Quantum anomalies & Generalized Parton Distributions

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University of Connecticut
21 May 2025

Based on:

Arxiv: 2210.13419, 2305.09431
with: **Yoshitaka Hatta (BNL)**
Werner Vogelsang (Tubingen U.)

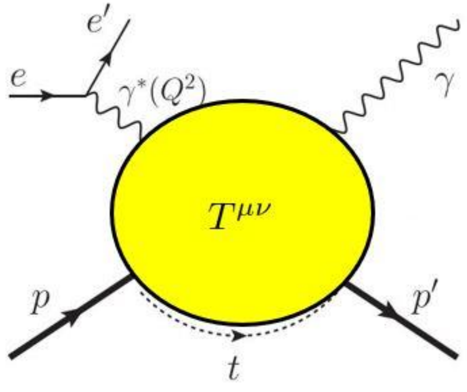
Arxiv: 2411.07024
with: **Yoshitaka Hatta (BNL)**
Jakob Schoenleber (BNL)

QCD EVOLUTION

Jefferson Lab

Outline

- **Motivation: Chiral & trace anomalies & GPDs**

- 

Perturbative calculations of box diagrams in DVCS

- **Non-perturbative relations between GPDs mediated by anomalies**

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- **Motivation: Chiral & trace anomalies & GPDs**

- **Perturbative calculations of box diagrams in DVCS**

- **Non-perturbative relations between GPDs mediated by anomalies**

Anomalies in QCD

Chiral anomaly

Classical:

- **U(1) axial symmetry:** QCD Lagrangian invariant under global chiral rotation of fermionic fields

- **Quantity conserved:** Axial-vector current $J_5^\mu = \sum_f \bar{\psi}_f \gamma^\mu \gamma_5 \psi_f$

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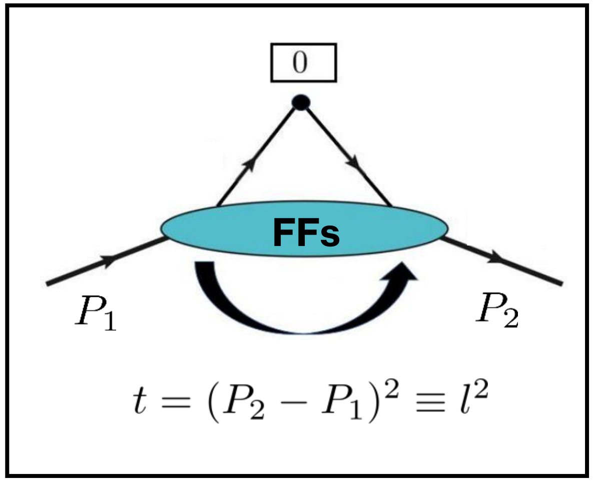
Consequence:



$$\langle P_2 | J_5^\mu | P_1 \rangle = \bar{u}(P_2) \left[\gamma^\mu \gamma_5 g_A(t) + \frac{l^\mu \gamma_5}{2M} g_P(t) \right] u(P_1)$$

Current conservation leads to (take $\partial_\mu J_5^\mu = 0$):

$$\boxed{\frac{g_P(t)}{2M} \approx -\frac{2M g_A(0)}{t} = -\frac{2M \Delta\Sigma}{t}}$$

Pole at t=0 from massless η_0 exchange

Anomalies in QCD

Chiral anomaly

Quantum mechanical:

U(1) axial symmetry is explicitly broken by chiral anomaly

Chiral anomaly equation: $$\boxed{\partial_\mu J_5^\mu = -\frac{n_f \alpha_s}{4\pi} F^{\mu\nu}\tilde{F}_{\mu\nu}}$$ $$\tilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$$

Anomalies in QCD

Chiral anomaly

Quantum mechanical:

U(1) axial symmetry is explicitly broken by chiral anomaly

Chiral anomaly equation:

$$\boxed{\partial_\mu J_5^\mu = -\frac{n_f \alpha_s}{4\pi} F^{\mu\nu} \tilde{F}_{\mu\nu}}$$

Consequence: In real QCD, there is no massless pole in form factor due to pole cancellation

$$\boxed{\frac{g_P(t)}{2M} = \frac{1}{t}\left[i\frac{\langle P_2|\frac{n_f\alpha_s}{4\pi}F\tilde{F}|P_1\rangle}{\bar{u}(P_2)\gamma_5 u(P_1)} - 2Mg_A(t)\right] \sim \frac{1}{t-m_{\eta'}^2}}$$

Eta-meson mass generation

(Witten-Veneziano, 1979)

Anomaly pole   **η_0 pole**

Anomalies in QCD

Chiral anomaly

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Anomaly pole $\qquad$ η_0 **pole**

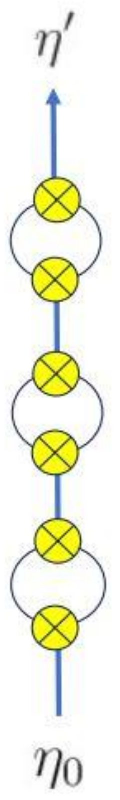
Eta-meson mass generation

(Witten-Veneziano, 1979)

Mass generation due to the topological fluctuation of the QCD vacuum

$$\frac{1}{t} + \frac{m_{\eta'}^2}{t^2} + \frac{m_{\eta'}^4}{t^3} + \cdots = \frac{1}{t - m_{\eta'}^2}$$

$$m_\eta^2 = -\frac{4n_f}{f_{\eta'}^2}\langle (F\tilde{F})^2\rangle$$



Anomalies in QCD

Trace anomaly

Classical:

- **Conformal symmetry:** **QCD Lagrangian invariant under scale transformation**

- **Quantity conserved:** **Dilatation current** $D^\mu = \Theta^{\mu\nu} x_\nu$ $\Theta^{\mu\nu}$: Energy Momentum Tensor (EMT)

Anomalies in QCD

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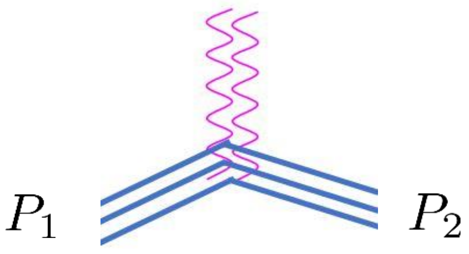

$$t = (P_2 - P_1)^2 \equiv l^2$$

$$\langle P_2 | \Theta^{\alpha\beta} | P_1 \rangle = \bar{u}(P_2) \left[A(t) \frac{P^{\alpha} P^{\beta}}{M} + (A(t) + B(t)) \frac{P^{(\alpha} i\sigma^{\beta)\lambda} l_{\lambda}}{2M} + D(t) \frac{l^{\alpha} l^{\beta} - g^{\alpha\beta} t}{4M} \right] u(P_1)$$

Current conservation leads to traceless EMT ($\Theta^{\alpha}_{\alpha} = 0$):

$$\boxed{\frac{3D(t)}{4} \approx \frac{M^2}{t}}$$

Pole at t=0 from massless particle exchange

Anomalies in QCD

Trace anomaly

Quantum mechanical:

Conformal symmetry is explicitly broken by trace anomaly

Trace anomaly equation:

$$\Theta^{\mu}_{\mu} = \frac{\beta(g)}{2g} F^{\mu\nu} F_{\mu\nu}$$

Anomalies in QCD

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Conformal symmetry is explicitly broken by trace anomaly

Trace anomaly equation:

$$\Theta^{\mu}_{\mu} = \frac{\beta(g)}{2g} F^{\mu\nu} F_{\mu\nu}$$

Consequence: In real QCD, there is no massless pole in form factor due to pole cancellation

See Mamo's talk

$$\frac{3D(t)}{4} \approx \frac{M^2}{t} \left(A(t) - \frac{\langle P_2 | \frac{\beta(g)}{2g} F^2 | P_1 \rangle}{M \bar{u}(P_2) u(P_1)} \right) \sim \frac{1}{t - m_G^2}$$

Massless pole **Anomaly pole**

Glueball mass dominance

(Mamo, Zahed, 2021/ Fujita et al, 2022)

Anomalies in QCD

Main message:

Anomalies constrain Form Factors

Form Factors are moments of GPDs:

$$\boxed{g_P(t) = \int_{-1}^{1} dx \tilde{E}(x,\xi,t)}$$

$$\boxed{A(t) + \xi^2 D(t) = \int_{-1}^{1} dx\, x\, H(x,\xi,t)}$$

Anomalies constrain GPDs

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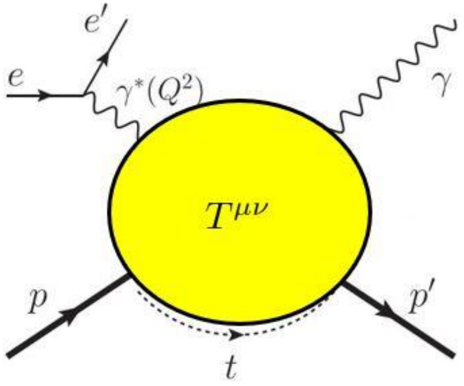
- 

Perturbative calculations of box diagrams in DVCS

- Non-perturbative relations between GPDs mediated by anomalies

Historical significance of the anomaly problem

THE ANOMALOUS GLUON CONTRIBUTION TO POLARIZED LEPTOPRODUCTION

G. ALTARELLI and G.G. ROSS[1]
CERN, CH-1211 Geneva 23, Switzerland

Received 29 June 1988

Gluonic contribution to g_1 and its relationship to the spin-dependent parton distributions

Geoffrey T. Bodwin and Jianwei Qiu*

THE ROLE OF THE AXIAL ANOMALY
IN MEASURING SPIN-DEPENDENT PARTON DISTRIBUTIONS

R.D. CARLITZ
Department of Physics and Astronomy, University of Pittsburgh, Pittsburgh, PA 15260, USA

J.C. COLLINS
Department of Physics, Illinois Institute of Technology, Chicago, IL 60616, USA

and

A.H. MUELLER
Department of Physics, Columbia University, New York, NY 10027, USA

Received 22 August 1988

The role of chiral anomaly in polarized DIS is a well-known old story

THE g_1 PROBLEM: DEEP INELASTIC ELECTRON SCATTERING
AND THE SPIN OF THE PROTON*

R.L. JAFFE and Aneesh MANOHAR**

Recent developments

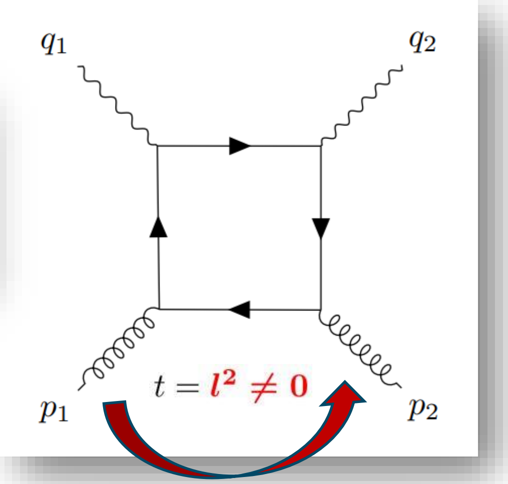
First calculation of box diagram with $t = l^2 \neq 0$:

The role of the chiral anomaly in polarized deeply inelastic scattering I: Finding the triangle graph inside the box diagram in Bjorken and Regge asymptotics

Andrey Tarasov[1,2] and Raju Venugopalan[3]

The role of the chiral anomaly in polarized deeply inelastic scattering II: Topological screening and transitions from emergent axion-like dynamics

Andrey Tarasov[1,2] and Raju Venugopalan[3]



Box diagram

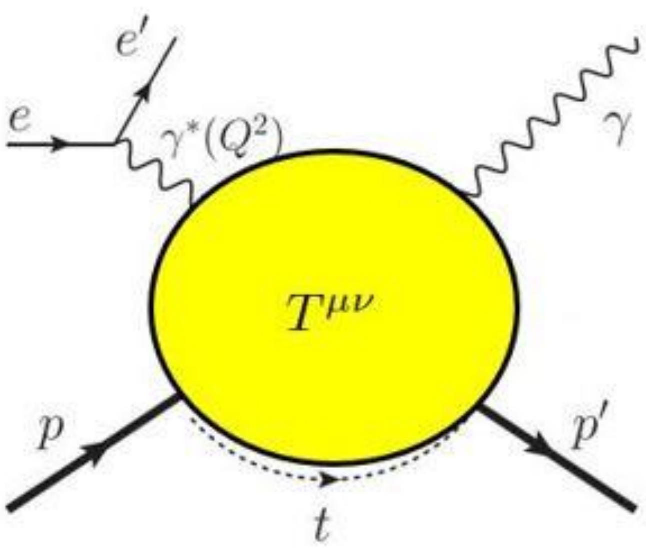
They computed polarized DIS using $t = l^2 \neq 0$ as a collinear regulator and revealed anomaly pole

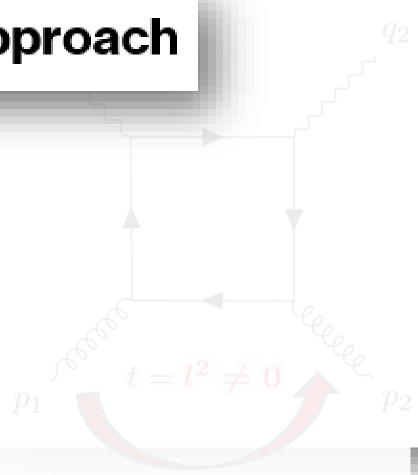
Imprint of Anomalies in QCD Compton scattering

Chiral and trace anomalies in Deeply Virtual Compton Scattering II: QCD factorization and beyond

Shohini Bhattacharya,[1,*] Yoshitaka Hatta,[2,1,†] and Werner Vogelsang[3,‡]

We explored the physics of anomaly in DVCS using Feynman-diagram approach





Deeply Virtual Compton Scattering (DVCS): Golden process to access GPDs

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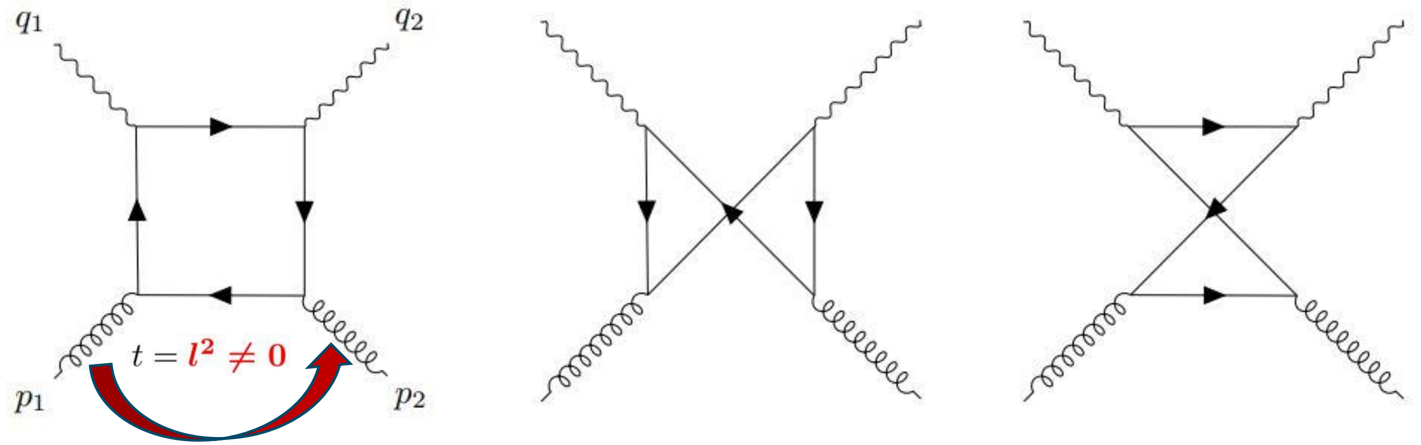

FIG. 1: Diagrams for the subprocess $\gamma^* g \to \gamma^* g$ in Compton scattering.

Gluon channel Diagrams

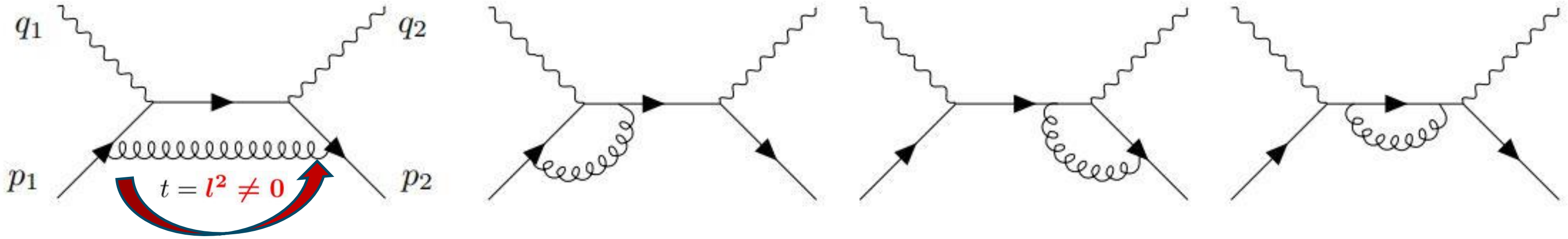

FIG. 2: Diagrams for the subprocess $\gamma^* q \to \gamma^* q$ in Compton scattering. Diagrams with photon lines crossed are not shown.

Quark channel Diagrams

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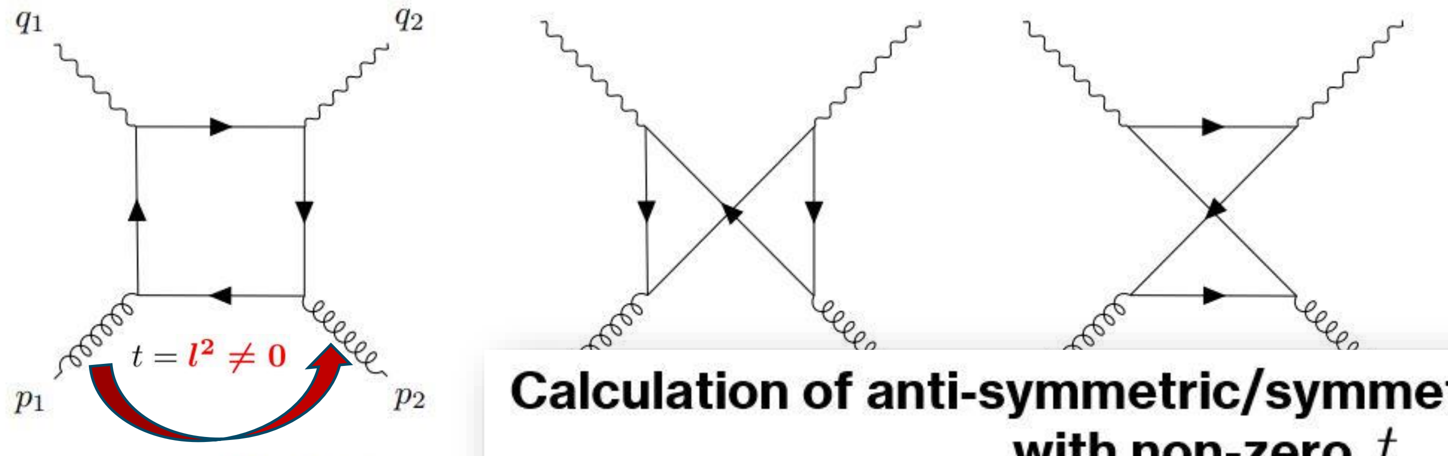


FIG. 1: Diagrams f

Calculation of anti-symmetric/symmetric (μ, ν) of Compton amplitude with non-zero t

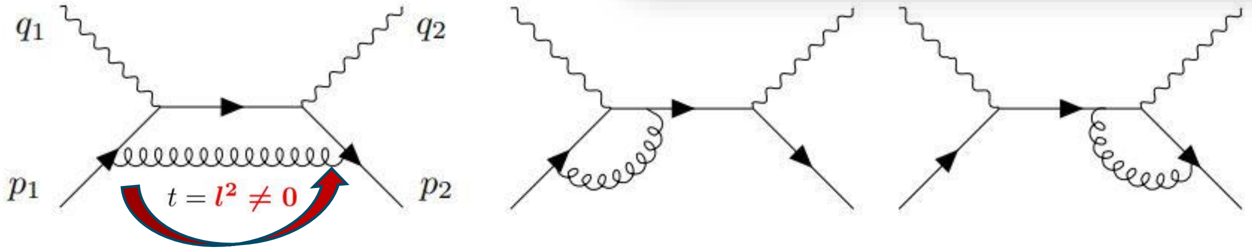
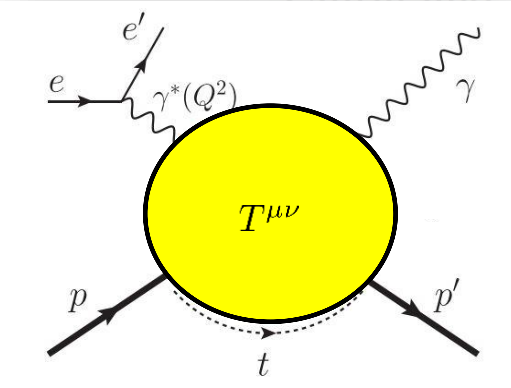


FIG. 2: Diagrams for the subprocess $\gamma^* q \to \gamma^* q$ in Compton scattering. Diagrams with photon l

