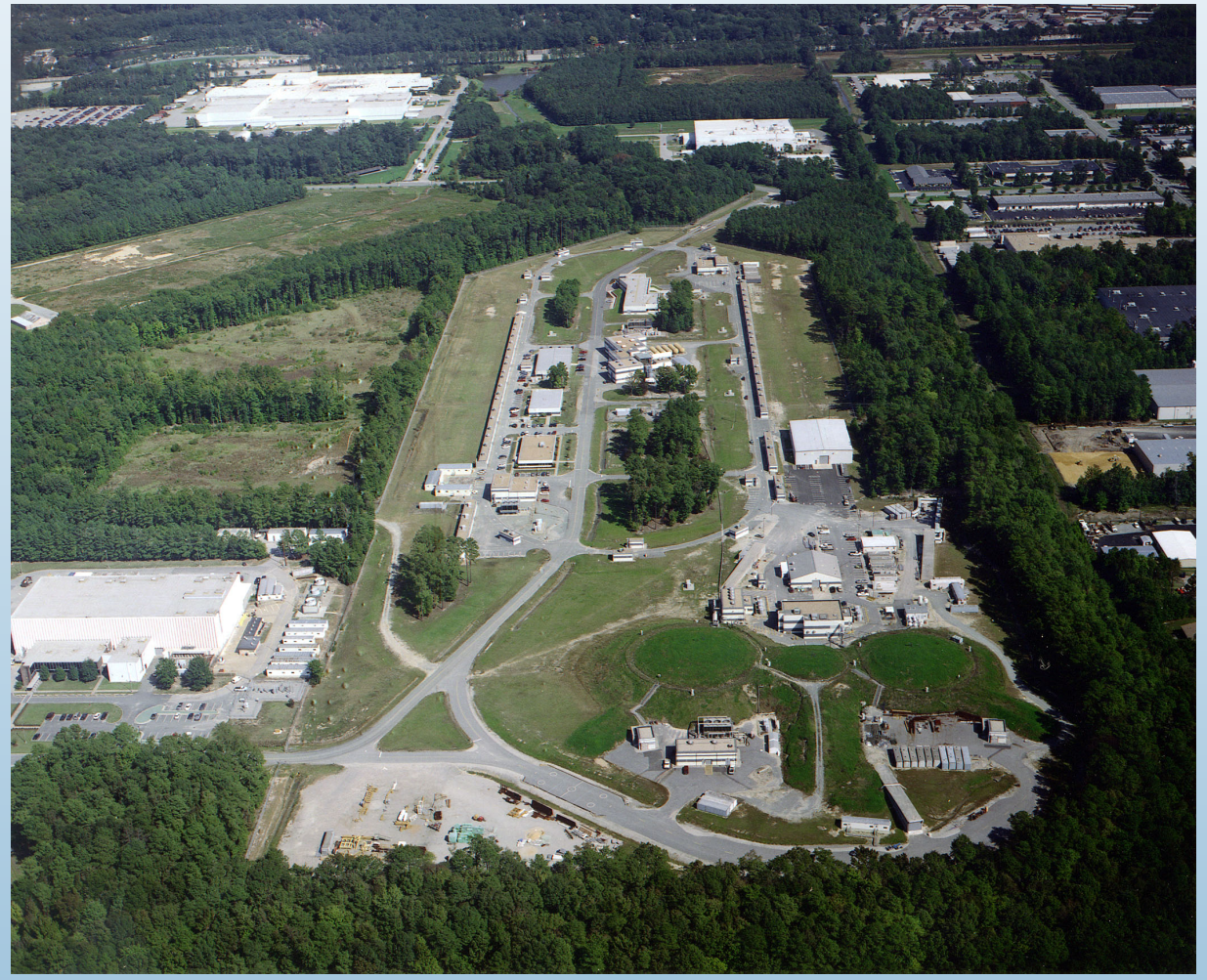
Global QCD Analysis of Proton's Transverse Spin Structure through TMDs and DiFFs

Chris Cocuzza

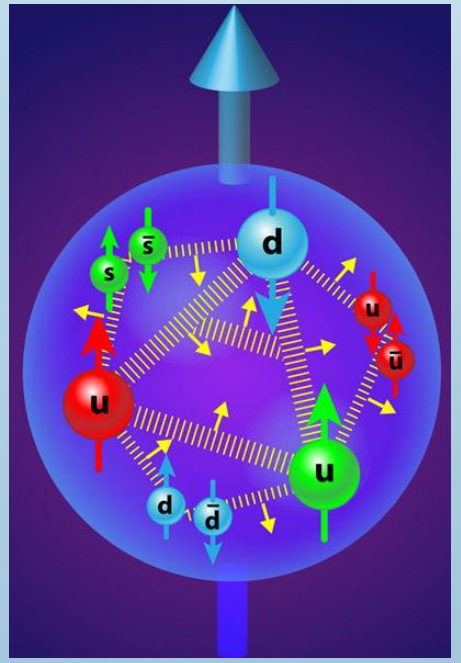
www.jlab.org/theory/jam

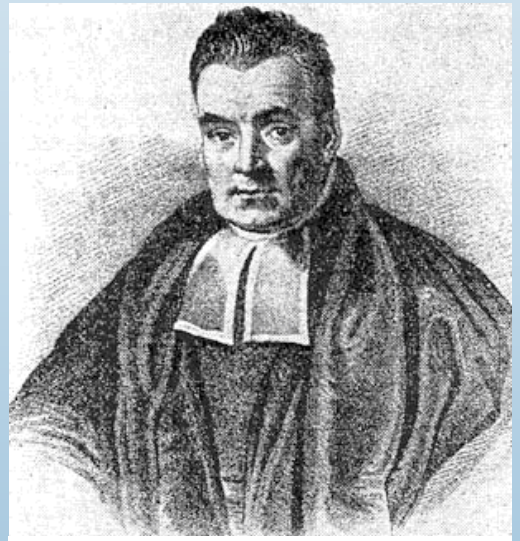


QCD Evolution

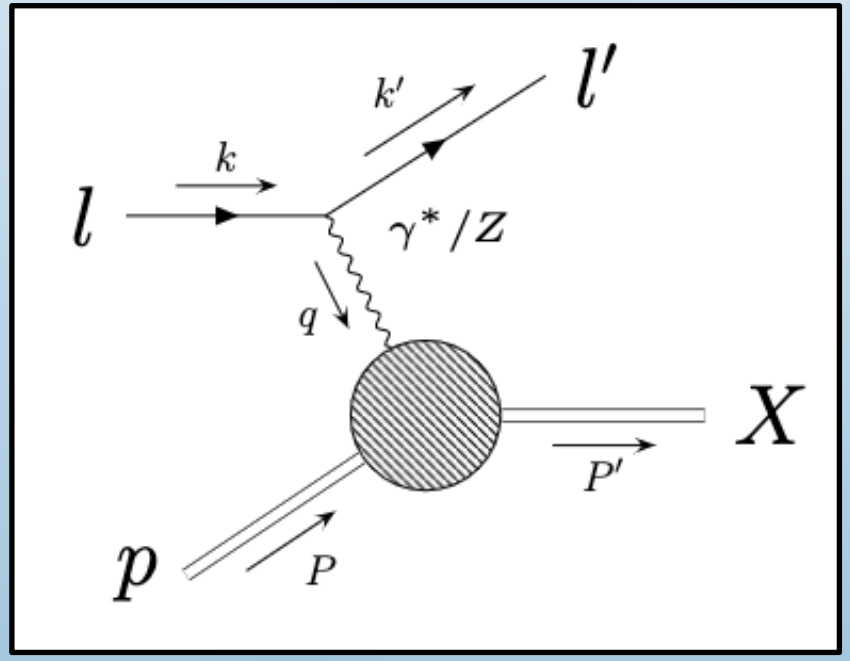
May 25, 2025

1. Introduction
2. Extraction of DiFFs
3. Extraction of Transversity PDFs
4. Extraction of Tensor Charges
5. Extraction with TMDs
6. Conclusions and Outlook





T. Bayes



Hadron Structure

Global QCD Analysis

Introduction

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Global QCD Analysis

USQCD

Extended Twisted Mass Collaboration

Hadron Structure

Global QCD Analysis

USQCD

Extended Twisted Mass Collaboration

Hadron Structure

$$\frac{\mathrm{d}}{\mathrm{d}\,\ln(\mu^2)} f_i(x,\mu) = \sum_j \int_x^1 \frac{dz}{z} P_{ij}(z,\mu) f_j(\frac{x}{z},\mu)$$

Param. + Evolve + Factorization

$$\sigma = \sum_{i,j} H_{ij} \otimes f_i \otimes f_j$$

Global QCD Analysis

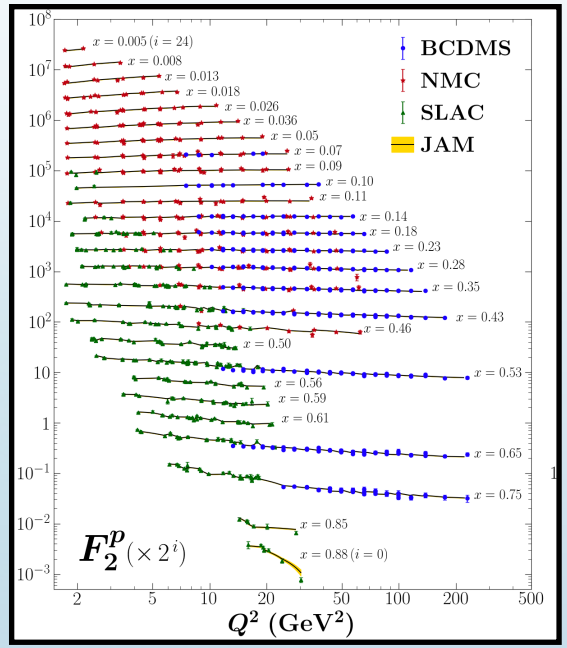
USQCD

Extended Twisted Mass Collaboration

Introduction

$$\chi^2(\boldsymbol{a}) = \sum_{i,e}\left(\frac{d_{i,e} - \sum_k r_e^k \beta_{i,e}^k - T_{i,e}(\boldsymbol{a})/N_e}{\alpha_{i,e}}\right)^2 + \sum_k \left(r_e^k\right)^2 + \left(\frac{1-N_e}{\delta N_e}\right)^2$$

χ^2 Minimization



$$\mathcal{L}(\boldsymbol{a}, \text{data}) = \exp\left(-\frac{1}{2}\chi^2(\boldsymbol{a}, \text{data})\right)$$

$$\mathcal{P}(\boldsymbol{a}|\text{data}) \sim \mathcal{L}(\boldsymbol{a}, \text{data})\,\pi(\boldsymbol{a})$$

Hadron Structure

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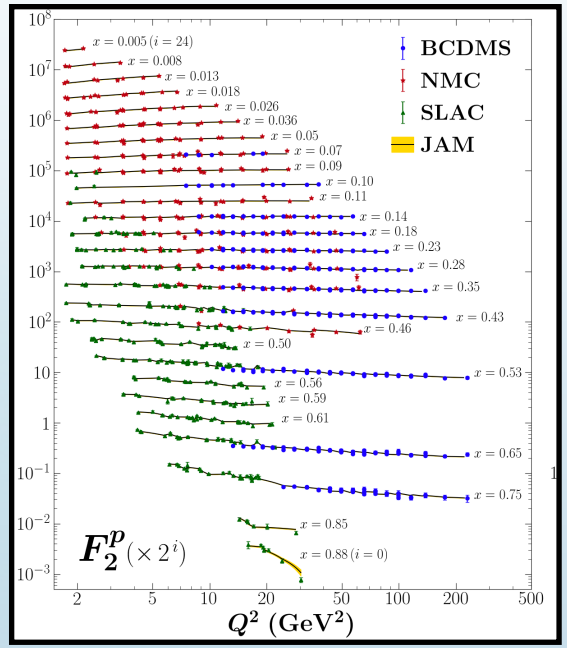

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Global QCD Analysis

USQCD

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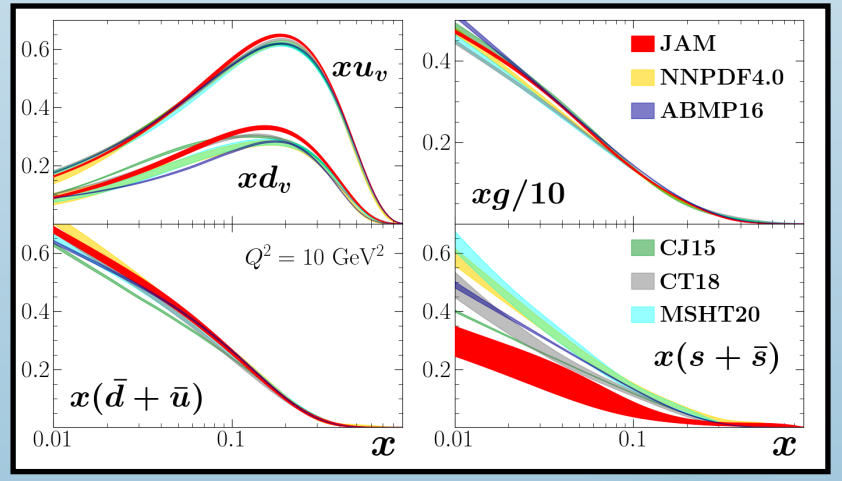

Data Resampling

$$\tilde{\sigma} = \sigma + N(0,1)\,\alpha$$

Approaches to Extract Transversity

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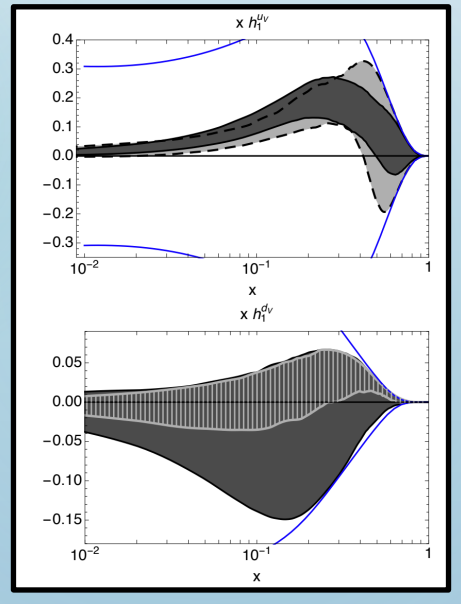
Dihadron Frag.

- Radici + Bacchetta (RB18)
- Benel + Courtoy + Ferro-Hernandez (2020)



M. Radici and A. Bacchetta,
Phys. Rev. Lett. **120**, no. 19, 192001 (2018)

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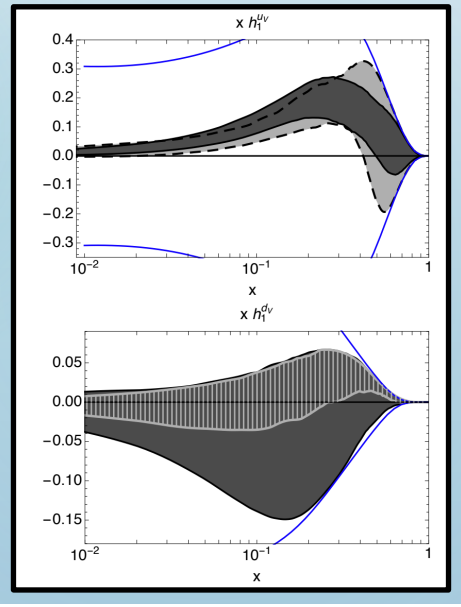

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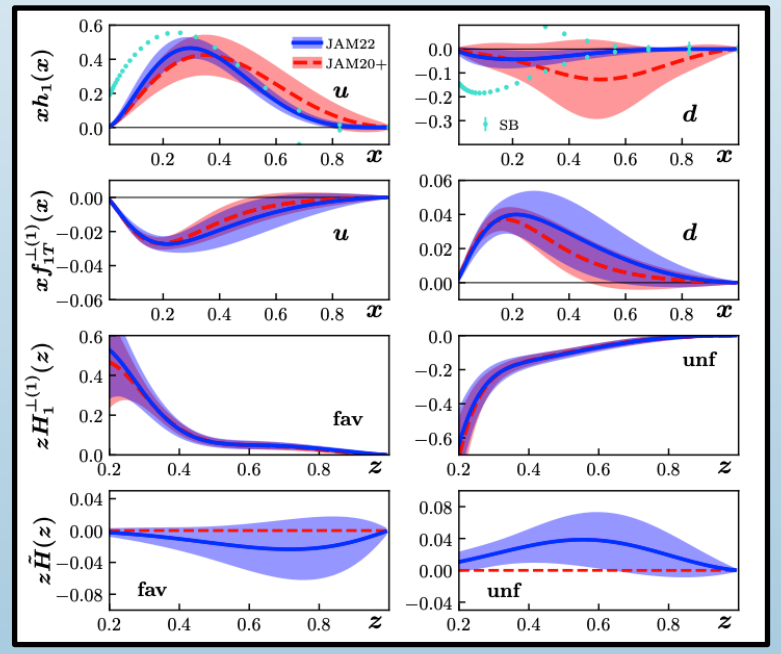
TMD + Collinear Twist-3

- JAM3D



L. Gamberg *et al.*, Phys. Rev. D **106**, no. 3, 034014 (2022)

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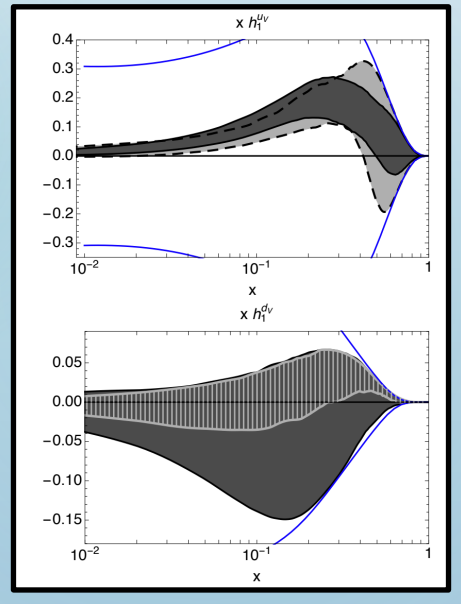

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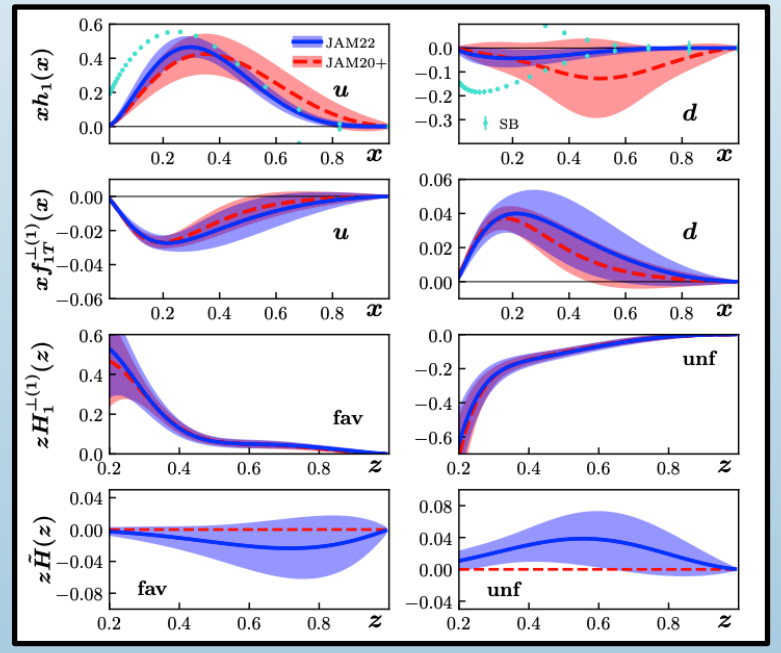
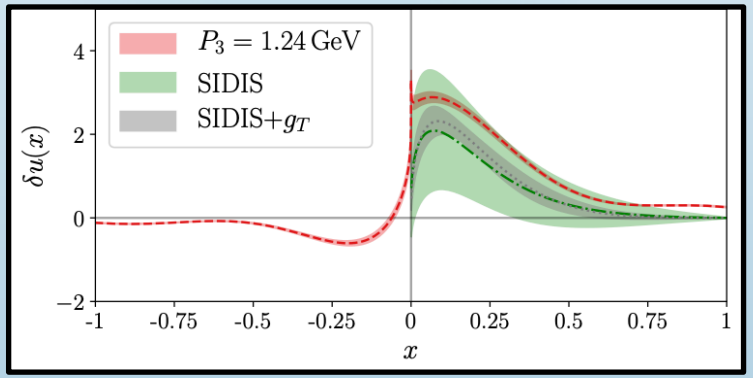


L. Gamberg *et al.*, Phys. Rev. D **106**, no. 3, 034014 (2022)

Lattice QCD

- ETMC Collaboration
- PNDME Collaboration
- LHPC Collaboration

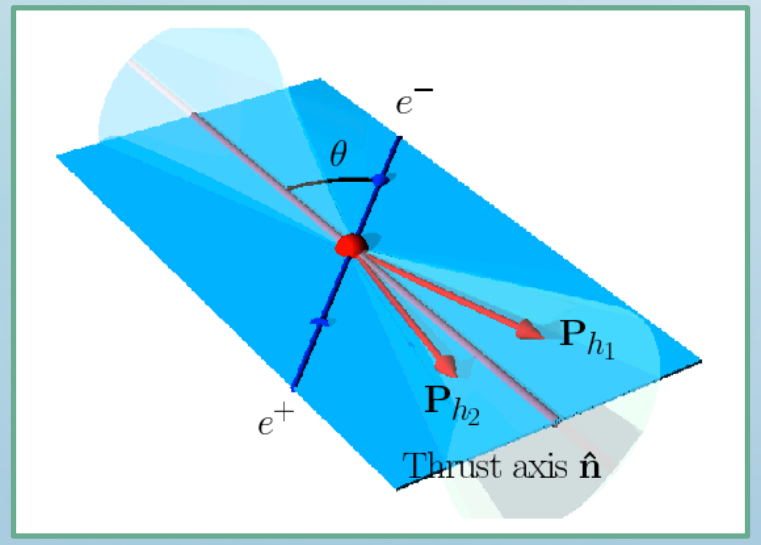


C. Alexandrou *et al.*, Phys. Rev. D **104**, no. 5, 054503 (2021)

JAM Global Analysis in the collinear DiFF Approach

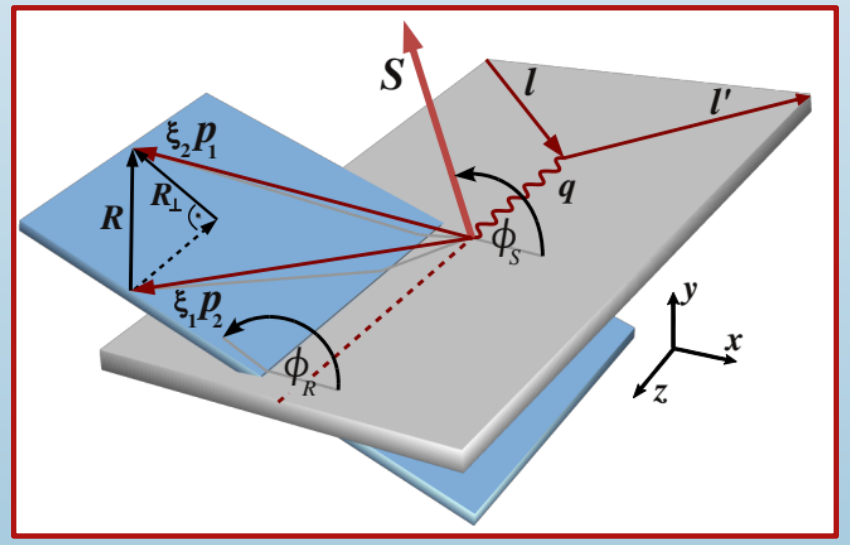
First s*imultaneous* extraction of $\pi^+\pi^-$ DiFFs (D_1^q), IFFs ($H_1^{\sphericalangle,q}$), and transversity PDFs (h_1^q) at LO

Semi-Inclusive Annihilation



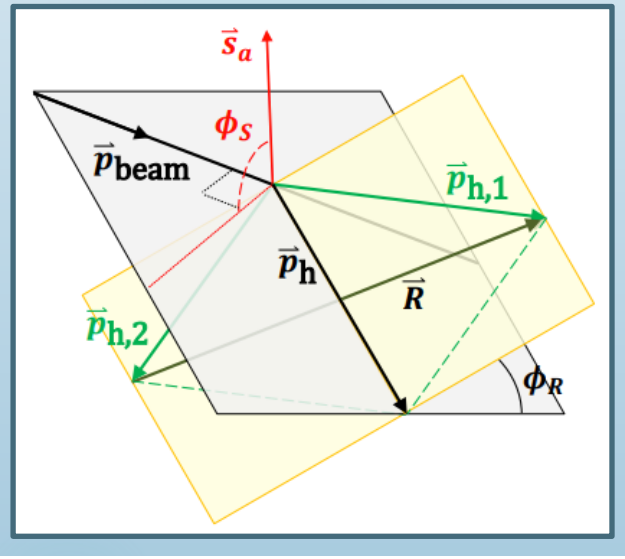
R. Seidl *et al.*, Phys. Rev. D **96**, no. 3, 032005 (2017)

Semi-Inclusive Deep Inelastic Scattering



C. Adolph *et al.*, Phys. Lett. B **713**, 10-16 (2012)

Proton-Proton Collisions



L. Adamczyk *et al.*, Phys. Rev. Lett. **115**, 242501 (2015)

Tensor Charges

$$\delta u \equiv \int_0^1 \mathrm{d}x (h_1^u - h_1^{\bar{u}}),$$

$$\delta d \equiv \int_0^1 \mathrm{d}x (h_1^d - h_1^{\bar{d}}),$$

$$g_T \equiv \delta u - \delta d,$$

Tensor Charges

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QCD Pheno for Transversity

Tensor Charges

Anselmino, *et al.* (2007, 2009, 2013, 2015);
Goldstein, *et al.* (2014);
Kang, *et al.* (2016);
D'Alesio, *et al.* (2020);
Cammarota, *et al.* (2020);
Gamberg, *et al.* (2022);
Zheng, *et al.* (2024);
Boglione, *et al.* (2024)

Radici, *et al.* (2013, 2015, 2018);
Benel, *et al.* (2020);
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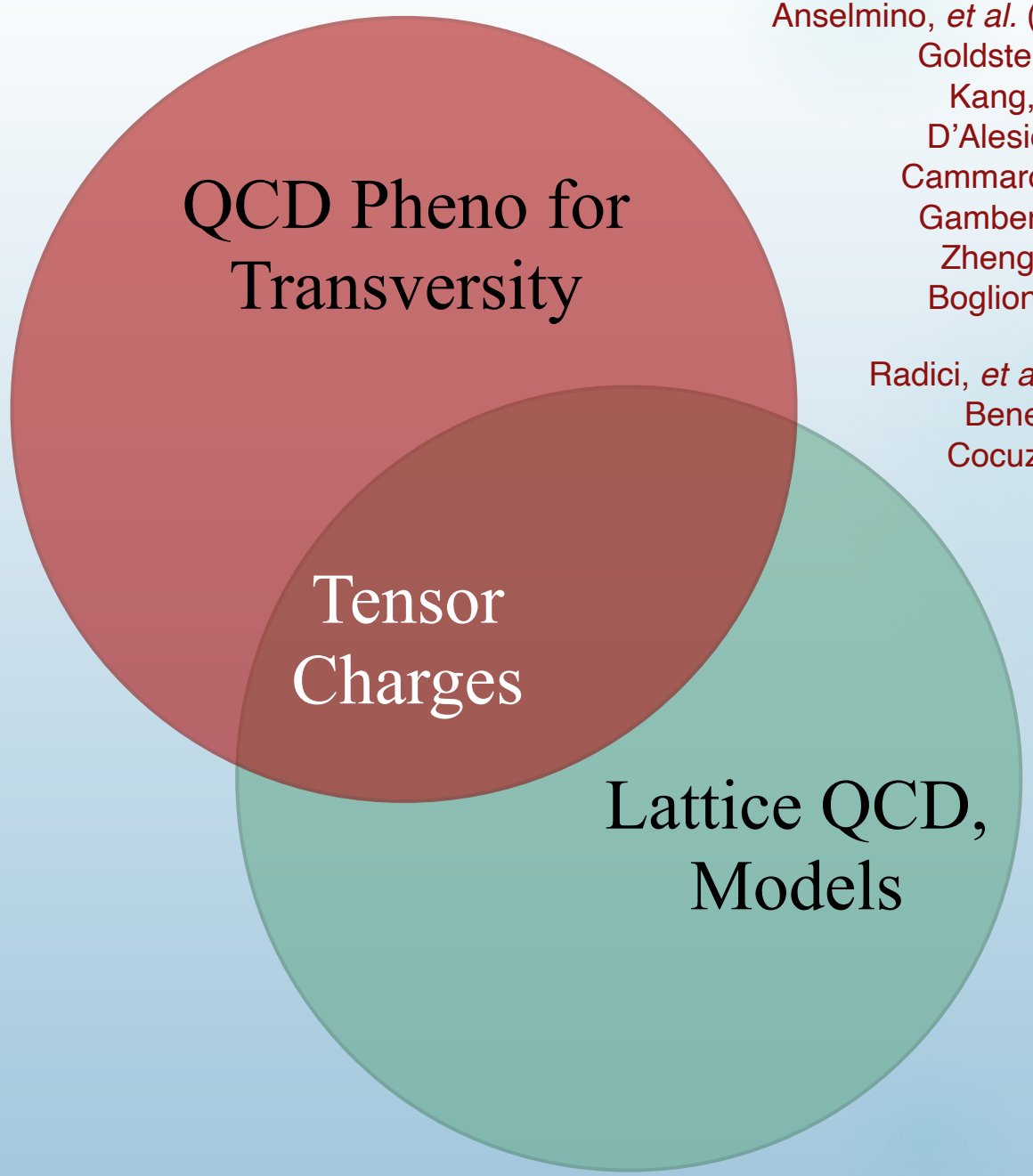

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Gamberg, Goldstein (2001);
Pasquini, *et al.* (2005);
Wakamatsu (2007);
Lorce (2009);
Gupta, *et al.* (2018);
Yamanaka, *et al.* (2018);
Hasan, *et al.* (2019);
Alexandrou, *et al.* (2019, 2023);
Yamanaka, *et al.* (2013);
Pitschmann, *et al.* (2015);
Xu, *et al.* (2015);
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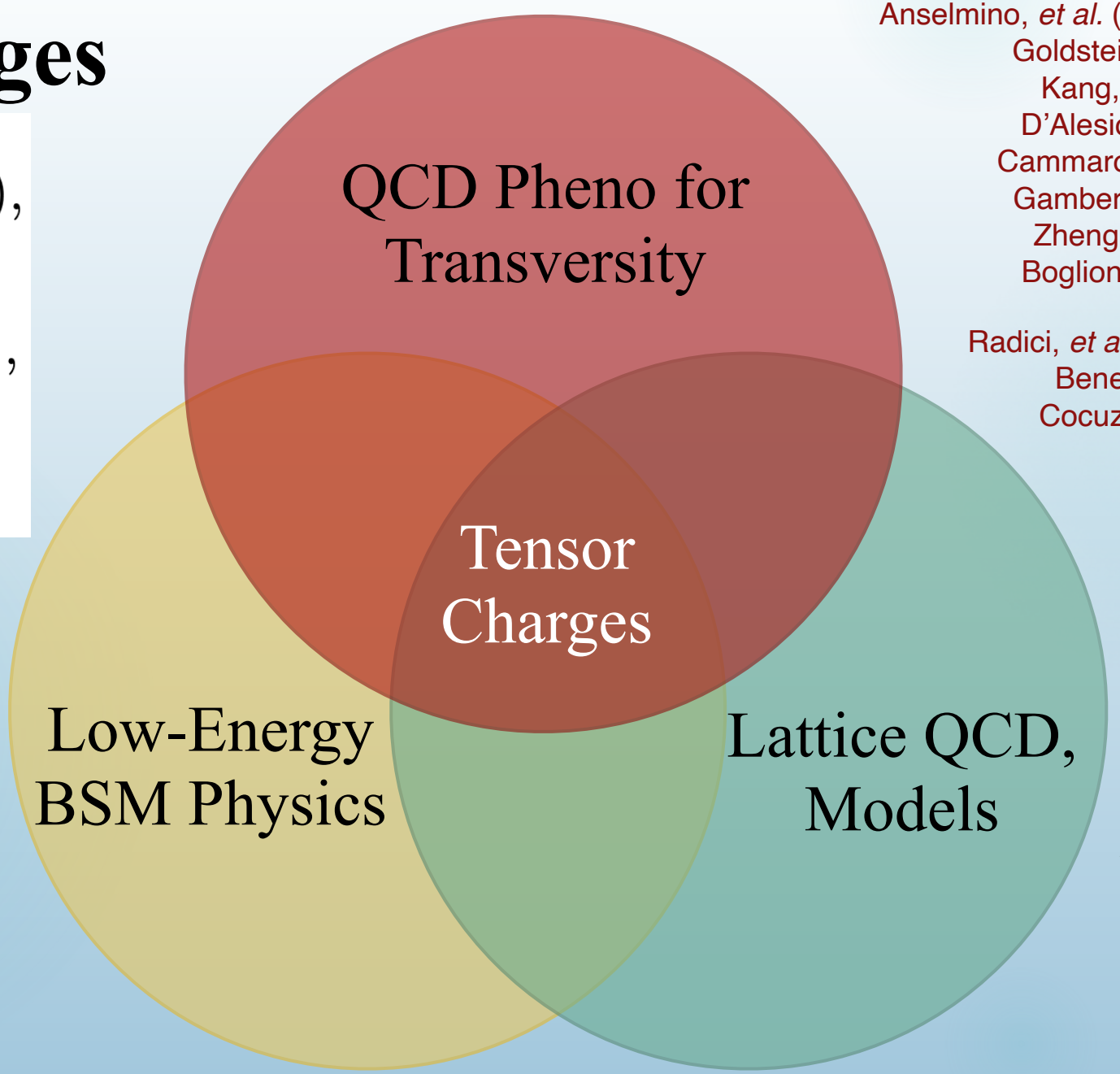

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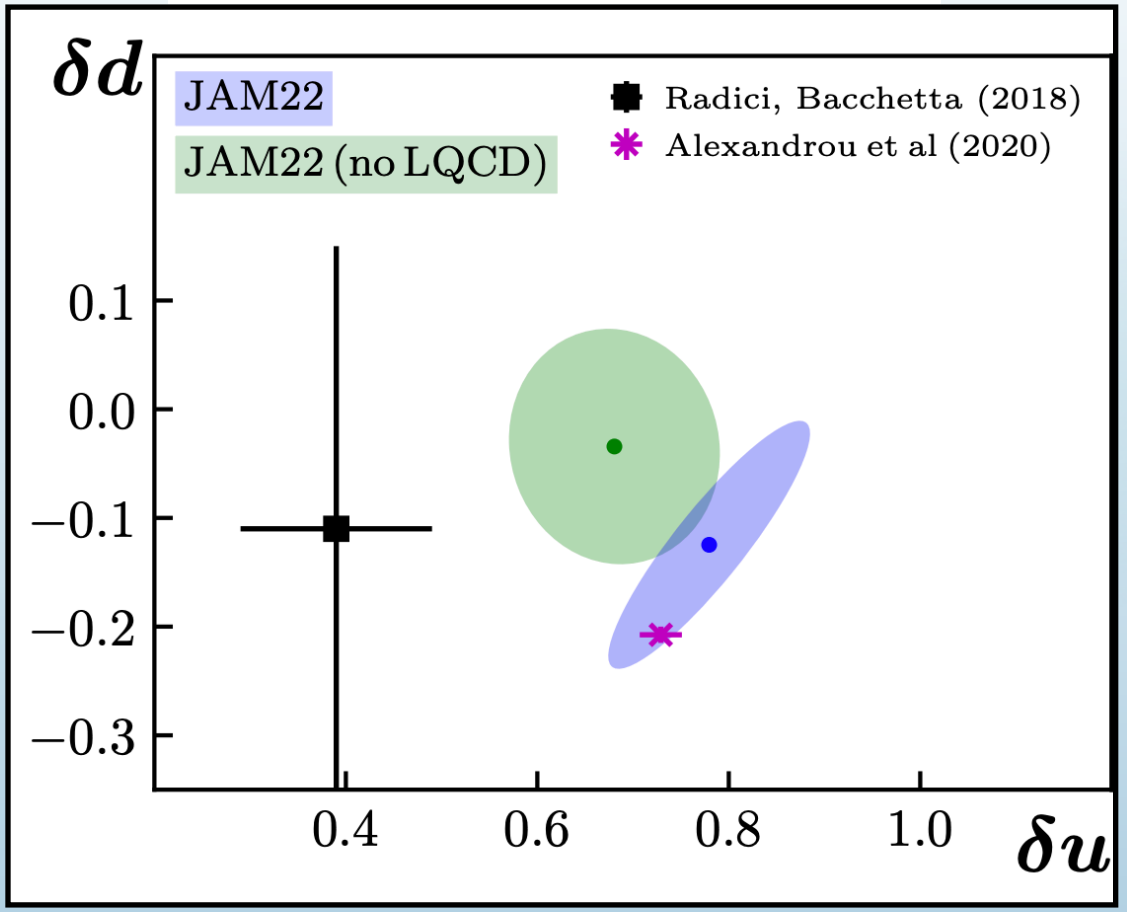
Herczeg (2001);
Erler, Ramsey-Musolf (2005);
Pospelov, Ritz (2005);
Severijns, *et al.* (2006);
Cirigliano, *et al.* (2013);
Courtoy, *et al.* (2015);
Yamanaka, *et al.* (2017);
Liu, *et al.* (2018);
Gonzalez-Alonso, *et al.* (2019)

The Transverse Spin Puzzle?

L. Gamberg *et al.*, Phys. Rev. D **106**, no. 3, 034014 (2022)



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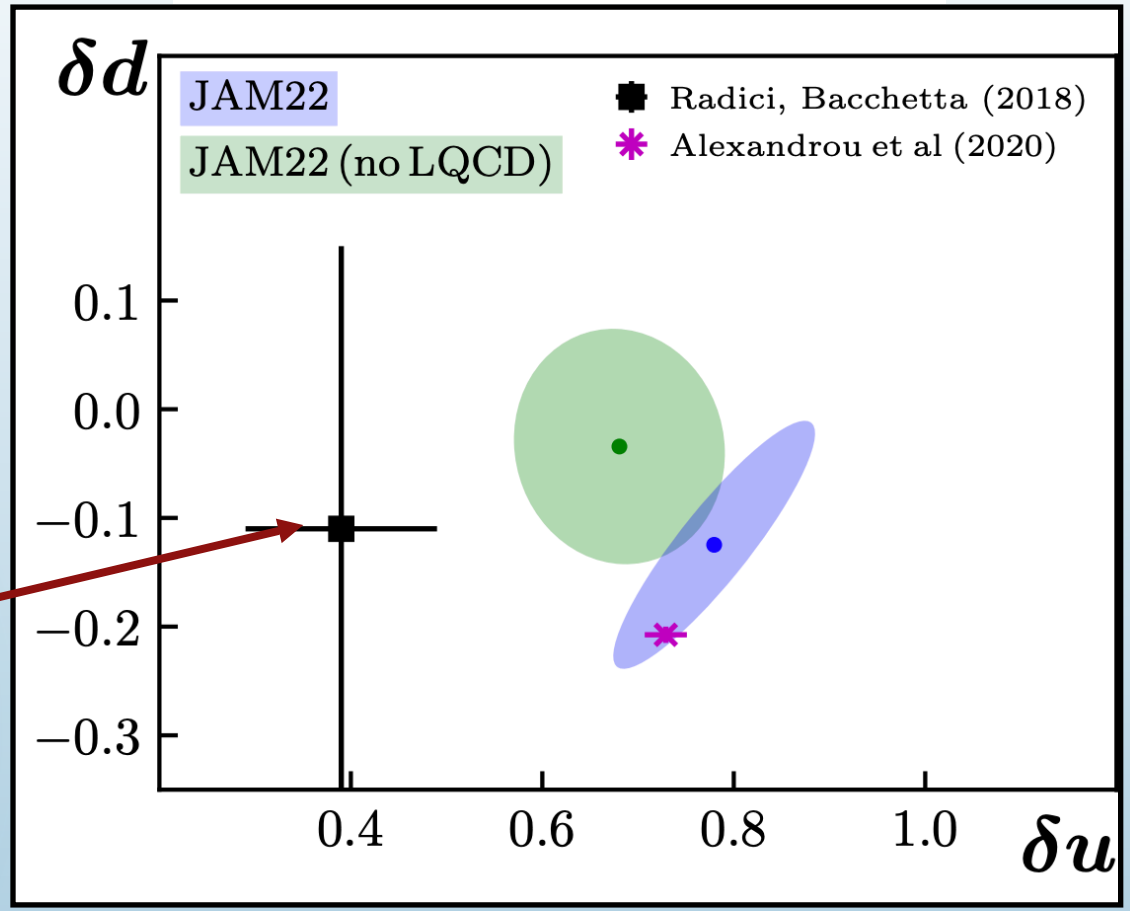
RB18



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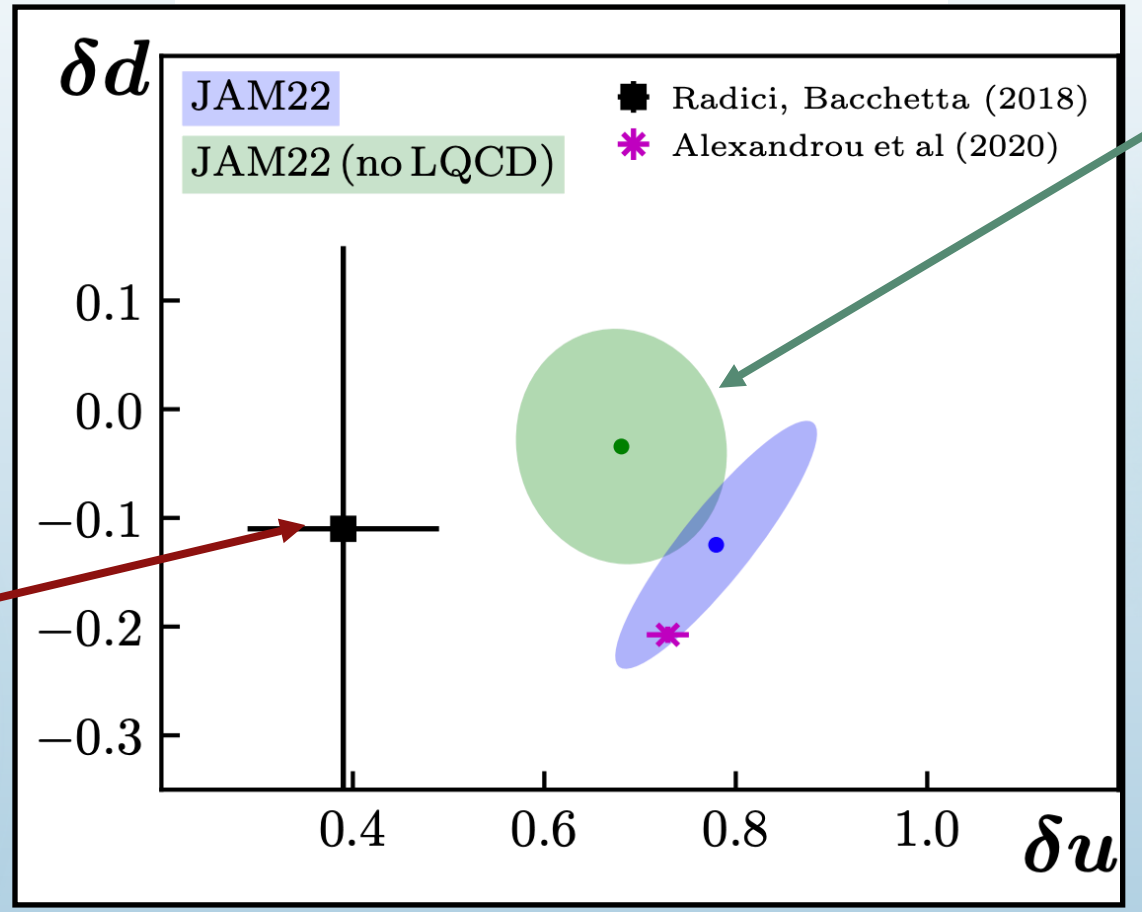

JAM3D (no LQCD)

RB18

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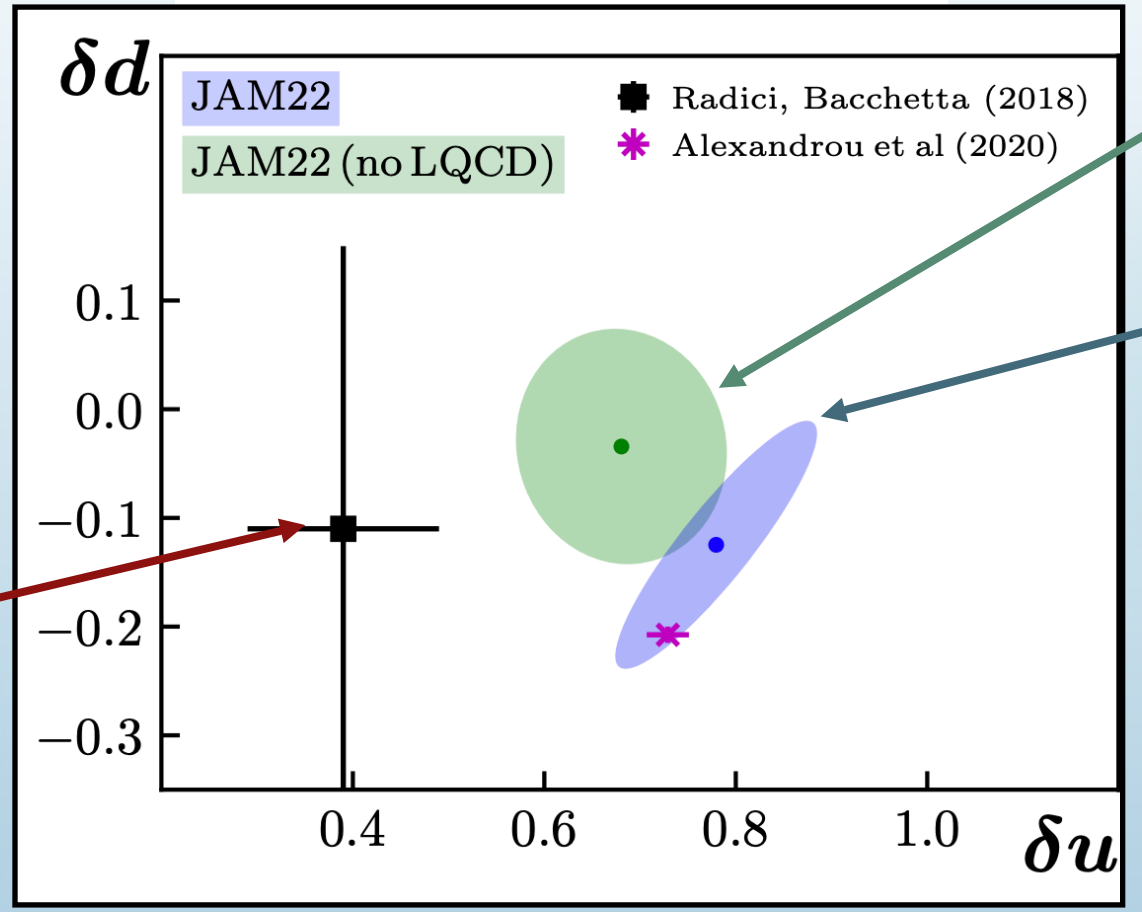

RB18

JAM3D (no LQCD)

JAM3D (w/ LQCD)

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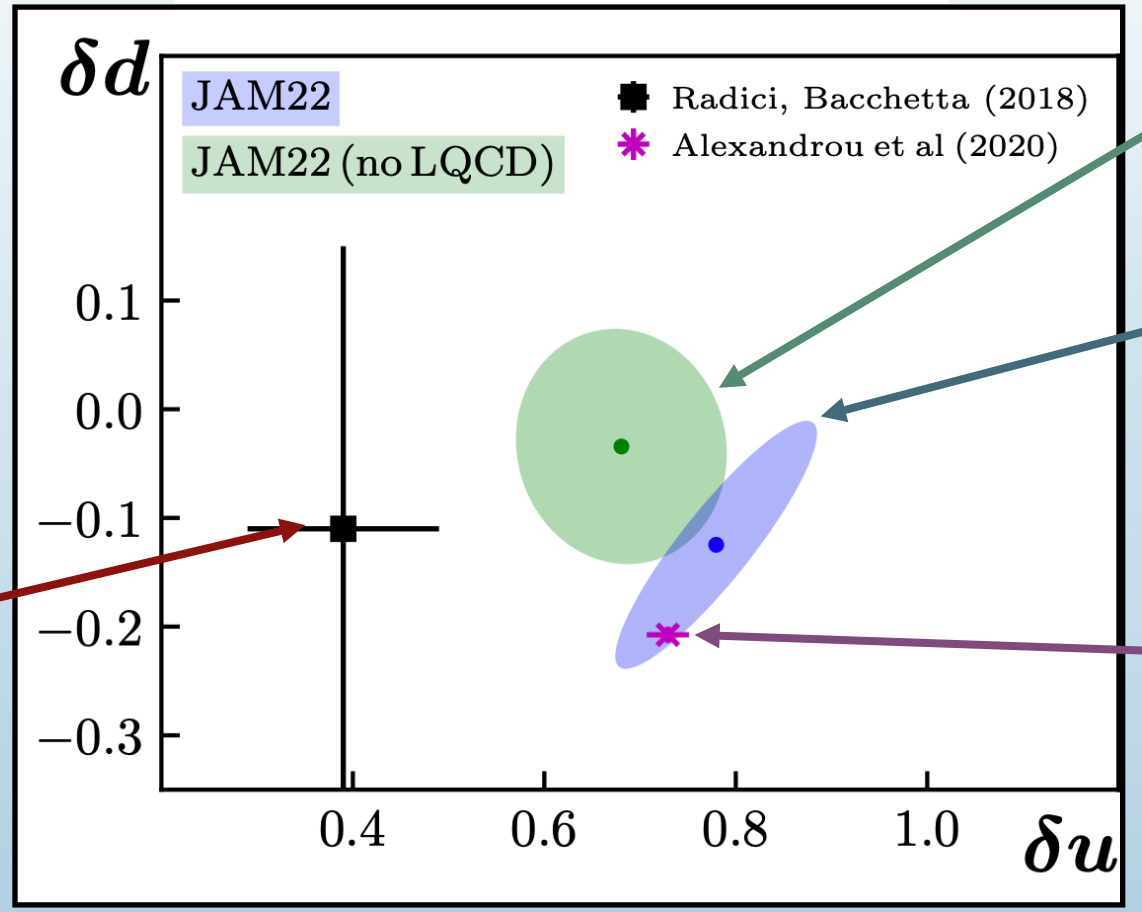

RB18

JAM3D (no LQCD)

JAM3D (w/ LQCD)

Lattice (ETMC)

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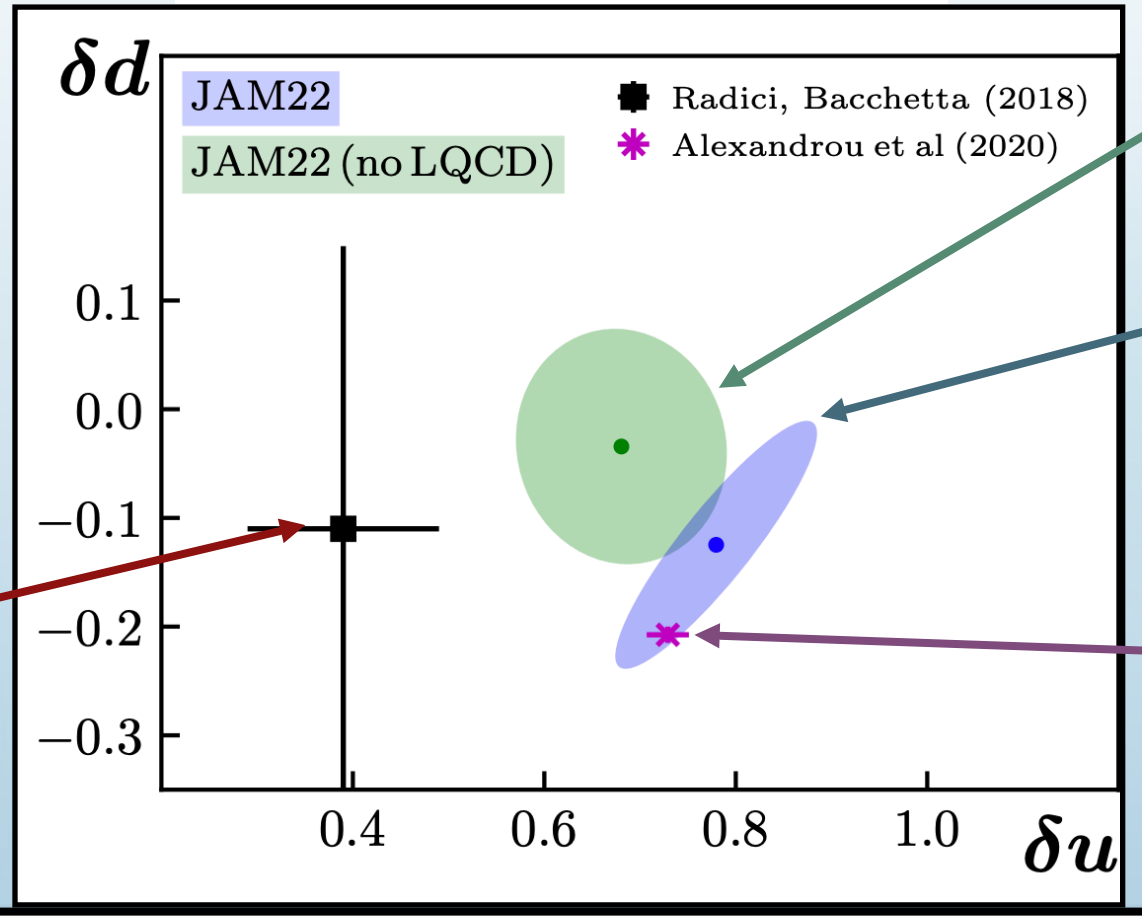

JAM3D (no LQCD)

JAM3D (w/ LQCD)

RB18

Lattice (ETMC)

Large disagreements between three approaches…
Can this be solved?

