INVESTIGATING THE TRANSVERSE STRUCTURE OF HADRONS USING PARTON PSEUDODISTRIBUTIONS

M GABRIEL SANTIAGO

WITH ANATOLY RADYUSHKIN

OVERVIEW

- Introduction to Parton Pseudodistributions
- ‘Primordial’ TMDs
- TMDs with staple gauge links

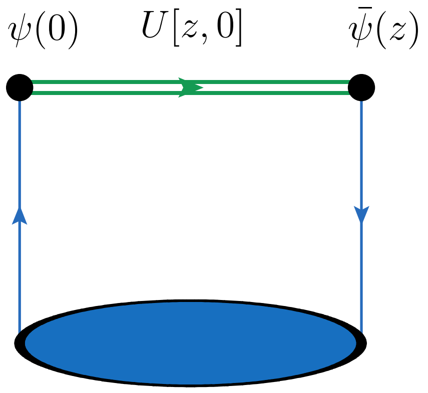
PARTON DISTRIBUTION FUNCTIONS

- The familiar quark PDFs from DIS are defined from a forward matrix element of two quark field operators separated along the light cone direction perpendicular to the large momentum of the hadron, with a Wilson line connecting the operators

$$\mathcal{M}^{\alpha}(z,P) = \langle P|\,\bar{\psi}(z)U[z,0]\Gamma^{\alpha}\psi(0)\,|P\rangle$$

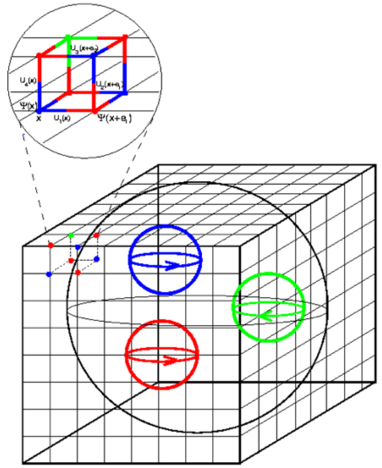
$$U[z,0] = \mathcal{P}\exp\left(ig\int_0^z \mathrm{d}s_\mu\, A^\mu(s)\right)$$

$$f(x,\mu^2) = \int \frac{\mathrm{d}z}{2\pi} e^{-ix(P\cdot z)}\mathcal{M}(z,P)\Big|_{z^2=0\to\mu^2}$$



PARTON DISTRIBUTION FUNCTIONS

- These quantities are non-perturbative objects and thus we cannot calculate them within QCD using analytical methods
- Lattice QCD allows one to perform first principles calculations of QCD matrix elements
- But, lattice QCD uses a discretized Euclidean spacetime - no access to objects separated along the light cone!



TOWARD LATTICE PDFS

- Lattice calculations can give us position space matrix elements , those entering the PDFs define *Ioffe time* distributions (ITDs)
- We can decompose the general matrix element based on the available vectors and indices as

$$\mathcal{M}^{\alpha}(z,P) = P^{\alpha}\mathcal{M}(\nu,-z^2) + z^{\alpha}\mathcal{M}_z(\nu,-z^2)$$

$\nu = -P \cdot z$ ~Ioffe time

$\mathcal{M}(\nu,-z^2)$ ~ITD

- For light cone PDFs we have $\alpha = +$ and $z^{\mu} = (0^{+}, z^{-}, 0_{\perp})$ so only ITD contributes!

PARTON PSEUDODISTRIBUTIONS

- Allowing the separation of the quark fields in the Ioffe time distributions to be off the light cone, one can define parton pseudodistributions as the corresponding Fourier transform from Ioffe time to x

$$\mathcal{P}(x,-z^2) = \int \frac{\mathrm{d}\nu}{2\pi} e^{-ix\nu} \mathcal{M}(\nu,-z^2)$$

- Without imposing constraints on z^2, one can show based on a general analysis of handbag type diagrams that the support of this Fourier transform is $-1 \leq x \leq 1$ (Radyushkin 2017)
- The ITD separates out the dependence on the longitudinal momentum coming from the parent hadron from the dependence on the momentum given by the Fermi motion of the quarks

$$\nu = -P \cdot z \rightarrow k^z/P^z$$

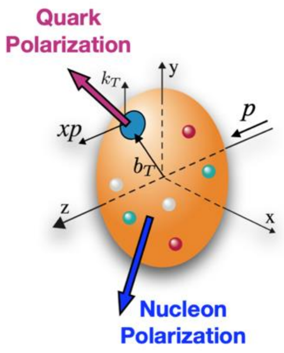
$$z^2 \rightarrow (k^z)^2 + \underline{k}^2$$