Our calculation covers regimes:

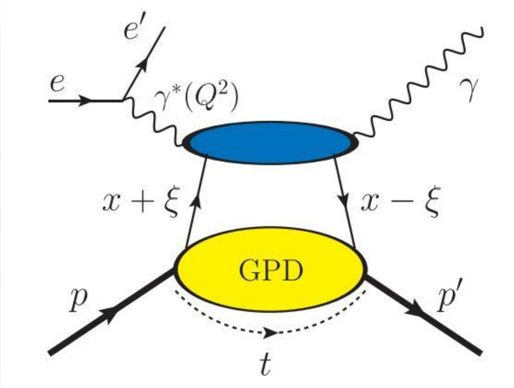
$$t = \Lambda^2_{\mathrm{QCD}} \ \& \ \Lambda^2_{\mathrm{QCD}} << t << Q^2$$

Remarks on DVCS:



Bjorken limit

$$t \sim \Lambda^2_{\rm QCD} \ll Q^2$$



The QCD factorization theorem: Collins, Freund; Ji, Osborne (1998)

$$T^{\mu\nu} = \sum_{a=q,g} \int \frac{dx}{x} C_a^{\mu\nu}\left(\frac{x_B}{x}, \frac{\xi}{x}\right) f_a(x,\xi,t) + \mathcal{O}(1/Q^2)$$

- **Original proofs implicitly assumed** $t = \Lambda^2_{\rm QCD}$
- **When** $\Lambda^2_{\rm QCD} << t << Q^2$ **, there are anomaly poles and Sudakov double logarithms that need to be addressed**

Our calculation covers regimes:

$t = \Lambda^2_{\rm QCD}$ & $\Lambda^2_{\rm QCD} << t << Q^2$

Imprint of Anomalies in QCD Compton scattering

Perturbative calculations of box diagrams

Main findings:

We computed one-loop Compton amplitude in all channels (quark/gluon, polarized/unpolarized) using t as a regulator and find:

- Collinear logs (complete GPD evolution kernel)
- Anomaly poles
- Sudakov double poles

Demonstrated that all the singularities can be absorbed into GPDs by infrared matching

Beyond factorization: What happens to the anomaly poles within GPDs?

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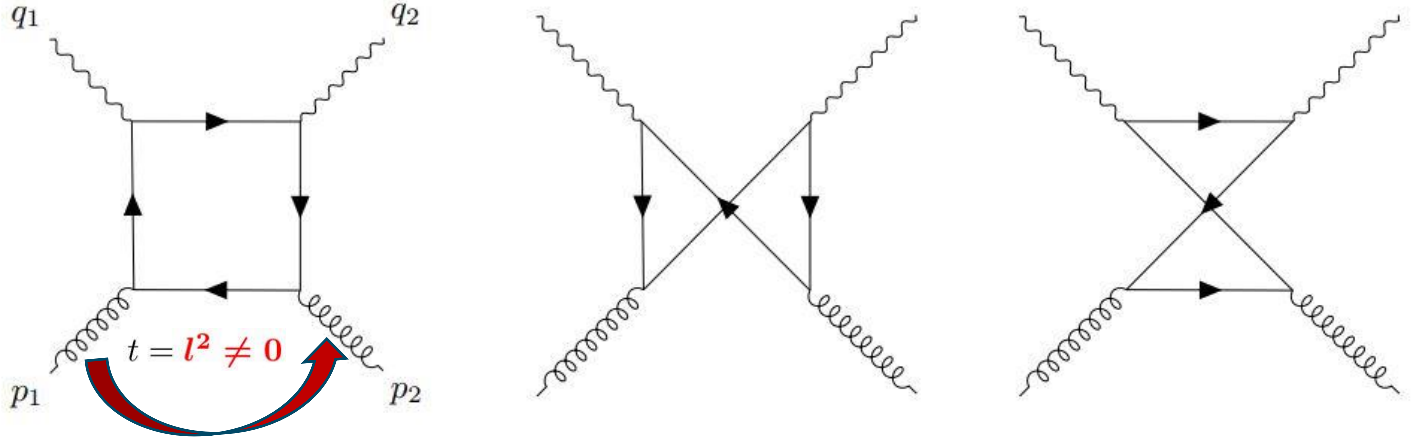

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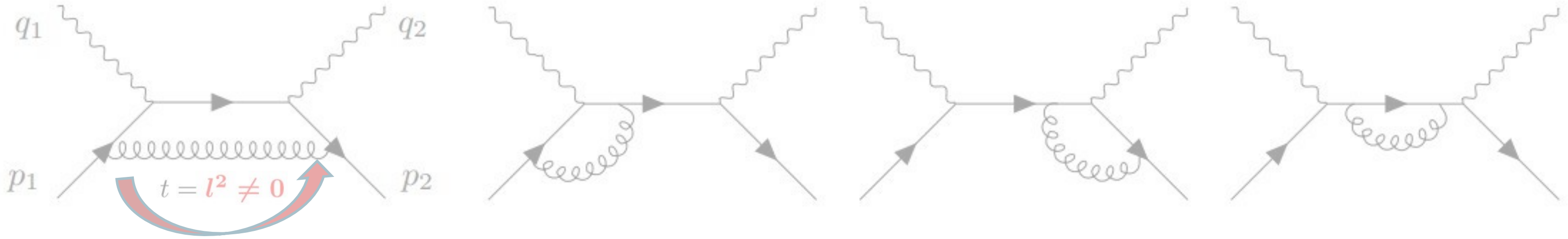
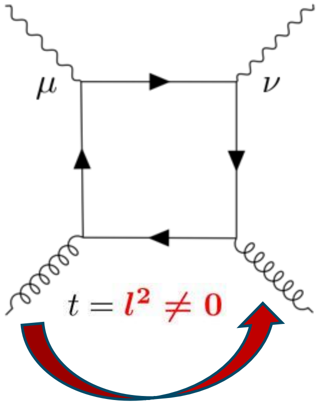


FIG. 2: Diagrams for the subprocess $\gamma^* q \to \gamma^* q$ in Compton scattering. Diagrams with photon lines crossed are not shown.

Imprint of Anomalies in QCD Compton scattering

Polarized DVCS & chiral anomaly



Antisymmetric part of Compton amplitude:

Collinear singularity regularized by l^2

$$-\epsilon^{\alpha\beta\mu\nu} P_\beta \times T^{\mathrm{asym}}_{\mu\nu} \approx \frac{1}{2}\frac{\alpha_s}{2\pi}\left(\sum_f e_f^2\right)\bar{u}(P_2)\left[\left(\tilde{\kappa}_{qg}\ln\frac{Q^2}{-l^2} + \delta\tilde{C}_g^{\mathrm{off}}\right)\otimes\langle F^{+\mu}\tilde{F}^{+}_{\mu}\rangle\gamma^\alpha\gamma_5 + \frac{l^\alpha}{l^2}\tilde{A}_g^{\mathrm{anom}}\otimes\tilde{\mathcal{F}}(x_B)\gamma_5\right]u(P_1)$$

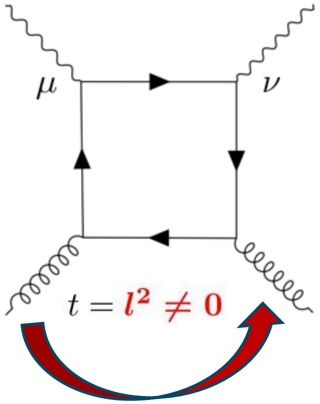
Reproduced the GPD evolution kernel

$$\boxed{\tilde{\kappa}_{qg}(\hat{x},\hat{\xi}) = \frac{2\hat{x}-1-\hat{\xi}^2}{(1-\hat{\xi}^2)^2}\ln\frac{\hat{x}-1}{\hat{x}} - 2\frac{\hat{x}-\hat{\xi}}{(1-\hat{\xi}^2)^2}\ln\frac{\hat{x}-\hat{\xi}}{\hat{x}} - (\hat{x}\to-\hat{x})}$$

Ji, Osborne; Belitsky, Mueller

Imprint of Anomalies in QCD Compton scattering

Polarized DVCS & chiral anomaly



Antisymmetric part of Compton amplitude:

Anomalous coefficient function

$$-\epsilon^{\alpha\beta\mu\nu}P_\beta \times T^{\mathrm{asym}}_{\mu\nu} \approx \frac{1}{2}\frac{\alpha_s}{2\pi}\left(\sum_f e_f^2\right)\bar{u}(P_2)\left[\left(\tilde{\kappa}_{qg}\ln\frac{Q^2}{-l^2}+\delta\tilde{C}_g^{\mathrm{off}}\right)\otimes\langle F^{+\mu}\tilde{F}^{+}_{\mu}\rangle\gamma^\alpha\gamma_5 + \frac{l^\alpha}{l^2}\tilde{A}_g^{\mathrm{anom}}\otimes\tilde{\mathcal{F}}(x_B)\gamma_5\right]u(P_1)$$

Twist 4 GPD

$$\tilde{\mathcal{F}}(x,\xi,t) = \frac{iP^+}{M}\int\frac{dz^-}{2\pi}e^{ixP^+z^-}\frac{\langle P_2|F^{\mu\nu}(-z^-/2)W\tilde{F}_{\mu\nu}(z^-/2)|P_1\rangle}{u(P_2)\gamma_5 u(P_1)}$$

(Non-local) chiral anomaly manifests itself in high-energy scattering amplitude

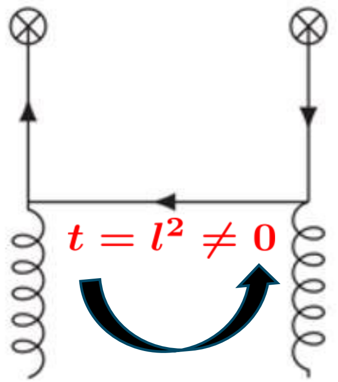
Imaginary part of pole term ($\xi = 0$) in agreement with Andrey & Raju

Anomalous coefficient function:

$$\tilde{A}_g^{\mathrm{anom}} = \frac{8T_R}{x}\frac{(1-\hat{x})\ln\frac{\hat{x}-1}{x}+(\hat{x}-\hat{\xi})\ln\frac{\hat{x}-\hat{\xi}}{\hat{x}}-(\hat{x}\to-\hat{x})}{1-\hat{\xi}^2}$$

Imprint of Anomalies in QCD Compton scattering

Justifying factorization: “Infrared subtraction”



GPD at one loop:

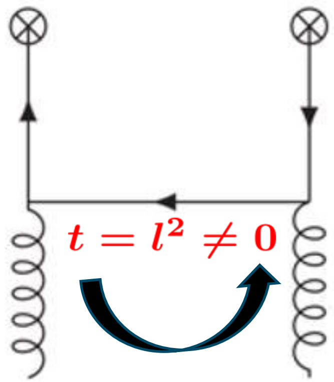
We computed quark GPD of a gluon & found the same pole structure in one-loop calculation:

$$
= \begin{cases}
\frac{\alpha_s T_R}{2\pi}\left[(1-\xi^2) i\epsilon^{+p\epsilon_2^*\epsilon_1}\left(\frac{2x-1-\xi^2}{(1-\xi^2)^2}\left(\frac{\left(\frac{\tilde{\mu}^2}{-l^2}\right)^{\epsilon}}{\epsilon_{\mathrm{UV}}} - \ln\frac{(1-x)^2}{1-\xi^2}\right) - 2\frac{1-x}{(1-\xi^2)^2}\right) + \boxed{\frac{2il^+\epsilon^{\epsilon_1\epsilon_2^* lp}}{l^2}}\frac{1-x}{1-\xi^2}\right] & x > \xi \\
\frac{\alpha_s T_R}{2\pi}\left[(1-\xi^2) i\epsilon^{+p\epsilon_2^*\epsilon_1}\frac{\left(\frac{\tilde{\mu}^2}{-l^2}\right)^{\epsilon}}{\epsilon_{\mathrm{UV}}}\frac{-1}{(1+\xi)^2} + \boxed{\frac{2il^+\epsilon^{\epsilon_1\epsilon_2^* lp}}{l^2}}\frac{1}{1+\xi} \right. & x < \xi \\
\left. (1-\xi^2) i\epsilon^{+p\epsilon_2^*\epsilon_1}\frac{1}{(1-\xi^2)^2}\left[-2\xi\ln(\xi^2-x^2)+(1+\xi^2)\ln(1-x^2)-2x\ln\frac{(1-x)(x+\xi)}{(1+x)(\xi-x)}-2(1+\xi^2)\ln(1+\xi)+4\xi\ln(2\xi)+2\xi-2\right]\right] &
\end{cases}
$$

The pole “belongs” to GPD

Imprint of Anomalies in QCD Compton scattering

Justifying factorization: “Infrared subtraction”



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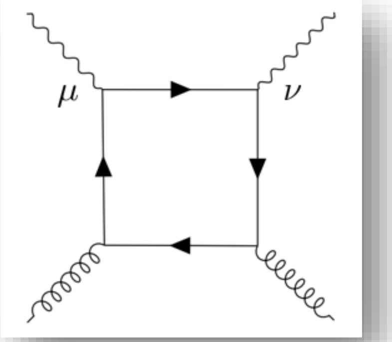
\dfrac{\alpha_s T_R}{2\pi}\left[(1-\xi^2) i\epsilon^{+p\epsilon_2^*\epsilon_1}\dfrac{\left(\frac{\tilde{\mu}^2}{-l^2}\right)^{\epsilon}}{\epsilon_{\mathrm{UV}}}\dfrac{-1}{(1+\xi)^2}+\boxed{\dfrac{2il^+\epsilon^{\epsilon_1\epsilon_2^* lp}}{l^2}}\dfrac{1}{1+\xi}\right] & x < \xi \\
\cdots)\ln(1+\xi)+4\xi\ln(2\xi)+2\xi-2\Big]\Big]
\end{cases}
$$

The pole “belongs” to GPD

Absorb the 1/t poles of Compton amplitude into twist-2 GPDs

Factorization restored

Comment on equivalence with $\overline{\text{MS}}$ scheme



Antisymmetric part of Compton amplitude

Coefficient function

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$$\begin{aligned}\delta\tilde{C}_1^g(\hat{x},\hat{\xi}) = &-\frac{2\hat{x}-1-\hat{\xi}^2}{(1-\hat{\xi}^2)^2}\ln\frac{\hat{x}-1}{\hat{x}}+2\frac{\hat{x}-\hat{\xi}}{(1-\hat{\xi}^2)^2}\ln\frac{\hat{x}-\hat{\xi}}{\hat{x}}\\ &+\frac{2\hat{x}-1-\hat{\xi}^2}{2(1-\hat{\xi}^2)^2}\ln^2\frac{\hat{x}-1}{\hat{x}}+\frac{\hat{\xi}}{1-\hat{\xi}^2}\ln^2\frac{\hat{x}-\hat{\xi}}{\hat{x}}-\frac{\hat{x}}{(1-\hat{\xi}^2)^2}\ln\frac{\hat{x}-\hat{\xi}}{\hat{x}}\ln\frac{\hat{x}+\hat{\xi}}{\hat{x}}\\ &+\frac{2\hat{\xi}}{(1-\hat{\xi}^2)^2}\mathrm{Li}_2\frac{-2\hat{\xi}}{\hat{x}-\hat{\xi}}+\frac{2\hat{x}-1-\hat{\xi}^2}{(1-\hat{\xi}^2)^2}\left(\mathrm{Li}_2\frac{1-\hat{\xi}}{1-\hat{x}}+\mathrm{Li}_2\frac{1+\hat{\xi}}{1-\hat{x}}\right)-(\hat{x}\to-\hat{x})\end{aligned}$$

After subtracting IR singularities and finite terms, $t \neq 0$ regularization is equivalent to $\overline{\text{MS}}$ scheme

This means that the result can be smoothly connected to the regime $t \sim \Lambda^2_{\text{QCD}}$ as considered in the works by Collins, Freund; Ji, Obsborne

Imprint of Anomalies in QCD Compton scattering

Perturbative calculations of box diagrams

Main findings:

We computed one-loop Compton amplitude in all channels (quark/gluon, polarized/unpolarized) using t as a regulator and find:

- Collinear logs (complete GPD evolution kernel)
- Anomaly poles
- Sudakov double poles

Demonstrated that all the singularities can be absorbed into GPDs by infrared matching

Beyond factorization: What happens to the anomaly poles within GPDs?

Imprint of Anomalies in QCD Compton scattering

Fate of anomaly pole

Chiral anomaly pole in GPD $\tilde{E}$:

$$\tilde{E}_f(x_B, l^2) + \tilde{E}_f(-x_B, l^2) = \frac{\alpha_s}{2\pi} \underbrace{\frac{2M}{l^2}}_{\text{Anomaly pole}} \tilde{A}_g^{\text{anom}} \otimes \tilde{\mathcal{F}}(x_B, l^2)$$

Remarks:

- We absorbed the $\frac{1}{t}$ pole in GPD $\tilde{E}$. What does this mean physically?

The GPD $\tilde{E}$ cannot have $\frac{1}{t}$ pole; instead, it should have $\frac{1}{t - m_{\eta'}^2}$. This shift $\frac{1}{t} \to \frac{1}{t - m_{\eta'}^2}$ is well-known to occur in form factor $g_P(t)$.

Can we discuss the same thing for GPD?

Imprint of Anomalies in QCD Compton scattering

Fate of anomaly pole

Chiral anomaly pole in GPD $\tilde{E}$:

$$\tilde{E}_f(x_B, l^2) + \tilde{E}_f(-x_B, l^2) = \frac{\alpha_s}{2\pi}\frac{2M}{l^2}\tilde{A}_g^{\mathrm{anom}} \otimes \tilde{\mathcal{F}}(x_B, l^2)$$

Anomaly pole

Remarks:

- We absorbed the $\frac{1}{t}$ pole in GPD $\tilde{E}$. What does this mean physically?

  The pole that we found exactly integrates to:

$$\frac{g_P(t)}{2M} = \boxed{\frac{1}{\color{red}{t}}\left[i\frac{\langle P_2|\frac{n_f\alpha_s}{4\pi}F\tilde{F}|P_1\rangle}{\bar{u}(P_2)\gamma_5 u(P_1)}\right.} - 2Mg_A(t)\Bigg] \sim \frac{1}{t-m_{\eta'}^2}$$

Anomaly pole η_0 **pole**

Imprint of Anomalies in QCD Compton scattering

Fate of anomaly pole

Chiral anomaly pole in GPD $\tilde{E}$:

$$\tilde{E}_f(x_B, l^2) + \tilde{E}_f(-x_B, l^2) = \underbrace{\frac{\alpha_s}{2\pi}\frac{2M}{l^2}}_{\text{Anomaly pole}} \tilde{A}_g^{\rm anom} \otimes \tilde{\mathcal{F}}(x_B, l^2)$$

Remarks:

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Can we make this correspondence more precise for GPD, nonperturbatively?

Imprint of Anomalies in QCD Compton scattering

Fate of anomaly pole

Chiral anomaly pole in GPD $\tilde{E}$:

$$\tilde{E}_f(x_B, l^2) + \tilde{E}_f(-x_B, l^2) = \frac{\alpha_s}{2\pi} \underbrace{\frac{2M}{l^2}}_{\text{Anomaly pole}} \tilde{A}_g^{\text{anom}} \otimes \tilde{\mathcal{F}}(x_B, l^2)$$

Remarks:

- **The GPD $\tilde{E}$ cannot have $\frac{1}{t}$ pole; instead, it should have $\frac{1}{t - m_{\eta'}^2}$.**
- Unlike the anomaly pole, the η_0 pole does not appear in perturbation theory. Instead, it can be addressed through an effective action approach, but later **we will explicitly derive the η_0 pole and find that its residue consists of the polarized GPD $\tilde{H}$ and twist-4 GPDs.**

Imprint of Anomalies in QCD Compton scattering

Fate of anomaly pole

Chiral anomaly pole in GPD $\tilde{E}$ (see later slides):

$$\tilde{E}_f(x_B,l^2)+\tilde{E}_f(-x_B,l^2) = \boxed{\tilde{E}_f^{\mathrm{c.t.}}(x_B,l^2)+\tilde{E}_f^{\mathrm{c.t.}}(-x_B,l^2)} + \frac{\alpha_s}{2\pi}\frac{2M}{l^2}\,\tilde{A}_g^{\mathrm{anom}}\otimes\tilde{\mathcal{F}}(x_B,l^2)$$

η_0 **pole**

Anomaly pole

$$\boxed{\sim \frac{1}{l^2 - m_{\eta'}^2}}$$

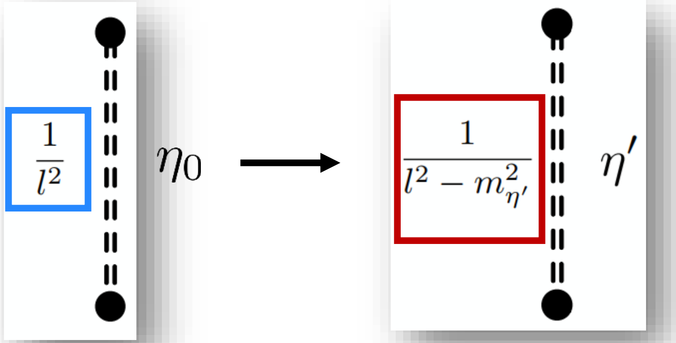
Connection between twist 2 & twist 4 GPDs due to anomaly