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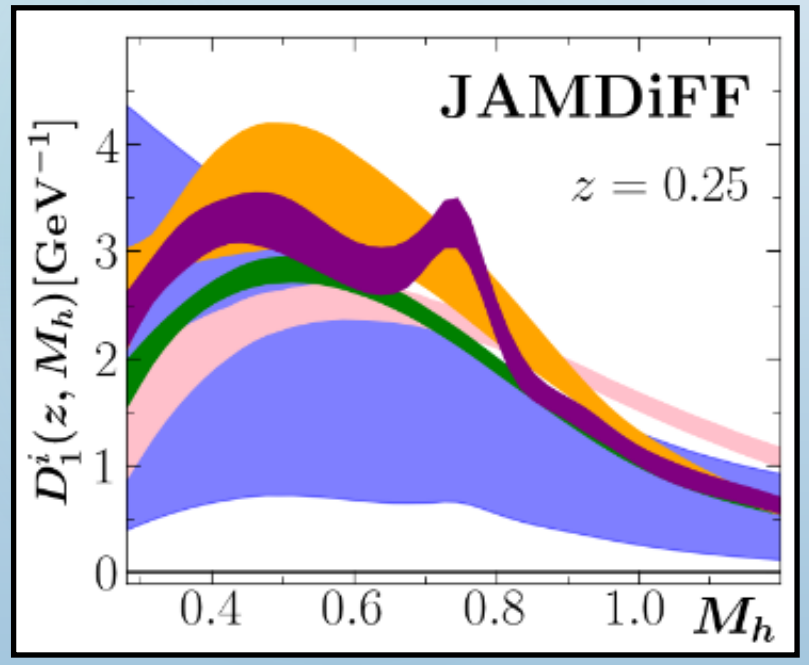


First simultaneous global QCD analysis of dihadron fragmentation functions and transversity parton distribution functions

Jefferson Lab Angular Momentum (JAM) Collaboration • C. Cocuzza (Temple U.) Show All(6)

Aug 28, 2023

34 pages

Published in: *Phys.Rev.D* 109 (2024) 3, 034024

Published: Feb 1, 2024

DiFF Theory

Talk by D. Pitonyak

Latest Developments in the Theory of Multi-Hadron Fragmentation Functions

Daniel Pitonyak

Lebanon Valley College, Annville, PA, USA

QCD Evolution Workshop

Jefferson Lab, Newport News, VA

May 19, 2025

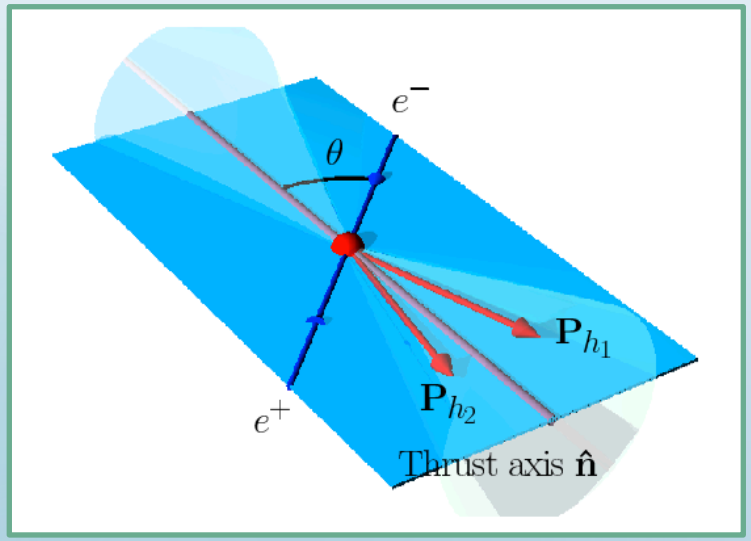
Based on

D. Pitonyak, C. Cocuzza, A. Metz, A. Prokudin and N. Sato,
"Number density interpretation of dihadron fragmentation functions,"
Phys. Rev. Lett. **132**, 011902 (2024) [arXiv:2305.11995 [hep-ph]].

D. Pitonyak, C. Cocuzza, A. Metz, A. Prokudin and N. Sato,
"Comment on "QCD factorization with multihadron fragmentation functions","
[arXiv:2502.15817 [hep-ph]], submitted to PRD.

Observables for DiFFs

SIA Cross Section



R. Seidl *et al.*, Phys. Rev. D **96**, no. 3, 032005 (2017)

$$\frac{\mathrm{d}\sigma}{\mathrm{d}z\,\mathrm{d}M_h} = \frac{4\pi\alpha_{\mathrm{em}}^2}{s}\sum_q e_q^2\, D_1^q(z, M_h)$$

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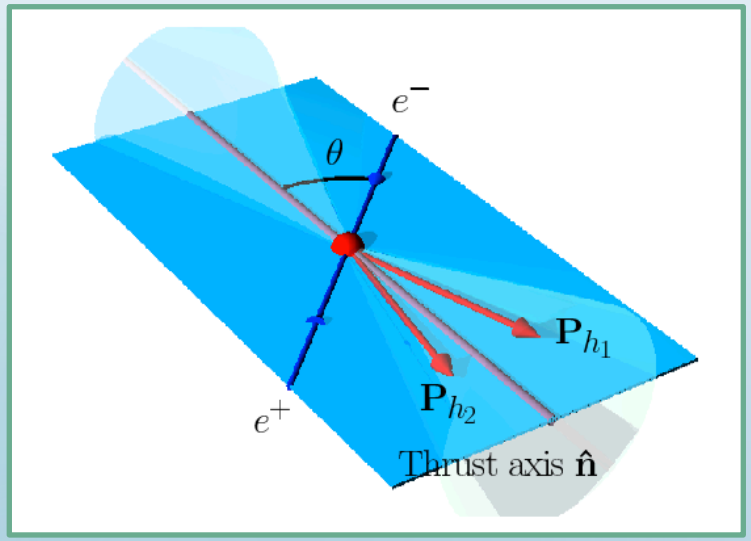
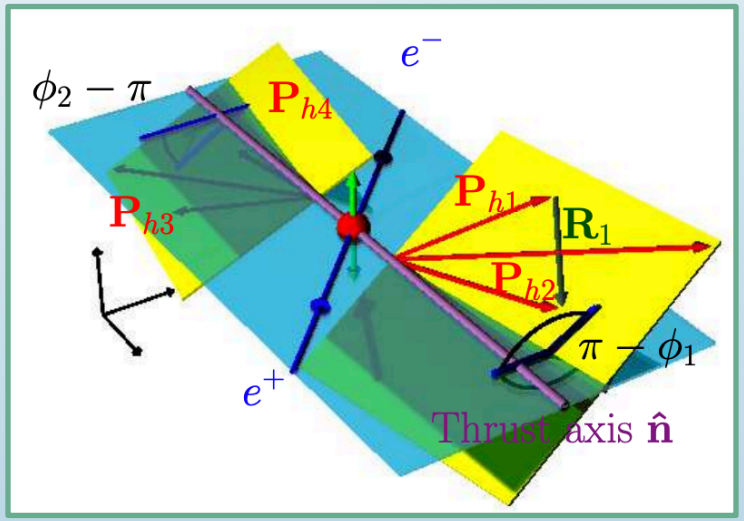


R. Seidl *et al.*, Phys. Rev. D **96**, no. 3, 032005 (2017)

SIA Artru-Collins Asymmetry



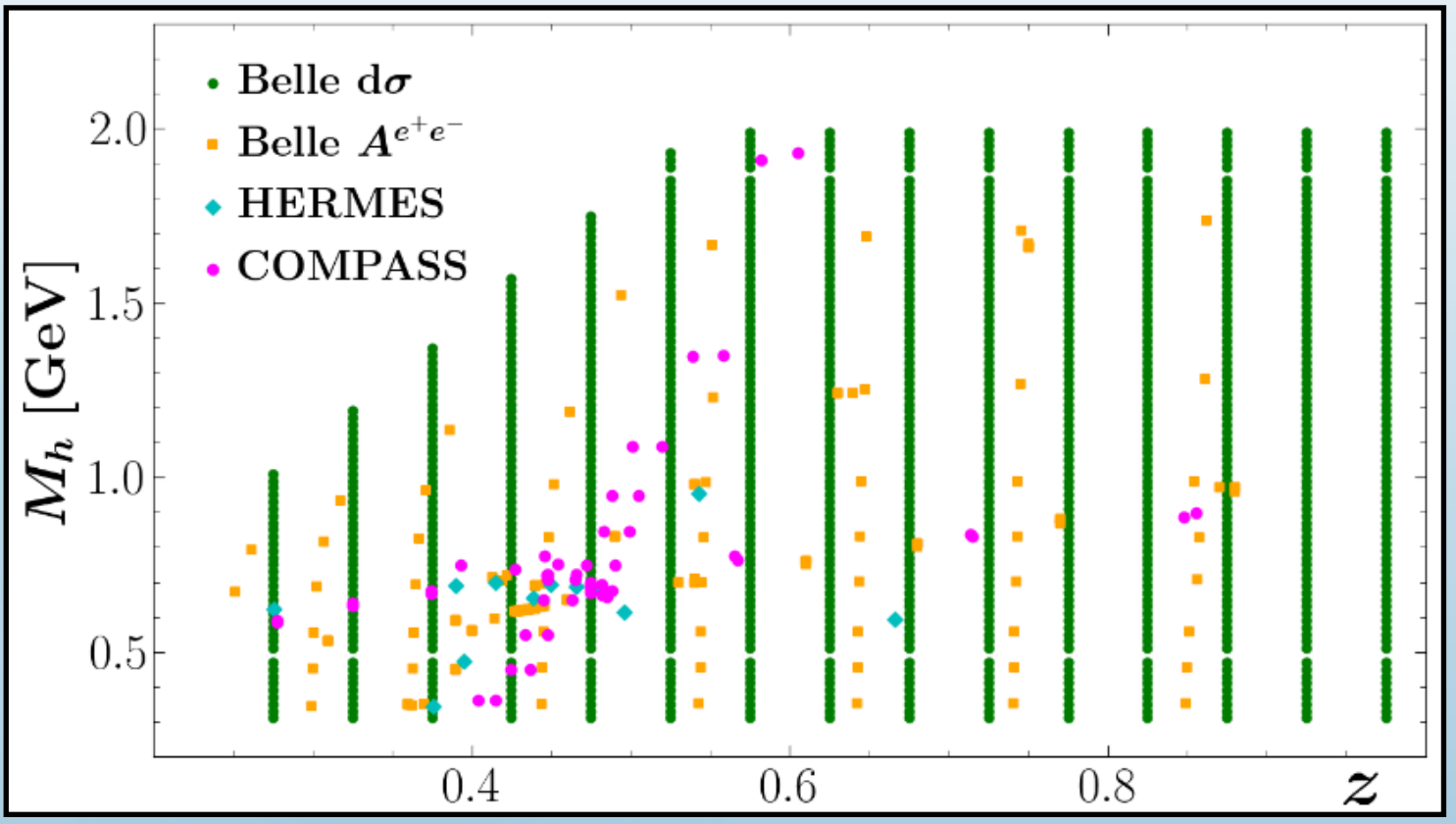
A. Vossen *et al.*, Phys. Rev. Lett. **107**, 072004 (2011)

$$\frac{\mathrm{d}\sigma}{\mathrm{d}z\,\mathrm{d}M_h} = \frac{4\pi\alpha_{\mathrm{em}}^2}{s}\sum_q e_q^2\, D_1^q(z, M_h)$$

$$A^{e^+e^-}(z, M_h, \bar{z}, \overline{M}_h) = \frac{\sin^2\theta \sum_q e_q^2\, H_1^{\sphericalangle,q}(z, M_h) H_1^{\sphericalangle,\bar{q}}(\bar{z}, \overline{M}_h)}{(1+\cos^2\theta)\sum_q e_q^2\, D_1^q(z, M_h) D_1^{\bar{q}}(\bar{z}, \overline{M}_h)}$$

Data for DiFFs

SIA cross section	Belle	1094 points
SIA Artru-Collins	Belle	183 points



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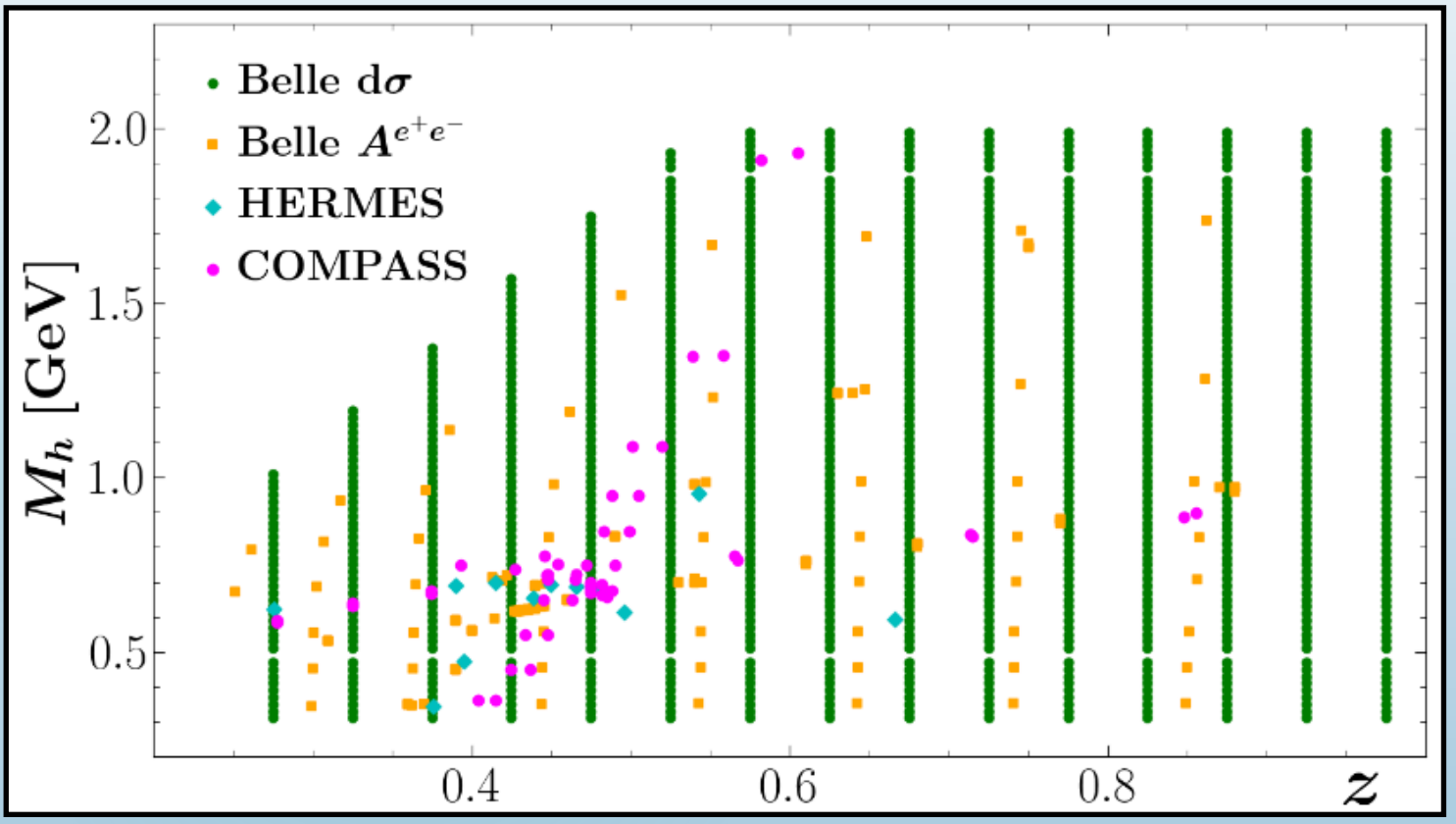


$\pi^+\pi^-$ DiFFs

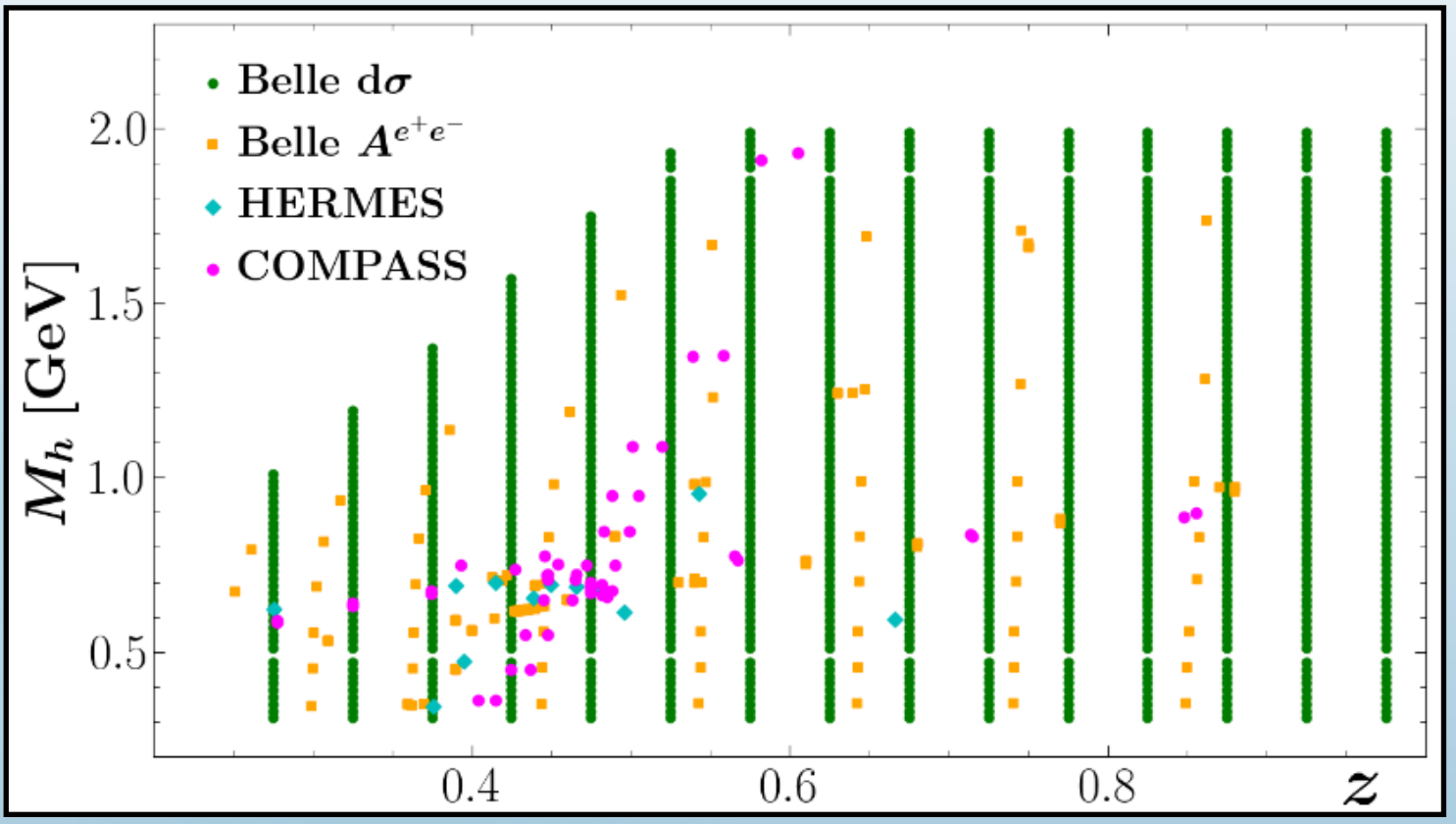
$$D_1^u = D_1^d = D_1^{\bar{u}} = D_1^{\bar{d}},$$

$$D_1^s = D_1^{\bar{s}}, \quad D_1^c = D_1^{\bar{c}}, \quad D_1^b = D_1^{\bar{b}},$$

5 independent functions (w/ D_1^g)
[supplement with PYTHIA data]

Data for DiFFs

SIA cross section	Belle	1094 points
SIA Artru-Collins	Belle	183 points



$\pi^+\pi^-$ DiFFs

$$D_1^u = D_1^d = D_1^{\bar{u}} = D_1^{\bar{d}},$$
$$D_1^s = D_1^{\bar{s}}, \quad D_1^c = D_1^{\bar{c}}, \quad D_1^b = D_1^{\bar{b}},$$

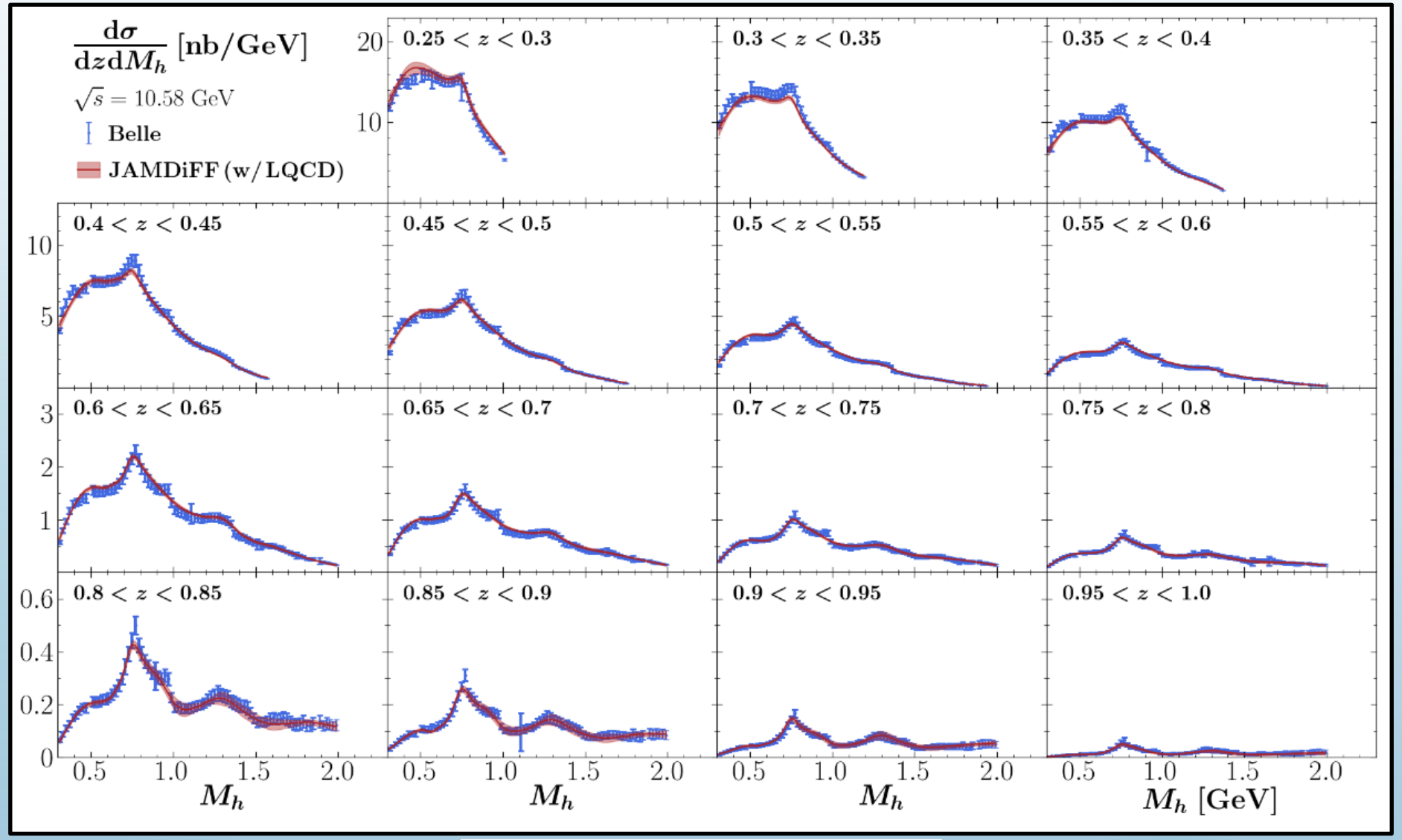
5 independent functions (w/ D_1^g)
[supplement with PYTHIA data]

$$H_1^{\sphericalangle,u} = -H_1^{\sphericalangle,d} = -H_1^{\sphericalangle,\bar{u}} = H_1^{\sphericalangle,\bar{d}},$$
$$H_1^{\sphericalangle,s} = -H_1^{\sphericalangle,\bar{s}} = H_1^{\sphericalangle,c} = -H_1^{\sphericalangle,\bar{c}} = 0,$$

1 independent function

A. Courtoy et al., Phys. Rev. D **85**, 114023 (2012)

Quality of Fit (Unpolarized Cross Section)



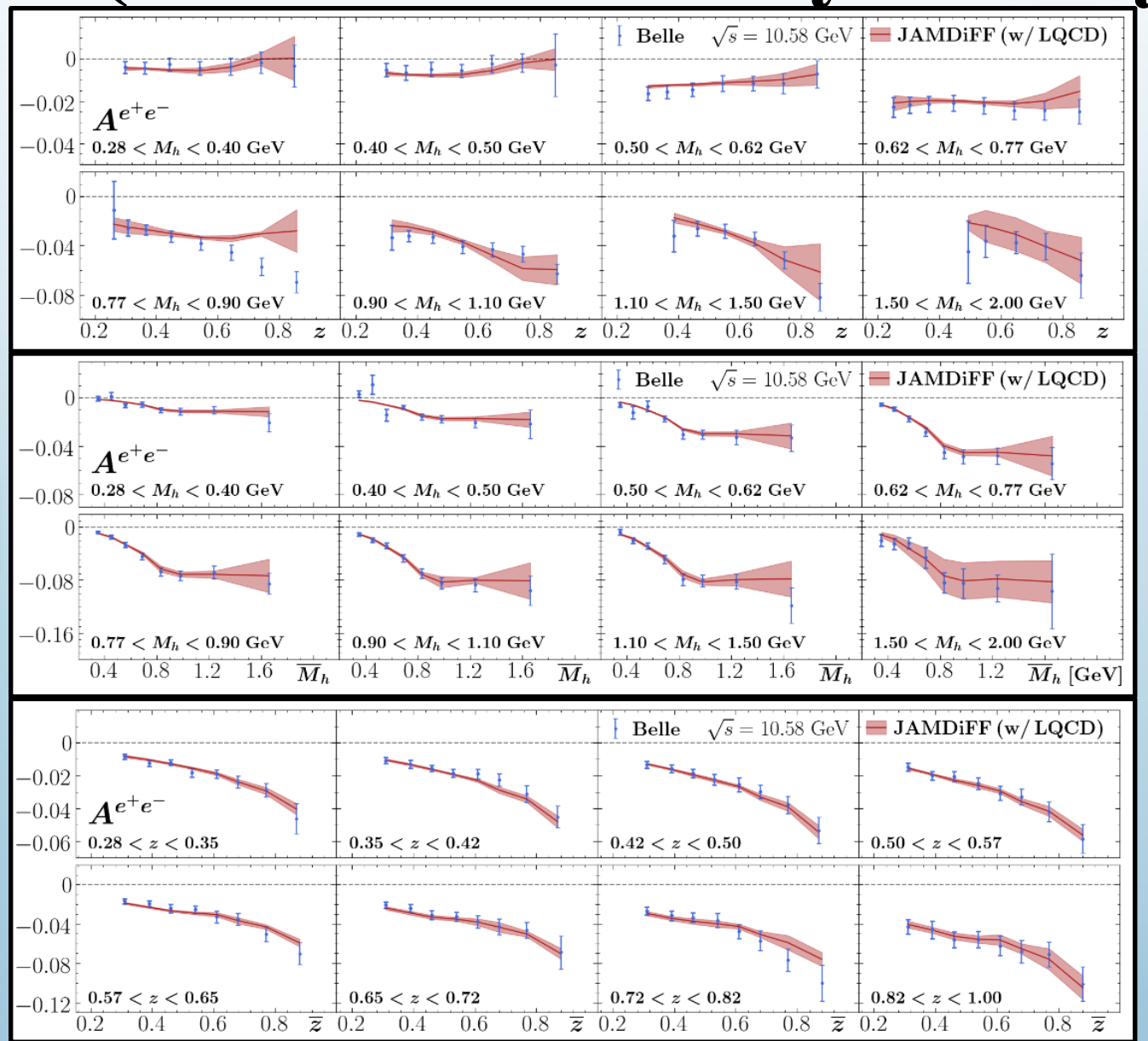
R. Seidl *et al.*, Phys. Rev. D **96**, 032005 (2017)

Quality of Fit (Artru-Collins Asymmetry)

(z, M_h) binning

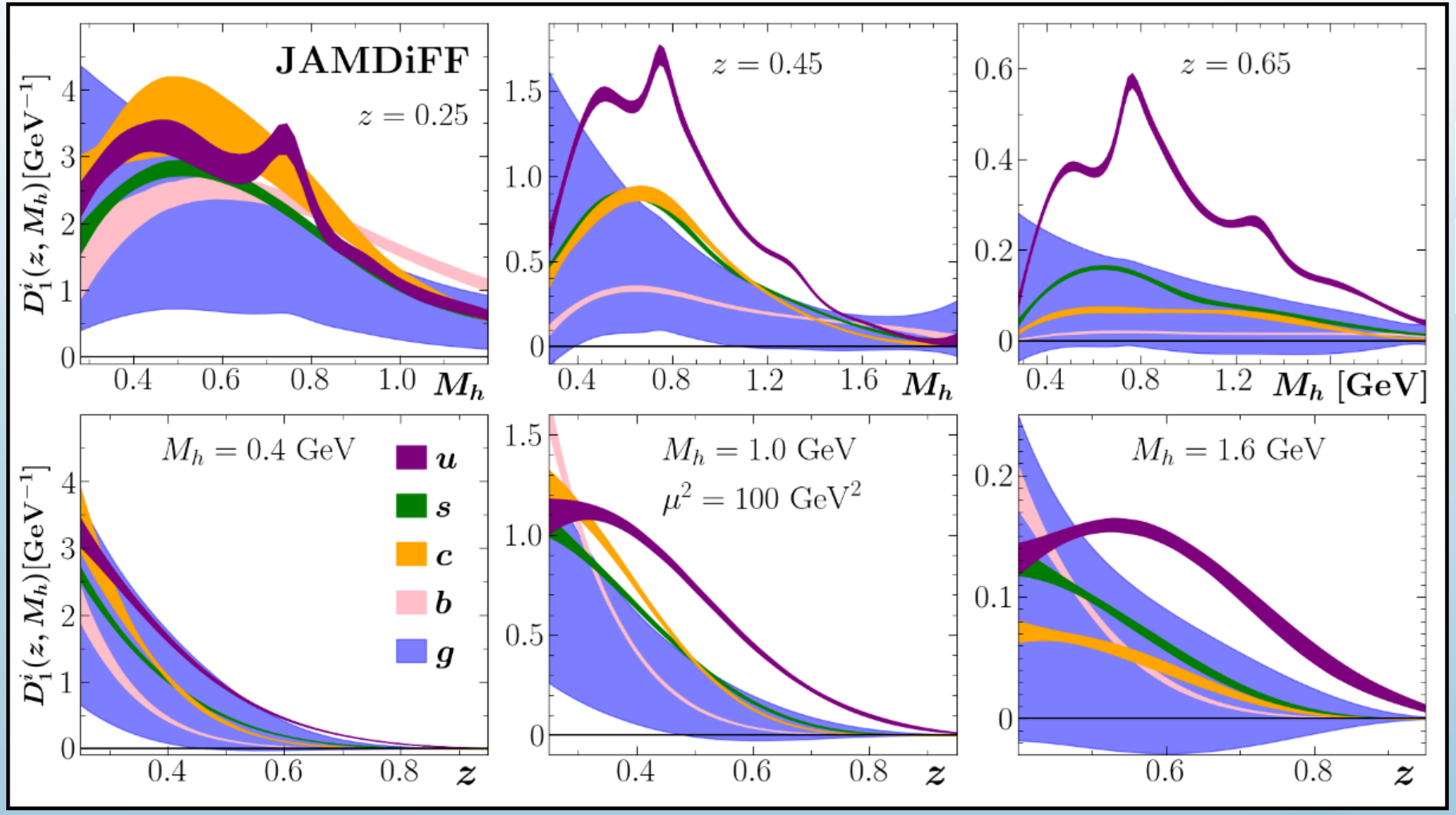
$(M_h, \overline{M}_h)$ binning

$(z, \bar{z})$ binning



A. Vossen *et al.*,
Phys. Rev. Lett. **107**, 072004 (2011)

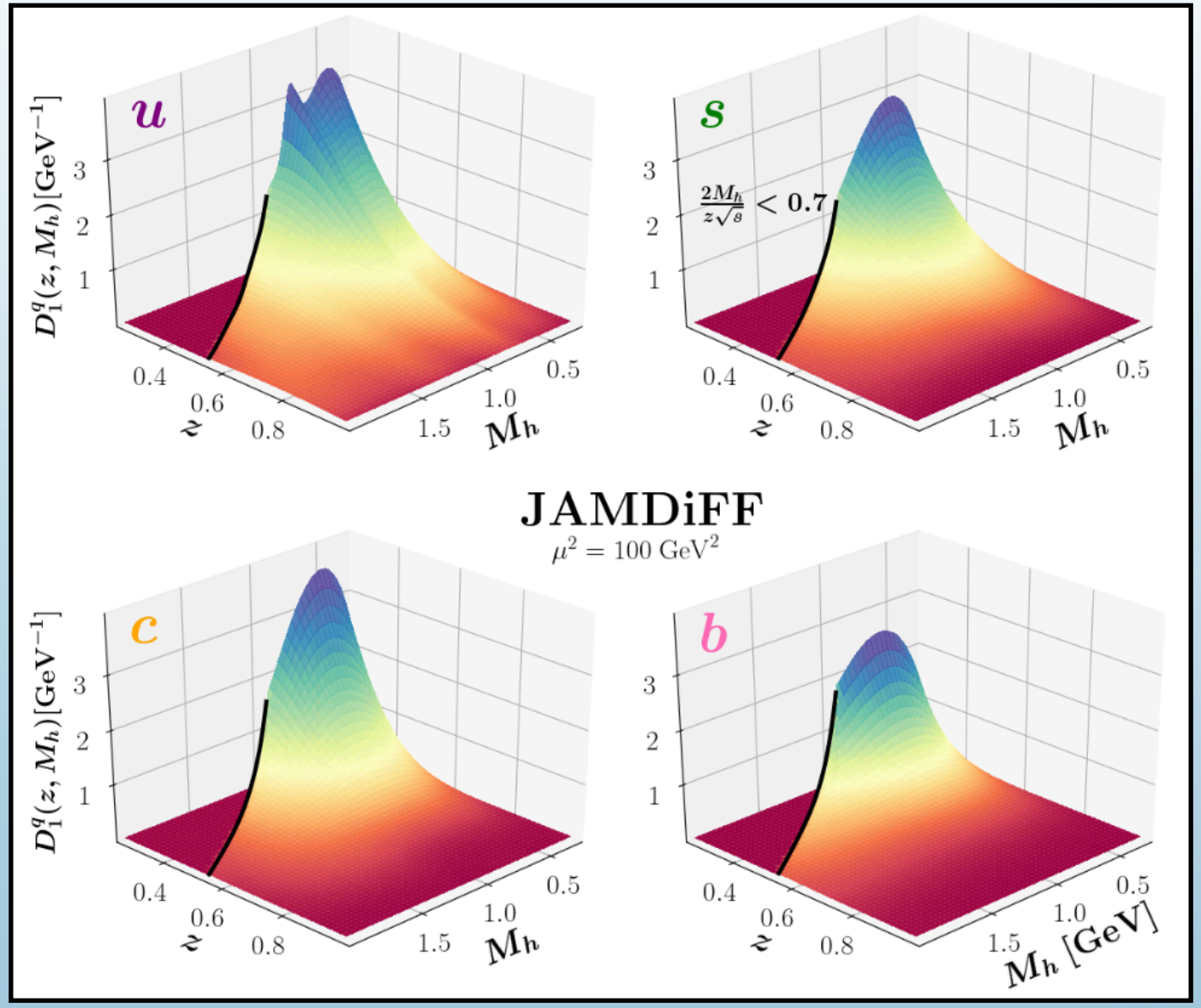
Extracted DiFFs



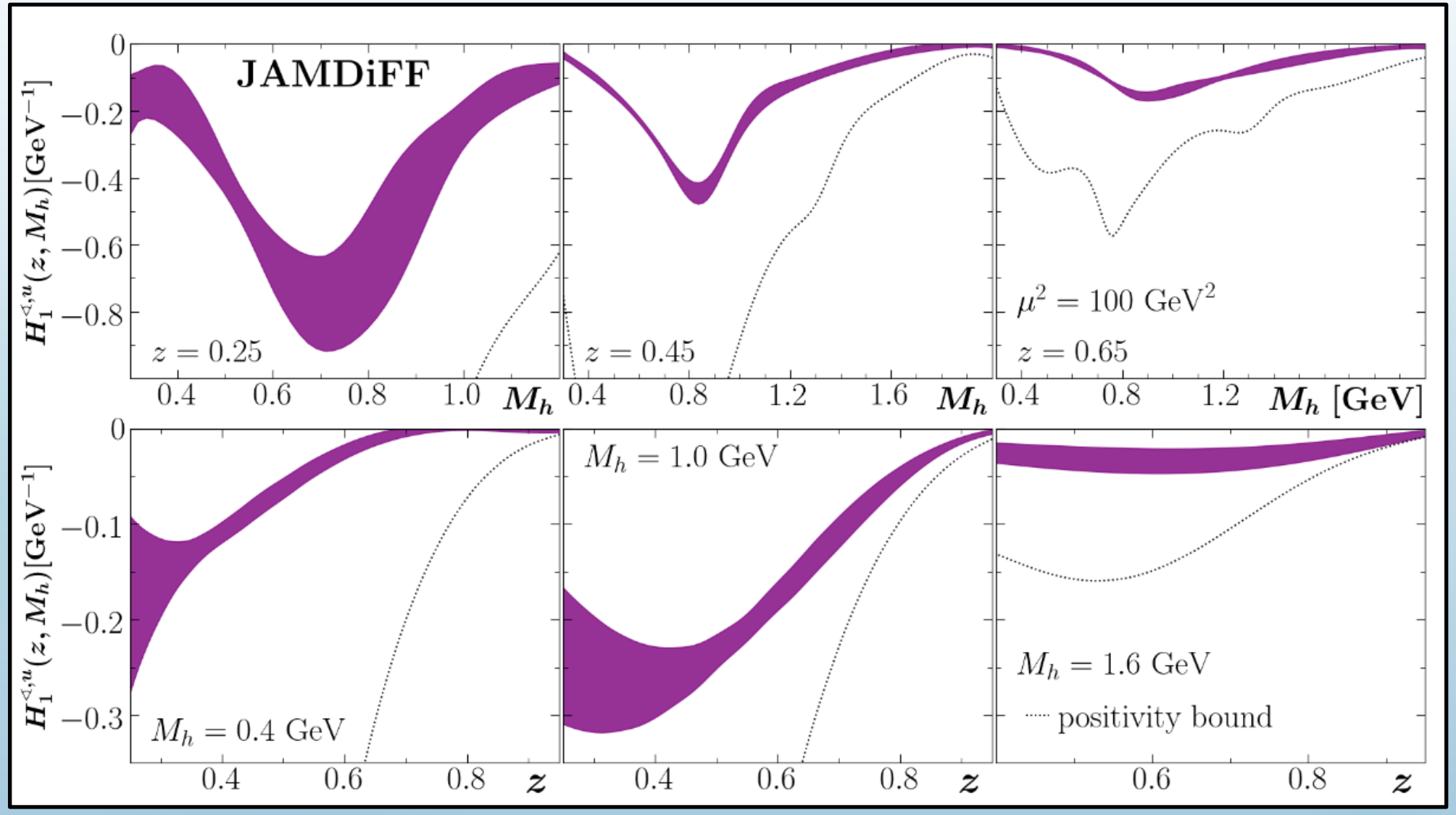
Bound: $D_1^q > 0$

A. Bacchetta and M. Radici, Phys. Rev. D **67**, 094002 (2003)

Extracted DiFFs (3D)



Extracted IFFs

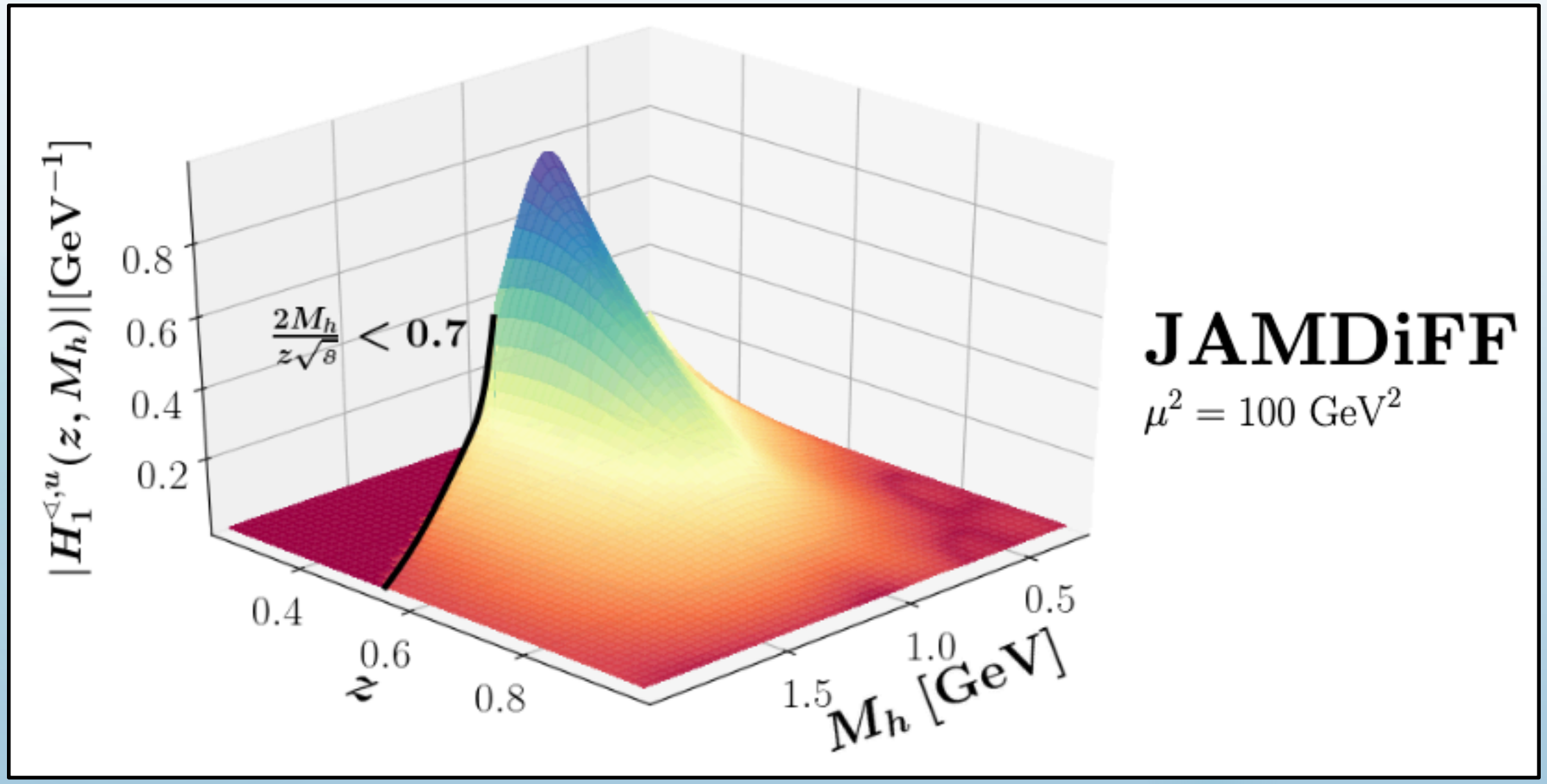


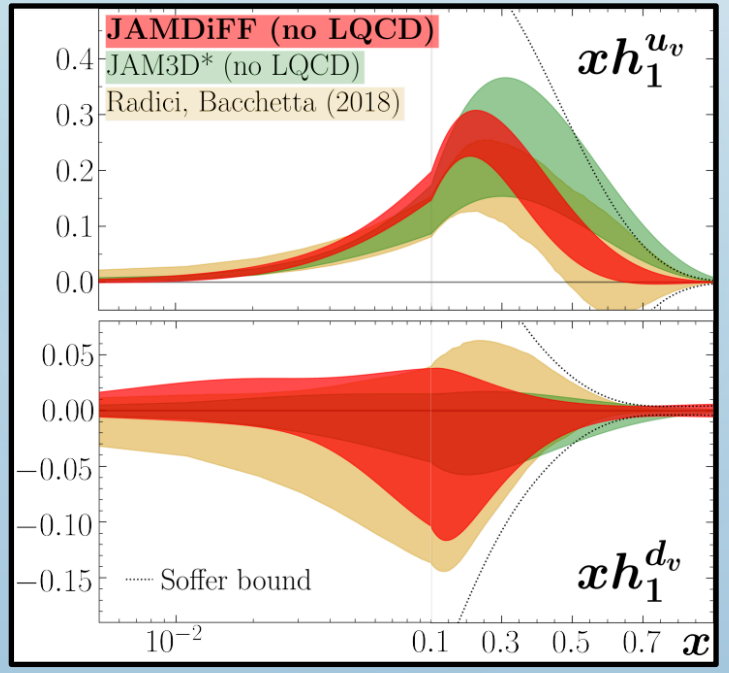
Bound:

$$|H_1^{\sphericalangle,q}| < D_1^q$$

A. Bacchetta and M. Radici, Phys. Rev. D **67**, 094002 (2003)

Extracted IFFs (3D)





