# Resummation of Flattened Jet Angularity Using Soft-Collinear Effective Theory

Yang-Ting Chien

QCD Evolution workshop, JLab
May 19th, 2025

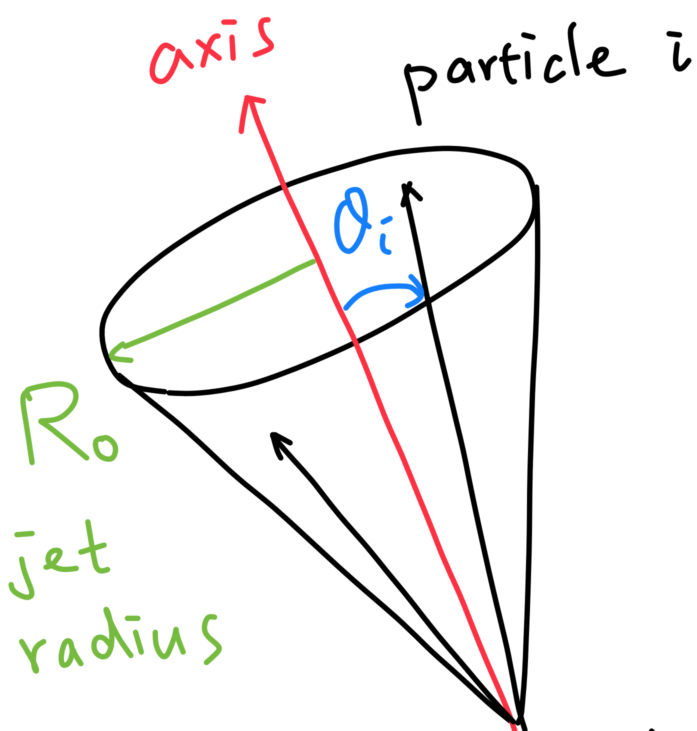
# Outline

- Introduction
  - Jet angularity and soft-drop grooming
  - Collinear-drop and flattened jet angularity
- Soft Collinear Effective Theory
  - Factorization and resummation
- Monte Carlo comparison
- Conclusion

# Jet angularity

Berger, Kucs, Sterman, PRD 68 (2003) 014012 ⟵ Introduced 22 years ago

Larkoski, Thaler, Waalewijn, JHEP 11 (2014) 129 ⟵ Generalized angularity



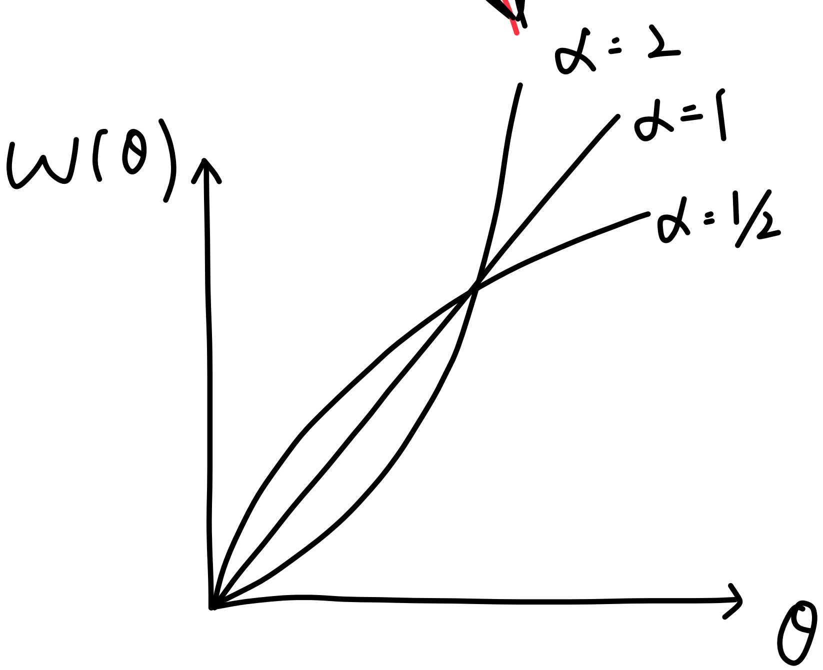

$$\lambda_\alpha^\kappa = \sum_{i \in jet} z_i^\kappa \boxed{\left(\frac{\theta_i}{R_0}\right)^\alpha} \quad w(\theta)$$

$\kappa = 1$ for IRC safety

$\alpha > 0$

momentum fraction $\quad z_i = \frac{p_t^{\,i}}{p_t^{\,jet}}$

SUM over per particle contribution weighed by angles ⇒ angularity



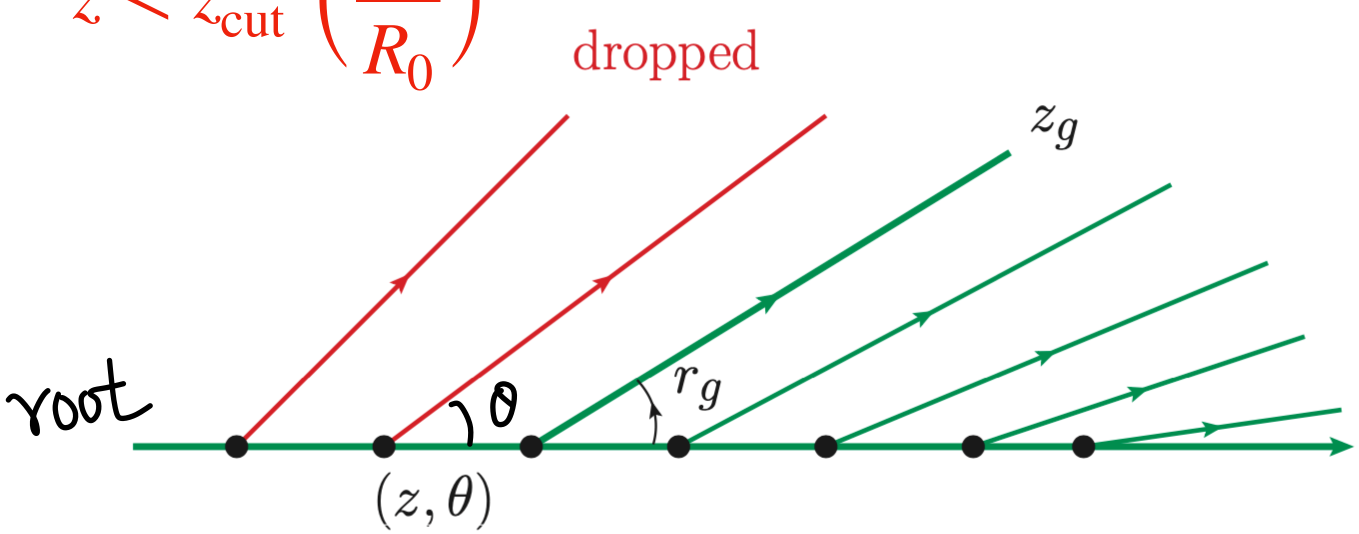

For $\alpha > 0$, $w(\theta)$ is monotonic therefore weighs wide angle particles the most. Suffers from contaminations such as underlying events, pileup, or even event-wide radiation

theoretical control challenging!