- Taking the fields to be at a spacelike separation allows us to compute the matrix elements on the lattice, then take the $z^2 \rightarrow 0$ limit to obtain the light cone PDFs once appropriate subtractions and matchings conditions are applied
- The pseudo-distributions come from off light cone matrix elements which can be calculated on the lattice!

'PRIMORDIAL' TMDS

- In fact, the Ioffe time distributions contain information on parton momentum that is separate from the longitudinal momentum inherited from the parent hadron, accounting for the Fermi motion of the partons
- One can take a specific separation with nonzero components along the direction of the hadron momentum and along perpendicular directions, obtaining a distribution in coordinate space which can be Fourier transformed to a transverse momentum dependent pseudodistribution

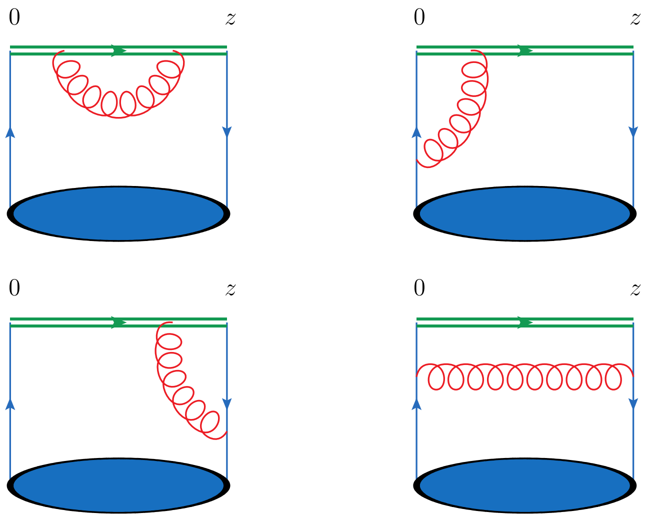
$$f(x, (k^z)^2 + \underline{k}^2) = \int \frac{\mathrm{d}\nu\, \mathrm{d}b_\perp}{(2\pi)^3} e^{-ix\nu} e^{i\underline{k}\cdot\underline{b}} \mathcal{M}(\nu, z^2 + \underline{b}^2)$$



- This distribution has no extra information about a scattering process which would be supplied by staple gauge links, so in a sense it is a 'primordial' TMD for the undisturbed hadron
- Note that this can still be written using a Ioffe time distribution with a *straight-line* gauge link, just like a PDF!

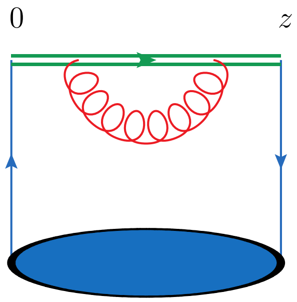
PERTURBATIVE CORRECTIONS

- The Wilson lines in these pseudo-distributions are not on the light cone, so new divergences arise which will need to be subtracted
- We study these divergences by calculating perturbative gluon exchanges within the operator



PERTURBATIVE CORRECTIONS

- The new contribution which arises for off light cone Wilson lines is the self energy diagram
- Working in Feynman gauge and regulating the gluon propagator with the Polyakov prescription $\frac{1}{z^2} \to \frac{1}{z^2 - a^2}$



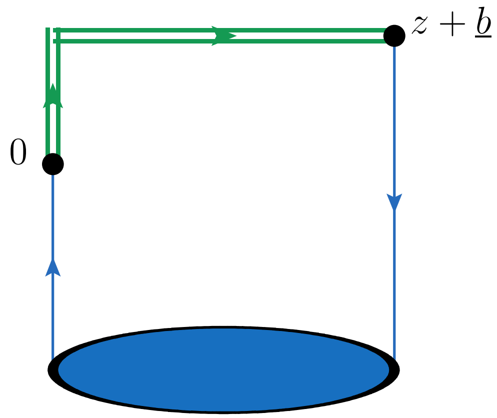
$$= -C_F \frac{\alpha_s}{2\pi} \left[2\frac{|z|}{a} \arctan\left(\frac{|z|}{a}\right) - \ln\left(1 + \frac{z^2}{a^2}\right) \right]$$