Transversity Distributions and Tensor Charges of the Nucleon: Extraction from Dihadron Production and Their Universal Nature

JAM Collaboration • C. Cocuzza (Temple U.) Show All(6)

Jun 22, 2023

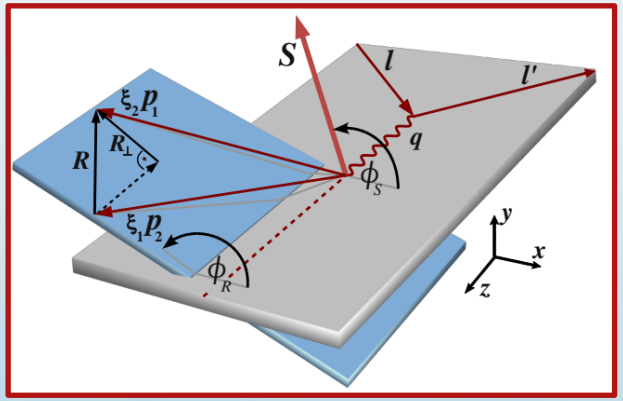
7 pages

Published in: *Phys.Rev.Lett.* 132 (2024) 9, 091901

Published: Feb 27, 2024

Observables for Transversity PDFs

SIDIS asymmetry (p and D)

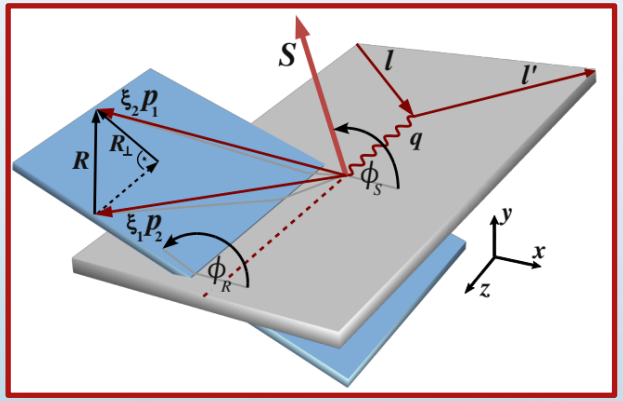


$$A_{UT}^{\text{SIDIS}} = c(y) \frac{\sum_q e_q^2 \, h_1^q(x) \, H_1^{\sphericalangle,q}(z, M_h)}{\sum_q e_q^2 \, f_1^q(x) \, D_1^q(z, M_h)}$$

C. Adolph *et al.*, Phys. Lett. B **713**, 10-16 (2012)

Observables for Transversity PDFs

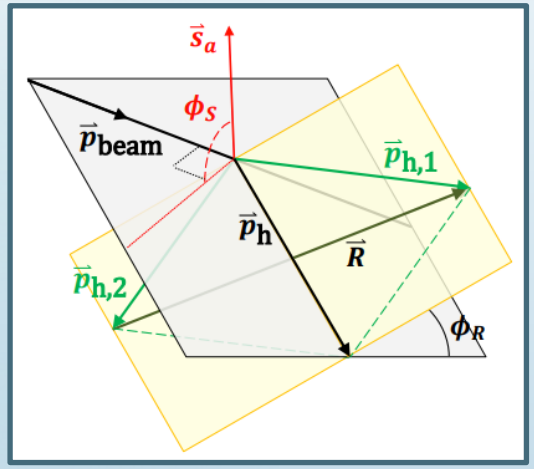
SIDIS asymmetry (p and D)



$$A_{UT}^{\text{SIDIS}} = c(y)\frac{\sum_q e_q^2\, h_1^q(x)\, H_1^{\sphericalangle,q}(z, M_h)}{\sum_q e_q^2\, f_1^q(x)\, D_1^q(z, M_h)}$$

C. Adolph *et al.*, Phys. Lett. B **713**, 10-16 (2012)

pp Asymmetry



L. Adamczyk *et al.*, Phys. Rev. Lett. **115**, 242501 (2015)

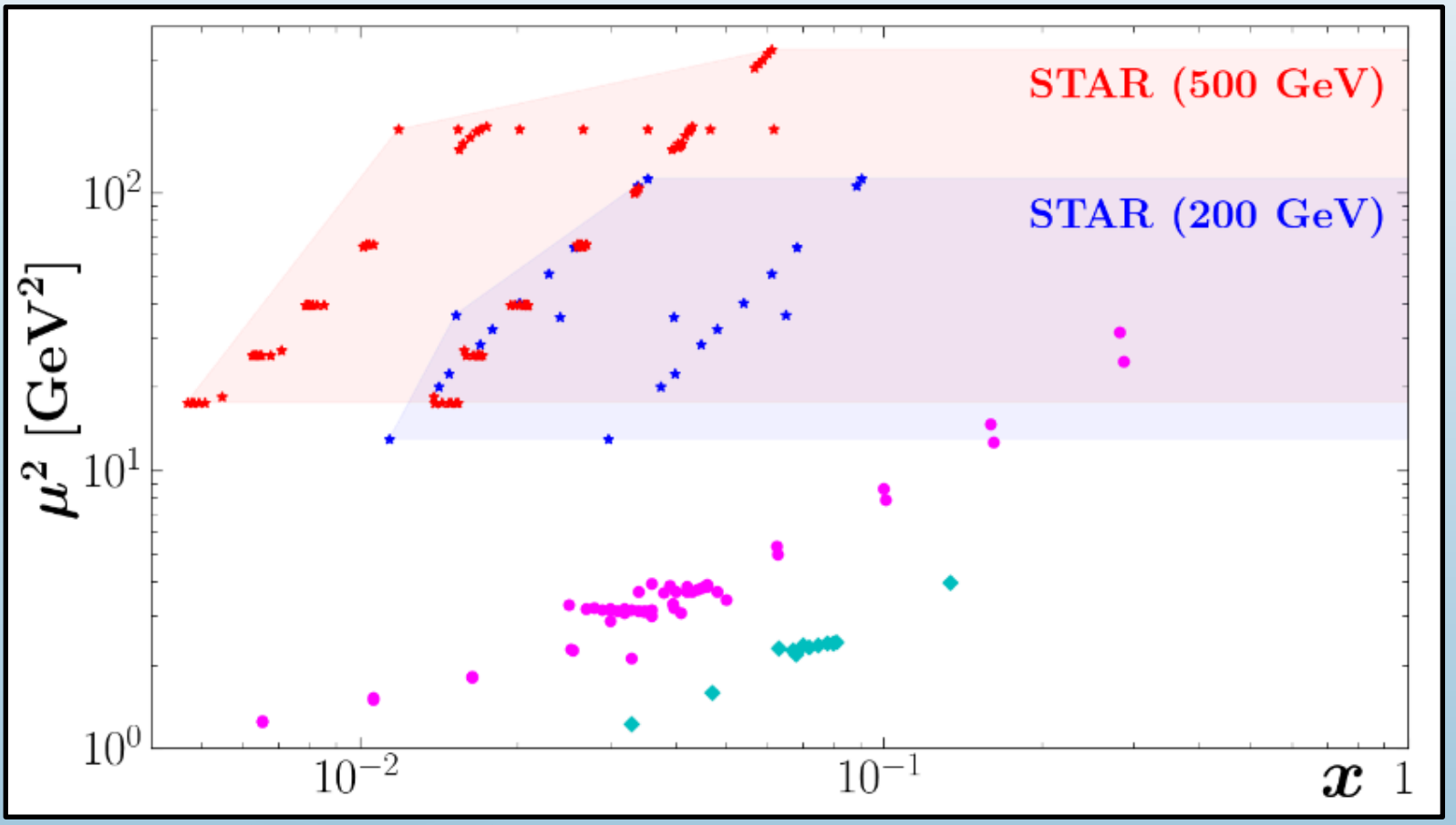
$$A_{UT}^{pp} = \frac{\mathscr{H}(M_h, P_{hT}, \eta)}{\mathscr{D}(M_h, P_{hT}, \eta)}$$

$$\mathscr{H}(M_h, P_{hT}, \eta) = 2P_{hT}\sum_i\sum_{a,b,c}\int_{x_a^{\min}}^1 \mathrm{d}x_a \int_{x_b^{\min}}^1 \frac{\mathrm{d}x_b}{z} f_1^a(x_a)\, h_1^b(x_b)\frac{\mathrm{d}\Delta\hat{\sigma}_{ab^\uparrow\to c^\uparrow d}}{\mathrm{d}\hat{t}}\, H_1^{\sphericalangle,c}(z, M_h)$$

$$\mathscr{D}(M_h, P_{hT}, \eta) = 2P_{hT}\sum_i\sum_{a,b,c}\int_{x_a^{\min}}^1 \mathrm{d}x_a \int_{x_b^{\min}}^1 \frac{\mathrm{d}x_b}{z} f_1^a(x_a) f_1^b(x_b)\frac{\mathrm{d}\hat{\sigma}_{ab\to cd}}{\mathrm{d}\hat{t}}\, D_1^c(z, M_h)$$

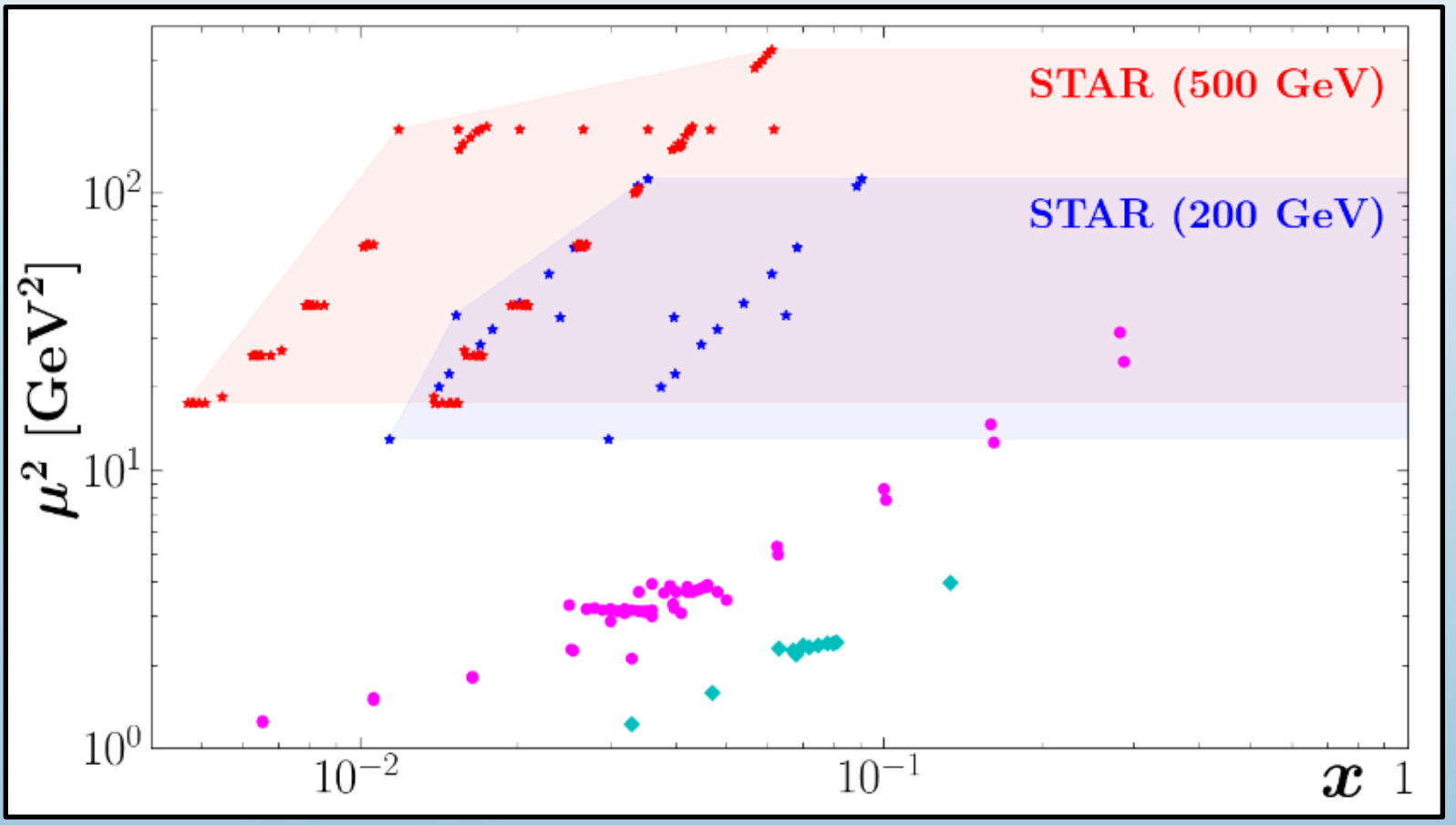
Data for PDFs

Process	Collaborations	Points
SIDIS (p, D)	COMPASS, HERMES	64
Proton-Proton	STAR	269



Data for PDFs

Process	Collaborations	Points
SIDIS (p, D)	COMPASS, HERMES	64
Proton-Proton	STAR	269



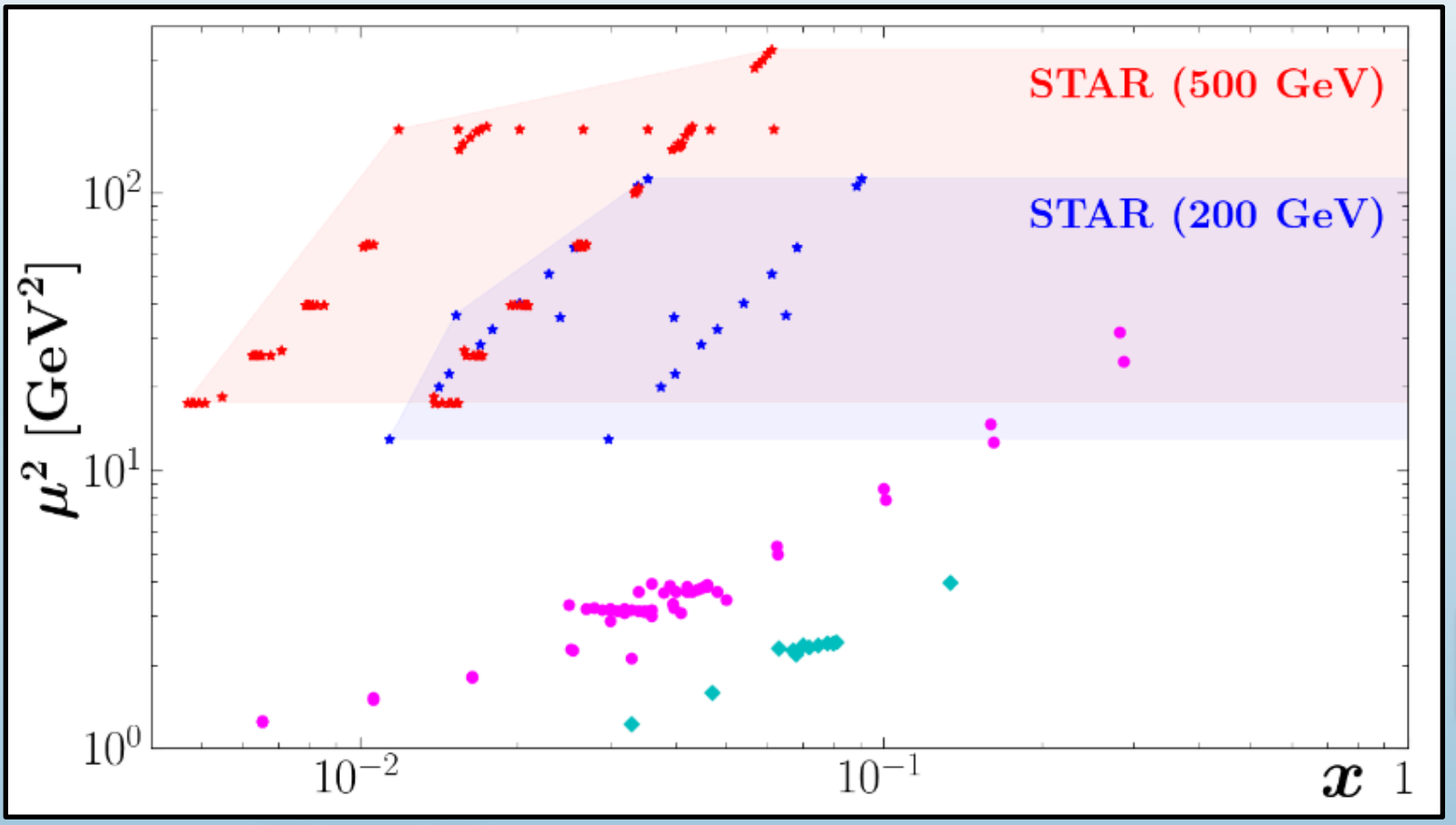
Parameterization Choices

3 independent observables
3 independent functions

$$h_1^{u_v}$$
$$h_1^{d_v}$$
$$h_1^{\bar{u}} = -\,h_1^{\bar{d}}$$

Data for PDFs

Process	Collaborations	Points
SIDIS (p, D)	COMPASS, HERMES	64
Proton-Proton	STAR	269



Parameterization Choices

3 independent observables
3 independent functions

$$h_1^{u_v}$$
$$h_1^{d_v}$$
$$h_1^{\bar{u}} = -\, h_1^{\bar{d}}$$

Prediction from large-N_c limit

P. V. Pobylitsa, arXiv:hep-ph/0301236 (2003)

Quality of Fit

Experiment	N_{dat}	χ^2_{red} w/ LQCD	χ^2_{red} no LQCD
Belle (cross section) [63]	1094	1.01	1.01
Belle (Artru-Collins) [92]	183	0.74	0.73
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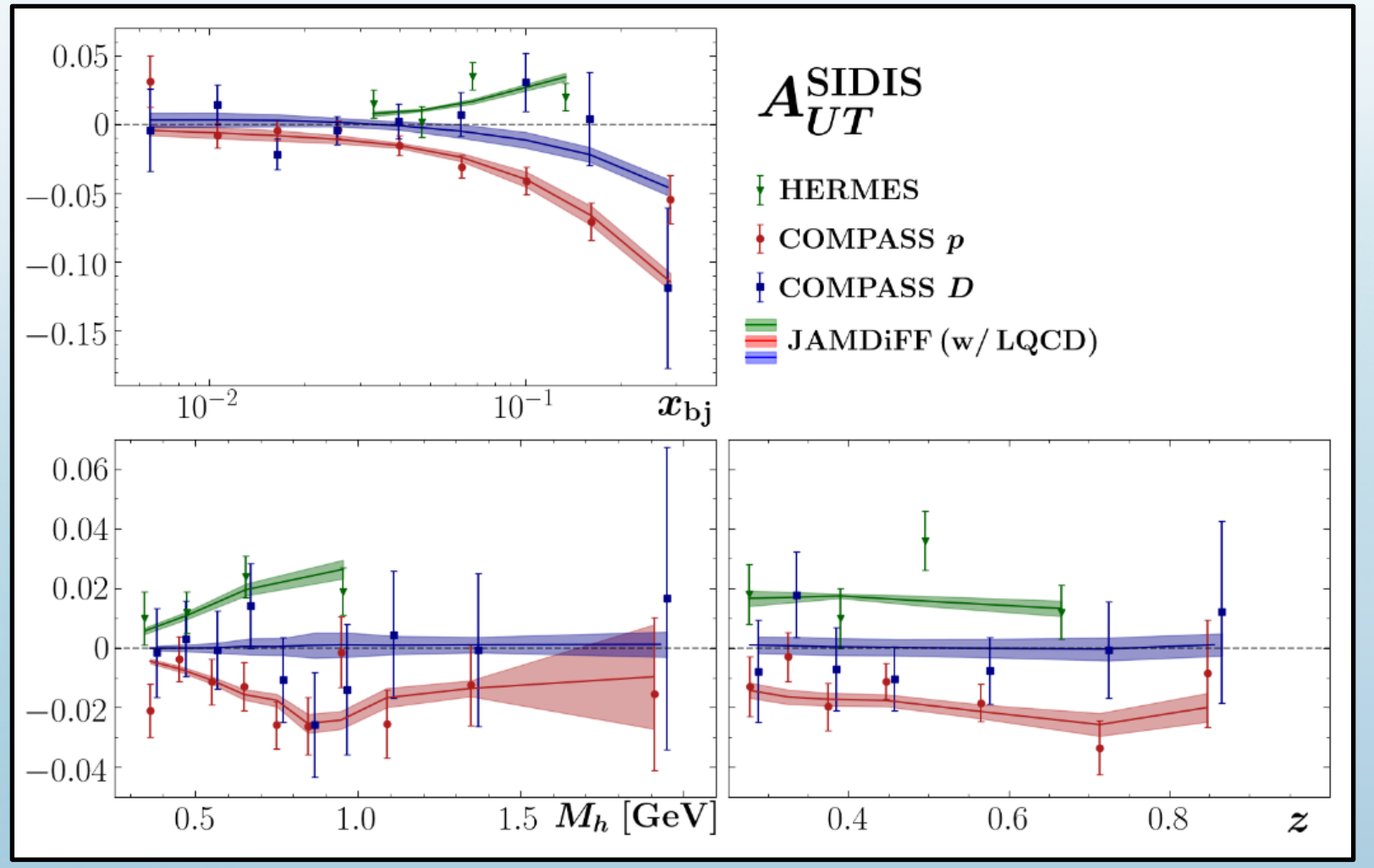
PNDME δu [25]	1	8.68	—
PNDME δd [25]	1	0.04	—
Total χ^2_{red} (N_{dat})		**1.01** (1475)	**0.98** (1471)

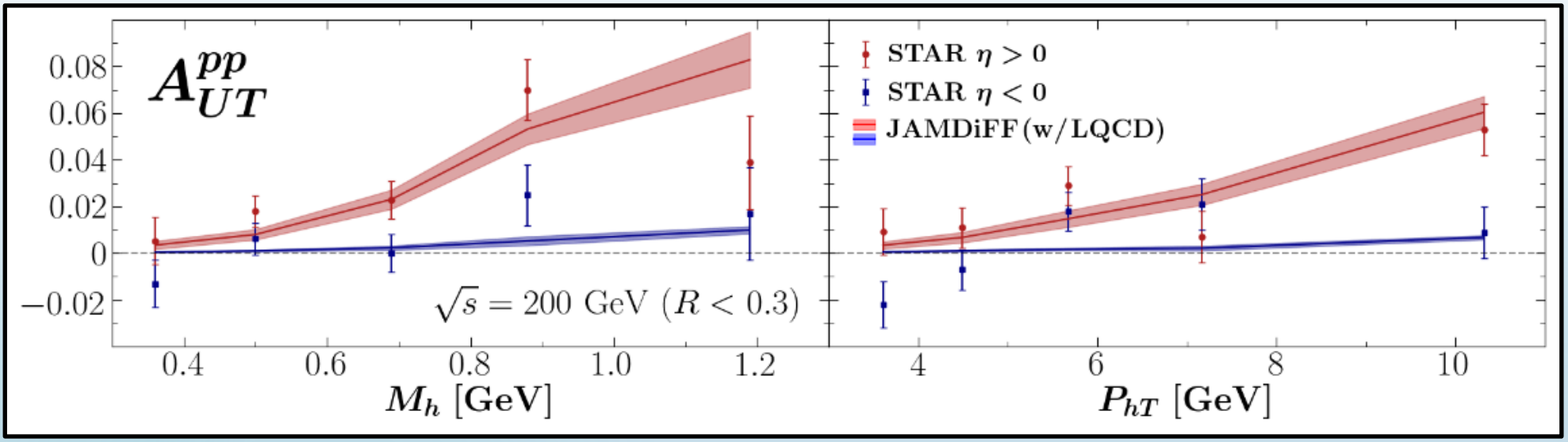
Quality of Fit (SIDIS)

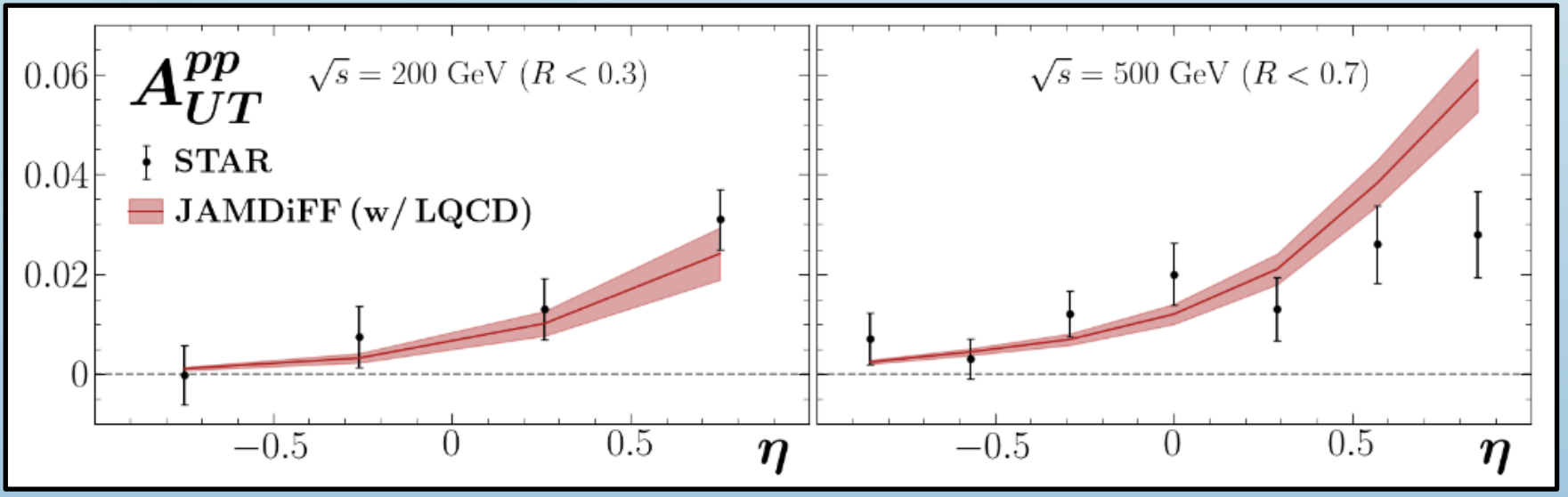


A. Airapetian *et al.*, JHEP **06**, 017 (2008)

COMPASS, arXiv:hep-ph/2301.02013 (2023)

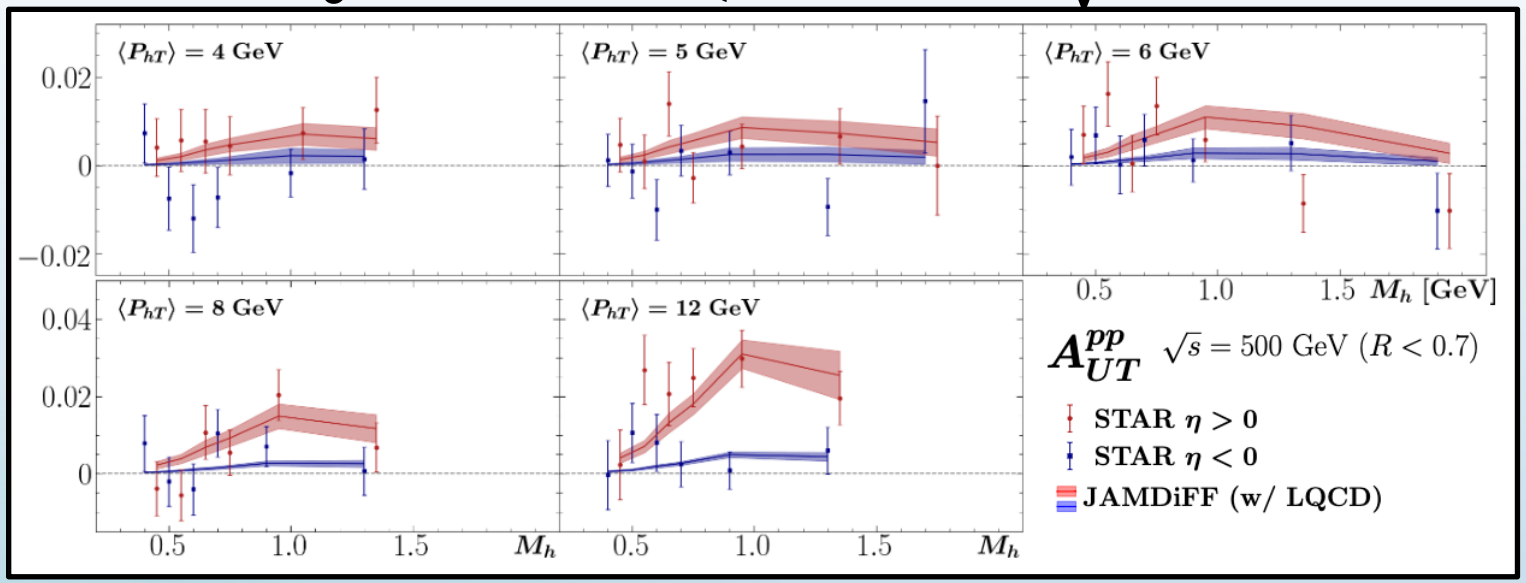
Quality of Fit (STAR $\sqrt{s} = 200$ GeV)

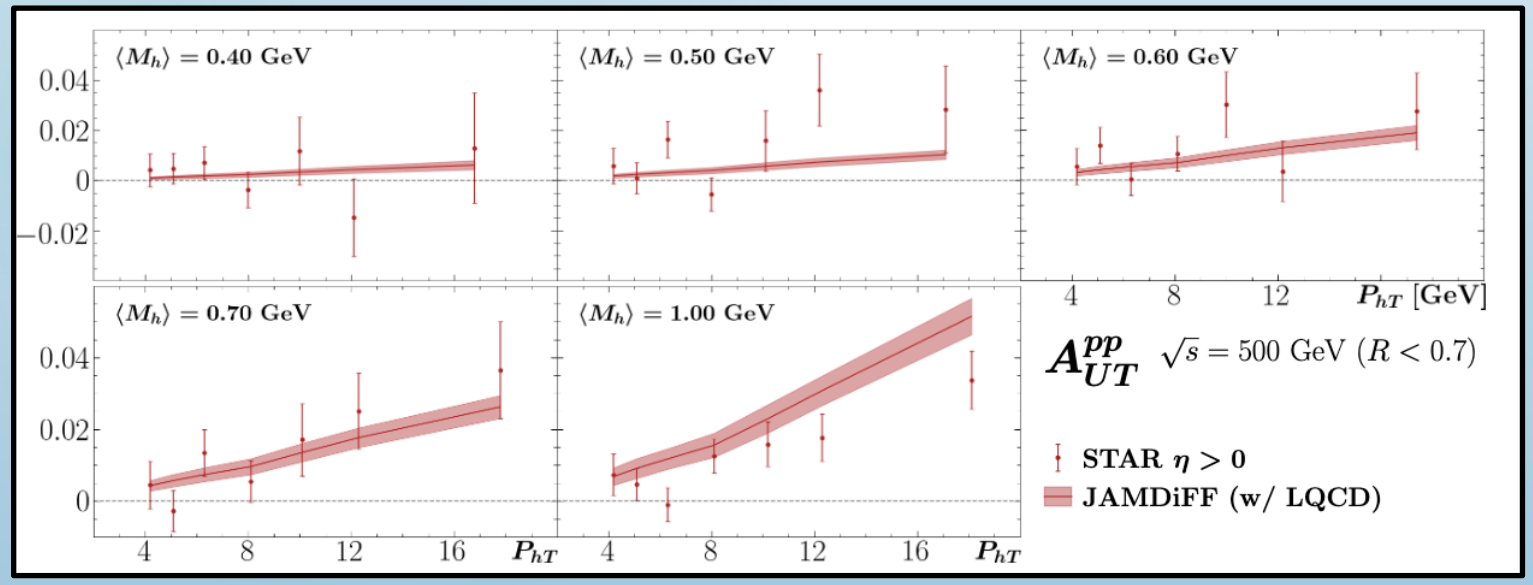




L. Adamczyk *et al.*, Phys. Rev. Lett. **115**, 24501 (2015)

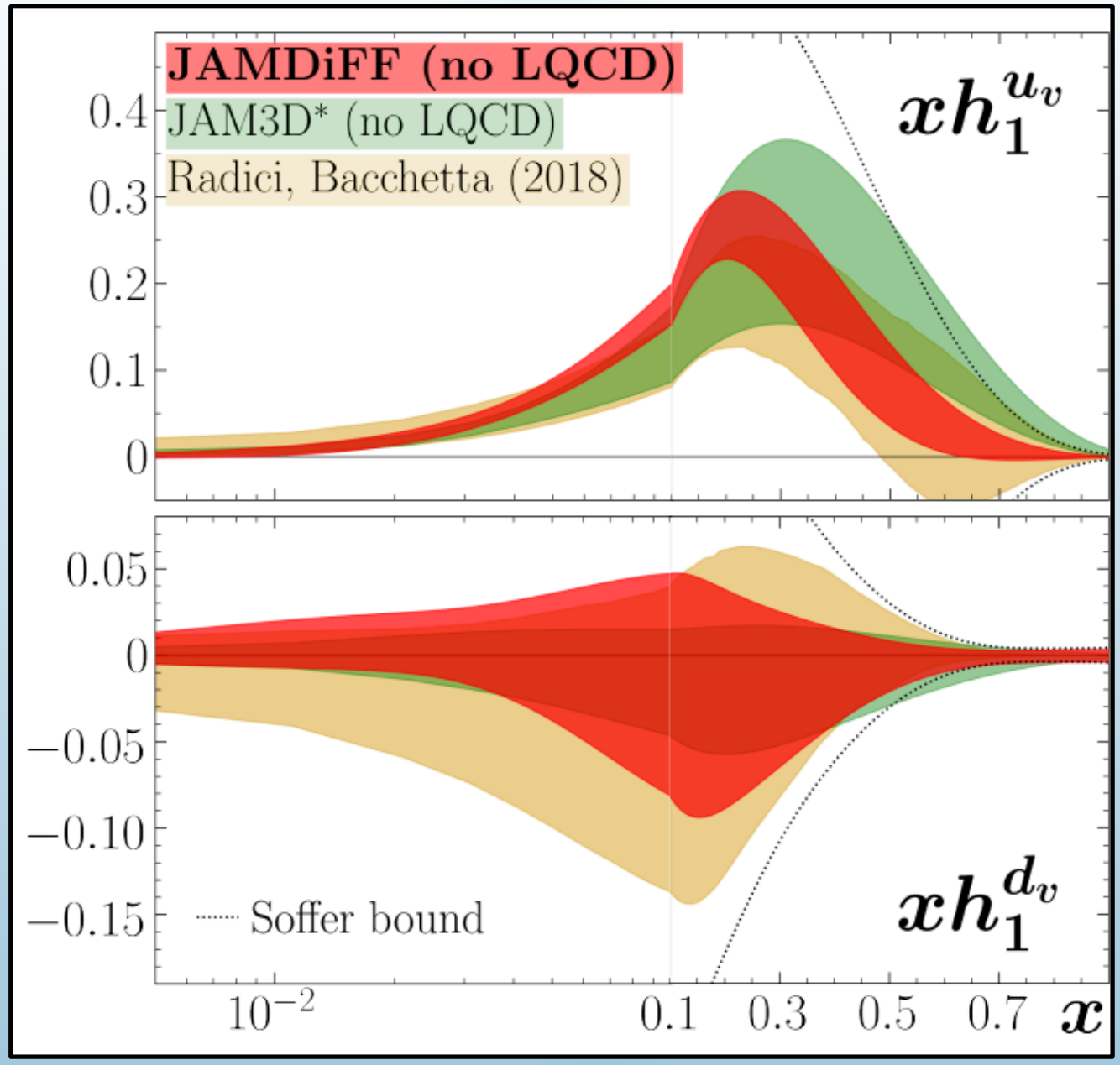
Quality of Fit (STAR $\sqrt{s} = 500$ GeV)





L. Adamczyk *et al.*, Phys. Rev. B **780**, 332-339 (2018)

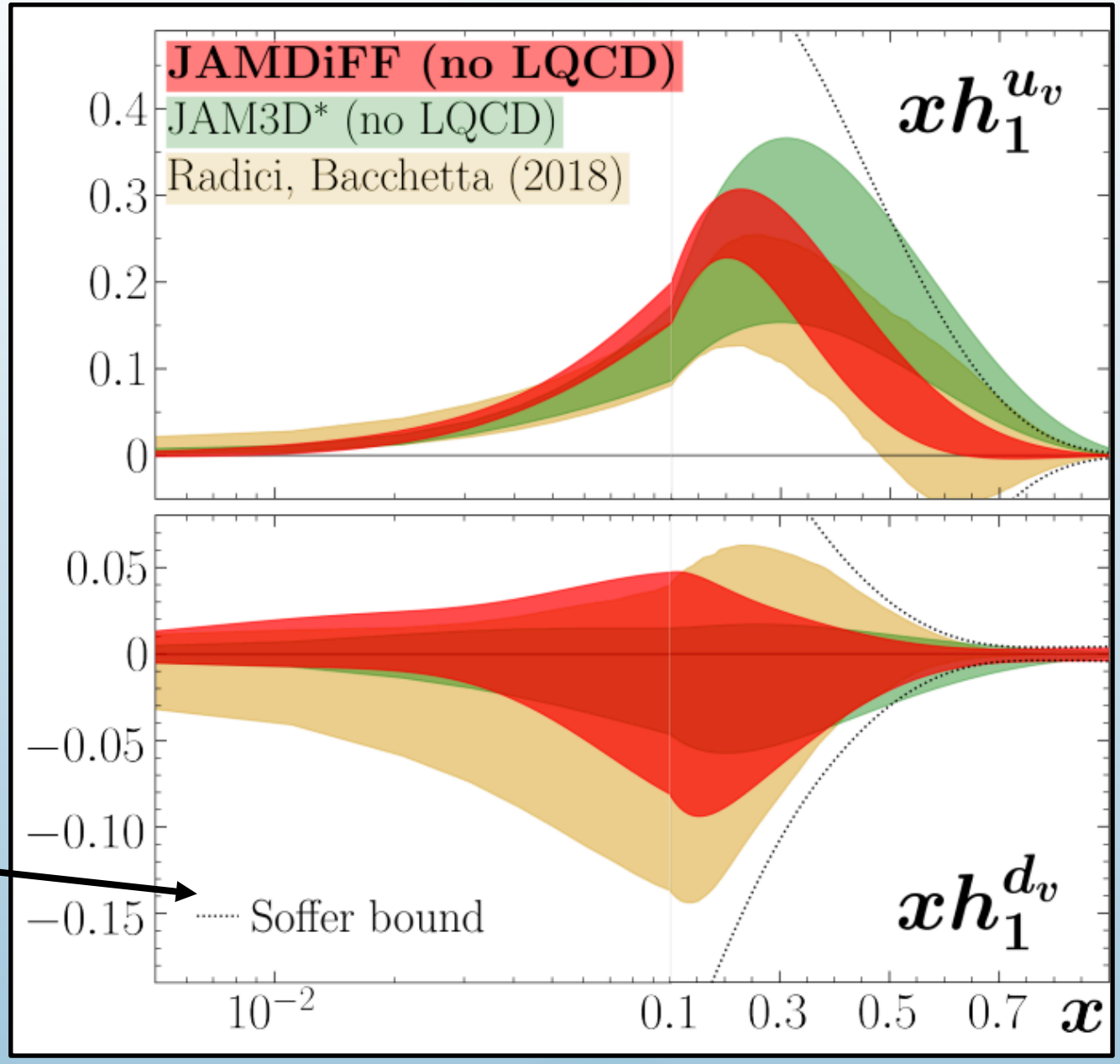
Transversity PDFs



Transversity PDFs

Soffer Bound: $|h_1^q| < \frac{1}{2}[f_1^q + g_1^q]$

J. Soffer, Phys. Rev. Lett. **74**, 1292-1294 (1995)

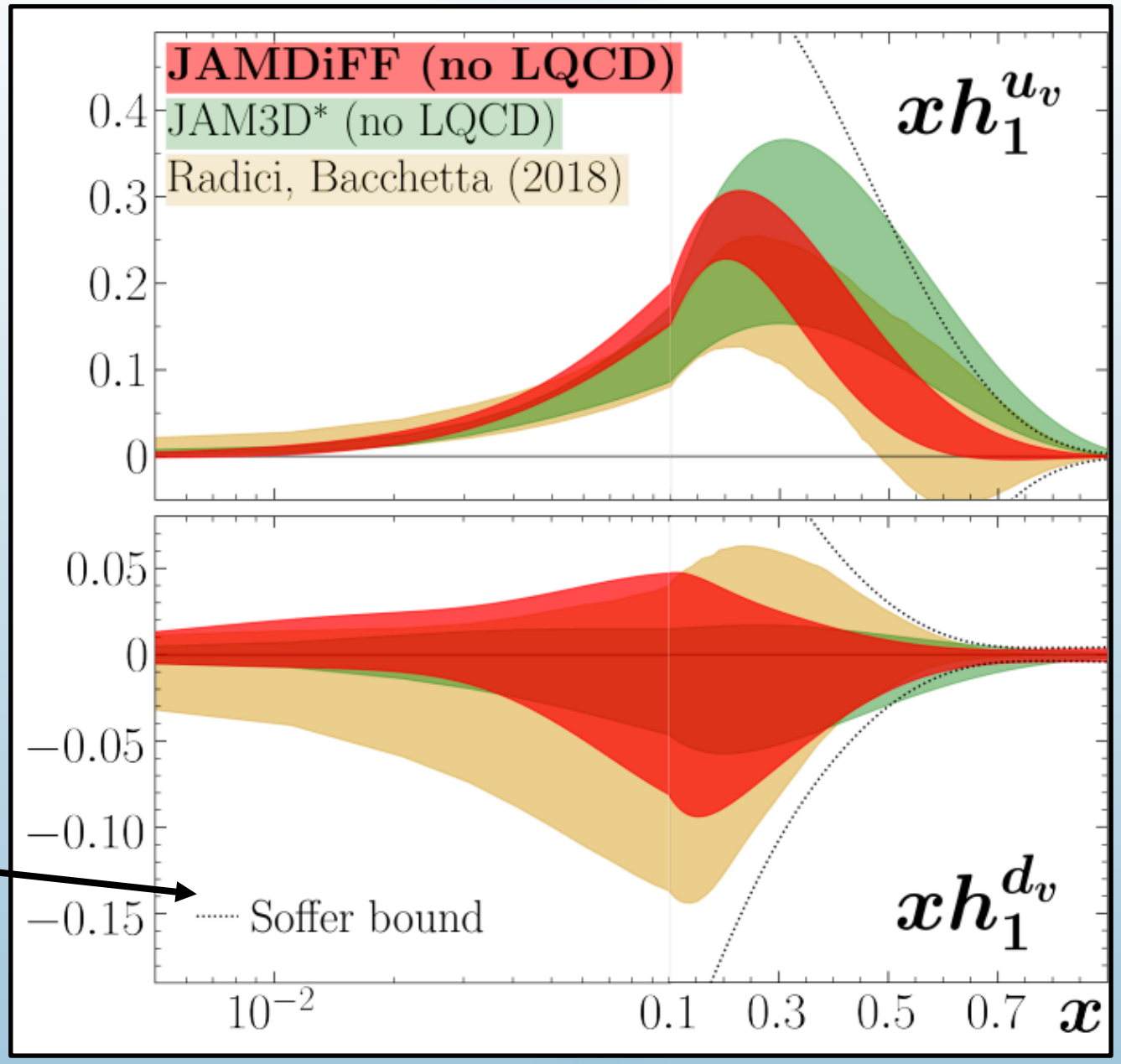


Transversity PDFs

JAM3D* = JAM3D-22 (no LQCD)
+ Antiquarks w/ $\bar{u} = -\bar{d}$
+ small-x constraint (see slide 27)