# Soft-drop grooming

Larkoski, Marzani, Soyez, Thaler, JHEP05(2014)146

$$z < z_{\mathrm{cut}} \left(\frac{\theta}{R_0}\right)^{\beta}$$ ← soft-drop condition




Soft-drop is one of the grooming techniques which can be calculated to high precision

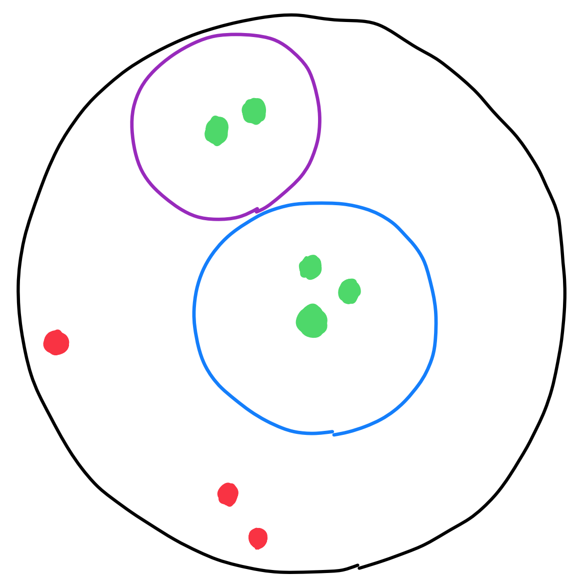
Soft-drop removes wide-angle, soft particles

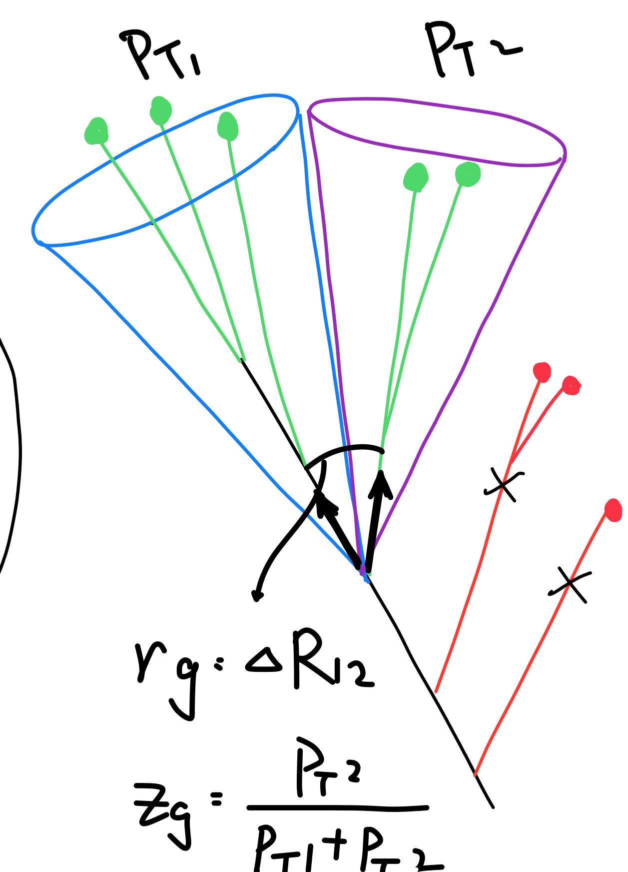
Procedure:

* Recluster jet using Cambridge-Aachen algorithm → angular ordering
* Starting from the "root" and check the i, j merging satisfy soft-drop condition $z < z_{cut}\left(\frac{\theta}{R_0}\right)^{\beta}$

