_{\overline{y}_0}^{y_0} dy\, K(a_S(\mu), L_N + y)} \mathcal{J}_j^{\text{sub}}(N;\mu,\overline{y}_0)$$

1. Natural choice



Arbitrary reference rapidity scales, bridged by CS-kernel

Factorization theorem

The factorization theorem becomes:

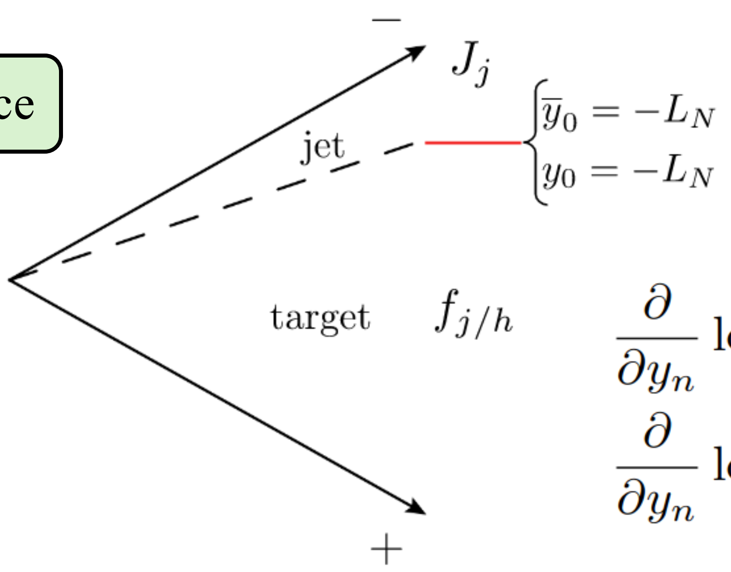
$$W^{\mu\nu} = \mathbf{u}^{\mu\nu} H(\mu, Q) \int \frac{dN}{2\pi i} x^{-N} \mathcal{F}_j(N; \mu, y_n) \mathcal{J}_j(N; \mu, y_n)$$

Where:

$$\mathcal{F}_j(N; \mu, y_n) \mathcal{J}_j(N; \mu, y_n) = \mathcal{F}_j^{\rm sub}(N; \mu, y_0) e^{\int_{\overline{y}_0}^{y_0} dy\, K(a_S(\mu), L_N + y)} \mathcal{J}_j^{\rm sub}(N; \mu, \overline{y}_0)$$

Arbitrary reference rapidity scales, bridged by CS-kernel

2. Matching choice



Magic value!

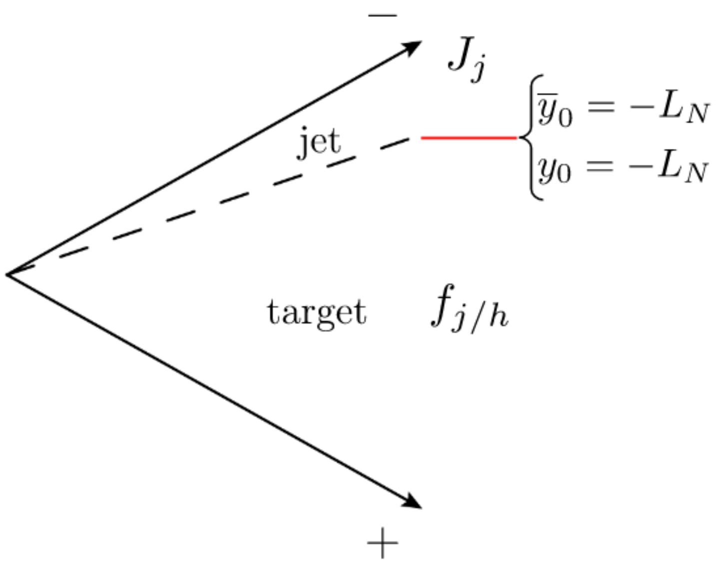
$$\frac{\partial}{\partial y_n} \log \mathcal{F}_j(N; \mu, y_n) = -K\big(a_S(\mu), L_N + y_n\big)$$

$$\frac{\partial}{\partial y_n} \log \mathcal{J}_j(N; \mu, y_n) = +K\big(a_S(\mu), L_N + y_n\big)$$

Lightcone factorization theorem

2. Matching choice

Magic value!



$$
\begin{cases}
\dfrac{\partial}{\partial y_n} \log \mathcal{F}_j(N;\mu,y_n) = -K\big(a_S(\mu), L_N + y_n\big) \\[2ex]
\dfrac{\partial}{\partial y_n} \log \mathcal{J}_j(N;\mu,y_n) = +K\big(a_S(\mu), L_N + y_n\big)
\end{cases}
$$

The evolution becomes trivial (up to a simple power series in the strong coupling).

Matching to lightcone operators:

$$\mathcal{J}_j(N;\mu,-L_N) = C(a_S(\mu), L_N)\, J_j(N;\mu),$$

$$\mathcal{F}_j(N;\mu,-L_N) = \frac{f_j(N;\mu)}{C(a_S(\mu), L_N)}.$$

Threshold PDF → $f_j(N,\mu)$

Jet Function → $J_j(N,\mu)$

$$W^{\mu\nu} = \mathbf{u}^{\mu\nu} H(\mu, Q) \int \frac{dN}{2\pi i} x^{-N} f_j(N,\mu) J_j(N,\mu)$$

Conclusions

- The study of the light-cone deviations is crucial for a better understanding of the physics
- The cancellation of the off light-cone effects is not true *a priori*. Always check!!
- Universality lies in the Collins–Soper kernel, not in the soft function: it reflects the geometry of opposite lightcone directions