- This gives a linear divergence as $a \to 0$ which can be removed by dividing a matrix element by the rest frame matrix element of the same operator
- Note that this correction vanishes in the light cone limit $z^2 \to 0$

ISOLATING TRANSVERSE SPATIAL DEPENDENCE

- To separate the transverse and longitudinal spatial separation dependence in the Ioffe time distribution, one can modify the gauge link to an L shaped Wilson line, then compute the resulting matrix elements

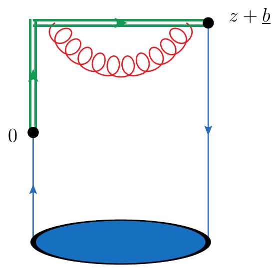
$$\langle P|\,\bar{\psi}(z+\underline{b})U[z+\underline{b},z]U[z,0]\Gamma^{\alpha}\psi(0)\,|P\rangle$$

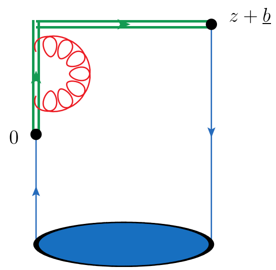


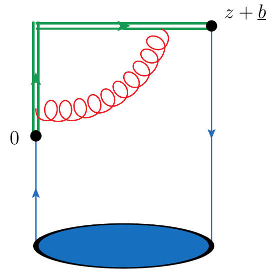
L-LINK PTMDS

- The L-shaped Wilson line introduces new perturbative divergences which need to be subtracted from the lattice calculations
- There are now three gluon exchanges among the gauge links to consider
- The exchange between the two perpendicular links vanishes
- The two self energy diagrams give the same contribution as in the straight link case, but with $z \to b$ in the transverse link

$$\Gamma_z(z,b) + \Gamma_b(z,b) = -C_F \frac{\alpha_s}{2\pi}\left[2\frac{|z|}{a}\arctan\left(\frac{|z|}{a}\right) + 2\frac{|b|}{a}\arctan\left(\frac{|b|}{a}\right) - \ln\left(1+\frac{z^2}{a^2}\right) - \ln\left(1+\frac{b^2}{a^2}\right)\right]$$

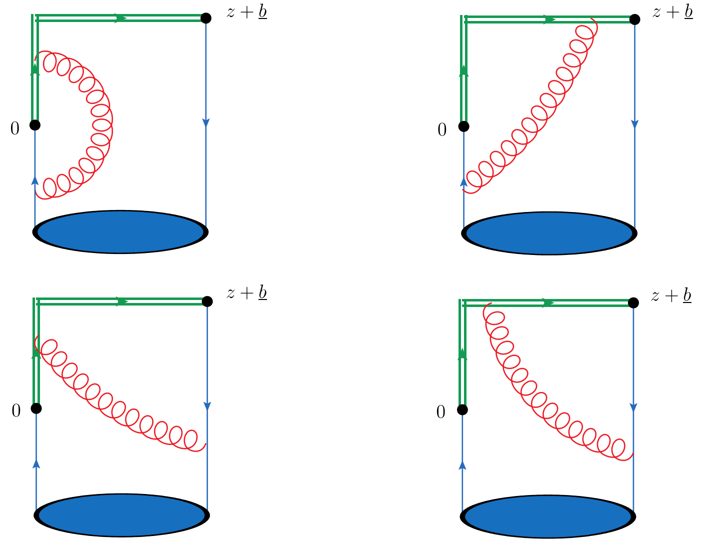






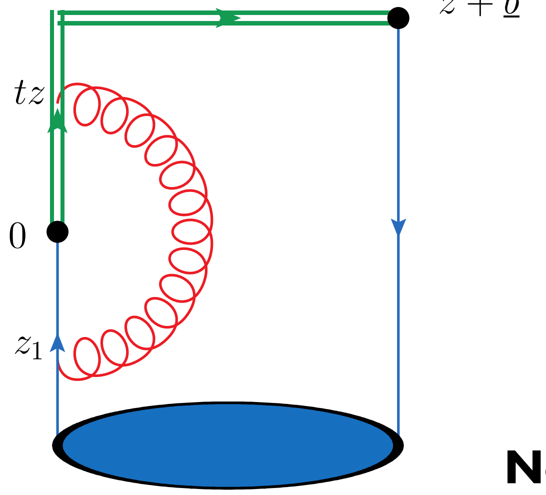
L-LINK PTMDS

- We also have new diagrams for the exchanges between the quark lines and the two straight Wilson lines
- These will give us the evolution logarithms



