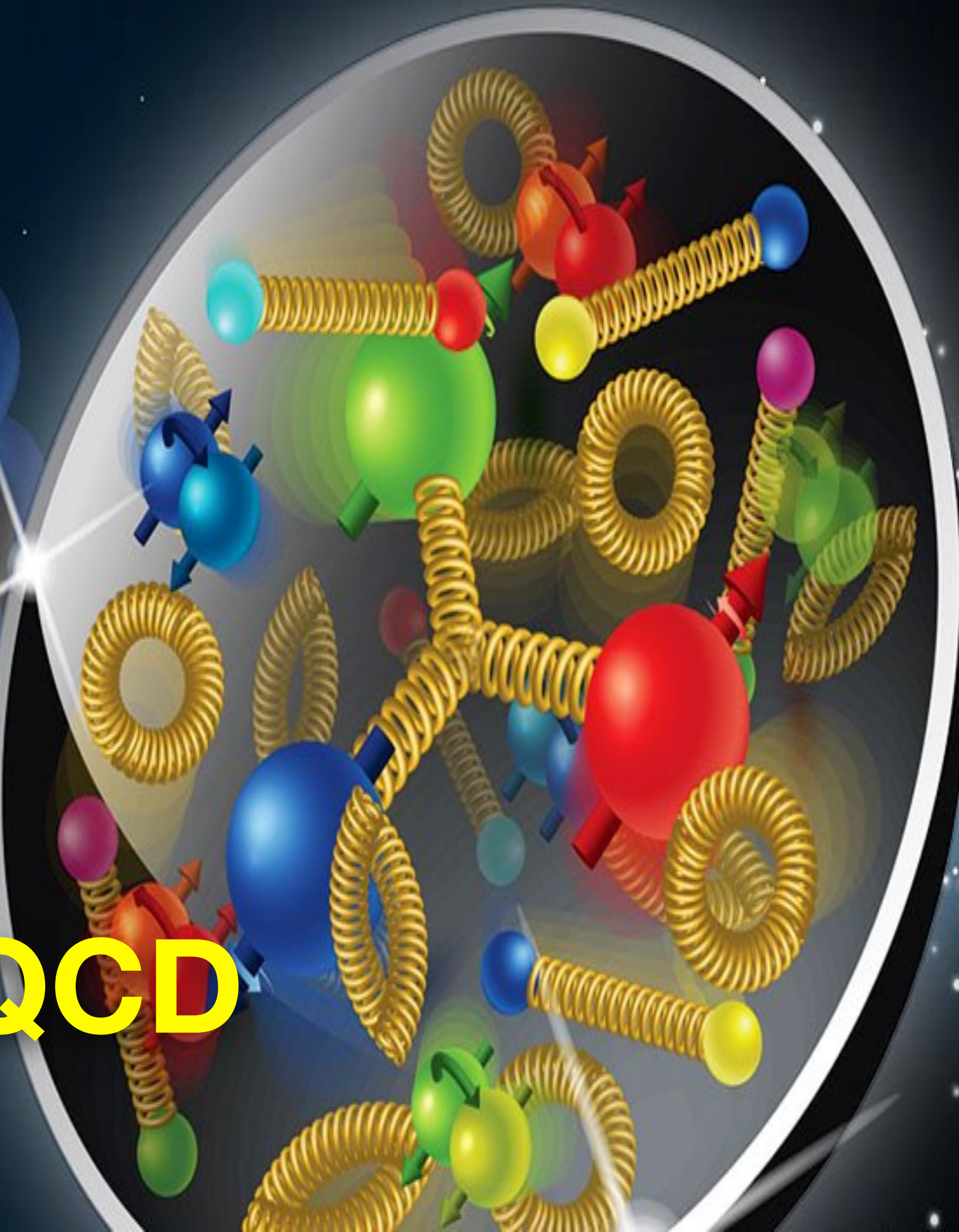
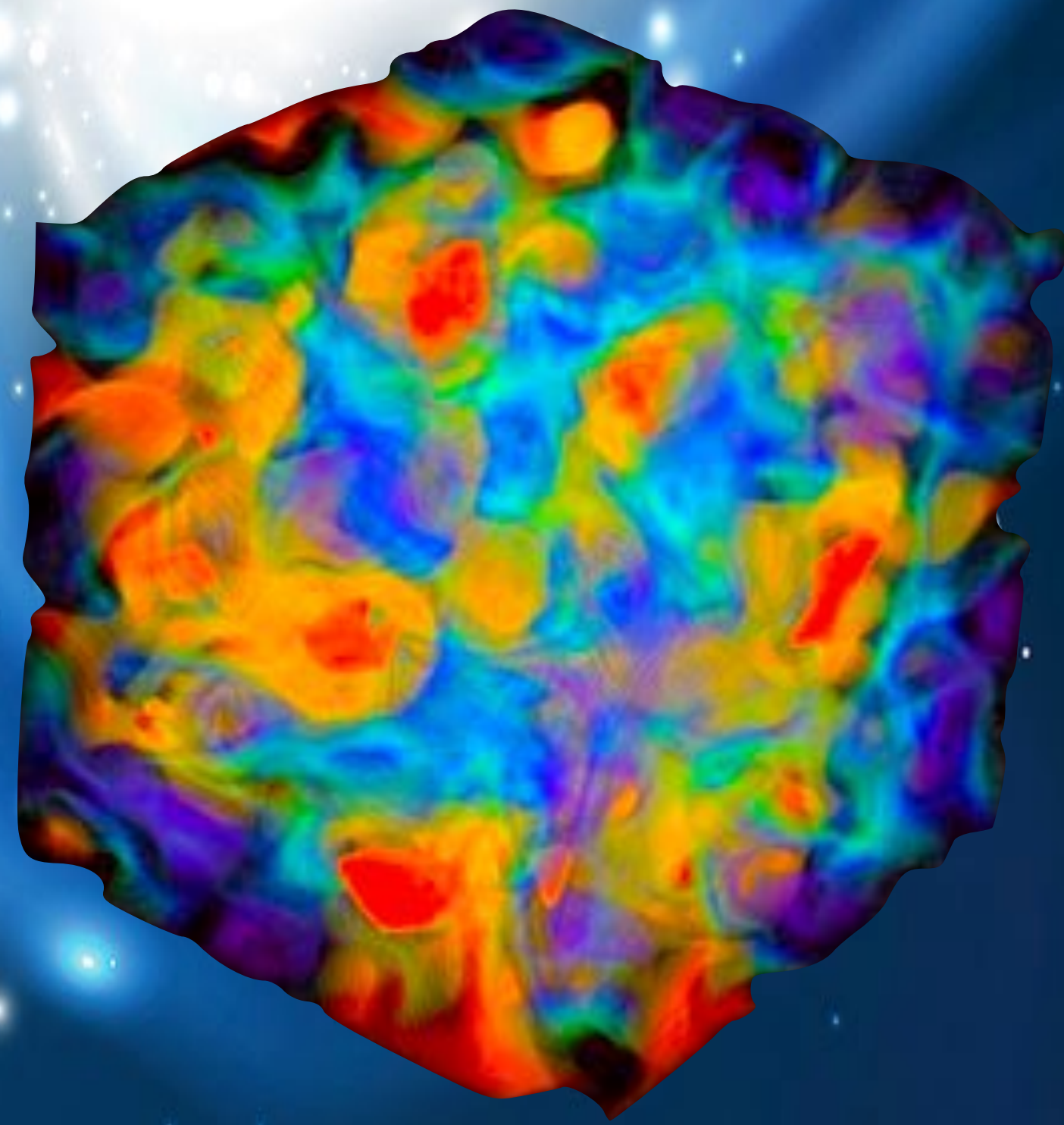




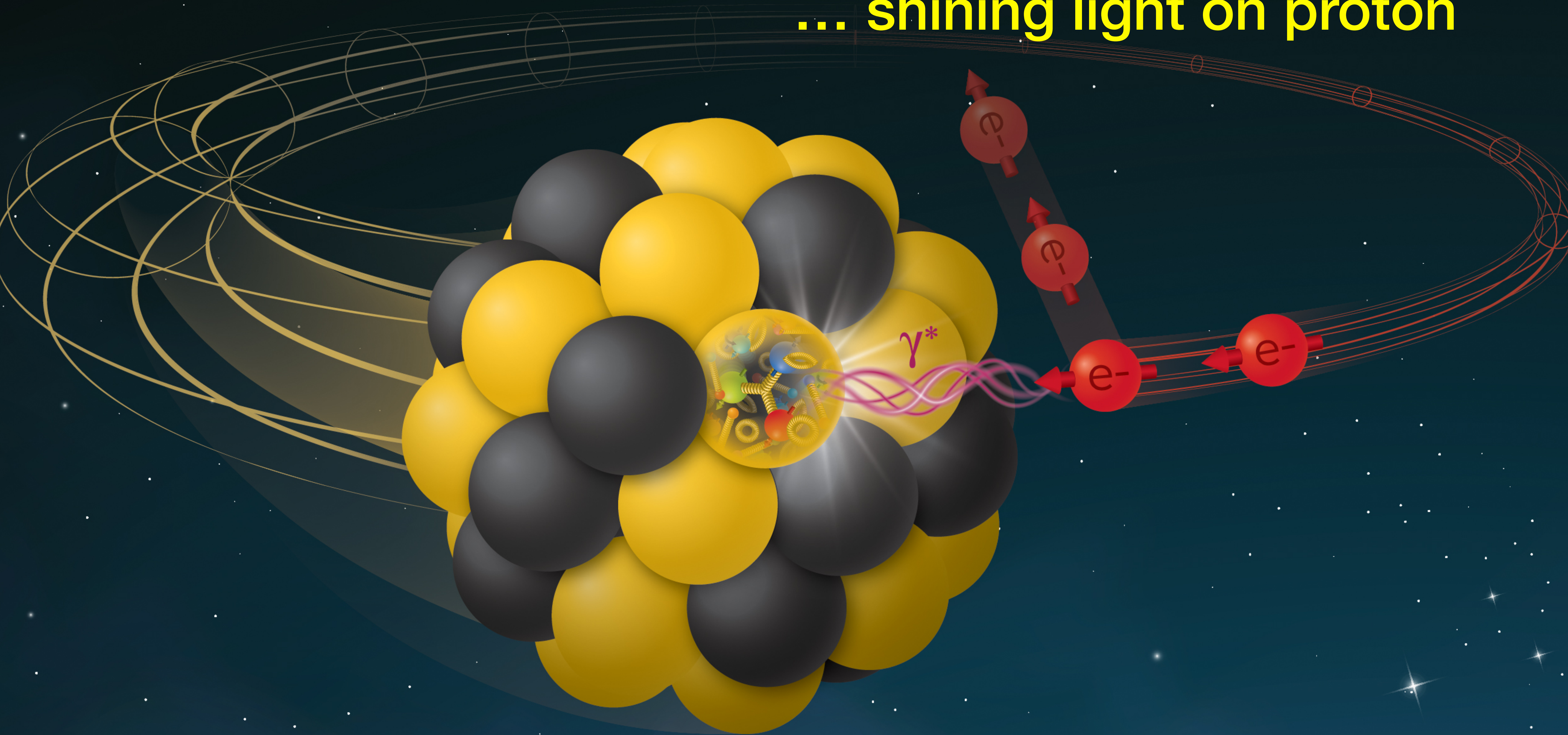
All About Pion from Lattice QCD

... and a little bit more

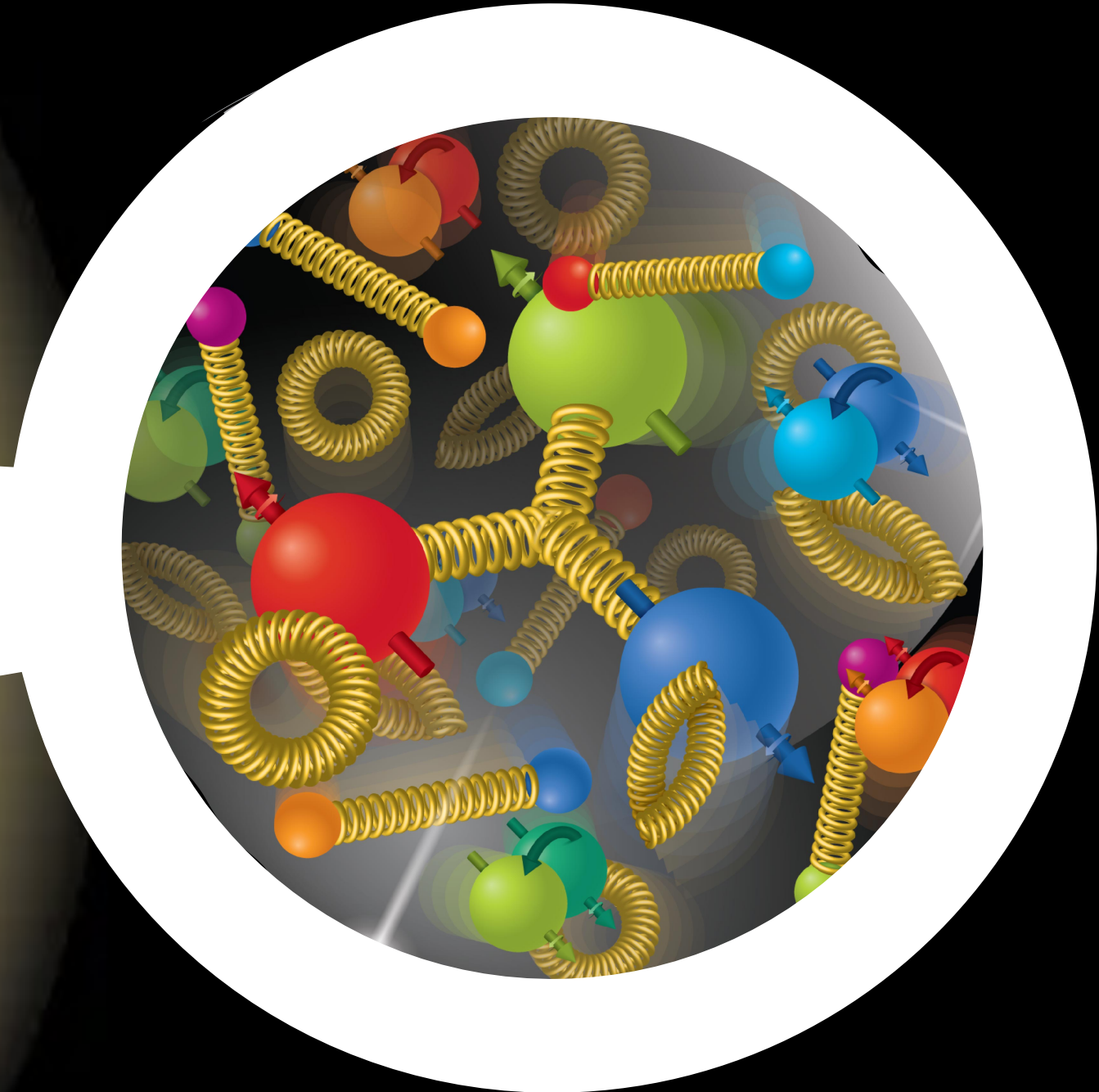
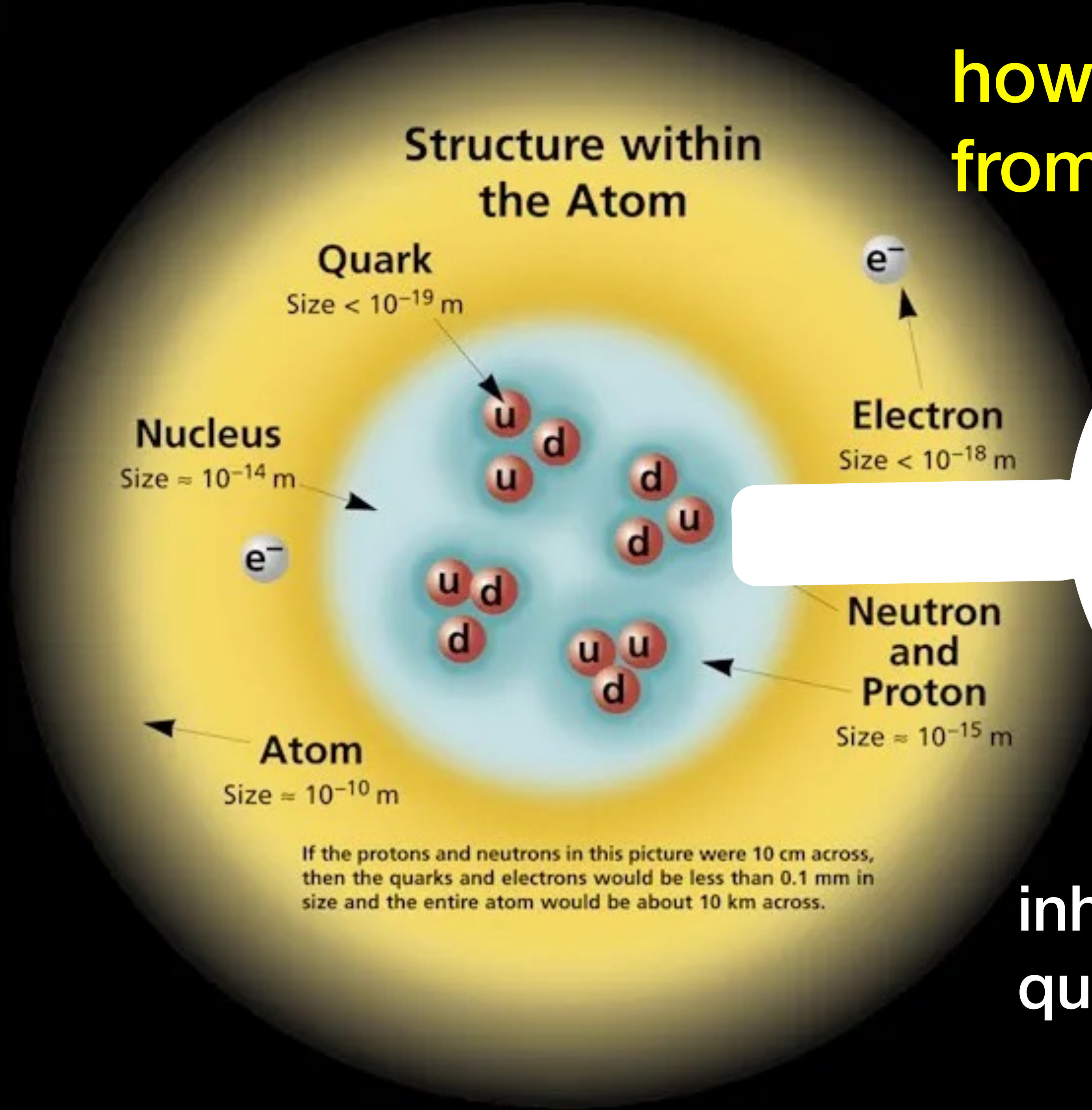
May 2025, Newport News,
Virginia, USA

