Status of the simultaneous global analysis of PDFs and TMDs

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QCD Evolution

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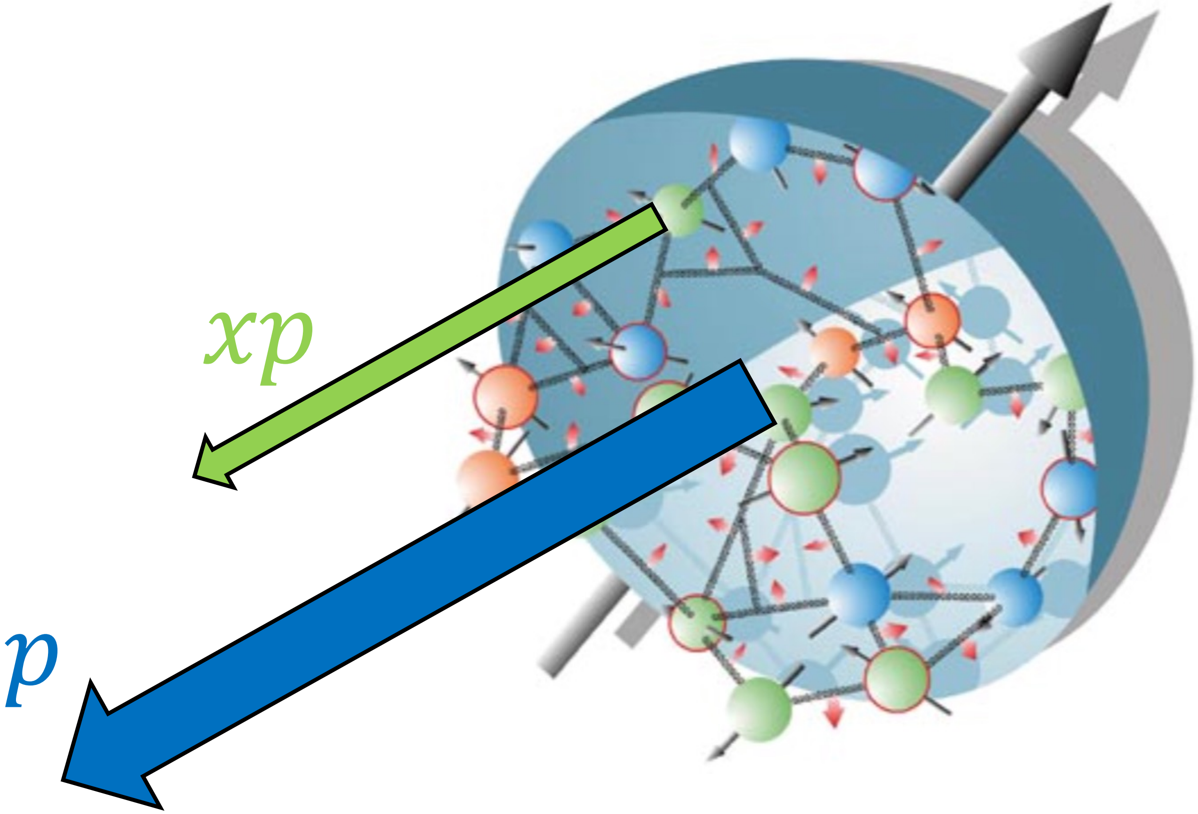
Collinear structure – parton distribution function (PDF)

- Describes the collinear momentum distributions of quarks and gluons
- Partons have momentum along the direction of the hadron
- Evolution is descriped through DGLAP

$$\frac{\partial f(x,\mu^2;\boldsymbol{\theta})}{\partial \log \mu^2} = \int_x^1 dz\, \mathcal{P}\left(\frac{x}{z}, \alpha_S(\mu^2)\right) f(z,\mu^2;\boldsymbol{\theta})$$



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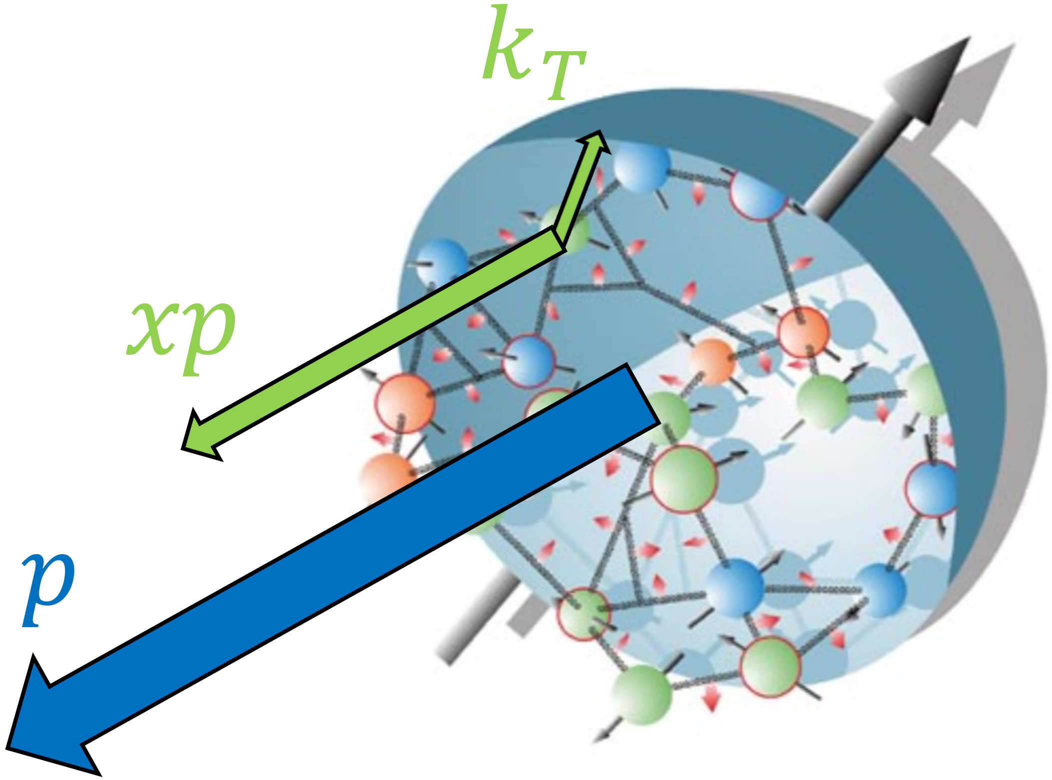
Transverse Momentum Dependent distributions (TMDs)

- Encode both the collinear and transverse momentum carried by partons
- TMDs are related to collinear **PDFs** via Operator Product Expansion
- Both TMDs and PDFs can be extracted from variety of experimentally measured processes where factorization is applicable, such as Drell-Yan (DY)



$$\tilde{f}(x, b_T; \mu, \zeta) = [C \otimes \boldsymbol{f}]\,(x, b_T; \mu_0, \zeta_0) \times e^{S_{\rm evo}(b_T; \mu, \mu_0, \zeta, \zeta_0)} f_{\rm NP}(x, b_T)$$

Collins, Soper, Sterman Nucl. Phys. B **250**, 199, (1985).
Collins, Cambridge University Press, (2011).