L-LINK PTMDS

- We keep our calculations in position space, writing the loop corrections in terms of the correlators which can be calculated on the lattice
  - We can straightforwardly convert these to x-space evolution kernels for comparison with other work
- Consider the exchange between the left quark line and the z direction gauge link



DGLAP type terms

m~IR regulator

$$\mathcal{O}_{Lz}(z,b) = C_F\frac{\alpha_s}{2\pi}\Bigg\{2\int_0^1 \mathrm{d}\beta\left(\left[\frac{(1-\beta)}{\beta}\right]_+ \mathrm{K}_0(|z|m) - \left[\frac{\ln(\beta)+1-\beta}{\beta}\right]_+\right)\bar{\psi}(z+\underline{b})\gamma^\alpha\psi(\beta z)$$

$$+\bar{\psi}(z+\underline{b})\gamma^\alpha\psi(0)\left[\left(1-\frac{a^2}{z^2}\right)\ln\left(1-\frac{z^2}{a^2}\right)-1\right]+\int_0^1 \mathrm{d}t\left[\frac{(1-t)}{t}\right]_+\bar{\psi}(z+\underline{b})\gamma^\alpha\psi(tz)\Bigg\}$$

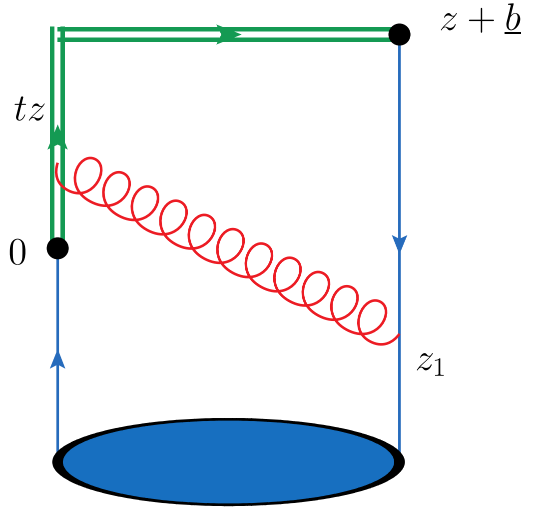
No $\underline{b}$ dependence!

Off light cone UV divergences

UV finite piece,

L-LINK PTMDS

- The exchange between the right quark line and the z direction gauge link gives us $\underline{b}$ dependence



$$
\begin{aligned}
\mathcal{O}_{Rz}(z,b) = -C_F \frac{\alpha_s}{2\pi} \Bigg\{ 2 \int_0^1 \mathrm{d}\beta \left[\frac{(1-\beta)}{\beta} \right]_+ \Bigg[& \mathrm{K}_0(|\underline{b}|m)\bar{\psi}(-\beta \underline{b} + z + \underline{b}) - \mathrm{K}_0(|z+\underline{b}|m)\bar{\psi}(-\beta z - \beta \underline{b} + z + \underline{b}) \\
& - \int_0^1 \mathrm{d}t \, \frac{tz^2}{t^2 z^2 + \underline{b}^2} \bar{\psi}(-\beta t z - \beta \underline{b} + z + \underline{b}) \Bigg] \gamma^\alpha \psi(0) \\
& + \frac{1}{2} \bar{\psi}(z+\underline{b}) \left[- \frac{2 \not{z} \not{b}}{|z||\underline{b}|} \arctan\left(\frac{|z|}{|\underline{b}|} \right) + \ln\left(\frac{\underline{b}^2}{\underline{b}^2 + z^2} \right) \right] \gamma^\alpha \psi(0) \\
& - \int_0^1 \mathrm{d}\beta \int_0^1 \mathrm{d}t \, \frac{tz^2 + \not{z}\not{b}}{t^2 z^2 + \underline{b}^2} \left[\bar{\psi}(-\beta t z - \beta \underline{b} + z + \underline{b}) - \bar{\psi}(z+\underline{b}) \right] \gamma^\alpha \psi(0) \Bigg\}
\end{aligned}
$$