Swagato Mukherjee

... shining light on proton

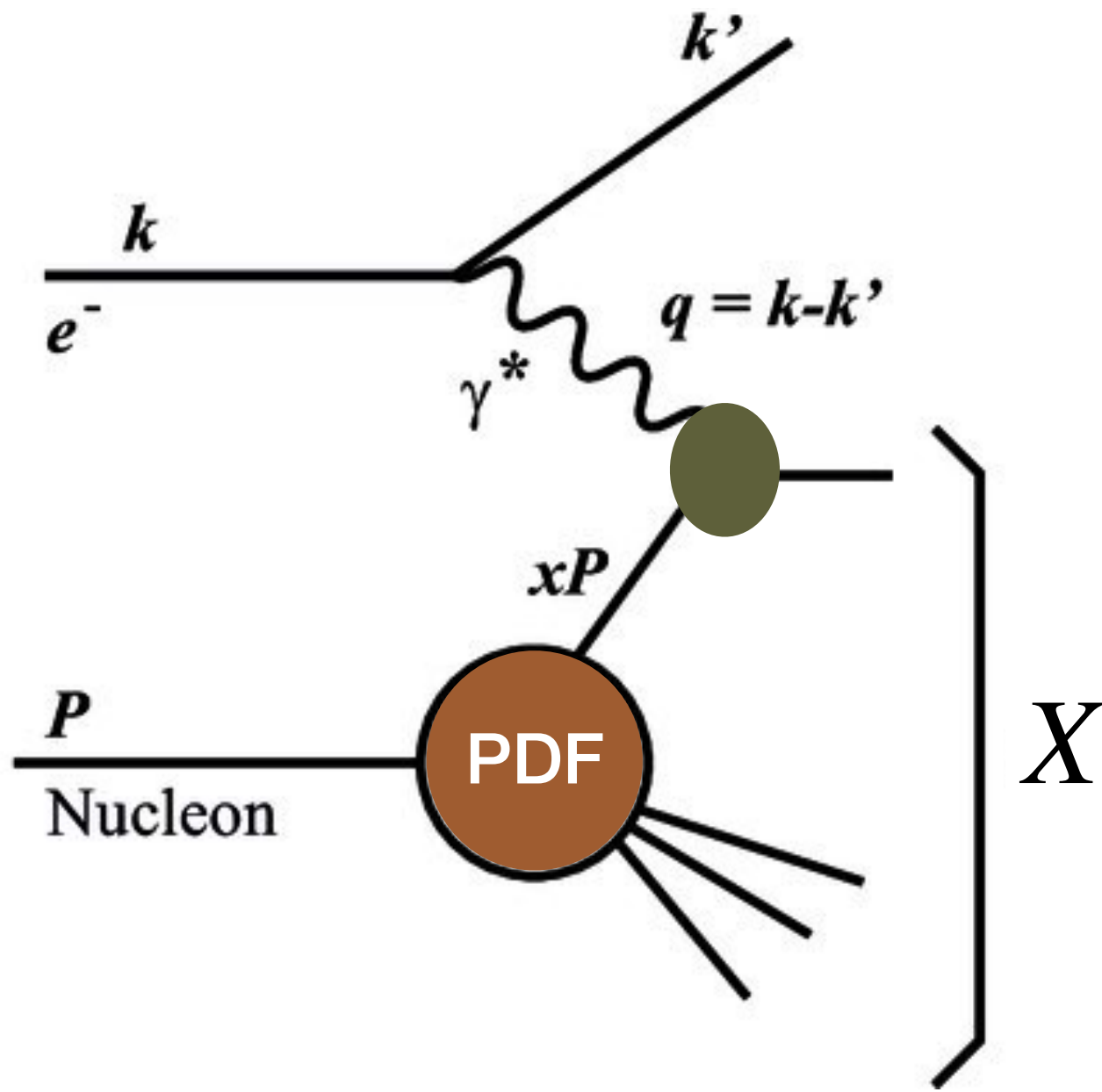


how proton emerges from quarks and gluons?

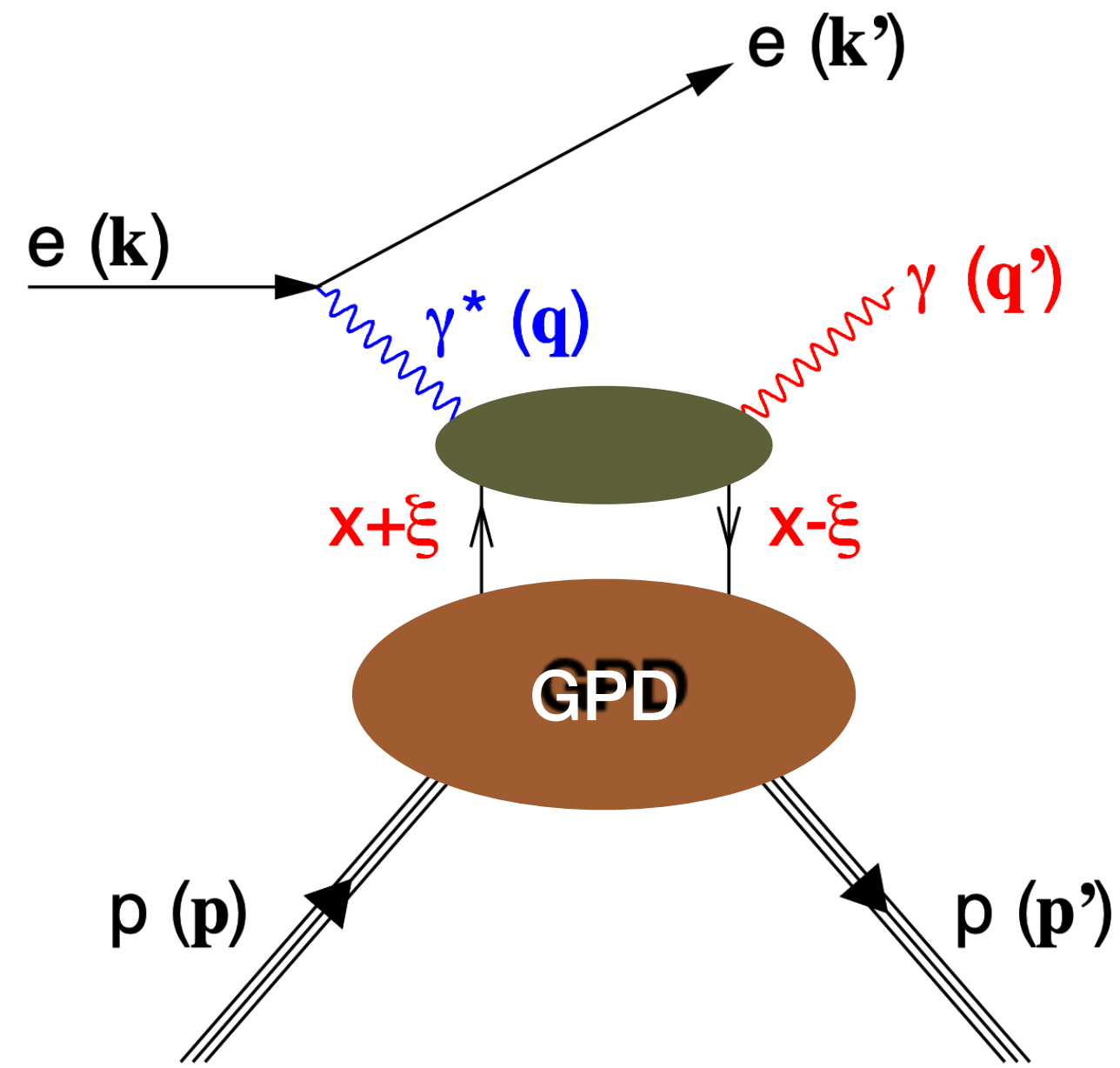


inherently nonperturbative question

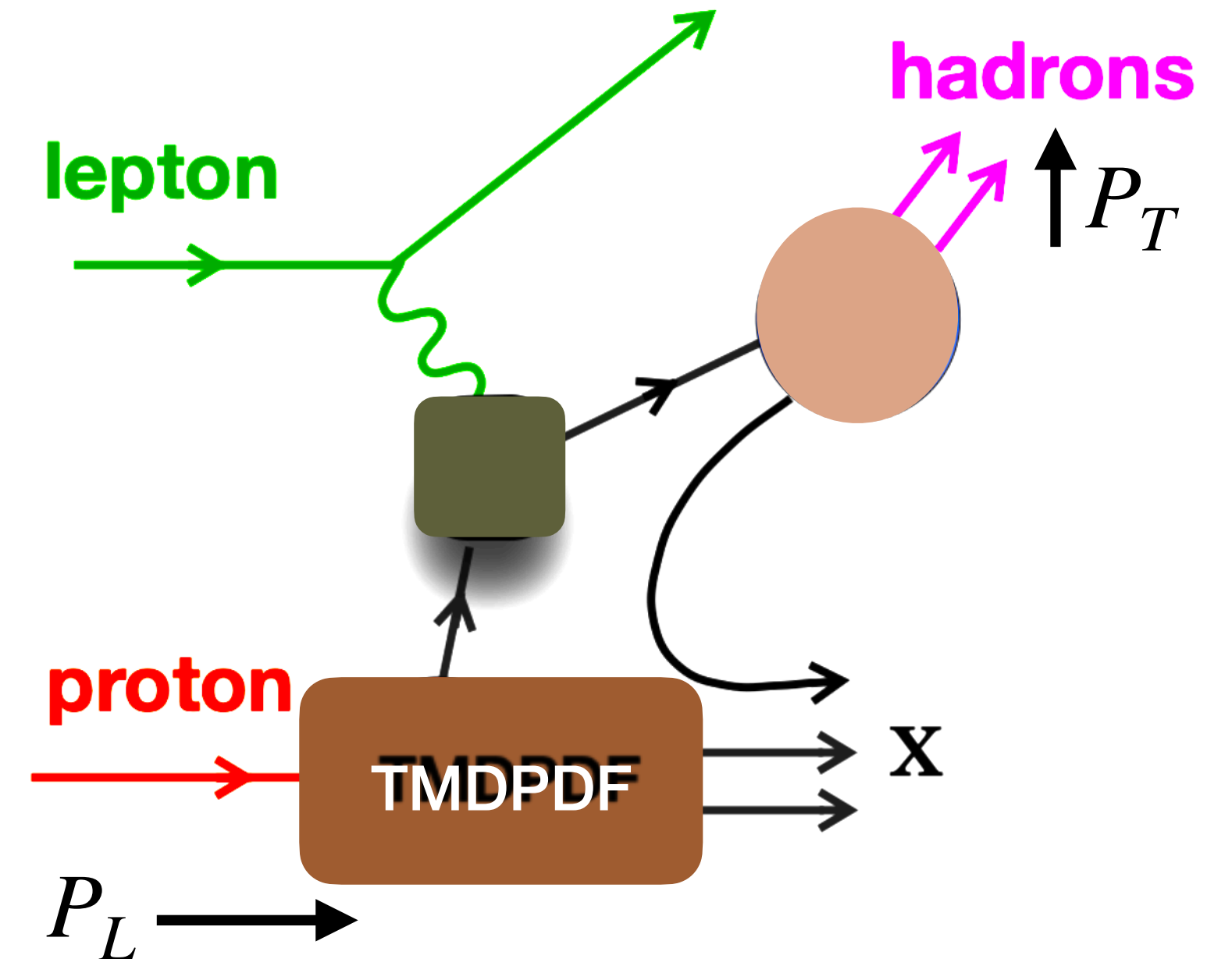
$$e + P \rightarrow e + X$$



$$e + P \rightarrow e + P' + \gamma$$



$$e + P \rightarrow e + H + X$$



$$\text{cross-section} \sim c(y, x, \mu) \otimes f(x, \mu)$$

perturbative QCD
analytically calculable

nonperturbative QCD

encodes proton's
quark-gluon structure

knowing proton ...

$$\text{cross-section} \sim c(y, x, \mu) \otimes f(x, \mu)$$

measure at JLAB12/EIC

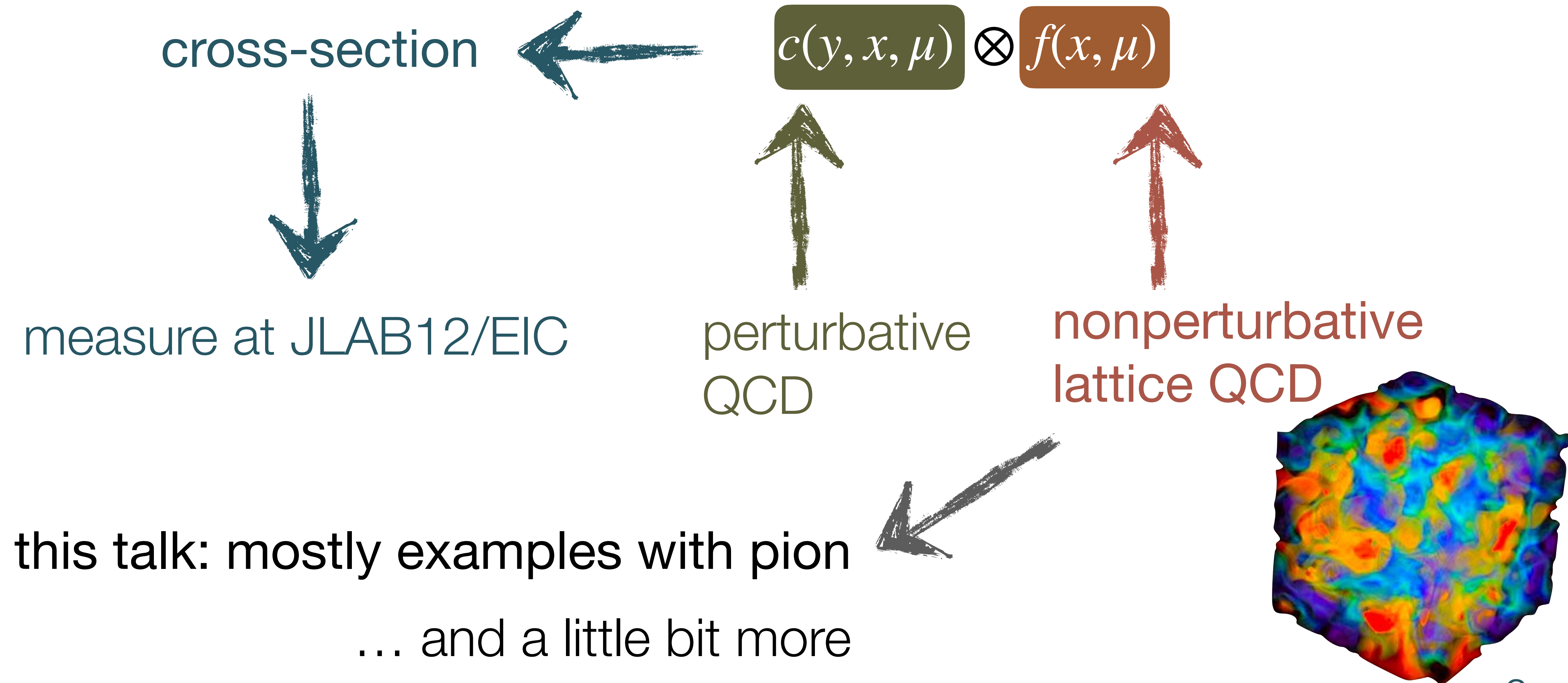
perturbative
QCD

nonperturbative
proton structure



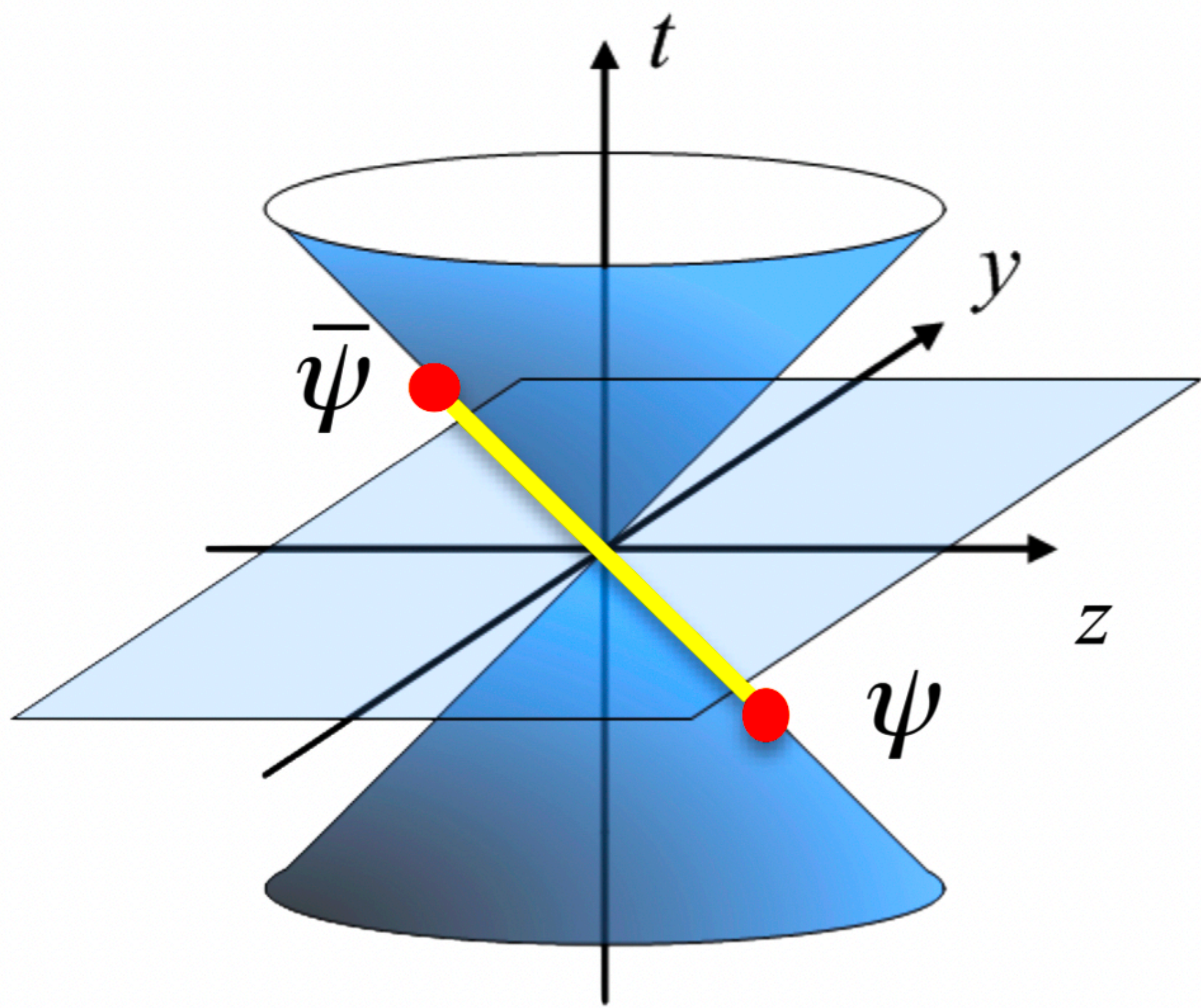
extract by fitting experimental data
using perturbative QCD inputs

dream: understanding proton ... from QCD to cross-section



nonperturbative partonic image of hadron

regularize QCD after taking the lightcone, $P_z \rightarrow \infty$ / $z^2 \rightarrow 0$, limit

