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Perturbative corrections to quark TMDPDFs in the background-field method

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S. Mukherjee, V. Skokov, A. Tarasov, S. Tiwari, Phys. Rev. D 109 (2024) 3, 034035;
arXiv:2502.15889; and in preparation

Transverse momentum dependent factorization

- The transverse momentum dependent (TMD) factorization scheme can be used for analysis of scattering with production of a final state of transverse momentum $p_\perp$ which is much smaller than a hard scale

$$p_\perp^2 \ll z^2 Q^2$$

TMD factorization limit for SIDIS

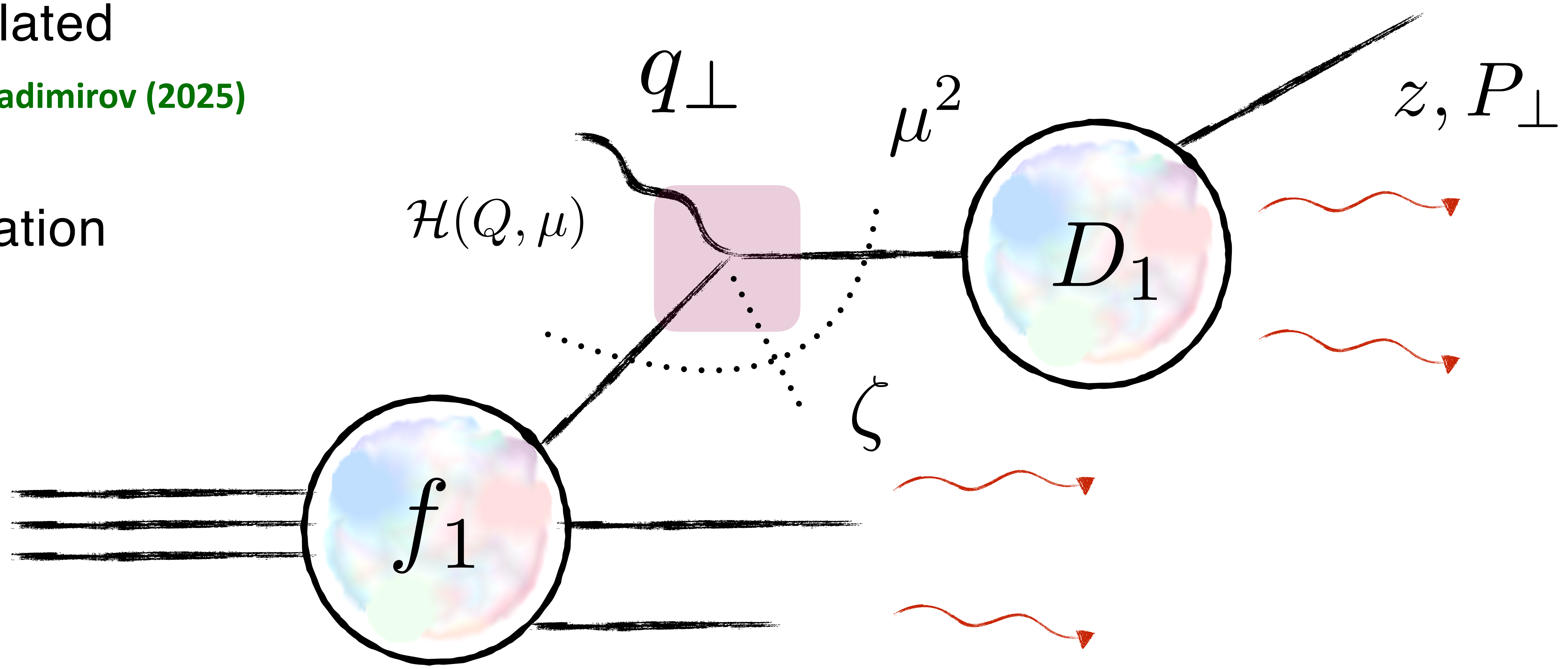
$$\frac{d\sigma}{dx dz dQ^2 dp_\perp^2} \propto \sum_f e_f^2 \mathcal{H}(Q,\mu) \int_0^\infty db_\perp b_\perp J_0\Big(\frac{b_\perp p_\perp}{z}\Big) f_1(x, b_\perp; \mu, \zeta) D_1(z, b_\perp; \mu, \bar{\zeta}) + \mathcal{O}\Big(\frac{p_\perp^2}{Q^2}\Big)$$

- The power corrections (i.e. terms of order $p_\perp^2/Q^2$) to the TMD factorization can be systematically calculated

see e.g. Balitsky, Tarasov (2017-2018); Arroyo-Castro, Scimemi, Vladimirov (2025)

- The region of applicability of the TMD factorization corresponds to relatively large values of $b_\perp$

$$\boxed{p_\perp^2/Q^2 \ll 1 \Rightarrow b_\perp \lesssim \Lambda_{QCD}^{-1}}$$



TMDPDFs as functions of four variables

- The TMDPDF is an intrinsically non-perturbative object

$$f_i(x, b_\perp; \mu, \zeta)$$

kinematic variables

factorization scales

- It is a function of four variables!

$$\frac{d}{d\ln\mu} f_i(x, b_\perp; \mu, \zeta) = \gamma_\mu^i(\mu, \zeta) f_i(x, b_\perp; \mu, \zeta)$$

$$\frac{d}{d\ln\zeta} f_i(x, b_\perp; \mu, \zeta) = \frac{1}{2}\gamma_\zeta^i(\mu, b_\perp) f_i(x, b_\perp; \mu, \zeta)$$

The anomalous dimensions are known up to 4 loops

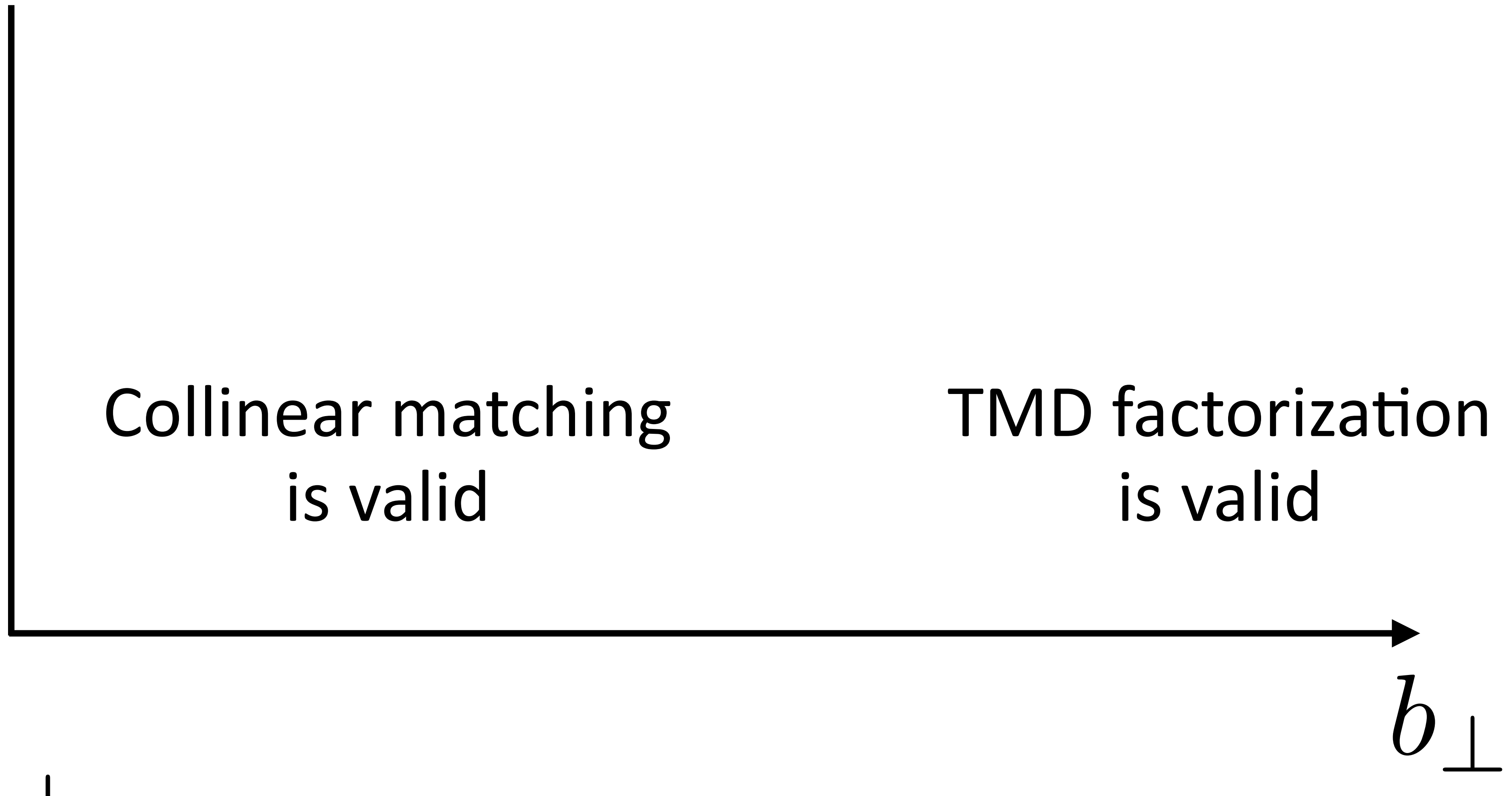
Duhr, Mistlberger, Vita (2022)
Manteuffel, Panzer, Schabinger (2020)

- To appropriately restrain this function we want to extract as much perturbative information as possible, including the dependence on the kinematic variables
- However, an appropriate calculation of the perturbative component of the function **in the region of applicability of the TMD factorization** (large $b_\perp \lesssim \Lambda_{QCD}^{-1}$) is challenging, and, in general, has not been done

Collinear matching

- Instead TMDPDFs have been directly calculated only in the region of small $b_\perp \ll \Lambda^{-1}_{QCD}$
- The conventional approach is to consider TMDPDFs in a small $b_\perp \ll \Lambda^{-1}_{QCD}$ approximation and expand them in terms of the collinear PDFs - collinear matching:

see e.g. Scimemi, Tarasov and Vladimirov (2019)



$$f_i(x, b_\perp, \mu, \zeta) = C_1 \otimes f_1(x, \mu) + b_\perp^2 C_2 \otimes f_2(x, \mu) + \ldots$$

Matching coefficients (perturbative), contains logs of IR origin

Collinear PDFs of rising twist

- However, the region of applicability of the TMD factorization corresponds to large $b_\perp \lesssim \Lambda^{-1}_{QCD}$. The TMDPDFs genuinely contain contribution of all collinear twists!

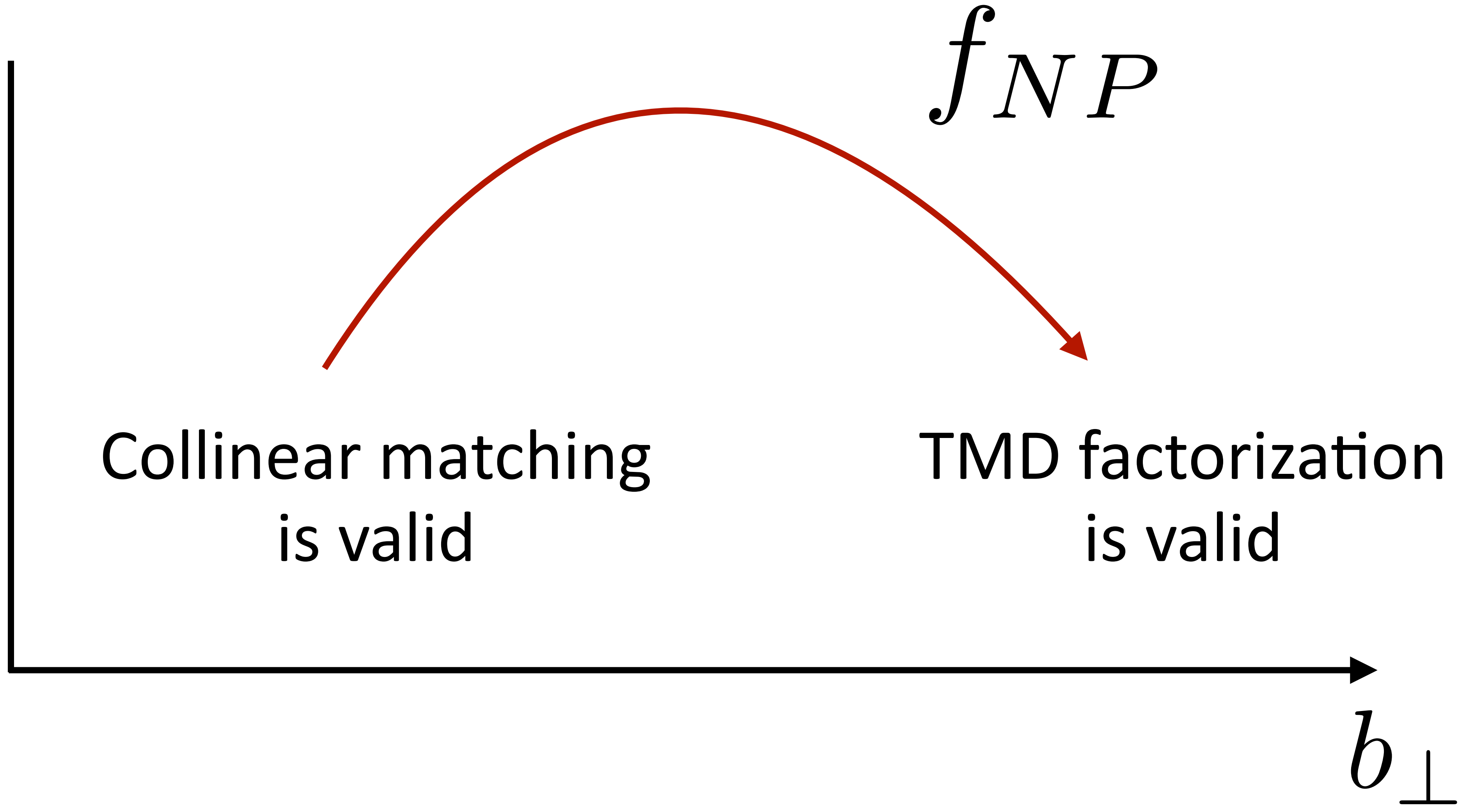
Problem: Only first few terms of the collinear matching are known. To obtain the correct structure of the TMDPDFs, which describes all collinear twist content of the distributions, one would need to resum all terms of the collinear expansion, which at the moment is not feasible!