Yes → drop the soft branch and move on to the hard branch

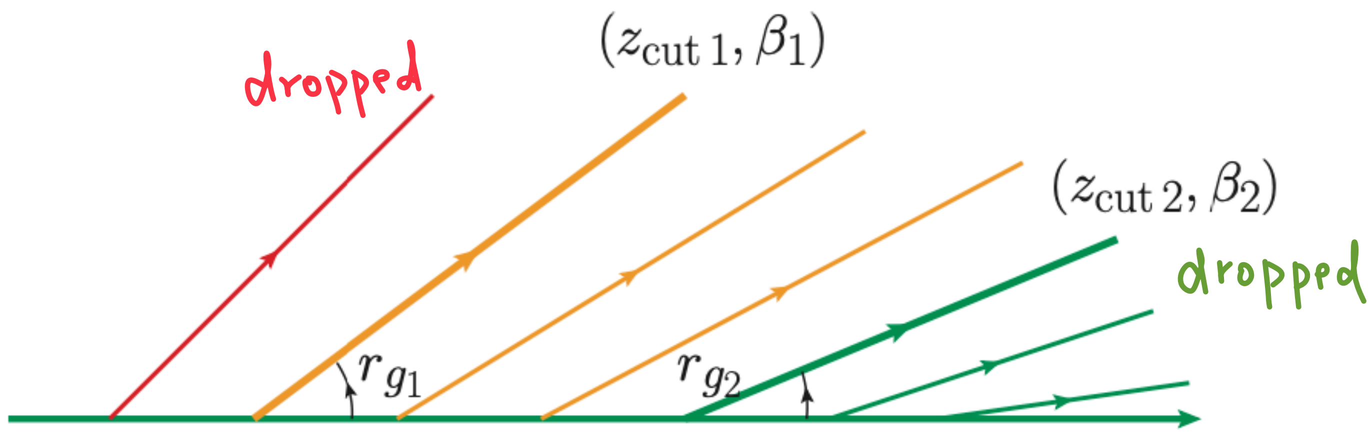
No → stop

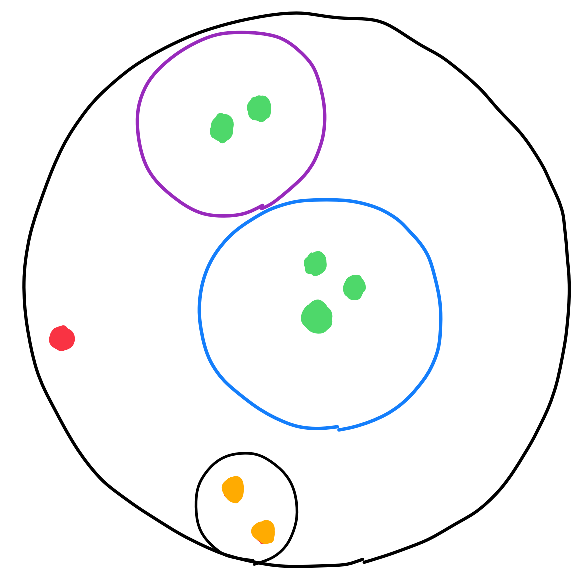


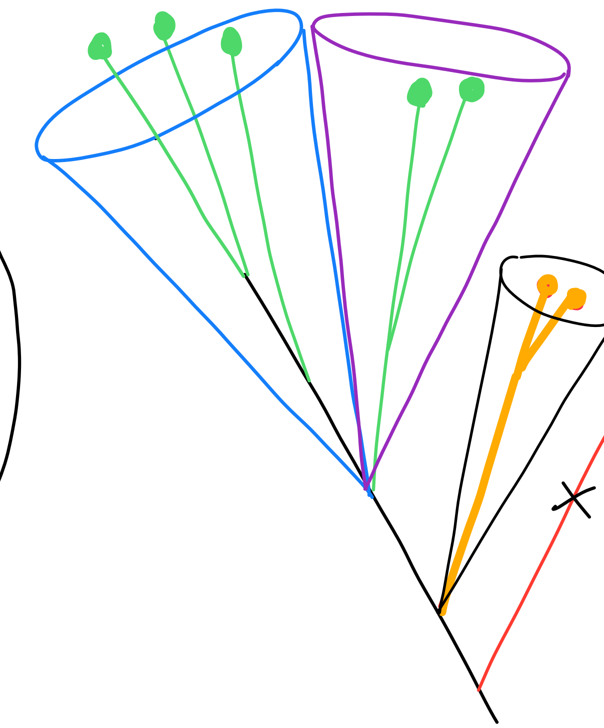


# Collinear-drop

Chien, Stewart, JHEP06(2020)064

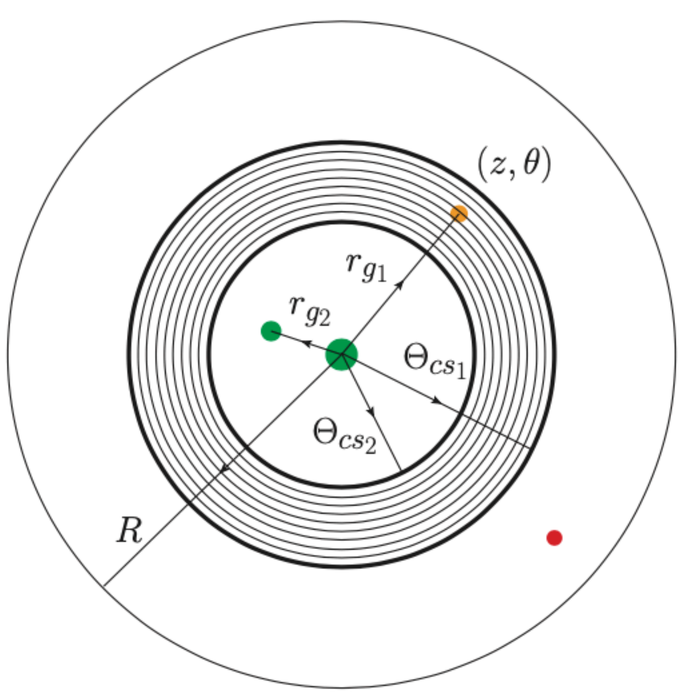




$$z_{cut_1}\left(\frac{\theta}{R_0}\right)^{\beta_1} < z < z_{cut_2}\left(\frac{\theta}{R_0}\right)^{\beta_2}$$

Collinear drop condition




- Conventionally only particles surviving soft drop are studied. However, one could study the dropped particles as well
- One could even pick out an intermediate branch with two soft drop conditions

An extension of soft drop mass is the collinear drop mass

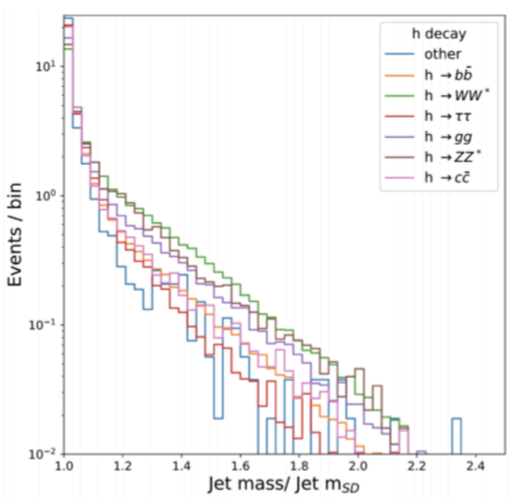
$$\boxed{\Delta m^2 = m^2_{SD_1} - m^2_{SD_2}}$$

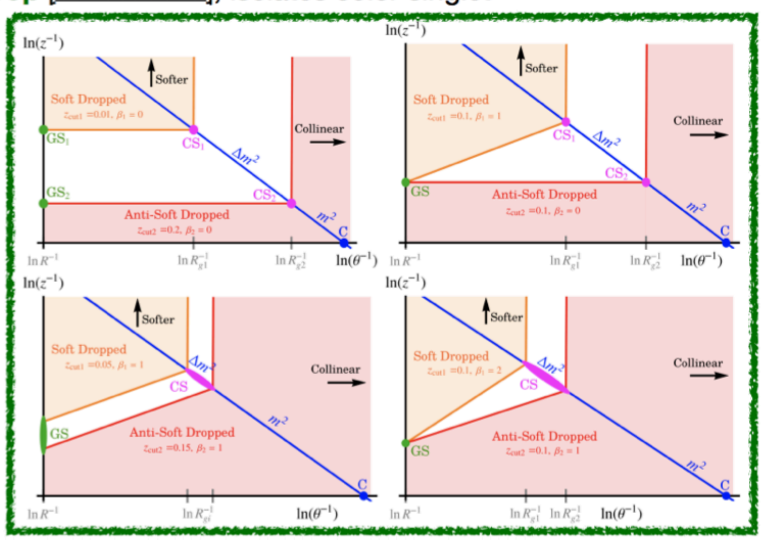
# Useful application

D. Rankin's talk at ML4Jets 2020, 1910.02082

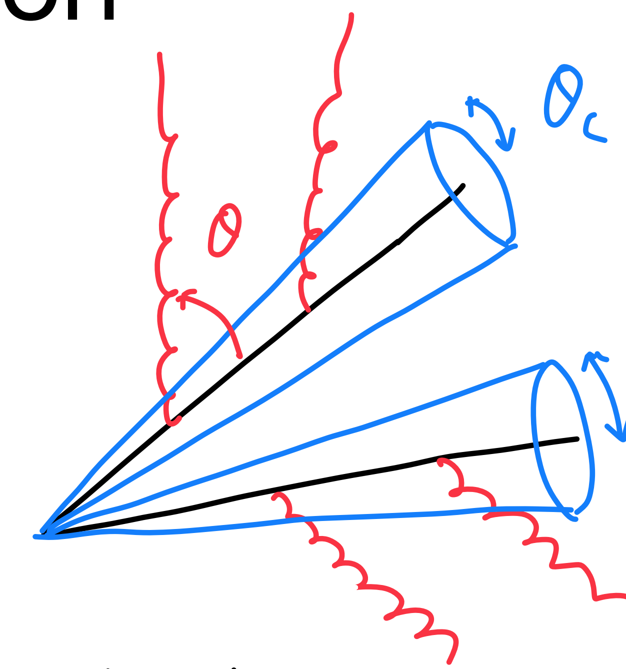
## Higgs Tagging - gg

- Particle content of h→gg jet similar to QCD jet
  - Dedicated GRU struggles to differentiate
- MLP with expert features - *jet mass ratios*
  - Similar concept to **collinear drop** [1907.11107], isolates color singlet




$\theta_c$ : medium resolution angle




Medium-induced, color decoherent radiation is not angular-ordered and may just be soft-dropped.