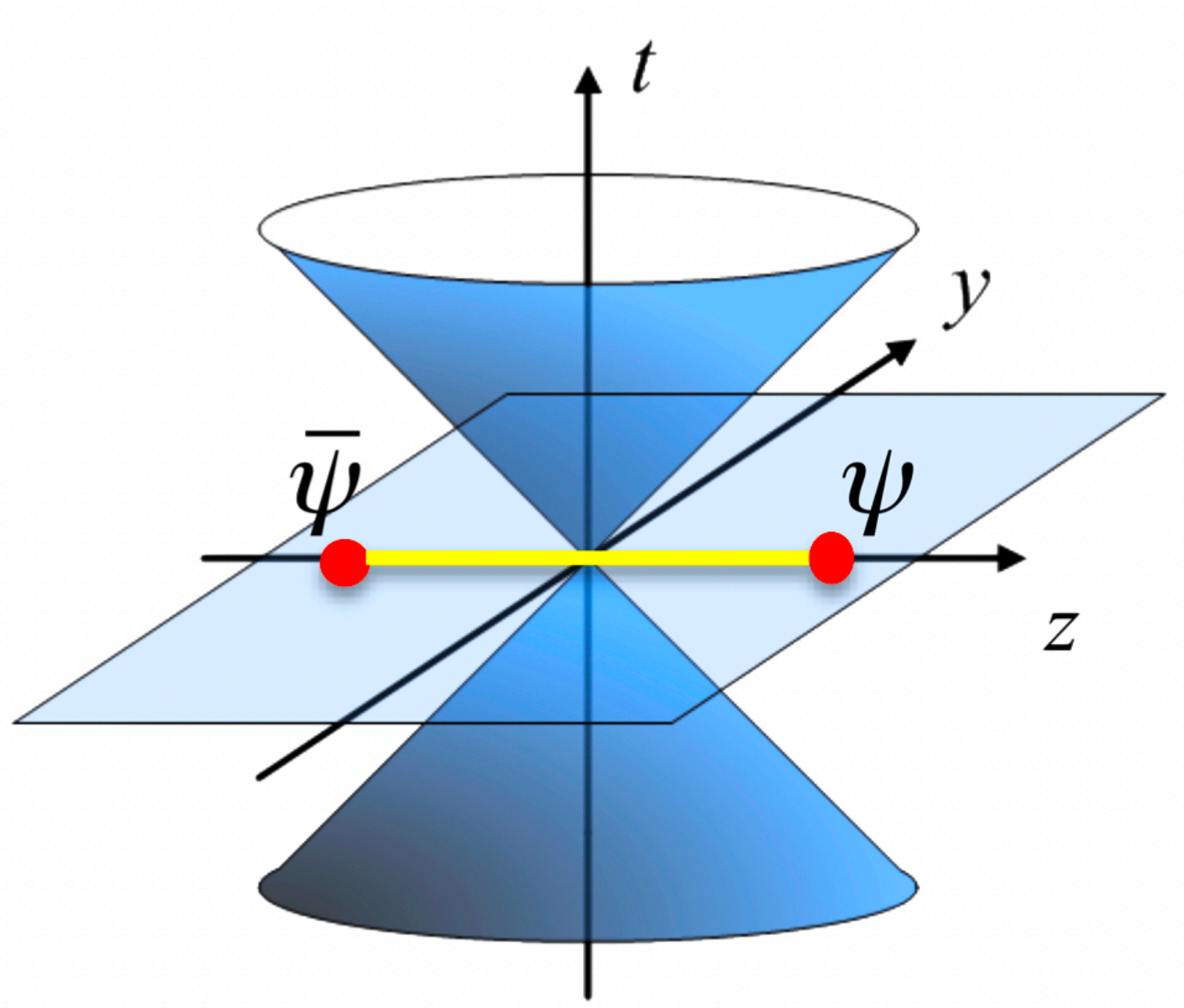
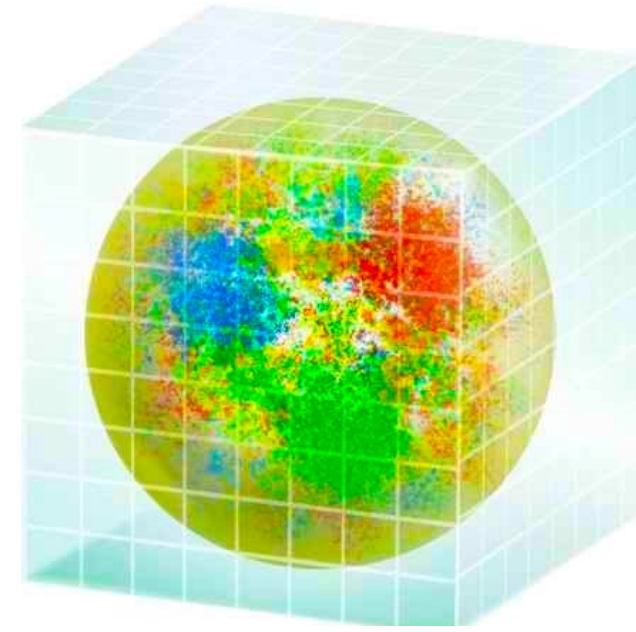


$$f(x, \mu) \sim \left\langle H(P_z) \left| \hat{O}(z^-, \mu) \right| H(P_z) \right\rangle$$

timelike separated bilocal operator

partonic structures from lattice QCD

hadron at rest

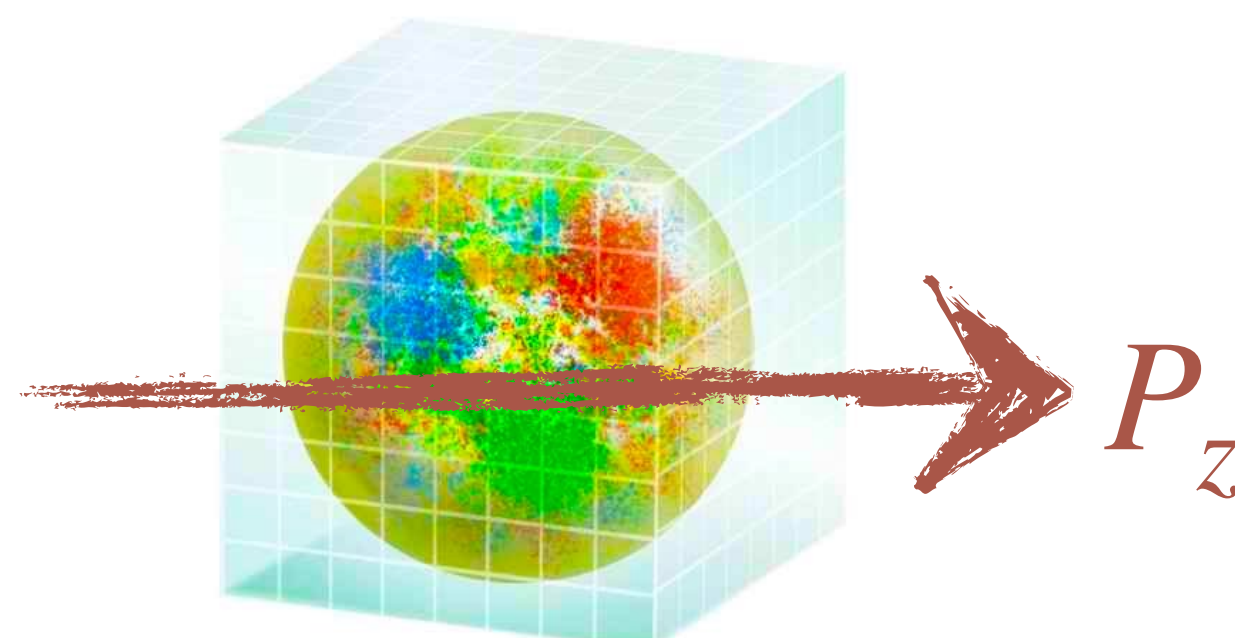


renormalize: scale μ

$$M(z^2, \mu) \sim \left\langle H(0) \left| \hat{O}(z, \mu) \right| H(0) \right\rangle$$

spacelike separated bilocal operator

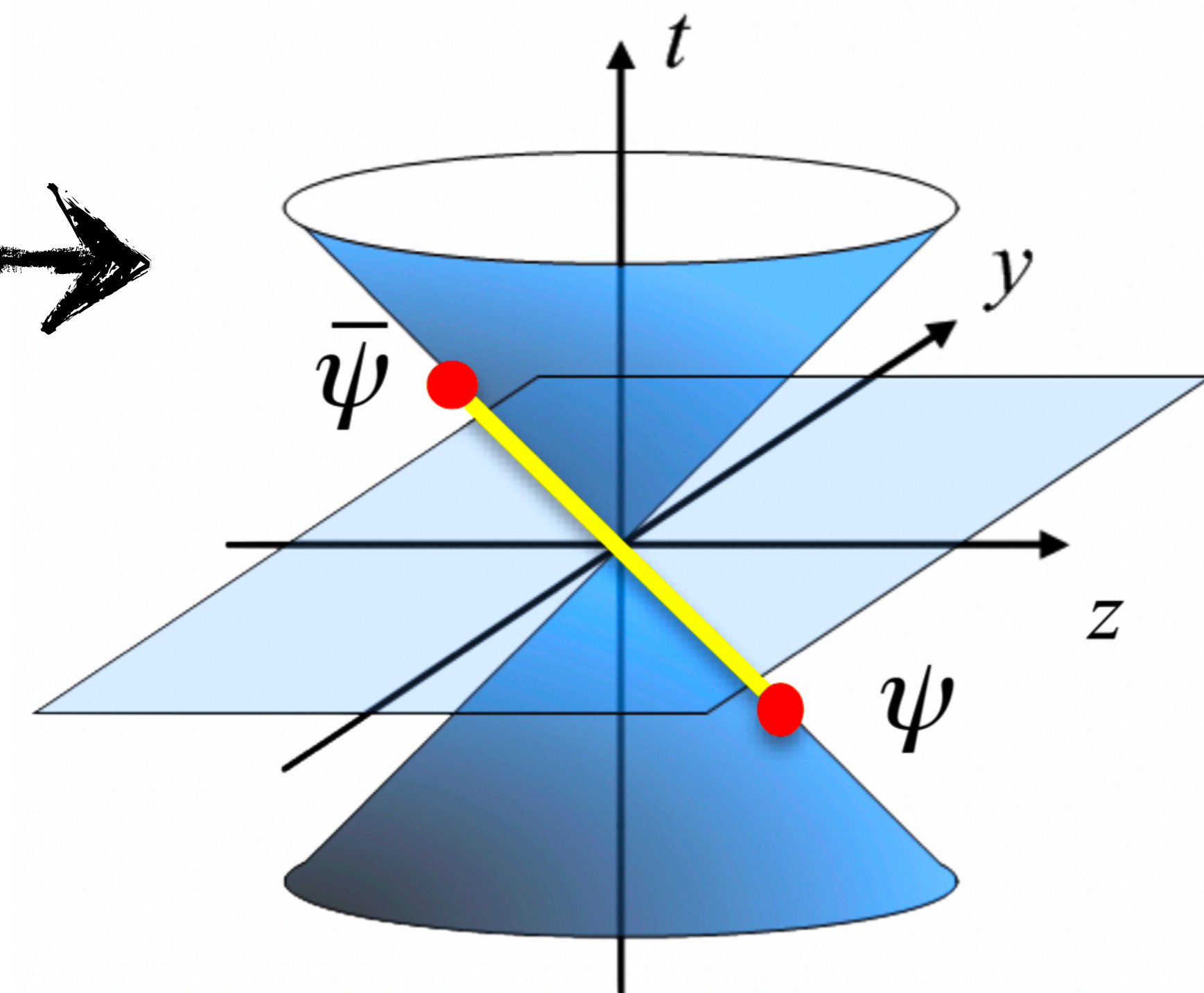
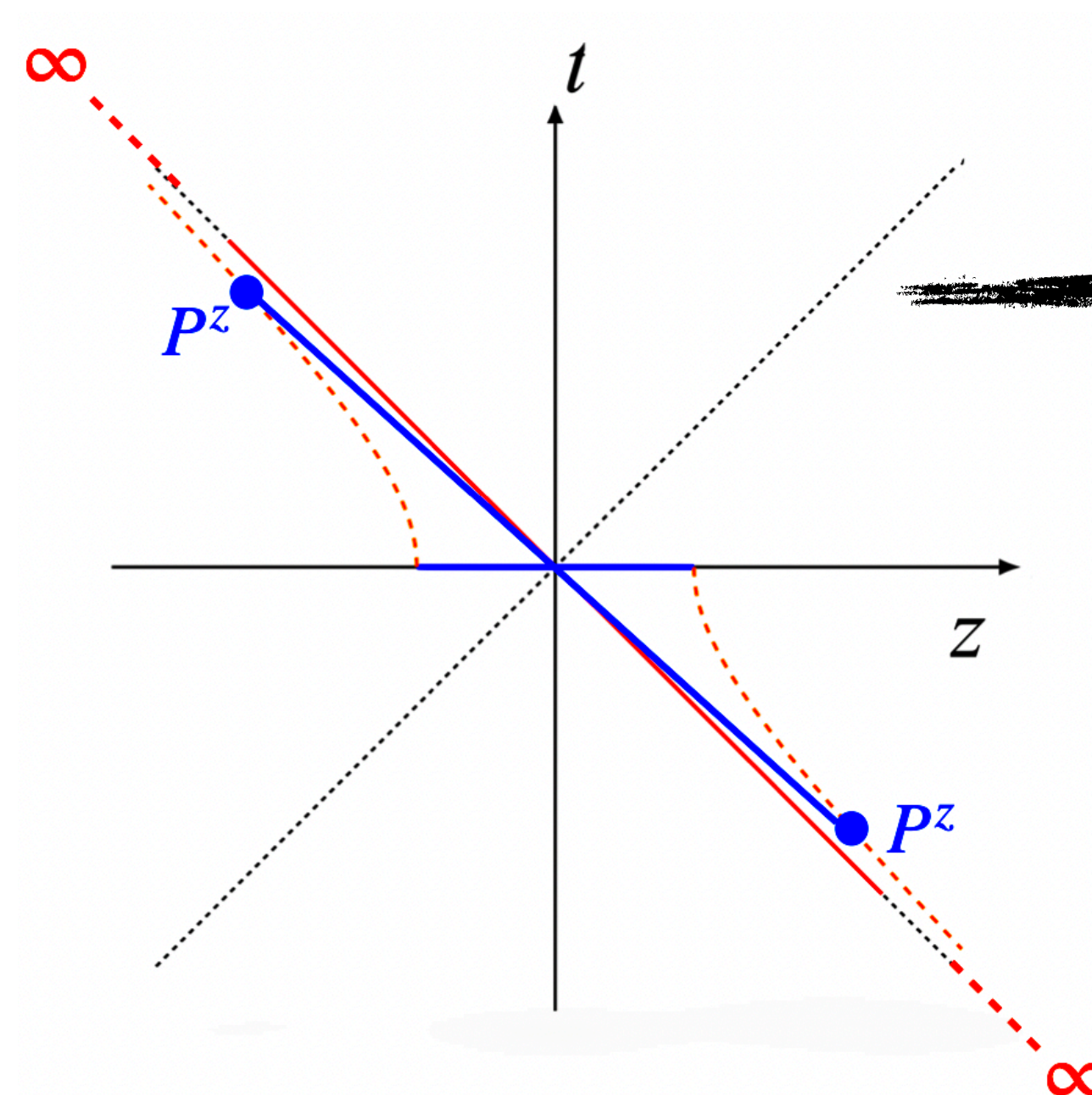
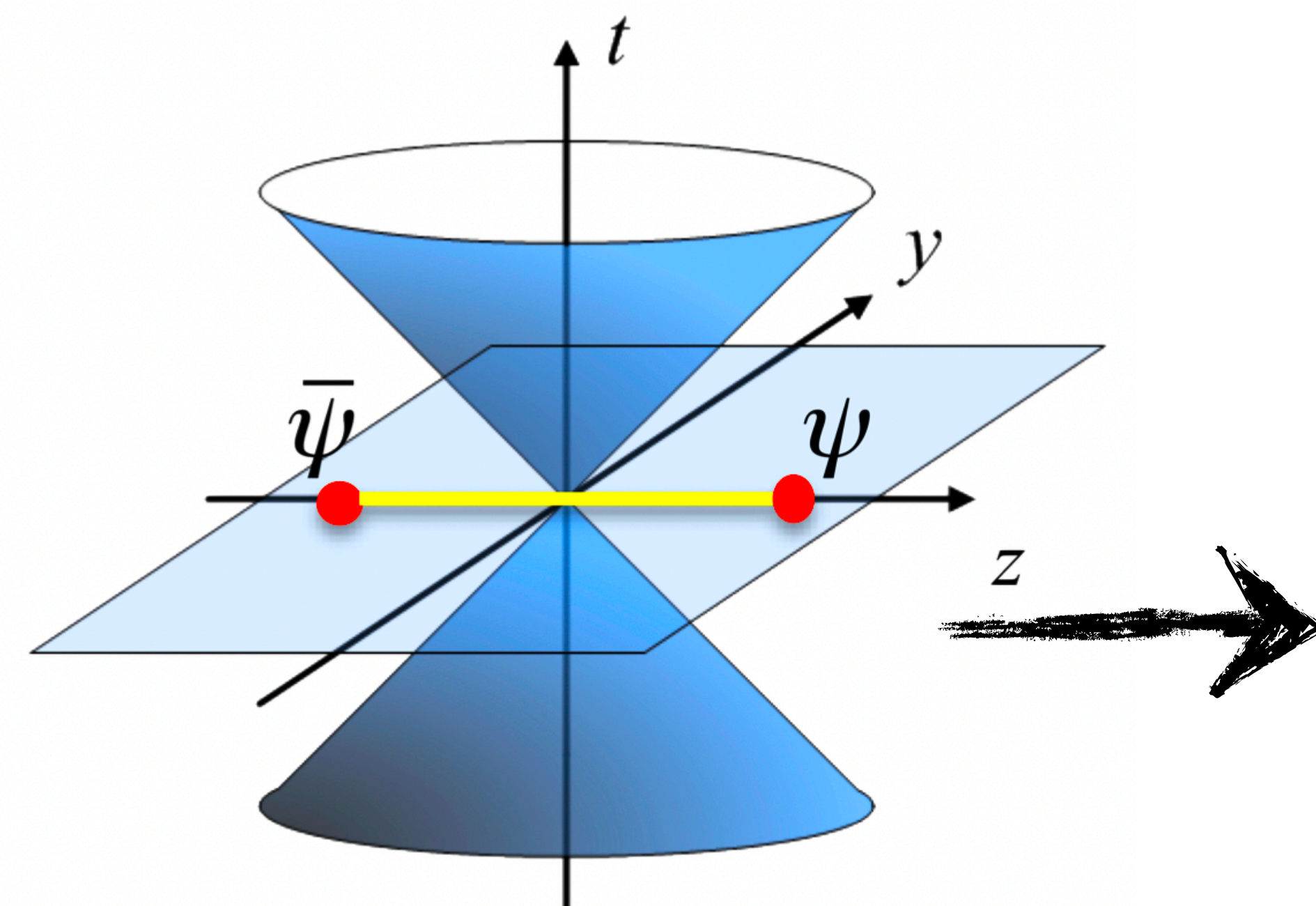
fast-moving hadron