Non-perturbative function

- In practice a phenomenological solution is used: higher twist terms are dropped

$$f_i(x, b_\perp, \mu, \zeta) = C_1 \otimes f_1(x, \mu) + \cancel{b_\perp^2 C_2 \otimes f_2(x, \mu) + \dots}$$

- A phenomenological function f_{NP} is introduced to extrapolate the result from $b_\perp \ll \Lambda_{QCD}^{-1}$ into the physical region $b_\perp \lesssim \Lambda_{QCD}^{-1}$



- The collinear matching "model" for unpolarized TMDPDF used in the phenomenology:

TMD distribution for the large $b_\perp \lesssim \Lambda_{QCD}^{-1}$

$$\hat{f}_1^a(x, b_\perp^2; \mu_f, \zeta_f)$$

$$= \underbrace{[C \otimes f_1](x, b_*; \mu_{b_*}, \mu_{b_*}^2)}_{\text{Collinear matching constructed in the region of small } b_\perp \ll \Lambda_{QCD}^{-1}} \underbrace{\exp\left\{ \int_{\mu_{b_*}}^{\mu_f} \frac{d\mu}{\mu} \gamma(\mu, \zeta_f) \right\} \left(\frac{\zeta_f}{\mu_{b_*}^2} \right)^{K(b_*, \mu_{b_*})/2}}_{\text{CSS evolution}} \underbrace{f_{1\mathrm{NP}}(x, b_\perp^2; \zeta_f, Q_0)}_{\text{Phenomenological function containing all collinear twist content of the distribution and describing its behavior at large } b_\perp \lesssim \Lambda_{QCD}^{-1}}$$

The procedure doesn't allow us to reveal all collinear twist content of the TMDPDFs which is important in the region of applicability of the TMD factorization

TMD factorization $\neq$ collinear factorization

Collinear vs. TMD factorization

- The discrepancies between collinear and TMD factorizations manifest themself in the phenomenology:

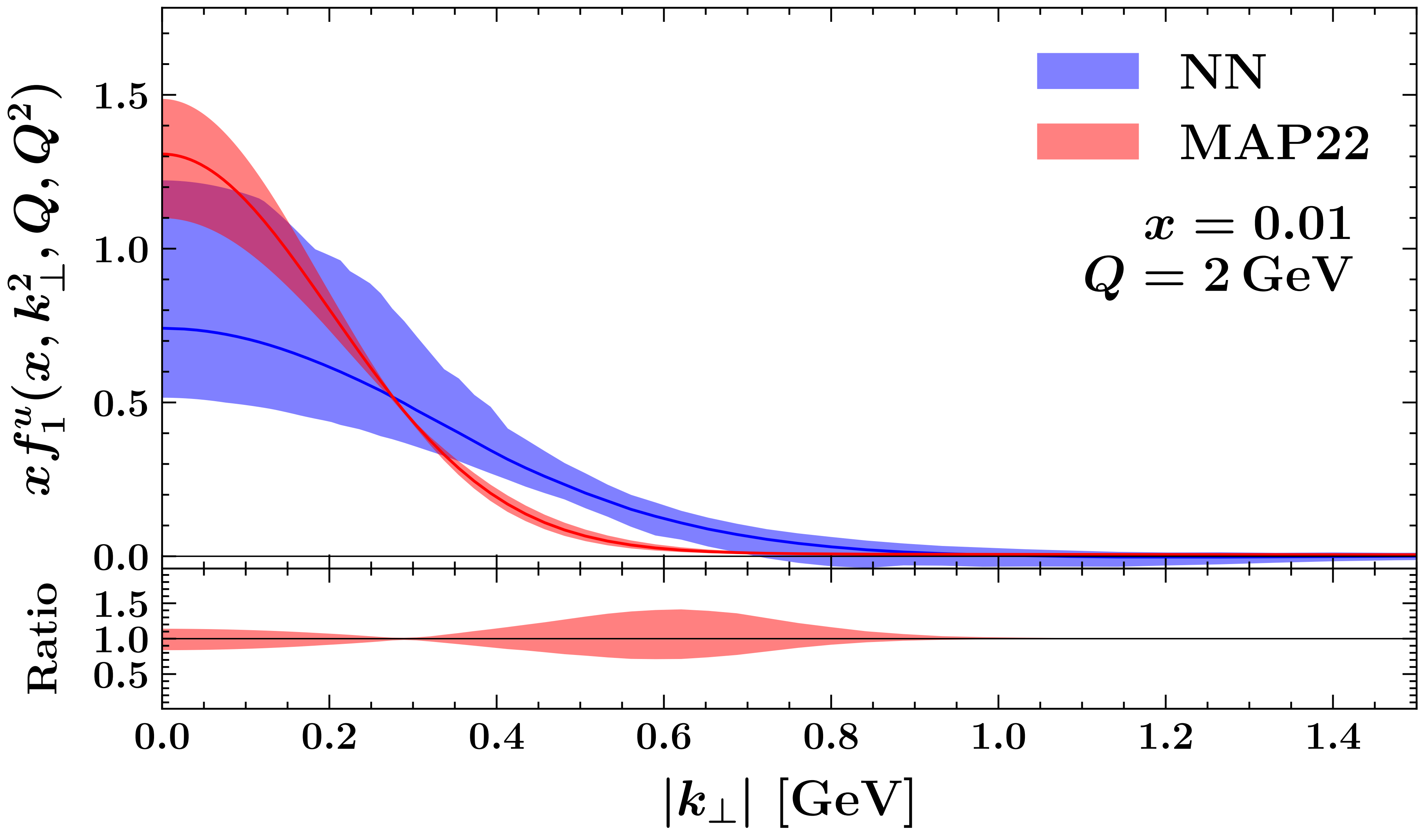
$$\frac{d\sigma_{\omega}^{\mathrm{SIDIS}}}{dx\,dz\,d|\boldsymbol{q}_T|\,dQ} = \omega(x,z,Q)\,\frac{d\sigma^{\mathrm{SIDIS}}}{dx\,dz\,d|\boldsymbol{q}_T|\,dQ}$$

At NLL, $\omega(x,z,Q) = 1$. Beyond NLL, the prefactor becomes larger than one and guarantees that the integral of the TMD part of the cross section reproduces most of the collinear cross section, as suggested by the data.

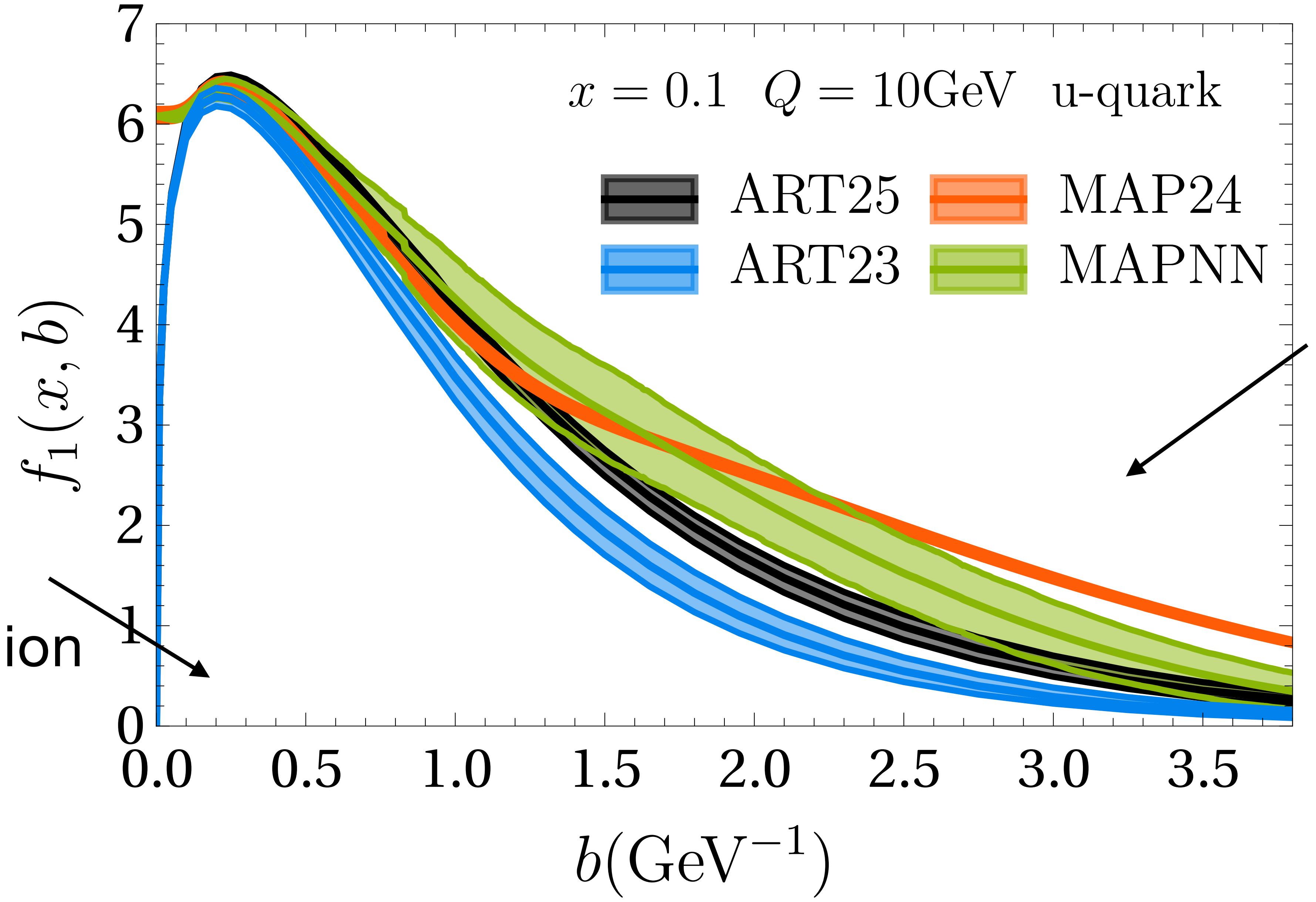
Bacchetta, Bertone, Bissolotti, Bozzi, Cerutti, Piacenza, Radici, Signori (2022)

- Significant dependence on the model for the non-perturbative function and wide spread of the function in the $b_\perp$ dependence

Moos, Scimemi, Vladimirov, Zurita (2025)



Bacchetta, Bertone, Bissolotti, Cerutti, Radici, Rodini, Rossi (2025)



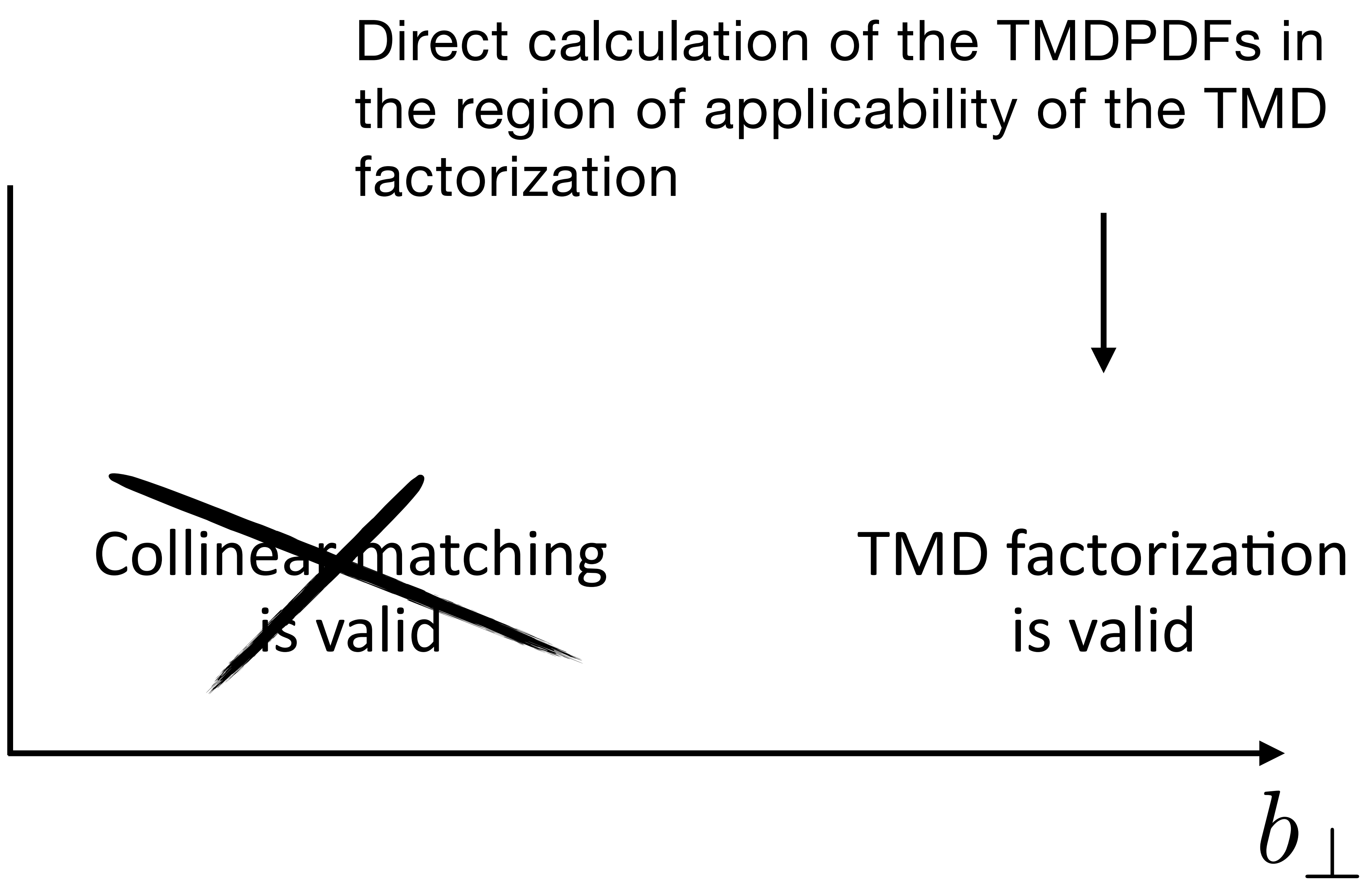
leading collinear twist region

all collinear twist region. Contribution of this region to observables is even more enhanced!

Can we better address the all collinear twist content of the TMDPDFs?