Color-singlet jet isolation

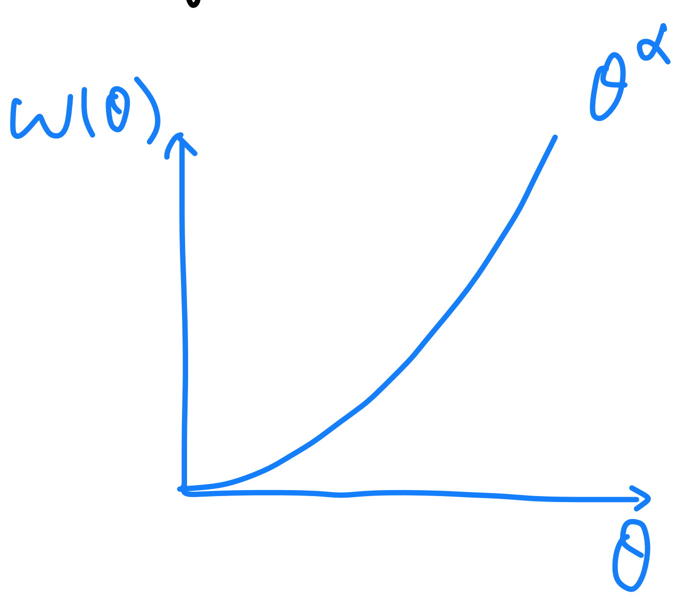
Chien et al PRD 101 (2020) 11, 114006
1711.11041
Telescoping jet substructure

# Flattened jet angularity

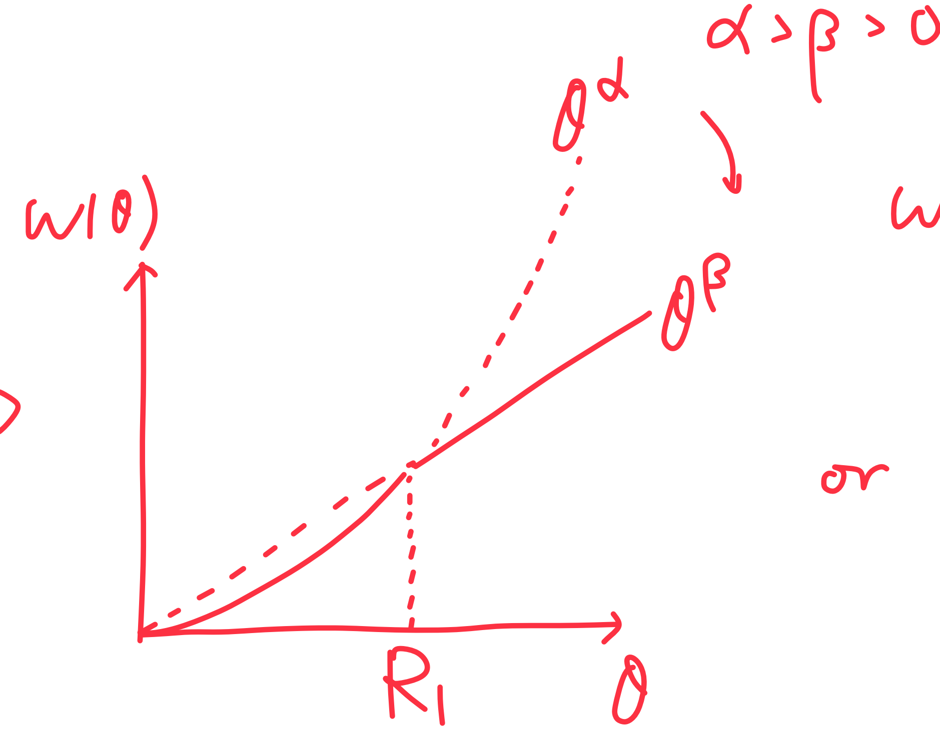
While groomed observables are mostly constructed using clustering algorithms, which often complicate theoretical calculations, in this talk we introduce a jet-shape based technique to suppress wide-angle radiation

Flattened jet angularity generalize the functional form of $w(\theta)$

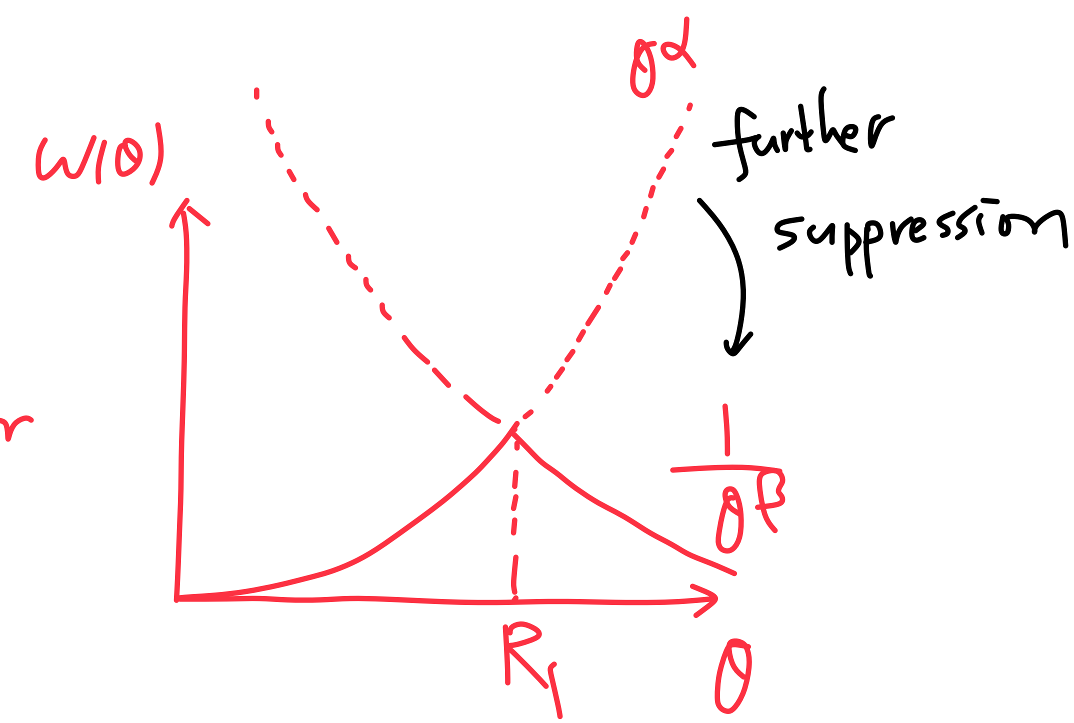
$$\sum_{i \in \text{jet}} z_i \, w(\theta)$$