$$P_z \approx E$$

$$\langle H(0) | \hat{O}(z, \mu) | H(0) \rangle$$

$$\langle H(P_z) | \hat{O}(z^-, \mu) | H(P_z) \rangle$$



factorization of $M(y, \mu, P_z) \sim$ perturbative $\otimes$ non-perturbative

$$\tilde{c}(y, x, \mu, P_z) \otimes \tilde{f}(x, \mu)$$

momentum space

$$\tilde{c}(y, x, \mu, z^2) \otimes \tilde{f}(x, \mu)$$

position space

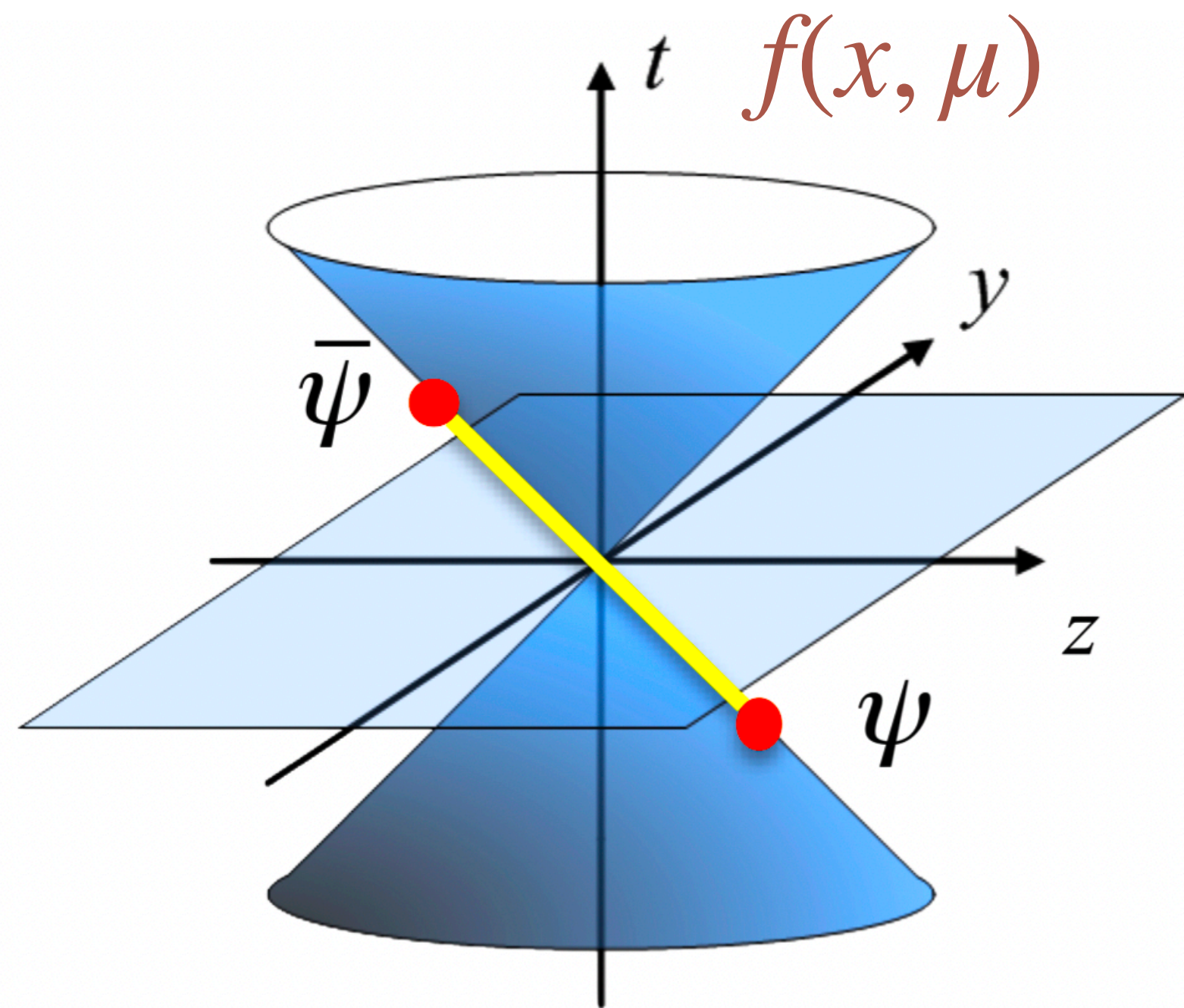
nonperturbative objects on the lightcone, $f(x, \mu)$, and from lattice QCD, $\tilde{f}(x, \mu)$, shares same infrared singularities, i.e. governed by same evolution equations

factorization: perturbative $\otimes$ non-perturbative

$$M(y, \mu, P_z) \sim \tilde{\sigma}(y, x, \mu, P_z) \otimes \tilde{f}(x, \mu) \qquad M(y, \mu, z^2) \sim \tilde{\sigma}(y, x, \mu, z^2) \otimes \tilde{f}(x, \mu)$$

- regularize QCD on a lattice, then $P_z \rightarrow \infty$ / $z^2 \rightarrow 0$
- opposite order of limits from light-cone quantization
- two limits don't commute
- difference is UV physics, can be taken care of through perturbative matching

parton physics



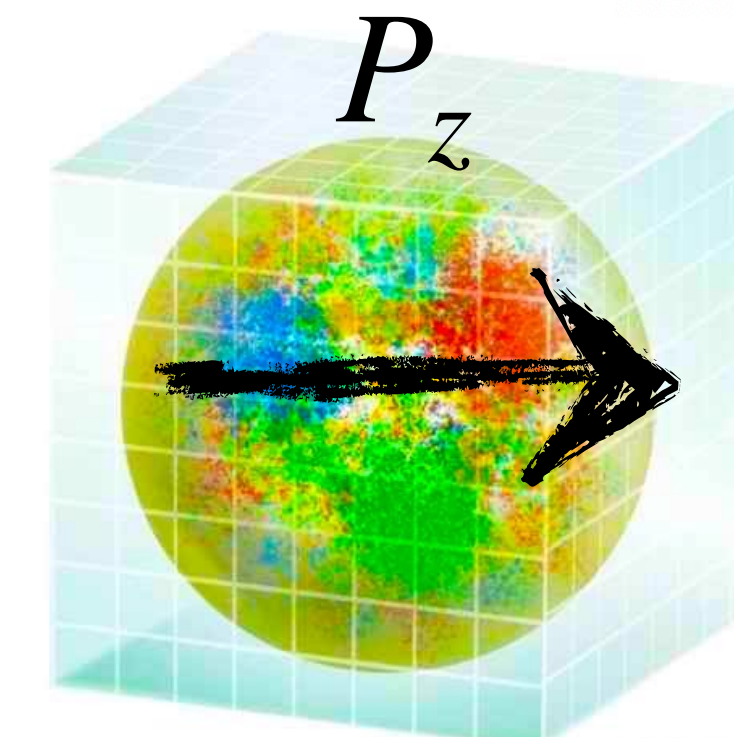
LO

perturbative matching

$$C(x, P_z, \mu) \otimes$$

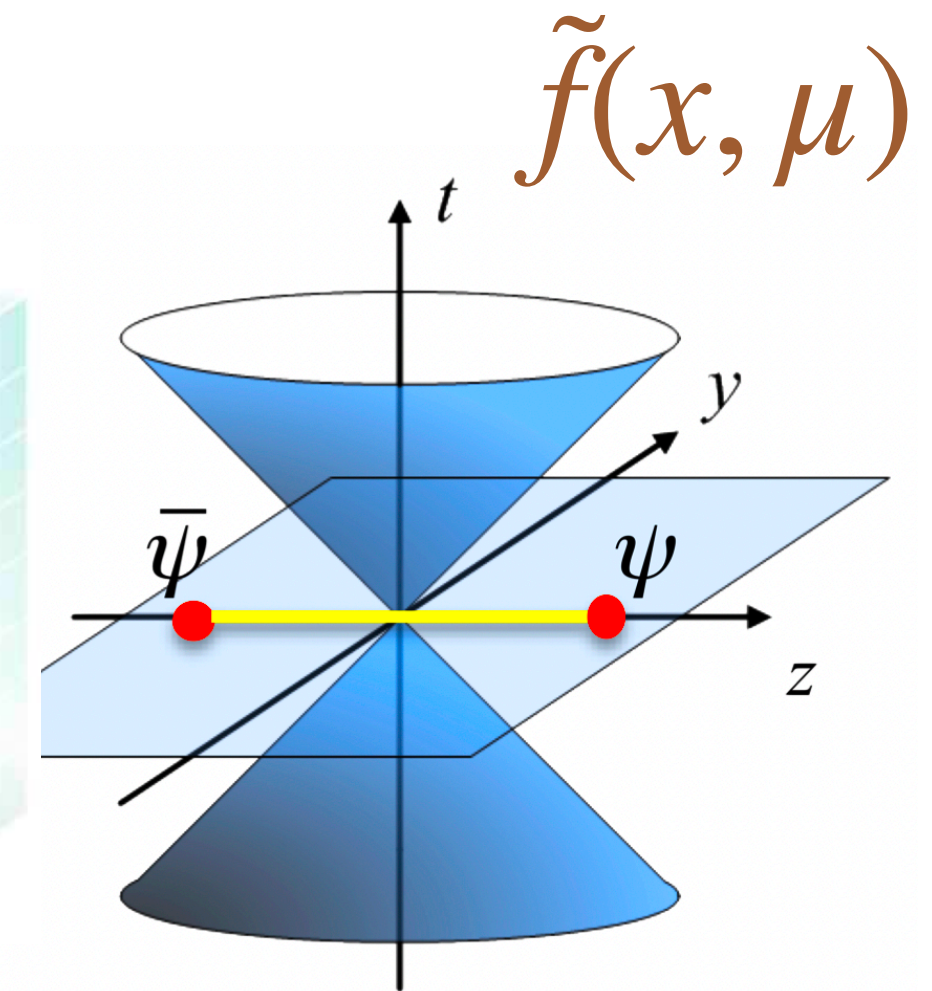
$$C(\alpha, z^2, \mu) \otimes$$

NLO



NNLO

lattice QCD



$$C(\mathcal{S}, \mu) \sim \alpha_s^0(\mu) + \alpha_s(\mu) f(\ln[\mathcal{S}\mu]) + \alpha_s^2(\mu) f(\ln[\mathcal{S}\mu]) + \dots$$

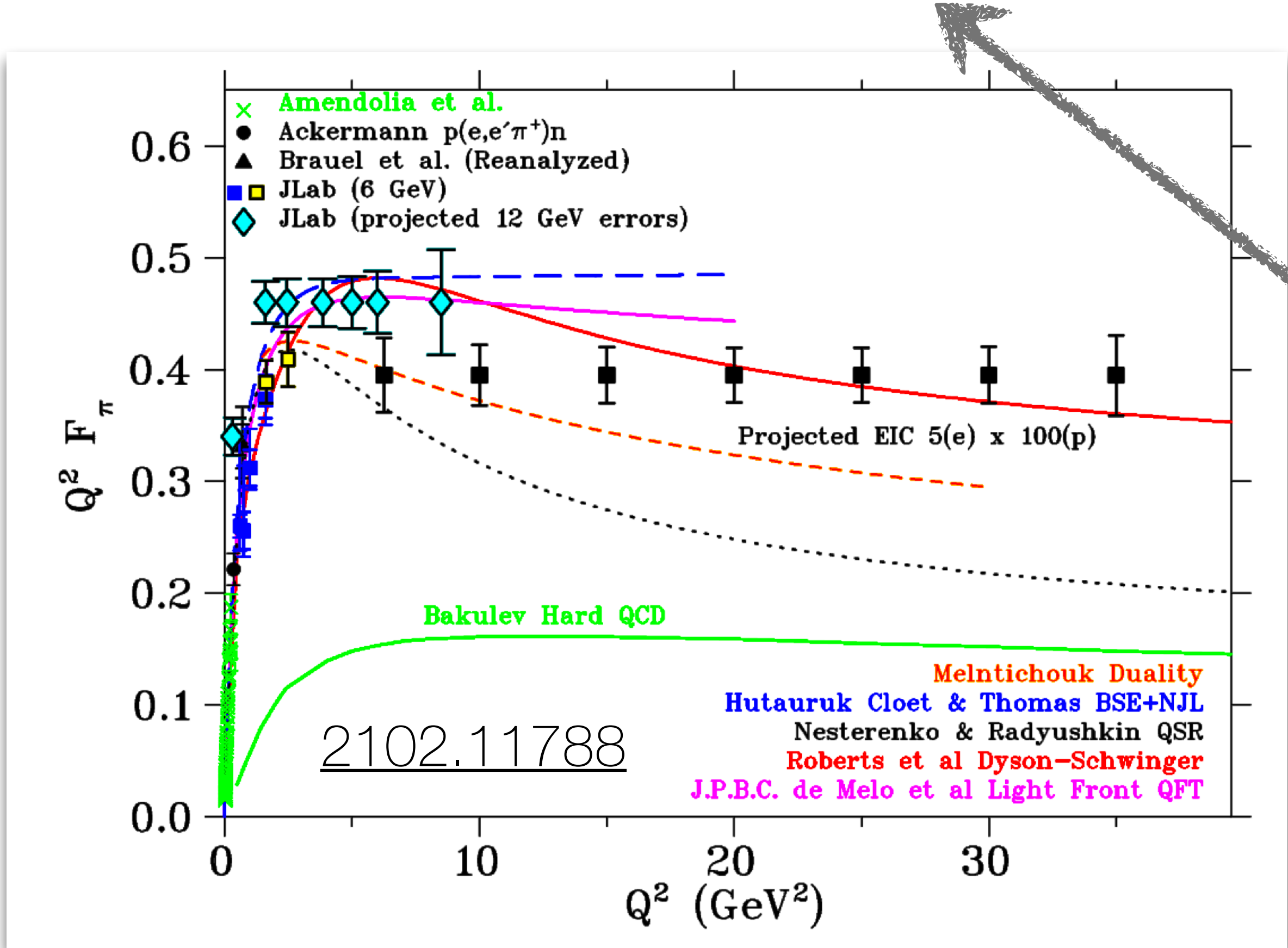
$$(\mathcal{S} = 2xP_z, z^2)$$

for more details see talk by Xiang Gao (Thursday, 4:30 pm)

from QCD to cross-section ...

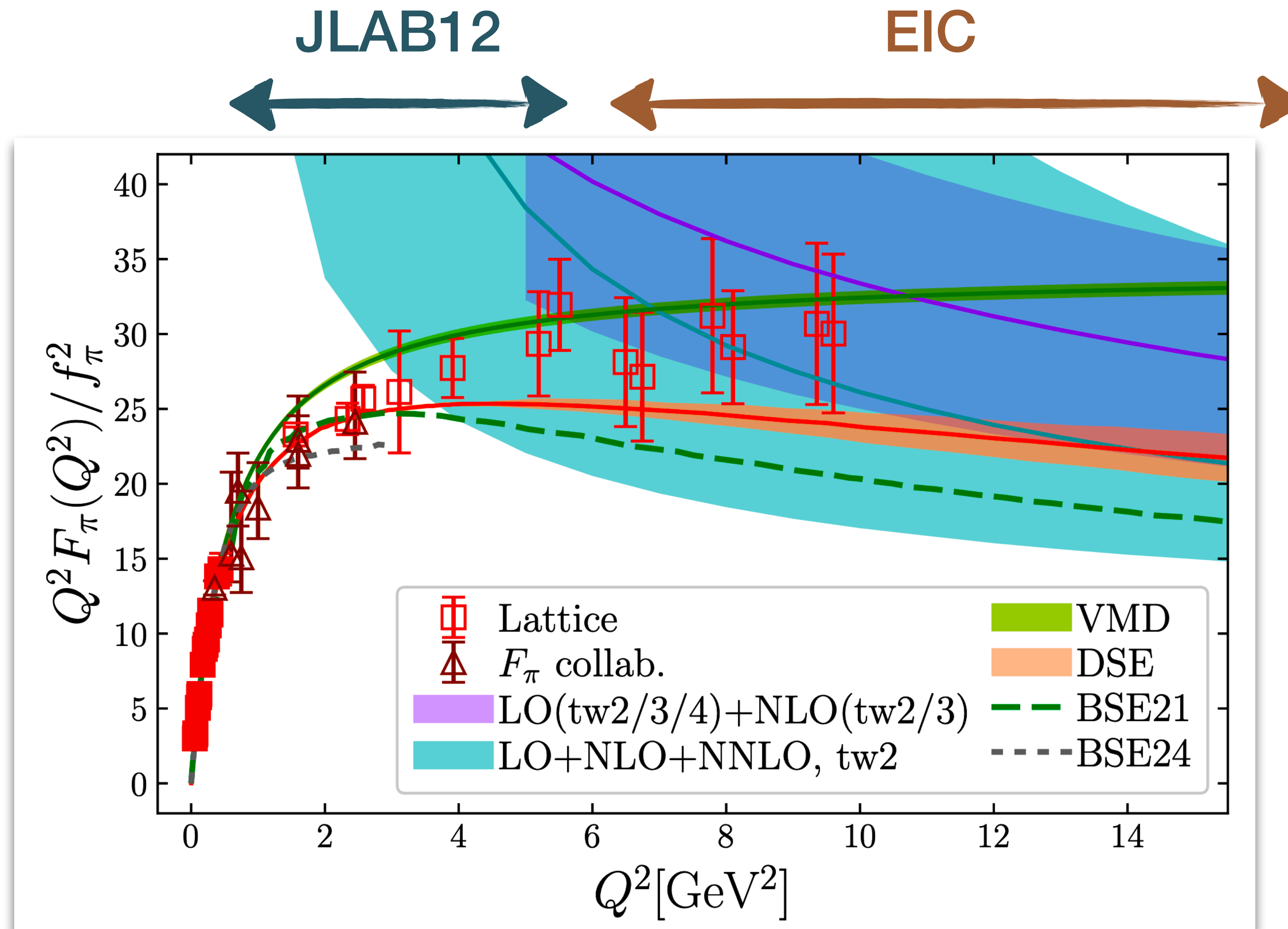
... example with pion/kaon EM form factor