Fate of anomaly pole:
Cancellation with non-perturbative pole arising due to η_0 exchange

(Witten-Veneziano scenario at the GPD level)

$$\boxed{\tilde{E}_f^{\mathrm{c.t.}}(x_B,l^2)+\tilde{E}_f^{\mathrm{c.t.}}(-x_B,l^2)} \approx -\frac{\alpha_s}{2\pi}\frac{2M}{l^2}\,\tilde{A}_g^{\mathrm{anom}}\otimes\tilde{\mathcal{F}}(x_B,l^2=0)$$

Eta-meson mass generation:



Imprint of Anomalies in QCD Compton scattering

Fate of anomaly pole

Chiral anomaly pole in GPD $\tilde{E}$ (see later slides):

$$\tilde{E}_f(x_B,l^2)+\tilde{E}_f(-x_B,l^2) = \underbrace{\tilde{E}_f^{\text{c.t.}}(x_B,l^2)+\tilde{E}_f^{\text{c.t.}}(-x_B,l^2)}_{\eta_0\ \textbf{pole}} + \frac{\alpha_s}{2\pi}\underbrace{\frac{2M}{l^2}}_{\textbf{Anomaly pole}}\tilde{A}_g^{\text{anom}}\otimes\tilde{\mathcal{F}}(x_B,l^2)$$

$$\sim \frac{1}{l^2-m^2_{\eta'}}$$

The physics of anomalies present at the level of Form Factor is established for the first time at the level of GPDs

Imprint of Anomalies in QCD Compton scattering

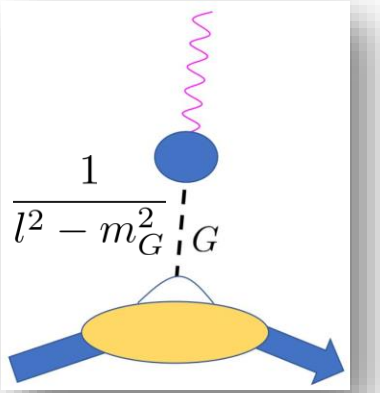
Unpolarized DVCS & trace anomaly

Trace anomaly pole in GPDs H & E :

$$H \sim -E \sim \frac{\alpha_s}{l^2} A \otimes \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \frac{\langle P_2|F^{\mu\nu}(-z^-/2)WF_{\mu\nu}(z^-/2)|P_1\rangle}{\bar{u}(P_2)u(P_1)}$$

(Non-local) trace anomaly manifests itself in the GPD

Fate of anomaly pole:
Pole cancellation results in the generation of glueball mass



Imprint of Anomalies in QCD Compton scattering

Chiral and trace anomalies in Deeply Virtual Compton Scattering : QCD factorization and beyond

Shohini Bhattacharya,[1,*] Yoshitaka Hatta,[2,1,†] and Werner Vogelsang[3,‡]

We explored the physics of anomaly in DVCS using Feynman-diagram approach

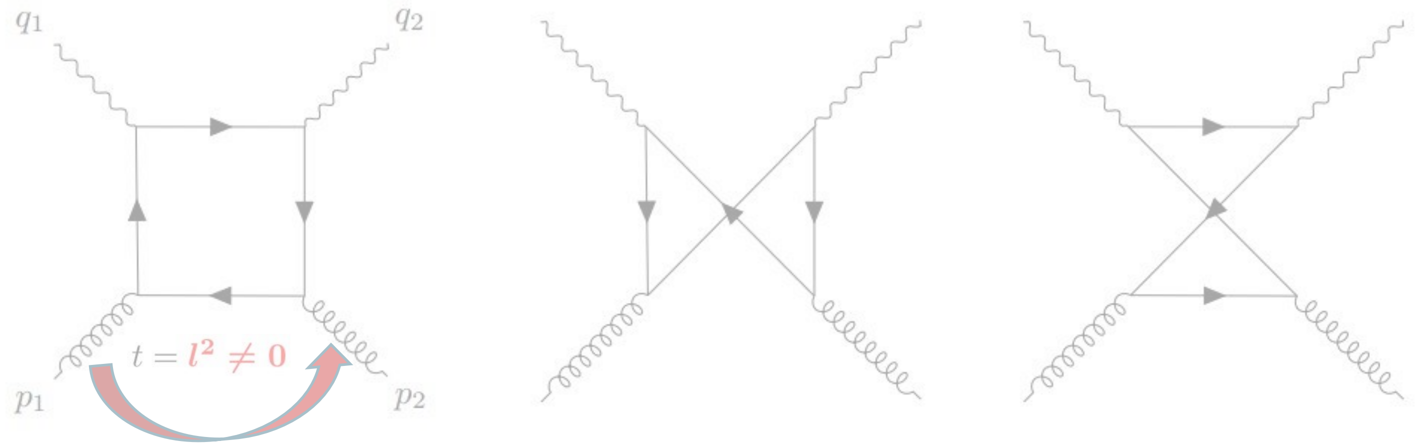


FIG. 1: Diagrams for the subprocess $\gamma^* g \to \gamma^* g$ in Compton scattering.

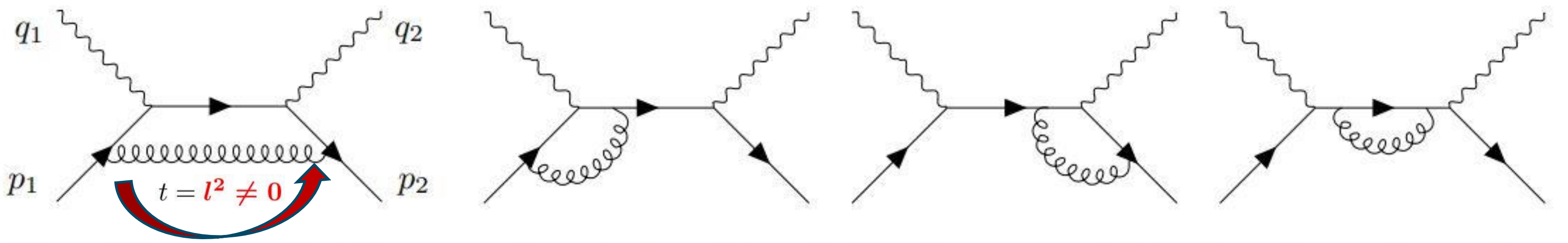
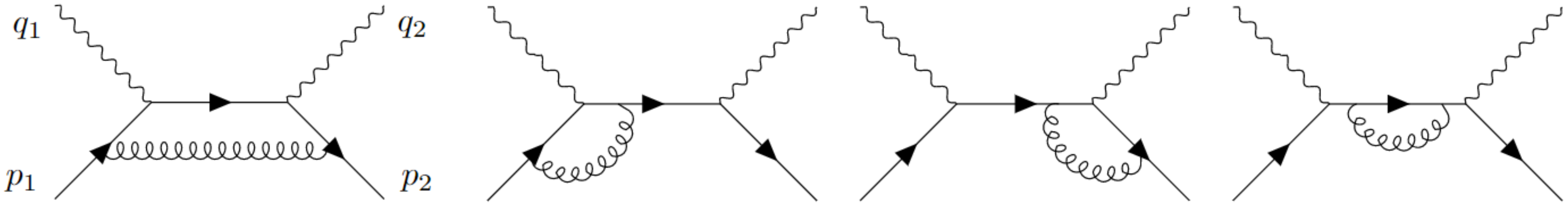


FIG. 2: Diagrams for the subprocess $\gamma^* q \to \gamma^* q$ in Compton scattering. Diagrams with photon lines crossed are not shown.

Quark channel Diagrams

Imprint of Anomalies in QCD Compton scattering

Quark-channel diagrams in DVCS



Example: Antisymmetric case

$$\sim \cancel{\frac{1}{l^2}} + \tilde{\kappa}_{qq}(\hat{x},\hat{\xi}) \ln \frac{Q^2}{-l^2} \qquad + \delta \tilde{C}_1^q(\hat{x},\hat{\xi})$$

No pole!

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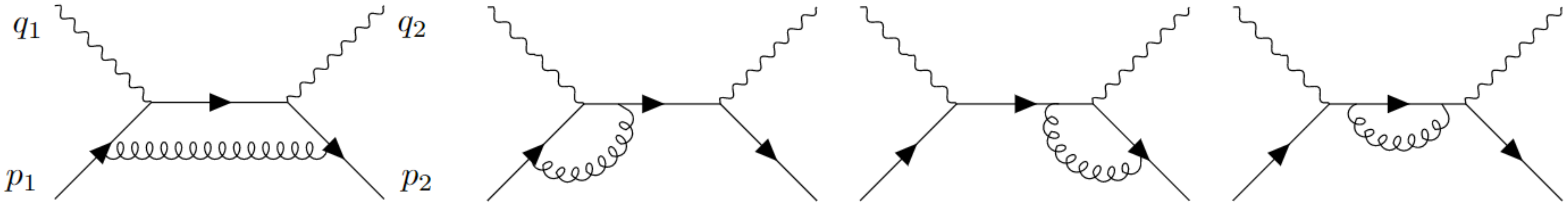
Quark-channel diagrams in DVCS



Example: Antisymmetric case

$$\sim \cancel{\frac{1}{l^2}} + \tilde{\kappa}_{qq}(\hat{x}, \hat{\xi}) \ln \frac{Q^2}{-l^2} + \delta \tilde{C}_1^q(\hat{x}, \hat{\xi})$$

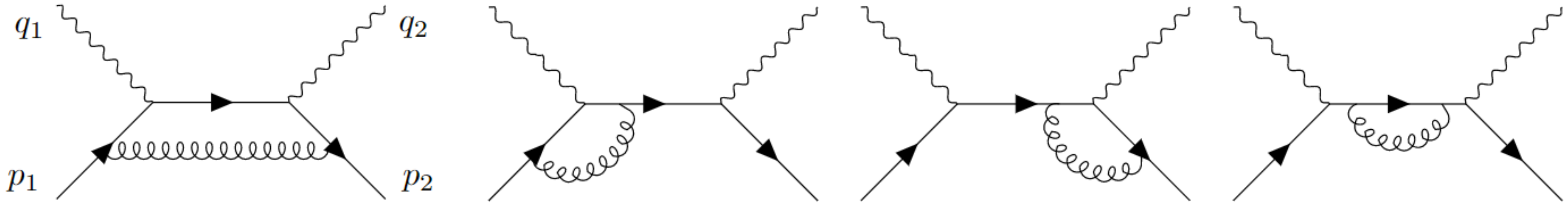
Reproduced the known logarithms from literature

$$\tilde{\kappa}_{qq}(\hat{x}, \hat{\xi}) = \frac{3}{2(1-\hat{x})} + \frac{\hat{x}^2 + 1 - 2\hat{\xi}^2}{(1-\hat{\xi}^2)(1-\hat{x})} \ln \frac{\hat{x}-1}{\hat{x}} - \frac{(\hat{x}-\hat{\xi})(1+\hat{x}^2+2\hat{x}\hat{\xi})}{(1-\hat{x}^2)(1-\hat{\xi}^2)} \ln \frac{\hat{x}-\hat{\xi}}{\hat{x}} - (\hat{x} \to -\hat{x})$$