$\Rightarrow$

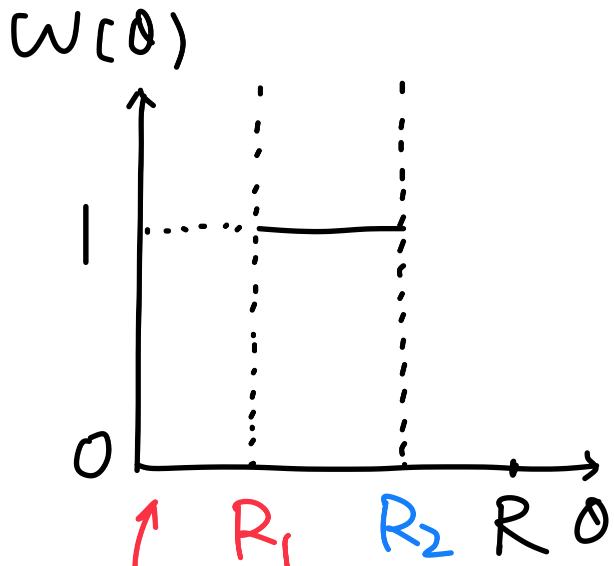



or



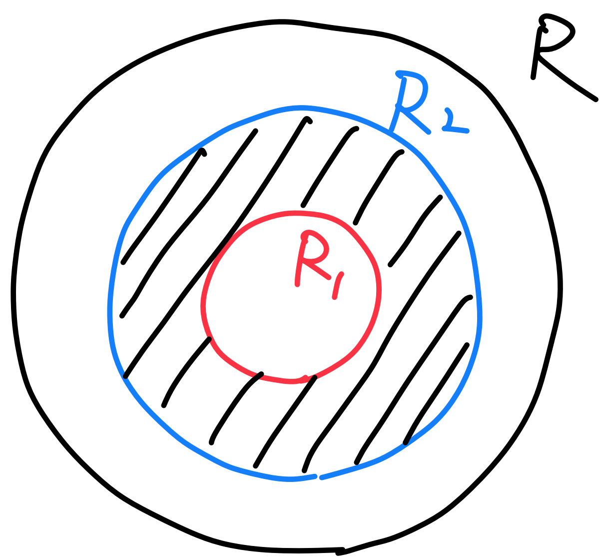

piecewise polynomial

# Flattened jet angularity

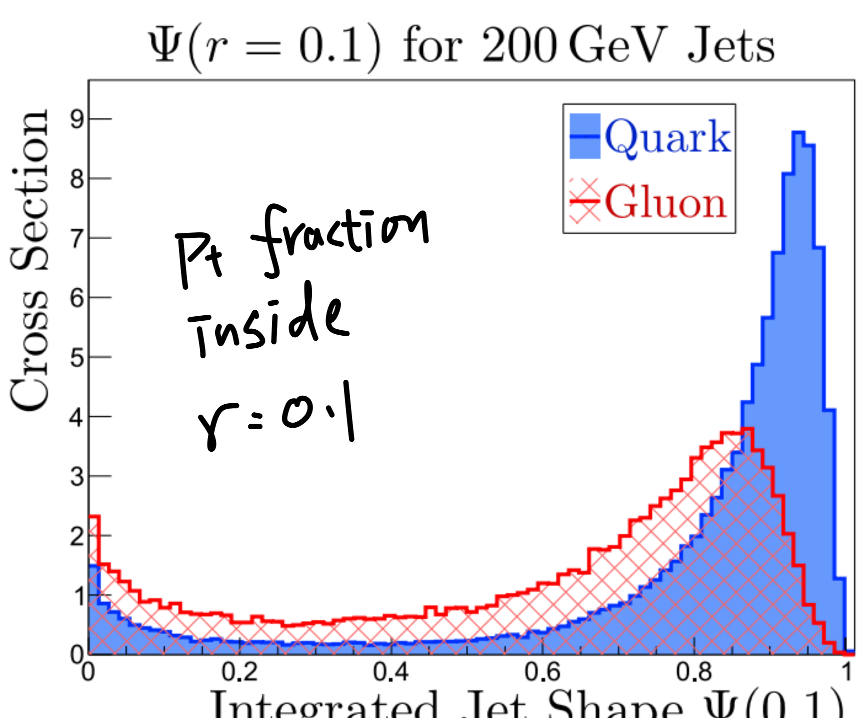



another implementation of collinear drop

Annulus $p_t$ fraction $\chi$



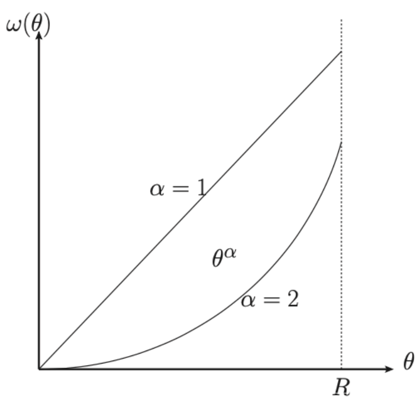

Similar observable: "color ring" for boosted boson tagging.

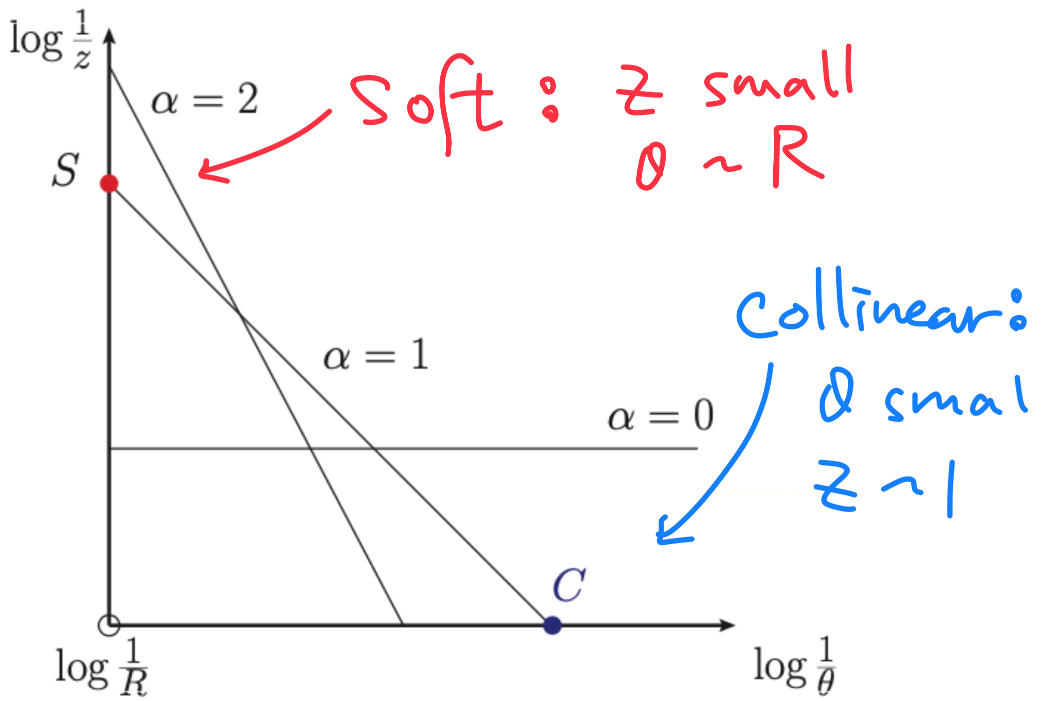



Gallicchio, Schwartz PRL 107 (2011) 172001

Instead of the average quantity, which is the conventional jet shape, this jet-by-jet distribution is shown to be sensitive to quark/gluon differences