Soffer Bound: $|h_1^q| < \frac{1}{2}[f_1^q + g_1^q]$

J. Soffer, Phys. Rev. Lett. **74**, 1292-1294 (1995)



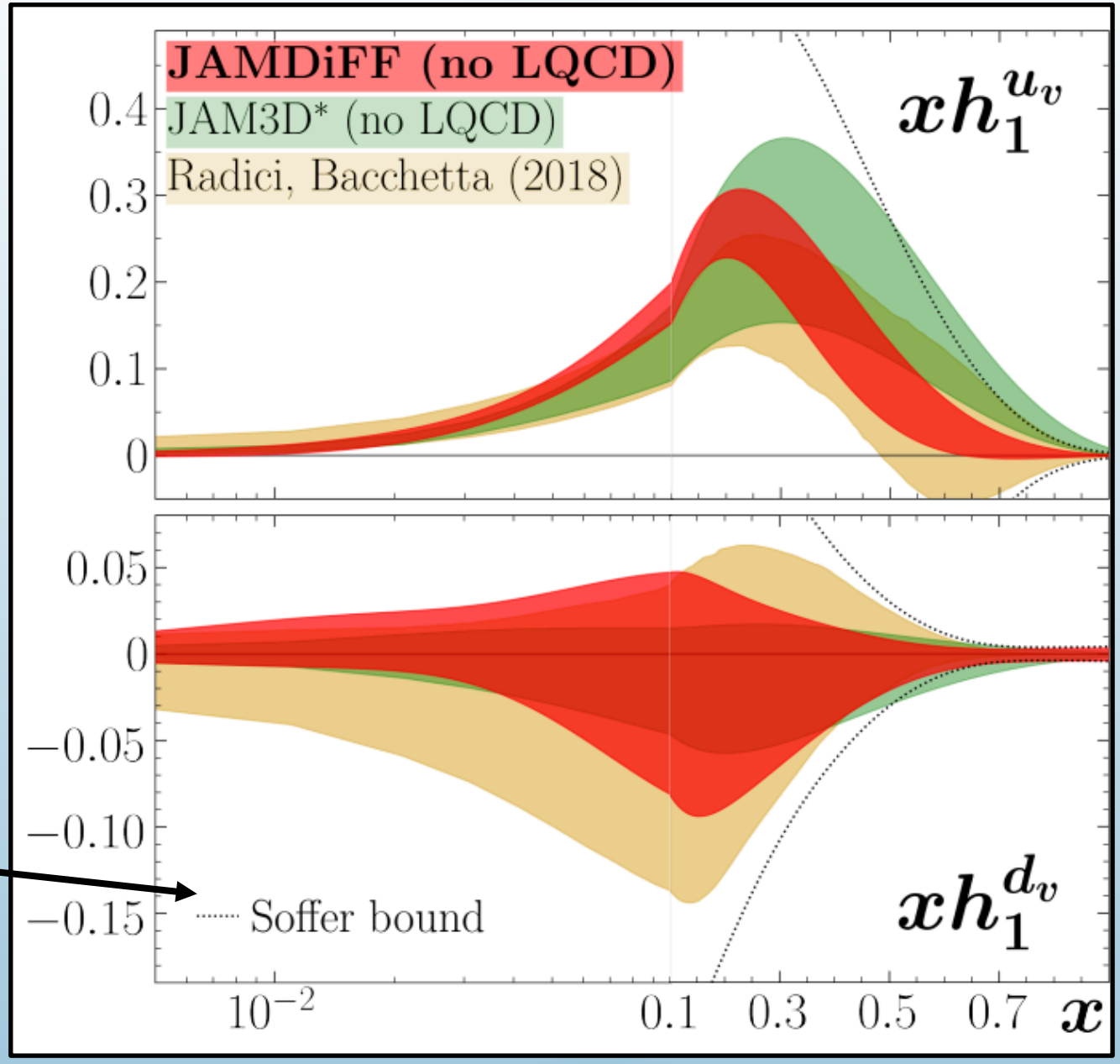
Transversity PDFs

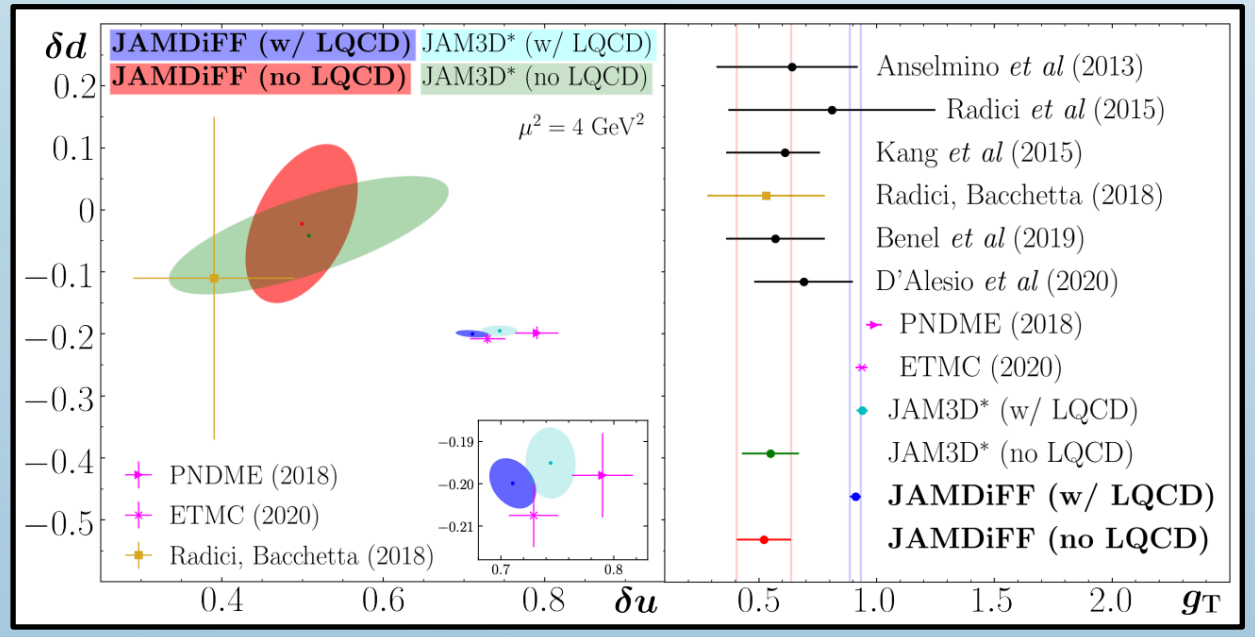
JAM3D* = JAM3D-22 (no LQCD)
+ Antiquarks w/ $\bar{u} = -\bar{d}$
+ small-x constraint (see slide 27)

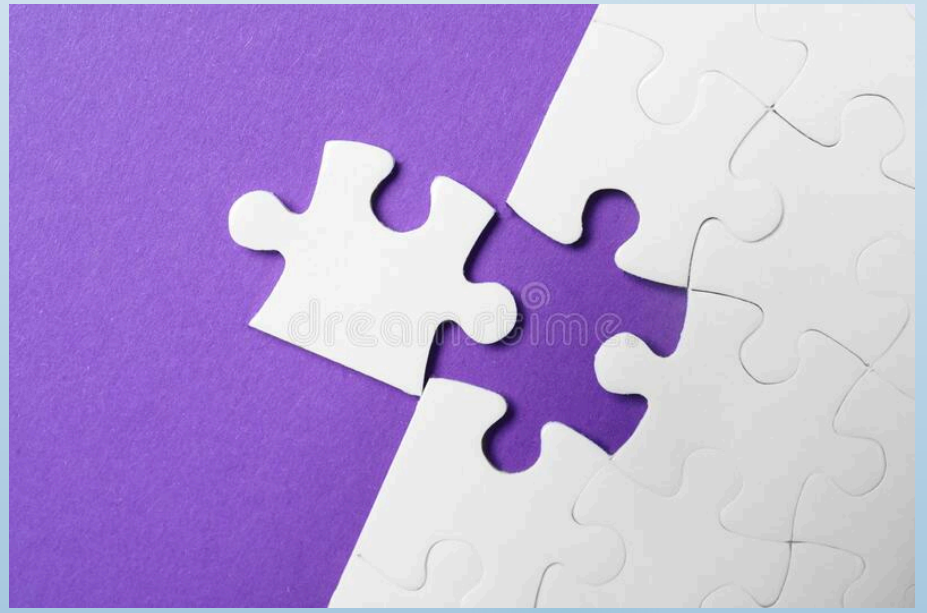
Agreement between all three analyses within errors

Soffer Bound: $|h_1^q| < \frac{1}{2}[f_1^q + g_1^q]$

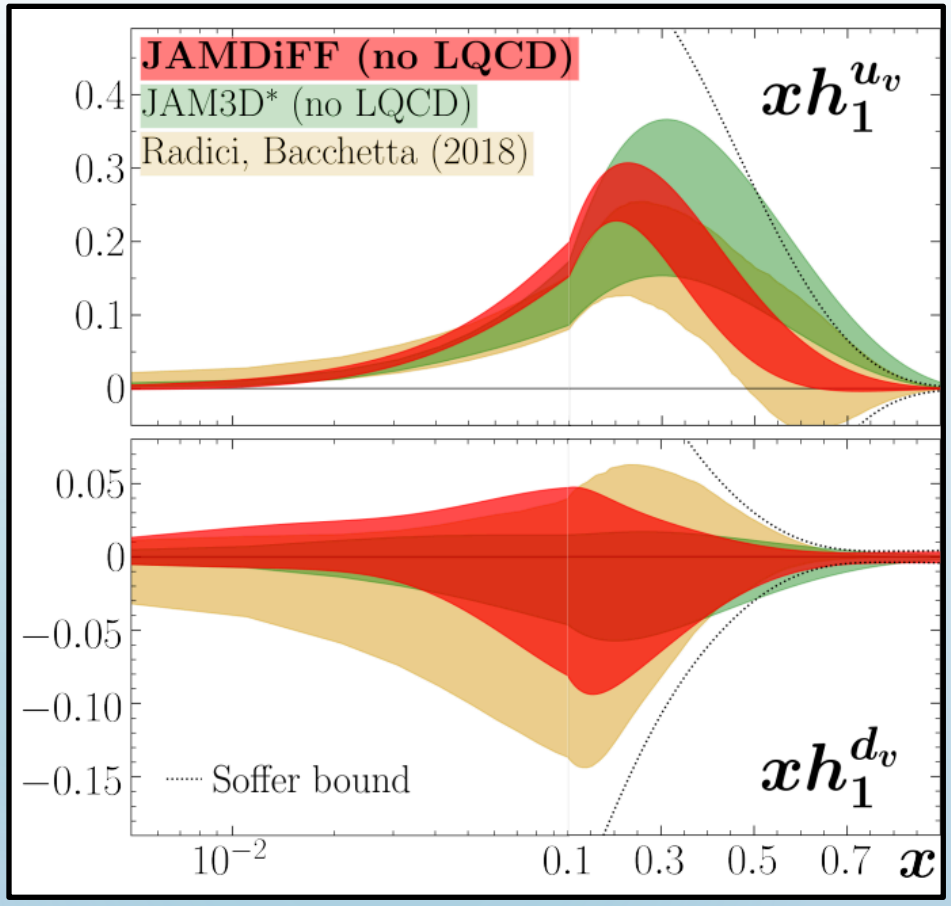
J. Soffer, Phys. Rev. Lett. **74**, 1292-1294 (1995)







Controlling Extrapolation

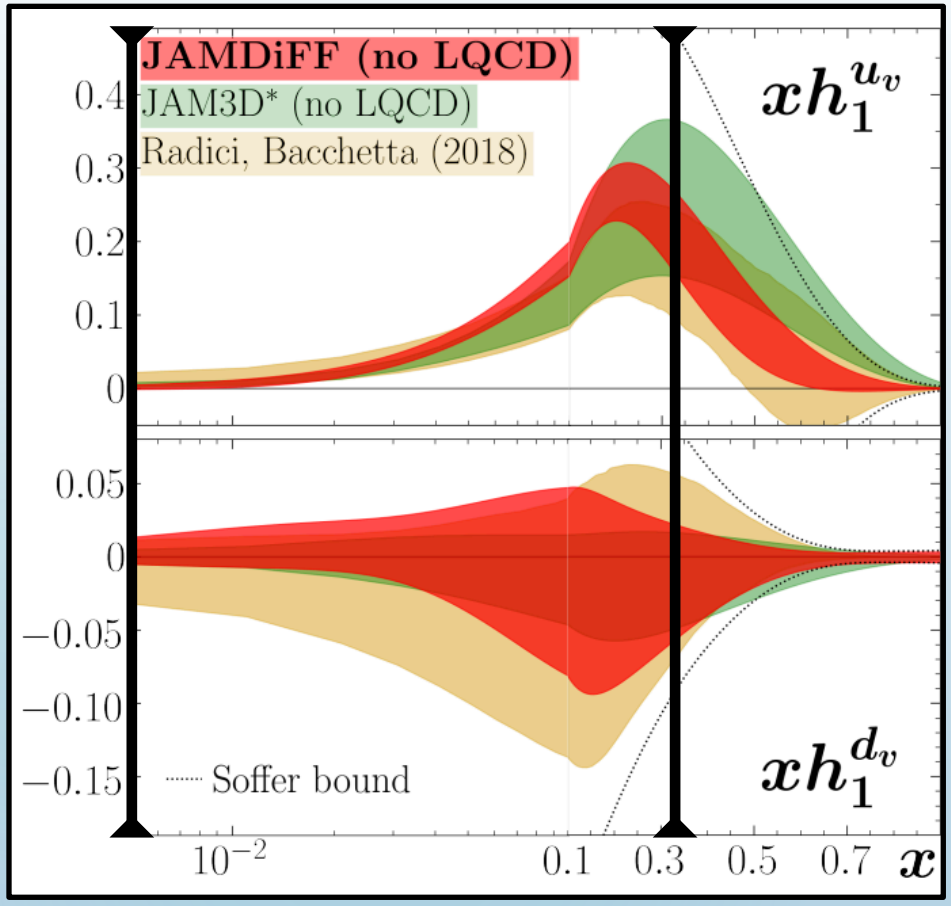


$$\delta u \equiv \int_0^1 \mathrm{d}x (h_1^u - h_1^{\bar{u}}),$$

$$\delta d \equiv \int_0^1 \mathrm{d}x (h_1^d - h_1^{\bar{d}}),$$

$$g_T \equiv \delta u - \delta d,$$

Controlling Extrapolation



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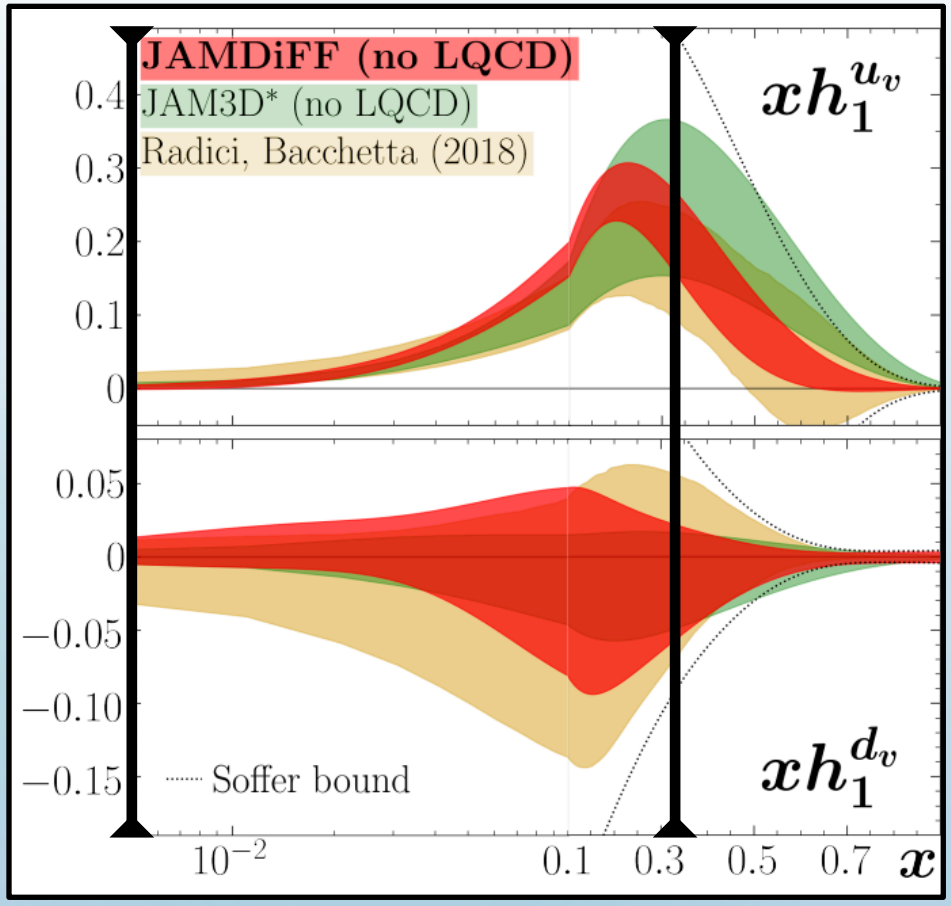
$$\delta d \equiv \int_0^1 \mathrm{d}x (h_1^d - h_1^{\bar{d}}),$$

$$g_T \equiv \delta u - \delta d,$$

Measured Region

Controlling Extrapolation



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$$\delta d \equiv \int_0^1 \mathrm{d}x (h_1^d - h_1^{\bar{d}}),$$

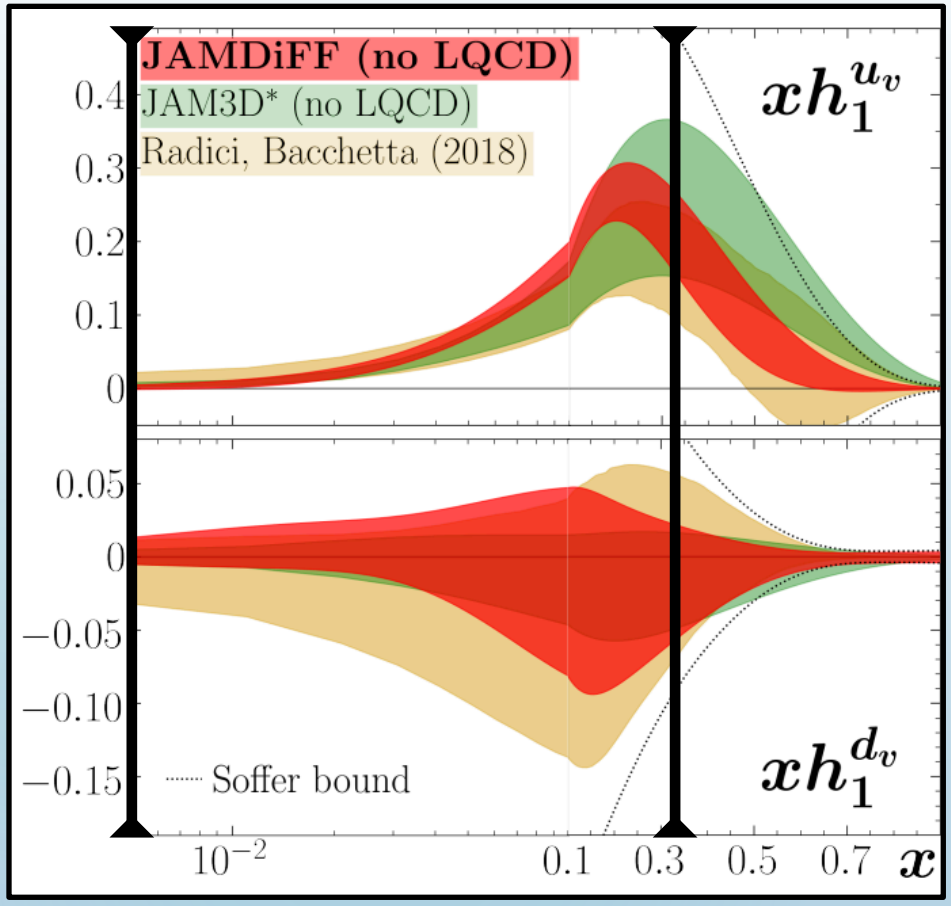
$$g_T \equiv \delta u - \delta d,$$

Large $x \gtrsim 0.3$

Soffer Bound: $|h_1^q| < \frac{1}{2}[f_1^q + g_1^q]$

J. Soffer, Phys. Rev. Lett. **74**, 1292-1294 (1995)

Controlling Extrapolation



Measured Region

$$\delta u \equiv \int_0^1 \mathrm{d}x (h_1^u - h_1^{\bar{u}}),$$

$$\delta d \equiv \int_0^1 \mathrm{d}x (h_1^d - h_1^{\bar{d}}),$$

$$g_T \equiv \delta u - \delta d,$$

Large $x \gtrsim 0.3$

Soffer Bound: $|h_1^q| < \frac{1}{2}\left[f_1^q + g_1^q\right]$

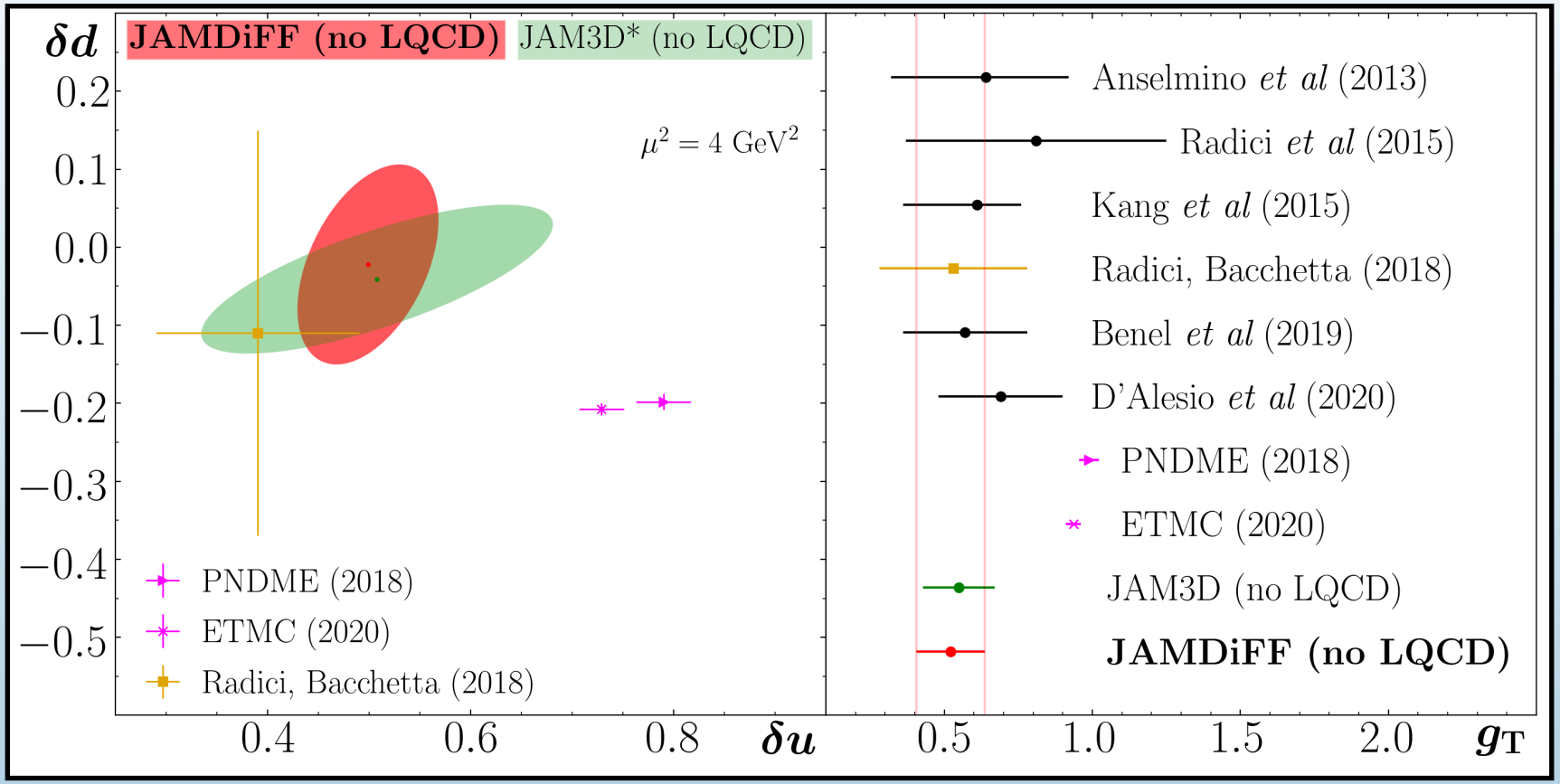
J. Soffer, Phys. Rev. Lett. **74**, 1292-1294 (1995)

Small $x \lesssim 0.005$

$$h_1^q \underset{x\to 0}{\longrightarrow} x^{\alpha_q} \qquad \alpha_q = 1 - 2\sqrt{\frac{\alpha_s N_c}{2\pi}} \approx 0.17 \pm 0.085$$

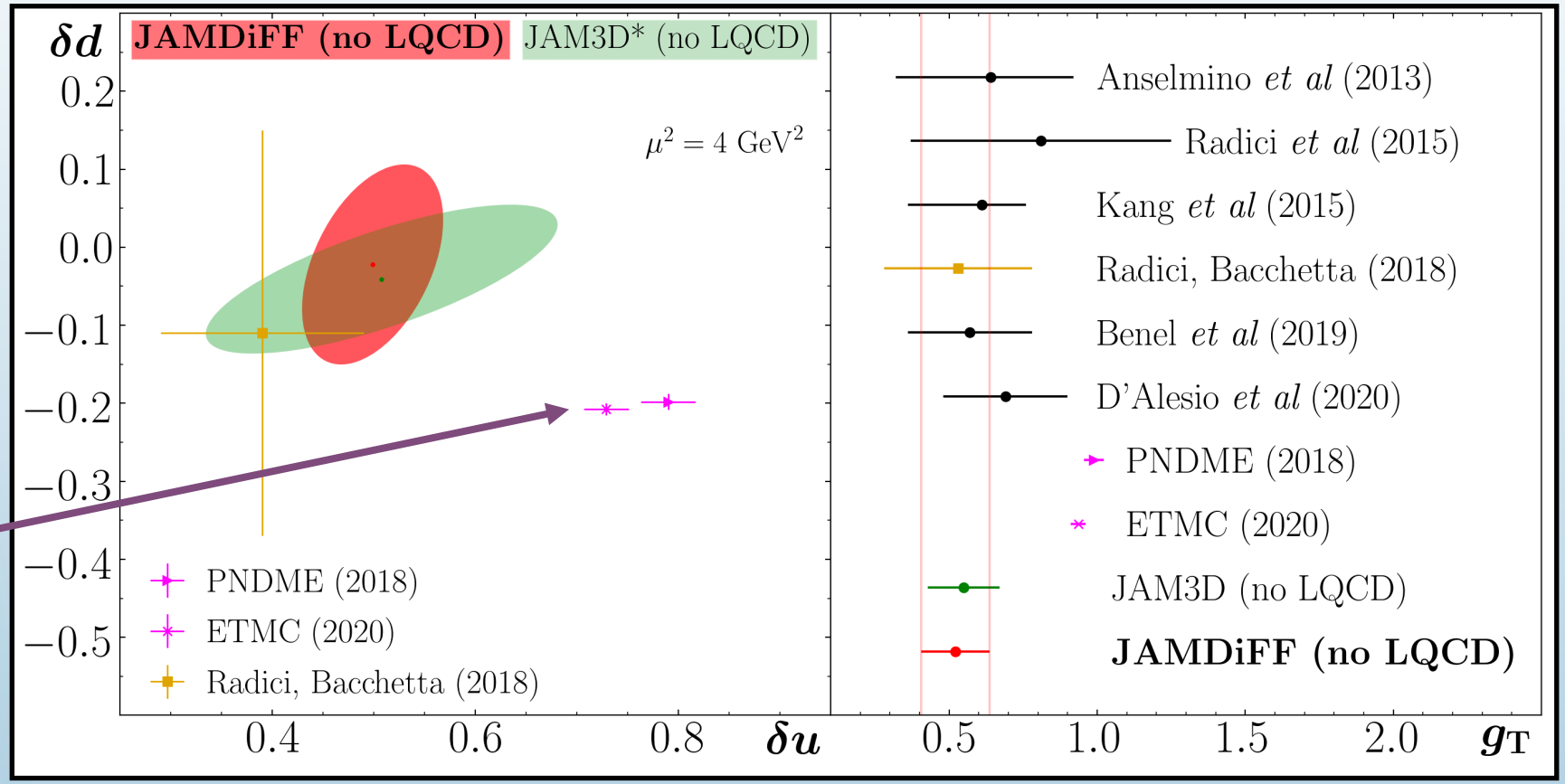
Y. V. Kovchegov and M. D. Sievert, Phys. Rev. D **99**, 054033 (2019)

Tensor Charges



Tensor Charges

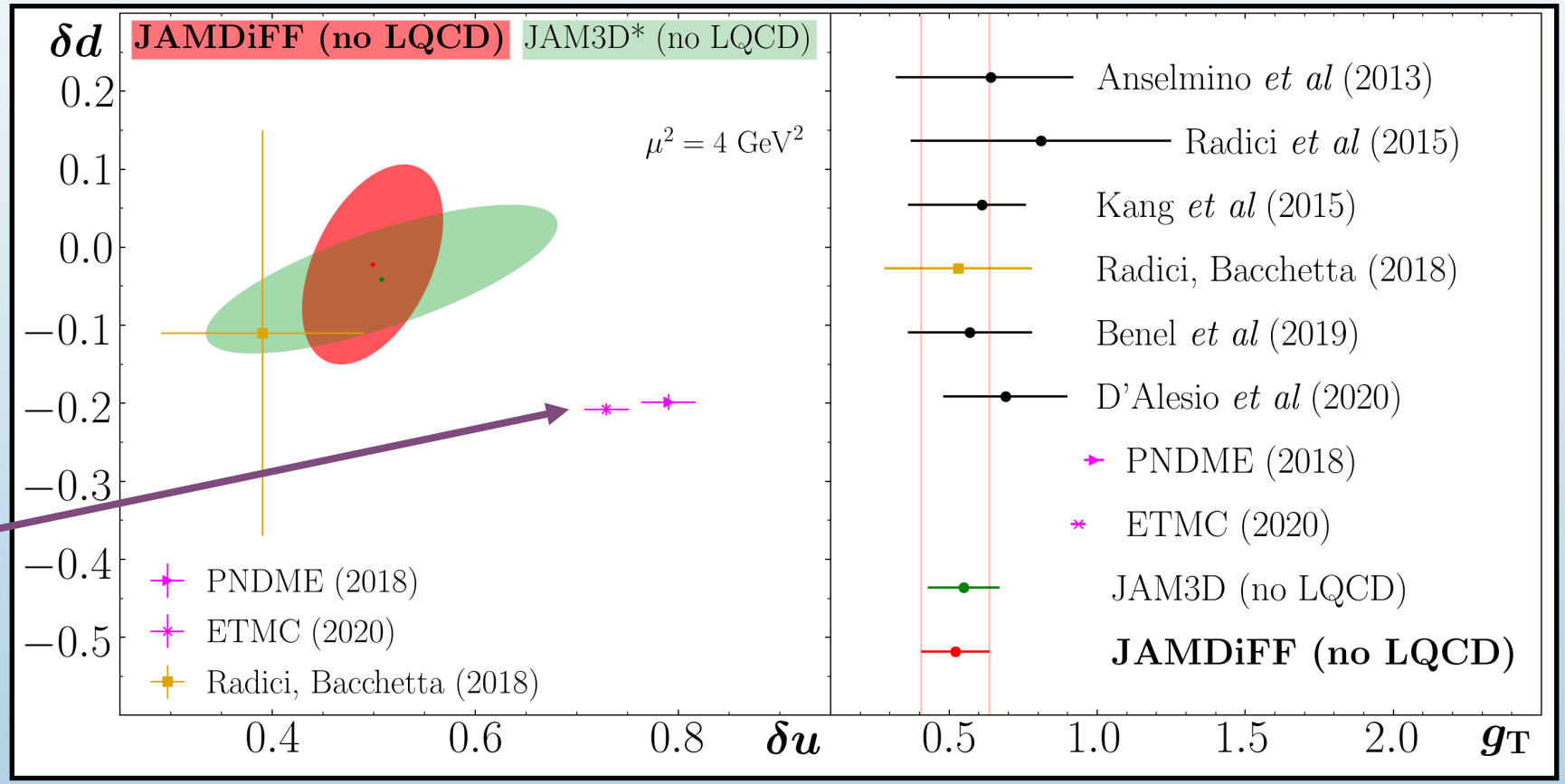
LQCD



R. Gupta *et al.*, Phys. Rev. D **98**, 091501 (2018)

C. Alexandrou *et al.*, Phys. Rev. D **102**, 054517 (2020)

Tensor Charges



LQCD

R. Gupta *et al.*, Phys. Rev. D **98**, 091501 (2018)

C. Alexandrou *et al.*, Phys. Rev. D **102**, 054517 (2020)

Consistent with RB18 and JAM3D* (no LQCD).
What happens if we include LQCD in the fit?

Quality of Fit

Experiment	N_{dat}	χ^2_{red} w/ LQCD	χ^2_{red} no LQCD
Belle (cross section) [63]	1094	1.01	1.01
Belle (Artru-Collins) [92]	183	0.74	0.73
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ETMC δu [28]	1	0.71	—
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PNDME δd [25]	1	0.04	—
Total χ^2_{red} (N_{dat})		**1.01** (1475)	**0.98** (1471)

Quality of Fit

Physical Pion Mass

$N_f = 2 + 1 + 1$

Use δu and δd instead of g_T

Experiment	N_{dat}	χ^2_{red} w/ LQCD	χ^2_{red} no LQCD
Belle (cross section) [63]	1094	1.01	1.01
Belle (Artru-Collins) [92]	183	0.74	0.73
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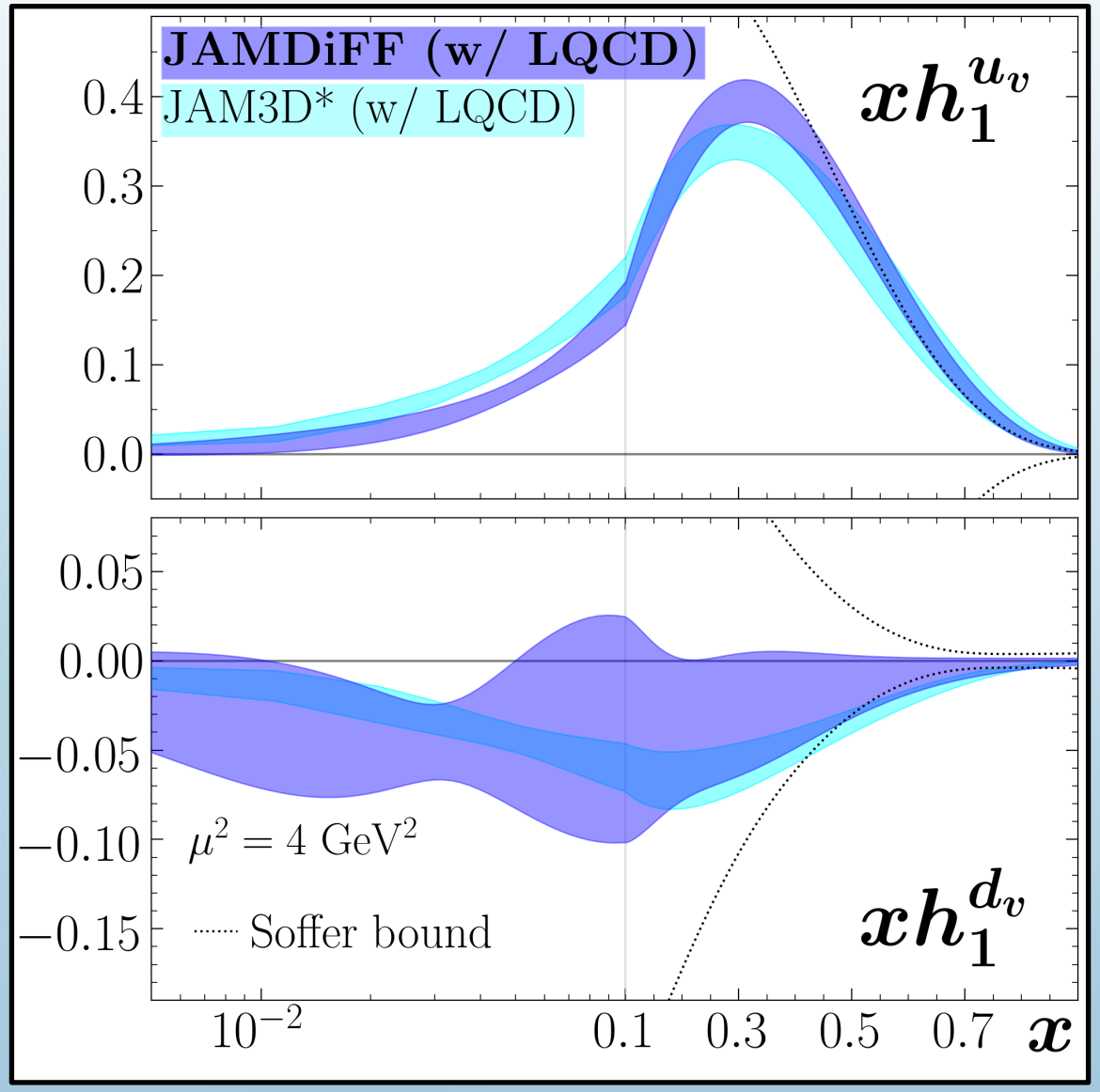
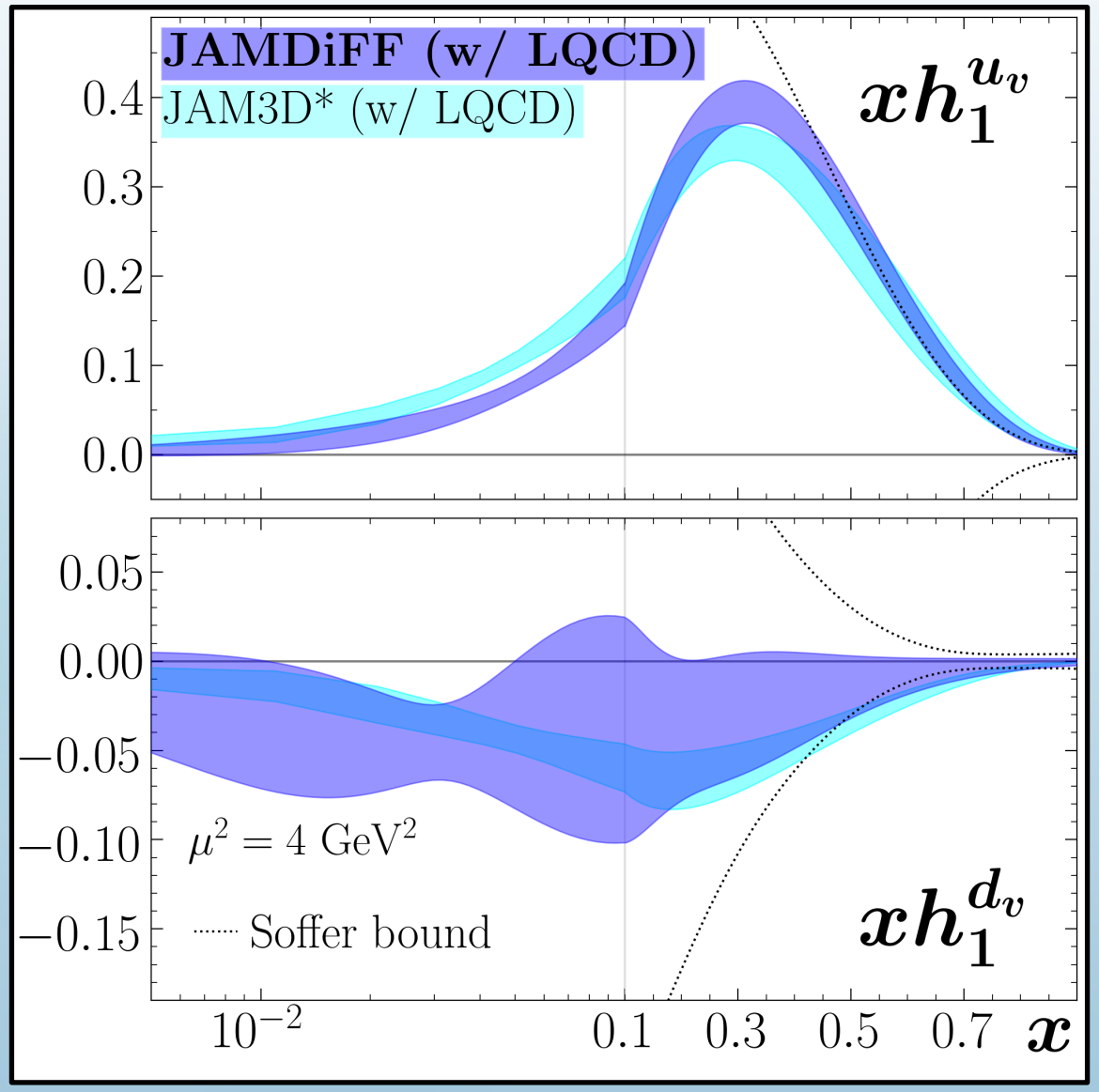
Transversity PDFs (w/ LQCD)

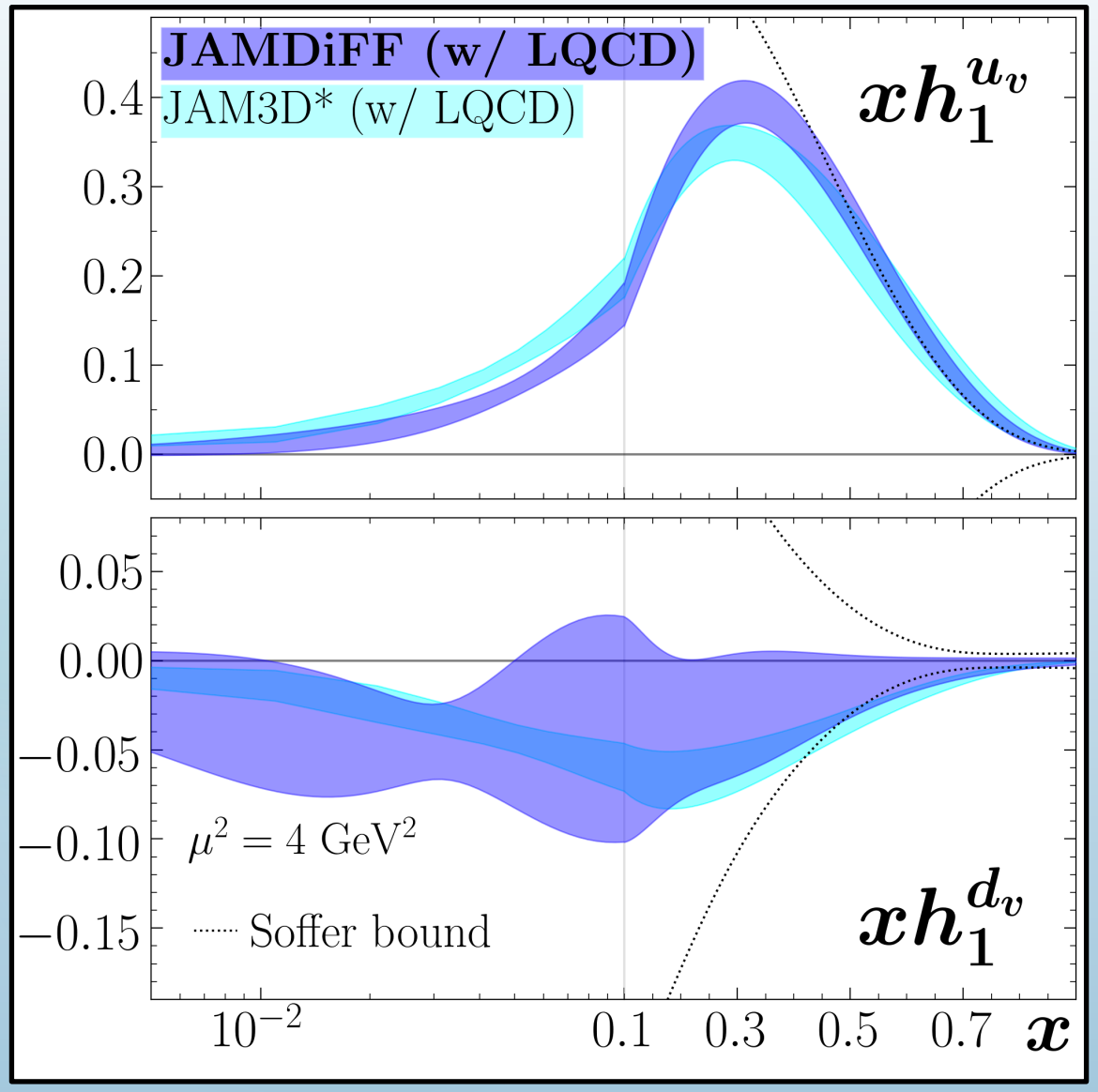


Transversity PDFs (w/ LQCD)



JAM3D* = JAM3D-22 (w/ LQCD)
+ Antiquarks w/ $\bar{u} = -\bar{d}$
+ small-x constraint (see slide 27)
+ $\delta u, \delta d$ from ETMC & PNDME
(instead of g_T from ETMC)

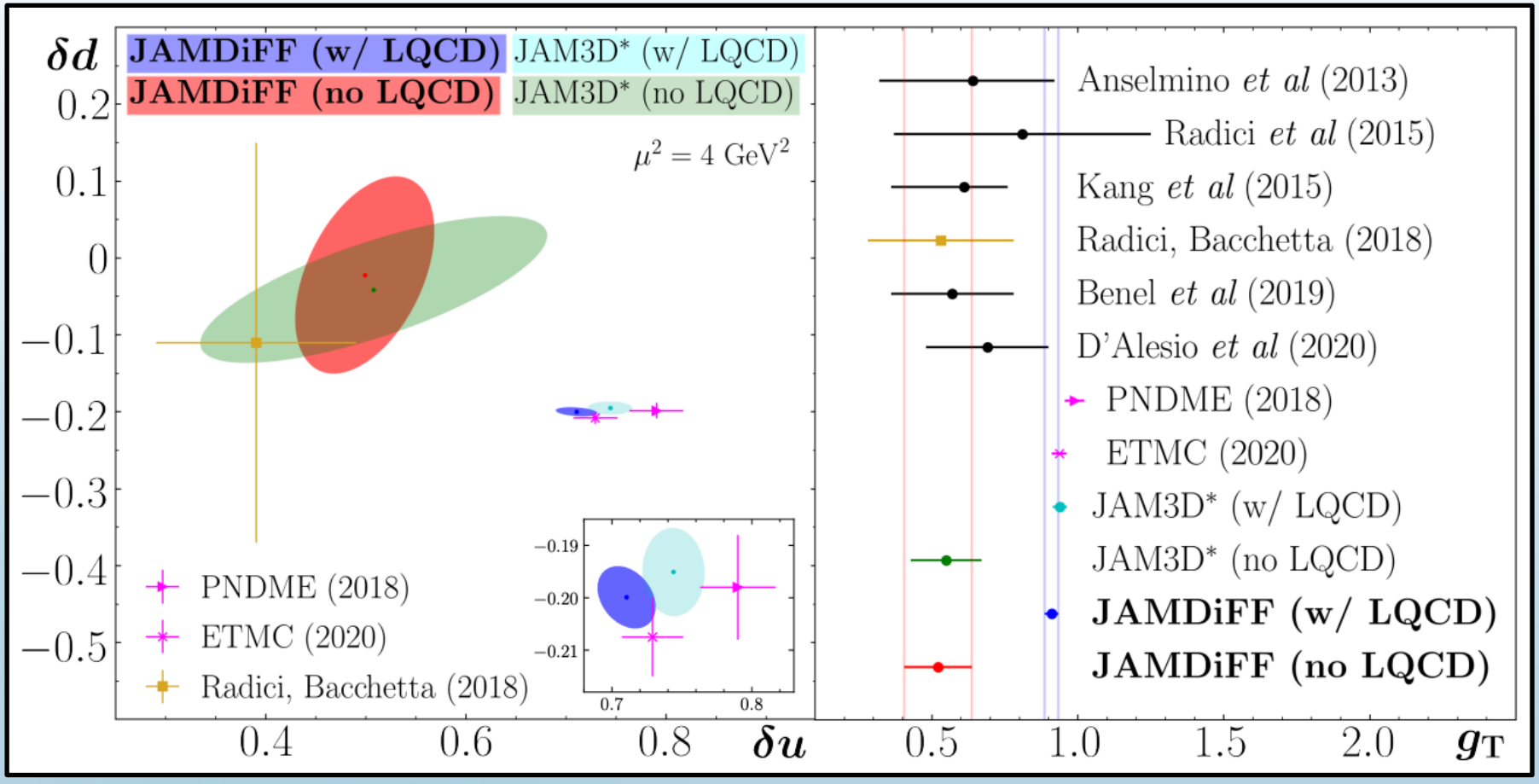
Transversity PDFs (w/ LQCD)



JAM3D* = JAM3D-22 (w/ LQCD)
+ Antiquarks w/ $\bar{u} = -\bar{d}$
+ small-x constraint (see slide 27)
+ $\delta u, \delta d$ from ETMC & PNDME
(instead of g_T from ETMC)

JAMDiFF (w/ LQCD) and JAM3D* (w/ LQCD) largely agree

Tensor Charges (w/ LQCD)