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Unpolarized TMD PDF

$$\tilde{f}_{q/\mathcal{N}}(x, b_T) = \int \frac{\mathrm{d}b^-}{4\pi} \, e^{-ixP^+b^-} \, \mathrm{Tr}\big[\langle \mathcal{N} \,|\, \bar{\psi}_q(b)\gamma^+ \mathcal{W}(b,0)\psi_q(0) \,|\, \mathcal{N} \rangle\big]$$

$$b \equiv (b^-, 0^+, \boldsymbol{b_T})$$

- $\boldsymbol{b_T}$ is the Fourier conjugate to the intrinsic transverse momentum of quarks in the hadron, $\boldsymbol{k_T}$
- Small $\boldsymbol{b_T}$: TMD can be described through the operator product expansion in terms of collinear PDFs
- Large $\boldsymbol{b_T}$: TMD has nonperturbative effects that must be determined from phenomenological analyses

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Input scale TMD

$$\tilde{f}_{q/\mathcal{N}}(x, b_T; \mu_0, \zeta_0) = f^{\rm NP}_{q/\mathcal{N}}(x, b_T) \sum_j \int_x^1 \frac{d\xi}{\xi} \tilde{\mathcal{C}}_{q/j}(x/\xi, b_T; \mu_0, \zeta_0) f_{j/\mathcal{N}}(\xi; \mu_0)$$

- $f^{\rm NP}$ describes the non-perturbative structure of the TMD at large-b_T
- Convolution is the operator-product expansion (OPE), which describes the small-b_T behavior
- Explicit dependence on the collinear PDF $f_{j/\mathcal{N}}$
- $\tilde{C}$ is perturbatively expanded in α_S
- Evolution in μ and ζ needs to take place to match with data

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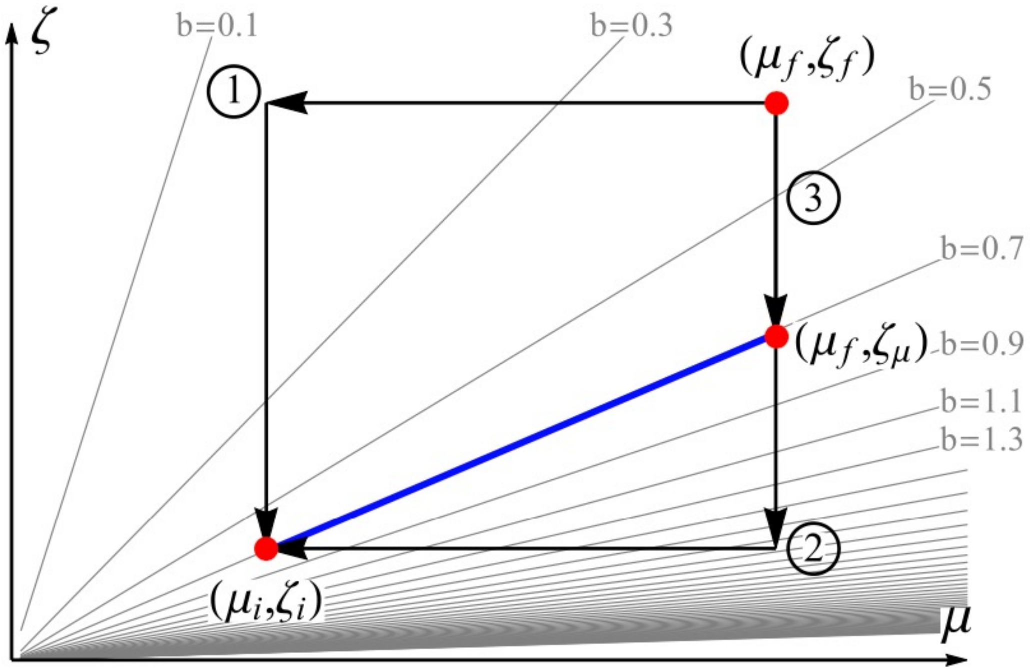
Building the TMD in the ζ-prescription

- We need to evolve the TMD $\tilde{f}(x, b_T; \mu_0, \zeta_0) \rightarrow \tilde{f}(x, b_T; \mu_f, \zeta_f)$
- A few choices:
  1. Evolve $\zeta_0 \rightarrow \zeta_f$ at a fixed μ_i, then evolve $\mu_0 \rightarrow \mu_f$ at a fixed ζ_f
  2. Evolve $\mu_0 \rightarrow \mu_f$ at a fixed ζ_i, then evolve $\zeta_0 \rightarrow \zeta_f$ at a fixed μ_f
  3. Evaluate the TMD along the **null-evolution line**, where $\tilde{f}(x, b_T; \mu_0, \zeta_0) = \tilde{f}(x, b_T; \mu_f, \zeta_\mu)$, then evolve $\zeta_\mu \rightarrow \zeta_f$ at a fixed μ_f



Scimemi and Vladimirov, EPJ **C 78**, 89 (2019).

$$\left(\mu^2 \frac{\partial}{\partial \mu^2} + \left(\frac{\mu^2}{\zeta}\frac{d\zeta}{d\mu^2}\right)\zeta\frac{\partial}{\partial \zeta}\right) F(x, \boldsymbol{b}; \mu, \zeta) = 0.$$

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TMD Evolution

- Since we evolve on the null-evolution line, no explicit evolution in μ has to be added, and we evolve in ζ according to

$$\tilde{f}_{q/\mathcal{N}}(x, b_T; \mu, \zeta = Q^2) = \left(\frac{Q^2}{\zeta_\mu(b_T)}\right)^{-\mathcal{D}(b_T,\mu)} \tilde{f}_{q/\mathcal{N}}(x, b_T; \mu_0, \zeta_0)$$

- $\mathcal{D}$ is the CS kernel, which has the following components

$$\mathcal{D}(b_T, \mu) = \mathcal{D}^{\text{pert}}(b_*, \mu_{b_*}) + \int_{\mu_{b_*}}^{\mu} \frac{d\mu'}{\mu'} \Gamma_{\text{cusp}}(\mu') + \mathcal{D}^{\text{NP}}(b_T)$$

$$b_* = \frac{b_T}{\sqrt{1 + \frac{b_T^2}{b_{\text{max}}^2}}}$$

$$\mu_{b_*} = \frac{2e^{-\gamma_E}}{b_*}$$

Described perturbatively

Non-perturbative description (large-b_T)

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Transverse momentum dependent DY

- Full cross section over all q_T

$$\frac{d\sigma}{dQ^2dydq_T^2} = W(\boldsymbol{q}_T, Q) + Y(\boldsymbol{q}_T, Q) + O((m/Q)^c)\,,$$

- At small q_T, $W(q_T, Q)$ should be the dominant term

$$W(\boldsymbol{q}_T, Q) = \int \frac{d^2\boldsymbol{b}_T}{(2\pi)^2}\, e^{i\boldsymbol{q}_T\cdot\boldsymbol{b}_T}\, \widetilde{W}(\boldsymbol{b}_T, Q)\,.$$

$$\frac{d^3\sigma}{dQ^2dydq_T^2} = \frac{4\pi^2\alpha_{\rm em}^2}{9Q^2s}\underbrace{\mathcal{P}}_{\text{Fiducial volume factor}}\sum_j \underbrace{c_j^2(Q)}_{\text{Electro-weak couplings}}\underbrace{H_{j\bar{j}}^{\rm DY}(\mu, Q)}_{\text{Hard factor for DY}}\int\frac{d^2\boldsymbol{b}_T}{(2\pi)^2}e^{i\boldsymbol{q}_T\cdot\boldsymbol{b}_T}\underbrace{\tilde{f}_{q_j/\mathcal{N}}(x_1, b_T;\mu,\zeta)}_{\text{TMD for the beam}}\underbrace{\tilde{f}_{\bar{q}_j/\mathcal{N}}(x_2, b_T;\mu,\zeta)}_{\text{TMD for the target}},$$