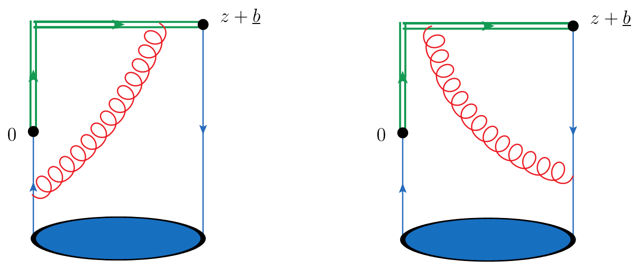
Evolution terms

Off light cone UV divergences

UV finite terms

L-LINK PTMDS

- We also have the exchange of each quark line with the transverse gauge link



$$
\begin{aligned}
\mathcal{O}_{Lb}(z,b) = C_F \frac{\alpha_s}{2\pi} \Bigg\{ & \int_0^1 \mathrm{d}\beta\, \bar{\psi}(z+\underline{b})\gamma^\alpha \left[\frac{(1-\beta)}{\beta}\right]_+ \Bigg[2\mathrm{K}_0(|\underline{b}+z|m)\psi(\beta\underline{b}+\beta z) - 2\mathrm{K}_0(|z|m)\psi(\beta z) \\
& + 2\int_0^1 \mathrm{d}t\, \frac{t\underline{b}^2}{t^2\underline{b}^2+z^2}\psi(\beta t\underline{b}+\beta z) \Bigg] \\
& + \bar{\psi}(z+\underline{b})\gamma^\alpha\psi(0)\frac{1}{2}\left[\frac{2\not{z}\not{\underline{b}}}{|z||\underline{b}|}\arctan\left(\frac{|z|}{|\underline{b}|}\right) + \ln\left(\frac{\underline{b}^2+z^2}{z^2}\right)\right] \\
& + \int_0^1 \mathrm{d}t \int_0^1 \mathrm{d}\beta\, \bar{\psi}(z+\underline{b})\gamma^\alpha \frac{t\underline{b}^2+\not{z}\not{\underline{b}}}{t^2\underline{b}^2+z^2}\left[\psi(\beta t\underline{b}+\beta z) - \psi(0)\right] \Bigg\}
\end{aligned}
$$

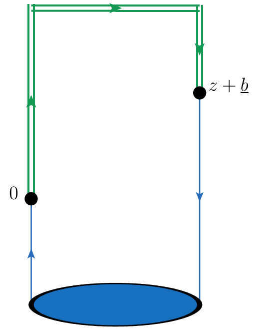
$$
\begin{aligned}
\mathcal{O}_{Rb}(z,b) = C_F \frac{\alpha_s}{2\pi} \Bigg\{ & \int_0^1 \mathrm{d}\beta \left[\frac{(1-\beta)}{\beta}\right]_+ \left[2\mathrm{K}_0(|\underline{b}|m)\bar{\psi}(-\beta\underline{b}+\underline{b}+z) + 2\int_0^1 \frac{\mathrm{d}t}{t}\bar{\psi}(-\beta t\underline{b}+\underline{b}+z)\right]\gamma^\alpha\psi(0) \\
& + \bar{\psi}(z+\underline{b})\gamma^\alpha\psi(0)\left[\left(1-\frac{a^2}{b^2}\right)\ln\left(1-\frac{b^2}{a^2}\right) - 1\right] + \int_0^1 \mathrm{d}t \left[\frac{(1-t)}{t}\right]_+ \bar{\psi}(-tz+z+\underline{b}) \Bigg\}
\end{aligned}
$$

'PRIMORDIAL' PSEUDO-TMD RESULTS

- We have the full perturbative corrections to these primordial pseudo-TMDs
- The divergences arising from the Wilson lines being off the light cone cannot simply be divided out with a rest frame element, but can still be subtracted order by order
- Thus, we have the theory tools to calculate the transverse momentum distribution of quarks in an undisturbed hadron on the lattice