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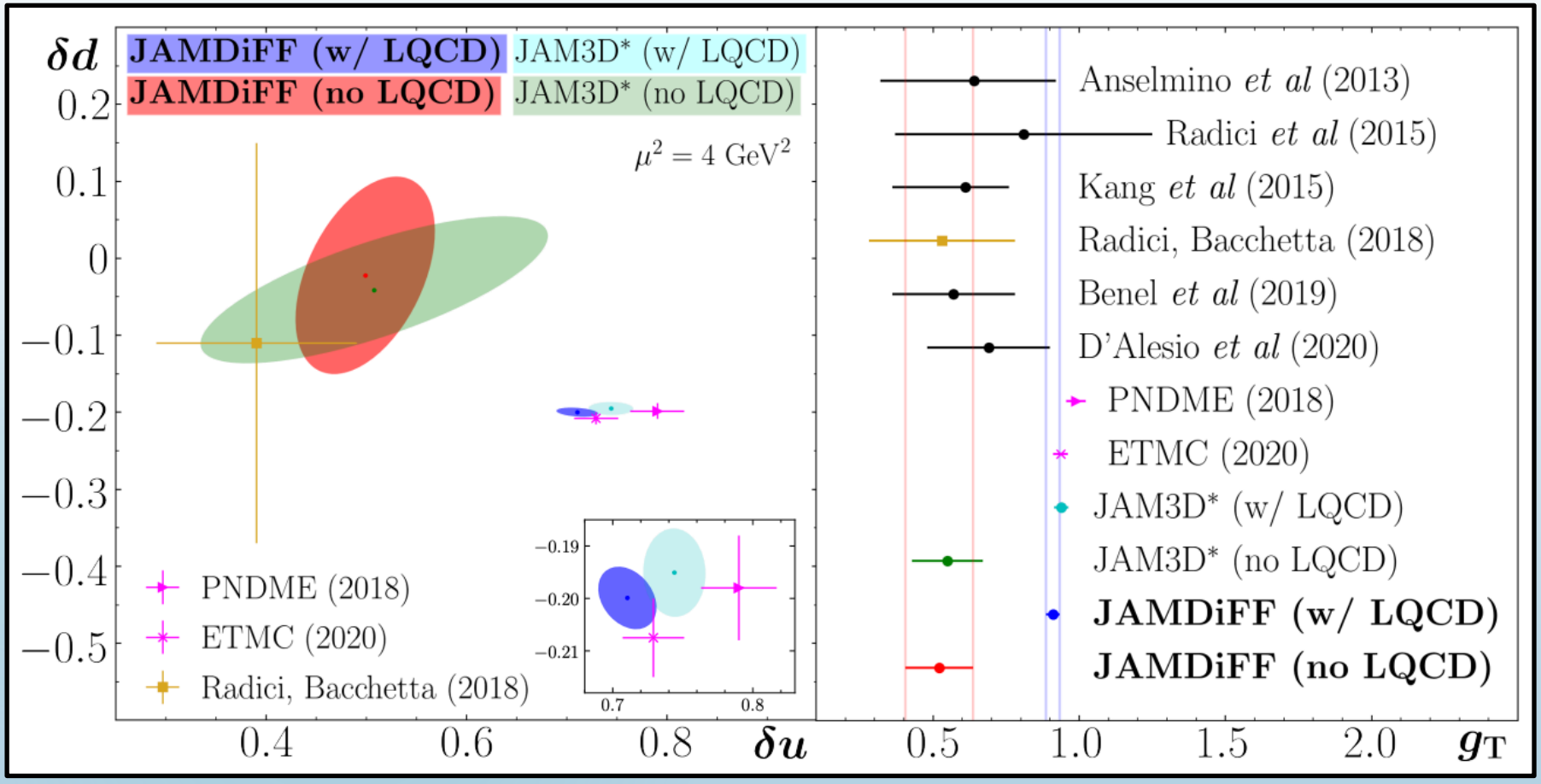
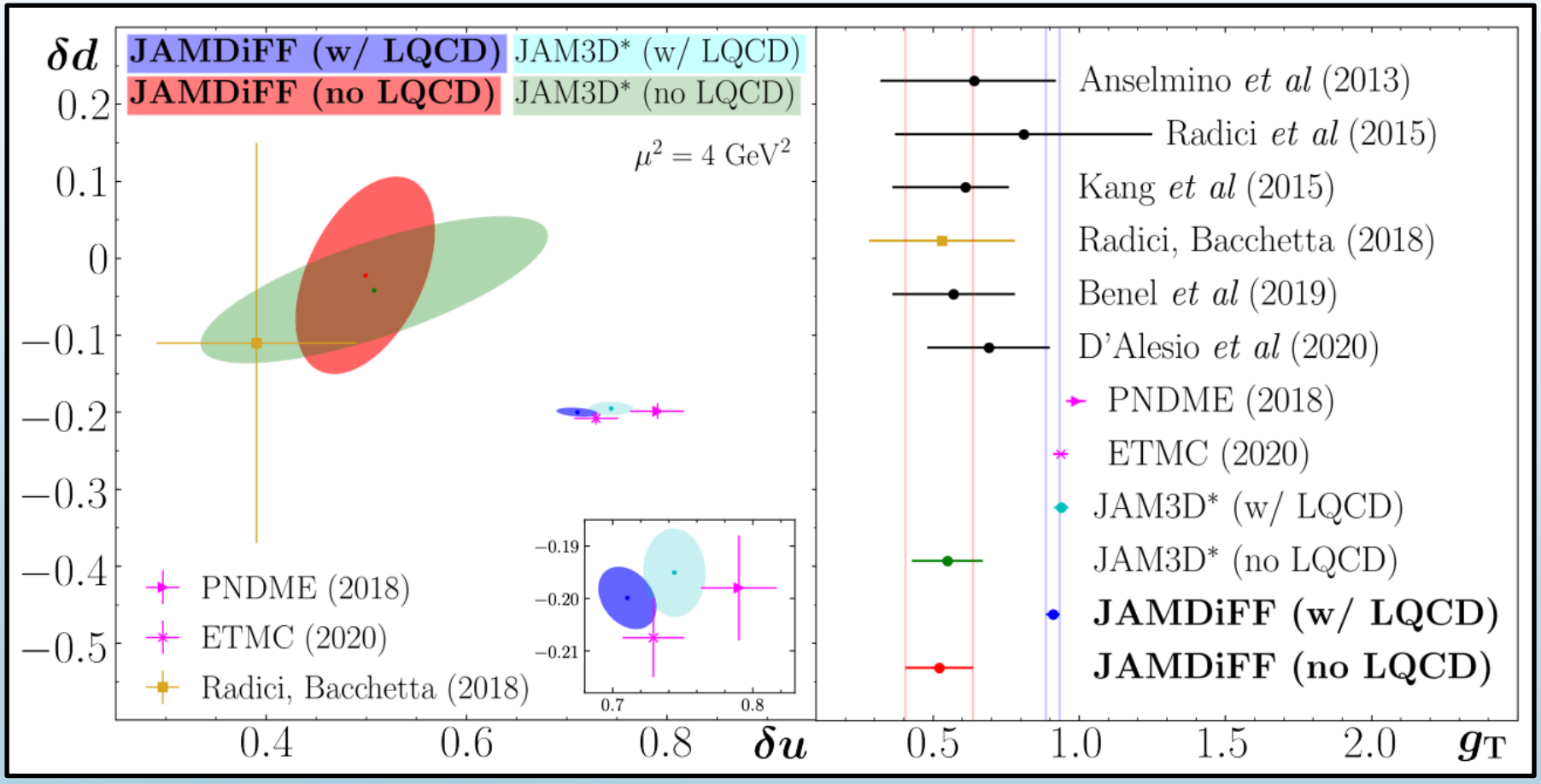


Noticeable shift from including lattice data

Tensor Charges (w/ LQCD)



Likelihood function $\mathscr{L} = \exp(-\chi^2/2)$ does not guarantee that errors overlap when using Monte Carlo method

Noticeable shift from including lattice data

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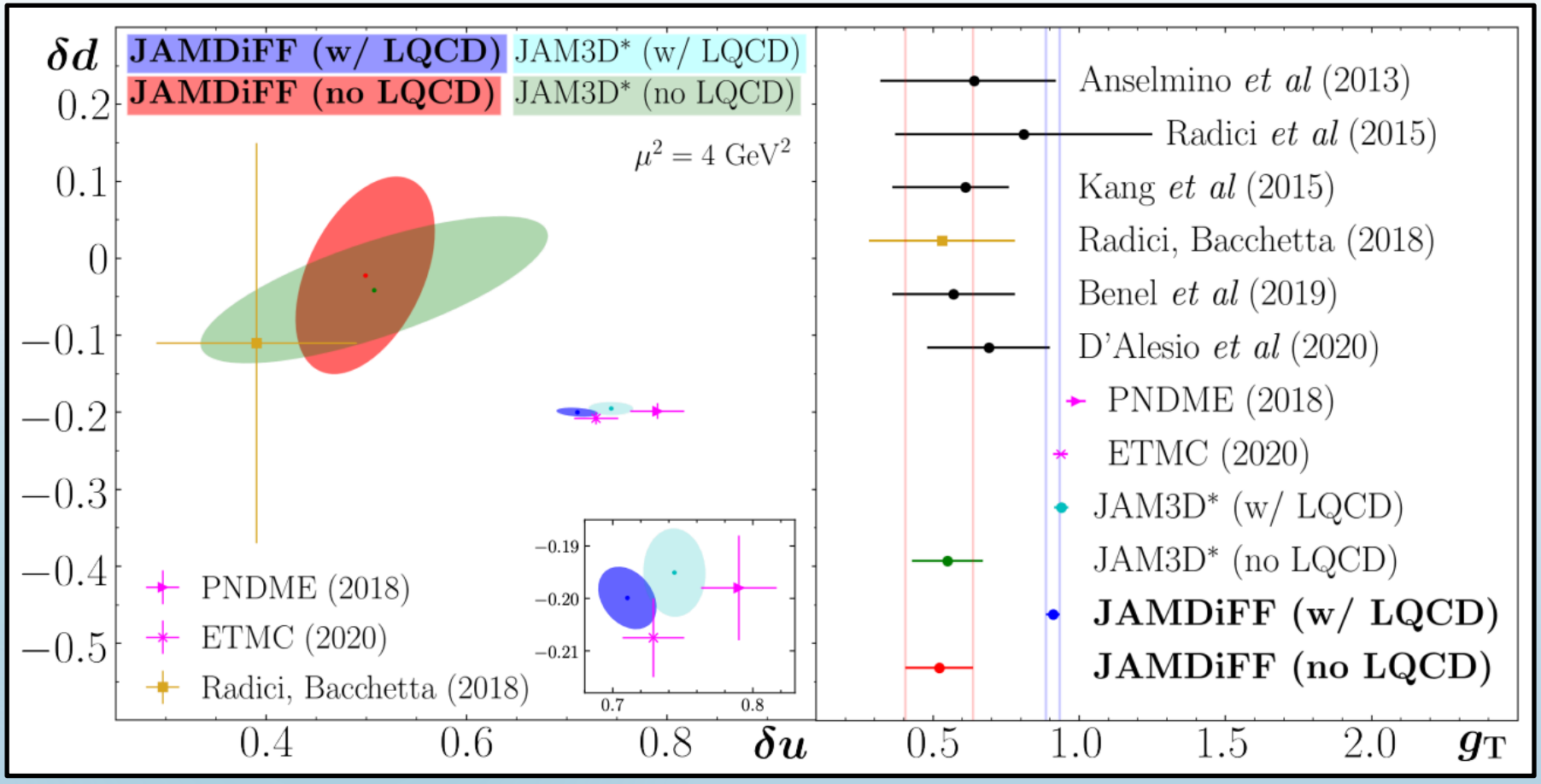

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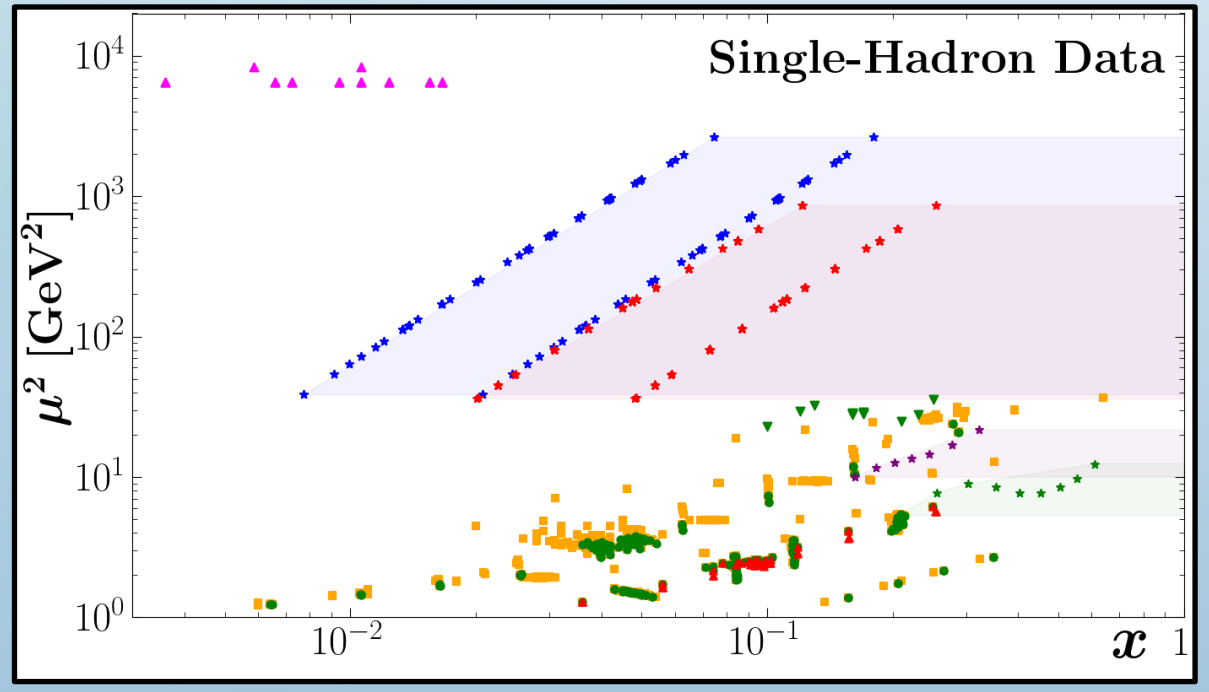
M.N. Constantini *et al.*, JHEP **12**, 064 (2024)

N.T. Hunt-Smith *et al.*, Comput. Phys. Commun. **296**, 109059 (2024)

N. T. Hunt-Smith *et al.*, Phys. Rev. D **106**, 036003 (2022)

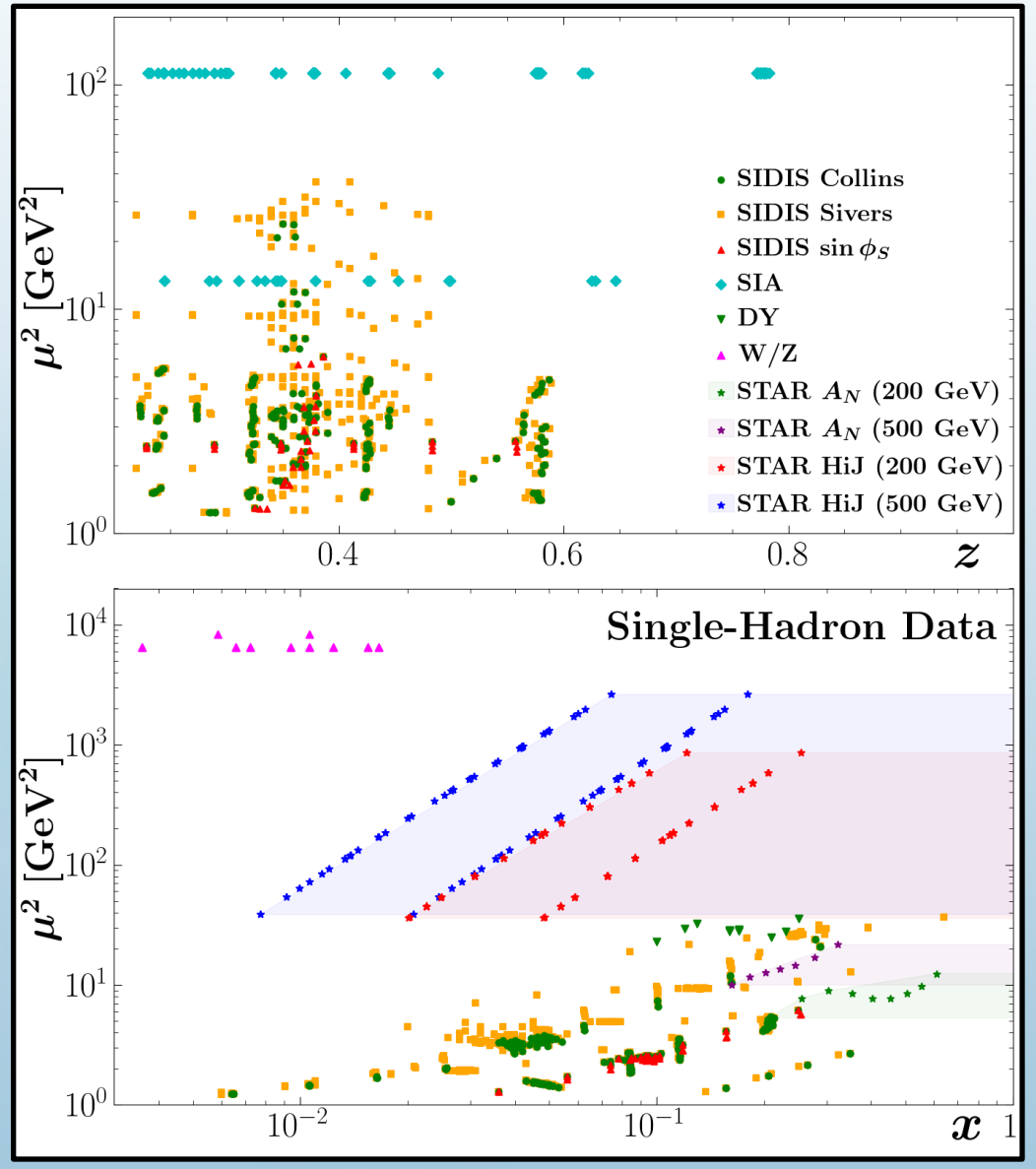
Noticeable shift from including lattice data

Currently looking into Markov Chain Monte Carlo to better assess uncertainties.



JAM3D +
JAM**DiFF**
=
JAM3**DiFF**

Kinematics and Functions



Process	Collaborations	Points
SIA	BaBaR, Belle, BESIII	176
SIDIS Asym.	COMPASS, HERMES	525
DY	COMPASS	15
W/Z	STAR	17
pp AN	STAR, AnDY	44
Hadron-in-jet	STAR	708

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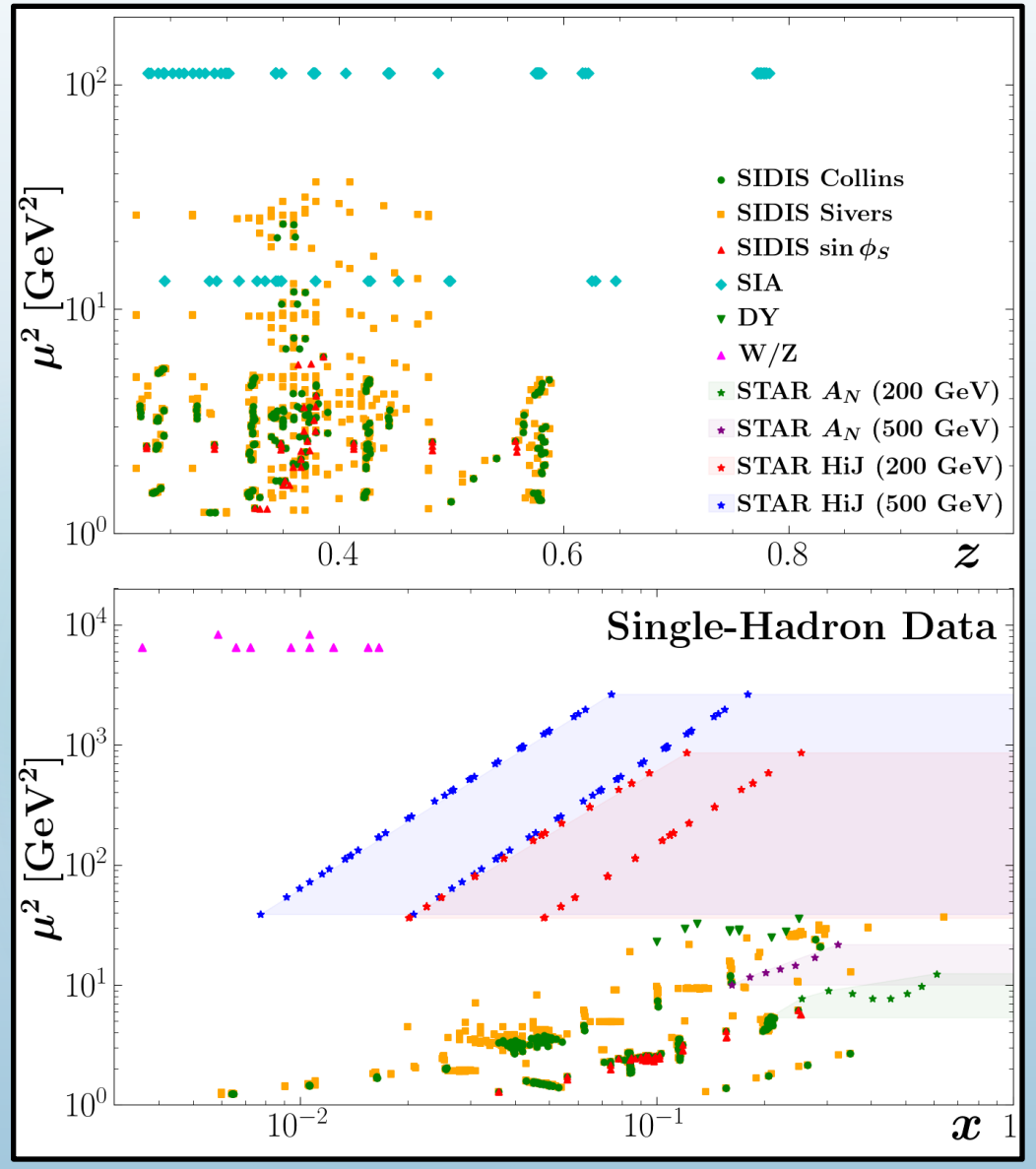

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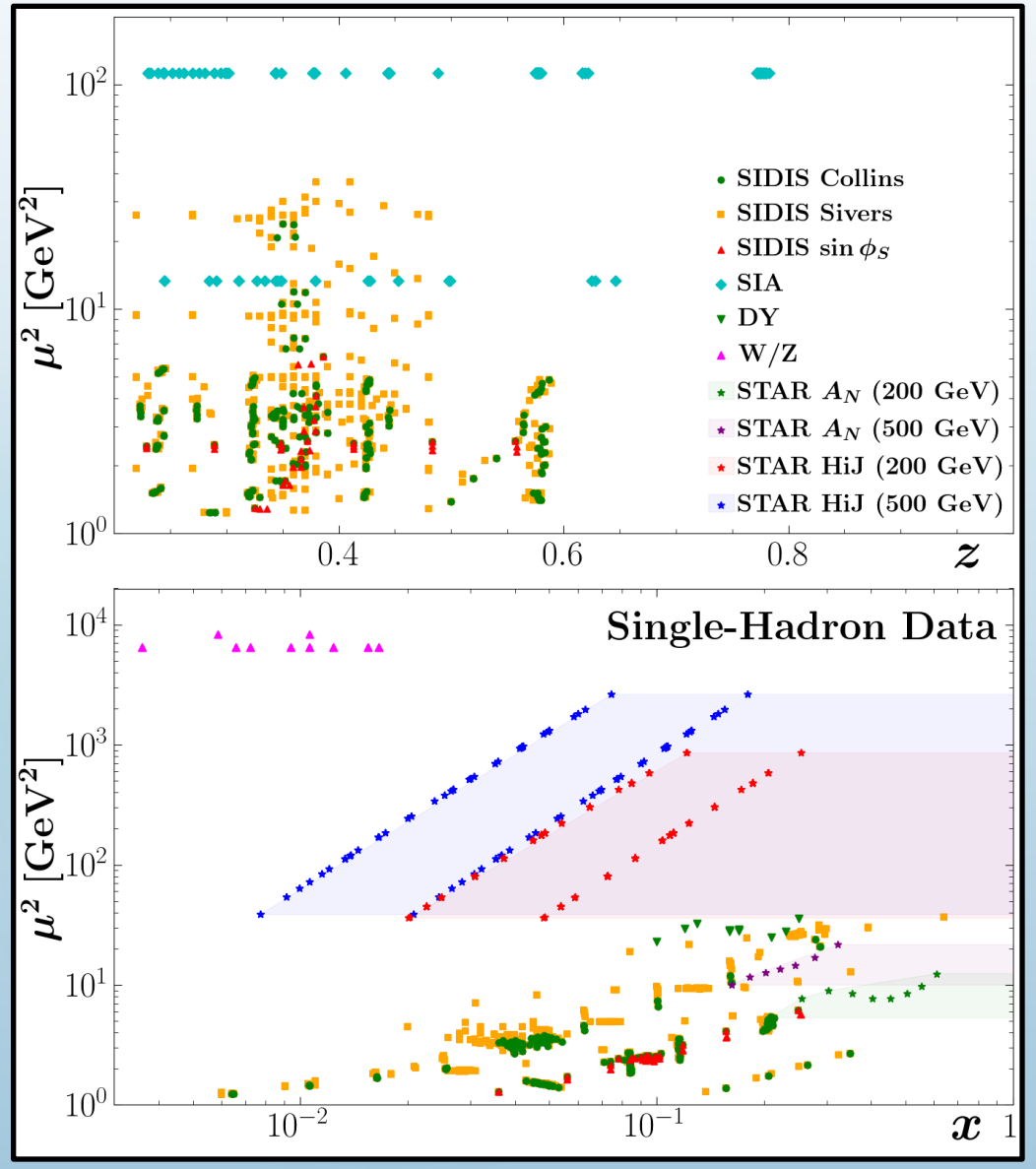
Transversity $h_1 : u, d, \bar{u}, \bar{d}$ + widths

Kinematics and Functions



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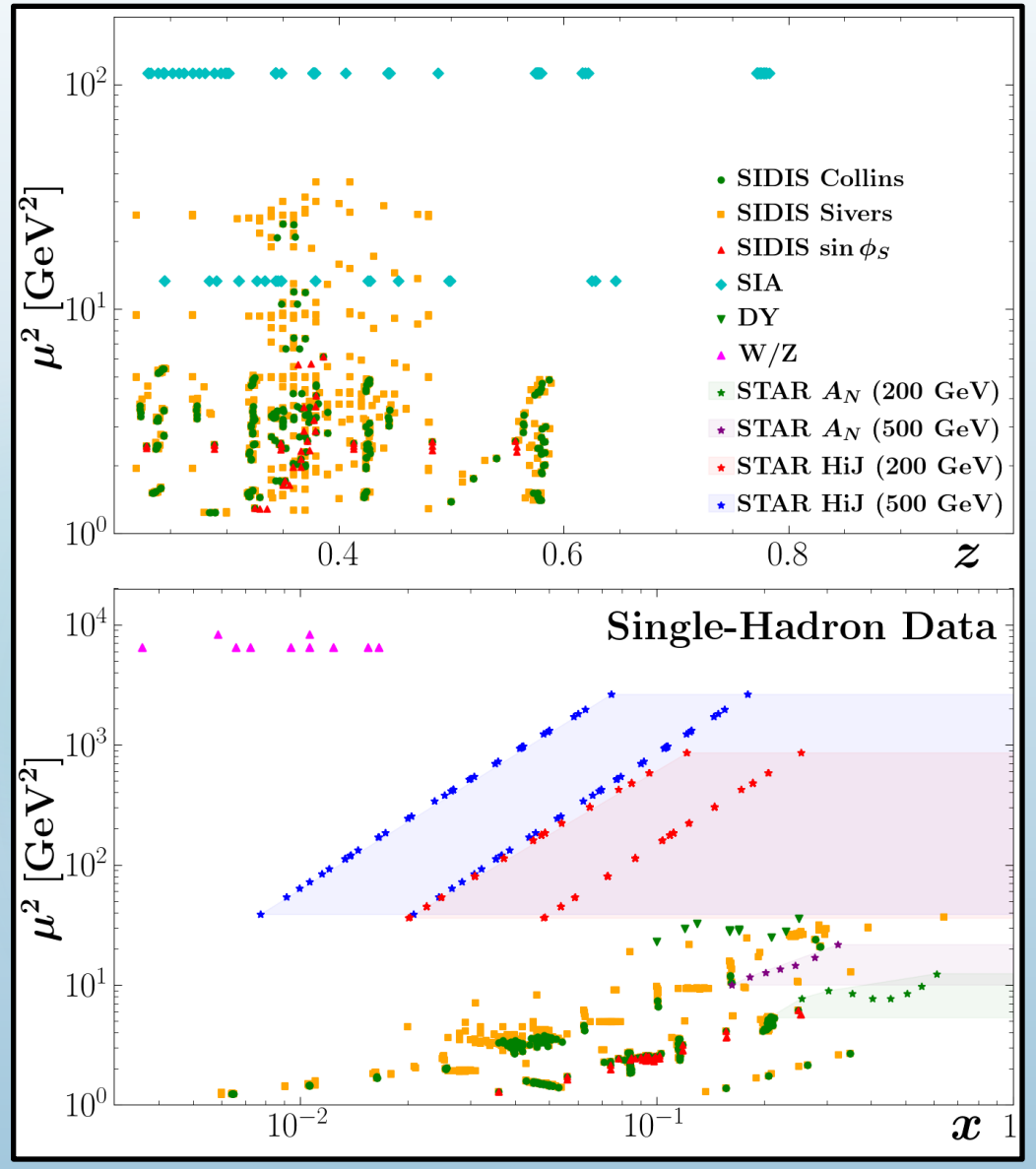
Sivers $f_{1T}^{\perp(1)} : u, d, \bar{u}, \bar{d}, s, \bar{s}$ + widths

Kinematics and Functions



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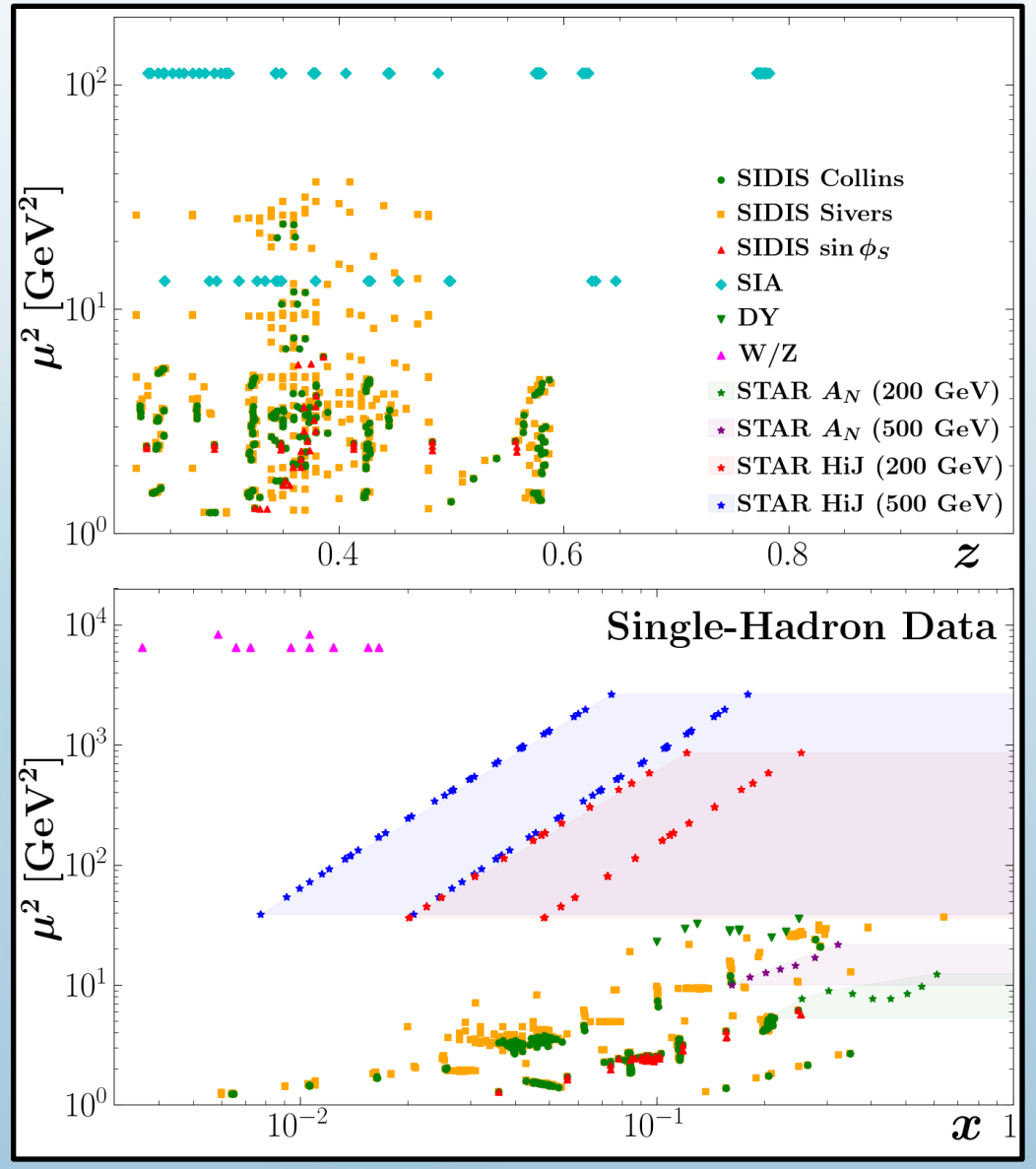
Collins (pion) $H_1^{\perp(1)}$: fav . , unfav. + widths

Kinematics and Functions



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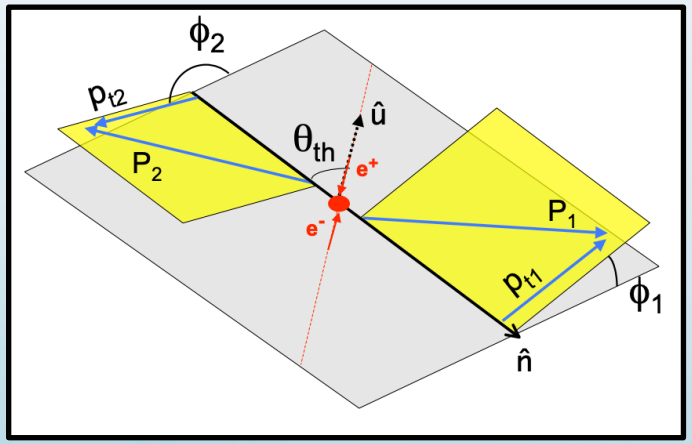
Collins (pion) $H_1^{\perp(1)}$: fav . , unfav. + widths

Twist-3 FF (pion) $\tilde{H}$: fav . , unfav .

3D/Twist-3 Observables

SIA



J.P. Lees *et al.*. Phys. Rev. D **90**. 052003 (2014)

$$F^{h_1 h_2}_{\cos 2\phi_0} \propto C\left[H_1^{\perp,h_1}\bar{H}_1^{\perp,h_2}\right]$$

3D/Twist-3 Observables

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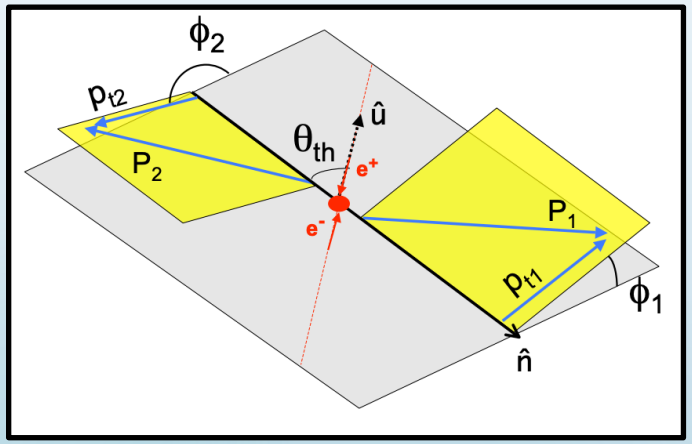

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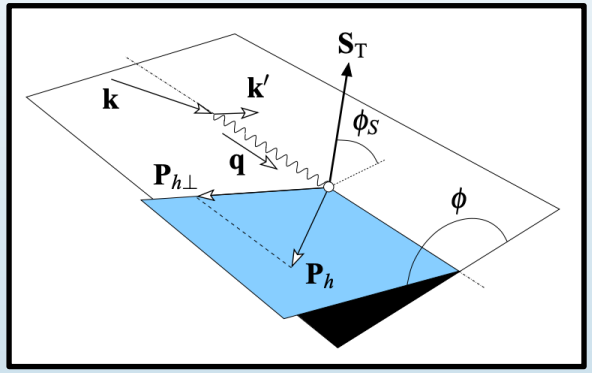


A. Airapetian *et al.*, JHEP **12**, 010 (2020)

$$F_{UU} = C\left[f_1 D_1\right] \quad \text{(constrains widths)}$$

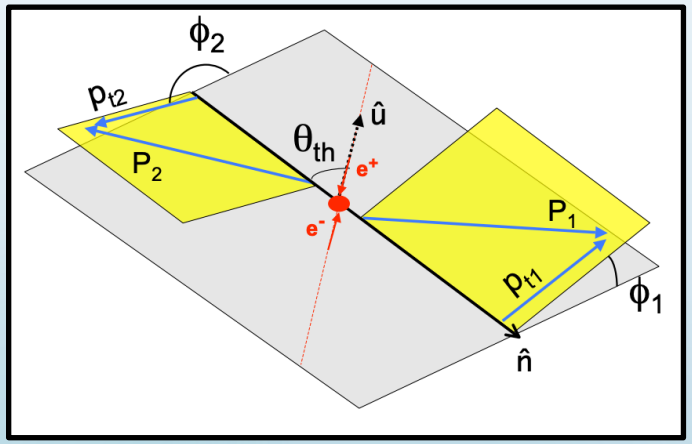
$$F^{\sin(\phi_h-\phi_s)}_{UT} \propto C\left[f_{1T}^{\perp} D_1\right]$$

$$F^{\sin(\phi_h+\phi_s)}_{UT} \propto C\left[h_1 H_1^{\perp}\right]$$

$$F^{\sin\phi_S}_{UT} \propto C\left[-\frac{M_h}{zM} h_1 \tilde{H} + \ldots\right]$$

3D/Twist-3 Observables

SIA



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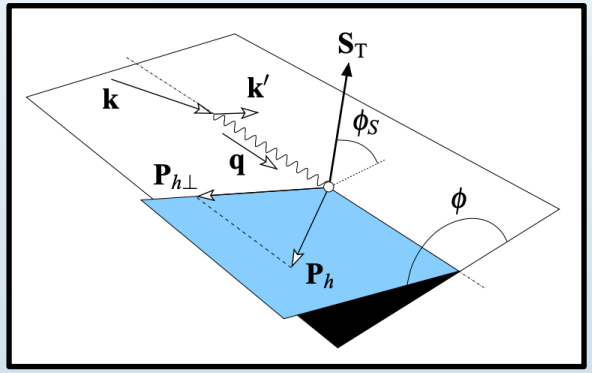

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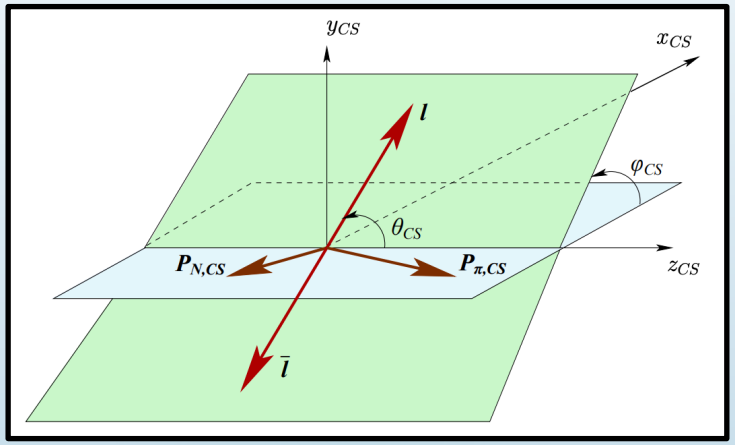
Drell-Yan



M. Aghasyan *et al.*, Phys. Rev. Lett. **119**, 112002 (2017)

$$F^{1}_{UT} \propto C\left[f_1^{\pi}\bar{f}_{1T}^{\perp}\right]$$

Pion PDF

3D/Twist-3 Observables

pp W/Z production

$$F_{UT}^{1} \propto C\left[f_{1T}^{\perp}\bar{f}_{1}\right]$$

3D/Twist-3 Observables

pp W/Z production

$$F_{UT}^{1} \propto C\left[f_{1T}^{\perp}\bar{f}_{1}\right]$$

pp A_N

$$d\Delta\sigma \propto \int_{z_{\min}}^{1}\frac{dz}{z^3}\int_{x_{\min}}^{1}\frac{dx}{xx'}\frac{1}{xS+U/z}f_1\left[M_h h_1 \mathscr{H}+\frac{M}{\hat{u}}D_1\mathscr{F}\right]$$

$$\mathscr{H}=\left[H_1^{\perp(1)}-z\frac{dH_1^{\perp(1)}}{dz}\right]\tilde{S}_{H_1^{\perp}}+\left[-2H_1^{\perp(1)}+\frac{1}{z}\tilde{H}\right]\tilde{s}_H$$

$$\mathscr{F}=\left[f_{1T}^{\perp(1)}-z\frac{df_{1T}^{\perp(1)}}{dz}\right]S_{F_{FT}}$$

3D/Twist-3 Observables

pp W/Z production

$$F_{UT}^{1} \propto C\left[f_{1T}^{\perp}\bar{f}_1\right]$$

pp hadron-in-jet

$$F_{UT}^{\sin(\phi_S-\phi_h)} \propto \int_{x_{\min}}^{1} \frac{dx}{xx'} \frac{1}{xS+U} \frac{2j_\perp z M_h}{\pi(\langle p_\perp^2\rangle_{H_1^\perp})^2} e^{-\frac{j_\perp^2}{\langle p_\perp^2\rangle_{H_1^\perp}}} f_1 h_1 H_1^{\perp(1)}$$

pp A_N

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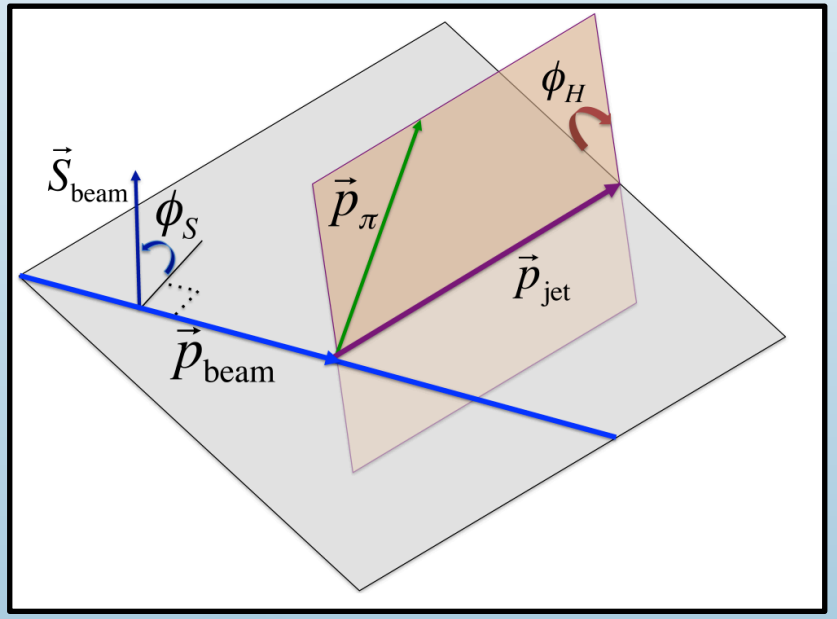


L. Adamczyk *et al.*, Phs. Rev. D **97**, 032004 (2018)

Previous JAM3D Analyses

Origin of single transverse-spin asymmetries in high-energy collisions

Jefferson Lab Angular Momentum Collaboration • Justin Cammarota (Coll. William and Mary and Lebanon Valley Coll.) Show All(8)

Feb 19, 2020

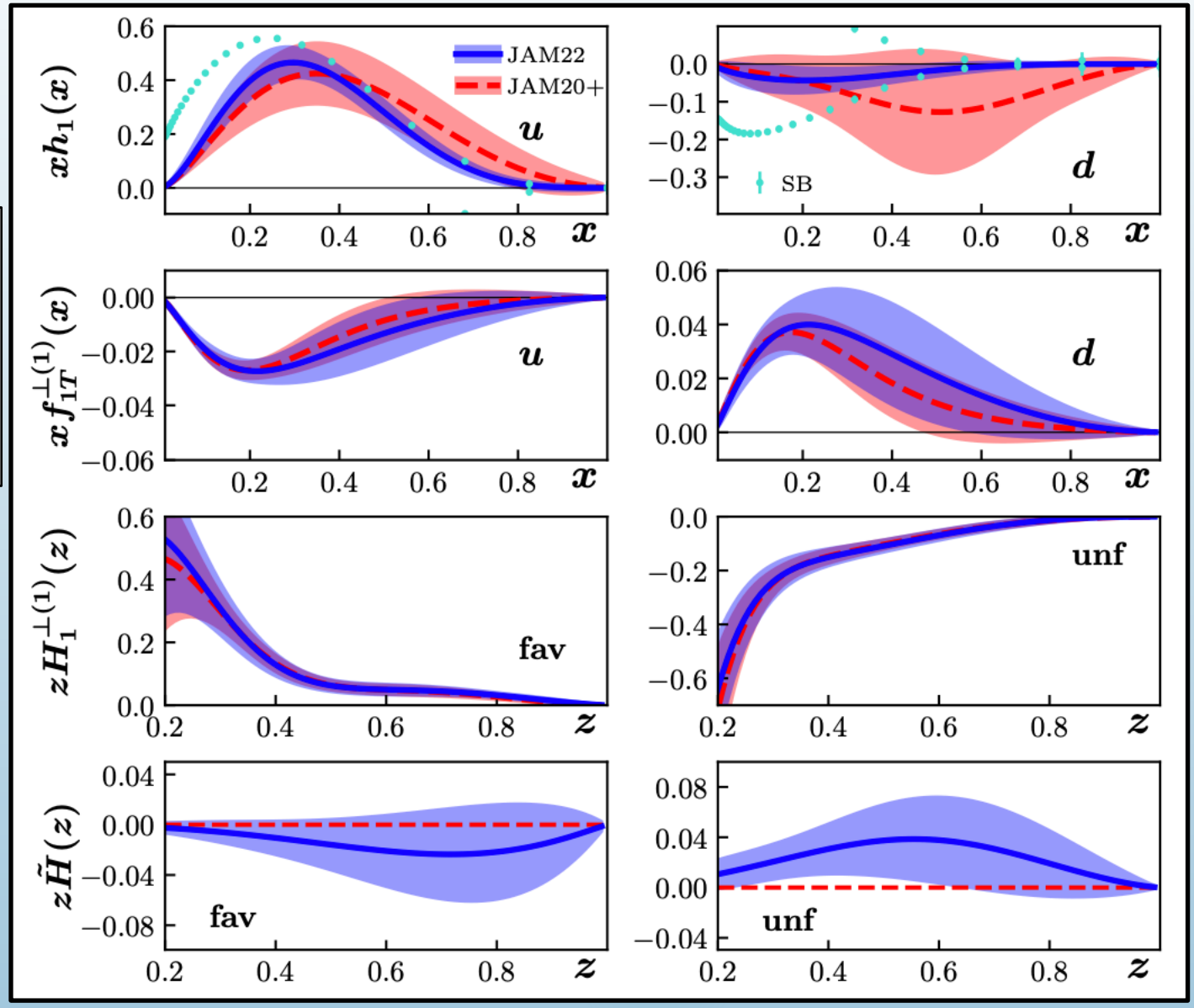
8 pages
Published in: *Phys.Rev.D* 102 (2020) 5, 054002
Published: Sep 8, 2020

Updated QCD global analysis of single transverse-spin asymmetries: Extracting $\tilde{H}$, and the role of the Soffer bound and lattice QCD

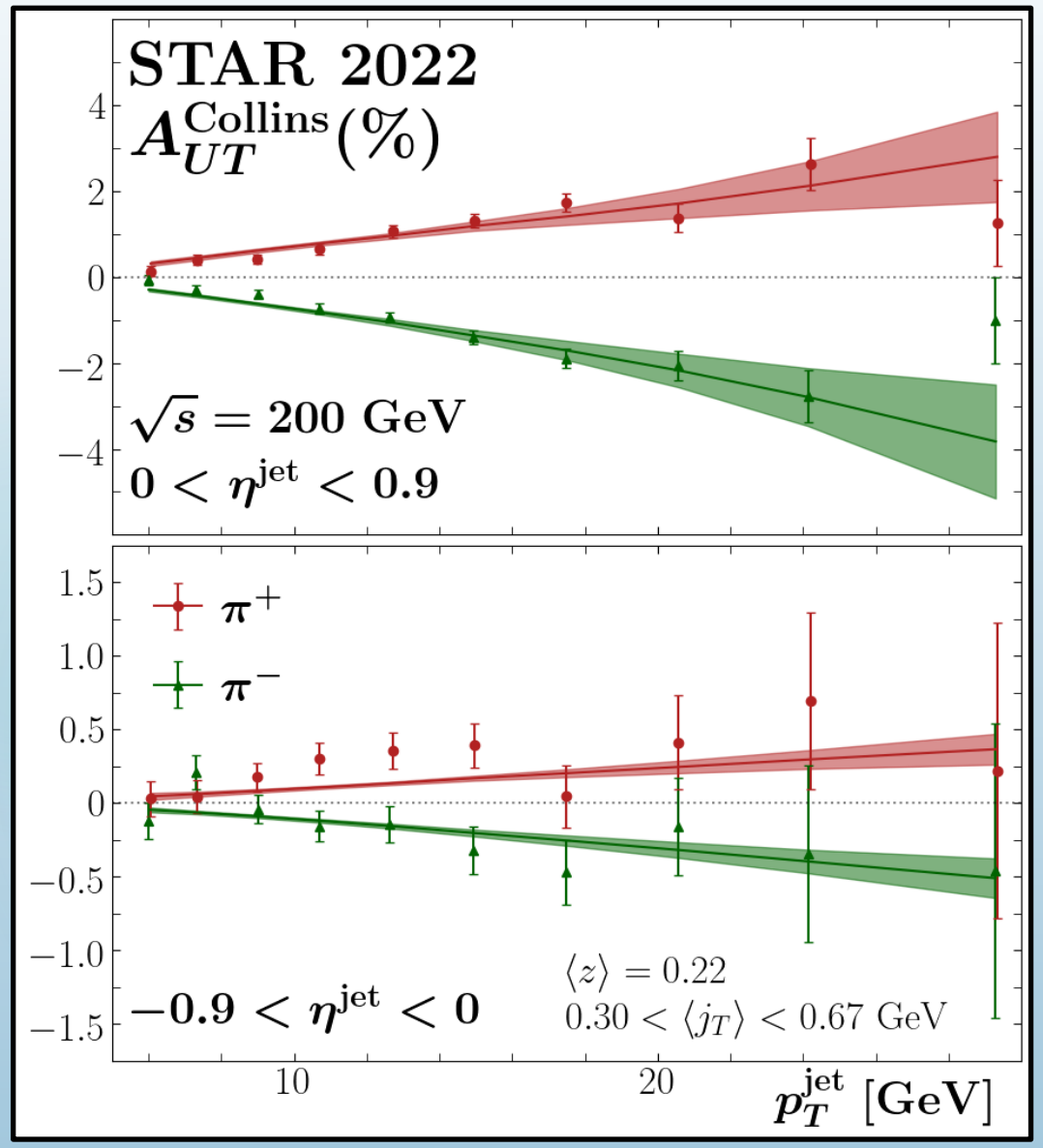
Jefferson Lab Angular Momentum (JAM) and Jefferson Lab Angular Momentum Collaborations • Leonard Gamberg (Penn State U., Berks-Lehigh Valley) Show All(6)