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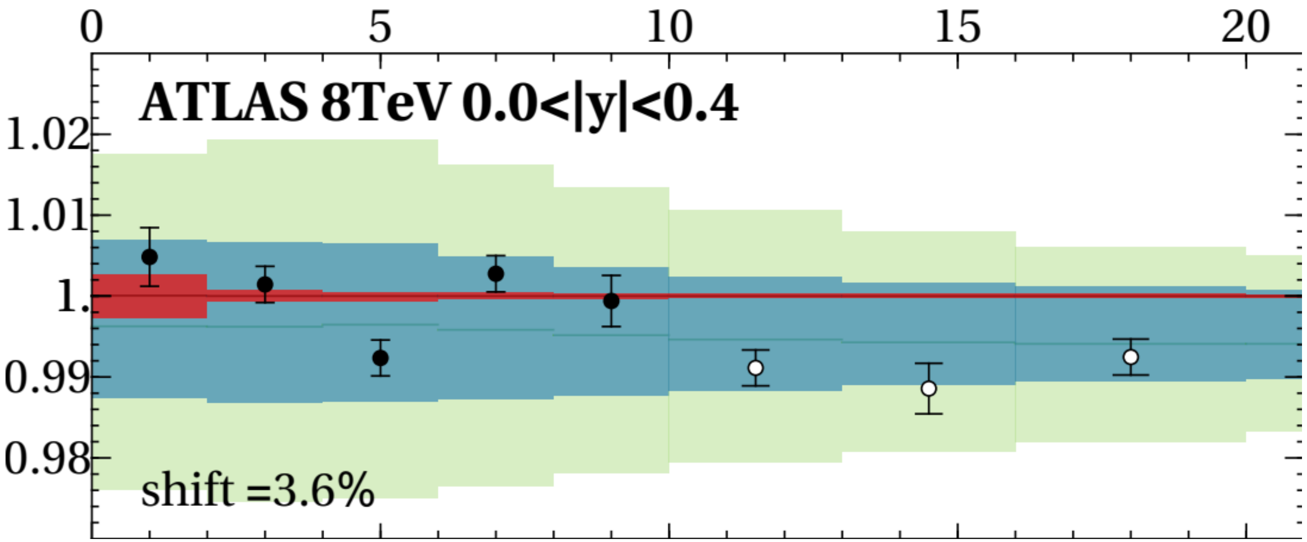
How sensitive are TMD observables to PDFs?

- Red: Bootstrapped fit with central PDFs
- Green: Unbootstrapped fit, varying the PDF replicas
- Blue: Weighted average
- **One needs to take a holistic approach and analyze both PDFs and TMDs simultaneously**

Bury, et al., JHEP **10**, 118 (2022).



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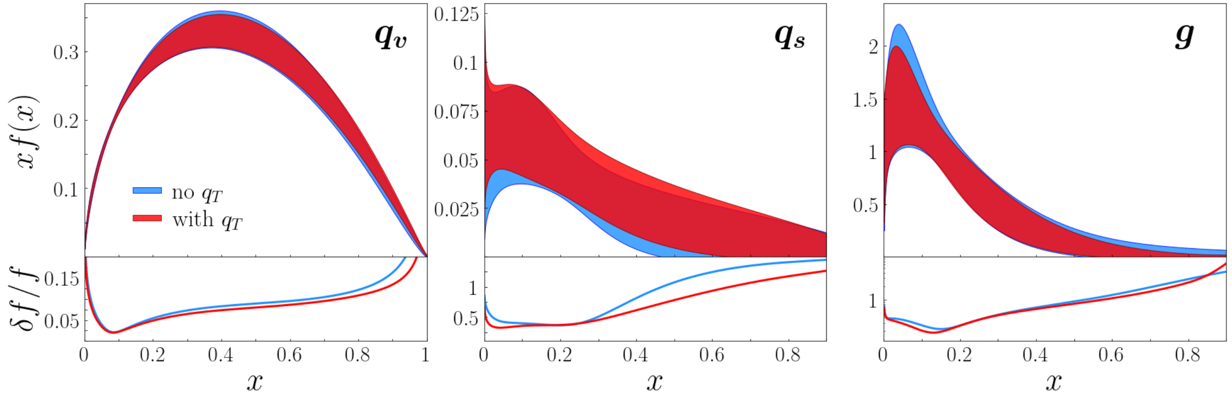
Can we learn about PDFs from TMD data?

- Viewing the uncertainties of the observables coming from the PDFs, there is potentially room for improvement on precision of PDFs
- How about for the pion?
- We extracted simultaneously the pion PDFs and TMDs



PCB, et al., PRD **108**, L091504 (2023).

- We found little change in the PDFs before and after the q_T-dependent DY data

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Prospects of high-energy data for protons

- There are two major reasons to have hope for improvement of PDFs in the proton sector

1. LHC data are much more precise than their fixed-target low-energy counterparts
    - Peaks of the cross-section in the Z-boson region gather high statistics
2. High-energy data shifts the peak of the b_T-spectrum into the small b_T region, where the operator product expansion and perturbative evolution dominates

- Have to perform the **simultaneous** extraction of PDFs and TMDs from high-energy data to find out!

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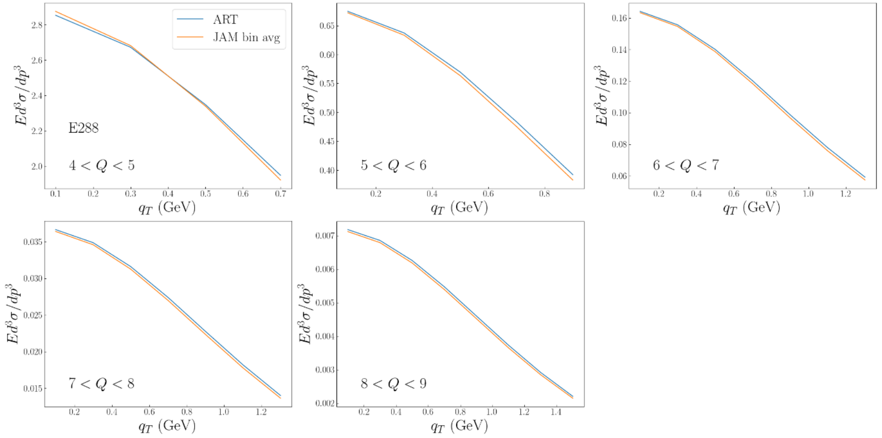
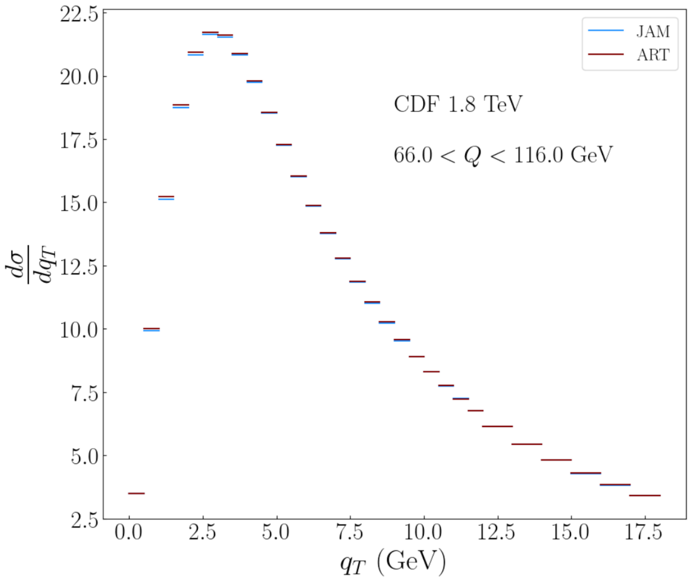
Implementing the ζ-prescription in JAM code

- We have spent time with the ART folks checking our JAM code against the arTeMiDe
- Examples here are all using MSHT20 PDF central values