# Lund plane and Soft Collinear Effective Theory



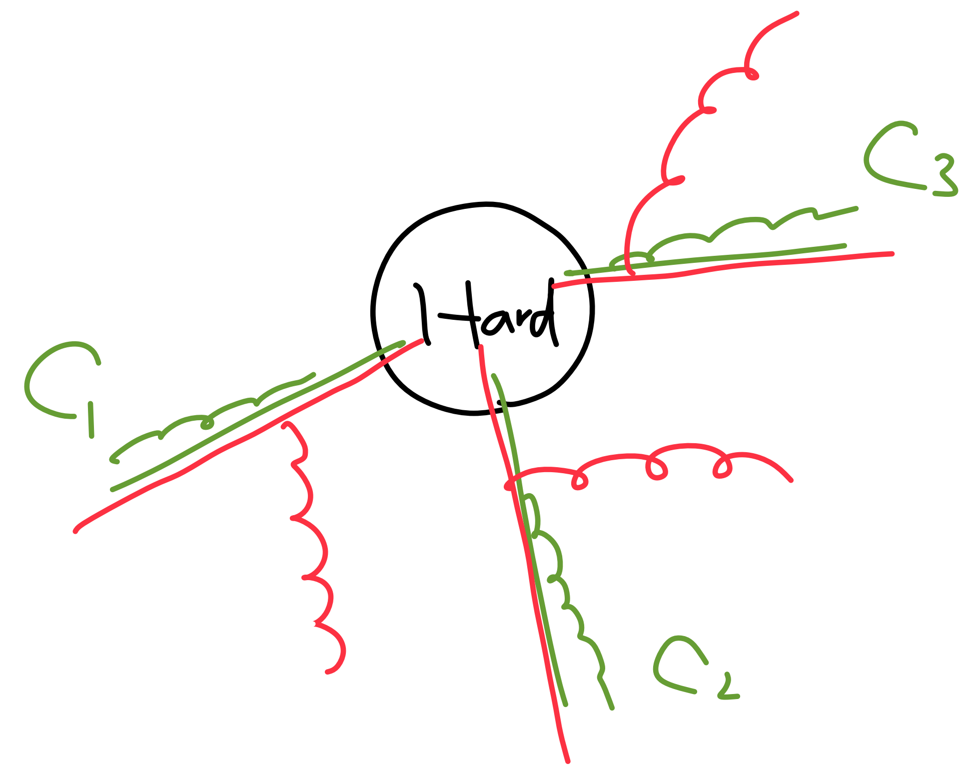




Soft-collinear effective theory factorizes QCD phase space into soft and collinear sectors, described by soft Wilson lines and collinear Wilson lines

$$\lambda_w : \sum_{i \in jet} z_i \theta_i^{\alpha}$$

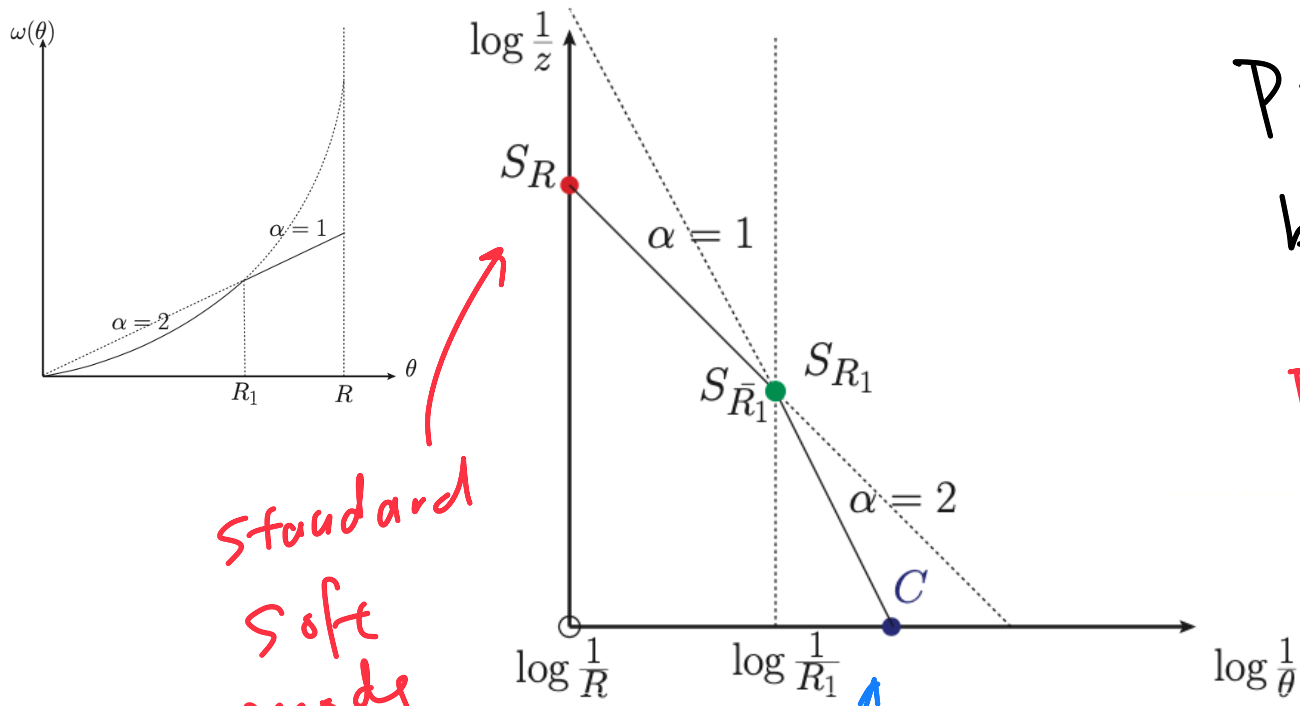
$$\log\frac{1}{\lambda_w} \sim \log\frac{1}{z} + \alpha \log\frac{1}{\theta}$$

Jet angularity measurement represents as a straight line on the Lund plane $\left(\log\frac{1}{\theta}, \log\frac{1}{z}\right)$ with slope $-\alpha$




Soft cross-talk

# Factorization of flattened jet angularity




standard soft mode for $\alpha = 1$

standard collinear mode for $\alpha = 2$

Piecewise polynomial angular weight $w(\theta)$ becomes piecewise straight lines (polygons)

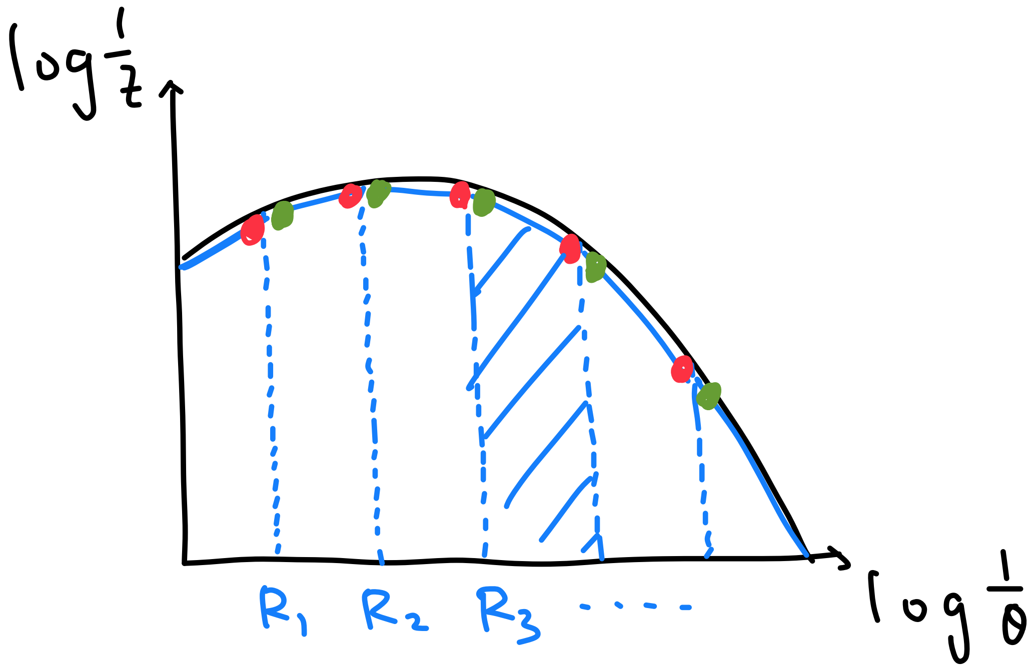
Each vertex requires two modes with exactly the same momentum scaling