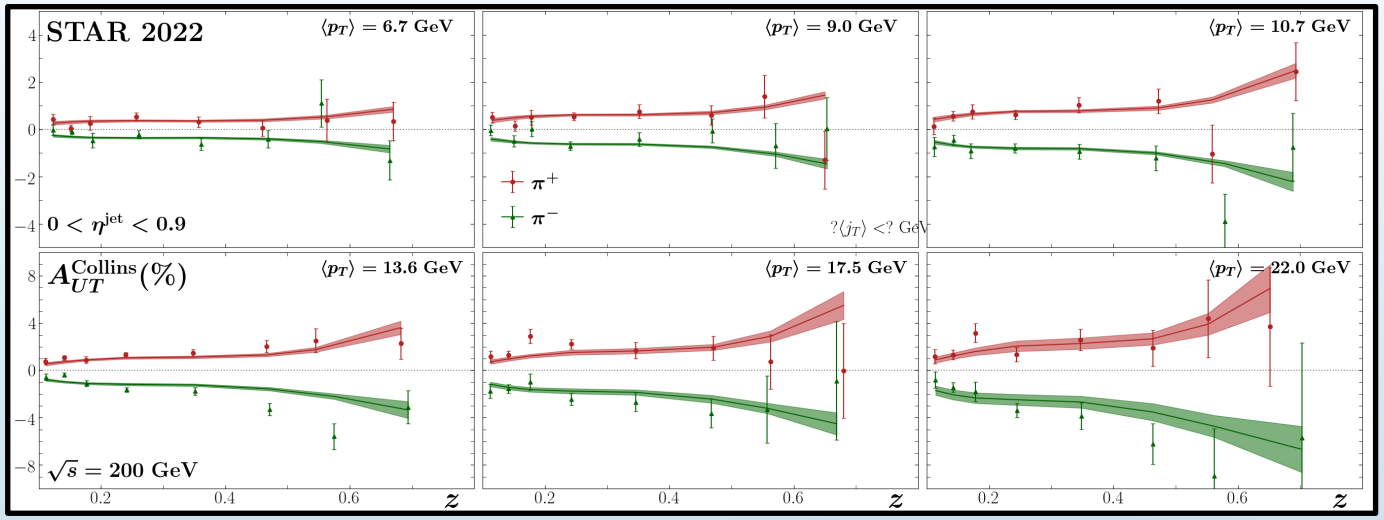
May 2, 2022

27 pages
Published in: *Phys.Rev.D* 106 (2022) 3, 034014
Published: Aug 1, 2022



Hadron-in-jet





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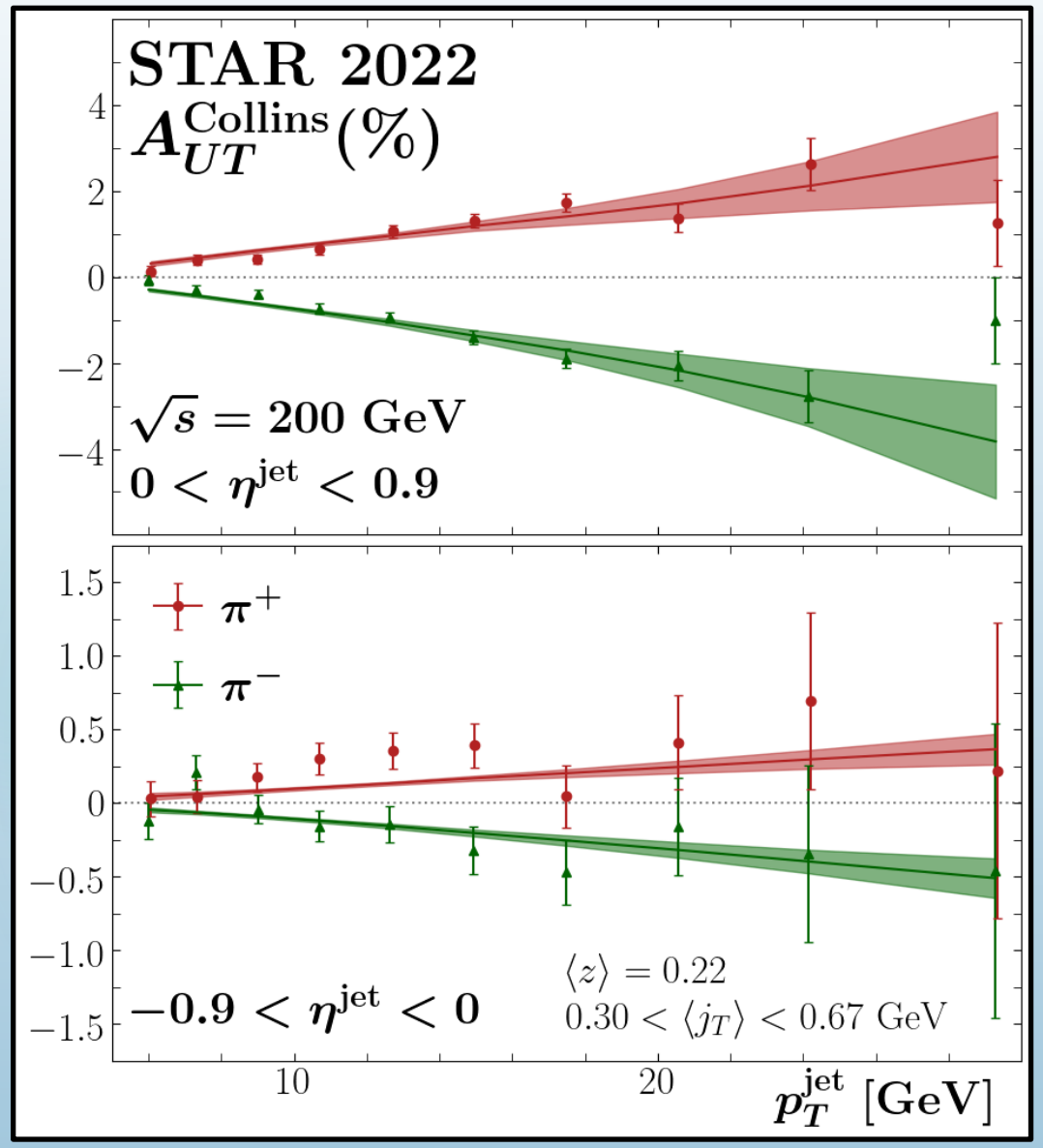
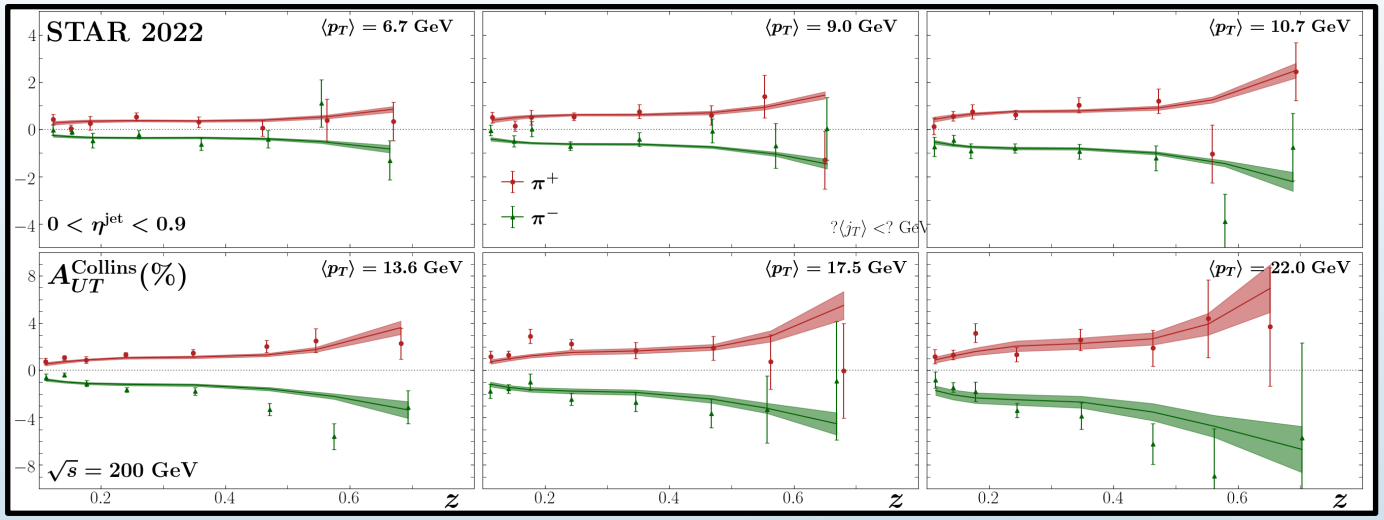
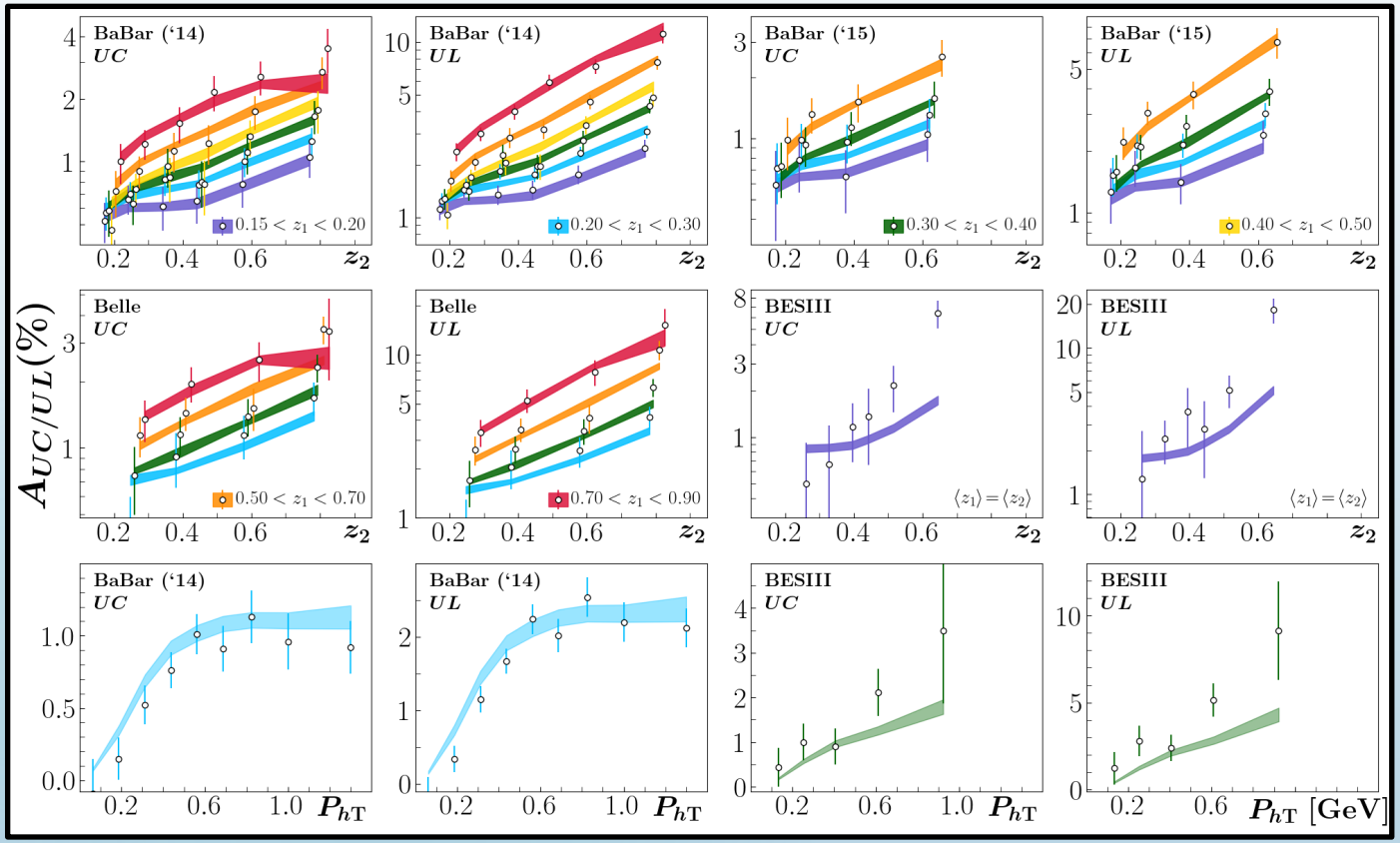
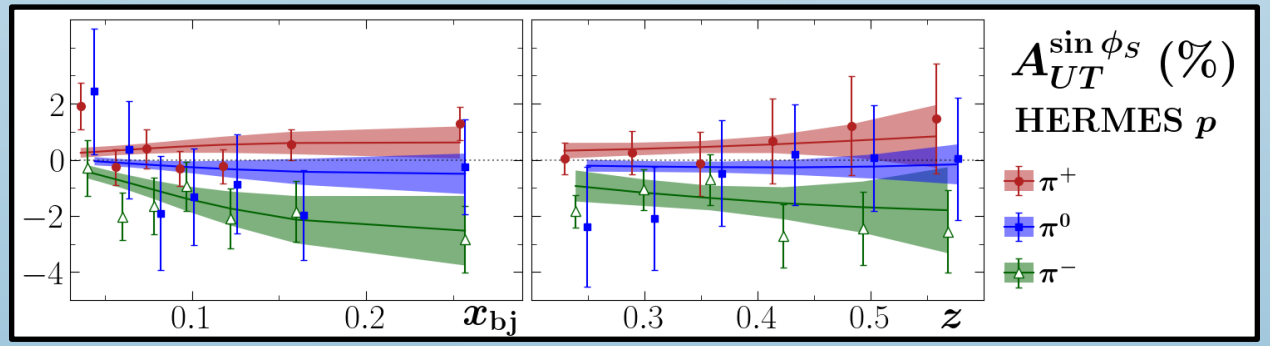


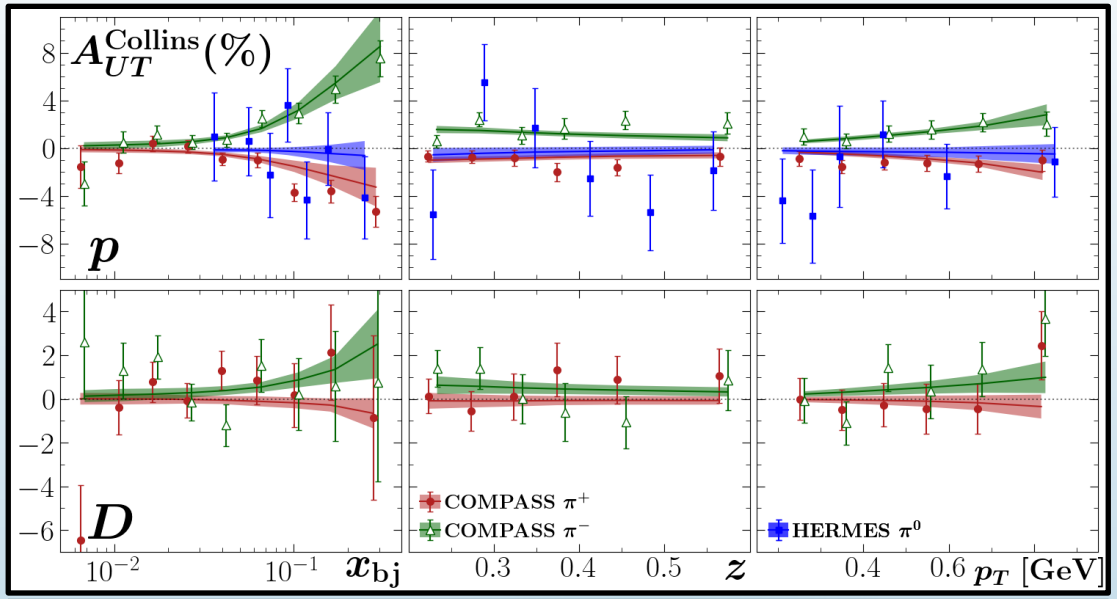


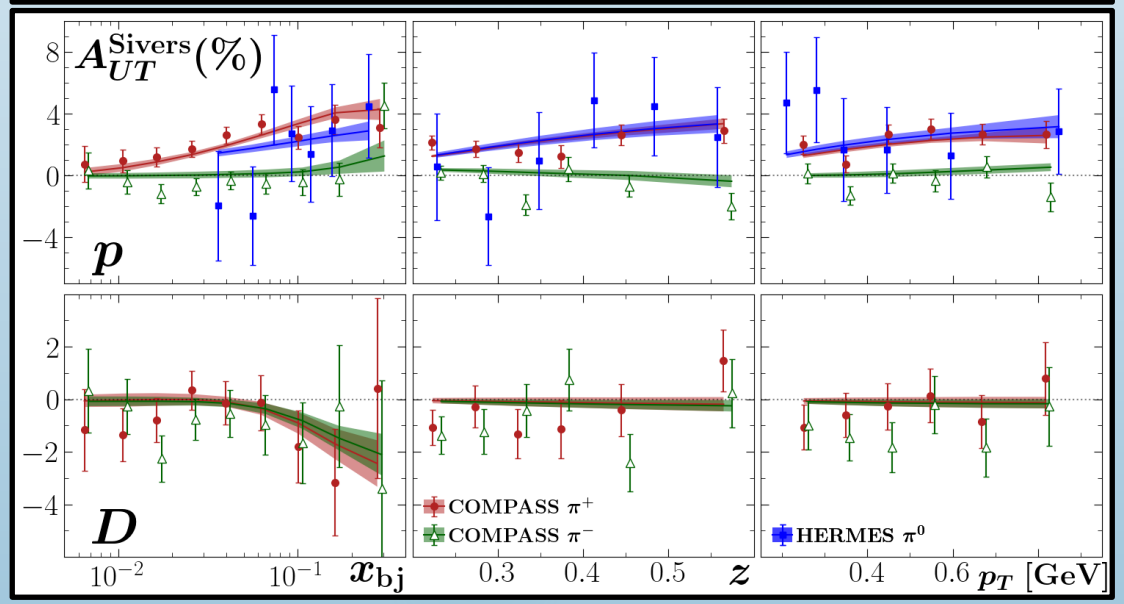
First global QCD analysis to include Hadron-in-jet data!

Data vs. Theory (SIA/SIDIS)

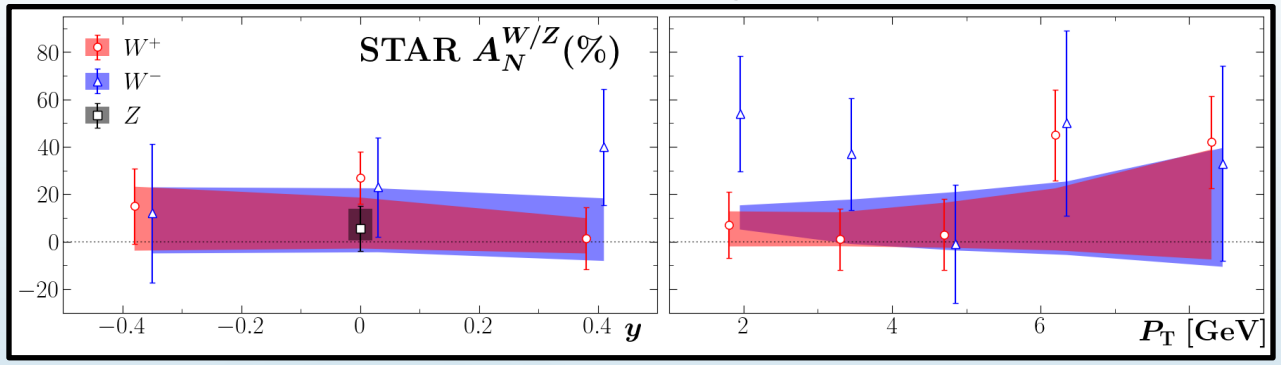


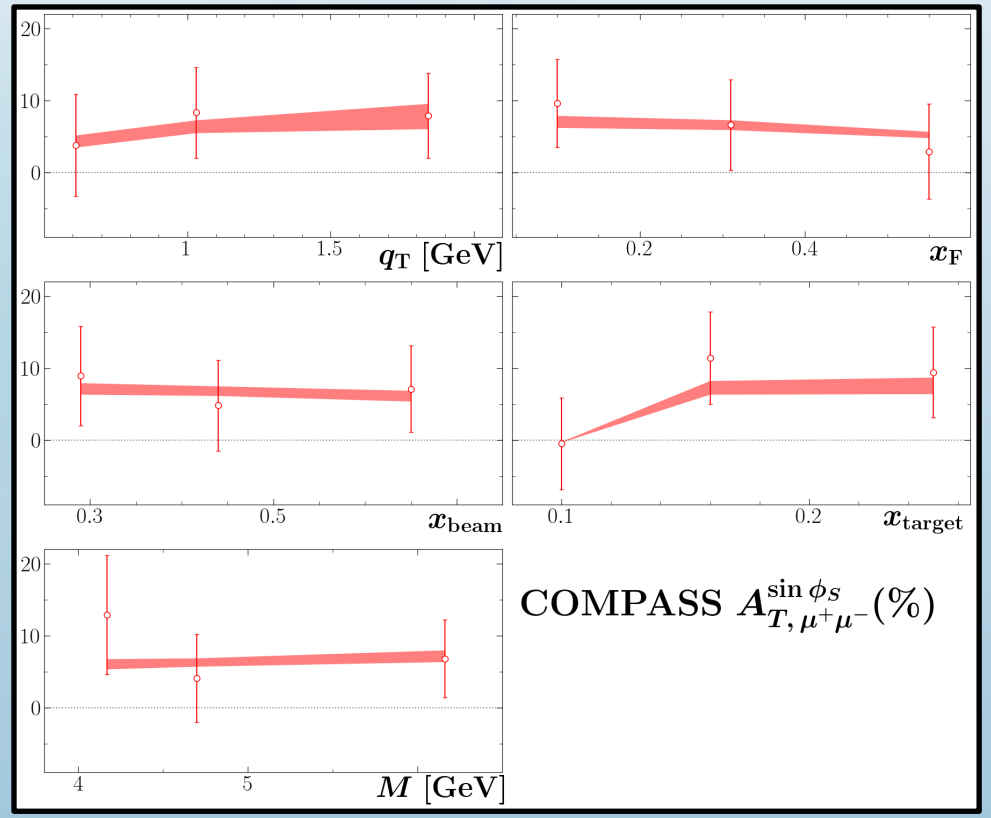


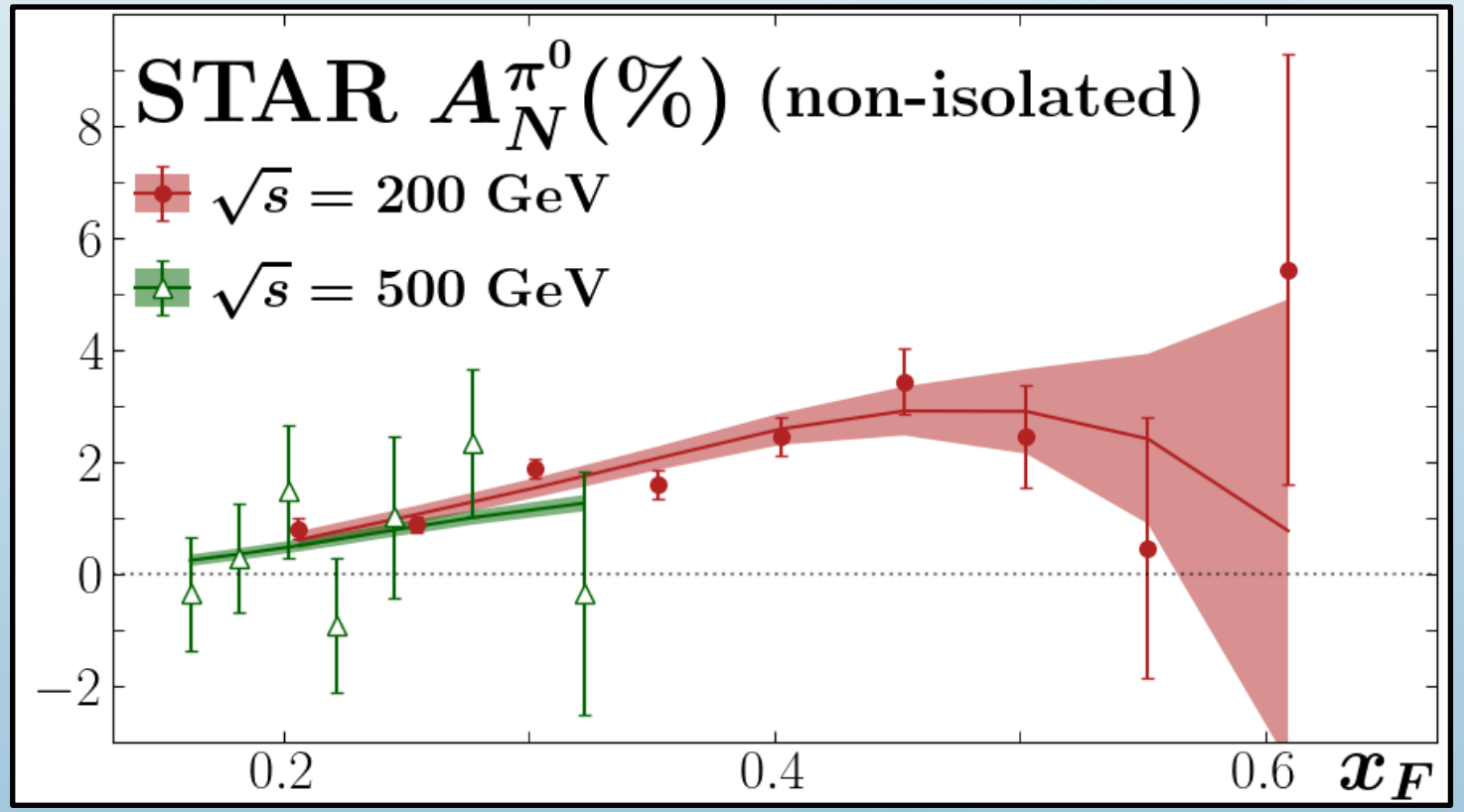




Data vs. Theory (DY, W/Z, A_N)







Quality of Fit and Inclusion of LQCD

Process	N_{dat}	χ^2_{red} (no LQCD)	χ^2_{red} (w/ LQCD)
SIA	176	1.09	1.15
SIDIS	1050	1.38	1.38
Drell-Yan	15	0.24	0.24
W/Z	17	1.71	1.68
AN	44	1.89	1.80
Hadron-in-jet	708	1.03	1.03
LQCD	4	—	0.92
Total	**2014**	**1.24**	**1.24**

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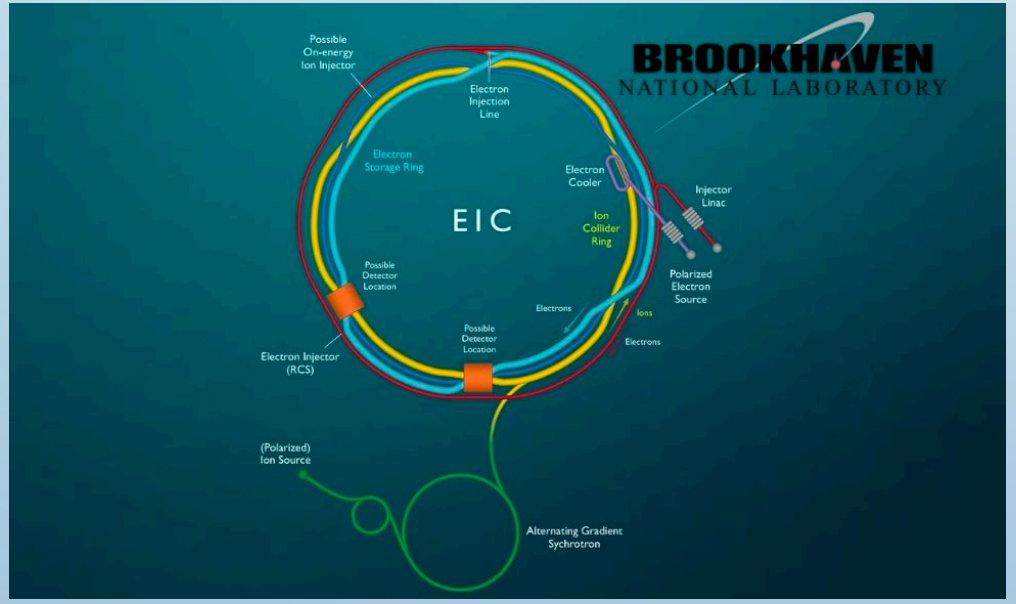
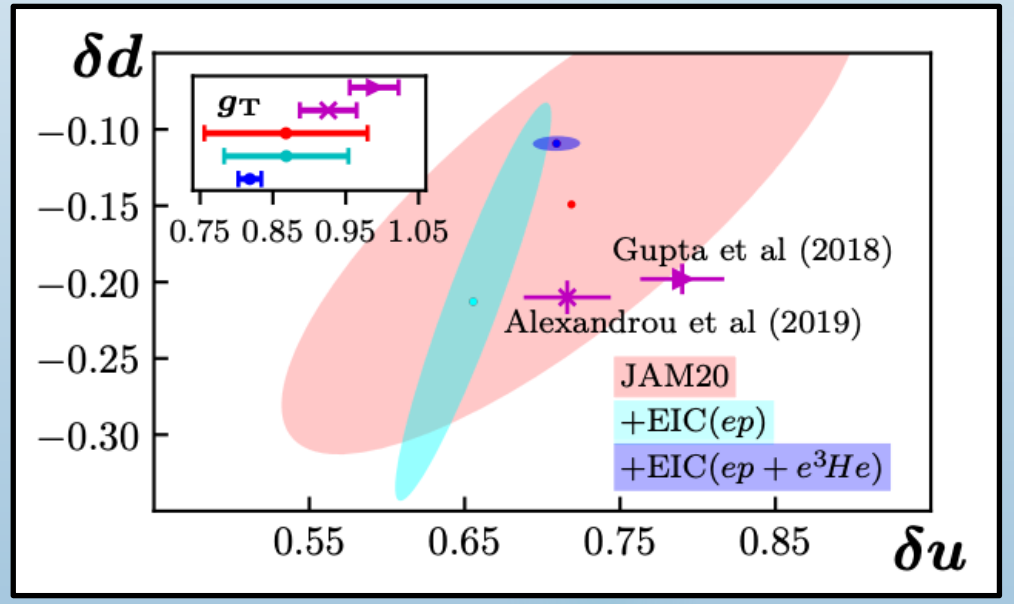
Inclusion of LQCD barely affects description of JAM3D data!

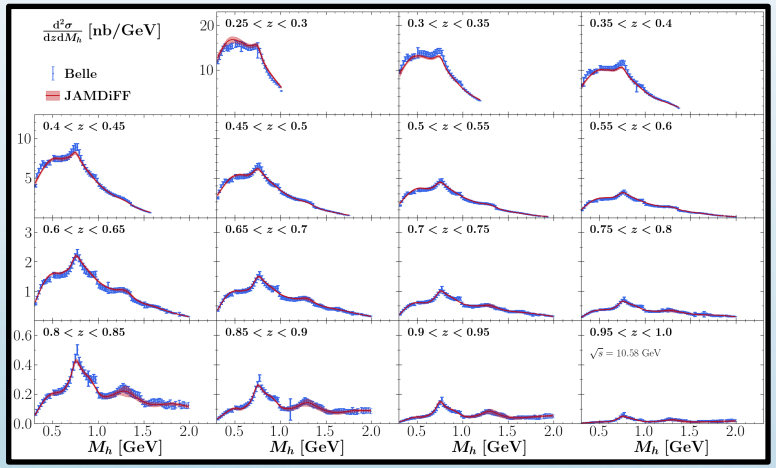
1. Introduction
2. Extraction of DiFFs
3. Extraction of Transversity PDFs
4. Extraction of Tensor Charges
5. Extraction with TMDs
6. Conclusions and Outlook





Comprehensive Analysis of DiFFs and Transversity

First inclusion of Belle cross section data



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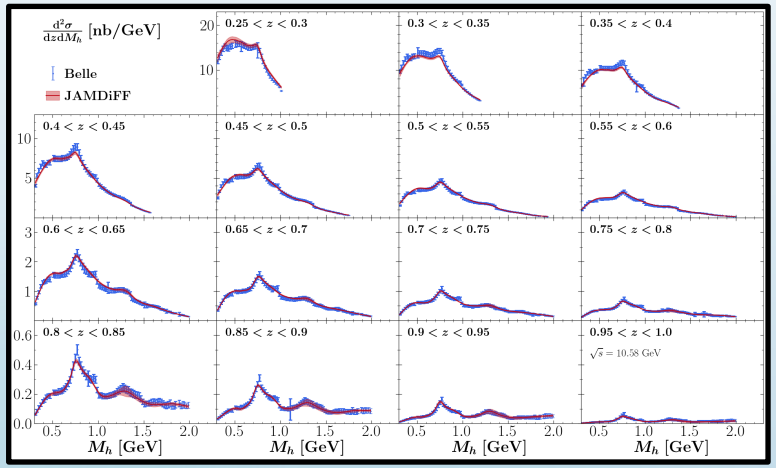
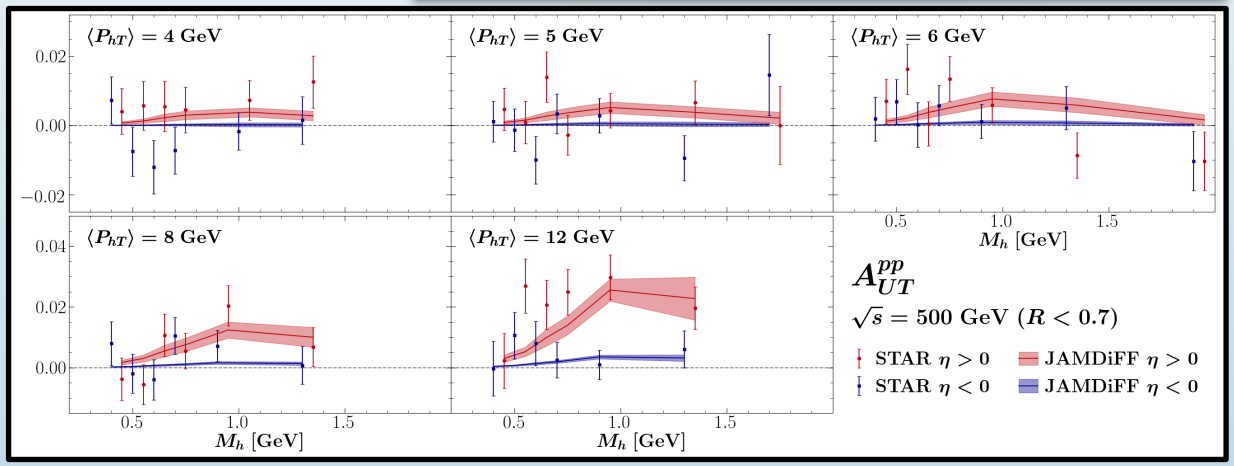


First inclusion of 500 GeV STAR data



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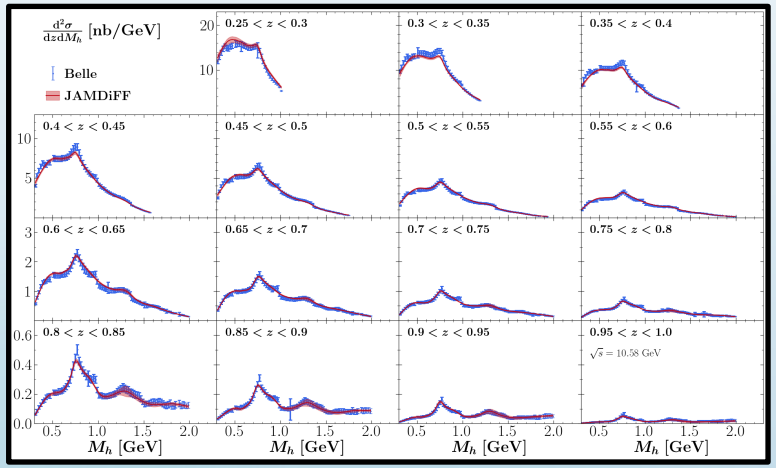

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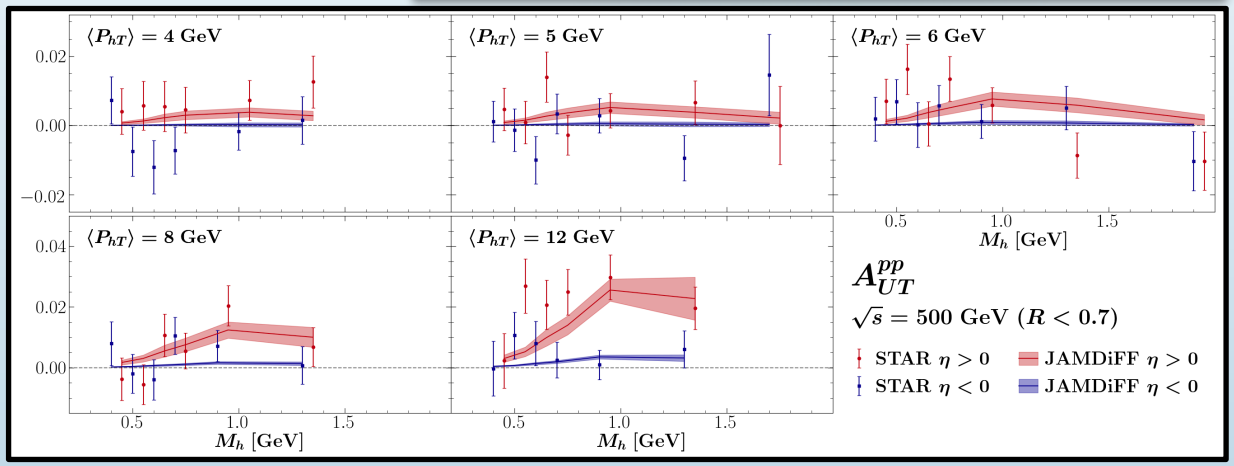
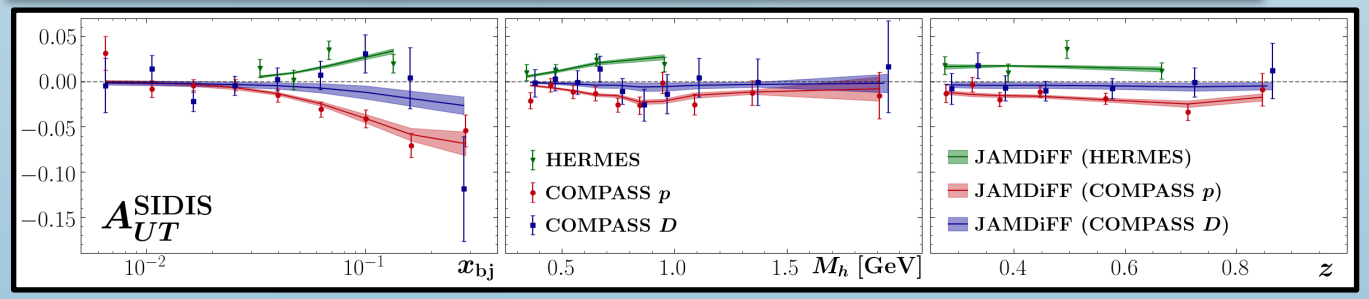
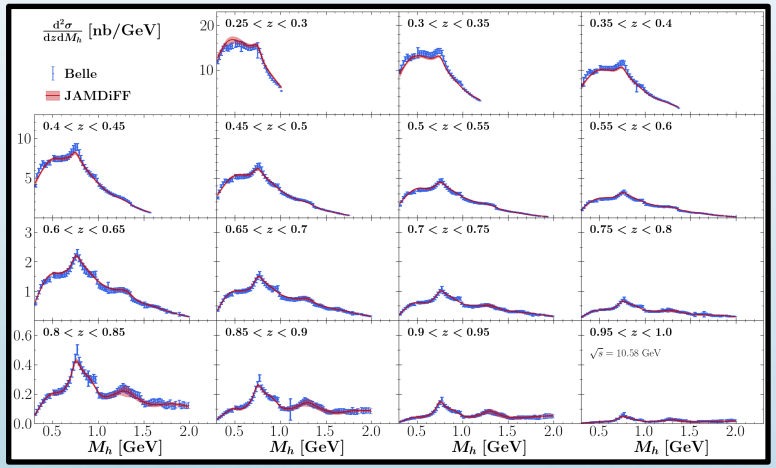


Utilized all binnings for Artru-Collins and SIDIS asymmetries

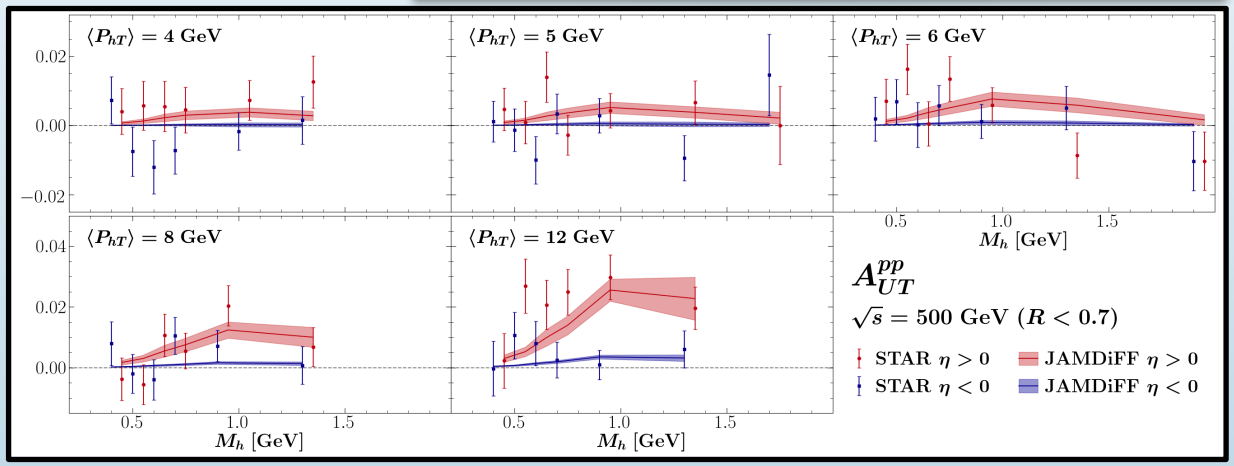


Comprehensive Analysis of DiFFs and Transversity

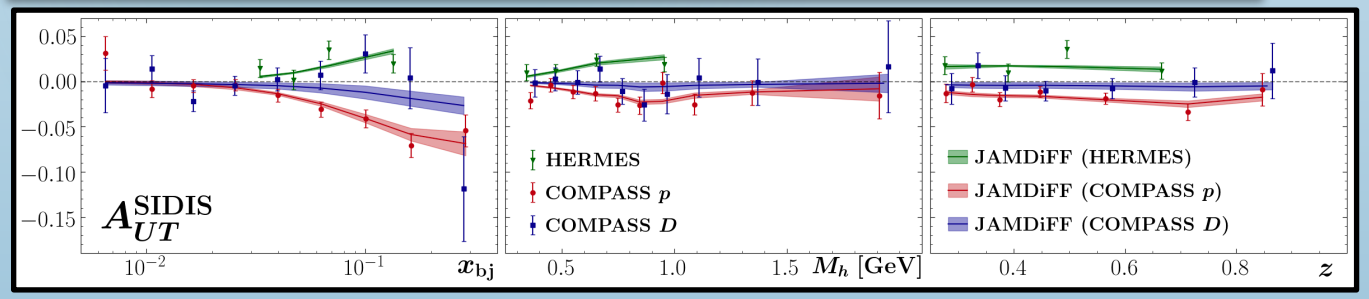
First inclusion of Belle cross section data



First inclusion of 500 GeV STAR data



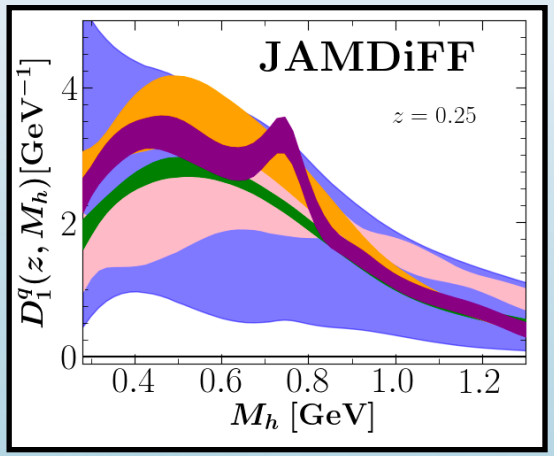
Utilized all binnings for Artru-Collins and SIDIS asymmetries

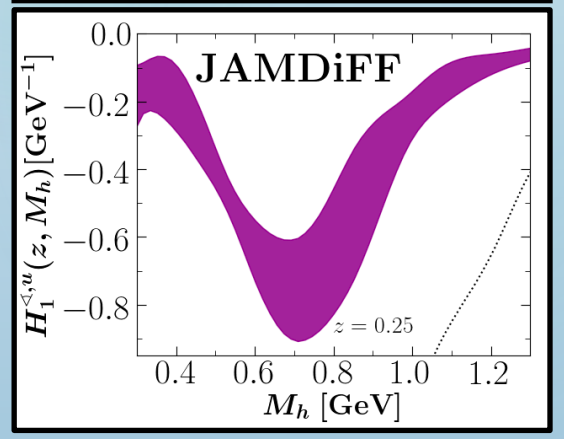


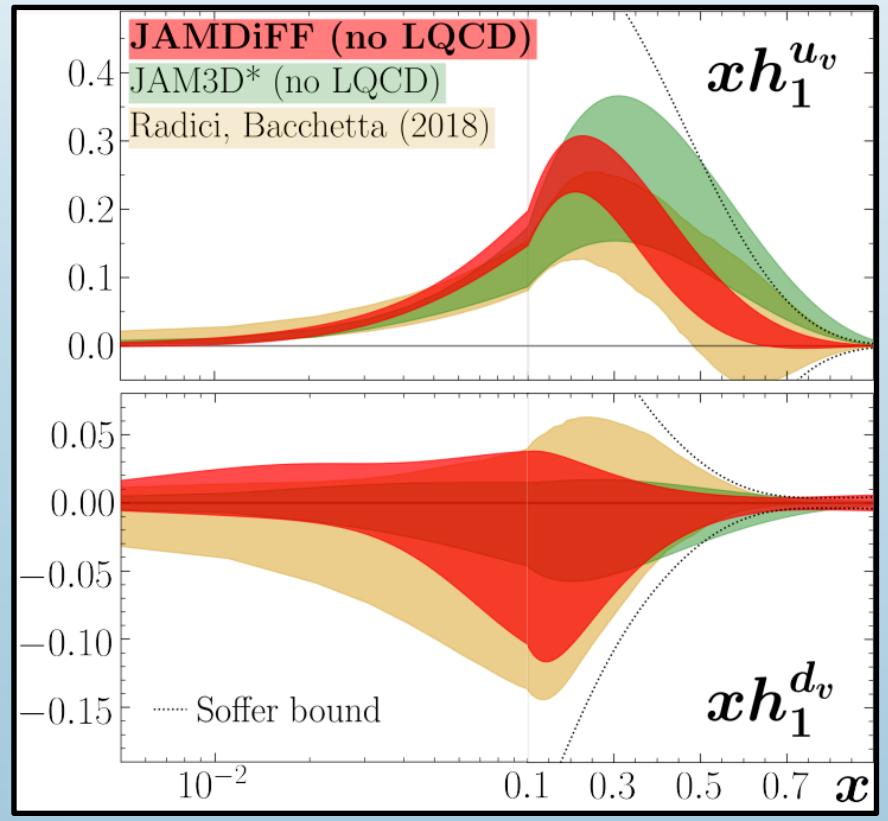
First simultaneous analysis of DiFFs and transversity PDFs