JLAB12/EIC $F(Q^2 \gg \Lambda_{QCD}) \sim$ NNLO pQCD $\otimes$ meson DA LQCD

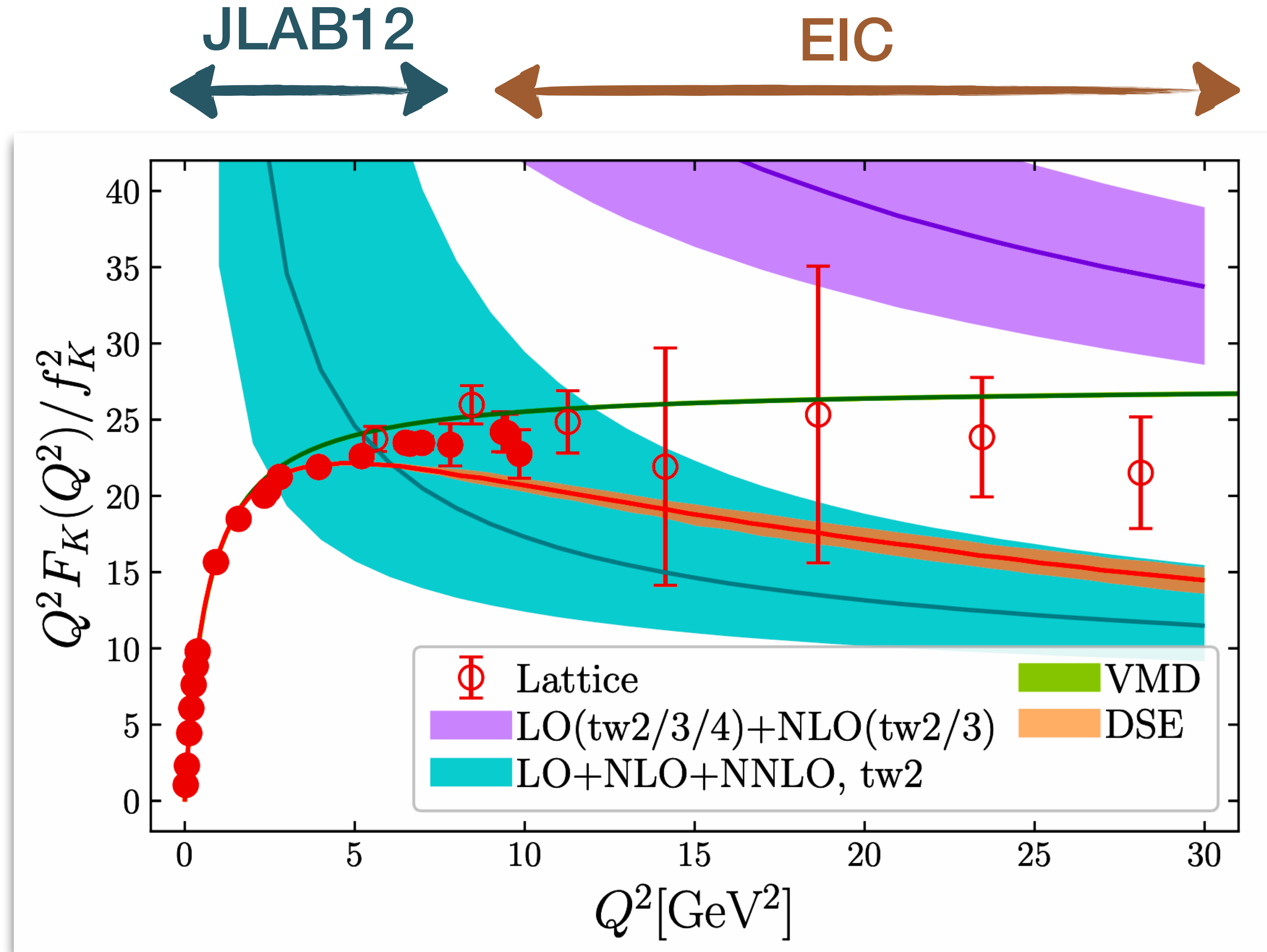


also can predict from LQCD !!!

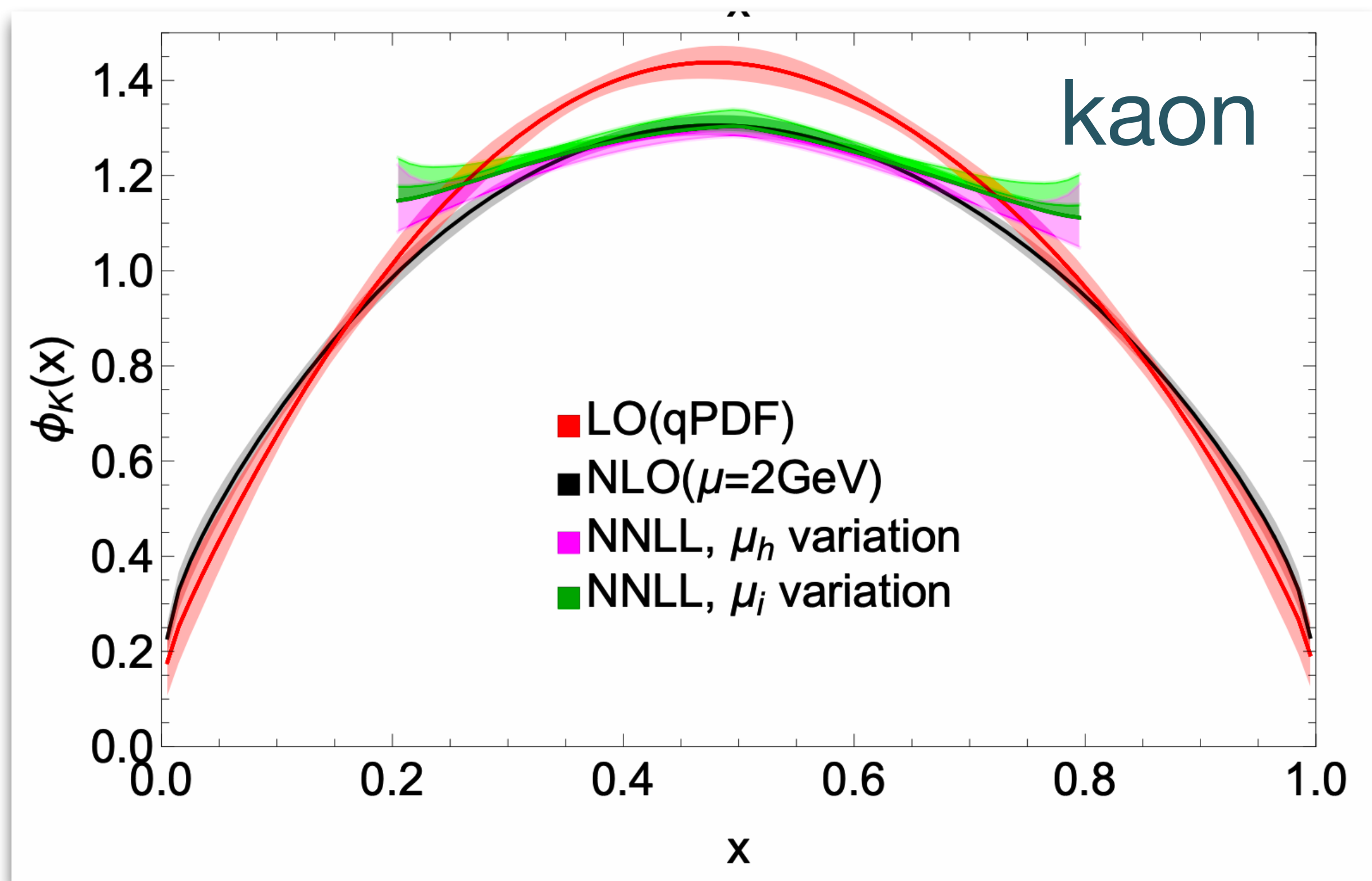
LQCD predictions for pion EM form factor



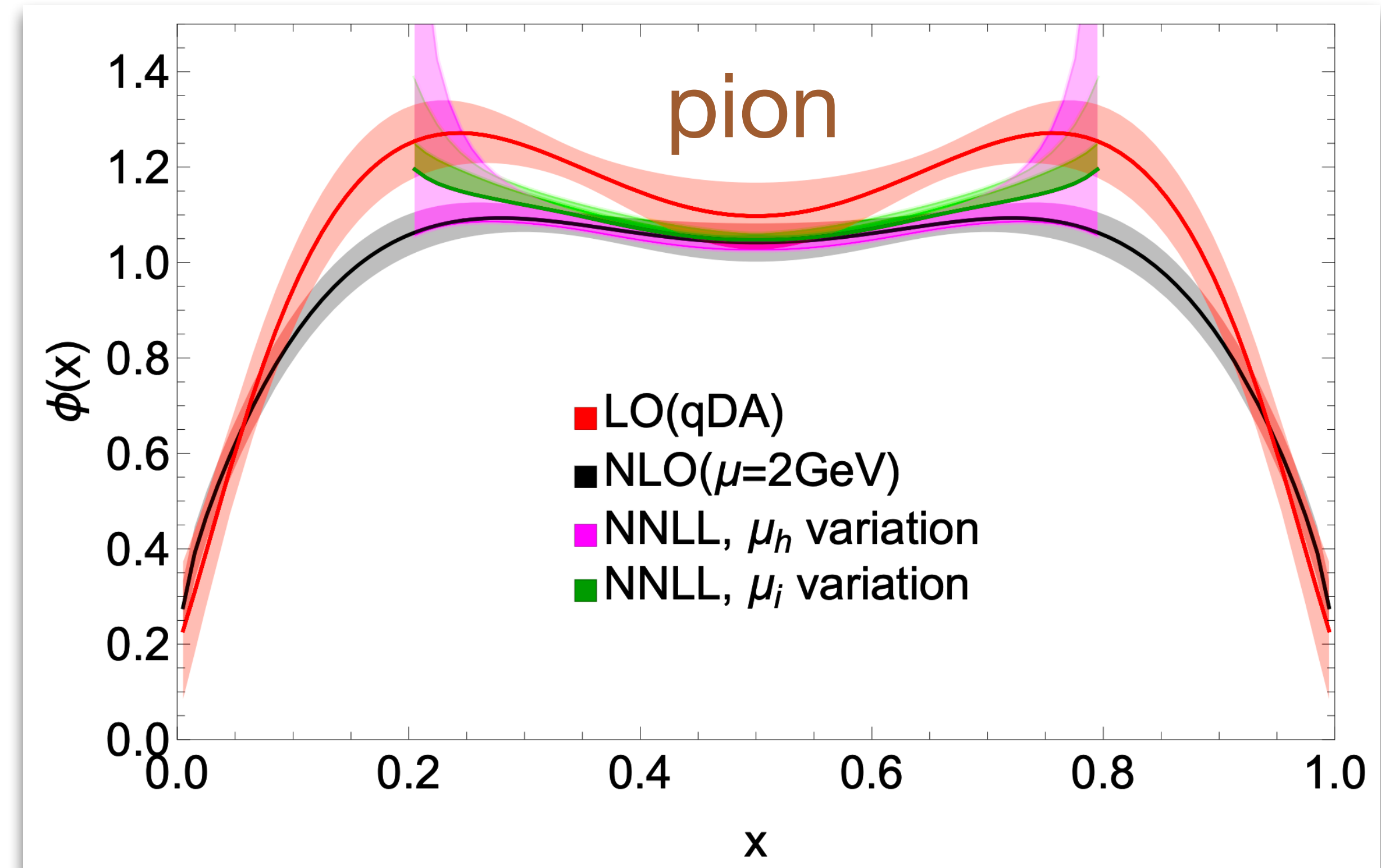
LQCD predictions for kaon EM form factor



meson distribution amplitudes from LQCD



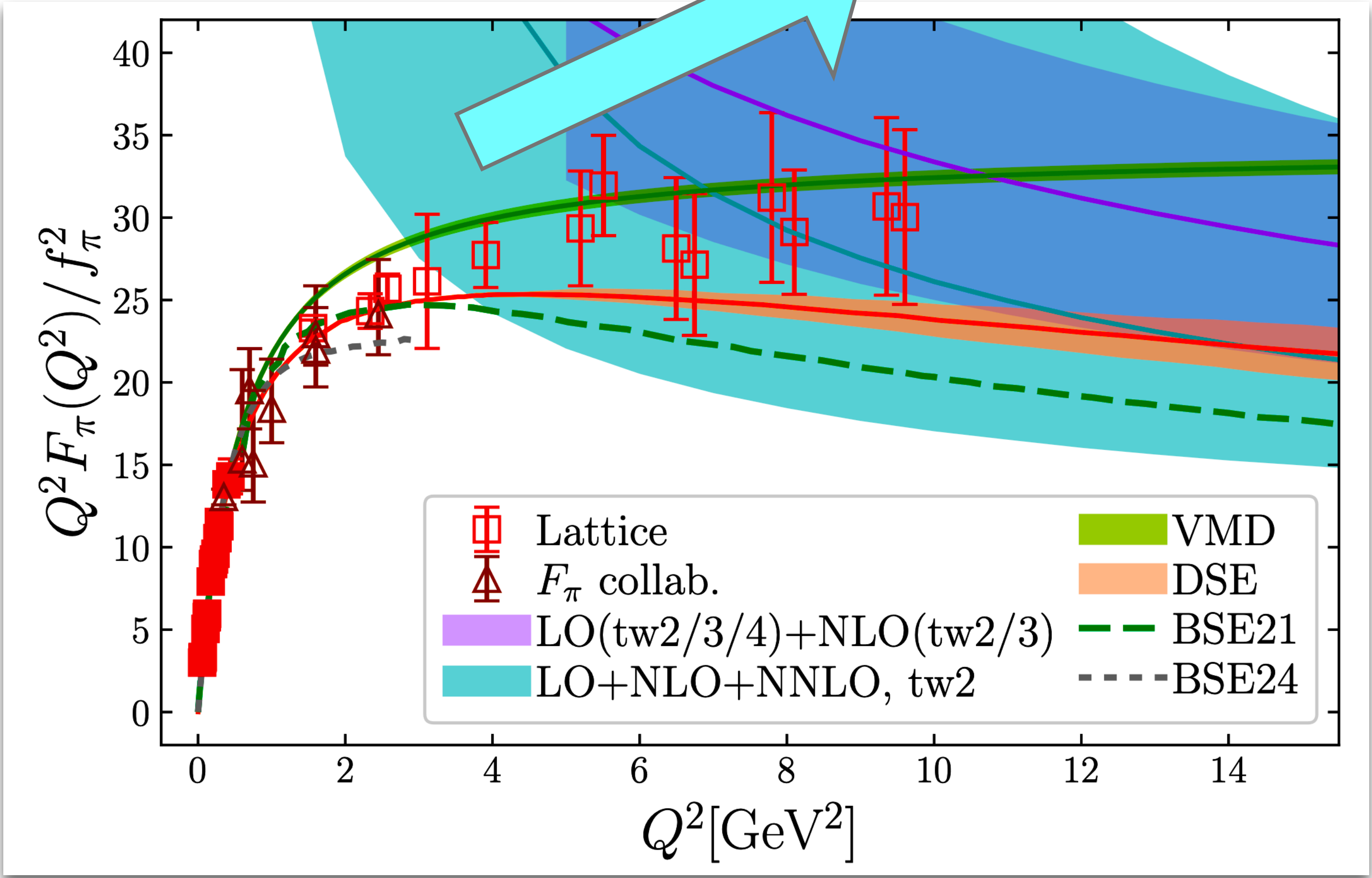
R. Zhang et al., [Phys. Rev. D 110, 114502 \(2024\)](#)



R. Zhang et al., [JHEP 07, 211 \(2024\)](#)

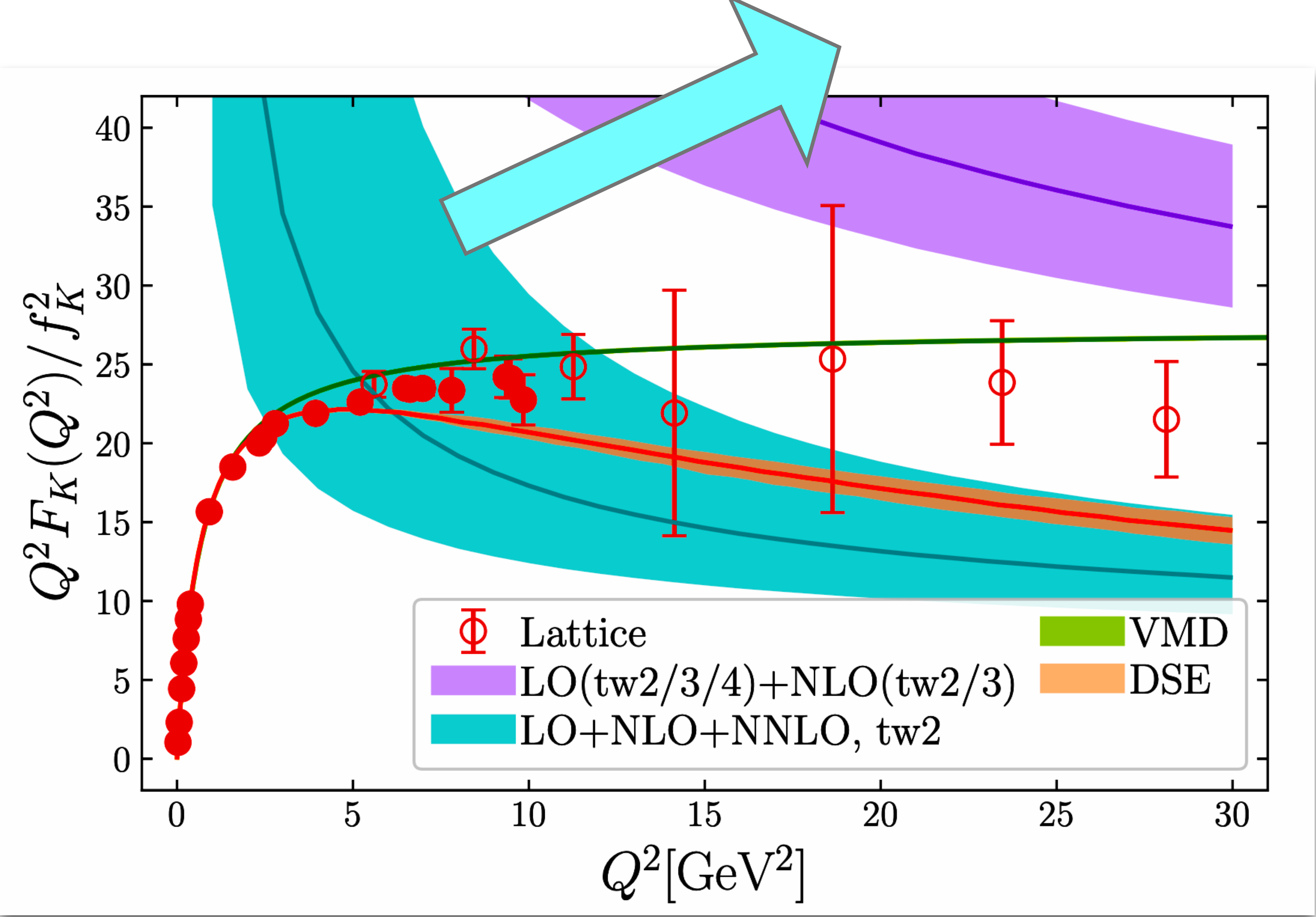
from QCD to cross-section...