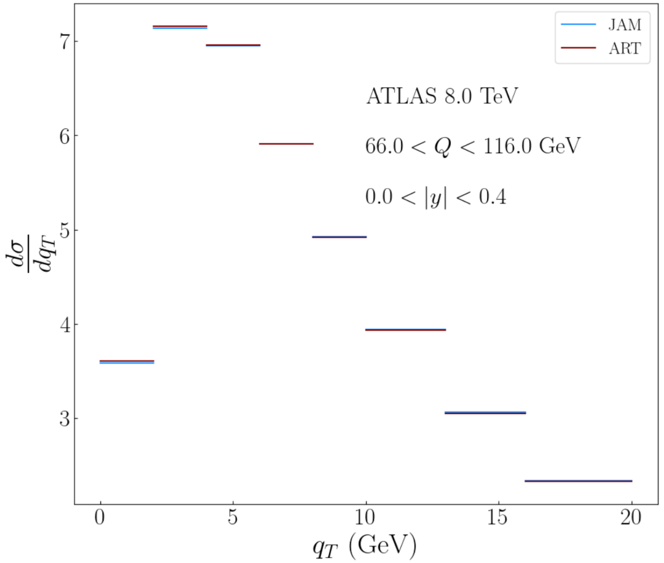
Low-energy regime



Collider - TeVatron

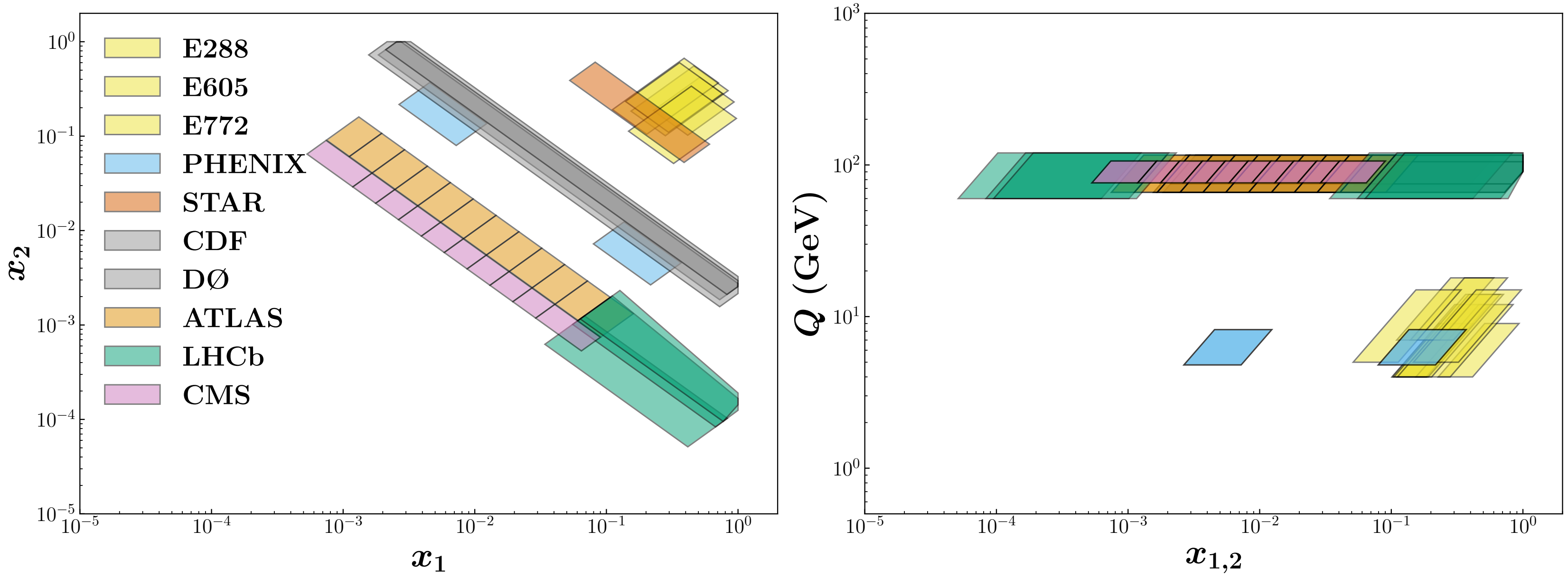


Collider - ATLAS



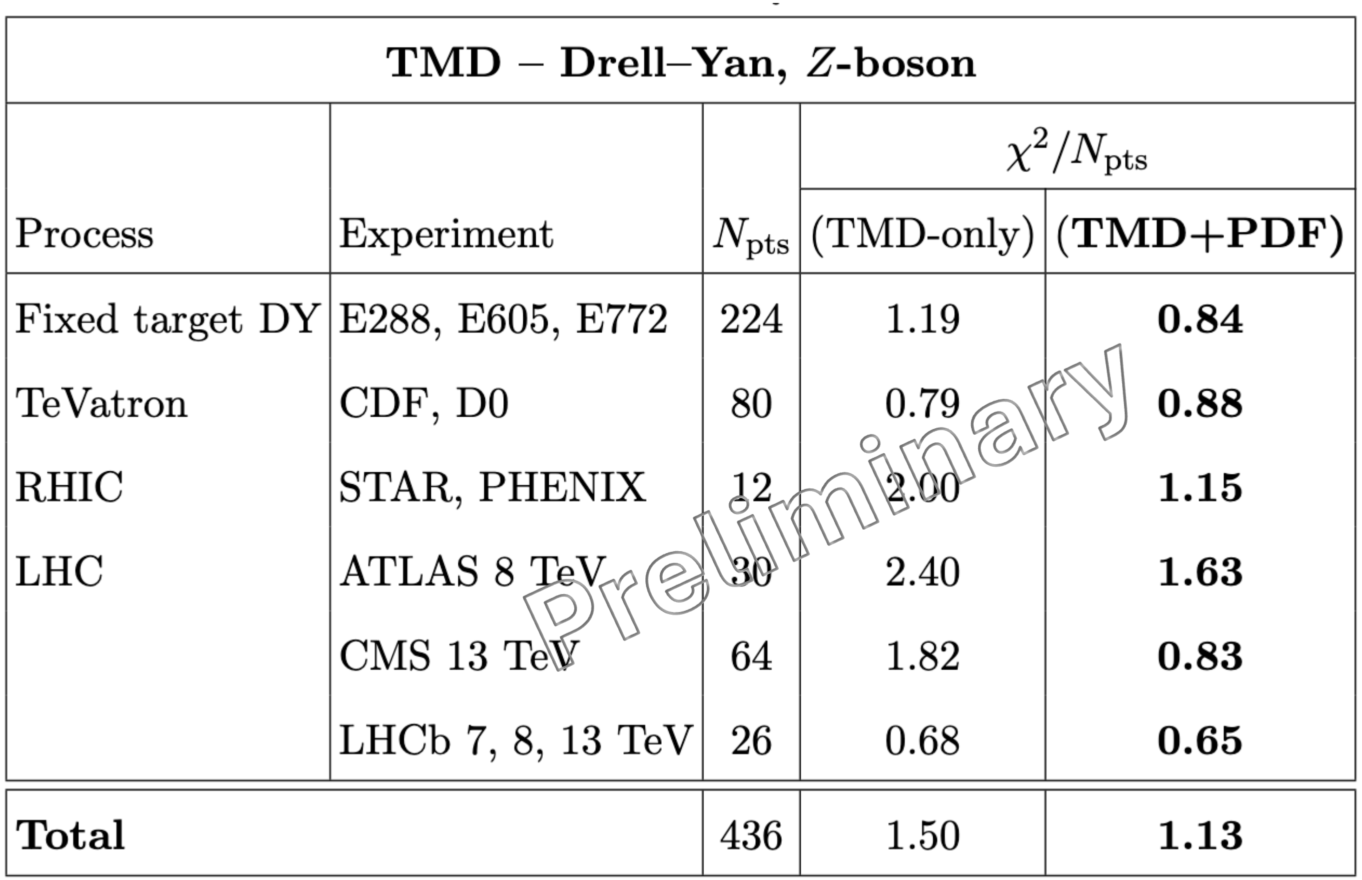
Datasets and kinematics

- Fixed-target low-energy datasets: more sensitivity to non-perturbative TMD structures
- Collider high-energy datasets: more sensitive to perturbative information while complementing the non-perturbative evolution in Q