Conclusions

Simultaneous extraction of DiFFs and transversity PDFs



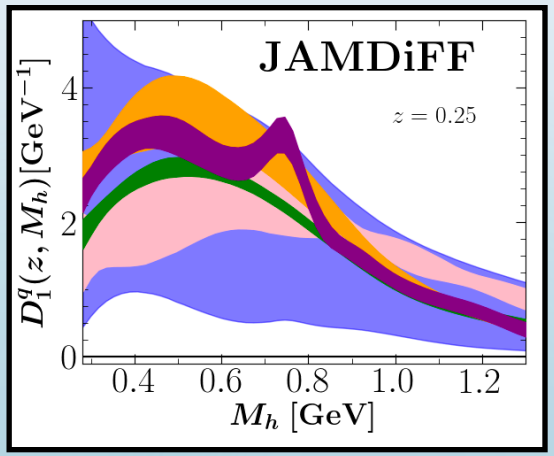


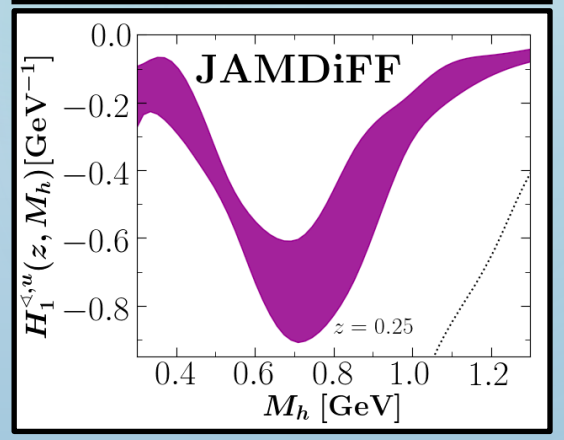


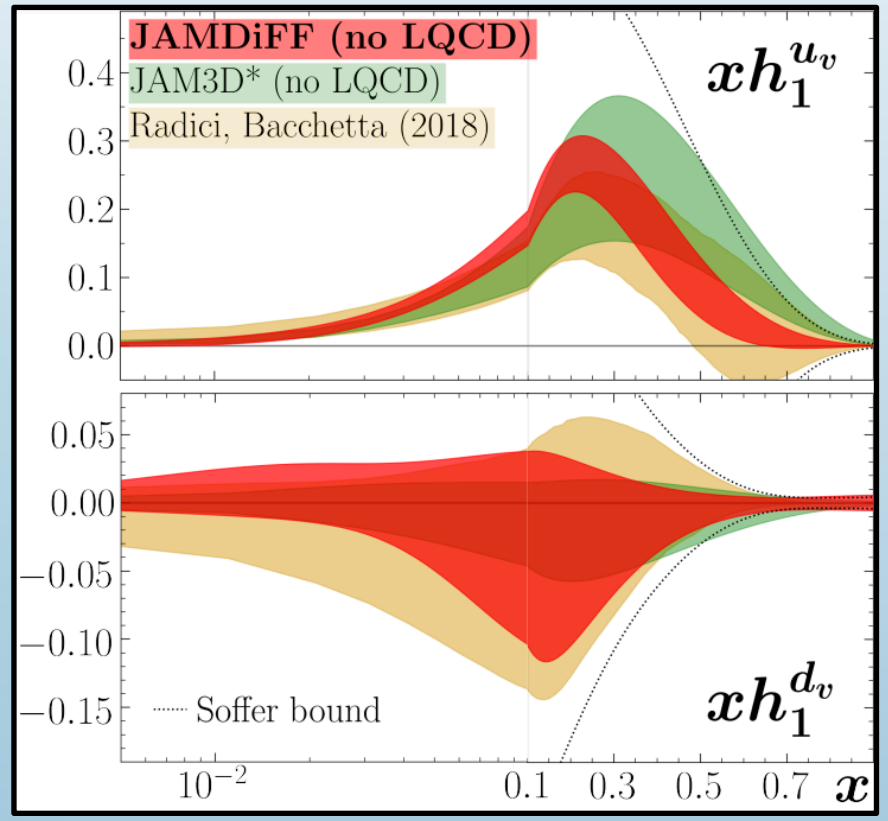
Conclusions

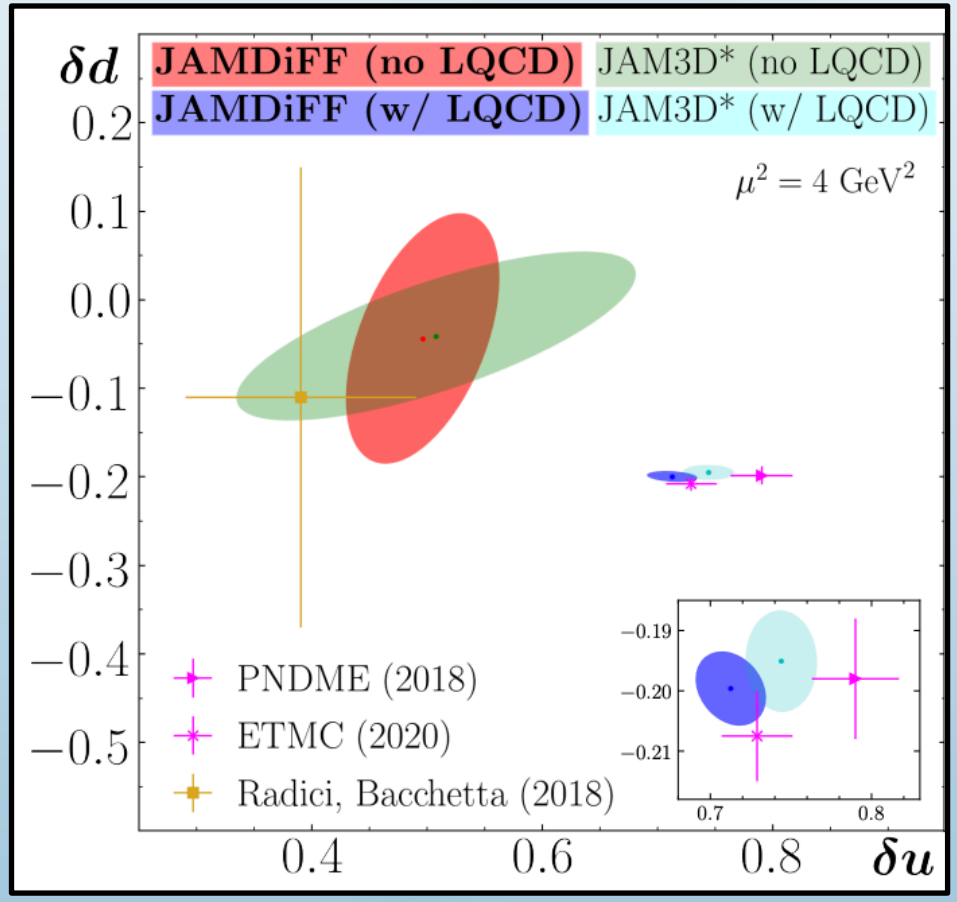
Simultaneous extraction of DiFFs and transversity PDFs

Universality of all available information on transversity









Outlook

More data from RHIC
Proton-proton cross section

Outlook

More data from RHIC
Proton-proton cross section

SIDIS multiplicities from COMPASS

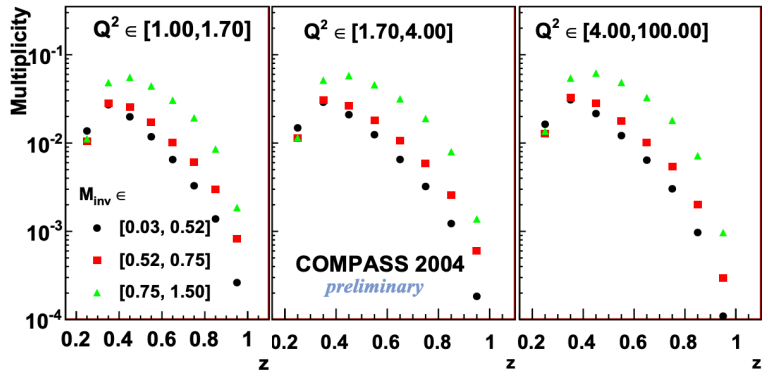


Figure 4: h^+h^- multiplicities measured versus z, M_{inv} and Q^2.

N. Makke, Phys. Part. Nucl. **45**, 138-140 (2014)

Outlook

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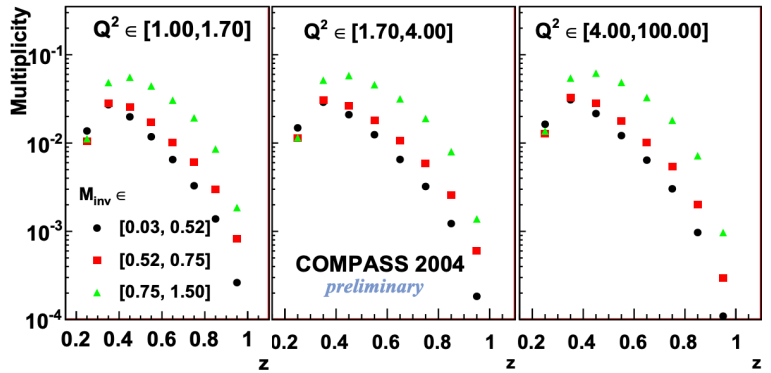

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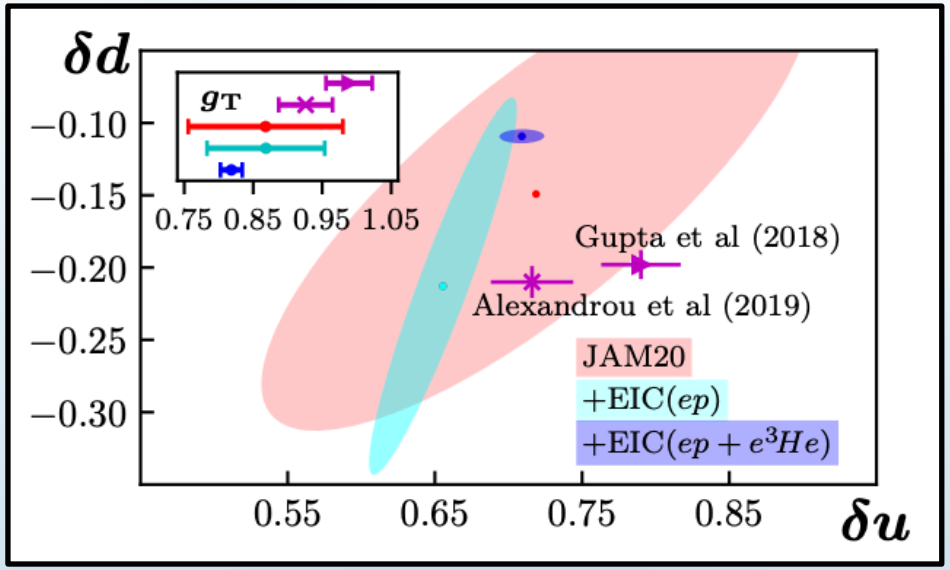
L. Gamberg *et al.*, Phys. Lett. B **816**, 136255 (2021)

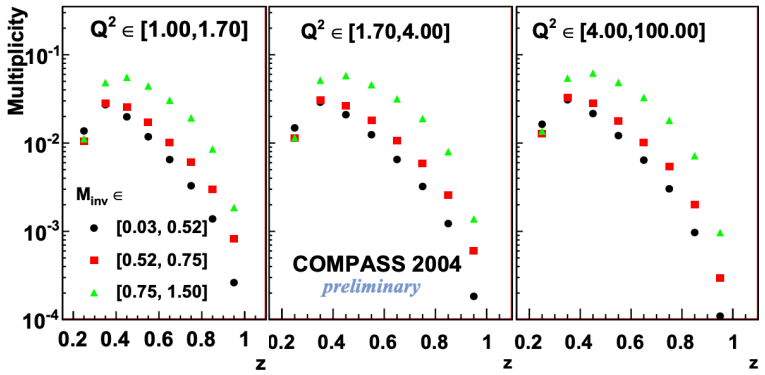


EIC can provide new information

Outlook

More data from RHIC
Proton-proton cross section

SIDIS multiplicities from COMPASS



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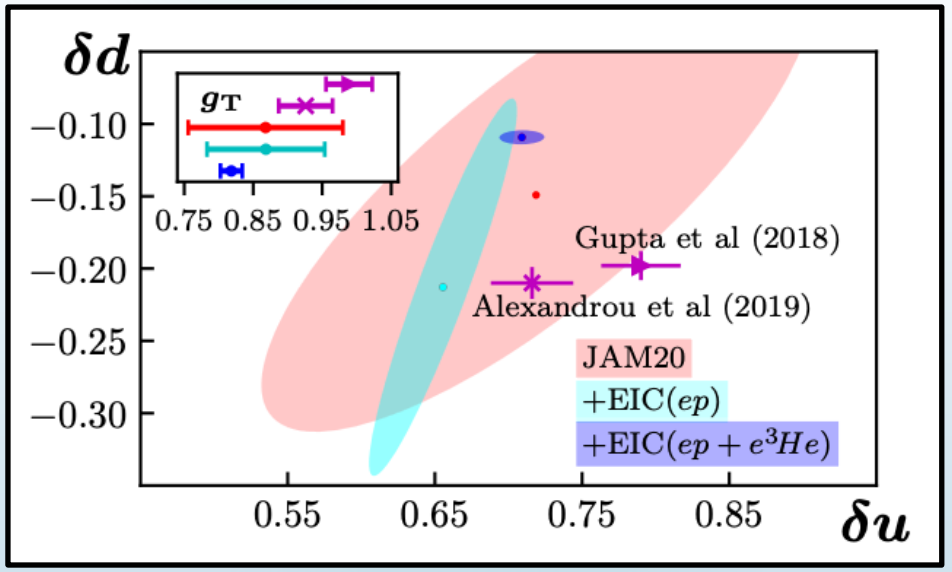


EIC can provide new information

Simultaneous fit of DiFF channel + TMD channel + Lattice QCD (**well under way!**)

Collaboration

Chris Cocuzza

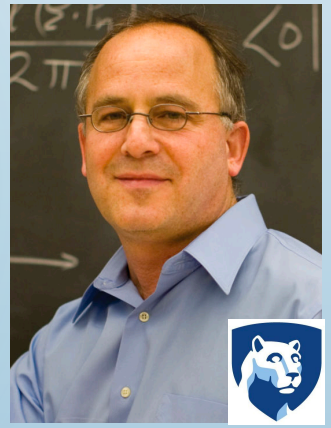
Nobuo Sato

Andreas Metz

Alexey Prokudin

Wally Melnitchouk

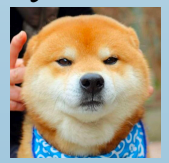
Leonard Gamberg

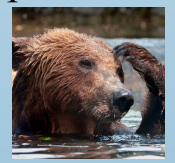


Daniel Pitonyak

Ralf Seidl

Thank you to Yiyu Zhou and
Patrick Barry for helpful discussions



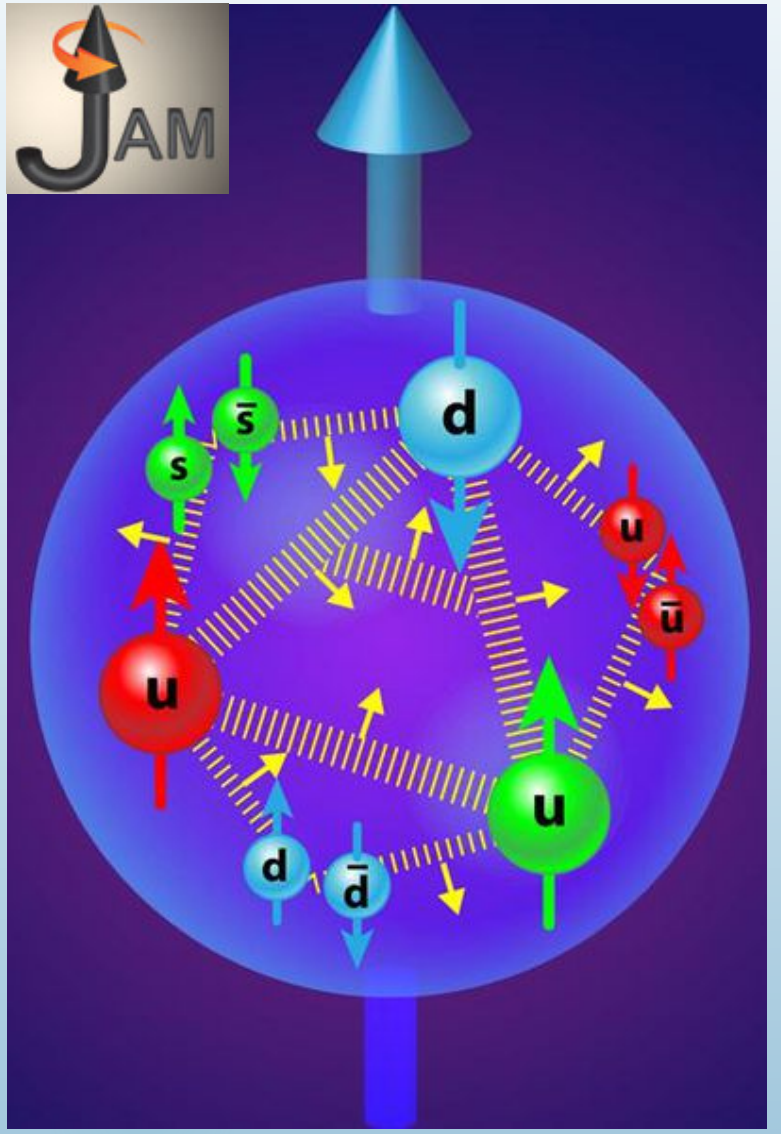


Extra Slides

JAM Collaboration

3-dimensional structure of nucleons:

- Parton distribution functions (PDFs)
- Fragmentation functions (FFs)
- Transverse momentum dependent distributions (TMDs)
- Generalized parton distributions (GPDs)

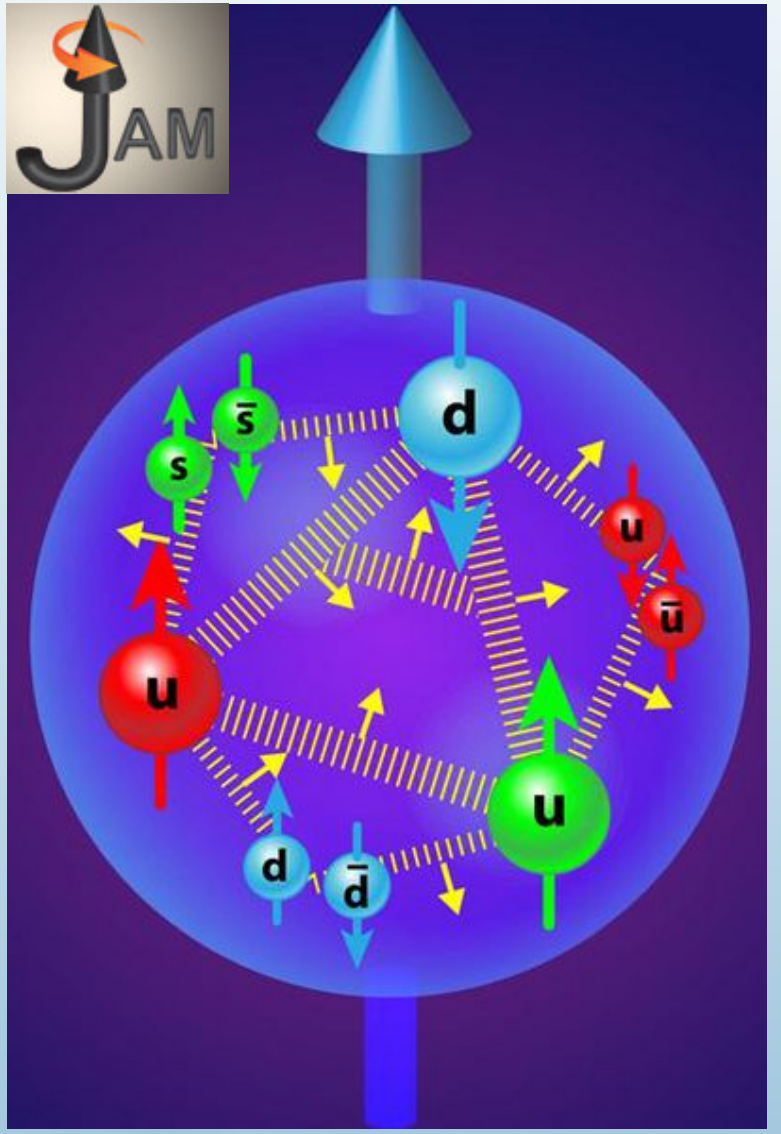


JAM Collaboration

3-dimensional structure of nucleons:

- Parton distribution functions (PDFs)
- Fragmentation functions (FFs)
- Transverse momentum dependent distributions (TMDs)
- Generalized parton distributions (GPDs)

- Collinear factorization in perturbative QCD
- Simultaneous determinations of PDFs, FFs, etc.
- Monte Carlo methods for Bayesian inference



Parameterize PDFs at input scale $Q_0^2 = m_c^2$

$$f_i(x) = Nx^{\alpha}(1-x)^{\beta}(1+\gamma\sqrt{x}+\eta x)$$

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Evolve PDFs using DGLAP

$$\frac{\mathrm{d}}{\mathrm{d}\,\ln(\mu^2)} f_i(x,\mu) = \sum_j \int_x^1 \frac{dz}{z} P_{ij}(z,\mu) f_j(\frac{x}{z},\mu)$$

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Calculate Observables

$$d\sigma^{pp} = \sum_{ij} H_{ij}^{pp} \otimes f_i \otimes f_j$$

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Mellin Space Techniques

$$\mathrm{d}\sigma^{pp} = \sum_{ijkl} \frac{1}{(2\pi i)^2} \int \mathrm{d}N \int \mathrm{d}M \tilde{f}_j(N,\mu_0)\tilde{f}_l(M,\mu_0)$$
$$\otimes \left[x_1^{-N} x_2^{-M} \tilde{\mathcal{H}}_{ik}^{pp}(N,M,\mu) U_{ij}^S(N,\mu,\mu_0) U_{kl}^S(M,\mu,\mu_0) \right]$$

Parameterize PDFs at input scale $Q_0^2 = m_c^2$

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$$\sigma = \sum_{ij} H_{ij} \otimes f_i \otimes f_j + \mathcal{O}(1/Q)$$

Experimentally measured cross-section

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“Hard part” (process dependent)
Cross-section at parton level
Calculated in perturbative QCD

Experimentally measured cross-section

“Soft part” (process independent)
Describes internal structure

$$\sigma = \sum_{ij} H_{ij} \otimes f_i \otimes f_j + \mathcal{O}(1/Q)$$

“Hard part” (process dependent)
Cross-section at parton level
Calculated in perturbative QCD

Now that the observables have been calculated…

$$\chi^2(\boldsymbol{a}) = \sum_{i,e} \left(\frac{d_{i,e} - \sum_k r_e^k \beta_{i,e}^k - T_{i,e}(\boldsymbol{a})/N_e}{\alpha_{i,e}} \right)^2 + \sum_k \left(r_e^k\right)^2 + \left(\frac{1 - N_e}{\delta N_e} \right)^2$$

Now that the observables have been calculated…

Data

$$\chi^2(\boldsymbol{a}) = \sum_{i,e} \left(\frac{d_{i,e} - \sum_k r_e^k \beta_{i,e}^k - T_{i,e}(\boldsymbol{a})/N_e}{\alpha_{i,e}} \right)^2 + \sum_k \left(r_e^k\right)^2 + \left(\frac{1 - N_e}{\delta N_e} \right)^2$$

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Uncorrelated Uncertainties

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Uncorrelated Uncertainties

Correlated Uncertainties

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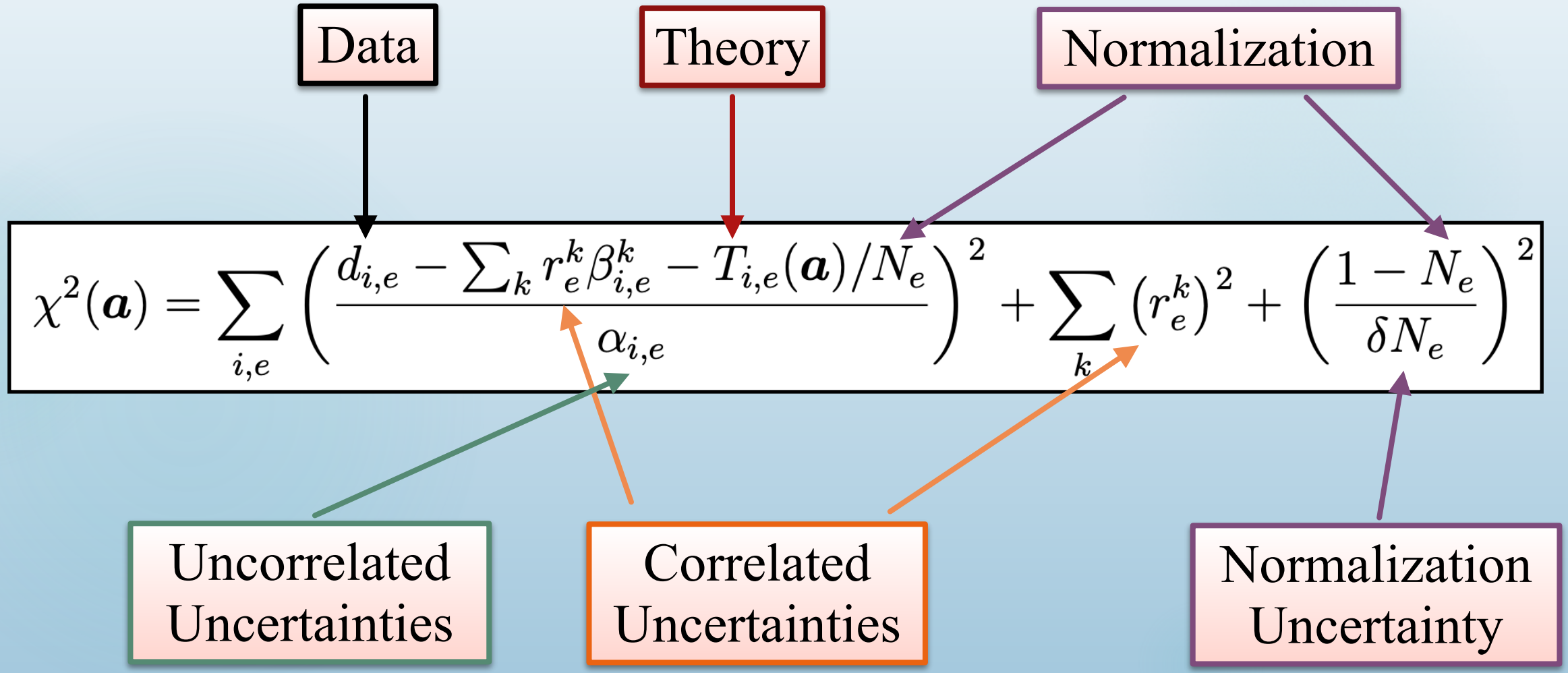

Now that we have calculated $\chi^2(\boldsymbol{a}, \text{data})\ldots$

Likelihood Function

$$\mathcal{L}(\boldsymbol{a}, \text{data}) = \exp\left(-\frac{1}{2}\chi^2(\boldsymbol{a}, \text{data})\right)$$

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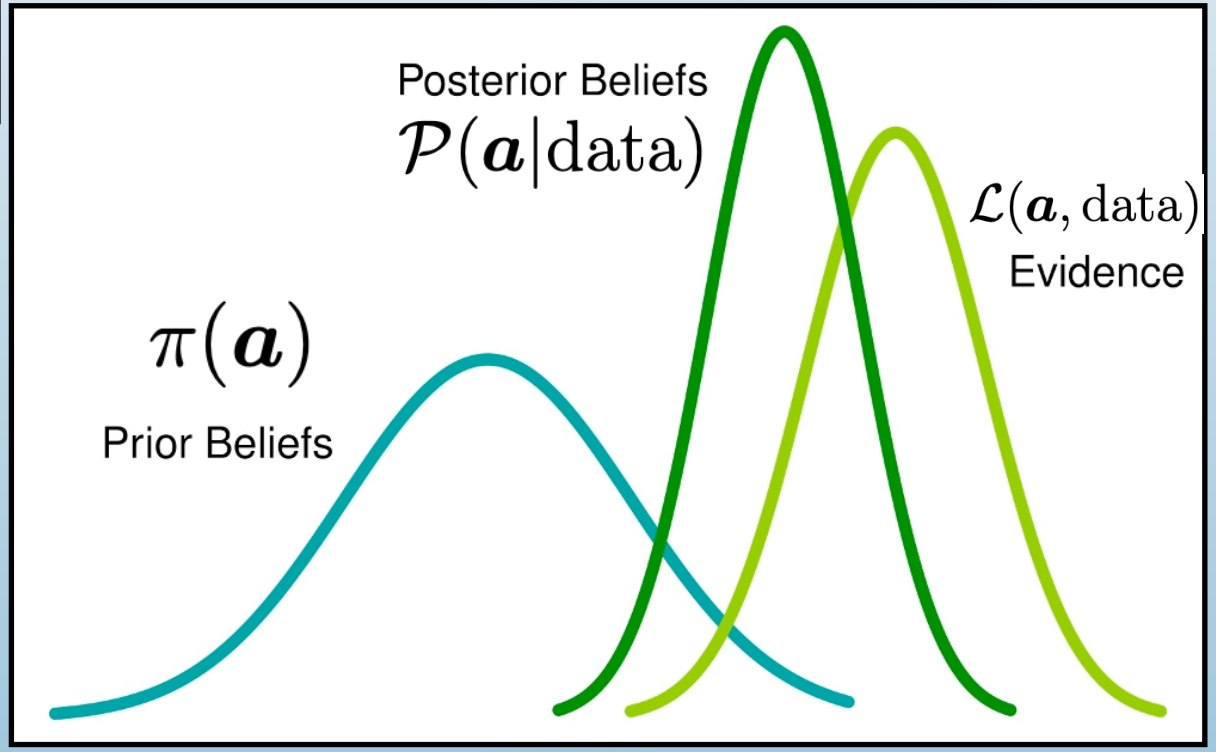
Bayes' Theorem

$$\mathcal{P}(\boldsymbol{a}|\text{data}) \sim \mathcal{L}(\boldsymbol{a}, \text{data})\,\pi(\boldsymbol{a})$$



$$\tilde{\sigma} = \sigma + N(0,1)\ \alpha$$

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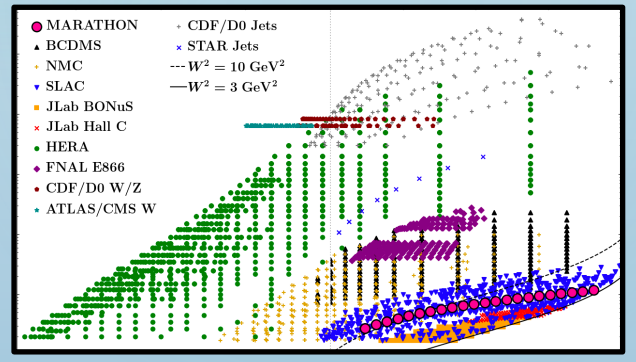
Data

Original Data



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Uncorrelated Uncertainties

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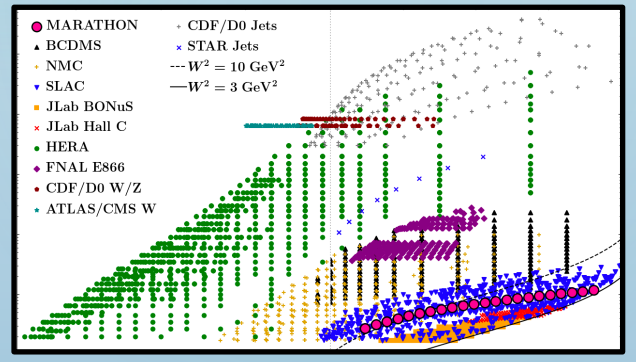
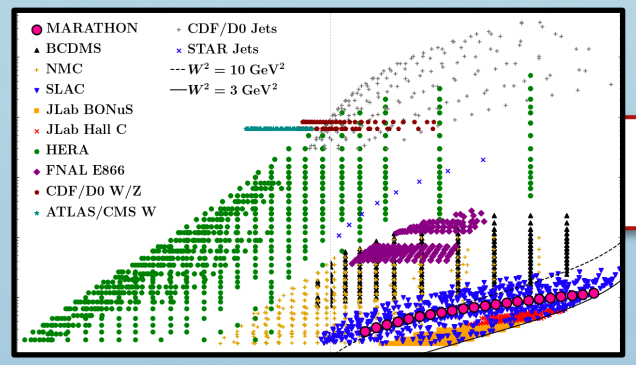
Original Data



Pseudo-Data → $\tilde{\sigma} = \sigma + N(0,1)\ \alpha$ ← Uncorrelated Uncertainties

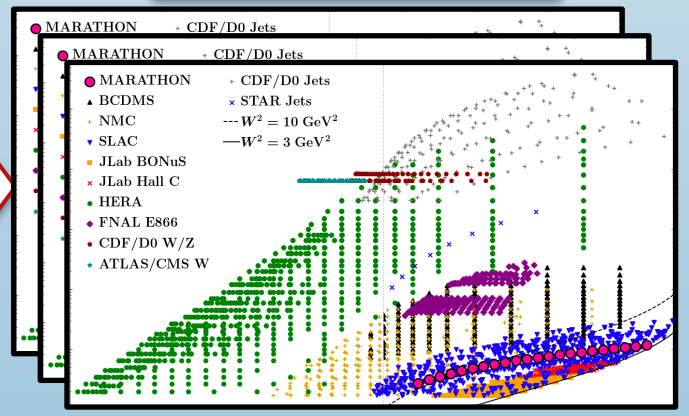
Data

Original Data



DR

Replica Data



Pseudo-Data

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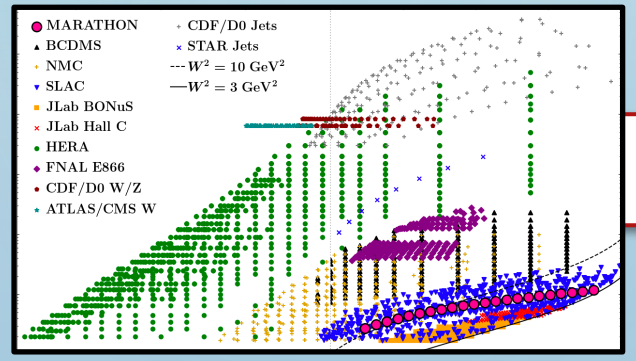
Uncorrelated Uncertainties

Data

Original Data



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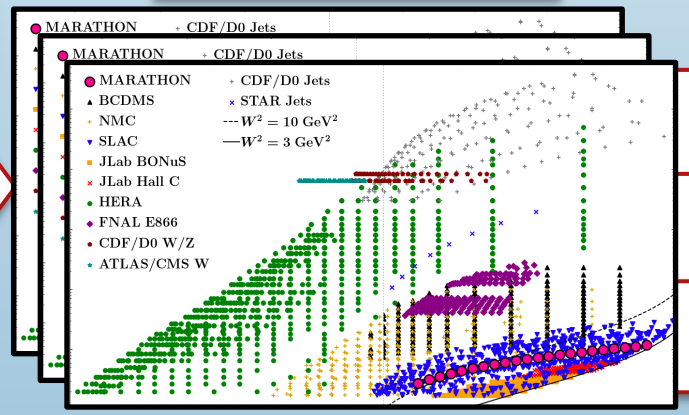
Replica Data

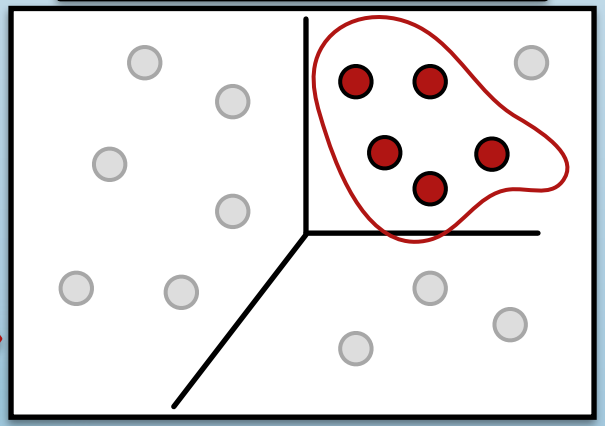


Maximum Likelihood

Maximum Likelihood

Maximum Likelihood

Parameter Space



For a quantity $O(\boldsymbol{a})$: (for example, a PDF at a given value of (x, Q^2))

$$E[O] = \int d^n a \; \rho(\boldsymbol{a}\,|\,data) \; O(\boldsymbol{a})$$

$$V[O] = \int d^n a \; \rho(\boldsymbol{a}\,|\,data) \; \left[O(\boldsymbol{a}) - E[O]\right]^2$$

Exact, but
$n = \mathcal{O}(100)$!

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Build an MC ensemble

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Average over k sets of the parameters (replicas)

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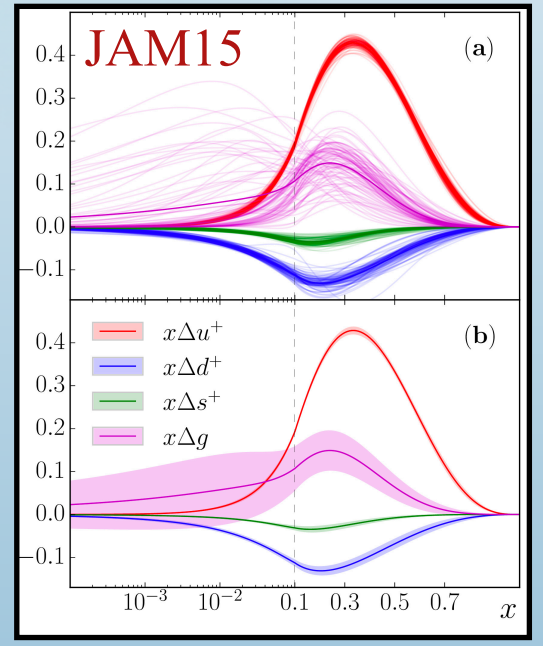

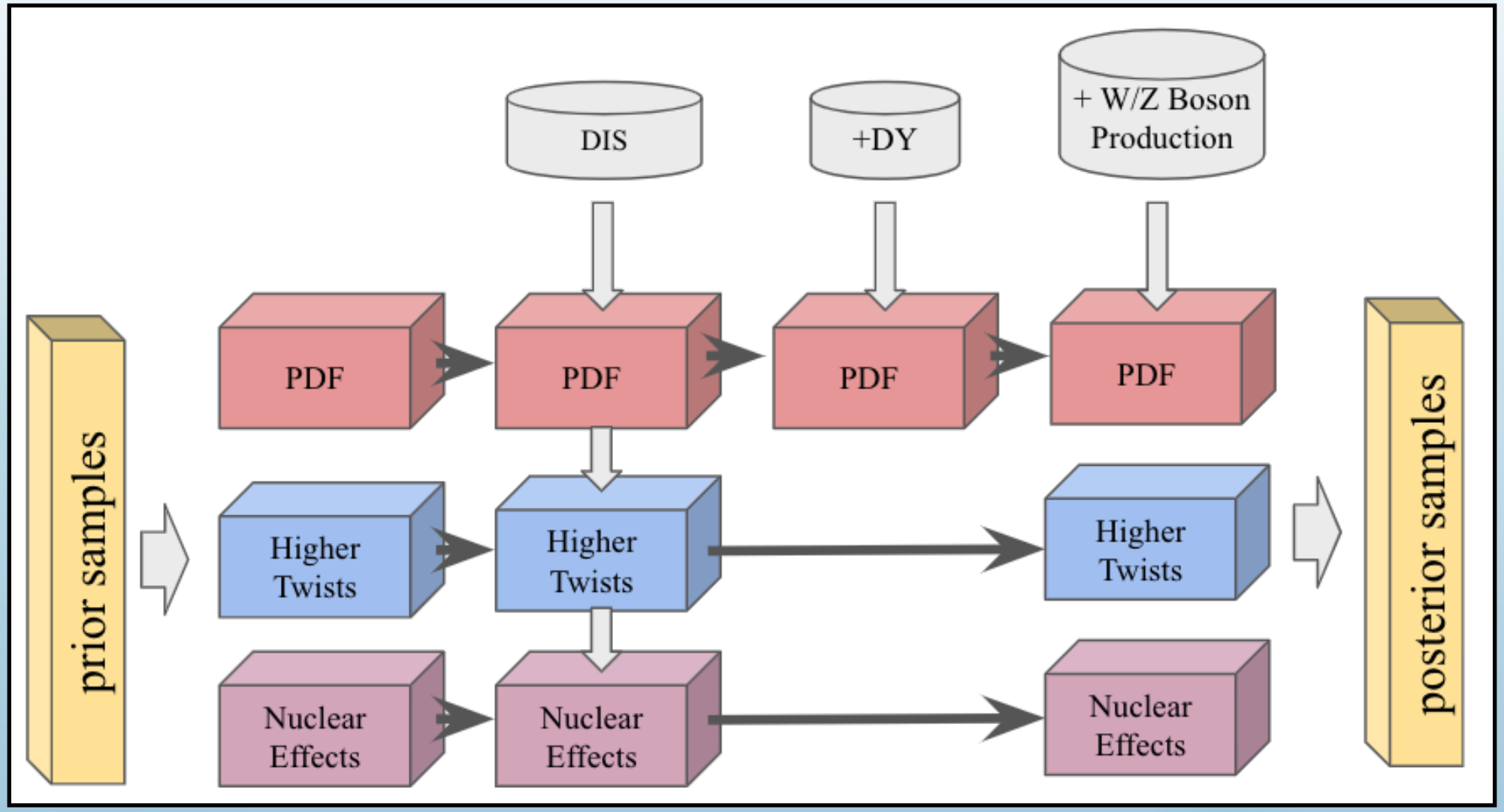
DIS
+DY
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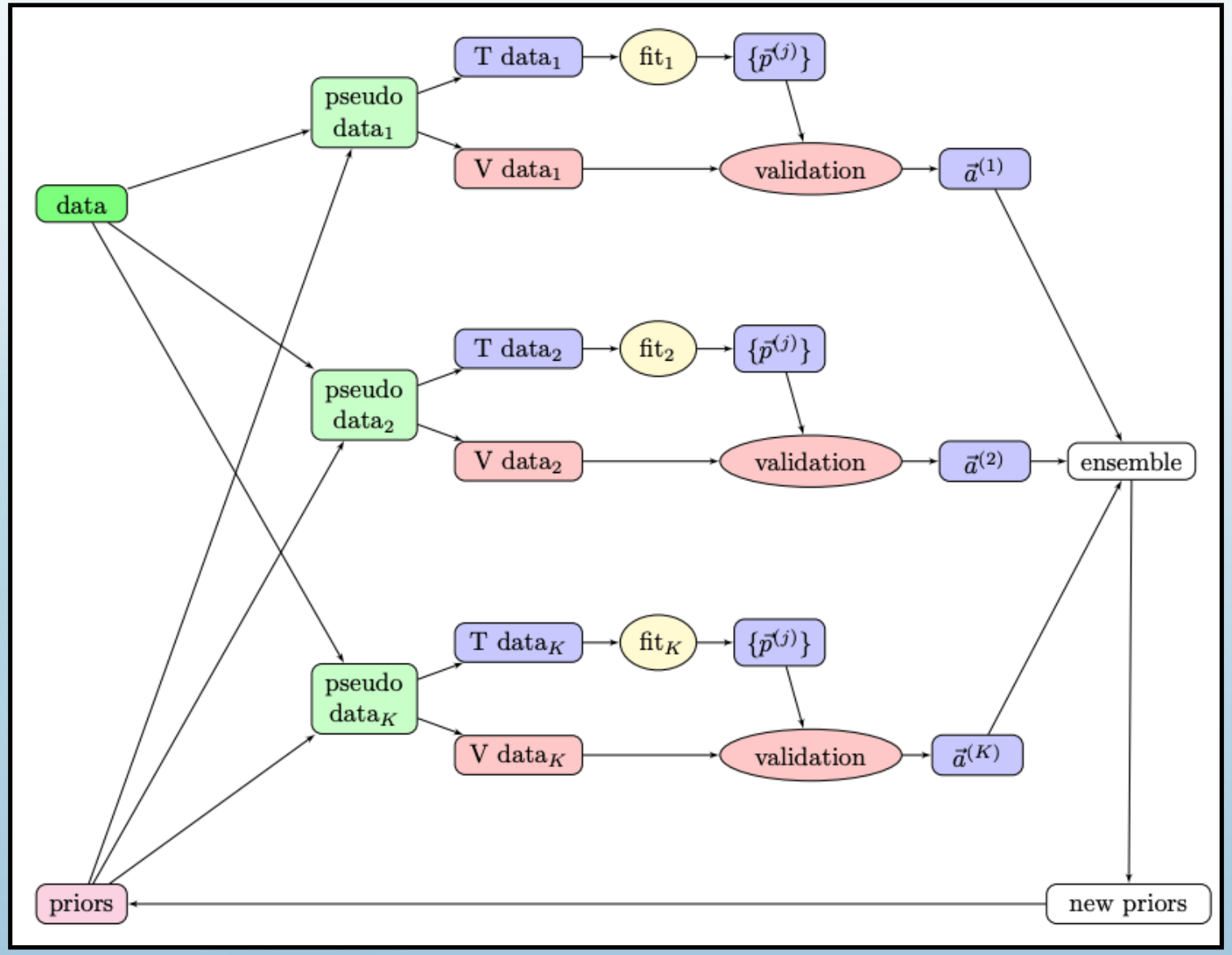
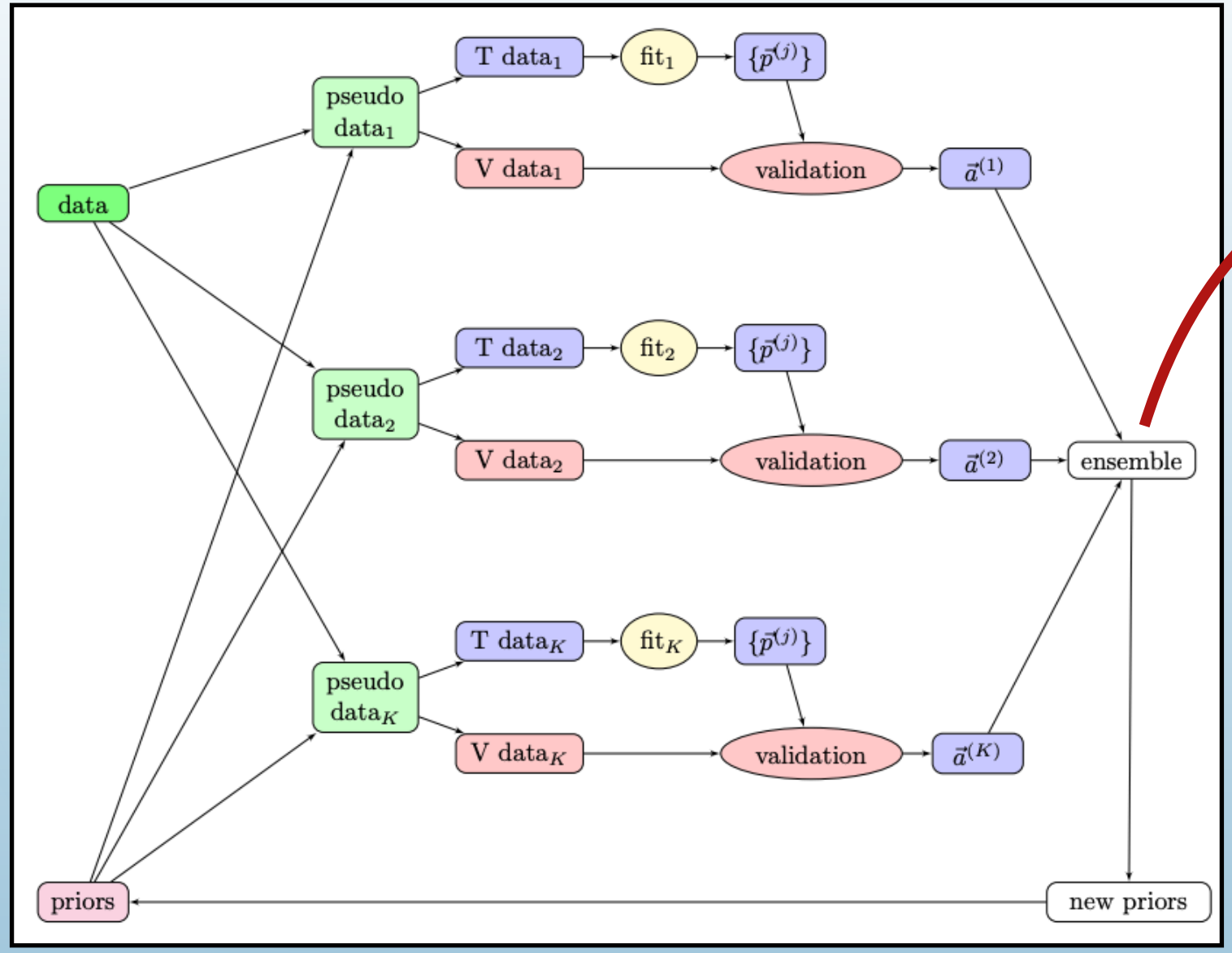
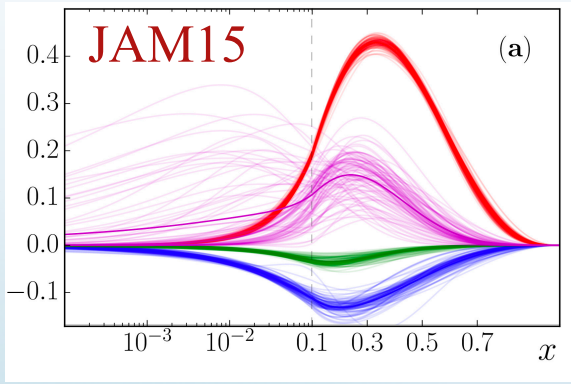
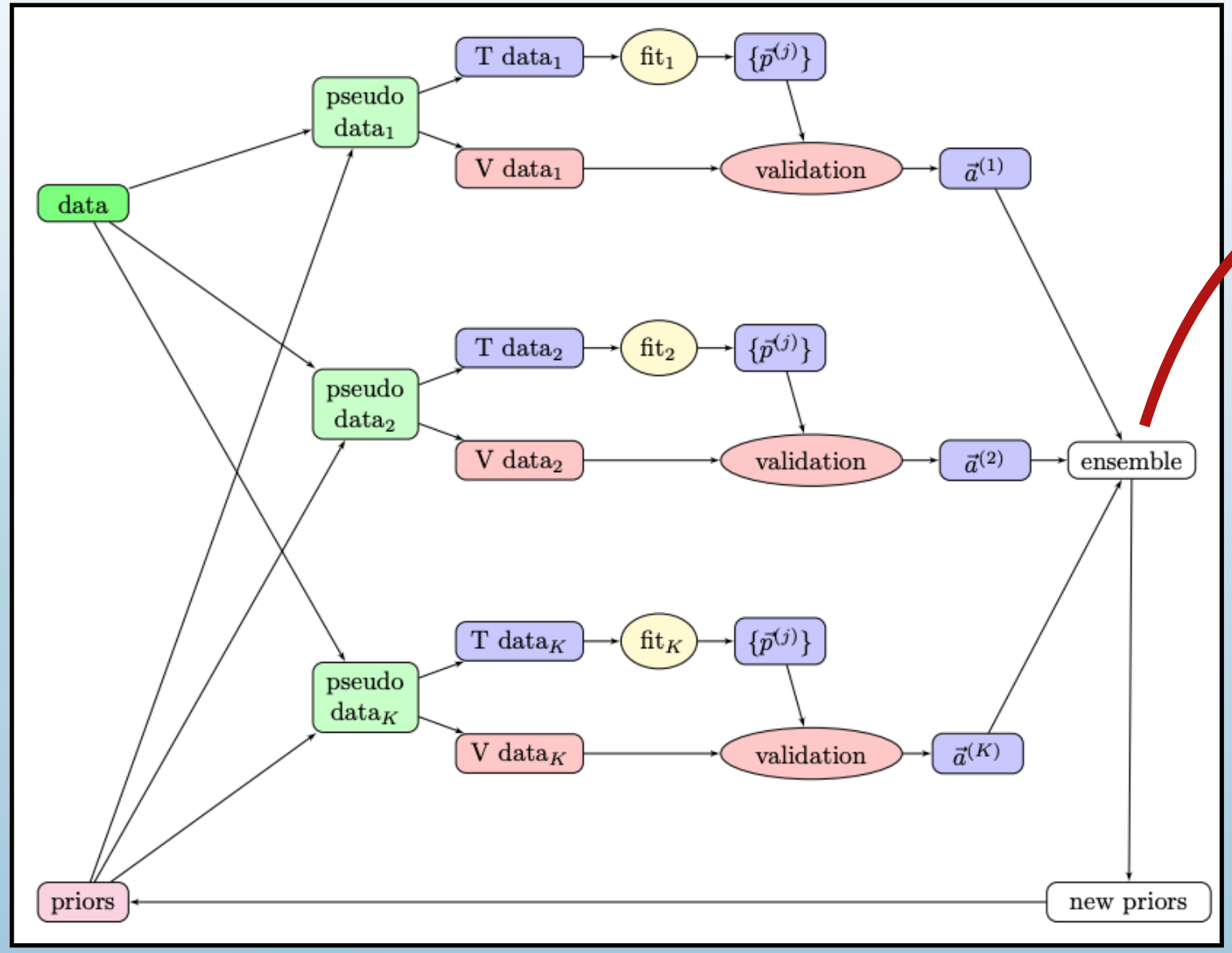
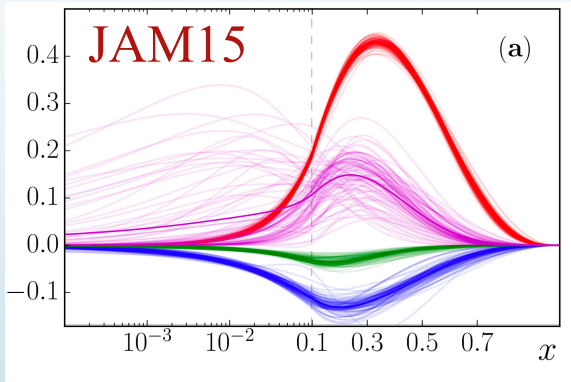
Nuclear Effects
Nuclear Effects
Nuclear Effects
posterior samples