STAPLE GAUGE LINKS

- To match the operator definition of the TMDs which enter factorization theorems, we need to include the full staple gauge links



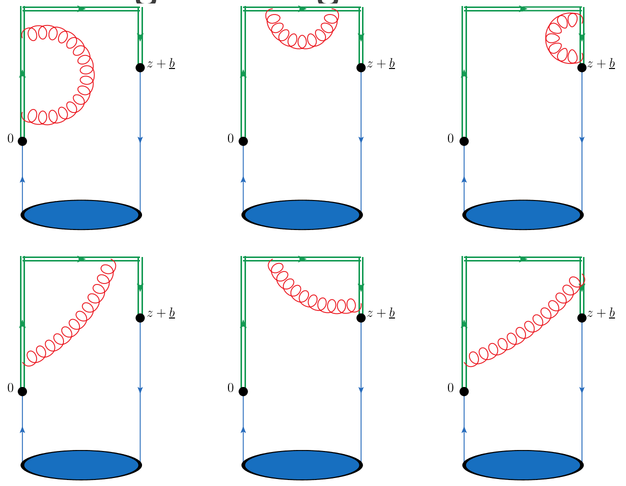
$$\langle P|\, \bar{\psi}(z+\underline{b})U[z+\underline{b}, Lz+\underline{b}]U[Lz+\underline{b}, Lz]U[Lz, 0]\Gamma^{\alpha}\psi(0)\,|P\rangle$$

- These modifications to the Wilson lines bring in information about the scattering process, manifesting in effects like the sign reversal for the T-odd Sivers and Boer-Mulders TMDs between SIDIS and DY

$$f_{1T}^{\perp,\mathrm{SIDIS}} = -f_{1T}^{\perp,\mathrm{DY}}$$

STAPLE GAUGE LINKS

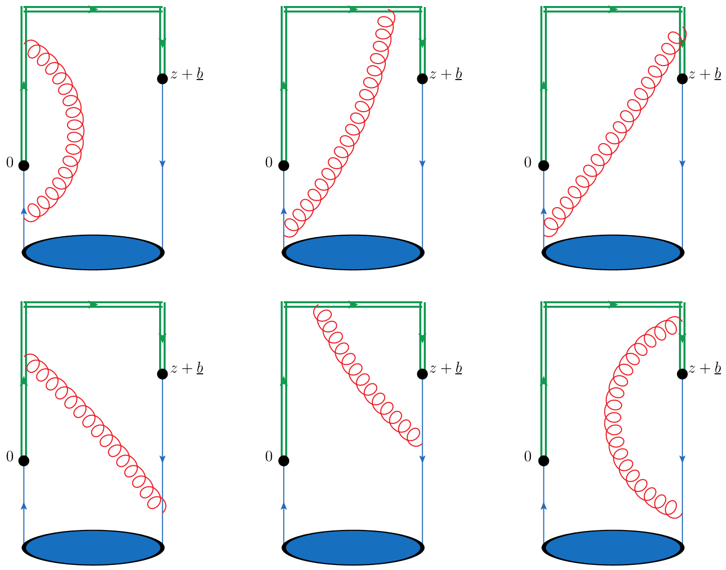
- Once again, we have new diagrams for exchanges among the Wilson lines



- These give similar linear divergence terms to the L shaped gauge link case

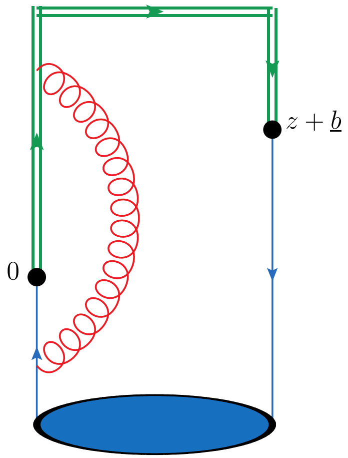
STAPLE GAUGE LINKS

- We also have new diagrams for the gluon exchanges between the quark lines and the Wilson lines