$f_{\pi}(Q^2) \sim$ NNLO pQCD $\otimes$ LQCD pion DA

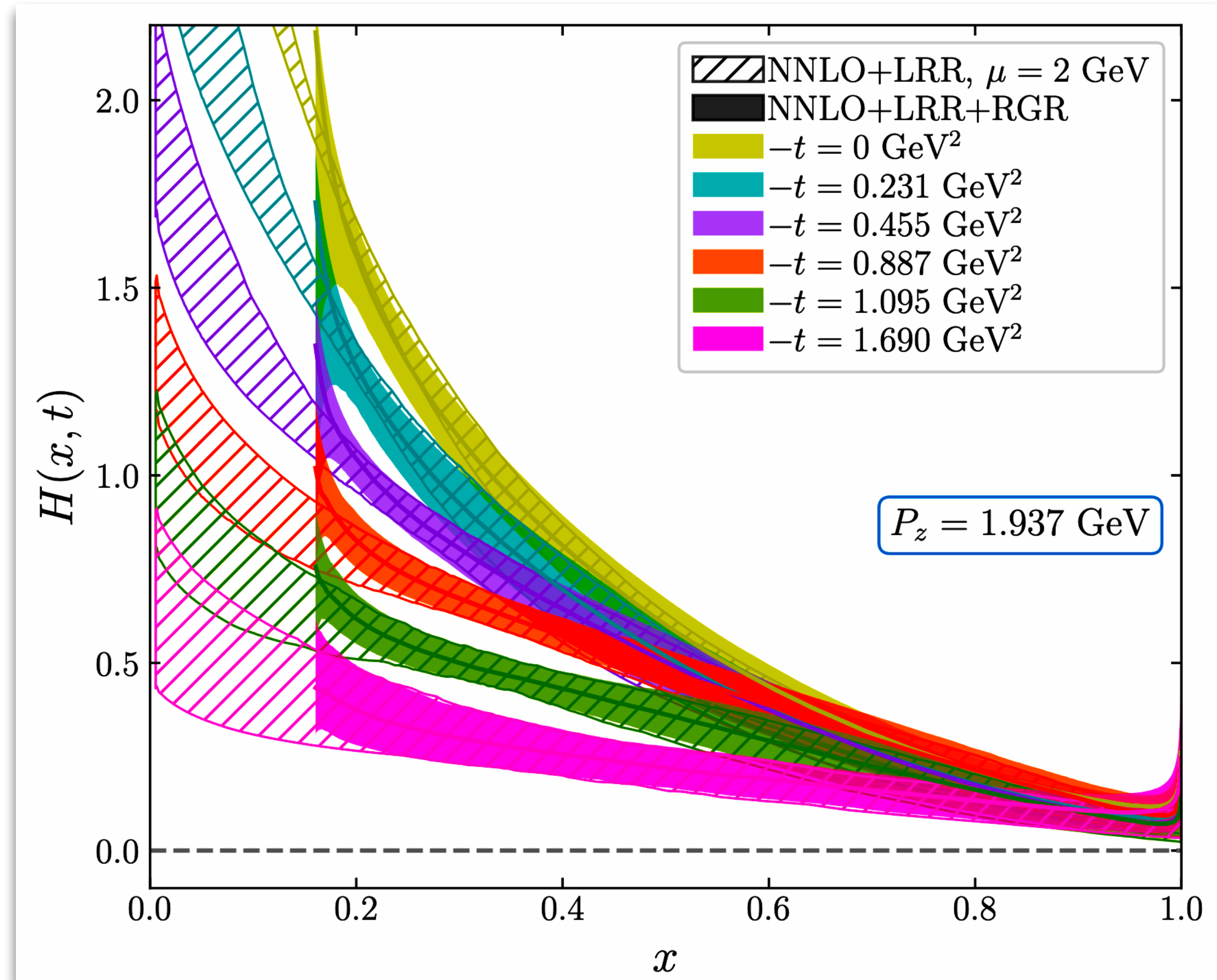


from QCD to cross-section...

$f_K(Q^2) \sim$ NNLO pQCD $\otimes$ LQCD kaon DA



generalized parton distribution of pion



impact parameter distribution: pion vs proton

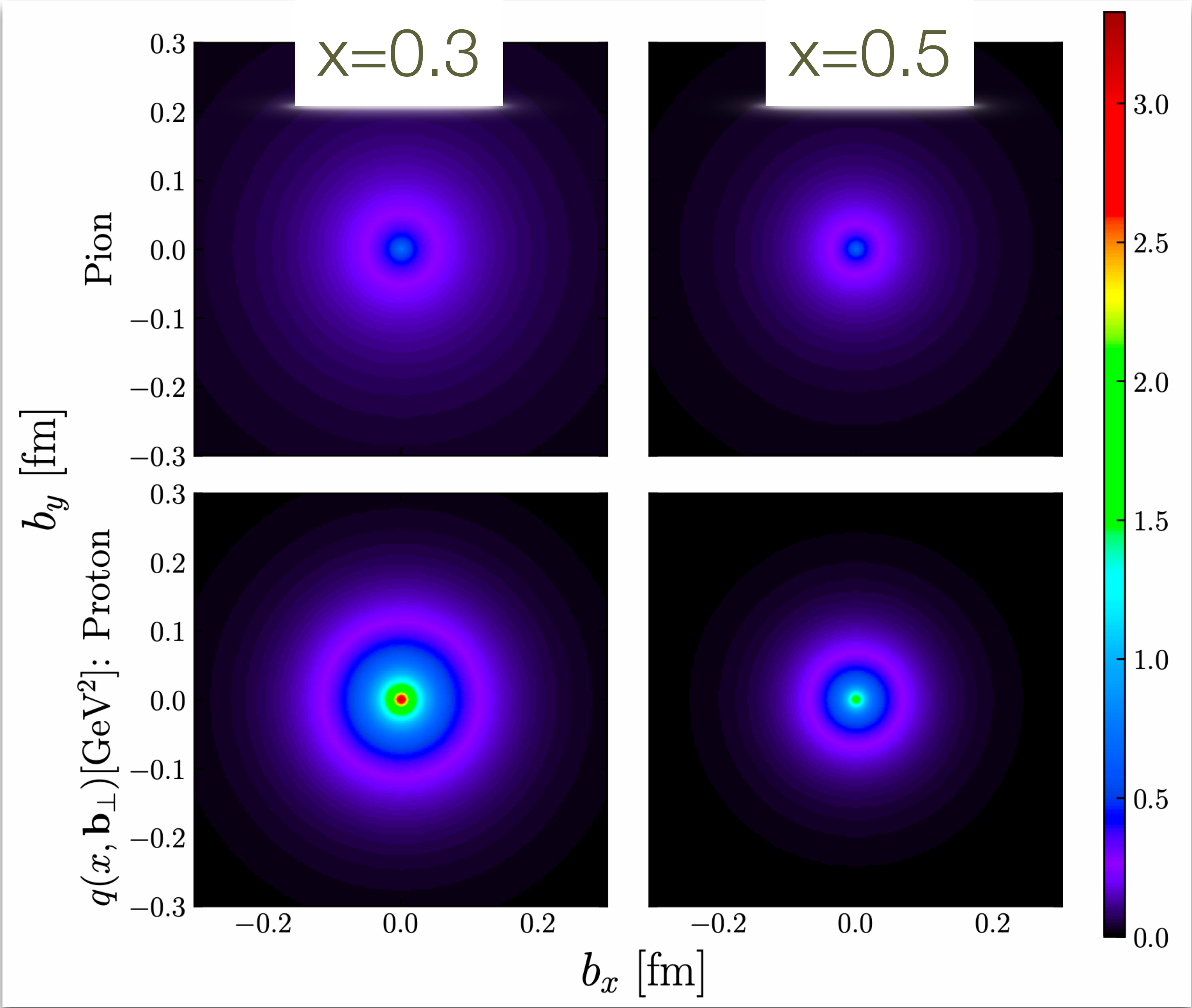
pion

Q. Shi et al., [JHEP 02, 056 \(2025\)](#)

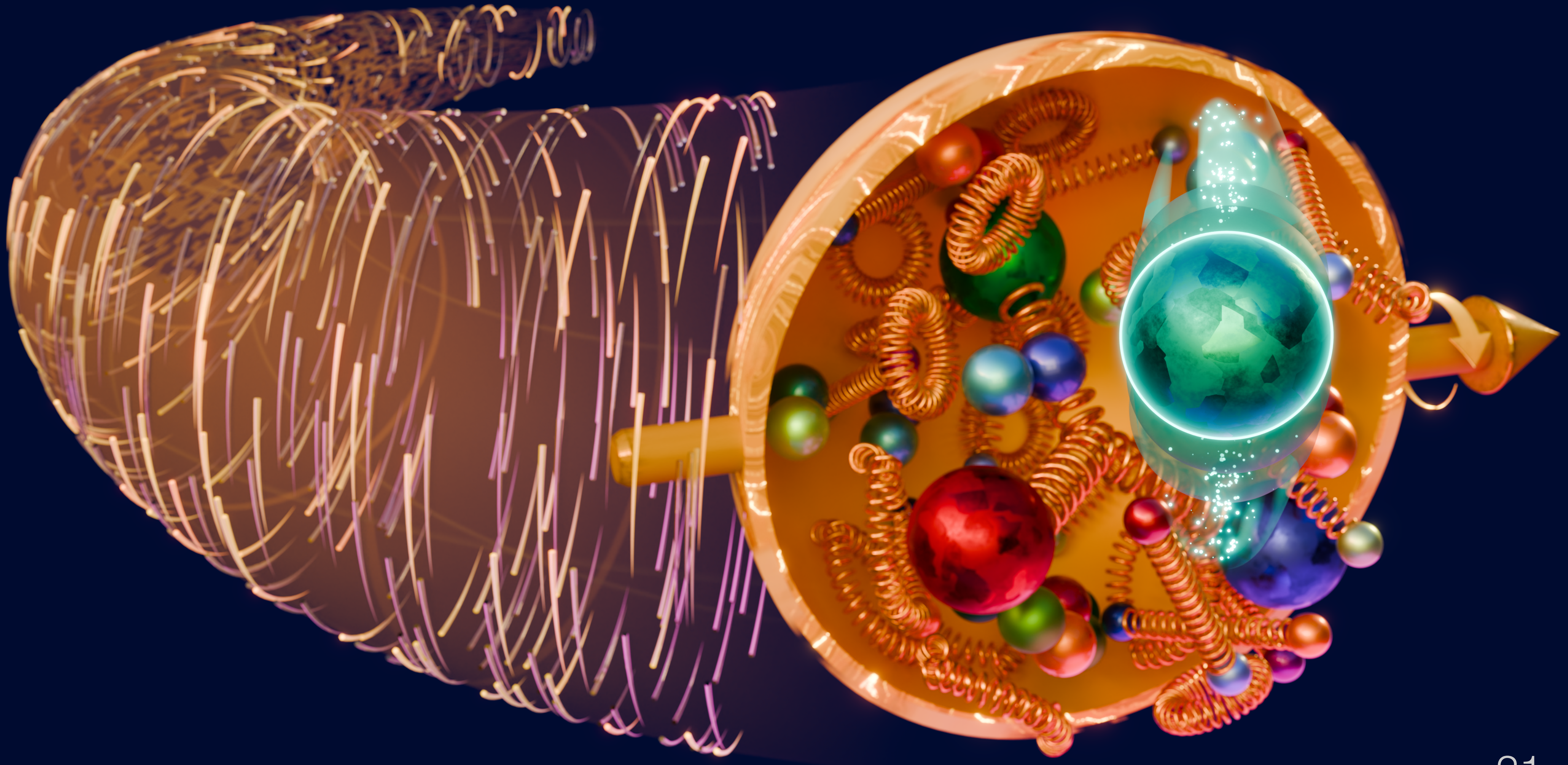
vs.

proton

S. Bhattacharya et al., [Phys. Rev. D 106, 1, 114512 \(2022\)](#)



TMDPDF: partonic image of hadron in 3-D momentum space



TMD factorization of LQCD beam function

LQCD

CS kernel

TMDPDF

$$\sqrt{S_I(b_T, \mu)} \cdot \tilde{f}(x, b_T, P_z, \mu) = H(x, P_z, \mu) \cdot \exp \left[\frac{1}{2} \ln \frac{(2xP_z)^2}{\zeta} \gamma^{\overline{\text{MS}}}(b_T, \mu) \right] \cdot f(x, b_T, \zeta, \mu)$$

intrinsic soft factor

pQCD kernel

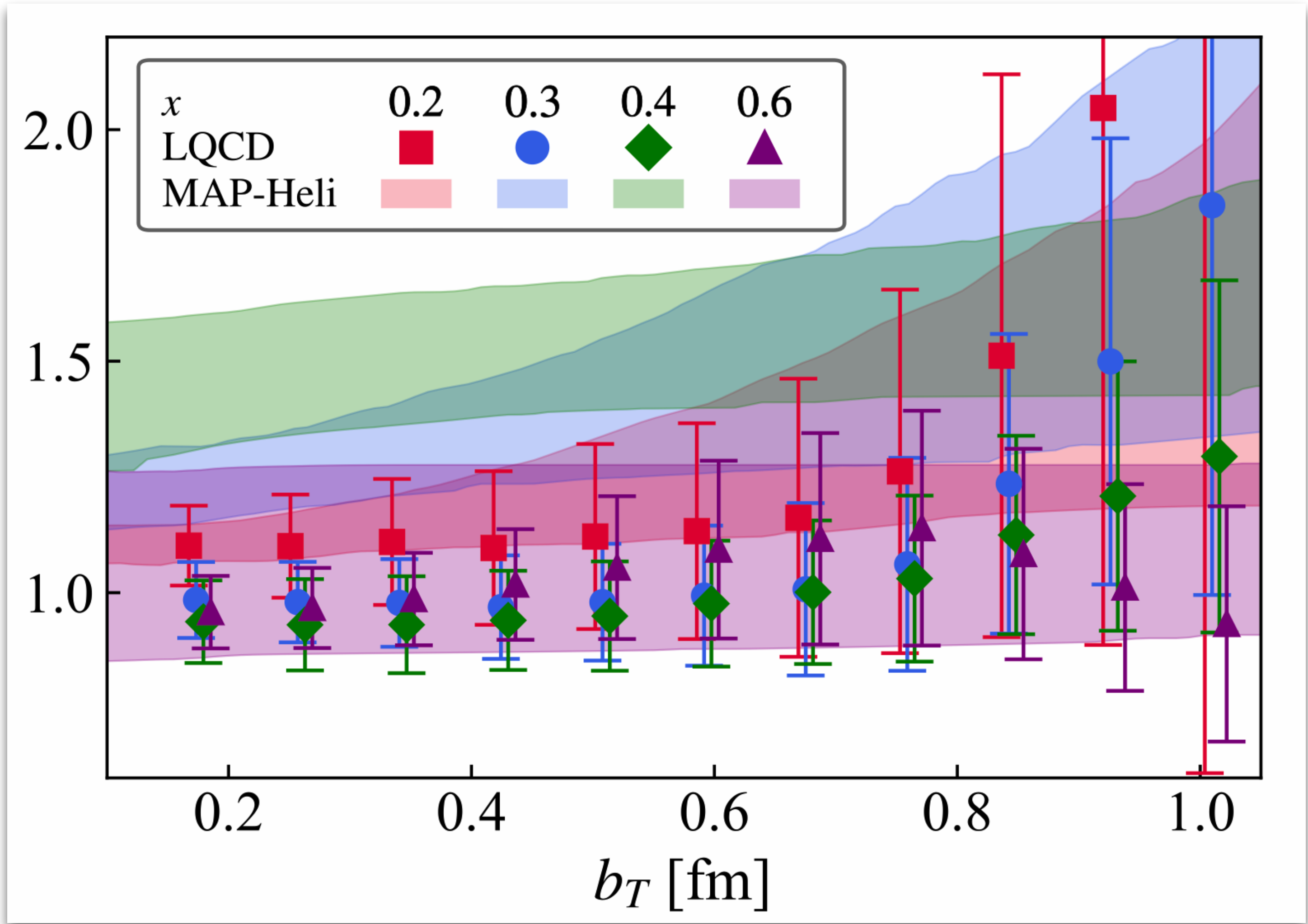
scale-independent ratios of TMDPDF

$$\frac{f_a(x, b_T, \zeta, \mu)}{f_b(x, b_T, \zeta, \mu)} = \frac{\tilde{f}_a(x, b_T, P_z, \mu)}{\tilde{f}_b(x, b_T, P_z, \mu)}$$

TMDPDF of proton: helicity to unpolarized TMDPDF

$$\frac{g_{1L}^{\Delta u_+ - \Delta d_+}(x, b_T, \zeta, \mu)}{g_A \cdot f_1^{u_v - d_v}(x, b_T, \zeta, \mu)}$$