data
pseudo data$_1$
T data$_1$
fit$_1$
$\{\vec{p}^{(j)}\}$
V data$_1$
validation
$\vec{a}^{(1)}$
pseudo data$_2$
T data$_2$
fit$_2$
$\{\vec{p}^{(j)}\}$
V data$_2$
validation
$\vec{a}^{(2)}$
ensemble
pseudo data$_K$
T data$_K$
fit$_K$
$\{\vec{p}^{(j)}\}$
V data$_K$
validation
$\vec{a}^{(K)}$
priors
new priors


data
pseudo data$_1$
T data$_1$
fit$_1$
$\{\vec{p}^{(j)}\}$
V data$_1$
validation
$\vec{a}^{(1)}$
pseudo data$_2$
T data$_2$
fit$_2$
$\{\vec{p}^{(j)}\}$
V data$_2$
validation
$\vec{a}^{(2)}$
ensemble
pseudo data$_K$
T data$_K$
fit$_K$
$\{\vec{p}^{(j)}\}$
V data$_K$
validation
$\vec{a}^{(K)}$
priors
new priors


JAM15
(a)
0.4
0.3
0.2
0.1
0.0
−0.1
10^{-3}
10^{-2}
0.1
0.3
0.5
0.7
x

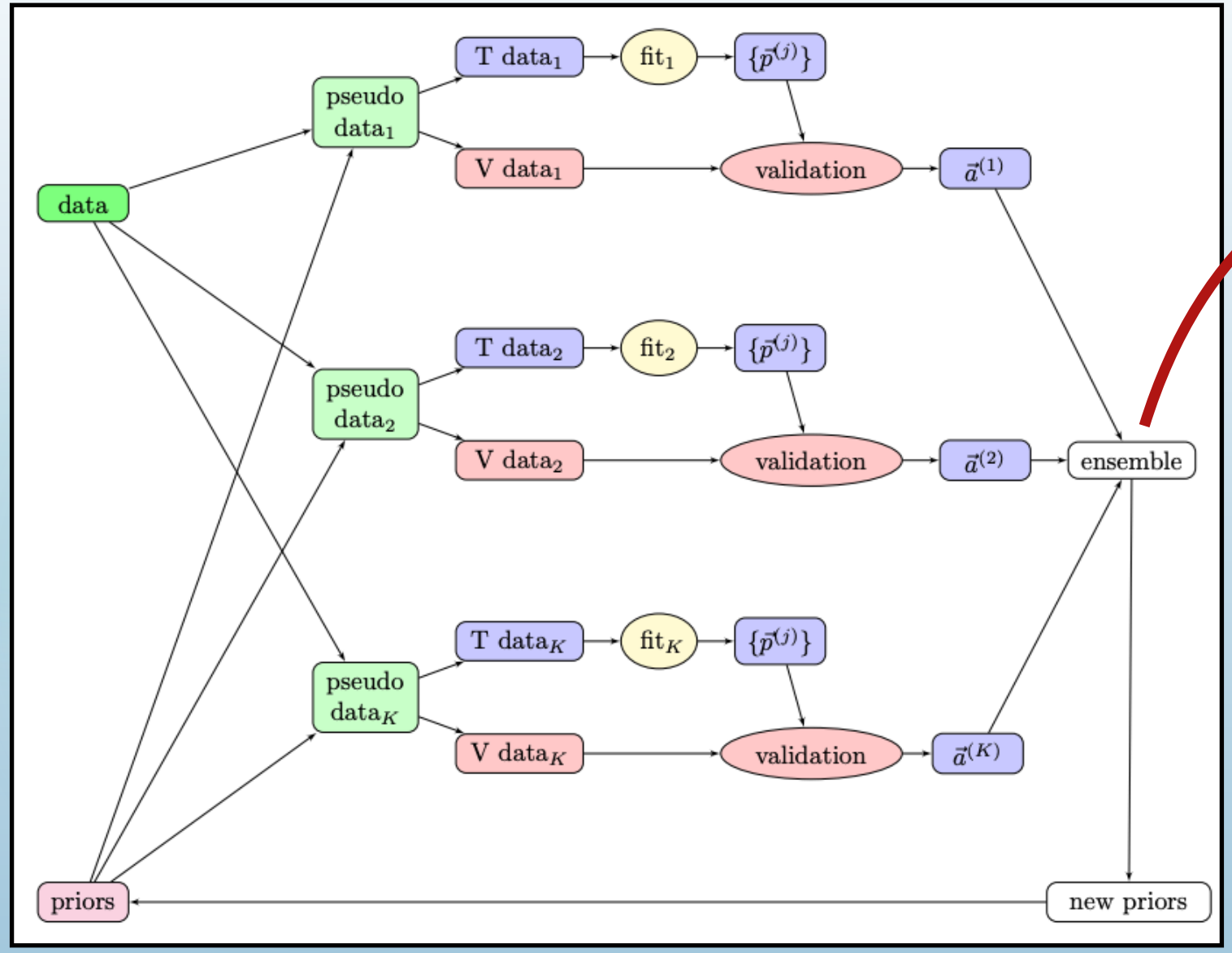


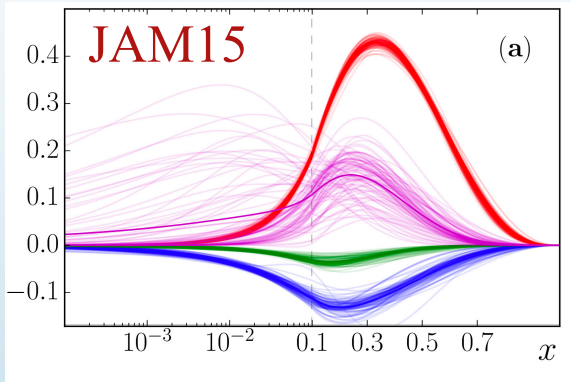


+

$$E[O] \approx \frac{1}{N}\sum_k O(\boldsymbol{a}_k)$$

$$V[O] \approx \frac{1}{N}\sum_k \left[O(\boldsymbol{a}_k) - E[O]\right]^2$$





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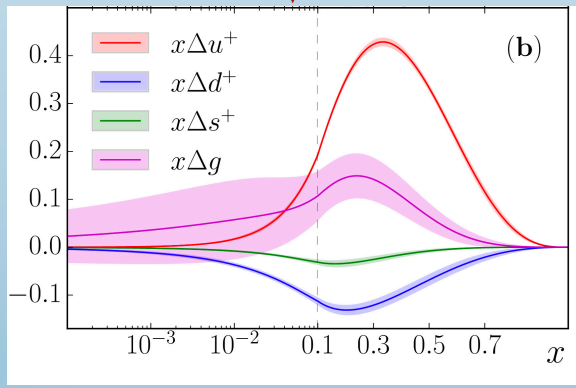
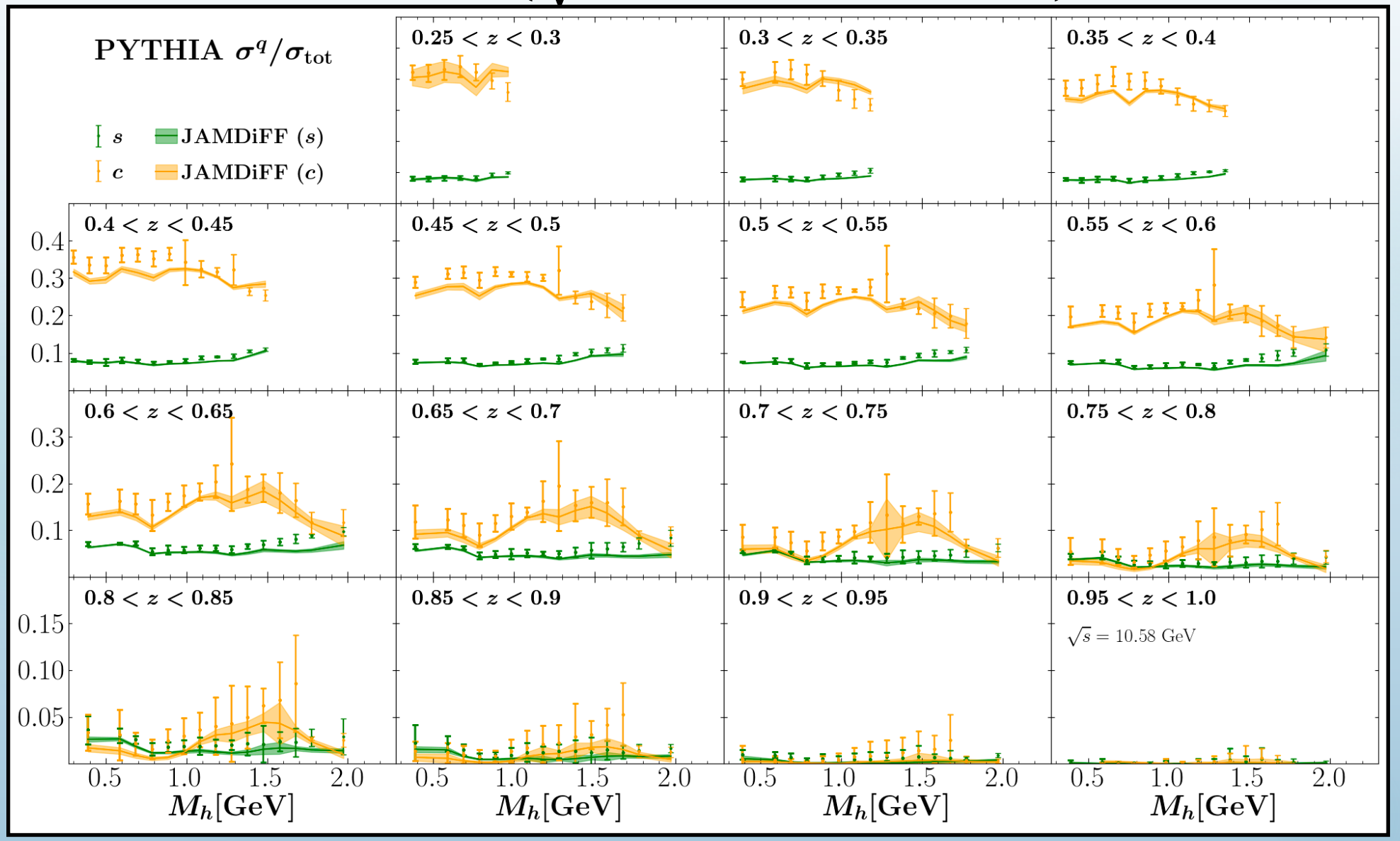
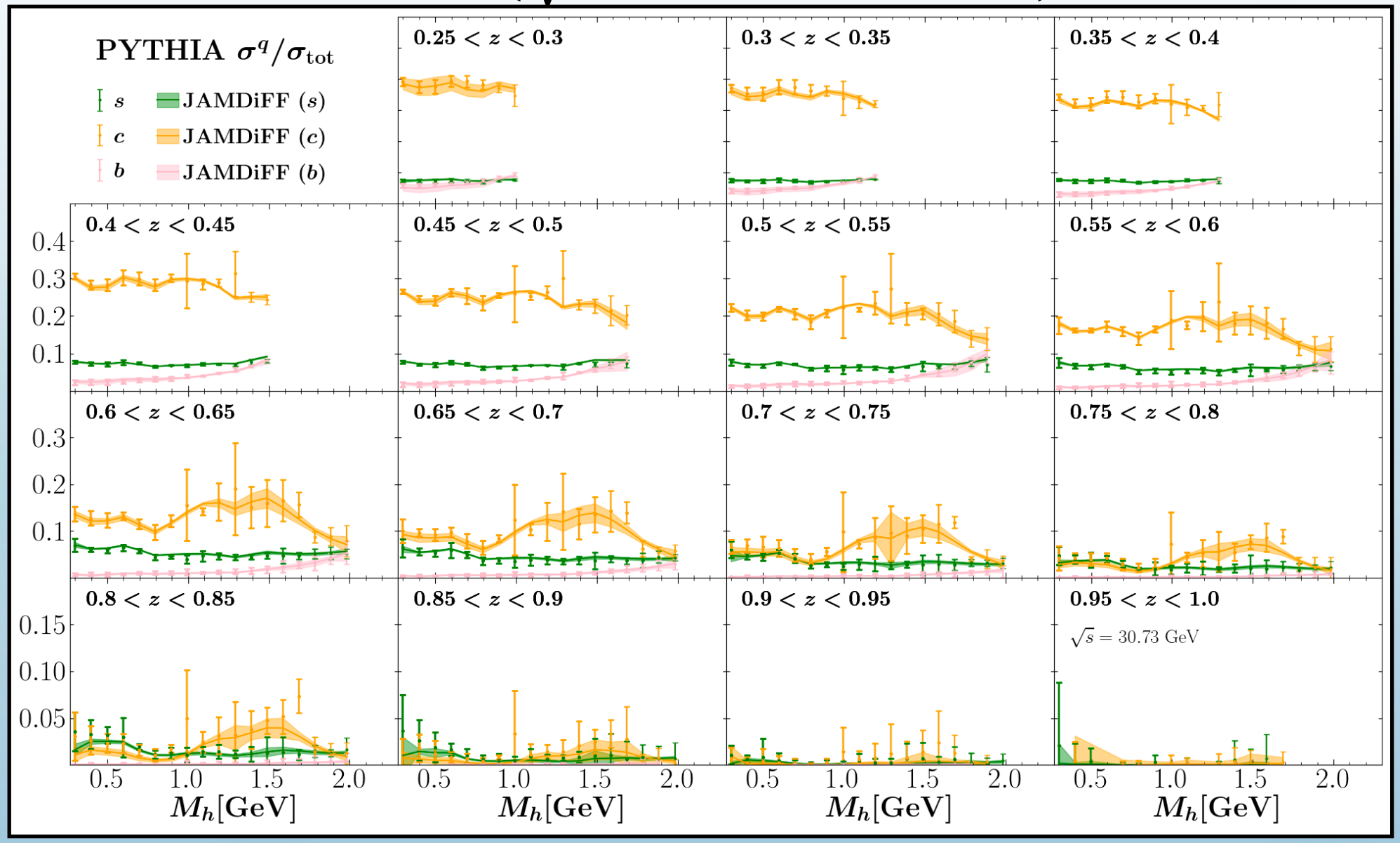


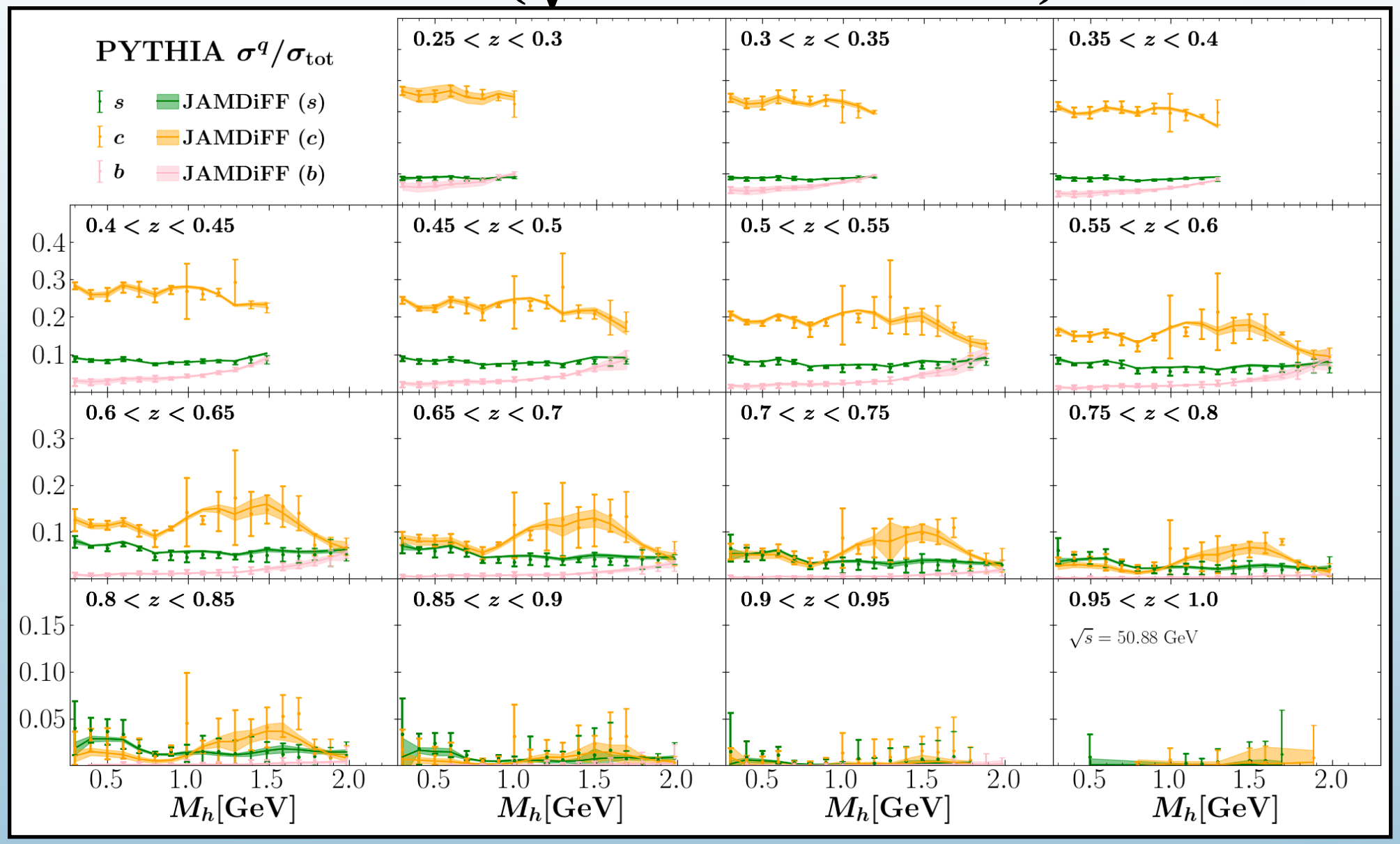
PYTHIA data ($\sqrt{s} = 10.58$ GeV)



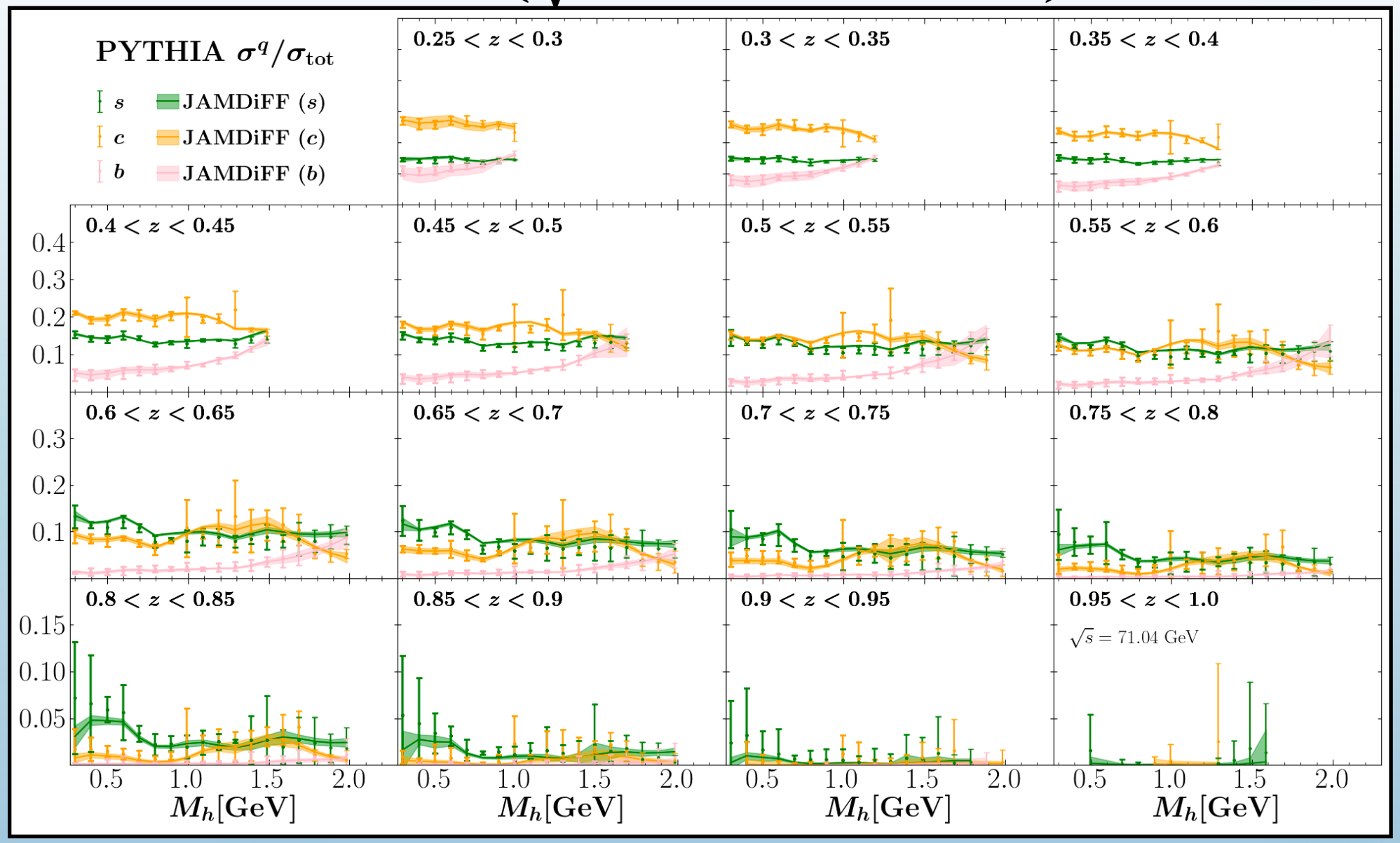
PYTHIA data ($\sqrt{s} = 30.73$ GeV)



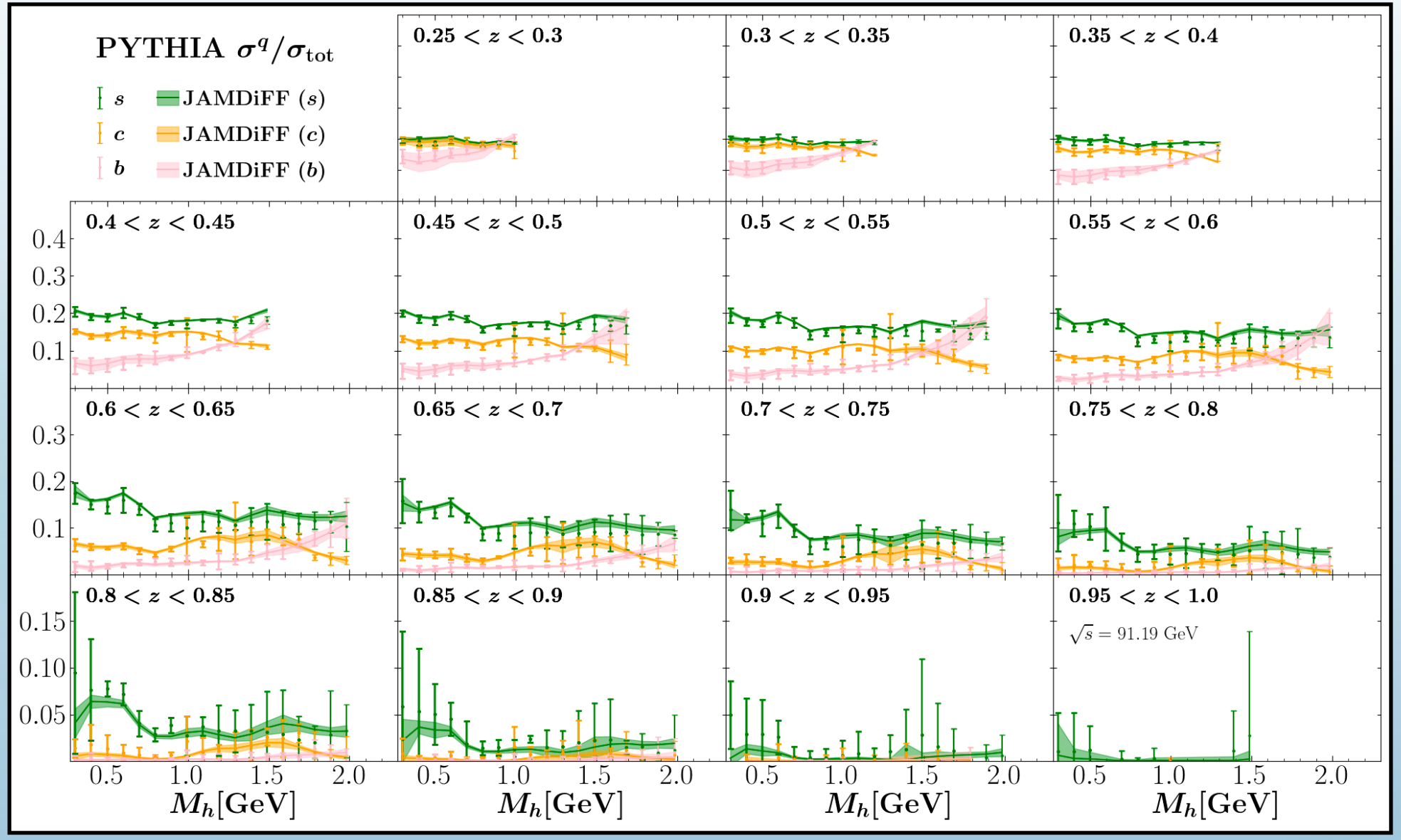
PYTHIA data ($\sqrt{s} = 50.88$ GeV)



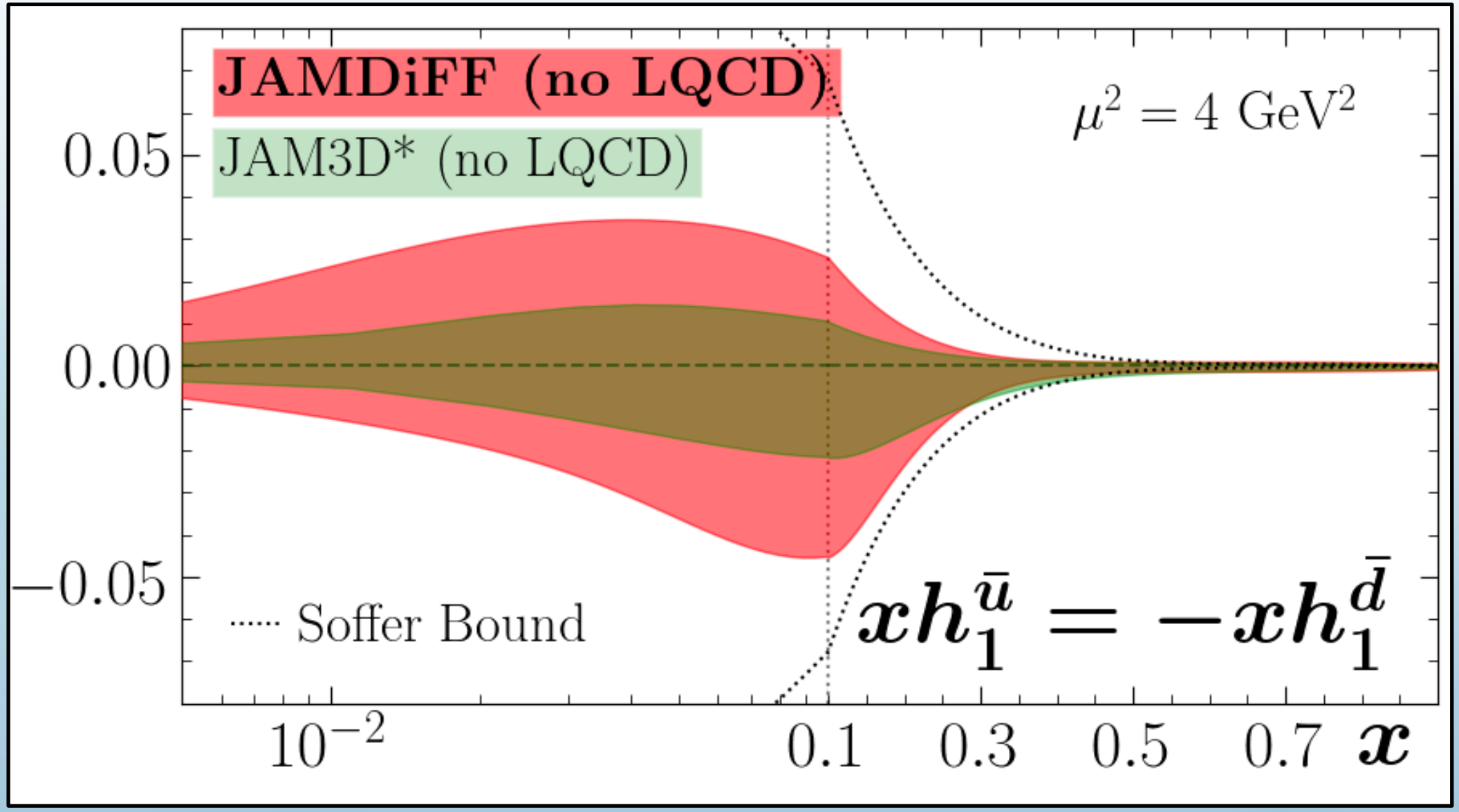
PYTHIA data ($\sqrt{s} = 71.04$ GeV)



PYTHIA data ($\sqrt{s} = 91.19$ GeV)



Transversity PDFs (antiquarks)



DiFF Parameterization

$$\mathbf{M}_h^u = [2m_\pi, 0.40, 0.50, 0.70, 0.75, 0.80, 0.90, 1.00, 1.20, 1.30, 1.40, 1.60, 1.80, 2.00] \text{ GeV}.$$

$$D_1^q(z, \mathbf{M}_h^{q,i}) = \sum_{j=1,2,3} \frac{N_{ij}^q}{\mathcal{M}_{ij}^q} z^{\alpha_{ij}^q} (1-z)^{\beta_{ij}^q},$$

204 parameters for D_1
48 parameters for $H_1^\sphericalangle$

PDF Parameterization

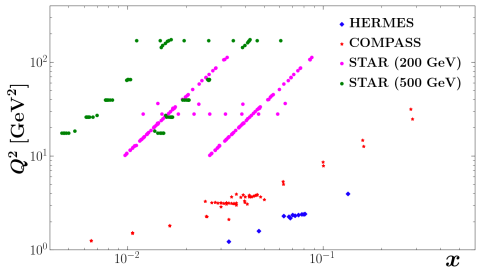
$$h_1^{u_v}$$
$$h_1^{d_v}$$
$$h_1^{\bar{u}} = -\, h_1^{\bar{d}}$$

$$f(x,\mu_0^2) = \frac{N}{\mathcal{M}}\, x^{\alpha}(1-x)^{\beta}(1+\gamma\sqrt{x}+\eta x),$$

15 parameters for h_1

Experiment + Lattice + Theory

EXPERIMENT
(measured region)



LATTICE
(full moments)

$$\delta u \equiv \int_0^1 \mathrm{d}x (h_1^u - h_1^{\bar{u}}),$$

$$\delta d \equiv \int_0^1 \mathrm{d}x (h_1^d - h_1^{\bar{d}}),$$

$$g_T \equiv \delta u - \delta d,$$

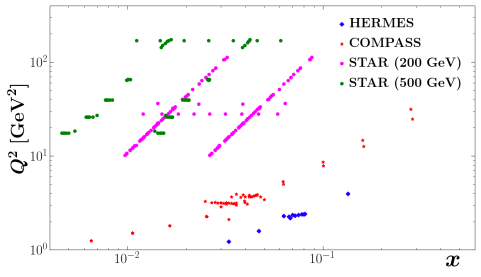
THEORY
(unmeasured regions)

$$|h_1^q| < \frac{1}{2}\left[f_1^q + g_1^q\right]$$

$$\alpha_q = 1 - 2\sqrt{\frac{\alpha_s N_c}{2\pi}}$$

Experiment + Lattice + Theory

EXPERIMENT
(measured region)



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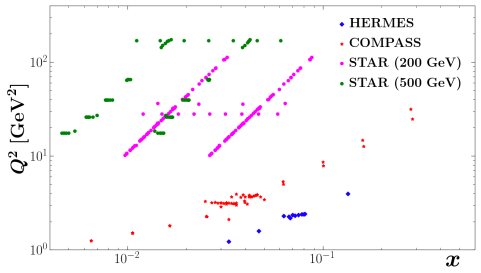
$$g_T \equiv \delta u - \delta d,$$

Presently, trivial to find compatibility between any two

Experiment + Lattice + Theory

EXPERIMENT
(measured region)



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Experiment + Lattice + Theory

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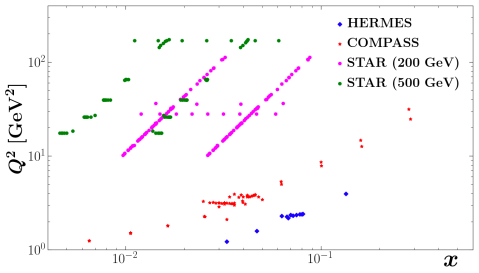
(measured region)



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Experiment + Lattice + Theory

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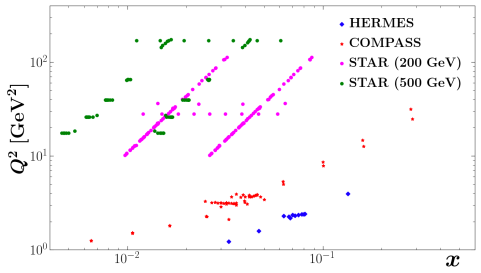
Experiment + Lattice + Theory

EXPERIMENT
(measured region)



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$$\delta d \equiv \int_0^1 \mathrm{d}x (h_1^d - h_1^{\bar{d}}),$$

$$g_T \equiv \delta u - \delta d,$$

Presently, trivial to find compatibility between any two

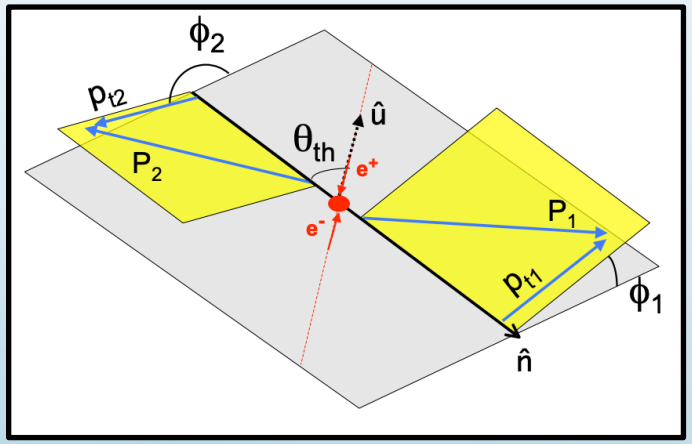
Only meaningful when all three are included

Tensor Charge Numbers

Fit	δu	δd	g_T
no LQCD	0.50(7)	-0.04(14)	0.54(12)
w/ LQCD	0.71(2)	-0.200(6)	0.91(2)

3D/Twist-3 Observables

SIA

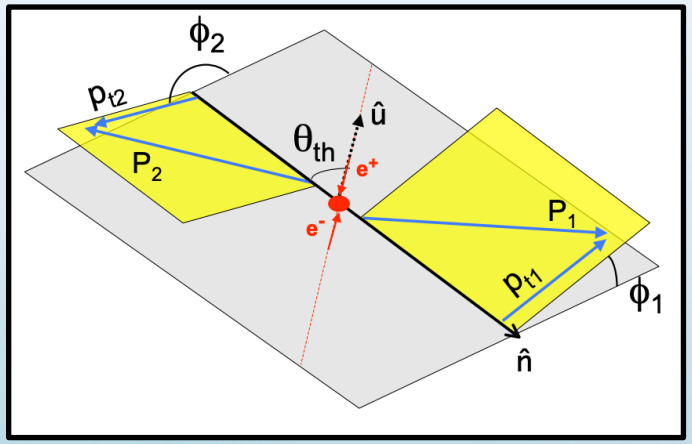


J.P. Lees *et al.*, Phys. Rev. D **90**, 052003 (2014)

$$F^{h_1 h_2}_{\cos 2\phi_0} = C\left[\frac{2(\hat{h}\cdot\vec{p}_{1T})(\hat{h}\cdot\vec{p}_{2T}) - \vec{p}_{1T}\cdot\vec{p}_{2T}}{M_{h_1}M_{h_2}} H_1^{\perp,h_1}\bar{H}_1^{\perp,h_2}\right]$$

3D/Twist-3 Observables

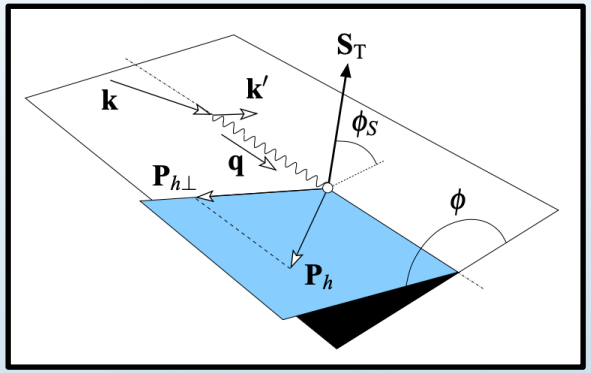
SIA



J.P. Lees *et al.*, Phys. Rev. D **90**, 052003 (2014)

$$F^{h_1h_2}_{\cos 2\phi_0} = C\Big[\frac{2(\hat{h}\cdot\vec{p}_{1T})(\hat{h}\cdot\vec{p}_{2T}) - \vec{p}_{1T}\cdot\vec{p}_{2T}}{M_{h_1}M_{h_2}} H_1^{\perp,h_1}\bar{H}_1^{\perp,h_2}\Big]$$

SIDIS



A. Airapetian *et al.*, JHEP **12**, 010 (2020)

$$F_{UU} = C\big[f_1 D_1\big] \quad \text{(constrains widths)}$$

$$F^{\sin(\phi_h-\phi_s)}_{UT} = -C\Big[\frac{\hat{h}\cdot\vec{k}_T}{M} f_{1T}^{\perp} D_1\Big]$$

$$F^{\sin(\phi_h+\phi_s)}_{UT} = -C\Big[\frac{\hat{h}\cdot\vec{p}_T}{M_h} h_1 H_1^{\perp}\Big]$$

$$F^{\sin\phi_S}_{UT} = \frac{2M}{Q} C\Big[-\frac{M_h}{zM} h_1\tilde{H} + \ldots\Big]$$

Observable Count!

Observable Count!

Transversity h_1 — **8** — Collins SIDIS [p ($\pi^{\pm,0}$), D ($\pi^{\pm}$)] + A_N π^0 + Hadron-in-jet $\pi^{\pm}$

Observable Count!

Transversity h_1 — **8** — Collins SIDIS [p $(\pi^{\pm,0})$, D $(\pi^{\pm})$] + A_N π^0 + Hadron-in-jet $\pi^{\pm}$

Sivers $f_{1T}^{\perp(1)}$ — **21** — Sivers SIDIS [p $(\pi^{\pm,0}, K^{\pm,0}, h^{\pm})$, D $(\pi^{\pm}, K^{\pm,0}, h^{\pm})$] + Drell-Yan + $A_N^{W^{\pm},Z}$ + A_N π^0 + A_N^{jet}

Observable Count!

Transversity h_1 — **8** — Collins SIDIS [p $(\pi^{\pm,0})$, D $(\pi^{\pm})$] + A_N π^0 + Hadron-in-jet $\pi^{\pm}$

Sivers $f_{1T}^{\perp(1)}$ — **21** — Sivers SIDIS [p $(\pi^{\pm,0}, K^{\pm,0}, h^{\pm})$, D $(\pi^{\pm}, K^{\pm,0}, h^{\pm})$] + Drell-Yan + $A_N^{W^{\pm},Z}$ + A_N π^0 + A_N^{jet}

Collins (pion) $H_1^{\perp(1)}$ — **10** — Collins SIDIS [p $(\pi^{\pm,0})$, D $(\pi^{\pm})$] + A_N π^0 + Hadron-in-jet $\pi^{\pm}$

Observable Count!

Transversity h_1 — **8** — Collins SIDIS [p $(\pi^{\pm,0})$, D $(\pi^{\pm})$] + A_N π^0 + Hadron-in-jet $\pi^{\pm}$

Sivers $f_{1T}^{\perp(1)}$ — **21** — Sivers SIDIS [p $(\pi^{\pm,0}, K^{\pm,0}, h^{\pm})$, D $(\pi^{\pm}, K^{\pm,0}, h^{\pm})$] + Drell-Yan + $A_N^{W^{\pm},Z}$ + A_N π^0 + A_N^{jet}

Collins (pion) $H_1^{\perp(1)}$ — **10** — Collins SIDIS [p $(\pi^{\pm,0})$, D $(\pi^{\pm})$] + A_N π^0 + Hadron-in-jet $\pi^{\pm}$

Twist-3 FF (pion) $\tilde{H}$ — **6** — $\sin\phi_S$ SIDIS [p $(\pi^{\pm,0}, h^{\pm})$] + A_N π^0

Observable Count!

Transversity h_1 — **8** — Collins SIDIS [p $(\pi^{\pm,0})$, D $(\pi^{\pm})$] + A_N π^0 + Hadron-in-jet $\pi^{\pm}$

Transversity h_1 also has **3** more observables from the dihadron side
LQCD also constrains h_1

Sivers $f_{1T}^{\perp(1)}$ — **21** — Sivers SIDIS [p $(\pi^{\pm,0}, K^{\pm,0}, h^{\pm})$, D $(\pi^{\pm}, K^{\pm,0}, h^{\pm})$] + Drell-Yan + $A_N^{W^{\pm},Z}$ + A_N π^0 + A_N^{jet}

Collins (pion) $H_1^{\perp(1)}$ — **10** — Collins SIDIS [p $(\pi^{\pm,0})$, D $(\pi^{\pm})$] + A_N π^0 + Hadron-in-jet $\pi^{\pm}$

Twist-3 FF (pion) $\tilde{H}$ — **6** — $\sin\phi_S$ SIDIS [p $(\pi^{\pm,0}, h^{\pm})$] + A_N π^0