STAPLE GAUGE LINKS

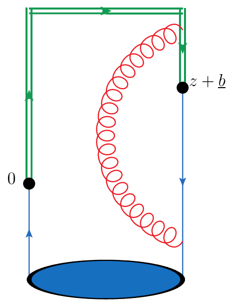
- Some of the diagrams do not change significantly, but now have L appearing in various logarithmic terms



$$
\begin{aligned}
\mathcal{O}_{LL}(z,b) = C_F\frac{\alpha_s}{2\pi}\Bigg\{ & 2\int_0^1 \mathrm{d}\beta\,\bar{\psi}(z+\underline{b})\gamma^\alpha\left(\left[\frac{(1-\beta)}{\beta}\right]_+ \mathrm{K}_0(|Lz||m)\psi(\beta L z)\right. \\
& \left. - \left[\frac{\ln(\beta)+1-\beta}{\beta}\right]_+ \psi(\beta z) + \left[\frac{(1-\beta)}{\beta}\right]_+ \int_1^L \frac{\mathrm{d}t}{t}\psi(\beta t z)\right) \\
& + \bar{\psi}(z+\underline{b})\gamma^\alpha\psi(0)\frac{1}{2}\left[\left(1-\frac{a^2}{L^2z^2}\right)\ln\left(1-\frac{L^2z^2}{a^2}\right)-1\right] \\
& + \int_0^L \mathrm{d}t\left[\frac{(1-t)}{t}\right]_+ \bar{\psi}(z+\underline{b})\gamma^\alpha\psi(tz)\Bigg\}
\end{aligned}
$$

STAPLE GAUGE LINKS

- New contributions emerge which contain contributions which contain two logarithmic integrals



$$\mathcal{O}_{RR}(z,b) = -C_F\frac{\alpha_s}{2\pi}\Bigg\{\int_0^1 \mathrm{d}\beta\left[\frac{(1-\beta)}{\beta}\right]_+\Bigg[\mathrm{K}_0(|(L-1)z|m)\bar{\psi}(\beta(L-1)z+z+\underline{b})\gamma^\alpha\psi(0)$$
$$+\int_1^{L-1}\frac{\mathrm{d}t}{t}\bar{\psi}(\beta tz+z+\underline{b})\gamma^\alpha\psi(0)\Bigg] - \int_0^1\mathrm{d}\beta\left[\frac{\ln(\beta)+1-\beta}{\beta}\right]_+\bar{\psi}(\beta z+z+\underline{b})\gamma^\alpha\psi(0)$$
$$+\bar{\psi}(z+\underline{b})\gamma^\alpha\psi(0)\frac{1}{2}\left[\left(1-\frac{a^2}{(L-1)^2z^2}\right)\ln\left(1-\frac{(1-L)^2z^2}{a^2}\right)-1\right]$$
$$+\int_0^{L-1}\mathrm{d}t\left[\frac{(1-t)}{t}\right]_+\bar{\psi}(tz+z+\underline{b})\gamma^\alpha\psi(0)\Bigg\}$$

- We find that the portion of the Wilson lines which runs off to infinity in the limit $L\to\infty$ generates the double logarithms expected from rapidity divergences

WHAT NEXT?

- Having obtained the one loop corrections, we need to extract the terms in the evolution kernels corresponding to the known CSS evolution kernels
- This will require choosing an appropriate regulator and scheme to make the CSS double logarithms manifest
- We need to cross check with calculations performed in the quasi-TMD framework (Ji et al 2018, Ebert et al 2019)
- We have calculated the 'bare TMD' or beam function and thus should apply this formalism to the soft function as well in order to match the TMD structure which enters factorization theorems
- In order to match the $\overline{MS}$ scheme we must construct a matching relation
- Finally, this formalism can be applied to gluon TMDs and spin-dependent TMDs to further information on the transverse momentum structure of hadrons from lattice QCD

CONCLUSIONS

- We have calculated the short distance perturbative corrections which will enter lattice calculations of pseudo-TMDs in position space
- Off light cone UV divergences need to be cancelled, while other divergences generate the evolution of the pseudo-TMDs
- We have seen that the transverse momentum structure of undisturbed hadrons can be calculated on the lattice using an L-shaped gauge link to construct 'primordial' pseudo-TMDs
- We have calculated the perturbative corrections arising from including a finite length, off staple gauge link, including double logarithmic terms which do not appear for the straight and L-shaped gauge links