$S_{R_1}$ and $S_{\bar{R}_1}$

the soft mode correspond to the measurement of $\alpha = 2$ for $\theta$ up to $R_1$

the dropped soft mode correspond to the measurement of $\alpha = 1$ for $\theta$ not below $R_1$

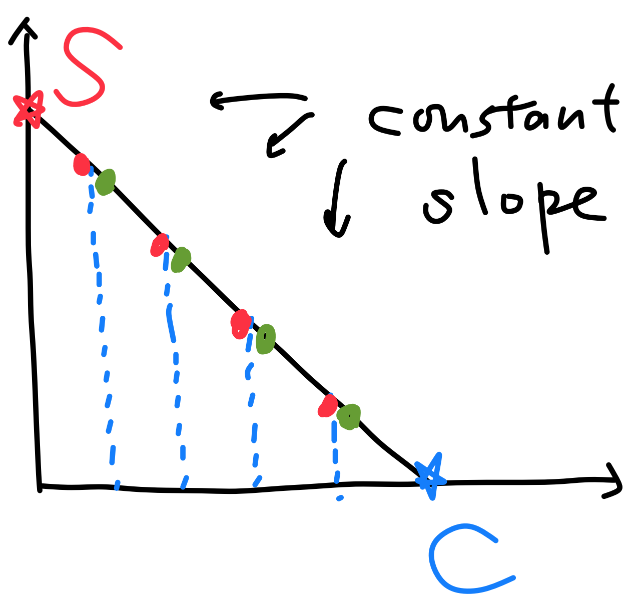
# Factorization of flattened jet angularity




One can approximate any arbitrary measurement contour with piecewise straight lines

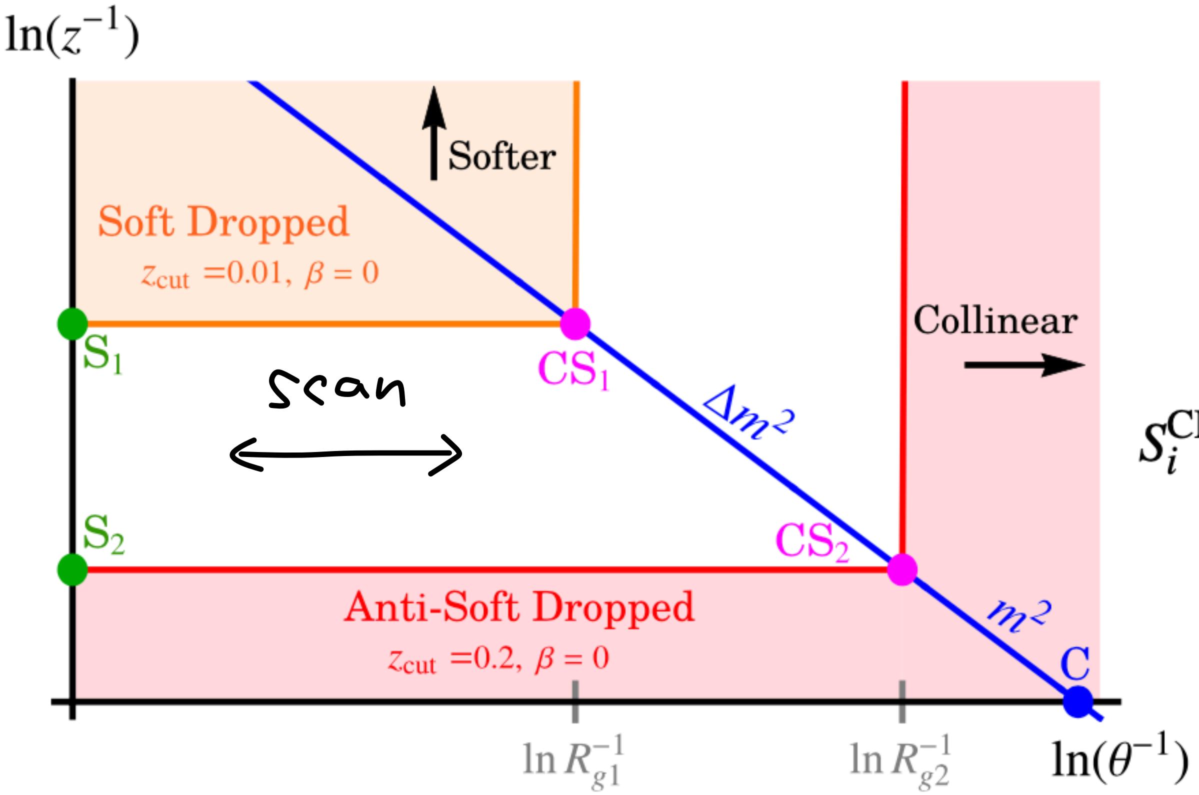
Therefore an annulus represents a differential building block of an EFT of a general jet substructure observable

special case




One can think of the classical jet angularity as factorized into an infinite number of modes, whose effects cancel except at the end points : soft & collinear modes.