MSTT(-erious) factorization: calculation of the TMDPDFs in the region of large $b_\perp$

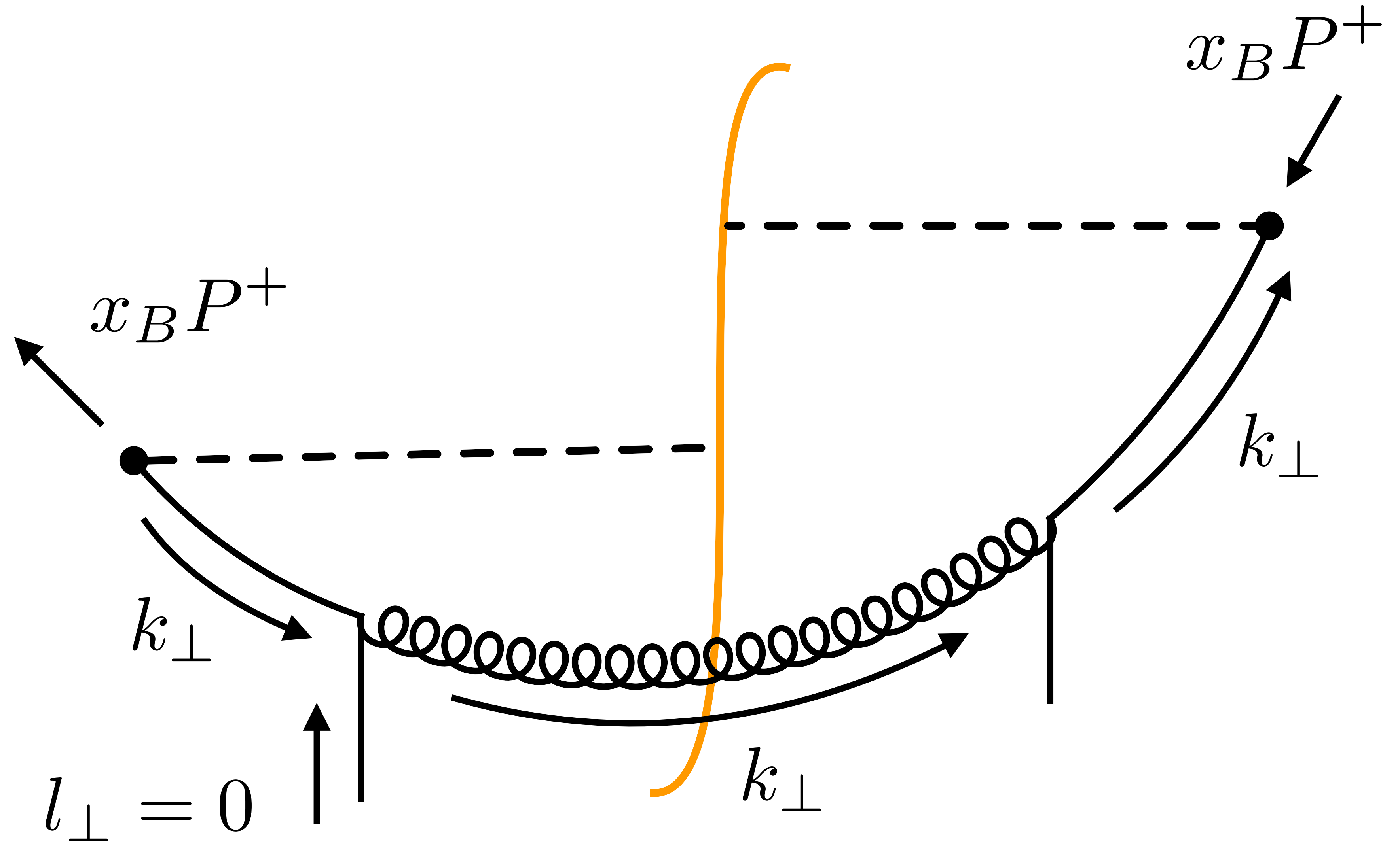
- The collinear matching procedure doesn't allows us to fully uncover the all collinear twist structure of the TMDPDFs
- To obtain this structure one has to perform calculation directly in the region of validity of the TMD factorization corresponding to large $b_\perp \lesssim \Lambda_{QCD}^{-1}$
- One needs calculation of TMDPDFs in the background field of general kinematics **Mukherjee, Skokov, Tarasov, Tiwari (2024) - MSTT factorization**



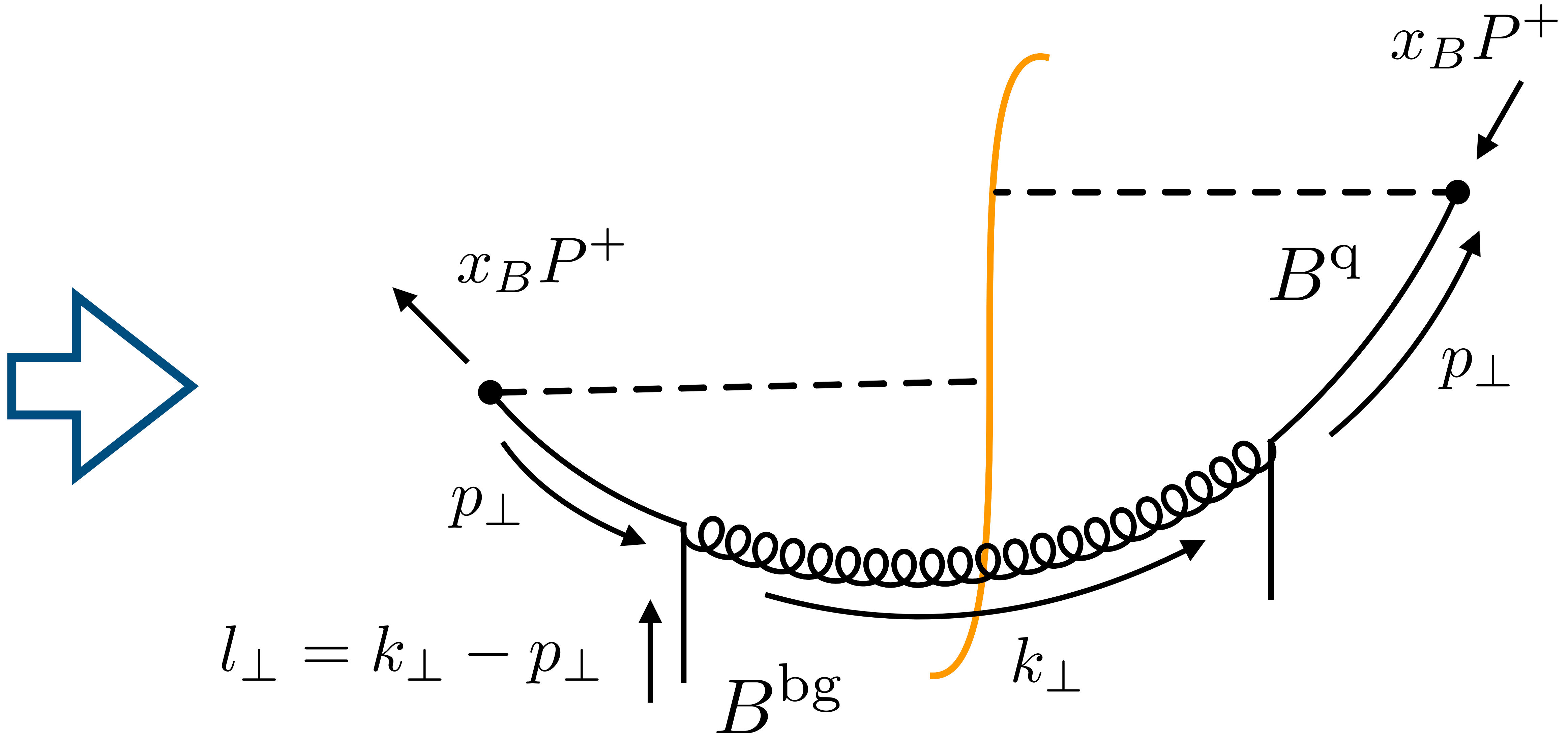
Direct calculation of the TMDPDFs in the region of applicability of the TMD factorization

~~Collinear matching is valid~~

TMD factorization is valid



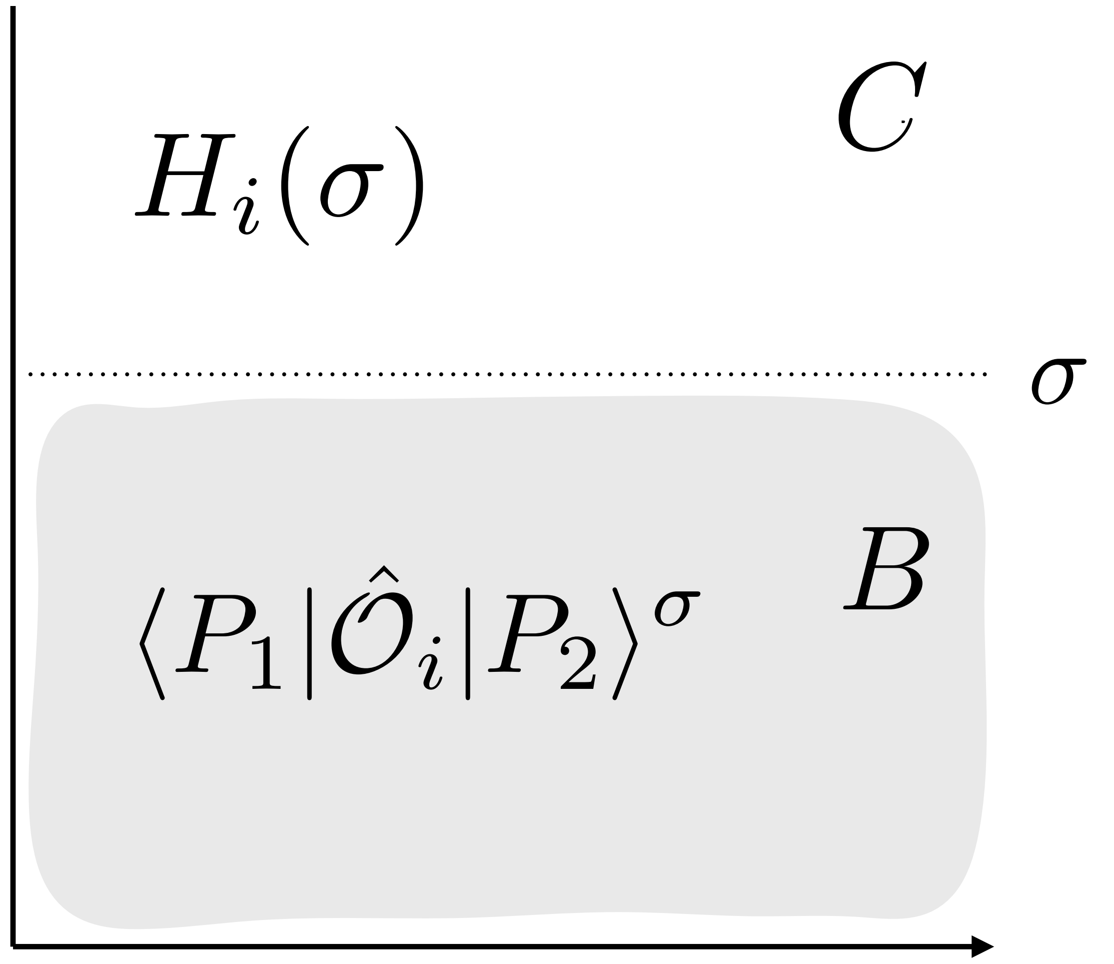
In the collinear limit we effectively assume strong separation between quantum fields and the background in transverse momentum ⇒ scattering in the background of collinear partons



At large $b_\perp \lesssim \Lambda_{QCD}^{-1}$ we have to discard the assumption about ordering in transverse momentum and consider scattering in a background $B^{\rm bg}$ of general kinematics

The background field method and the QCD factorization

- The calculation in general kinematics can be efficiently performed in the background field method
- In the background field method the QCD factorization is defined by a separation of the QCD fields (i.e. $A \to C + B$) into different modes separated by factorization scales σ