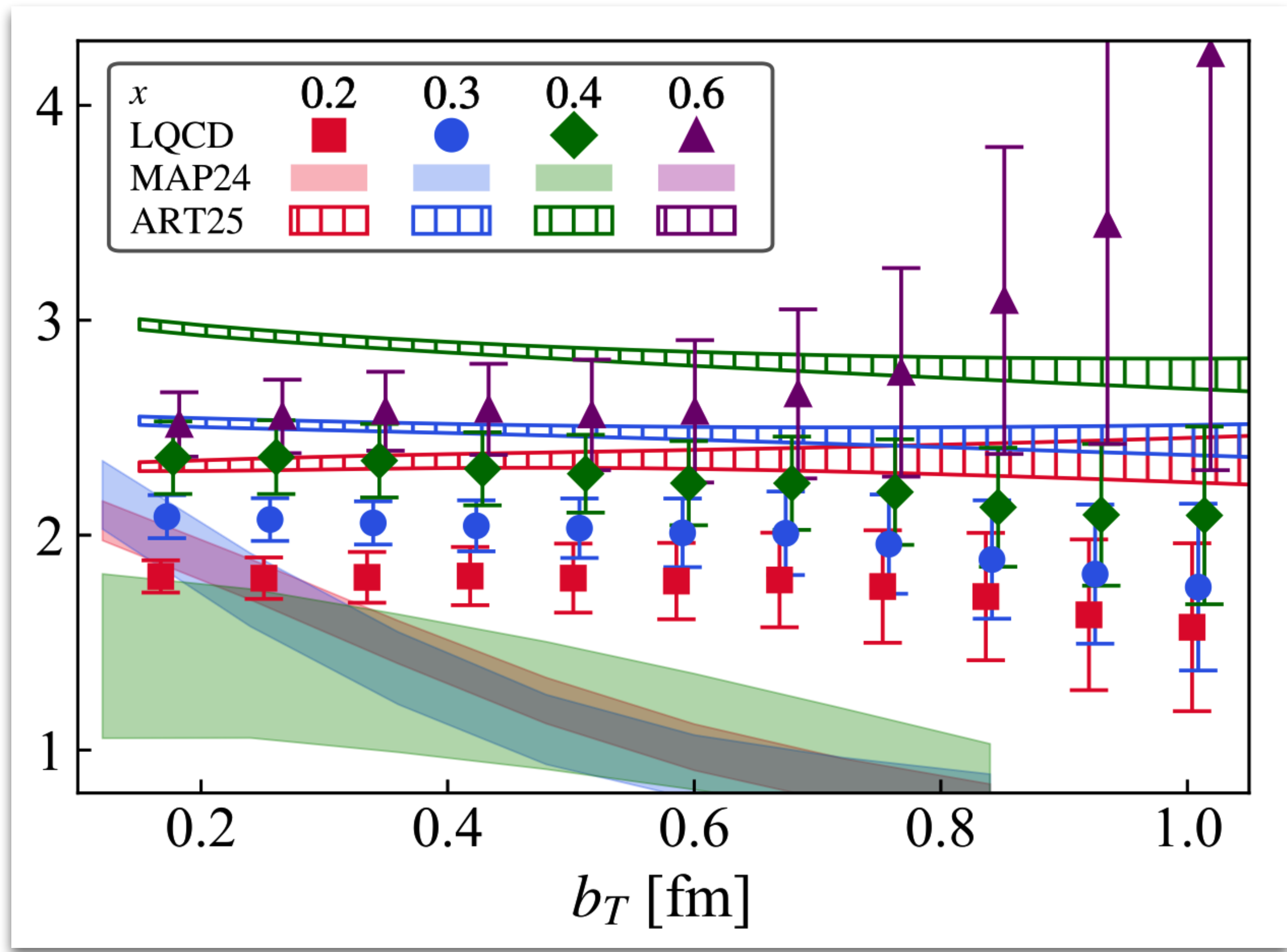
unitary chiral quarks,
physical mass



TMDPDF of proton: up to down unpolarized TMDPDF

$$\frac{f_1^{u_v}(x, b_T, \zeta, \mu)}{f_1^{d_v}(x, b_T, \zeta, \mu)}$$

unitary chiral quarks,
physical mass



nonperturbative Collins-Soper kernel from LQCD

LQCD

CS kernel

TMDPDF

$$\sqrt{S_I(b_T, \mu)} \cdot \tilde{f}(x, b_T, P_z, \mu) = H(x, P_z, \mu) \cdot \exp \left[\frac{1}{2} \ln \frac{(2xP_z)^2}{\zeta} \gamma^{\overline{\text{MS}}}(b_T, \mu) \right] \cdot f(x, b_T, \zeta, \mu)$$

intrinsic soft factor

pQCD kernel

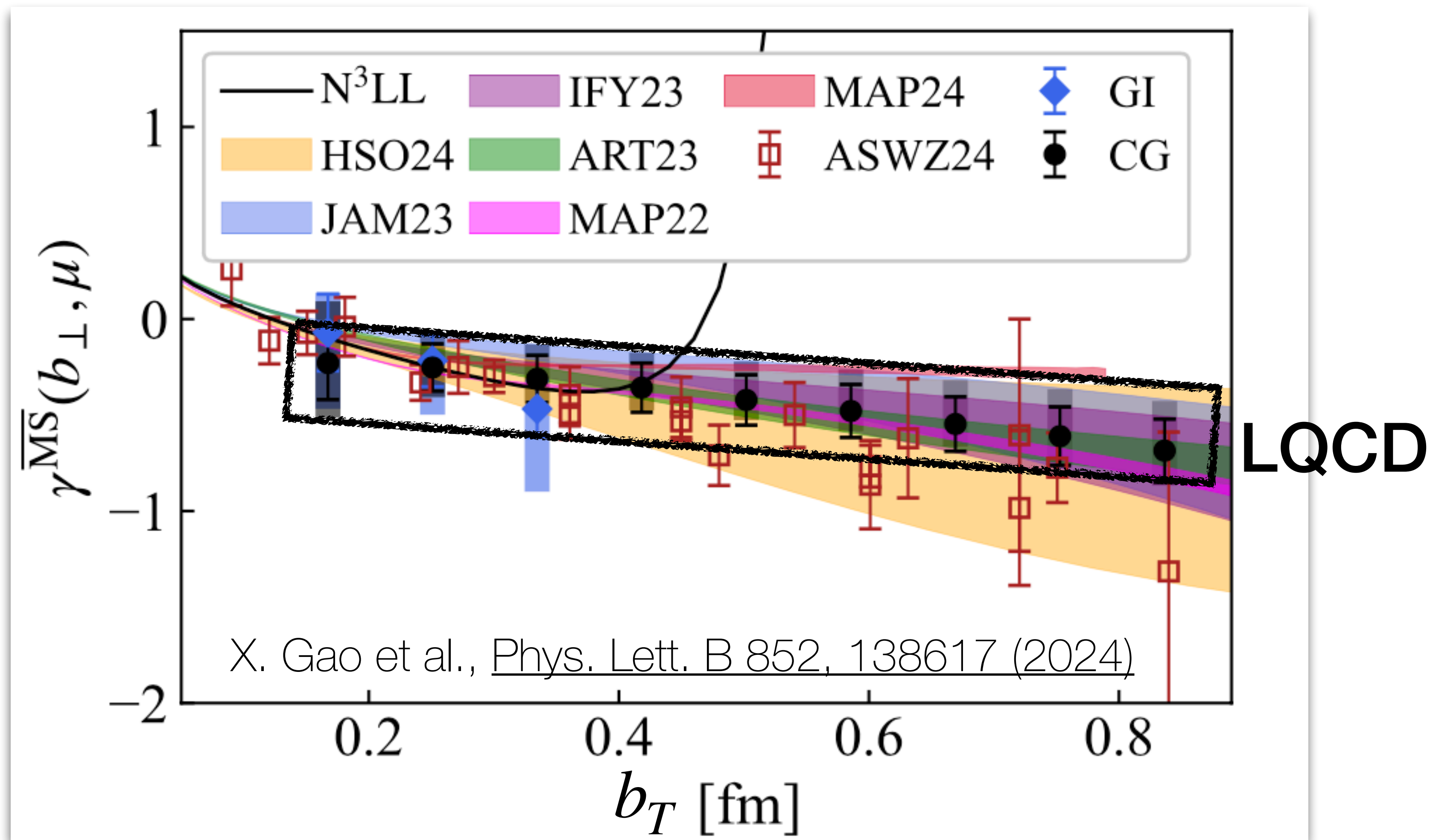
universal CS kernel

$$\gamma^{\overline{\text{MS}}}(b_T, \mu) = \frac{1}{\ln(P_2/P_1)} \ln \left[\frac{\tilde{f}(x, b_T, P_2, \mu)}{\tilde{f}(x, b_T, P_1, \mu)} \right] + \delta\gamma^{\overline{\text{MS}}}(b_T, \mu, P_1, P_2)$$

LQCD

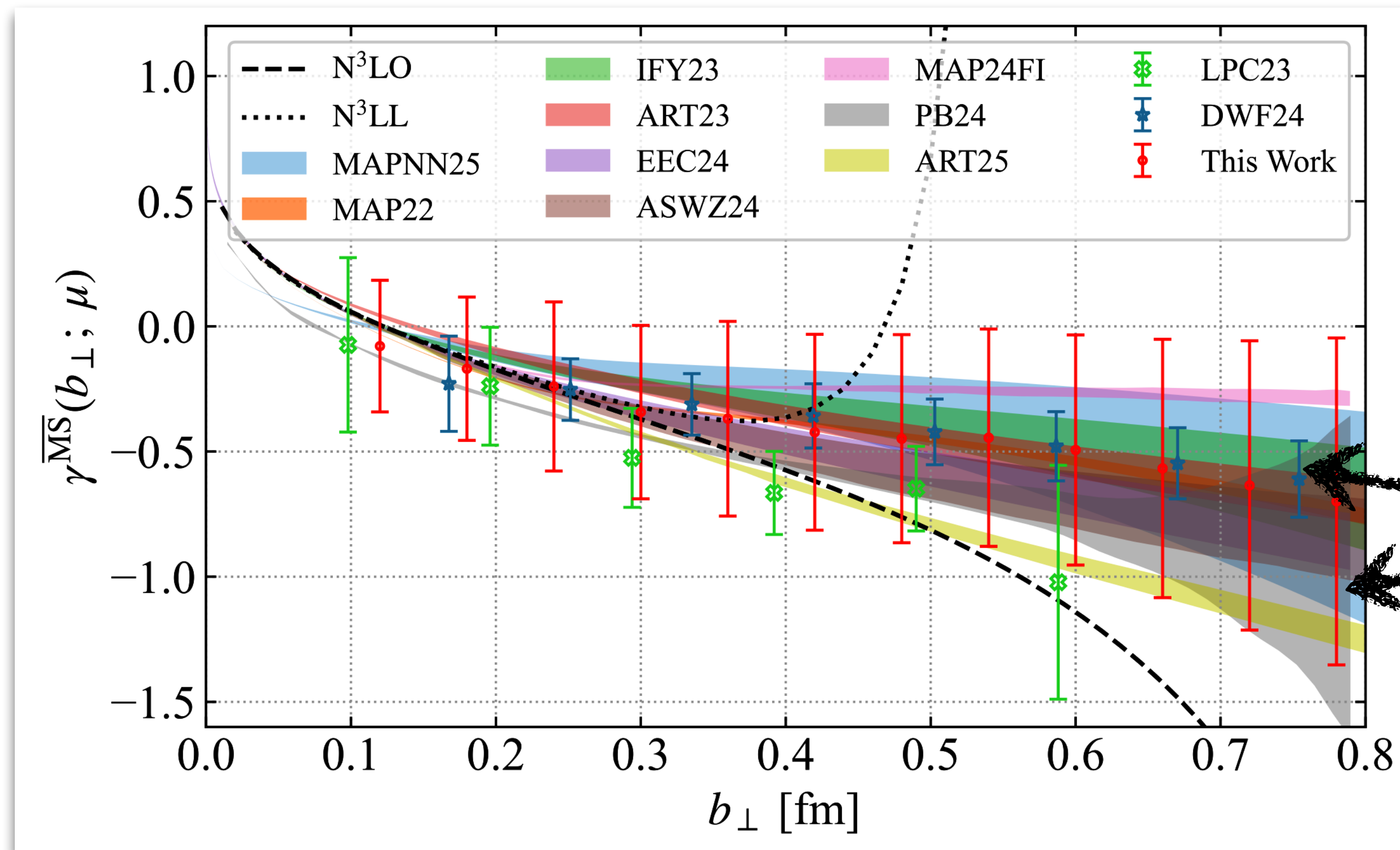
pQCD kernel

nonperturbative Collins-Soper kernel from LQCD



unitary chiral quarks, physical mass

nonperturbative Collins-Soper kernel from LQCD



J. C. He et al., [arXiv:2504.04625](https://arxiv.org/abs/2504.04625)

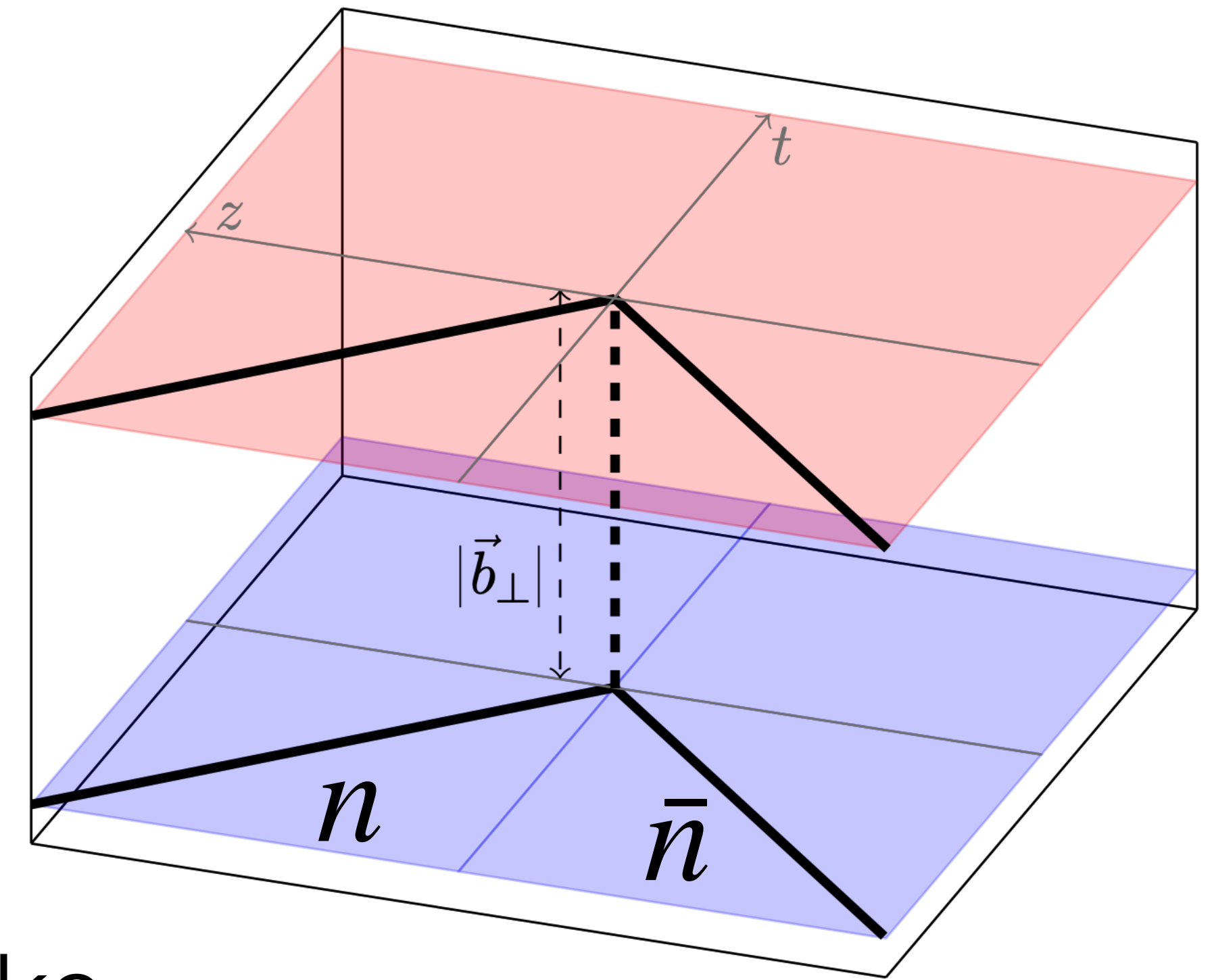
check universality through independent calculations to larger momenta, larger pion mass, and different lattice discretization

intrinsic soft factor

operator involves two lightcone directions

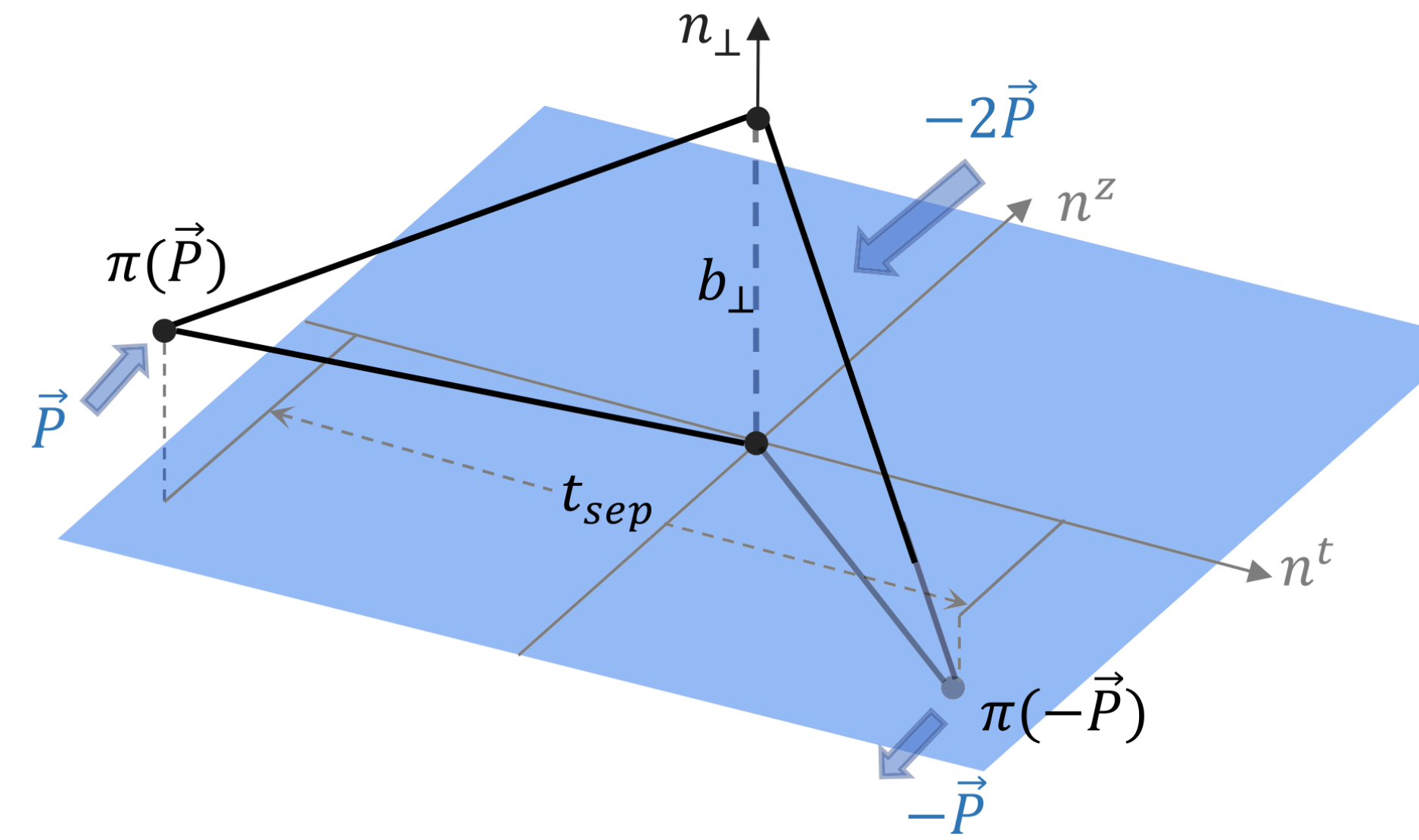
impossible to obtain by computing a space-like operator within a hadron boosted in a direction

need an alternative indirect approach



form factor of two transversely-separated currents within boosted pions from LQCD

$$F(b_T, P_z) \sim \langle P_z | [\bar{q}(b_T) \gamma_T q(b_T)] [\bar{q}(0) \gamma_T q(0)] | -P_z \rangle$$