Fit results

- Using NLO+N2LL accuracy, we performed fits with a JAM replica (Anderson, Melnitchouk, and Sato, 2501.00665 [hep-ph]) by
    1. Fixing the PDF and fitting TMDs only
    2. Opening the PDF and the collinear datasets
- Flexibility of the collinear PDF allowed for an improved fit

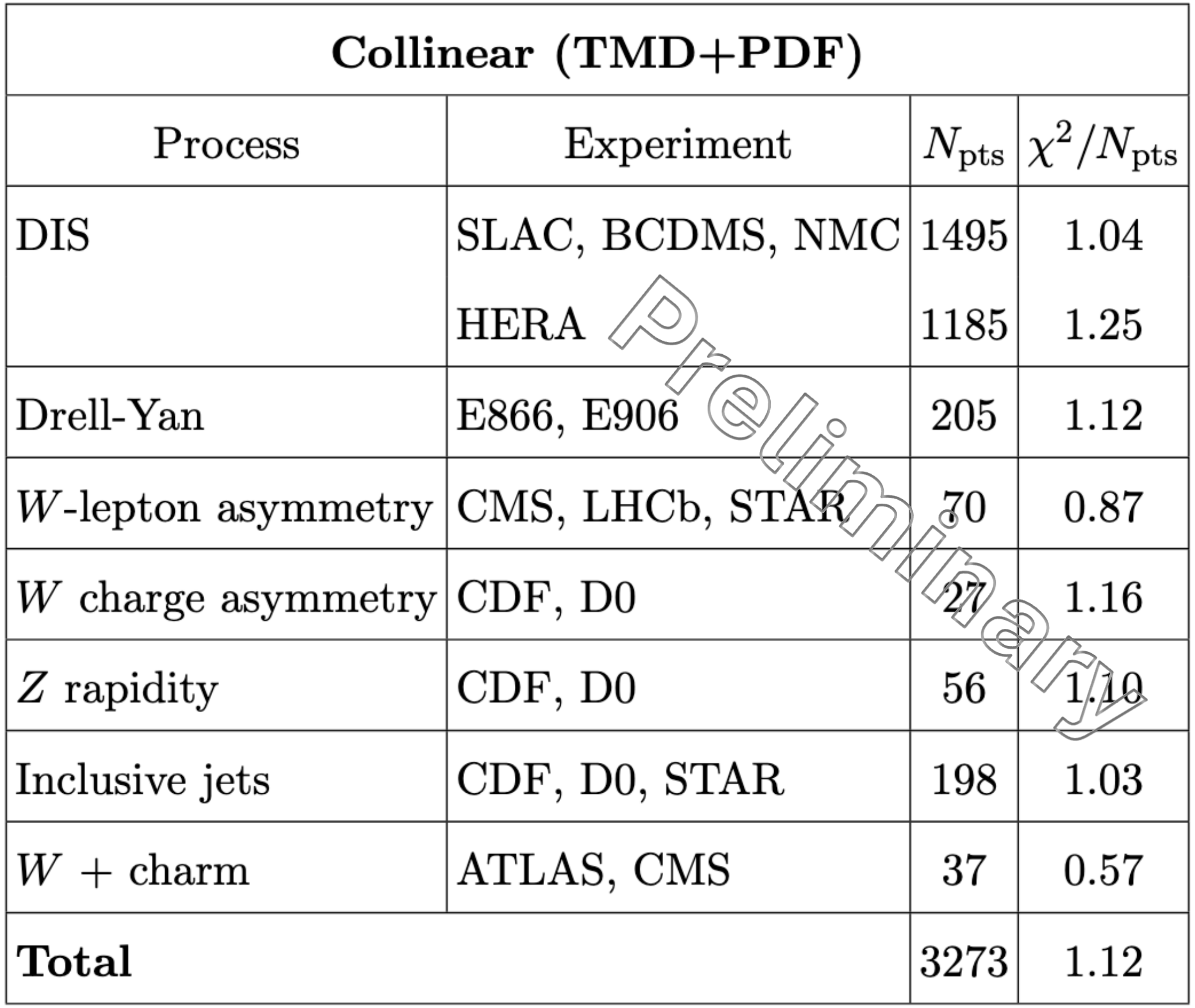


TMD – Drell–Yan, Z-boson

Process	Experiment	N_{pts}	χ^2/N_{pts} (TMD-only)	χ^2/N_{pts} **(TMD+PDF)**
Fixed target DY	E288, E605, E772	224	1.19	**0.84**
TeVatron	CDF, D0	80	0.79	**0.88**
RHIC	STAR, PHENIX	12	2.00	**1.15**
LHC	ATLAS 8 TeV	30	2.40	**1.63**
	CMS 13 TeV	64	1.82	**0.83**
	LHCb 7, 8, 13 TeV	26	0.68	**0.65**
Total		436	1.50	**1.13**

Fit results – collinear

- Use the datasets sensitive only to PDFs from the prior
- Good agreement with all collinear datasets



Collinear (TMD+PDF)

Process	Experiment	N_{pts}	χ^2/N_{pts}
DIS	SLAC, BCDMS, NMC	1495	1.04
	HERA	1185	1.25
Drell-Yan	E866, E906	205	1.12
W-lepton asymmetry	CMS, LHCb, STAR	70	0.87
W charge asymmetry	CDF, D0	27	1.16
Z rapidity	CDF, D0	56	1.10
Inclusive jets	CDF, D0, STAR	198	1.03
W + charm	ATLAS, CMS	37	0.57
Total		3273	1.12

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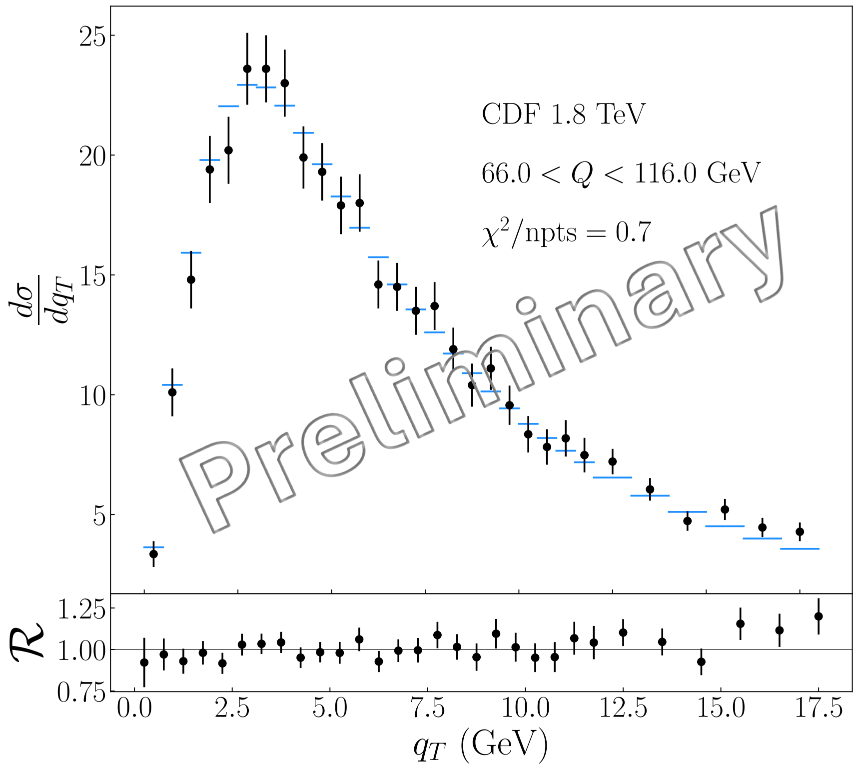
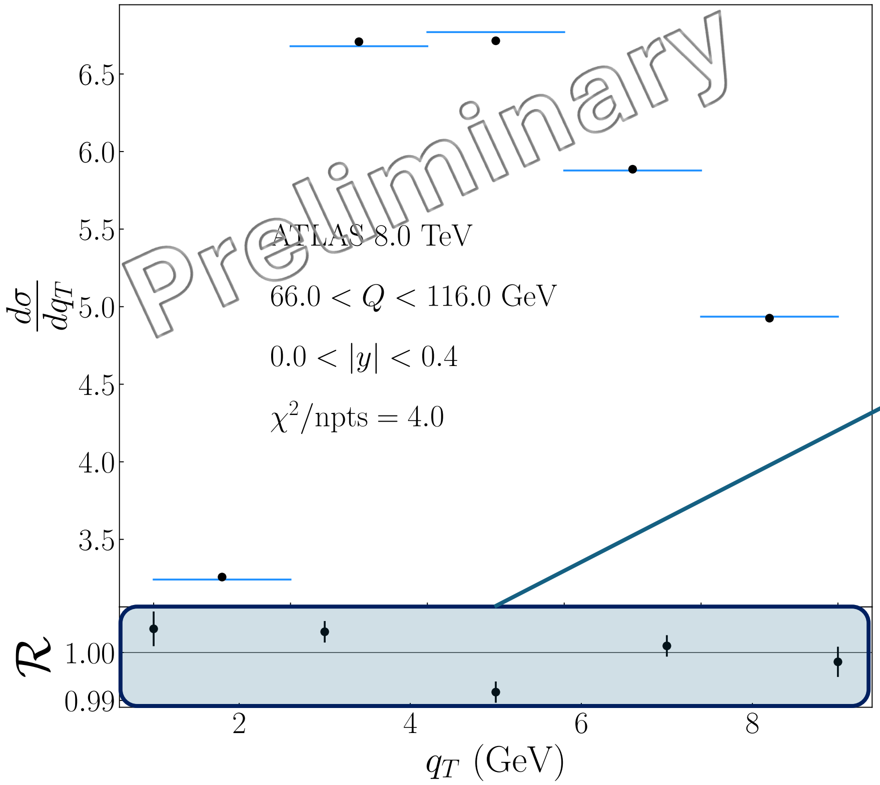
Agreement with the collider data