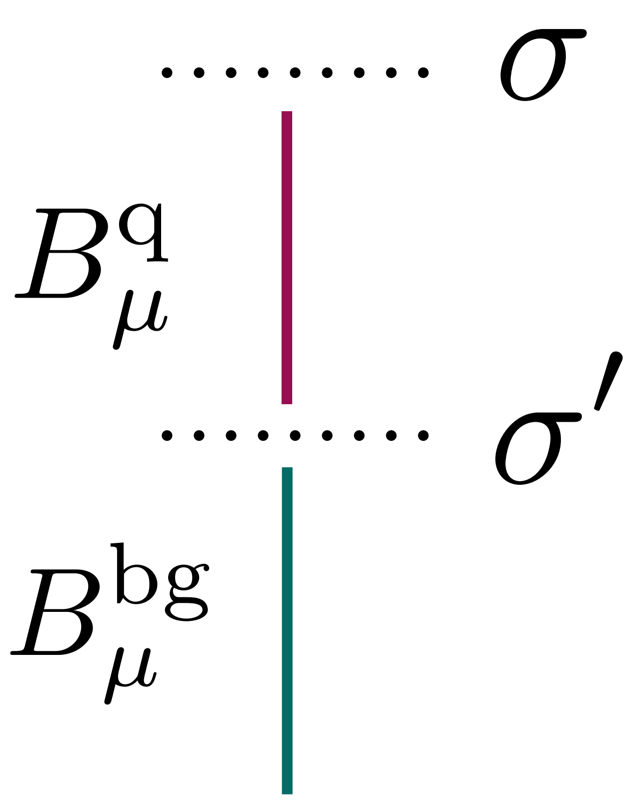


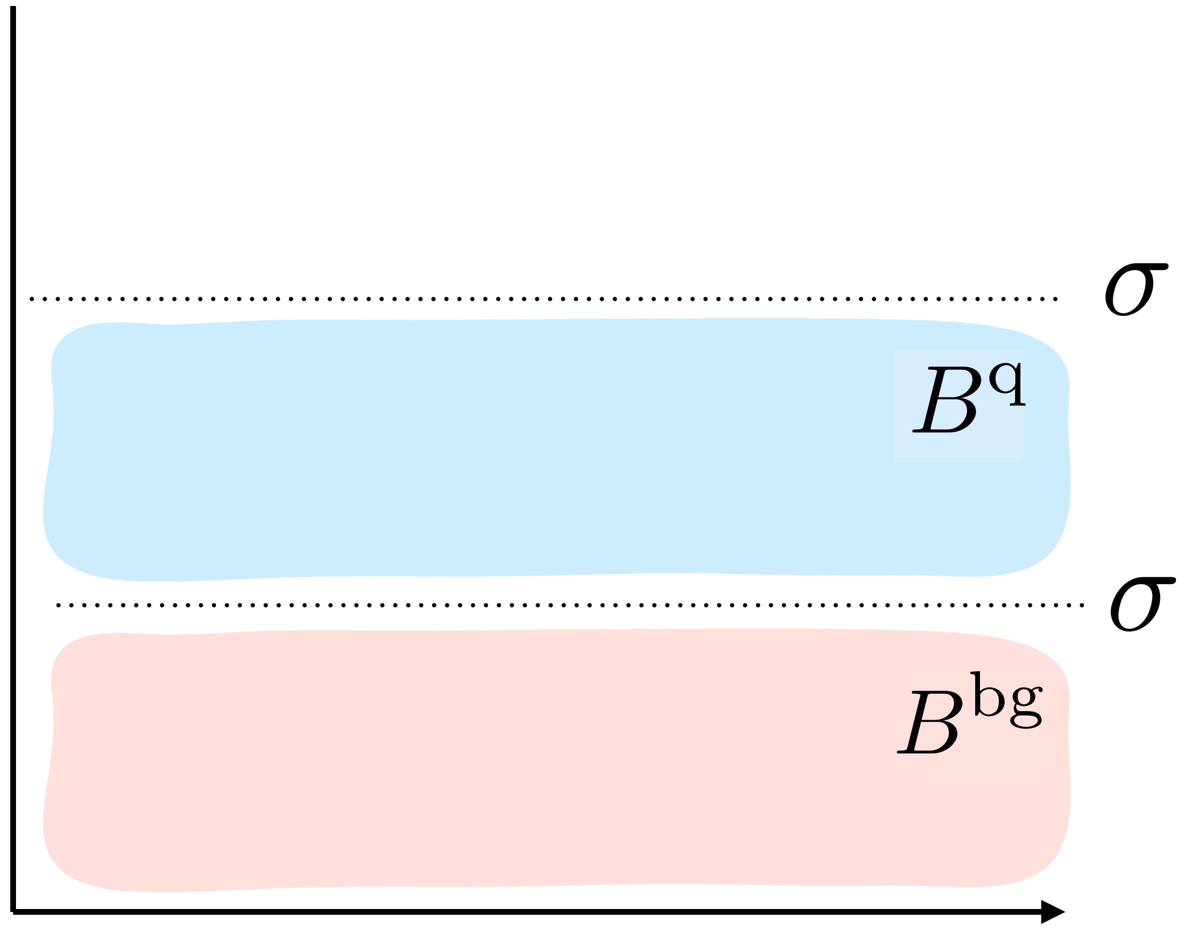
$$d\sigma = H_i(\sigma) \otimes \langle P_1|\hat{\mathcal{O}}_i|P_2\rangle^\sigma$$

$$\langle P_1|\hat{\mathcal{O}}_i|P_2\rangle^\sigma = \int \mathcal{D}B \; \Psi^*_{P_1}(\vec{B}(t_f))\mathcal{O}_i(B)\Psi_{P_2}(\vec{B}(t_i))e^{iS_{QCD}(B)}$$

- In the context of the TMD factorization $\sigma = (\mu^2, \zeta)$ and $\hat{\mathcal{O}}$ is a TMD operator
- Evaluation of the functional integral over B fields is hard in general, so, following the logic of the background field method, one can split the field into different components introducing IR scales σ' and integrate over quantum component

$$B_\mu \to \underset{\substack{\uparrow \\ \text{“quantum” fields} \\ \text{e.g. perturbative mode}}}{B^{\rm q}_\mu} + \underset{\substack{\uparrow \\ \text{background fields} \\ \text{e.g. non-perturbative mode}}}{B^{\rm bg}_\mu}$$





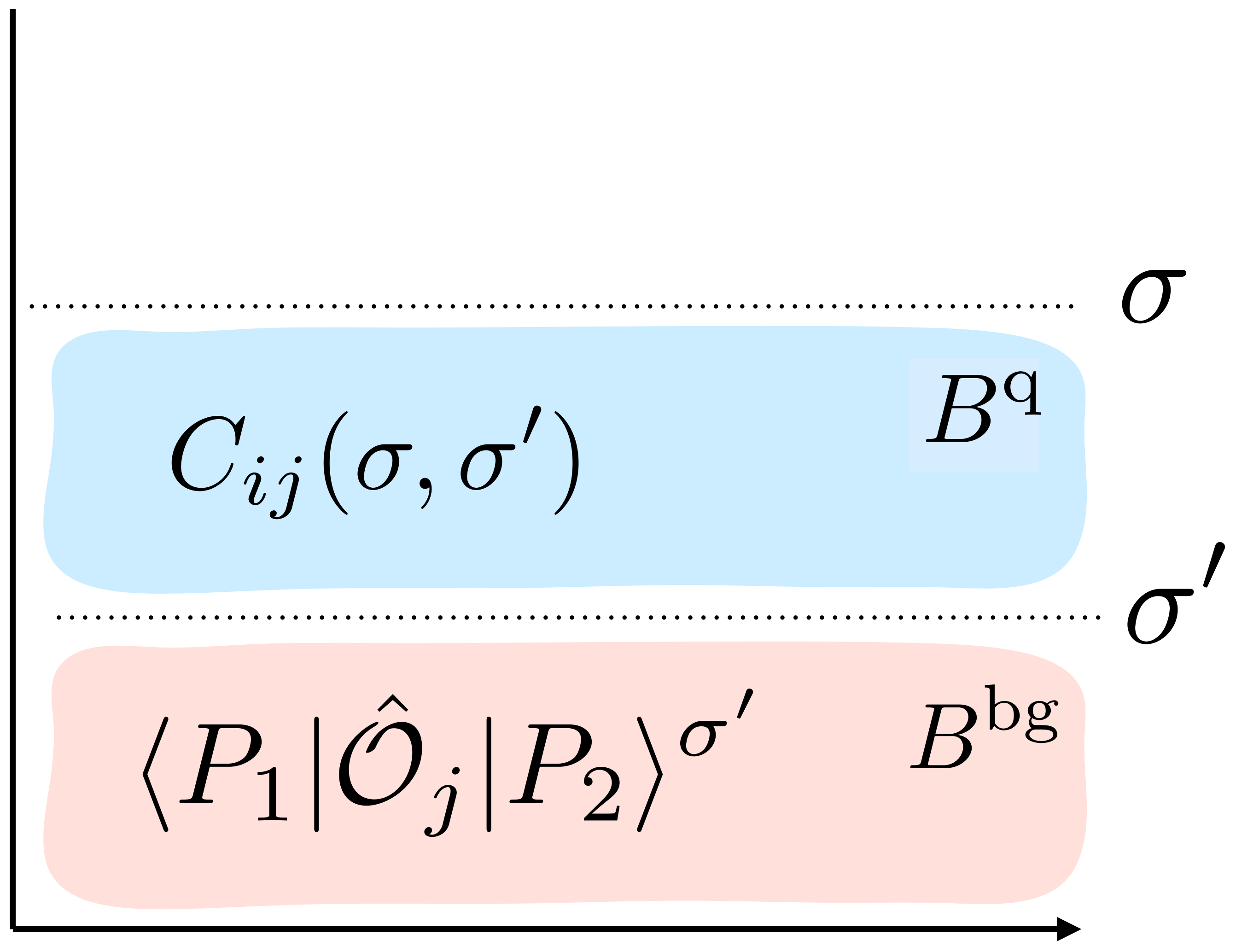
Background field method and the QCD evolution

- After splitting of the field B into two components, we aim to integrate over quantum modes B^{q} in a fixed background of B^{bg} fields. The integration over B^{q} fields is done perturbatively. The result has the following structure:

$$\langle P_1|\hat{\mathcal{O}}_i|P_2\rangle^{\sigma} = \sum_j C_{ij}(\sigma,\sigma') \otimes \langle P_1|\hat{\mathcal{O}}_j|P_2\rangle^{\sigma'}$$

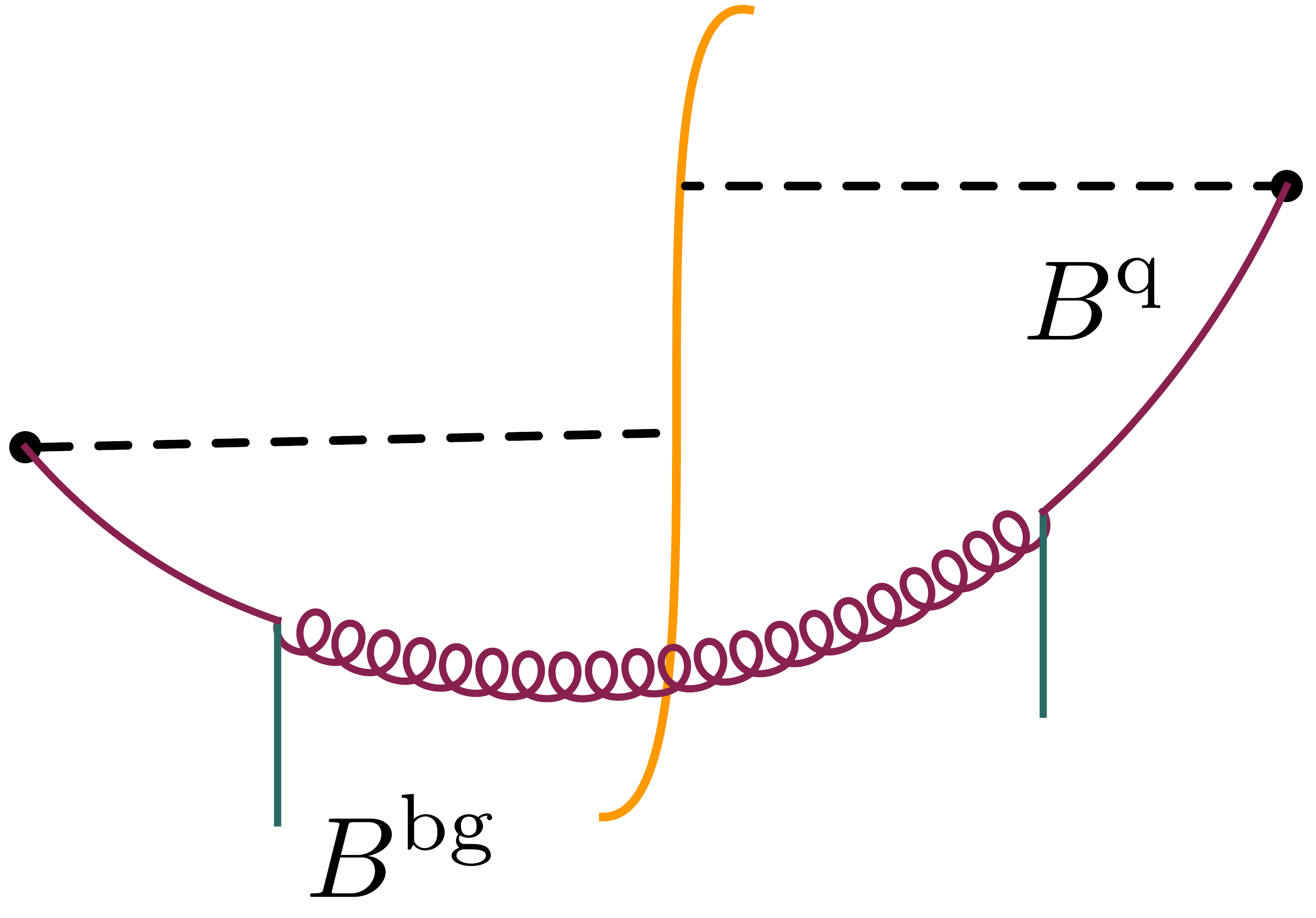
result of integration over B^{q} fields

operator constructed from B^{bg} fields



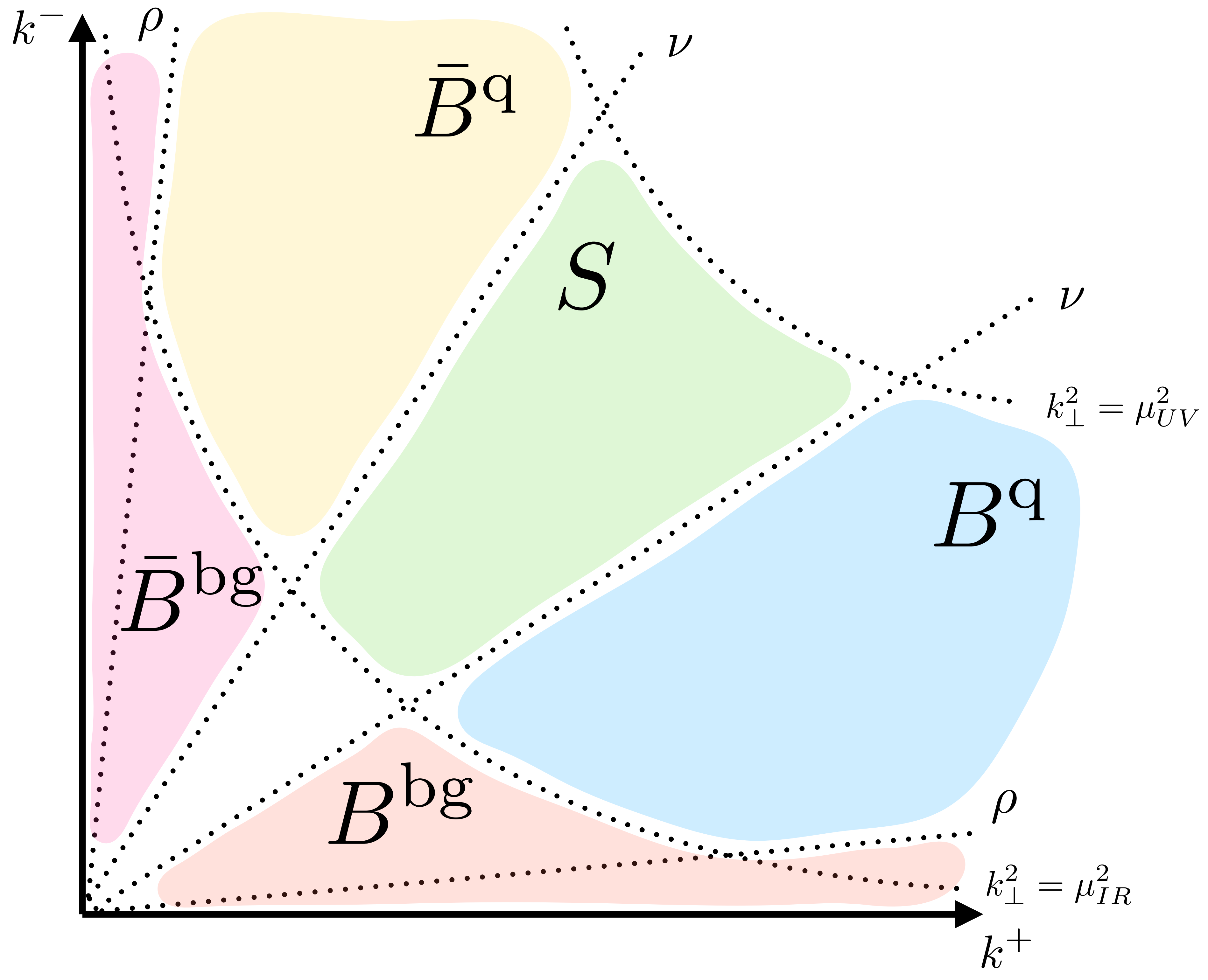
- The coefficient function C_{ij} describes dependence on the factorization scales σ (UV) and σ' (IR), and contains information on the perturbative component of the TMDPDFs. We aim to perturbatively calculate this function