factorizes into pion TMD wave function

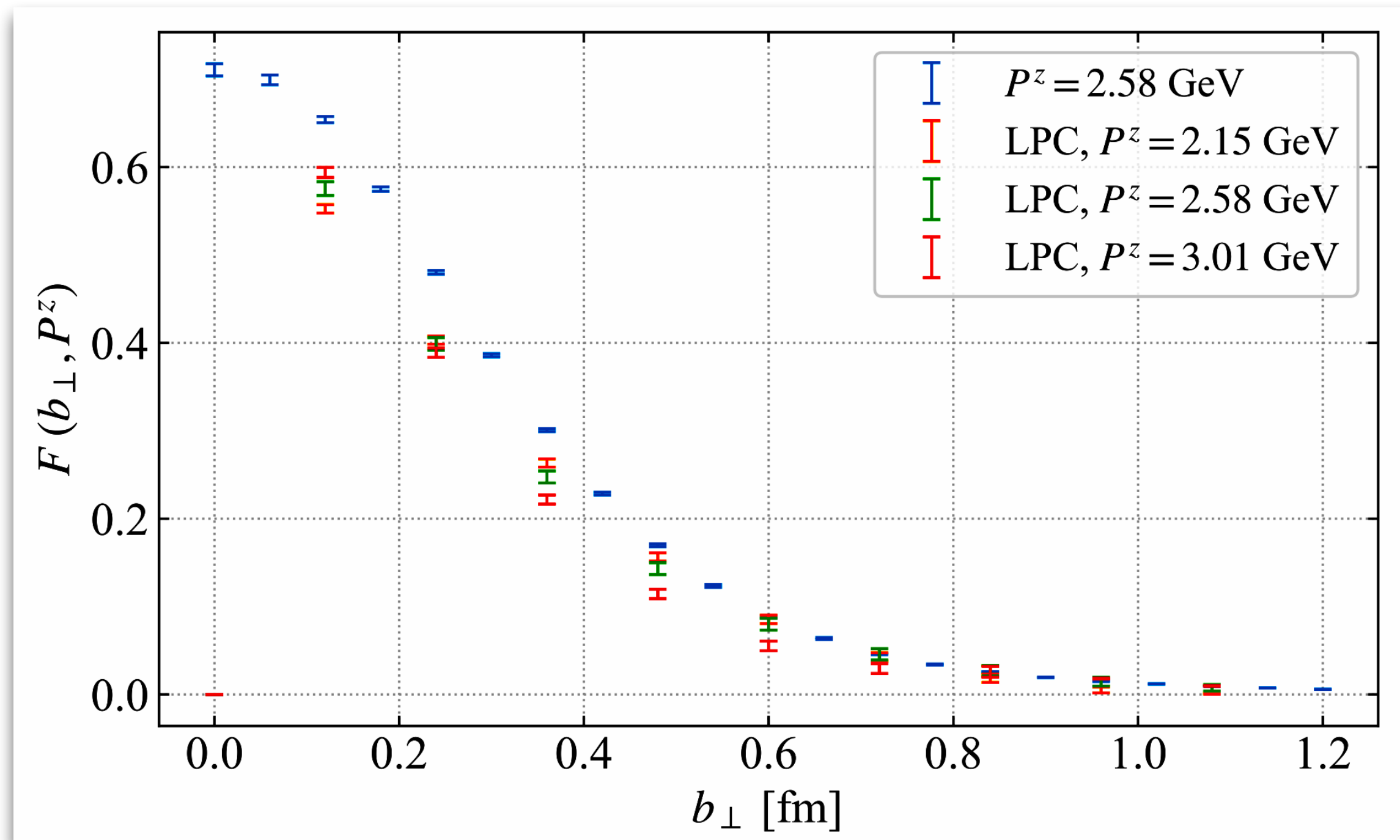
$$F(b_T, P_z) \sim \underset{\text{pQCD}}{H_F(x_1, x_2, P_z, \mu)} \otimes \phi^\dagger(x_1, b_T, \mu, \zeta_1, \bar{\zeta}_1) \otimes \phi(x_2, b_T, \mu, \zeta_2, \bar{\zeta}_2)$$

pion TMD wave function from LQCD

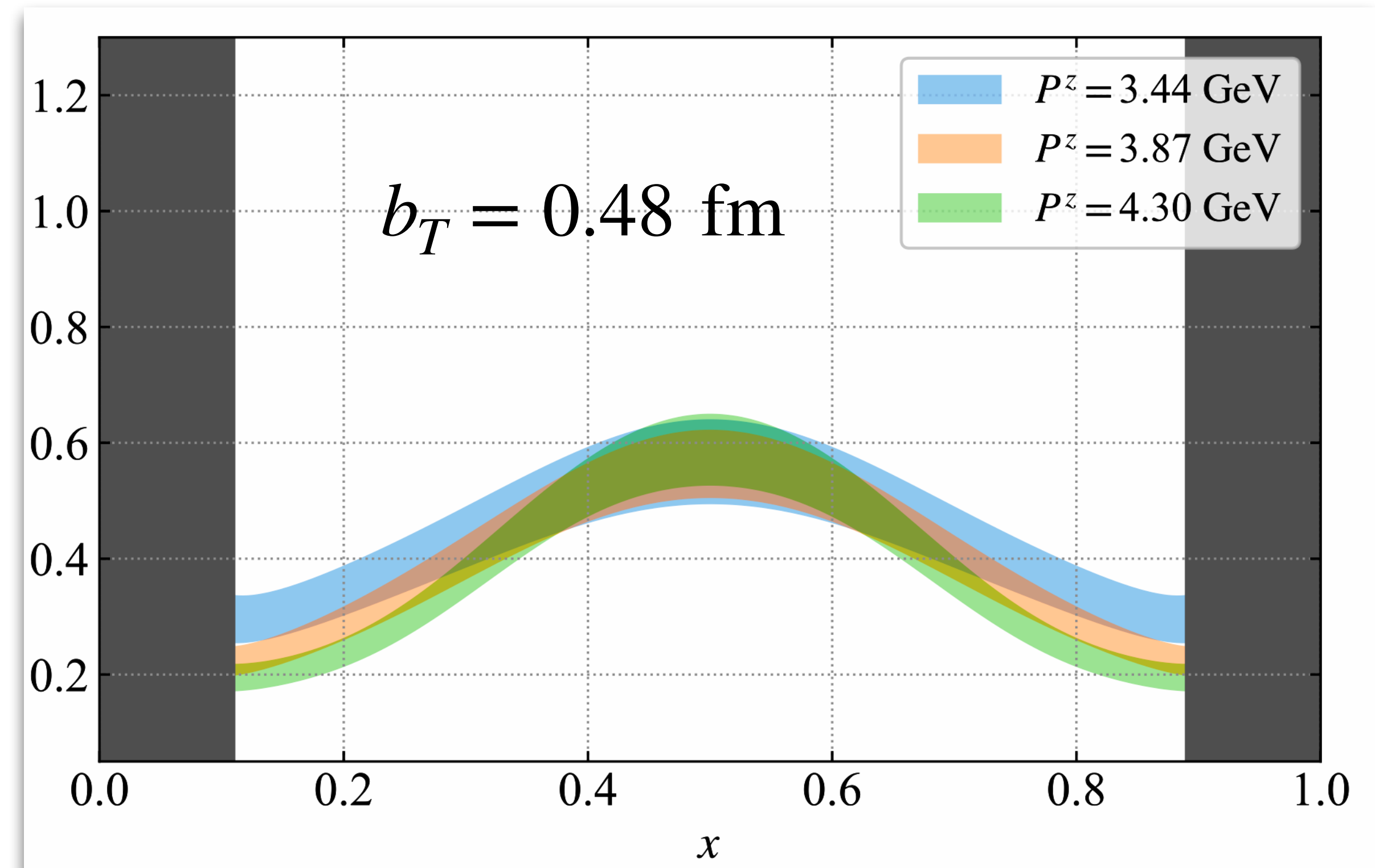
$$\sqrt{S_I(b_T, \mu)} \cdot \tilde{\phi}(x, b_T, P_z, \mu) = \underset{\text{pQCD}}{H_\phi(x, \bar{x}, P_z, \mu)} \cdot \phi(x, b_T, \mu, \zeta, \bar{\zeta})$$

intrinsic soft factor

form factor

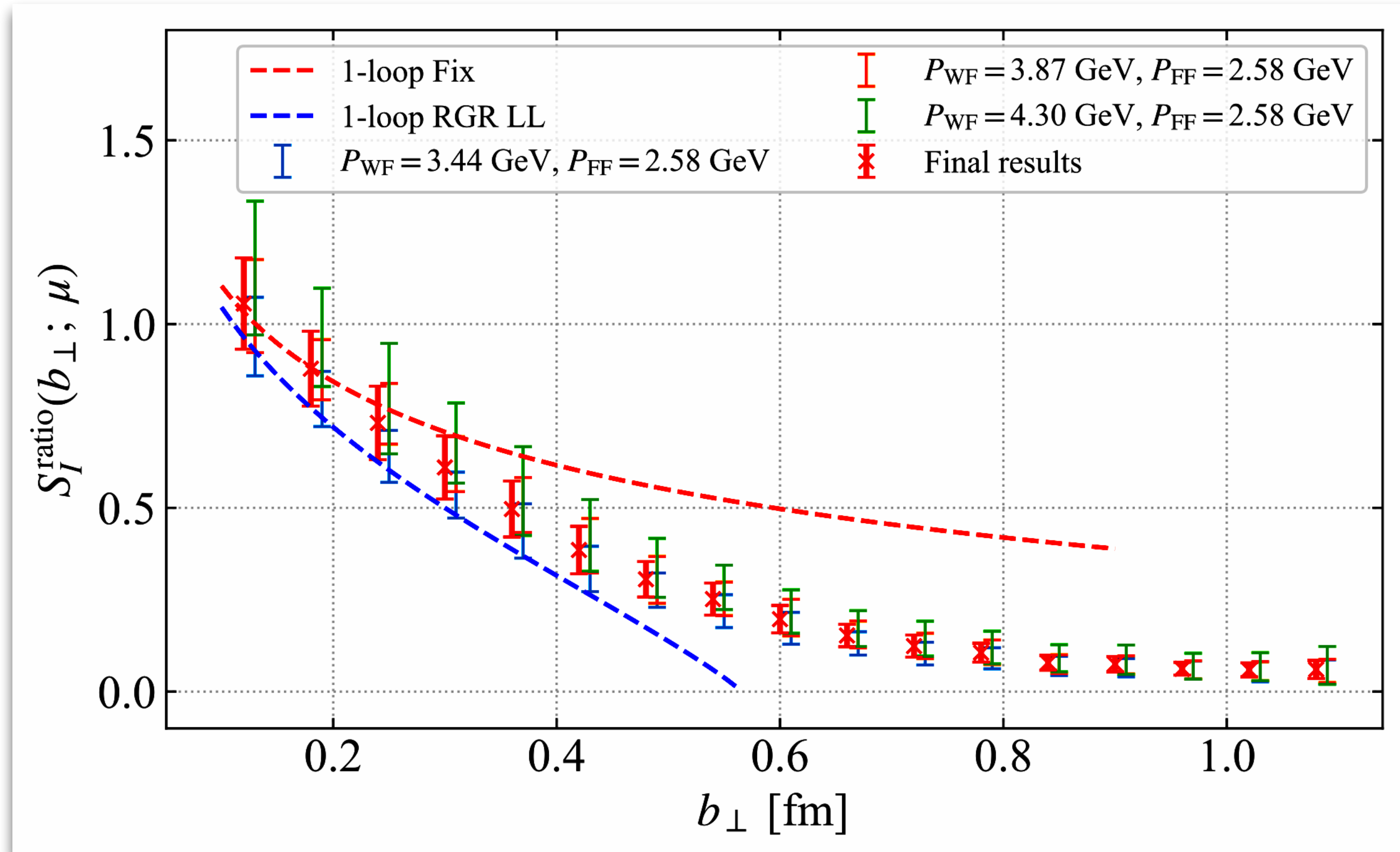


pion TMD wave function



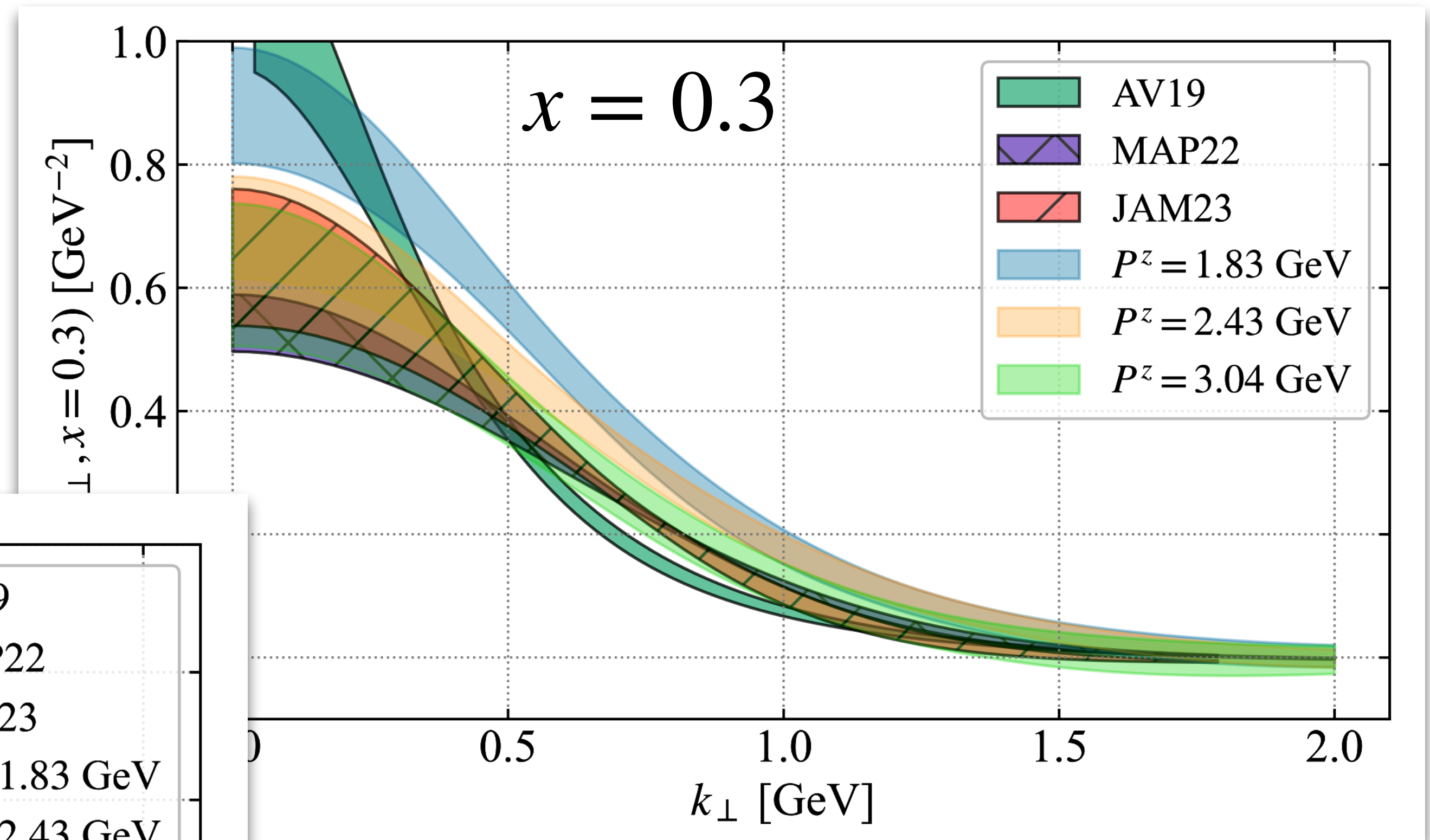
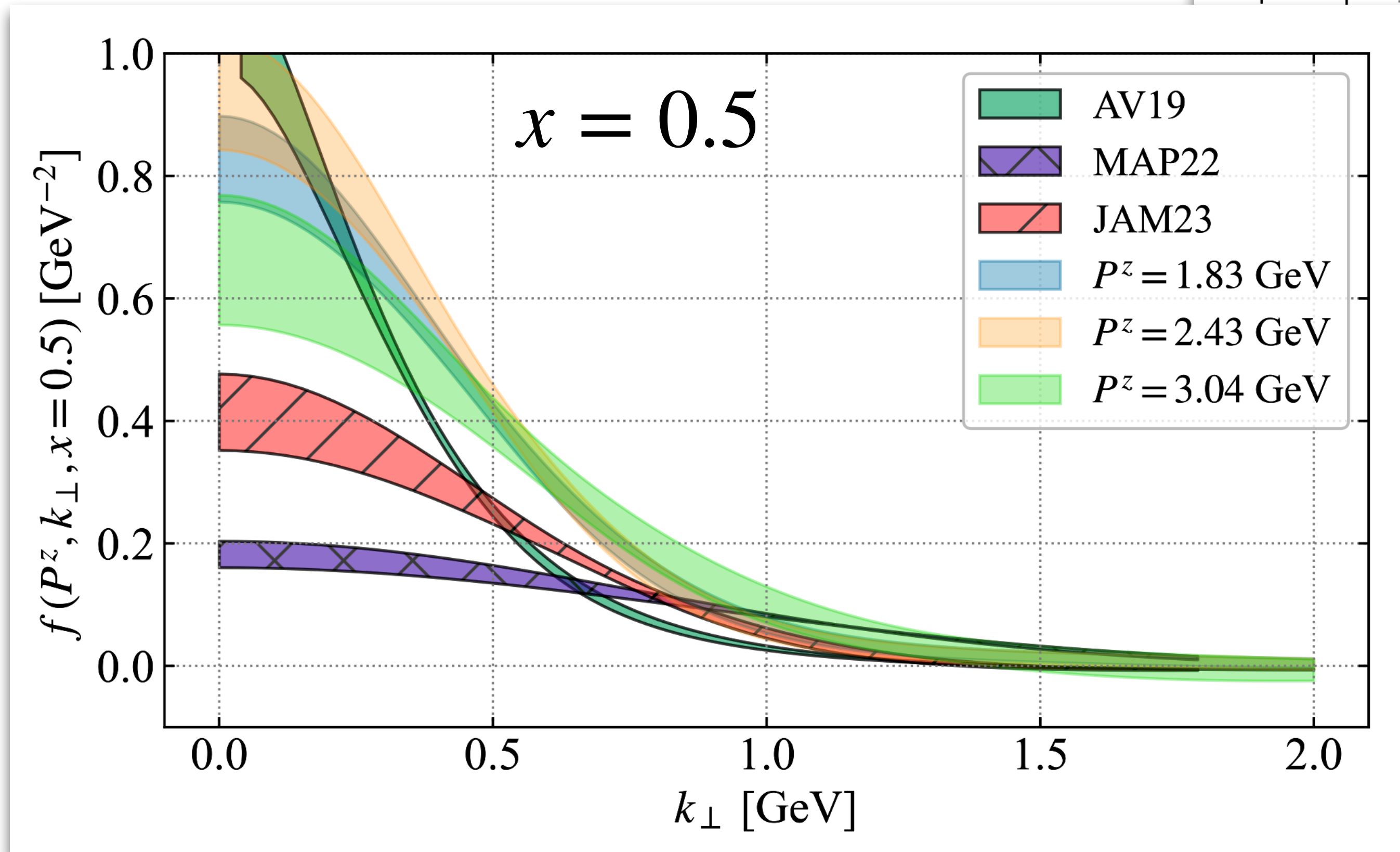
J. C. He et al., [arXiv:2504.04625](https://arxiv.org/abs/2504.04625)

intrinsic soft factor



scheme dependent

pion TMDPDF from LQCD



J. C. He et al., [arXiv:2504.04625](https://arxiv.org/abs/2504.04625)