- Results of the combined fit; $\mathcal{R}$ is the ratio of data to theory





Sub-percent precision!

Extremely sensitive in the fit.

Can we improve our PDFs because of precision of data?

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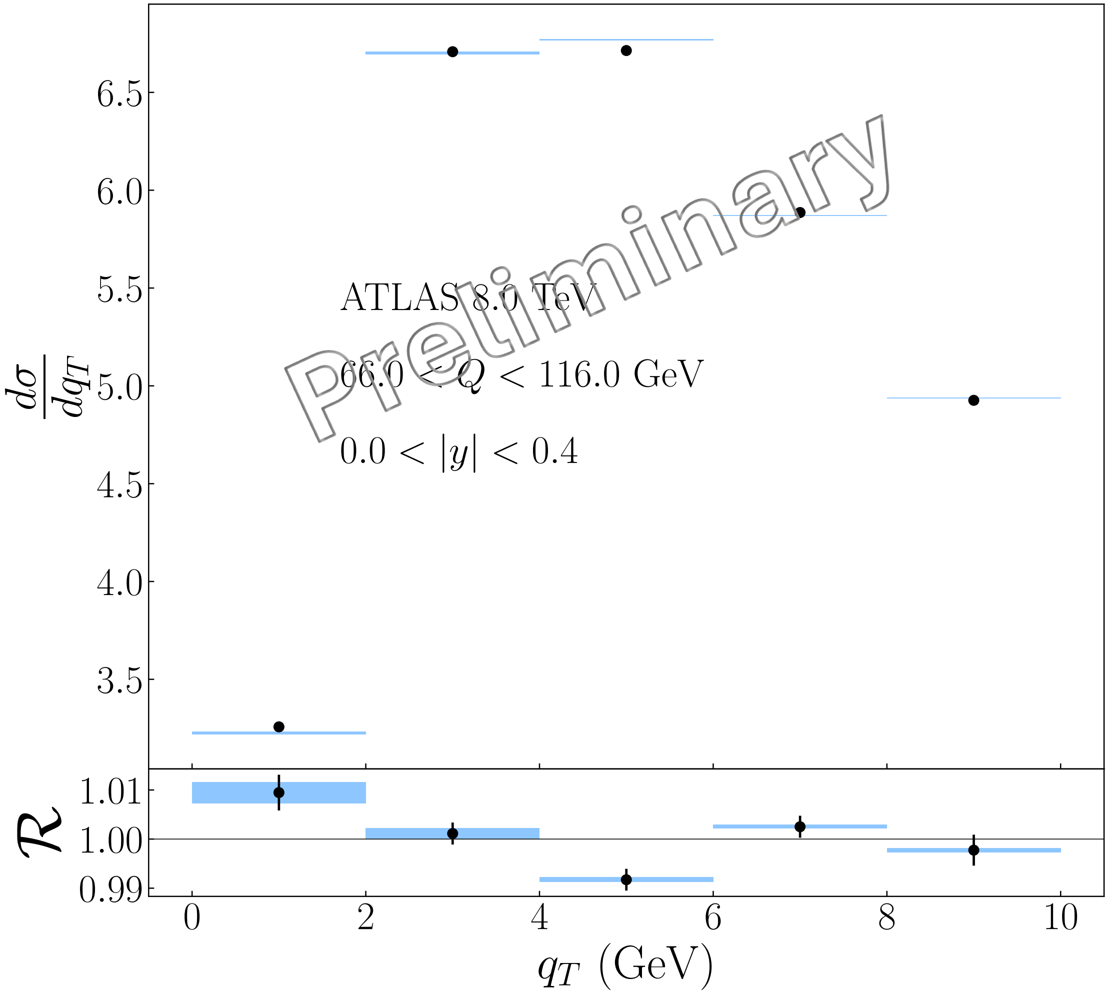
Uncertainties of JAM PDFs

- Run over JAM PDFs
- Band shows roughly the uncertainty from the PDFs
- **Not** reflective of an uncertainty from bootstrapping the data (next steps!)