- The result of calculation is defined by the structure of the factorization scales σ and σ', i.e. the definition of the factorization scheme



Separation of the field modes

- One can separate the field modes using explicit cut-offs (e.g. high-energy rapidity factorization), but this is usually technically hard



- Instead one can use the renormalization approach. The divergent integrals are regularized and the corresponding scales play the role of the cut-offs, while the singular poles should be appropriately subtracted
- We use the dimensional regularization for the transverse integrals
- The rapidity divergent integrals are regularized as

$$\int_0^\infty \frac{dk^-}{k^-} \to \nu^\eta \int_0^\infty \frac{dk^-}{k^-}|k^+|^{-\eta}$$

J. Chiu, A. Jain, D. Neill, I. Rothstein (2013)

- To resolve potential double counting due to overlap between field modes in rapidity the matrix element constructed from fields B^{q} and B^{bg} is to be multiplied by a soft factor

$$f_i(x_B, b_\perp) = \sqrt{\mathcal{S}(b_\perp)}\mathcal{B}_i(x_B, b_\perp)$$

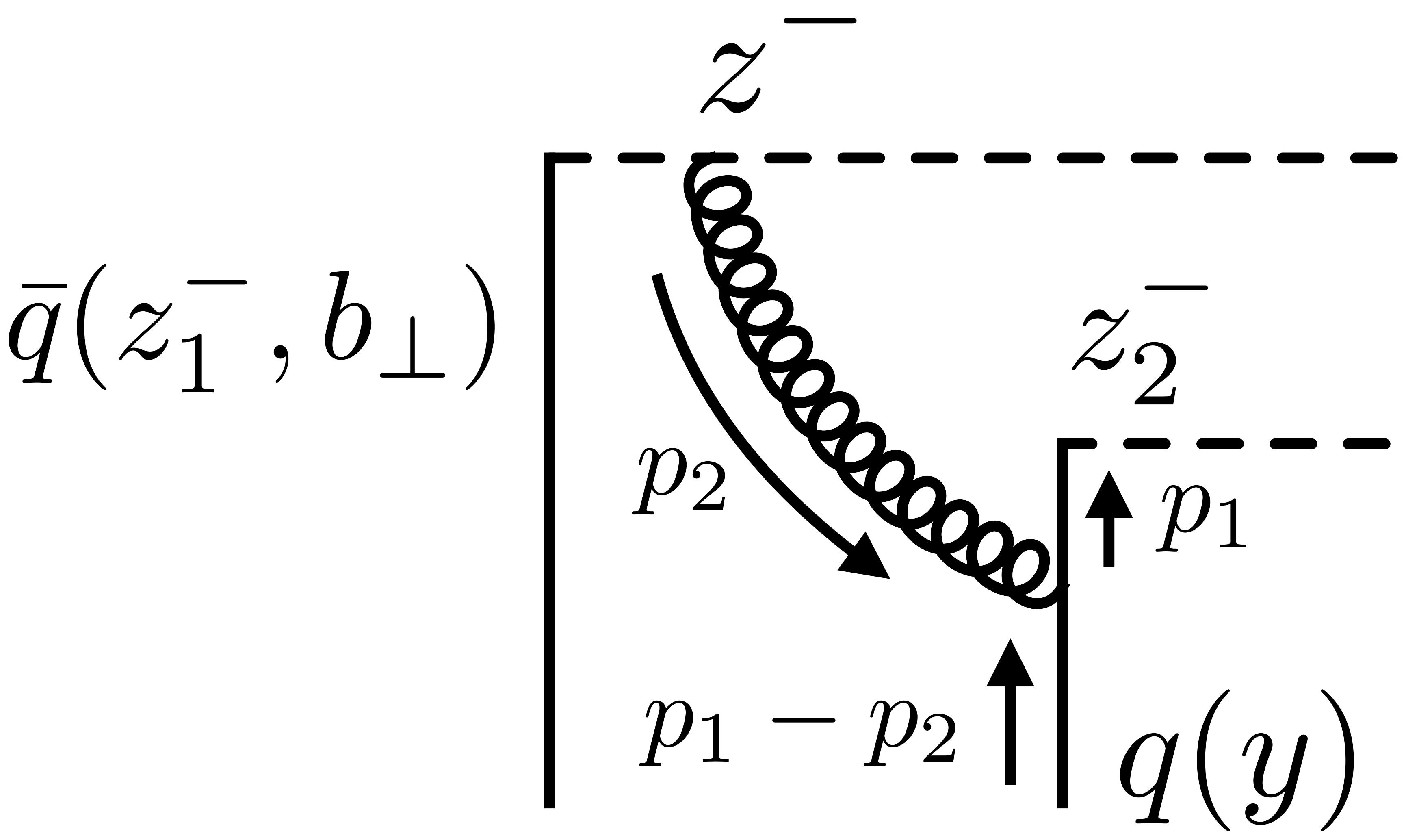
Calculation of quark TMDPDFs in the background field method

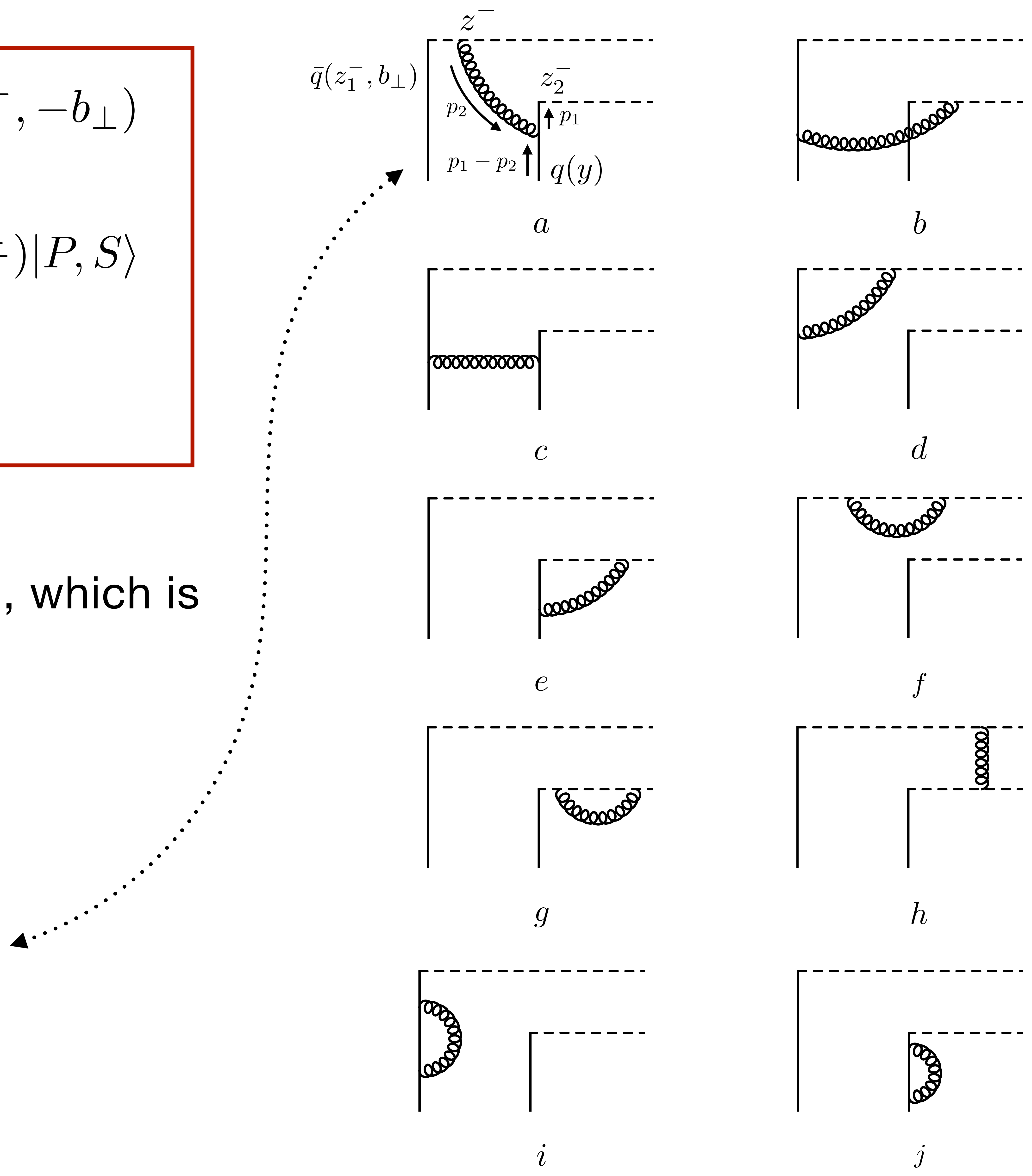
$$\mathcal{U}^{[\gamma^+]}(z_1^-, z_2^-, b_\perp) = \bar{q}(z_1^-, b_\perp)[z_1^-, \infty]_b \gamma^+ [\infty, z_2^-]_{-b}\, q(z_2^-, -b_\perp)$$

$$\Phi^{[\gamma^+]}(x, b_\perp) = \int \frac{dz^-}{2\pi} e^{-2iz^- xP^+} \langle P, S|\mathcal{U}^{[\gamma^+]}(z^-, -z^-, \frac{b_\perp}{2})|P, S\rangle$$

$$\Phi^{[\gamma^+]}(x, b_\perp) = f_1(x, b_\perp) + i b_k \tilde{s}^k M f_{1T}^{\perp}(x, b_\perp)$$

- We integrate over B^{q} fields at the NLO order in a background field, e.g. $q(y)$ field in the diagram (a), which is defined by the IR scales (μ_{IR}^2, ρ)





The structure of the result

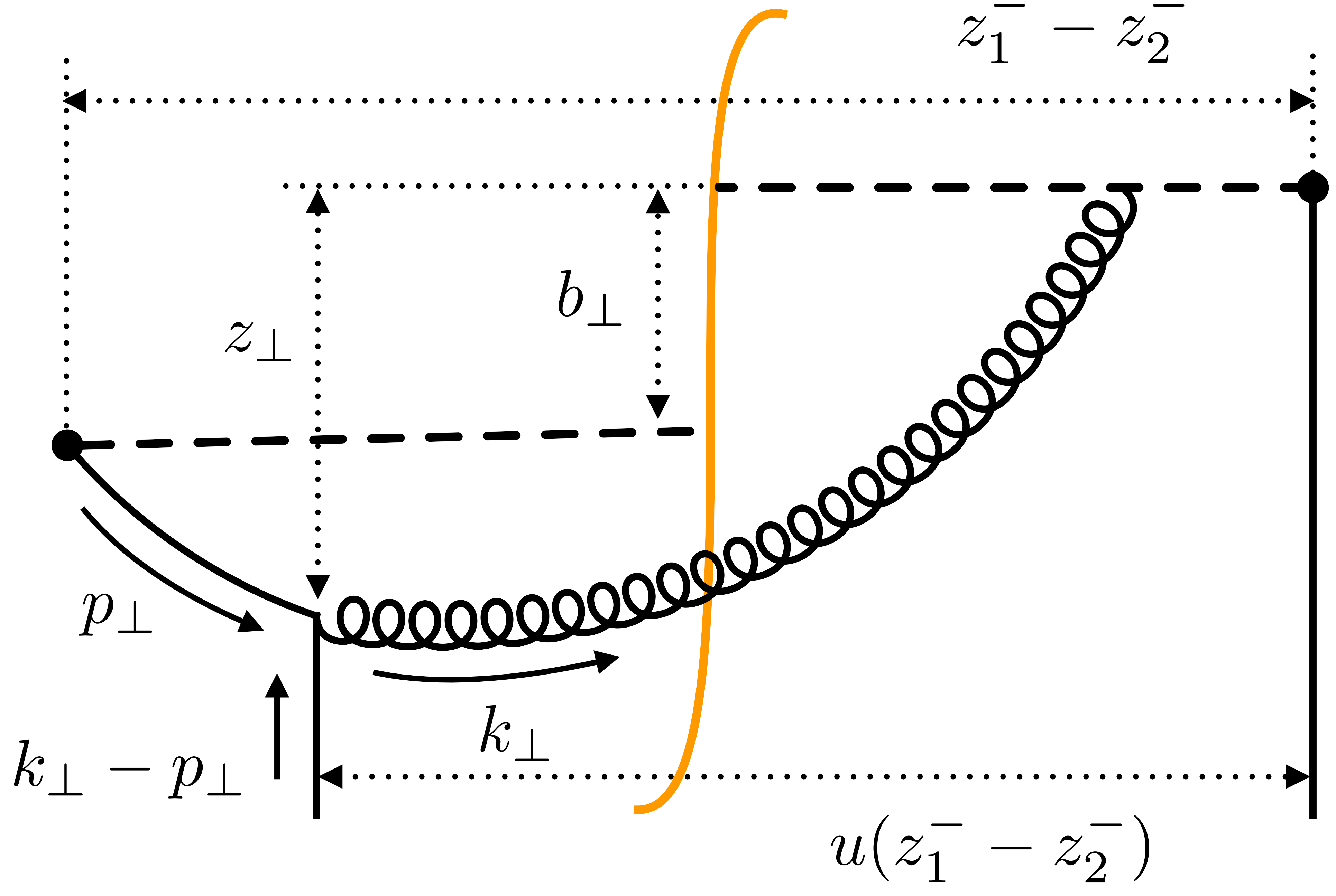
$$\mathcal{U}^{[\gamma^+]}(z_1^-, z_2^-, \frac{1}{2}b_\perp)\Big|_a = \frac{g^2 C_F}{2\pi}\int d^2 z_\perp \int {\mathchar'26\mkern-12mu d}^2 p_\perp e^{ip_\perp b_\perp} \int {\mathchar'26\mkern-12mu d}^2 k_\perp e^{i(k_\perp - p_\perp)z_\perp}$$

$$\times \int_0^1 du \left[\frac{u}{1-u}\right]_+ \frac{1}{(1-u)p_\perp^2 + uk_\perp^2}\mathcal{U}^{[\gamma^+]}\left(uz_1^-, uz_2^-, \frac{1}{2}z_\perp\right) + \frac{g^2\mu_0^{2\epsilon}C_F}{2\pi}\int \frac{{\mathchar'26\mkern-12mu d}^{2-2\epsilon}k_\perp}{k_\perp^2}e^{ik_\perp b_\perp}\int_0^1 du\frac{u}{1-u}\,\mathcal{U}^{[\gamma^+]}\left(z_1^-, z_2^-, \frac{1}{2}b_\perp\right)$$

finite term regulated by the non-zero value of the background transverse momentum $k_\perp - p_\perp$