Ji, Osborne; Belitsky, Mueller

Imprint of Anomalies in QCD Compton scattering

Quark-channel diagrams in DVCS



Example: Antisymmetric case

$$\sim \cancel{\frac{1}{l^2}} + \tilde{\kappa}_{qq}(\hat{x},\hat{\xi}) \ln \frac{Q^2}{-l^2} \quad + \delta\tilde{C}_1^q(\hat{x},\hat{\xi})$$

Coefficient function

$$\delta\tilde{C}_1^q(\hat{x},\hat{\xi}) = -\frac{\left(\frac{Q^2}{-l^2}\right)^{\epsilon_{IR}}}{\epsilon_{IR}^2(1-\hat{x})} - \frac{3\left(\frac{Q^2}{-l^2}\right)^{\epsilon_{IR}}}{2\epsilon_{IR}(1-\hat{x})} + \frac{-1+2\hat{x}-4\hat{x}^2+3\hat{\xi}^2}{2(1-\hat{x})(1-\hat{\xi}^2)} \ln\frac{\hat{x}-1}{\hat{x}} + \frac{(\hat{x}-\hat{\xi})(1+2\hat{x}^2+3\hat{x}\hat{\xi})}{(1-\hat{x}^2)(1-\hat{\xi}^2)} \ln\frac{\hat{x}-\hat{\xi}}{\hat{x}}$$

$$\cdots \ln\frac{\hat{x}-\hat{\xi}}{\hat{x}} \ln\frac{\hat{x}+\hat{\xi}}{\hat{x}} + \frac{\pi^2-54}{12(1-\hat{x})}$$

$$+ \frac{\hat{\xi}}{1-\hat{\xi}^2}\mathrm{Li}_2 \cdots + \frac{1+\hat{x}^2}{(1-\hat{x})} \cdots$$

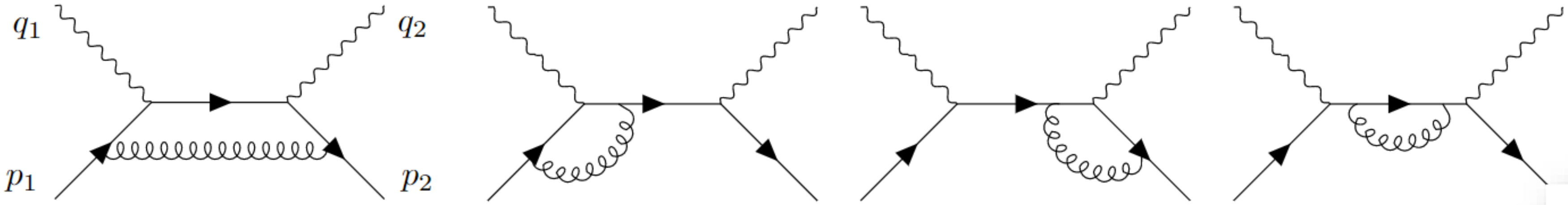
Unexpected double IR pole

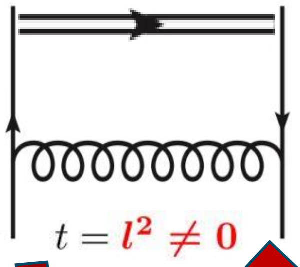
Unexpected single IR pole

Sudakov logs! $\ln\left(\frac{Q^2}{-l^2}\right), \ln^2\left(\frac{Q^2}{-l^2}\right)$

Imprint of Anomalies in QCD Compton scattering

Quark-channel diagrams in DVCS





But, when you compute GPD itself, you find the same double, single IR poles!
These poles can be systematically absorbed into GPD

Factorization restored

$$\delta\tilde{C}_1^q(\hat{x},\hat{\xi}) = -\frac{\left(\frac{Q^2}{-l^2}\right)^{\epsilon_{IR}}}{\epsilon_{IR}^2(1-\hat{x})} - \frac{3\left(\frac{Q^2}{-l^2}\right)^{\epsilon_{IR}}}{2\epsilon_{IR}(1-\hat{x})} + \frac{-1+2\hat{x}-4\hat{x}^2+3\hat{\xi}^2}{2(1-\hat{x})(1-\hat{\xi}^2)}\ln\frac{\hat{x}-1}{\hat{x}} + \frac{(\hat{x}-\hat{\xi})(1+2\hat{x}^2+3\hat{x}\hat{\xi})}{(1-\hat{x}^2)(1-\hat{\xi}^2)}\ln\frac{\hat{x}-\hat{\xi}}{\hat{x}}$$

$$\cdots\frac{\cdots+\hat{x}^2-2\hat{\xi}^2}{\cdots\hat{x})(1-\hat{\xi}^2)}\ln^2\frac{\hat{x}-1}{\hat{x}}\cdots\frac{\cdots+\hat{x}^2-2\hat{\xi}^2)}{\cdots\hat{x}^2)(1-\hat{\xi}^2)}\ln\frac{\hat{x}-\hat{\xi}}{\hat{x}}\ln\frac{\hat{x}+\hat{\xi}}{\hat{x}} + \frac{\pi^2-54}{12(1-\hat{x})}$$

$$+\frac{\hat{\xi}}{1-\hat{\xi}^2}\mathrm{Li}_2\frac{-2\hat{\xi}}{\hat{x}-\hat{\xi}} + \frac{1+\hat{x}^2-2\hat{\xi}^2}{(1-\hat{x})(1-\hat{\xi}^2)}\left(\mathrm{Li}_2\frac{1-\hat{\xi}}{1-\hat{x}} + \mathrm{Li}_2\frac{1+\hat{\xi}}{1-\hat{x}}\right) - (\hat{x}\to-\hat{x})$$

Unexpected double IR pole

Unexpected single IR pole

Outline

- Motivation: Chiral & trace anomalies & GPDs

-

Perturbative calculations of box diagrams in DVCS

- **Non-perturbative relations between GPDs mediated by anomalies**

Anomaly-mediated relations between twist-2,3,4 GPDs

Nonlocal chiral anomaly and generalized parton distributions

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Anomaly-mediated relations between twist-2,3,4 GPDs

Main findings:

- **Derived** η_0 pole of nonperturbative origin to cancel the 1/t anomaly pole:

$$\tilde{E}_f(x_B, l^2) + \tilde{E}_f(-x_B, l^2) = \underbrace{\boxed{\tilde{E}_f^{\text{c.t.}}(x_B, l^2) + \tilde{E}_f^{\text{c.t.}}(-x_B, l^2)}}_{\eta_0 \textbf{ pole}} + \frac{\alpha_s}{2\pi} \underbrace{\frac{2M}{l^2}}_{\textbf{Anomaly pole}} \tilde{A}_g^{\text{anom}} \otimes \tilde{\mathcal{F}}(x_B, l^2)$$

Anomaly-mediated relations between twist-2,3,4 GPDs

Main findings:

- **Derived** η_0 pole of nonperturbative origin to cancel the 1/t anomaly pole.

- Typical critiques for perturbative calculations: The 1/t poles had been identified in **partonic scattering amplitudes** and GPDs were evaluated using **partonic matrix elements**. It had been argued that they must be 'promoted' to **proton matrix elements** in order to be consistent with form factor relation:

$$\boxed{g_A(t) + \frac{t}{4M^2} g_P(t) = \frac{i}{2M} \frac{\langle p' | \frac{n_f \alpha_s}{4\pi} F\tilde{F} | p \rangle}{\bar{u}(p')\gamma_5 u(p)}}$$

We have now derived the distributions as well as the 't' at the proton (not partonic) level.

Anomaly-mediated relations between twist-2,3,4 GPDs

Main findings

Non-local chiral anomaly equation at the operator level:

$$\mathcal{D}_\mu\left[\bar{\psi}(z_2^-)W\gamma^\mu\gamma_5\psi(z_1^-)\right] = O_F(z_2^-, z_1^-) - \frac{n_f\alpha_s}{2\pi}\int_0^1 d\alpha \int_0^{1-\alpha} d\beta F^{\mu\nu}(z_{12}^{\beta-})\tilde{W}\tilde{F}_{\mu\nu}\left(z_{21}^{\alpha-}\right) + \ldots$$

Twist-4 GPD: $$O_F(z_2, z_1) \equiv iz^\nu \int_0^1 d\alpha \bar{\psi}(z_2)\gamma^\mu\gamma_5 W g F_{\mu\nu}(z_{21}^\alpha)W\psi(z_1),$$

$$z^\mu \equiv z_1^\mu - z_2^\mu, \qquad (z_{21}^\alpha)^\mu \equiv \alpha z_2^\mu + (1-\alpha)z_1^\mu$$

- The full non-Abelian contribution of $F\tilde{F}$ and the Wilson line is included, unlike in perturbative calculations.
- Agreement with Mueller, Teryaev (1997); but we provided a **more complete derivation** following Agaev, Braun, Offen, Porkert, Schafer (2014).

Anomaly-mediated relations between twist-2,3,4 GPDs

Main findings

Sketch of the key steps in the derivation:

Anomaly-mediated relations between twist-2,3,4 GPDs

Main findings

Sketch of the key steps in the derivation:

1) Approach the light-cone from the space-like region

$$z^2 < 0$$

where naïve equation of motion holds.

$$\mathcal{D}_\mu \left[\bar{\psi}(z_2)\gamma^\mu\gamma_5 W_{z_2,z_1}\psi(z_1)\right] = iz^\nu \int_0^1 d\alpha \bar{\psi}(z_2)\gamma^\mu\gamma_5 W g F_{\mu\nu}(z_{21}^\alpha) W \psi(z_1)$$

If the separation is light-like,

$$z^\mu = \delta^\mu_- z^-$$

the anomaly is still there even when $z^- \neq 0$

D. Muller and Teryaev (1997)

Anomaly-mediated relations between twist-2,3,4 GPDs

Main findings

Sketch of the key steps in the derivation:

2) Then take $z^2 \to 0$

Main findings

Sketch of the key steps in the derivation:

3) Nonlocal operator product expansion

$$\psi(z_1)\bar{\psi}(z_2) = \frac{i\not{z}}{2\pi^2(z^2)^2}W_{z_1,z_2} \boxed{-\frac{iz^\rho}{8\pi^2 z^2}\int_0^1 d\beta W_{z_1,z_{12}^\beta} g\tilde{F}_{\rho\lambda}(z_{12}^\beta)W_{z_{12}^\beta,z_2}\gamma^\lambda\gamma_5}$$
$$+\frac{i}{32\pi^2}\left(\frac{1}{\epsilon_{IR}}+\ln\frac{-z^2\mu_{IR}^2 e^{2\gamma_E}}{4}\right)\Bigg[g\int_0^1 d\alpha\alpha(1-\alpha)z_\mu D^2\tilde{F}^{\mu\nu}(z_{12}^\alpha)\gamma_\nu\gamma_5 + ig^2 z_\mu\int_0^1 d\alpha\int_0^\alpha d\beta$$
$$\times\left\{(1-2\alpha+2\beta)F^{\mu\lambda}(z_{12}^\alpha)\tilde{F}_{\lambda\rho}(z_{12}^\beta)\gamma^\rho + \tilde{F}^{\mu\lambda}(z_{12}^\alpha)F_{\lambda\rho}(z_{12}^\beta)\gamma^\rho + \beta F^{\rho\nu}(z_{12}^\alpha)\tilde{F}_{\rho\nu}(z_{12}^\beta)\gamma^\mu\right\}\gamma_5+\cdots\Bigg]$$

Balitsky, Braun (1989)

Schouten identity :

$$2z^\nu\left(F_{\nu\mu}W\tilde{F}^{\mu\rho}+\tilde{F}_{\nu\mu}WF^{\mu\rho}\right)z_\rho = -z^2F^{\mu\nu}W\tilde{F}_{\mu\nu}$$

Anomaly-mediated relations between twist-2,3,4 GPDs

Main findings

Sketch of the key steps in the derivation:

4) UV matching

The light-cone limit $z^2 \to 0$ is not smooth. One needs operator matching

$$A_i(z^2) = \sum_j C_{ij}(\ln(-z^2\mu_{UV}^2)) \otimes A_j(z^2 = 0, \mu_{UV}^2) + \mathcal{O}(z^2),$$

Nowadays familiar in quasi-PDF, pseudo-PDF business

Anomaly-mediated relations between twist-2,3,4 GPDs

Main findings

Non-local chiral anomaly equation at the operator level:

$$\mathcal{D}_\mu\left[\bar{\psi}(z_2^-)W\gamma^\mu\gamma_5\psi(z_1^-)\right] = O_F(z_2^-,z_1^-) - \frac{n_f\alpha_s}{2\pi}\int_0^1 d\alpha\int_0^{1-\alpha} d\beta F^{\mu\nu}(z_{12}^{\beta-})\tilde{W}\tilde{F}_{\mu\nu}\left(z_{21}^{\alpha-}\right) + \ldots$$

Byproduct:

In the **local limit**, one can make use of the following identity: (**Hint:** use the Schouten identity)

$$4z^\nu F_{\nu\mu}\tilde{F}^{\mu\rho}z_\rho = -z^2F^{\mu\nu}\tilde{F}_{\mu\nu}$$

This showed us that the symmetric limit procedure $\lim_{x\to 0}\frac{x^\mu x^\nu}{x^2} \to \frac{g^{\mu\nu}}{4}$ is actually unnecessary even in Peskin's textbook derivation of the chiral anomaly.

Anomaly-mediated relations between twist-2,3,4 GPDs

Main findings

Now take the proton matrix element $\langle P'|...|P\rangle$ of the operator-level equation:

$$\Delta_\mu P^+ \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle p'|\bar{\psi}(-z^-/2) W \gamma^\mu \gamma_5 \psi(z^-/2)|p\rangle = \frac{n_f \alpha_s M}{2\pi} \tilde{C}^{anom} \otimes \tilde{\mathcal{F}}(x,\xi,t) + O_F(x,\xi,t)$$

$$iO_F(x,\xi,t) = P^+ \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle p'|O_F(-z^-/2, z^-/2)|p\rangle$$

Anomaly-mediated relations between twist-2,3,4 GPDs

Main findings

1) Relations between the twist-2 and twist-4 GPDs of the proton mediated by the chiral anomaly:

$$\tilde{E} + \tilde{E}_4 = \boxed{\frac{4M^2}{t} \left(\frac{n_f \alpha_s}{2\pi} \tilde{C}^{anom} \otimes (\tilde{\mathcal{F}}_2 + \tilde{\mathcal{F}}_4) \right.} \left. - \tilde{H} - \tilde{H}_4 + O_{2F} + O_{4F} \right), \qquad \left(t = \text{Hadron-level Variable} \right)$$

This is the **anomaly pole** that has been identified in the partonic level calculations of the DVCS amplitude & GPDs (SB, Hatta, Vogelsang) & in (Tarasov, Venugopalan) for polarized DIS

Anomaly-mediated relations between twist-2,3,4 GPDs

Main findings

1) Relations between the twist-2 and twist-4 GPDs of the proton mediated by the chiral anomaly:

$$\tilde{E} + \tilde{E}_4 = \boxed{\frac{4M^2}{t}} \left(\frac{n_f \alpha_s}{2\pi} \tilde{C}^{anom} \otimes (\tilde{\mathcal{F}}_2 + \tilde{\mathcal{F}}_4) \underbrace{\boxed{- \tilde{H} - \tilde{H}_4 + O_{2F} + O_{4F}}}_{\eta_0 \text{ pole}} \right), \qquad \left(t = \text{Hadron-level Variable} \right)$$

$$O_F(x, \xi, t) = \bar{u}(p') \left[\Delta^- \gamma^+ \gamma_5 O_{F2} + \Delta_i \gamma^i \gamma_5 O_{F3} + \Delta^+ \gamma^- \gamma_5 O_{F4} \right] u(p)$$

Thus, we have explicitly derived the η_0 pole and find that its residue consists of the polarized GPD $\tilde{H}$ and twist-four GPDs O_F

Anomaly-mediated relations between twist-2,3,4 GPDs

Main findings

1) Relations between the twist-2 and twist-4 GPDs of the proton mediated by the chiral anomaly:

$$\tilde{E} + \tilde{E}_4 = \frac{4M^2}{t}\left(\frac{n_f \alpha_s}{2\pi}\tilde{C}^{anom} \otimes (\tilde{\mathcal{F}}_2 + \tilde{\mathcal{F}}_4) - \tilde{H} - \tilde{H}_4 + O_{2F} + O_{4F}\right), \qquad \left(t = \text{Hadron-level Variable}\right)$$

$$\Big\downarrow \int dx$$

$$g_A(t) + \frac{t}{4M^2} g_P(t) = \frac{i}{2M}\frac{\langle p' | \frac{n_f \alpha_s}{4\pi} F\tilde{F} | p\rangle}{\bar{u}(p')\gamma_5 u(p)}$$

Upon integrating, we exactly reproduce the pole-cancellation expression for the form factor

Anomaly-mediated relations between twist-2,3,4 GPDs

Main findings

1) Relations between the twist-2 and twist-4 GPDs of the proton mediated by the chiral anomaly:

$$\tilde{E} + \tilde{E}_4 = \frac{4M^2}{t}\left(\frac{n_f \alpha_s}{2\pi}\tilde{C}^{anom} \otimes (\tilde{\mathcal{F}}_2 + \tilde{\mathcal{F}}_4) - \tilde{H} - \tilde{H}_4 + O_{2F} + O_{4F}\right), \qquad \left(t = \text{Hadron-level Variable}\right)$$

2) Relations between the twist-3 and twist-4 GPDs of the proton mediated by the chiral anomaly:

$$\tilde{E}_3 = \frac{4M^2}{t}\left(\frac{n_f \alpha_s}{2\pi}\tilde{C}^{anom} \otimes \tilde{\mathcal{F}}_3 - \tilde{H}_3 + O_{F3}\right), \qquad \left(t = \text{Hadron-level Variable}\right)$$

The result provides a nonperturbative foundation for the anomaly pole previously identified in perturbation theory, further strengthening all our conclusions.

Anomaly-mediated relations between twist-2,3,4 GPDs

How does turning on quark masses modify the results?

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Remarks:

- As discussed, in perturbative calculations with **massless fermions**, the **anomaly manifests** as a pole at $t = 0$.
- When the **fermion has a finite mass**, the **anomaly pole disappears**. Instead of a pole, the singularity is replaced by a **branch cut** in the time like region ($t > 0$). This corresponds to the threshold for real fermion-antifermion pair production. (This is **relevant for QED**; see Adler, Bardeen, 69; Coriano, et al, 2013-Present; Castelli, et al, 24)

See Castelli's talk

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- So, what happens?

Nucleon isovector axial form factor & GPD:
(Nambu, 1960; Penttinen, Polyakov, Goeke, 2000)

$$g_P^{(3)}(t) \sim \frac{1}{t} \xrightarrow[\substack{\text{quark mass} \\ (m_q \neq 0)}]{\text{switch on}} \frac{1}{t - m_\pi^2}$$

$$\tilde{E}^{(3)}(t) \sim \frac{1}{t - m_\pi^2}$$

Non-perturbative shift in pole:

$$m_\pi^2 \propto m_q$$

Still a pole!

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(Witten Veneziano 1979;
SB, Hatta, Schoenleber, 2024;
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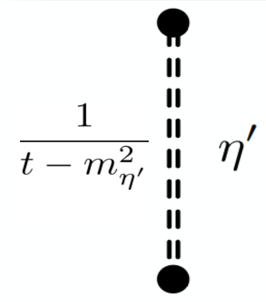
$$g_P(t) \sim \frac{1}{t} \xrightarrow{(m_q \neq 0)} \frac{1}{t - m_\pi^2} \xrightarrow{\text{Resummation}} \frac{1}{t - m_\pi^2 + \frac{4n_f}{f_{\eta'}^2}\chi} = \frac{1}{t - m_{\eta'}^2}$$

$$\tilde{E}(t) \sim \frac{1}{t - m_{\eta'}^2}$$ (similar **non-perturbative** shift in pole)

Summary

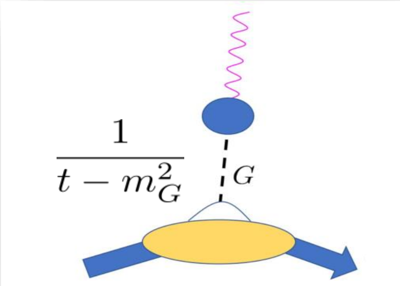
1) We calculated one-loop Compton amplitude in all channels (quark/gluon, polarized/unpolarized) using **t** as a regulator and demonstrated **factorization**

2) The physics of anomalies present at the level of Form Factor is established for the first time at the level of GPDs



Eta meson mass generation:

$$\tilde{E}(x) \sim \frac{1}{t - m_{\eta'}^2}$$



Glueball mass generation:

$$H(x), E(x) \sim \frac{1}{t - m_G^2}$$

GPDs encode profound aspects of QCD such as symmetry breakings and mass generations:

Reach out to a broader QCD community

Backup slides

Imprint of Anomalies in QCD Compton scattering

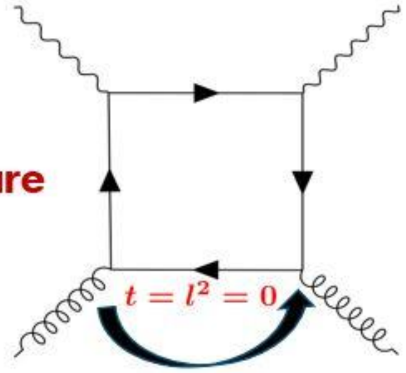
Elusive pole

Box diagram in off-forward kinematics

Setup of calculation:

Calculated box diagrams with $t = l^2 \neq 0$ **whereas all existing works in the literature set** $t = 0\,(\xi \neq 0)$

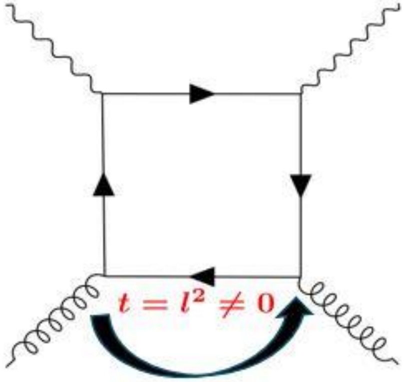
GPD literature



$$p_1 = p - \frac{l}{2} \approx p(1+\xi)$$

Only longitudinal momentum transfer

This work



Transverse momentum transfer essential to capture physics of anomalies

$$O_F(z_2,z_1) = -iz^\nu \int_0^1 d\alpha \mathrm{Tr}\Big[\psi(z_1)\bar{\psi}(z_2)\gamma^\mu\gamma_5 W g F_{\mu\nu}(z_{21}^\alpha) W\Big]. \tag{13}$$

When $z = z_1 - z_2$ approaches the light-cone $z^\mu \to \delta_-^\mu z^-$, the quark bilinear $\psi(z_1)\bar{\psi}(z_2)$ develops singularities which can be expanded in $1/z^2$ in $d = 4-2\epsilon$ dimensions [18, 19]

$$\begin{aligned}\psi(z_1)\bar{\psi}(z_2) &= \frac{i\not{z}}{2\pi^2(z^2)^2}W_{z_1,z_2} - \frac{iz^\rho}{8\pi^2 z^2}\int_0^1 d\beta W_{z_1,z_{12}^\beta}\, g\tilde{F}_{\rho\lambda}(z_{12}^\beta)W_{z_{12}^\beta,z_2}\gamma^\lambda\gamma_5 \\ &+\frac{i}{32\pi^2}\left(\frac{1}{\epsilon_{IR}} + \ln\frac{-z^2\mu_{IR}^2 e^{2\gamma_E}}{4}\right)\Bigg[g\int_0^1 d\alpha\alpha(1-\alpha)z_\mu D^2\tilde{F}^{\mu\nu}(z_{12}^\alpha)\gamma_\nu\gamma_5 + ig^2 z_\mu\int_0^1 d\alpha\int_0^\alpha d\beta \\ &\times\Big\{(1-2\alpha+2\beta)F^{\mu\lambda}(z_{12}^\alpha)\tilde{F}_{\lambda\rho}(z_{12}^\beta)\gamma^\rho + \tilde{F}^{\mu\lambda}(z_{12}^\alpha)F_{\lambda\rho}(z_{12}^\beta)\gamma^\rho + \beta F^{\rho\nu}(z_{12}^\alpha)\tilde{F}_{\rho\nu}(z_{12}^\beta)\gamma^\mu\Big\}\gamma_5 + \cdots\Bigg] + \mathcal{O}(z^2),\end{aligned} \tag{14}$$

where we neglected the quark mass and μ_{IR}^2 is the $\overline{\mathrm{MS}}$ scheme scale parameter associated with the infrared divergence. Apart from the leading term, we have kept only the terms that contain a γ_5 so that they survive when inserted into (13). For simplicity, we omitted Wilson lines in the logarithmic terms $\ln z^2$. When substituted into (13), these terms constitute the renormalization group evolution kernel of the twist-four operator $O_F \sim \bar{\psi}F^{+\mu}\gamma_\mu\gamma_5\psi$. (14) only includes the $g \to q$ splitting kernel of the evolution exhibiting the mixing with three-gluon, twist-four operators such as $F^{+\mu}F^{+\lambda}\tilde{F}_{\mu\lambda}$. In principle, at this order one has to include the complete evolution kernel including also the $q \to q$ kernel and other contributions [19, 20].

Let us focus on the $1/z^2$ term. We substitute it into (13) and find

$$\begin{aligned}&\frac{n_f g^2}{4\pi^2 z^2}\int_0^1 d\alpha\int_0^1 d\beta \mathrm{Tr}\left[\tilde{F}_{\rho\mu}(z_{12}^\beta)WF^{\mu\nu}(z_{21}^\alpha)W + F_{\rho\mu}(z_{12}^\beta)W\tilde{F}^{\mu\nu}(z_{21}^\alpha)W\right]z^\rho z_\nu \\ &= -\frac{n_f\alpha_s}{2\pi}\int_0^1 d\alpha\int_0^1 d\beta \mathrm{Tr}\left[F^{\mu\nu}(z_{12}^\beta)W\tilde{F}_{\mu\nu}(z_{21}^\alpha)W\right] \\ &= -\frac{n_f\alpha_s}{4\pi}\int_0^1 d\alpha\int_0^1 d\beta F^{\mu\nu}(z_{12}^\beta)\tilde{W}\tilde{F}_{\mu\nu}(z_{21}^\alpha), \\ &= -\frac{n_f\alpha_s}{2\pi}\int_0^1 d\alpha\int_0^{1-\alpha} d\beta F^{\mu\nu}(z_{12}^\beta)\tilde{W}\tilde{F}_{\mu\nu}(z_{21}^\alpha),\end{aligned} \tag{15}$$

where $\tilde{W}$ is the Wilson line in the adjoint representation. In the first line, we have used the cyclicity of the trace and the symmetry of the α,β-integrals. This allows us to utilize the formula[1]

$$2z^\nu\left(F_{\nu\mu}W\tilde{F}^{\mu\rho} + \tilde{F}_{\nu\mu}WF^{\mu\rho}\right)z_\rho = -z^2F^{\mu\nu}W\tilde{F}_{\mu\nu}, \tag{17}$$

$$\boxed{\tilde{H}_3 + \frac{t}{4M^2}\tilde{E}_3 = \frac{n_f\alpha_s}{2\pi}\tilde{C}^{anom} \otimes \tilde{\mathcal{F}}_3 + O_{F3},}$$ **(42)**

$\gamma_5 u = \bar{u}\not{\Delta}\gamma_5 u = 2M\bar{u}\gamma_5 u$. This result is valid for $\xi = 0$ but arbitrary t.
relation (2) among the form factors due to (40) and the relations such

$$\int_{-1}^{1} dx\tilde{H}_3(x,\xi,t) = g_A(t), \qquad \int_{-1}^{1} dx\tilde{E}_3(x,\xi,t) = g_P(t).$$

ase $\Delta_i = 0$ where

$$t = 2\Delta^+\Delta^- = -\frac{4\xi^2}{1-\xi^2}M^2.$$

entities

$$\Delta^+\bar{u}(p')\gamma^-\gamma_5 u(p) = \Delta^-\bar{u}(p')\gamma^+\gamma_5 u(p),$$
$$2M\bar{u}(p')\gamma_5 u(p) = \bar{u}(p')\not{\Delta}\gamma_5 u(p) = 2\Delta^-\bar{u}(p')\gamma^+\gamma_5 u(p),$$

$$\boxed{\tilde{H} + \tilde{H}_4 + \frac{t}{4M^2}(\tilde{E} + \tilde{E}_4) = \frac{n_f\alpha_s}{2\pi}\tilde{C}^{anom} \otimes (\tilde{\mathcal{F}}_2 + \tilde{\mathcal{F}}_4) + O_{F2} + O_{F4}.}$$ **(46)**

$$\Delta^- P^+ \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle p'|\bar{\psi}(-z/2)\gamma^+\gamma_5\psi(z/2)|p\rangle = \Delta^- \bar{u}(p') \left[\tilde{H}\gamma^+\gamma_5 + \tilde{E}\frac{\Delta^+}{2M}\gamma_5\right] u(p), \tag{33}$$

$$\Delta_i P^+ \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle p'|\bar{\psi}(-z/2)\gamma^i\gamma_5\psi(z/2)|p\rangle$$

$$= \Delta_i \bar{u}(p') \left[\tilde{H}_3\gamma^i\gamma_5 + \tilde{E}_3\frac{\Delta^i}{2M}\gamma_5 + \tilde{G}_3\frac{\Delta^i}{P^+}\gamma^+\gamma_5 + i\tilde{G}'_3\epsilon^{ij}\frac{\Delta_j}{P^+}\gamma^+\right] u(p),$$

$$\Delta^+ P^+ \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle p'|\bar{\psi}(-z/2)\gamma^-\gamma_5\psi(z/2)|p\rangle = \Delta^+ \bar{u}(p') \left[\tilde{H}_4\gamma^-\gamma_5 + \tilde{E}_4\frac{\Delta^-}{2M}\gamma_5\right] u(p).$$

All the GPDs are functions of x, ξ and t (and the renormalization scale), $\tilde{H} = \tilde{H}(x,\xi,t)$, etc. Also, the summation over quark flavors is implied, $\tilde{H} = \sum_q \tilde{H}_q$, etc. The twist-3 GPDs are from [24] where we redefined $\tilde{H} + \tilde{G}_2 \to \tilde{H}_3$ and $\tilde{E} + \tilde{G}_1 \to \tilde{E}_3$. (We also redefined $\tilde{G}_4 \to \tilde{G}'_3$.) The twist-4 GPDs are parametrized differently from [25] but the two parametrizations are equivalent in the present frame $P^i = 0$. On the right hand side, the twist-4 pseudoscalar GPD (25) is parametrized as [11]

$$\tilde{\mathcal{F}}(x,\xi,t) = \frac{1}{M}\bar{u}(p')\left[\Delta^-\gamma^+\gamma_5\tilde{\mathcal{F}}_2 + \Delta_i\gamma^i\gamma_5\tilde{\mathcal{F}}_3 + \Delta^+\gamma^-\gamma_5\tilde{\mathcal{F}}_4\right] u(p). \tag{34}$$

For phenomenological purposes, one may implement various approximations. If one ignores the differences due to different twists, namely $\tilde{H}_{3,4} \approx \tilde{H}$, $\tilde{\mathcal{F}}_{3,4} \approx \tilde{\mathcal{F}}_2$, etc., from (32) one immediately obtains

$$\tilde{E}(x,\xi,t) \approx \frac{4M^2}{t}\left(\frac{n_f\alpha_s}{2\pi}\tilde{C}^{anom} \otimes \tilde{\mathcal{F}}_2 - \tilde{H} + O_{F2}\right). \tag{49}$$