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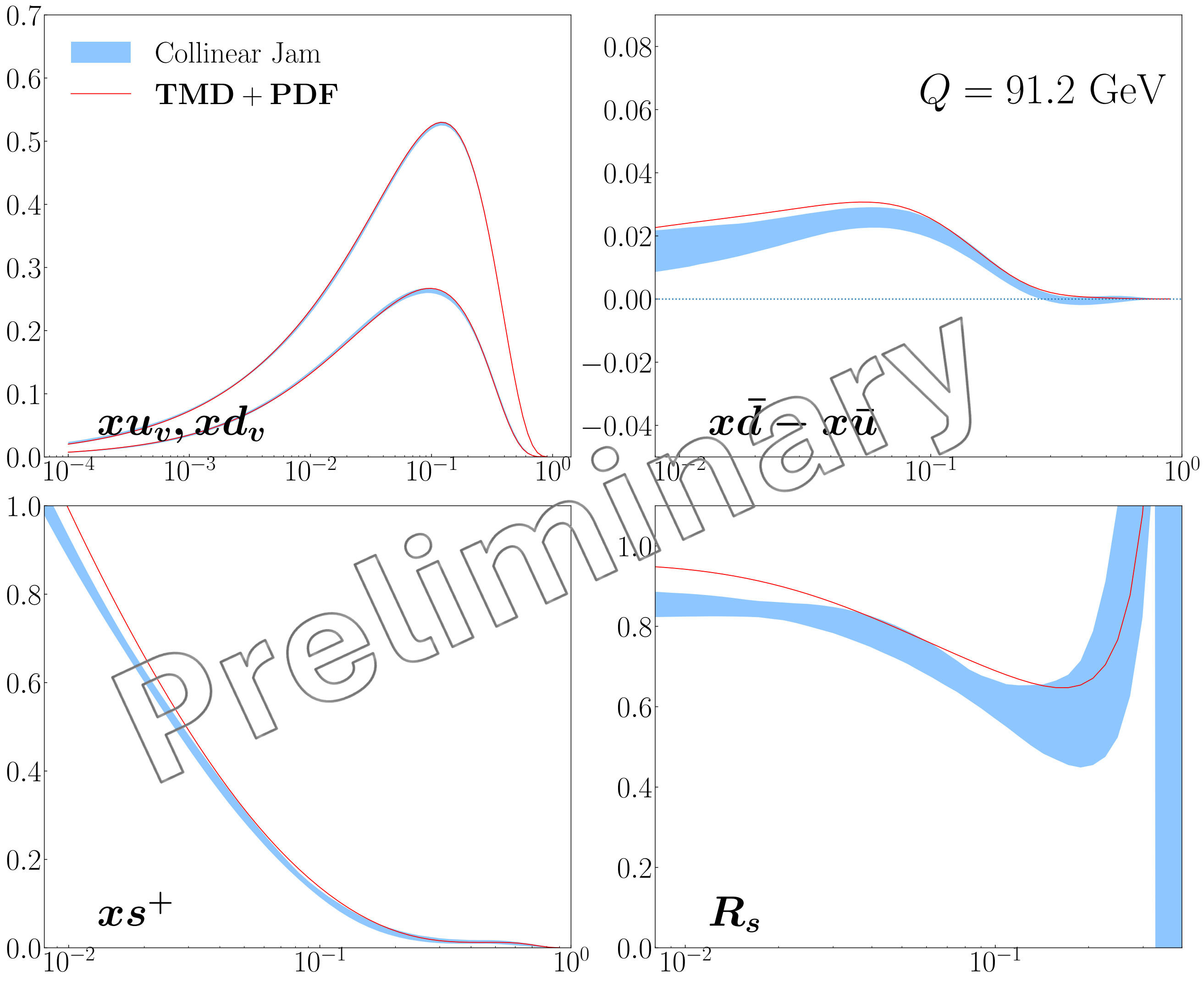
Single PDF result

- **Combined** fit suggests a larger s^+ and R_s at small x
- Remaining PDFs are consistent with previous fit
- Shown here are the 95% confidence interval for the priors

$$R_s = \frac{s+\bar{s}}{\bar{u}+\bar{d}}$$



Summary

- We have demonstrated agreement in our codes with the ART collaboration
- We have performed preliminary fits to the low-energy and high-energy q_T-dependent Drell-Yan data simultaneously along with collinear data

<u>Next Steps</u>

- Perform the full Monte Carlo bootstrapped analysis to obtain reliable uncertainties on PDFs and TMDs

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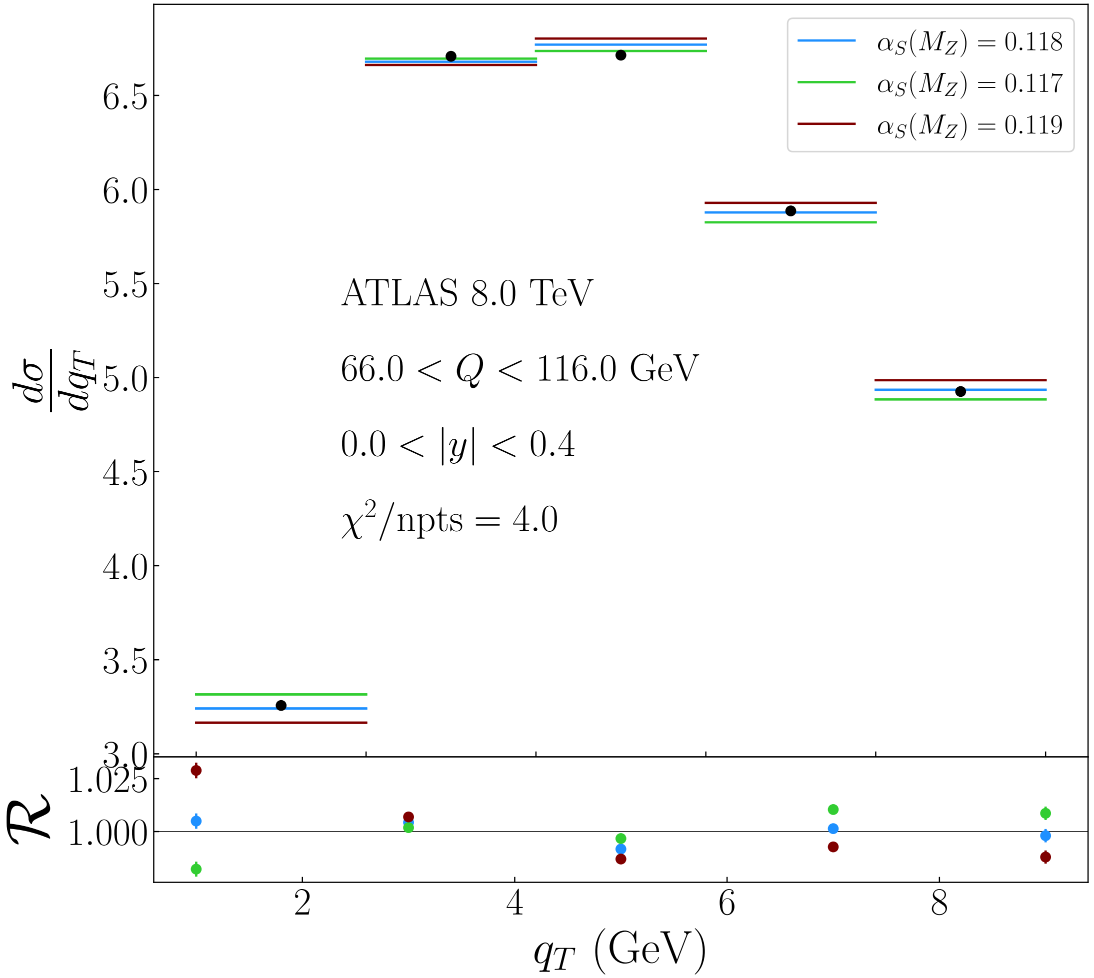
Future considerations – what can we do next?

- Some quantities in the standard model are not very precise
- Here, we run over a few values of $\alpha_S(M_Z)$ and see a large variation in the resulting curves
- We could also analyze the M_W since there are W-boson production TMD data



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Backup Slides

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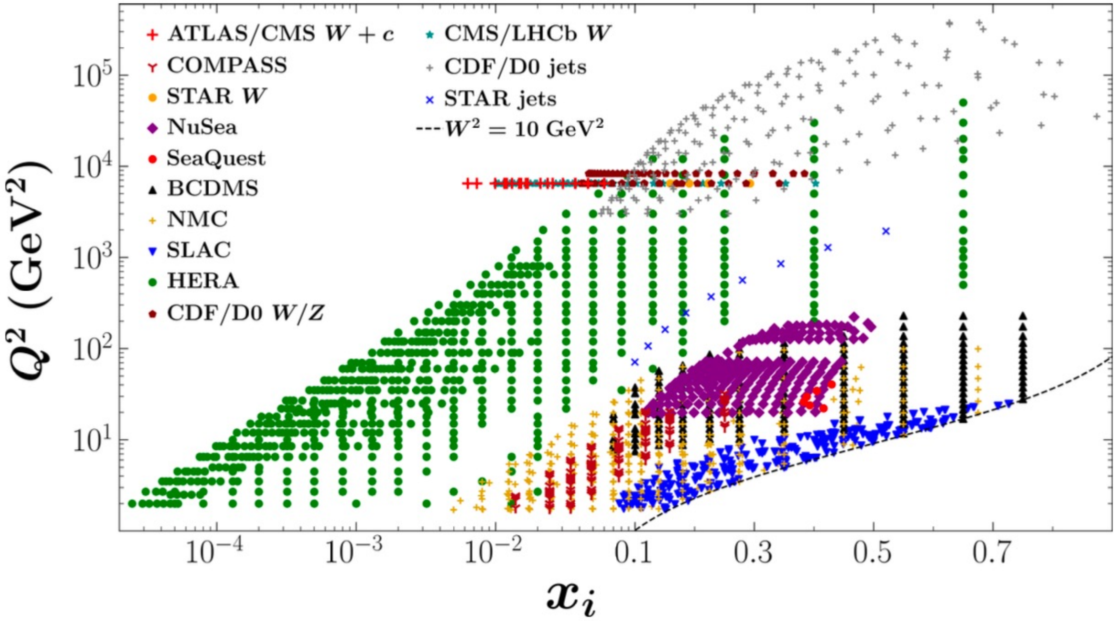
Technical considerations