TMD operator of size $z_\perp$, instead of its collinear counterpart

usual UV divergencies leading to the CSS evolution



$$\int_0^1 \frac{du}{1-u} \to \left(\frac{\nu}{x_B P^+}\right)^\eta \int_0^1 \frac{du}{1-u}\left(\frac{1-u}{u}\right)^{-\eta}$$

η is a regulator of the UV rapidity divergence and ν is the corresponding scale

- Similar expressions can be easily obtained for all other diagrams

The structure of the result. Distribution functions

$$\Phi^{[\gamma^+]}(x,b_\perp)\Big|_a = \frac{\alpha_s C_F}{2\pi}\Big(\frac{1}{\eta}+1+\ln(\frac{\nu}{x_B P^+})\Big)\Big(\frac{1}{\epsilon_{UV}}+L_b\Big)\Phi^{[\gamma^+]}(x,b_\perp) \longleftarrow$$

usual UV divergencies leading to the CSS evolution

$$+\,2\alpha_s C_F\int d^2z_\perp \int {\mathchar'26\mkern-12mu d}^2p_\perp e^{ip_\perp b_\perp}\int {\mathchar'26\mkern-12mu d}^2k_\perp e^{i(k_\perp-p_\perp)z_\perp}\int_0^1\frac{du}{u}\Big[\frac{u}{1-u}\Big]_+ \frac{1}{(1-u)p_\perp^2+uk_\perp^2}\Phi^{[\gamma^+]}(\frac{x}{u},z_\perp)$$

finite terms don’t develop a singularity $1/\epsilon_{IR}$ and doesn’t have logarithms $L_b^{\mu_{IR}} \equiv \ln\Big(\frac{b_\perp^2\mu_{IR}^2}{4e^{-2\gamma_E}}\Big)$ ↑

- The UV poles can be removed in the usual way by the soft factor and the UV renormalization factor

$$S^q = \frac{\alpha_s C_F}{2\pi}\Big[\frac{2}{\epsilon_{UV}^2}+4\Big(\frac{1}{\epsilon_{UV}}+L_b^{\mu_{UV}}\Big)\Big(-\frac{1}{\eta}+\ln\frac{\mu_{UV}}{\nu}\Big)-L_b^{\mu_{UV}\,2}-\frac{\pi^2}{6}\Big] \longleftarrow$$

soft factor

UV renormalization ⟶ $Z_{UV} = 1-\frac{\alpha_s C_F}{2\pi}\Big[\frac{1}{\epsilon_{UV}^2}+\frac{1}{\epsilon_{UV}}\Big(\frac{3}{2}+\ln(\frac{\mu_{UV}^2}{(x_B P^+)^2})\Big)\Big]$

- The result is the same for both unpolarized and Sivers functions

$$\Phi^{[\gamma^+]}(x,b_\perp) = f_1(x,b_\perp)+ib_k\tilde{s}^k M f_{1T}^\perp(x,b_\perp)$$

- The second term describes all collinear twist content of the TMDPDFs. Let’s see it more explicitly

Collinear matching procedure

The result for the diagram in the MSTT factorization scheme:

$$\mathcal{U}^{[\gamma^+]}(z_1^-, z_2^-, \frac{1}{2}b_\perp)\Big|_a = \frac{\alpha_s C_F}{2\pi}\Big(\frac{1}{\eta} + 1 + \ln(\frac{\nu}{x_B P^+})\Big)\Big(\frac{1}{\epsilon_{UV}} + L_b^{\mu_{UV}}\Big)\mathcal{U}^{[\gamma^+]}(z_1^-, z_2^-, \frac{1}{2}b_\perp)$$

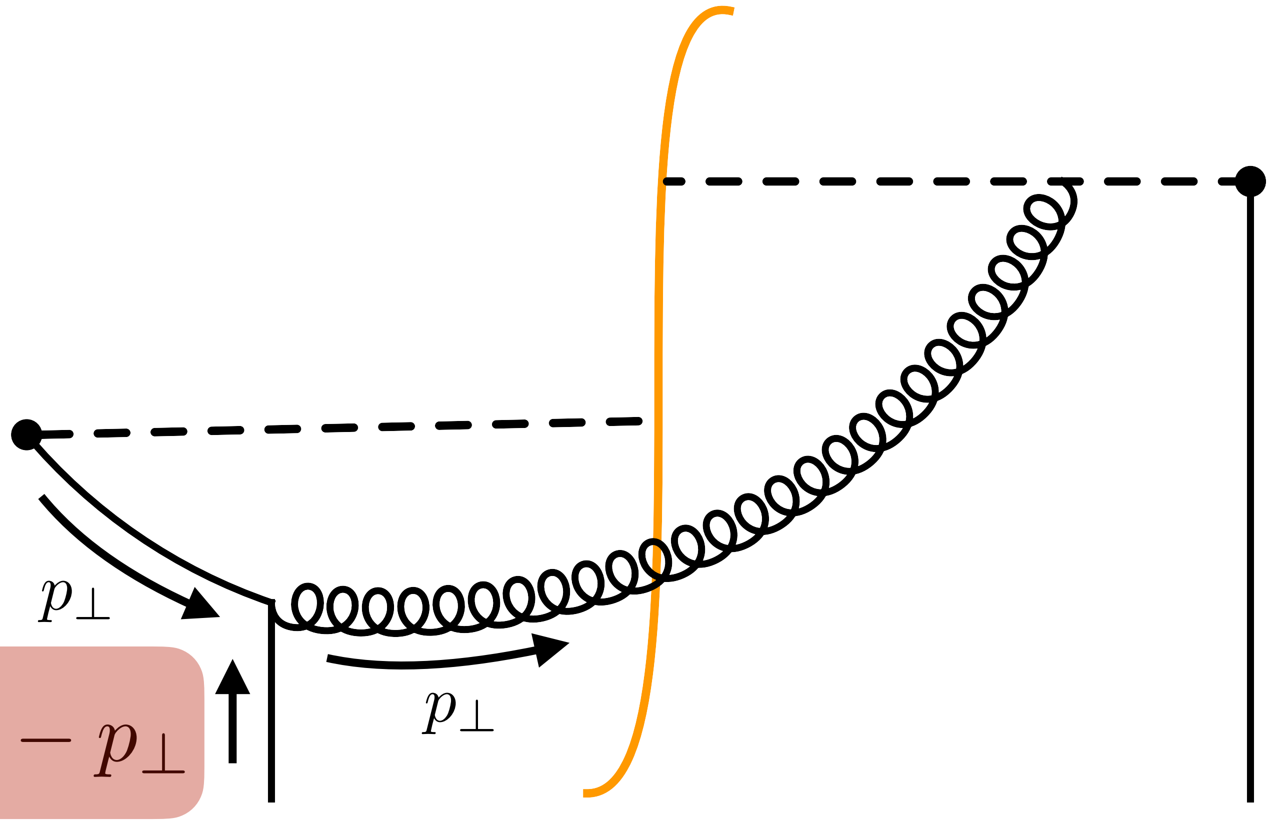
$$+ 2\alpha_s C_F \int d^2 z_\perp \int \text{đ}^2 p_\perp e^{ip_\perp b_\perp} \int \text{đ}^2 k_\perp e^{i(k_\perp - p_\perp) z_\perp} \int_0^1 du \Big[\frac{u}{1-u}\Big]_+ \frac{1}{(1-u)p_\perp^2 + u k_\perp^2}\, \mathcal{U}^{[\gamma^+]}(uz_1^-, uz_2^-, \frac{1}{2}z_\perp)$$

no ordering in transverse variables

- How is the result related to the collinear matching expansion? Let's expand the TMD operators in r.h.s. in terms of **collinear operators**:

$$\mathcal{U}^{[\gamma_\mu]}(uz_1^-, uz_2^-, \frac{1}{2}z_\perp) = \mathcal{O}^{[\gamma_\mu]}(uz_1^-, uz_2^-) + \frac{1}{2}z_k \mathcal{O}^k_{[\gamma_\mu]}(uz_1^-, uz_2^-) + \dots$$

- Each term of the expansion develops a new singularity and a delta-function for the background transverse momentum



$0_\perp = k_\perp - p_\perp$

$$\mathcal{U}^{[\gamma^+]}(z_1^-, z_2^-, \frac{1}{2}b_\perp)\Big|_a = \frac{\alpha_s C_F}{2\pi}\Big(\frac{1}{\eta} + 1 + \ln(\frac{\nu}{x_B P^+})\Big)\Big(\frac{1}{\epsilon_{UV}} + L_b^{\mu_{UV}}\Big)\mathcal{O}^{[\gamma^+]}(z_1^-, z_2^-)$$

$$+ 2\alpha_s C_F \int_0^1 du \Big[\frac{u}{1-u}\Big]_+ \int \frac{\text{đ}^2 p_\perp}{p_\perp^2} e^{ip_\perp b_\perp} \int \text{đ}^2 k_\perp (2\pi)^2 (k_\perp - p_\perp)\mathcal{O}^{[\gamma^+]}(uz_1^-, uz_2^-)$$

The collinear matching expansion introduces an additional ordering in transverse momentum between the quantum modes and the background

Ordering of transverse momenta in the collinear matching procedure

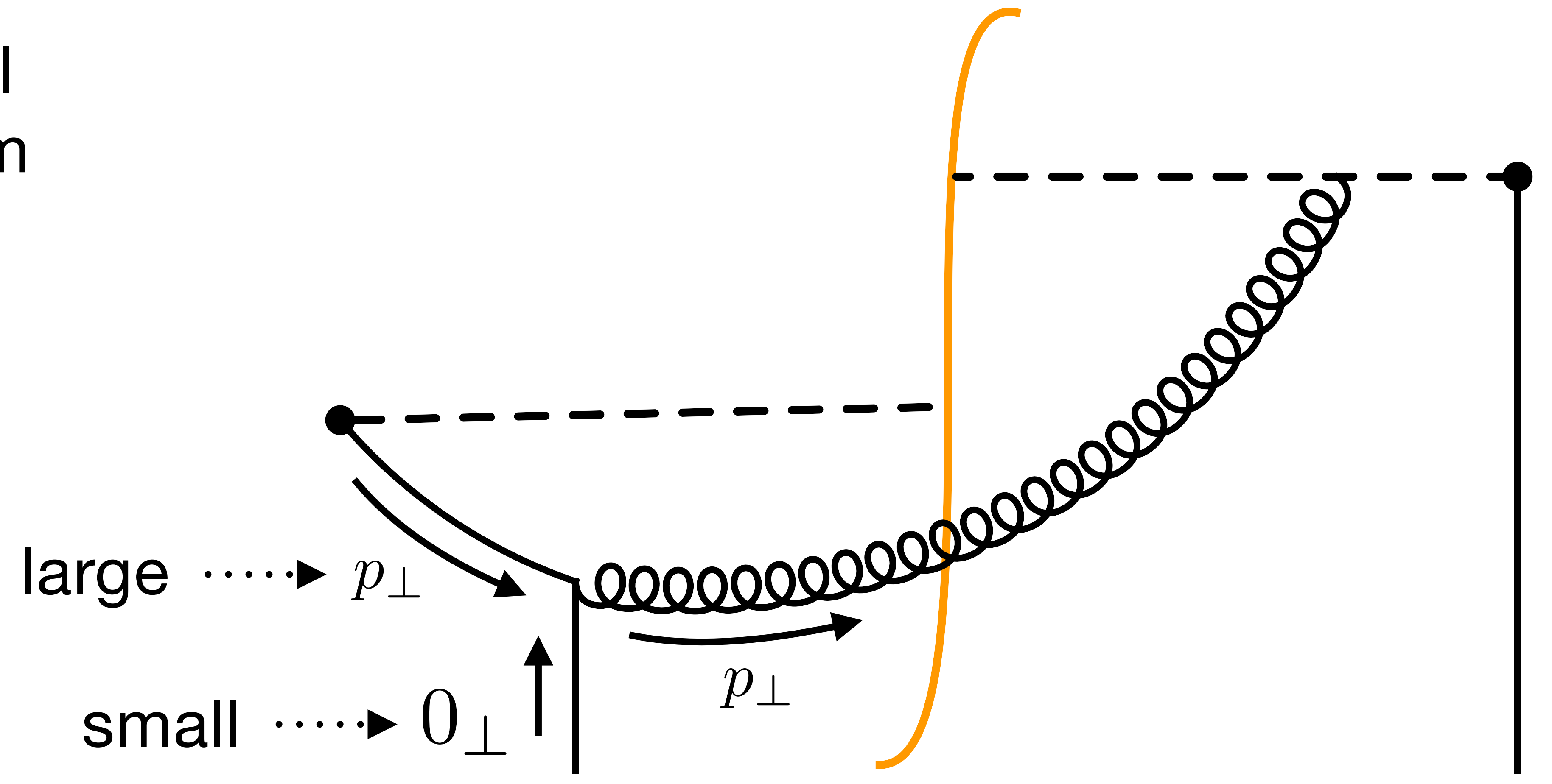
$$\mathcal{U}^{[\gamma^+]}(z_1^-, z_2^-, \frac{1}{2}b_\perp)\Big|_a = \frac{\alpha_s C_F}{2\pi}\Big(\frac{1}{\eta} + 1 + \ln(\frac{\nu}{x_B P^+})\Big)\Big(\frac{1}{\epsilon_{UV}} + L_b^{\mu_{UV}}\Big)\mathcal{O}^{[\gamma^+]}(z_1^-, z_2^-)$$

$$- \frac{\alpha_s C_F}{2\pi}\Big(\frac{1}{\epsilon_{IR}} + L_b^{\mu_{IR}}\Big)\int_0^1 du \left[\frac{u}{1-u}\right]_+ \mathcal{O}^{[\gamma^+]}(uz_1^-, uz_2^-) + \mathcal{O}(b)$$