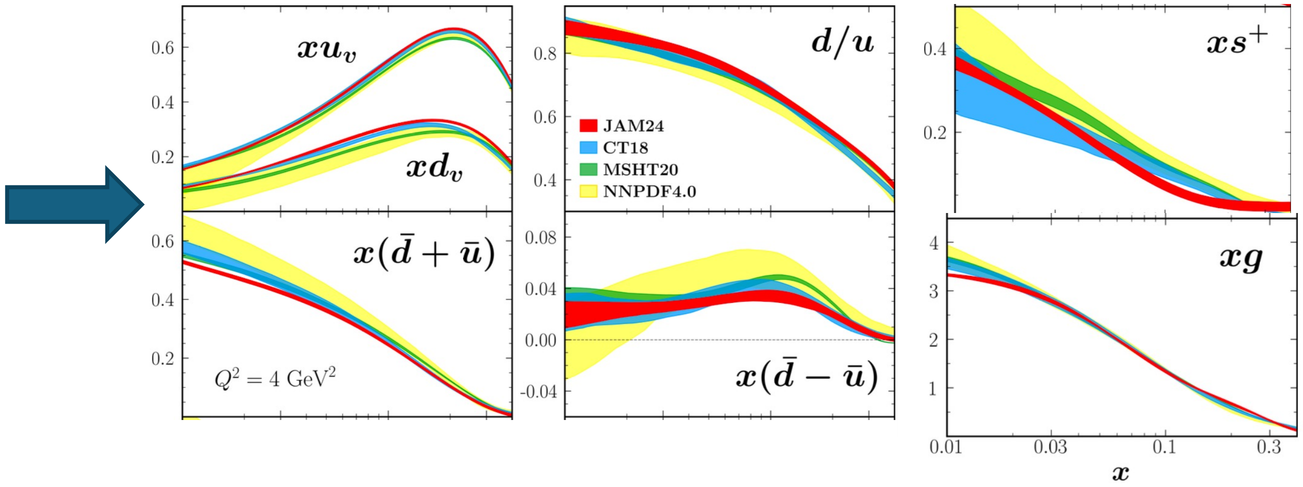
- Electroweak corrections use a running coupling for α_{EM}
- Fiducial volumes in collider experiments
  - ART uses an added q_T/Q power correction – makes little difference in our kinematics
- We perform Mellin space DGLAP evolution for collinear PDFs
  - We also use Mellin-space coefficients in the OPE
- Implementation of parallelization and code optimization in `python` with the help of the “1000 Scientists AI Jam”

1,000 SCIENTIST AI JAM

What do we know about structures?

- Most well-known structure is through longitudinal structure of hadrons, particularly protons





Anderson, Melnitchouk, and Sato, 2501.00665 [hep-ph]

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Non-perturbative models for TMDs

- Fit λ_1 and λ_2 to this functional form for each of the following flavors: $u, d, \bar{u}, \bar{d}$, and $sea = s = \bar{s} = c = \bar{c} = b = \bar{b}$

$$f_{NP}^{f}(x,b) = \frac{1}{\cosh\left(\left(\lambda_1^f(1-x) + \lambda_2^f x\right) b\right)},$$

- For the CS kernel, we fit two additional parameters, c_0 and c_1 according to this functional form

$$\mathcal{D}_{\rm NP}(b) = bb^* \left[c_0 + c_1 \ln\left(\frac{b^*}{B_{\rm NP}}\right)\right],$$

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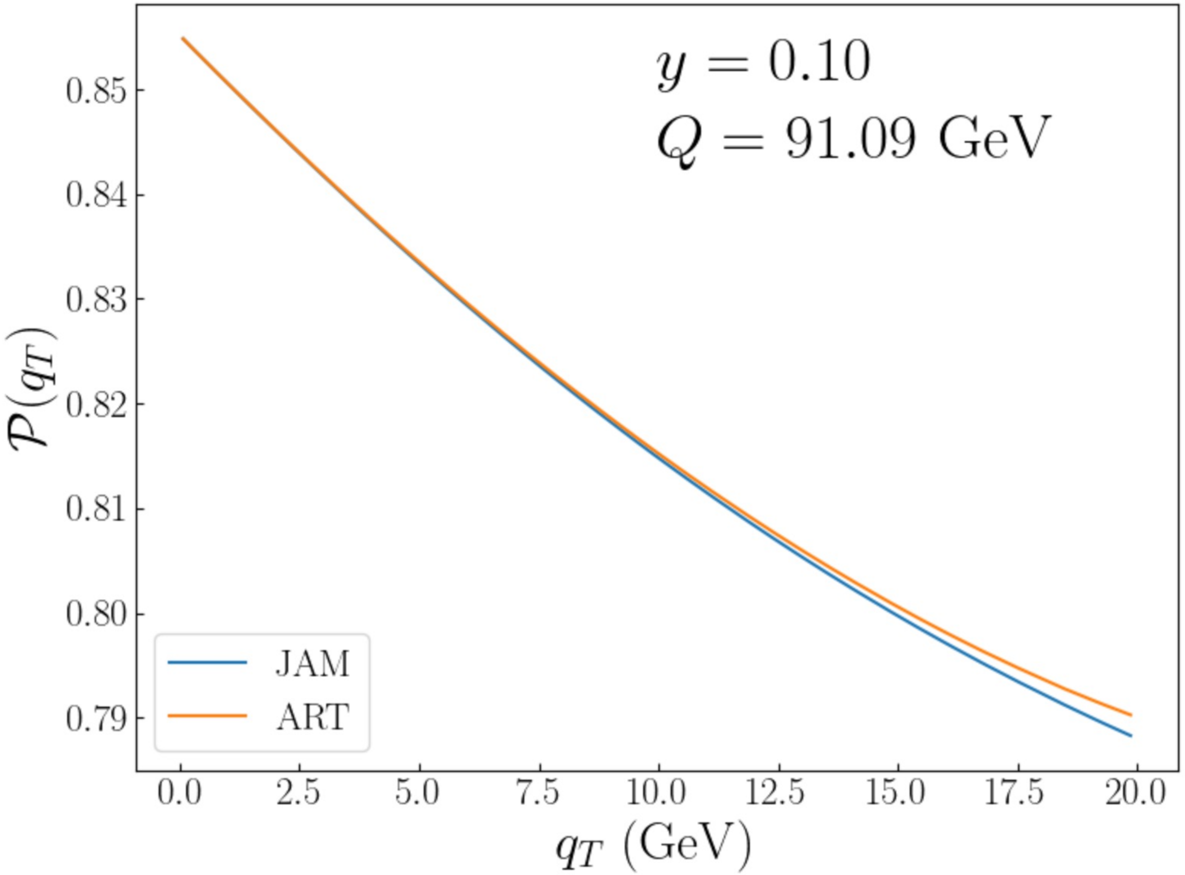
Fiducial cut comparisons



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