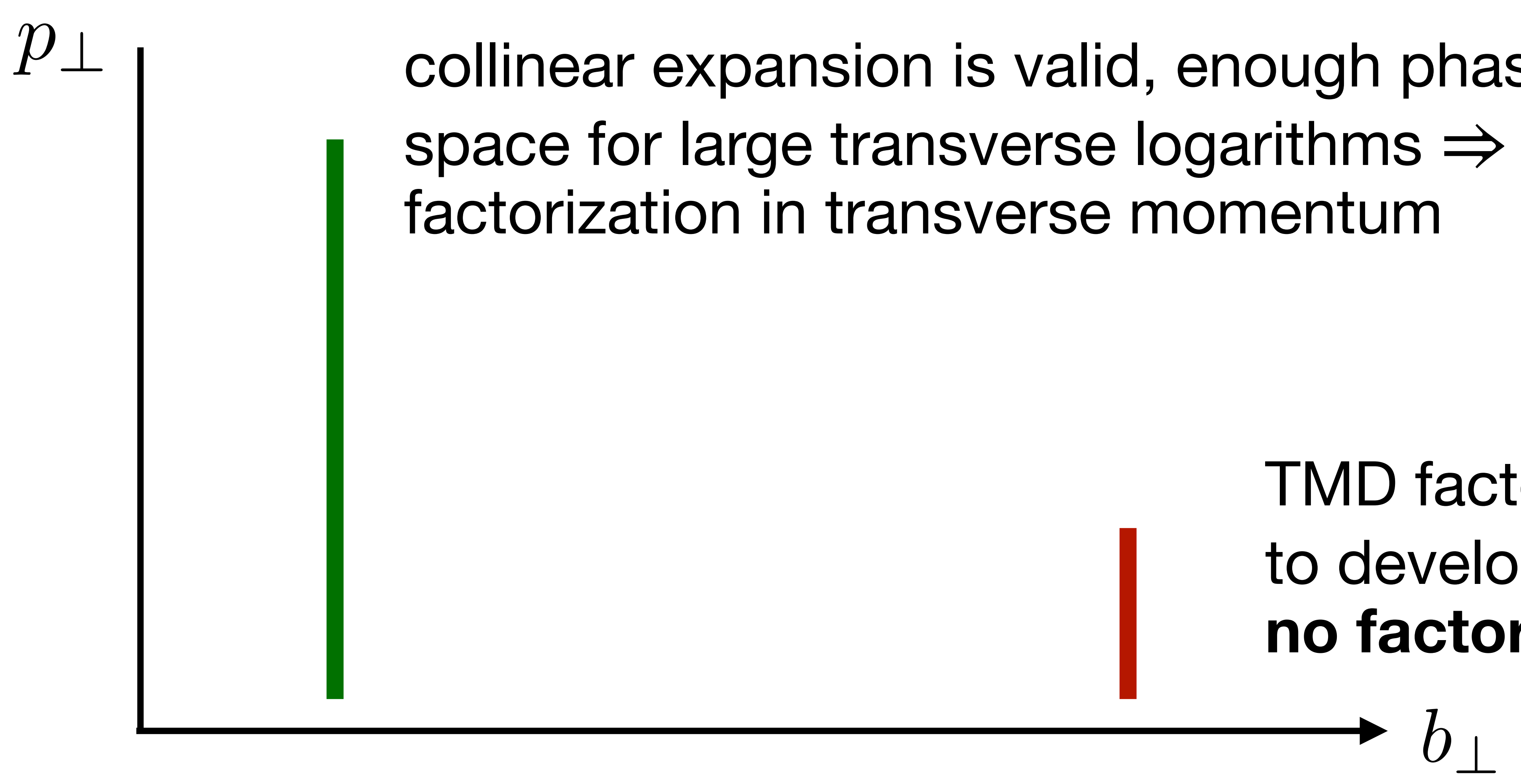
Part of the DGLAP kernel generated by the diagram

IR singularity in all terms of the expansion onto the collinear PDFs ⇒ factorization

- The presence of a new $1/\epsilon_{IR}$ manifests the fact that the collinear matching procedure introduces a new factorization condition defined by the strong ordering of the transverse momentum
- The exact MSTT result doesn't have this ordering, i.e. the factorization condition for transverse momentum
- The ordering is justified only in the region of small $b_\perp \ll \Lambda_{QCD}^{-1}$



wide separation in transverse momentum ⇒ factorization



collinear expansion is valid, enough phase space for large transverse logarithms ⇒ factorization in transverse momentum

TMD factorization is valid, no phase space to develop large transverse logarithms ⇒ **no factorization** in transverse momentum

Leading collinear twist

$$\mathcal{U}^{[\gamma^+]}(z_1^-, z_2^-, \frac{1}{2}b_\perp)\Big|_a = \frac{\alpha_s C_F}{2\pi}\Big(\frac{1}{\eta} + 1 + \ln(\frac{\nu}{x_B P^+})\Big)\Big(\frac{1}{\epsilon_{UV}} + L_b^{\mu_{UV}}\Big)\mathcal{U}^{[\gamma^+]}(z_1^-, z_2^-, \frac{1}{2}b_\perp)$$
$$+ 2\alpha_s C_F \int d^2 z_\perp \int \bar{d}^2 p_\perp e^{ip_\perp b_\perp} \int \bar{d}^2 k_\perp e^{i(k_\perp - p_\perp)z_\perp} \int_0^1 du \Big[\frac{u}{1-u}\Big]_+ \frac{1}{(1-u)p_\perp^2 + uk_\perp^2}\mathcal{U}^{[\gamma^+]}(uz_1^-, uz_2^-, \frac{1}{2}z_\perp)$$

- The MSTT result develops exact logarithms when the factorization condition for transverse momentum is imposed in the region of small $b_\perp \ll \Lambda_{QCD}^{-1}$
- The result matches IR logarithms of **all order terms** of the collinear expansion

$$f_1(x, b_\perp)\Big|_a = \frac{\alpha_s C_F}{2\pi}\Big(\frac{1}{\eta} + 1 + \ln(\frac{\nu}{x_B P^+})\Big)\Big(\frac{1}{\epsilon_{UV}} + L_b^{\mu_{UV}}\Big)f_1(x)$$
$$- \frac{\alpha_s C_F}{2\pi}\Big(\frac{1}{\epsilon_{IR}} + L_b^{\mu_{IR}}\Big)\int_0^1 \frac{du}{u}\Big[\frac{u}{1-u}\Big]_+ f_1(\frac{x}{u}) + \mathcal{O}(b^2)$$

in full agreement with the known result for the collider matching

MSTT contains contribution of the higher order collinear twists as well

note that the expansion converge only at small $b_\perp \ll \Lambda_{QCD}^{-1}$

- The MSTT result perfectly matches with known terms of the collinear expansion but contains contribution of all collinear terms which is essential for the region of validity of the TMD factorization large $b_\perp \lesssim \Lambda_{QCD}^{-1}$

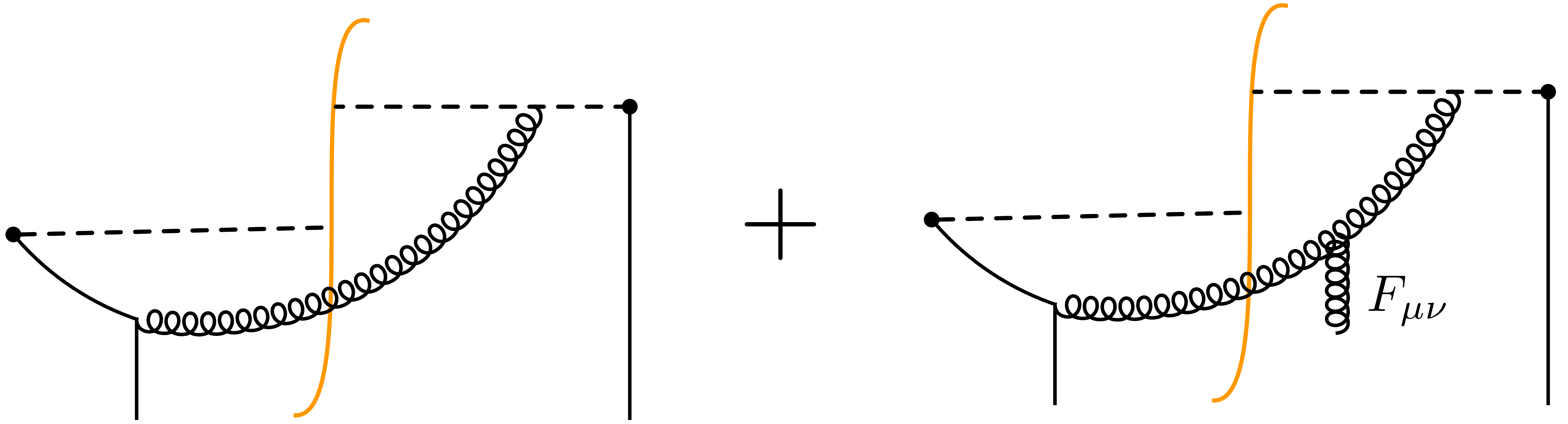
Higher collinear twists

- The MSTT result for the sum of all diagrams partially reproduces contribution of higher order collinear twists, but contain contribution of all of them

$$f_{1T}^{\perp}(x,b_\perp)\Big|_a = \frac{\alpha_s C_F}{2\pi}\Big(\frac{1}{\eta}+1+\ln(\frac{\nu}{x_B P^+})\Big)\Big(\frac{1}{\epsilon_{UV}}+L_b^{\mu_{UV}}\Big)\pi T(-x,0,x)$$
$$-\frac{\alpha_s C_F}{2\pi}\Big(\frac{1}{\epsilon_{IR}}+L_b^{\mu_{IR}}\Big)\int_0^1\frac{du}{u}\Big\{\Big[\frac{u}{1-u}\Big]_+ - u\Big\}\pi T(-\frac{x}{u},0,\frac{x}{u}) + \mathcal{O}(b^2)$$

in full agreement with the known result for the contribution of this diagram to the collider matching result for the Sivers function

- The MSTT result contains only mixing with leading twist TMDPDFs. To reproduce higher collinear twist terms one has to consider mixing with higher twist TMD operators



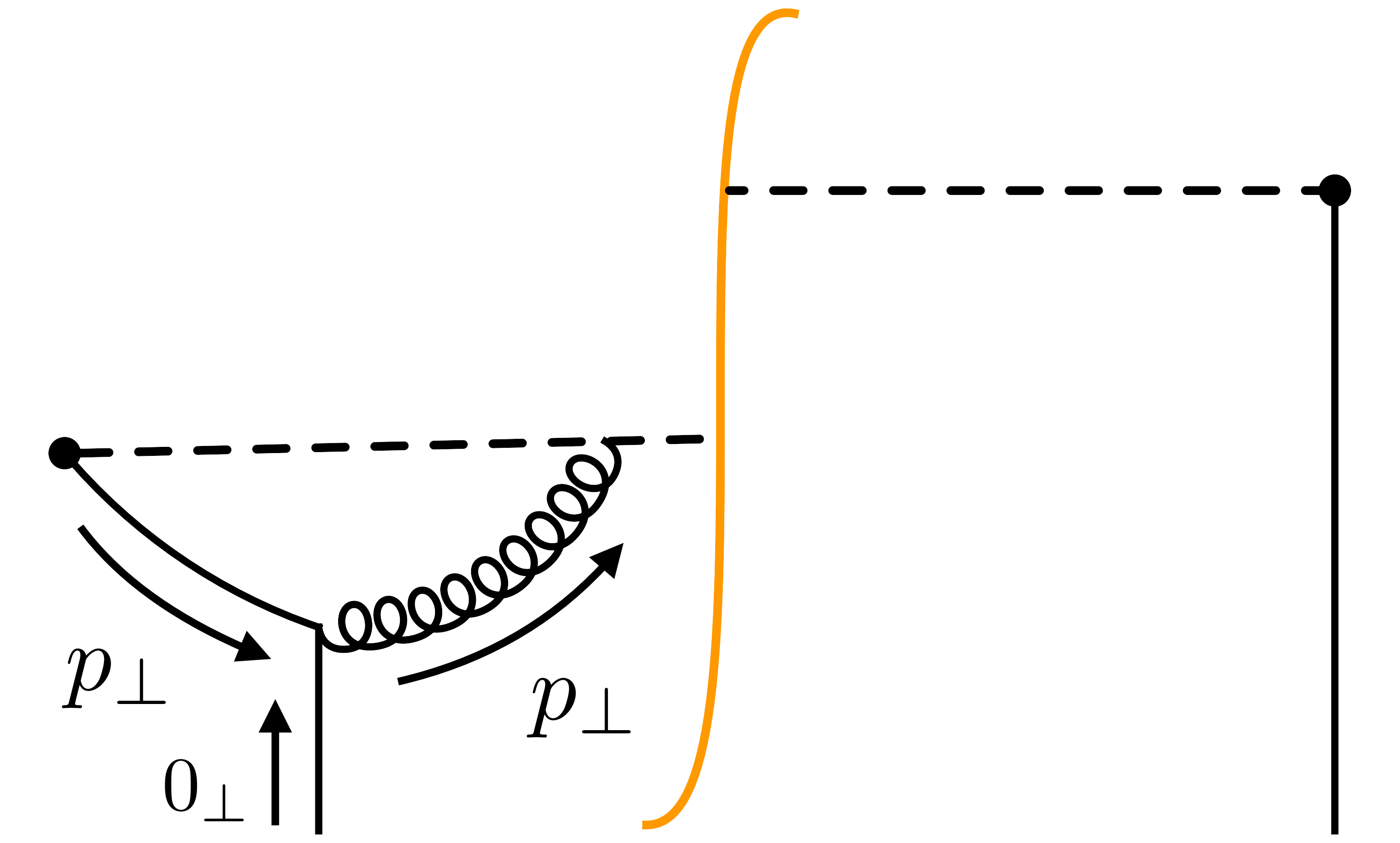
mixing with higher twist TMD operators can be systematically analyzed as well

Mukherjee, Skokov, Tarasov, Tiwari, arXiv:2502.15889

- The result of the collinear matching for the Sivers function contain contribution of the mixing with higher twist TMD operators even in the leading term

Virtual diagrams

- virtual diagram in the collinear matching procedure. Expanding the MSTT result:

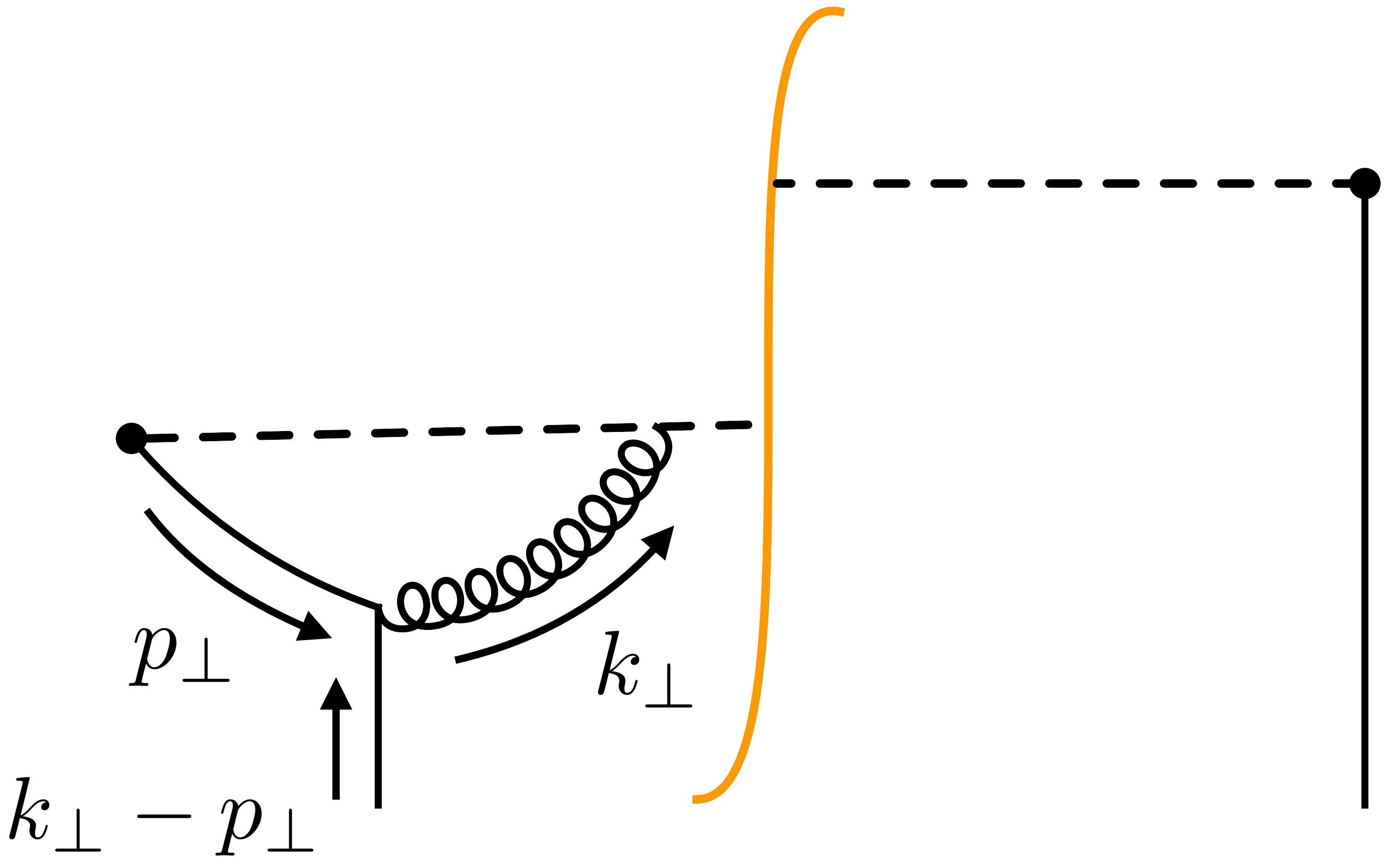


$$\mathcal{U}^{[\gamma^+]}(z_1^-, z_2^-, \frac{1}{2}b_\perp)\Big|_e = -\frac{g^2\mu_0^{2\epsilon}C_F}{2\pi}\int \frac{\bar{d}^{\,2-2\epsilon}k_\perp}{k_\perp^2}\int_0^1 du\,\frac{u}{1-u}\Big(\mathcal{O}^{[\gamma^+]}(z_1^-, z_2^-) + \dots\Big)$$

scaleless integral $\frac{1}{\epsilon_{UV}} - \frac{1}{\epsilon_{IR}} = 0$

- in the collinear matching, when the factorization in transverse momentum is imposed, **all terms** of the collinear expansion are trivial in agreement with the standard result

- In the MSTT approach, without the factorization in transverse momentum the diagram generates a finite term



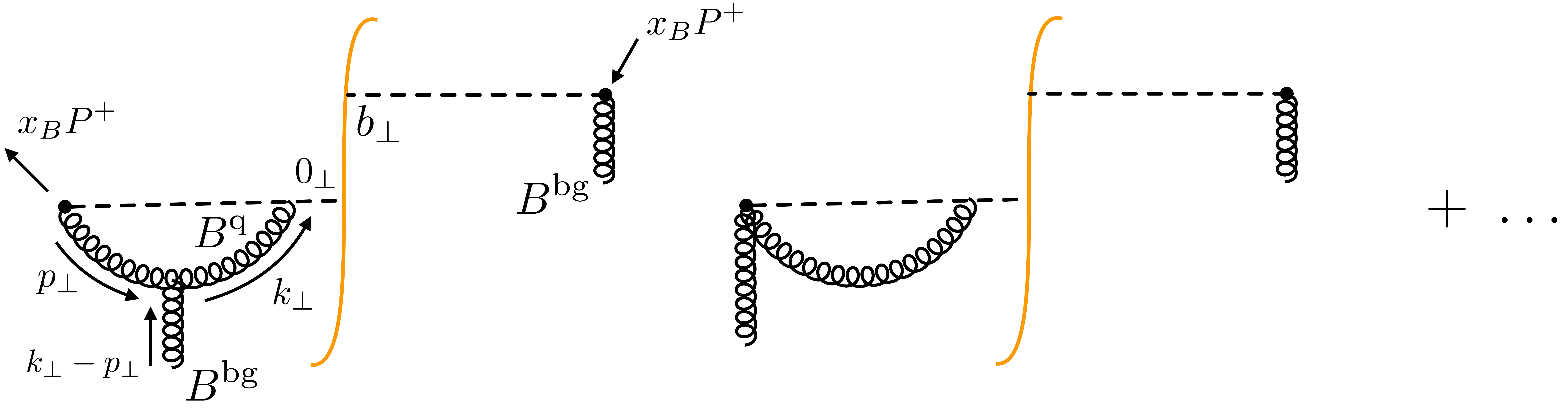
$$\mathcal{U}^{[\gamma^+]}(z_1^-, z_2^-, \frac{1}{2}b_\perp)\Big|_e = -\frac{g^2\mu_0^{2\epsilon}C_F}{2\pi}\int \frac{\bar{d}^{\,2-2\epsilon}k_\perp}{k_\perp^2}\int_0^1 du\,\frac{u}{1-u}\,\mathcal{U}^{[\gamma^+]}(z_1^-, z_2^-, \frac{1}{2}z_\perp)$$

$$-\frac{g^2C_F}{2\pi}\int d^2z_\perp \int \bar{d}^{\,2}p_\perp e^{ip_\perp(b_\perp - z_\perp)}\int \bar{d}^{\,2}k_\perp e^{ik_\perp(z_\perp - b_\perp)}$$

$$\times \int_0^1 du\, u\,\frac{k_\perp^2 - p_\perp^2}{k_\perp^2\{(1-u)p_\perp^2 + uk_\perp^2\}}\mathcal{U}^{[\gamma^+]}(z_1^-, z_2^-, \frac{1}{2}z_\perp)$$

non-trivial dynamics in the MSTT factorization!

Virtual diagrams for the gluon TMD operator



- In the conventionally used collinear matching approach these diagrams contain scaleless integrals, which are zero in the dimensional regularization. In the MSTT approach we observe a non-trivial structure

rapidity divergence on the IR type!

$$\mathcal{B}_{ij}^{\mathrm{q}(1)+\mathrm{bg;virt}}(x_B, b_\perp) = -2\alpha_s N_c \int_0^1 \frac{dz}{z} \int \bar{d}^2 p_\perp e^{ip_\perp b_\perp} \int \bar{d}^2 k_\perp e^{-ik_\perp b_\perp} \mathcal{V}_{ij;lm}(z, p_\perp - k_\perp, k_\perp)$$

$$\times \int d^2 z_\perp e^{i(k-p)_\perp z_\perp} \mathcal{B}_{lm}^{\mathrm{bg}}(x_B, z_\perp) - 4\alpha_s N_c \int_0^1 \frac{dz}{1-z} \int \frac{\bar{d}^2 k_\perp}{k_\perp^2} \mathcal{B}_{ij}^{\mathrm{bg}}(x_B, b_\perp)$$

This doesn't depend on the value of x_B

BFKL dynamics

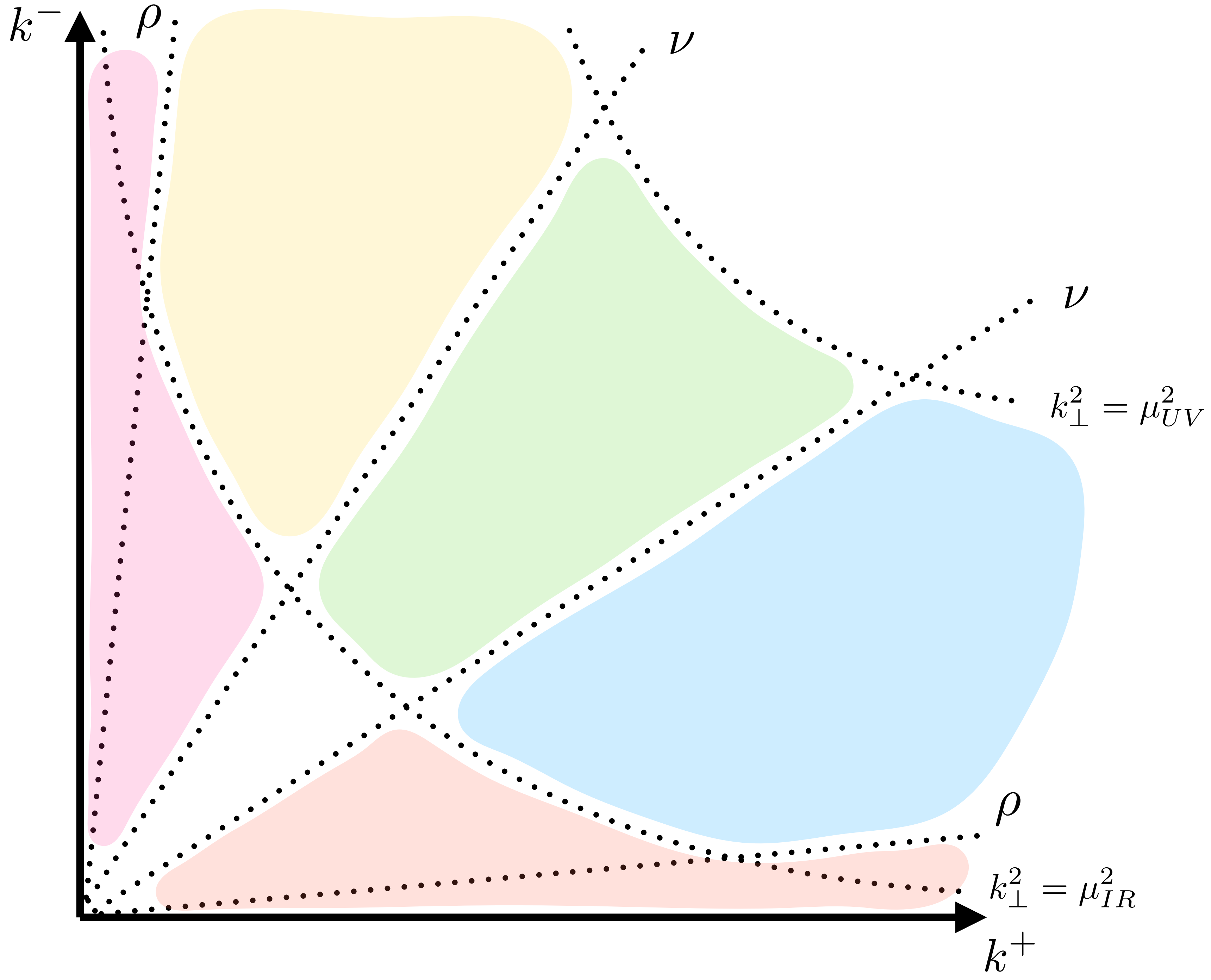
logarithmic dependence on ρ; “virtual” part of the BFKL evolution kernel

$$\mathcal{B}_{ij}^{\mathrm{q}(1)+\mathrm{bg;virt}}(x_B,b_\perp) = -\frac{\alpha_s N_c}{2\pi}\left(\frac{1}{\epsilon_{\mathrm{IR}}^2}+\frac{1}{\epsilon_{\mathrm{IR}}}\left(\frac{1}{\xi}+\ln\left(\frac{\rho}{x_B P^+}\right)\right)-\frac{\pi^2}{12}\right)$$

$$\times \int d^2 z_\perp \int {\mathchar'26\mkern-12mu d}^2 p_\perp e^{ip_\perp(b-z)_\perp}\left(\frac{\mu_{\mathrm{IR}}^2}{p_\perp^2}\right)^{\epsilon_{\mathrm{IR}}} \frac{g_{il}p_jp_m+p_ip_lg_{mj}}{p_\perp^2}\mathcal{B}^{\mathrm{bg}}_{lm}(x_B,z_\perp)$$

$$+\frac{\alpha_s N_c}{2\pi}\left(\frac{1}{\epsilon_{\mathrm{UV}}}\frac{\beta_0}{2N_c}+\frac{67}{18}-\frac{5N_f}{9N_c}\right)\int d^2 z_\perp \int {\mathchar'26\mkern-12mu d}^2 p_\perp e^{ip_\perp(b-z)_\perp}\left(\frac{\mu_{\mathrm{UV}}^2}{p_\perp^2}\right)^{\epsilon_{\mathrm{UV}}}\mathcal{B}^{\mathrm{bg}}_{ij}(x_B,z_\perp)$$

- We observe that the virtual diagrams have a non-trivial structure. This is different from the conventional approach
- The gluon TMDPDFs at **large** x_B contain logarithms of the BFKL type!
- To reveal this structure one has to take into account all-collinear twist content of the TMDPDFs in the region of large $b_\perp \lesssim \Lambda_{QCD}^{-1}$ using the MSTT approach



Thank you for your attention!