# Compare factorization of collinear drop



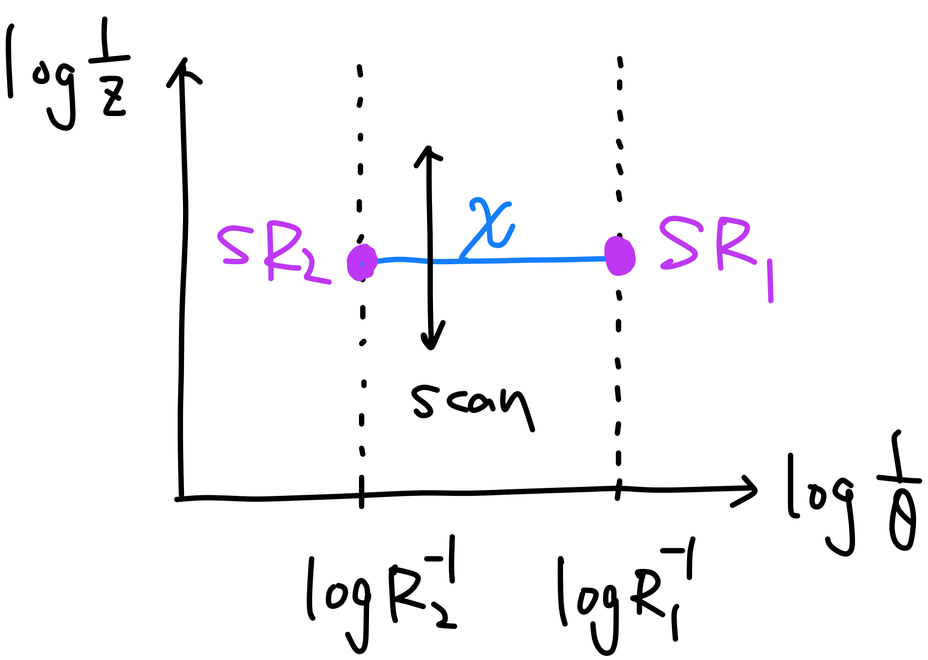

Chien, Stewart, JHEP06(2020)064

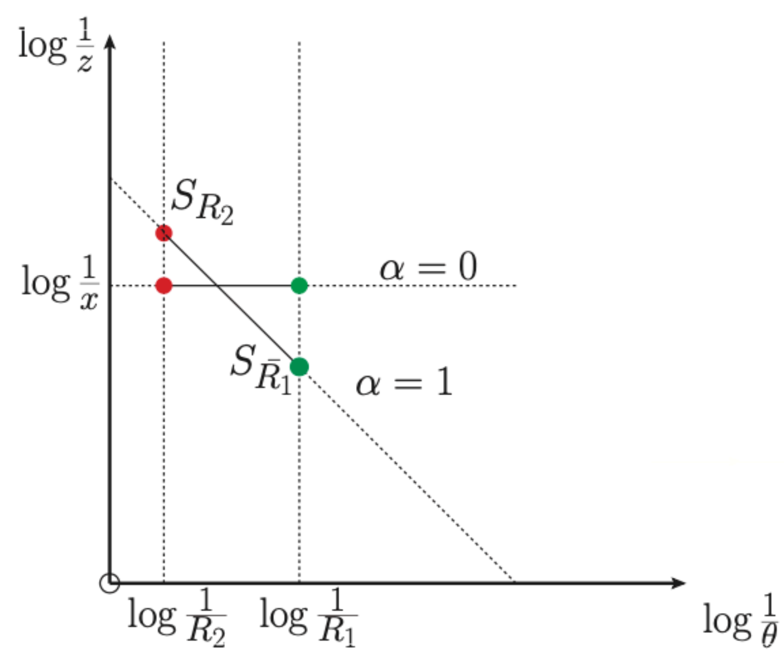
$$\frac{d\sigma}{d\Delta m^2} = \sum_{i=q,g} N_i(\mu) J^{\rm SD}_{{\rm un},i}(z_{{\rm cut}\,2}, \beta_2, \mu) S_i^{\rm CD}(\Delta m^2, z_{{\rm cut}\,i}, \beta_i, \mu)$$

$$S_i^{\rm CD}(\Delta m^2, \mu) = \int dk_i \overline{S_{C_2,i}}(k_2, \mu) S_{C_1,i}(k_1, \mu) \delta(\Delta m^2 - 2E_J(k_1 + k_2))$$

For studies using CAESAR framework and non perturbative studies, see Oleh's talk on Thursday

annulus energy fraction




$P_{S_R} \sim E_J x (R^2, 1, R)$

light cone momentum scaling

$$P_i^{\rm FA}(x, R_1, R_2) = \int dx_1 dx_2 \; S_{\bar{R}_1 i}(x_1, \mu) \; S_{R_2 i}(x_2, \mu) \delta(x - x_1 - x_2)$$

# Resummation using RG evolution

RG evolution kernels using anomalus dimension

$$\frac{P_i^{\mathrm{FA}}(x,\mu) J^{\mathrm{un}}_{R_1 i}(\mu)}{J^{\mathrm{un}}_{R_2 i}(\mu)} = \exp\Big[2C_i S(\mu_{R_1},\mu_{J_{R_1}}) - 2C_i S(\mu_{R_2},\mu_{J_{R_1}}) + 2C_i S(\mu_{J_{R_2}},\mu_{J_{R_1}}) + 2A_{S_{\overline{R}_1 i}}(\mu_{R_1},\mu_{J_{R_1}}) + 2A_{S_{R_2 i}}(\mu_{R_2},\mu_{J_{R_2}}) + 2A_{Ji}(\mu_{J_{R_1}},\mu_{J_{R_2}})\Big] \frac{J^{\mathrm{un}}_{R_1 i}(\mu_{J_{R_1}})}{J^{\mathrm{un}}_{R_2 i}(\mu_{J_{R_2}})} \left[\frac{\mu_{R_2}}{\mu_{R_1}}\frac{\mu_{J_{R_1}}}{\mu_{J_{R_2}}}\right]^{\eta_{R_1}} \tilde{S}_{R_2 i}(\partial\eta,\mu_{R_2}) \tilde{S}_{\overline{R}_1 i}\Big(\partial\eta + \ln\frac{\mu_{R_2}}{\mu_{R_1}}\frac{\mu_{J_{R_1}}}{\mu_{J_{R_2}}},\mu_{R_1}\Big)\frac{1}{x}\left(\frac{\mu_{J_{R_2}} x}{\mu_{R_2}}\right)^{\eta}\frac{e^{-\gamma_E \eta}}{\Gamma(\eta)}$$

One-loop expression for the soft and dropped soft function

$$S_{\overline{R}_1 i}(x_1,\mu) = \frac{2E_J g^2 C_i \mu^{2\epsilon} e^{\epsilon\gamma_E}\Omega_{d-2}}{(2\pi)^{d-1}(4\pi)^\epsilon}\int \frac{dq^+ dq^-}{(q^+q^-)^{1+\epsilon}}\,\delta(q^- - 2E_J x_1)\,\Theta\Big(\frac{q^+}{q^-} - \frac{R_1^2}{4}\Big)$$

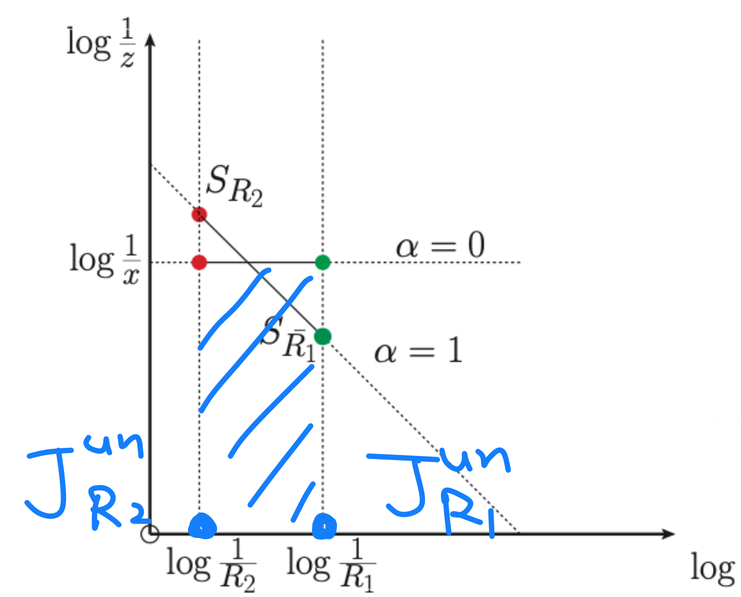
$$S_{R_2 i}(x_2,\mu) = \frac{2E_J g^2 C_i \mu^{2\epsilon} e^{\epsilon\gamma_E}\Omega_{d-2}}{(2\pi)^{d-1}(4\pi)^\epsilon}\int \frac{dq^+ dq^-}{(q^+q^-)^{1+\epsilon}}\,\delta(q^- - 2E_J x_2)\,\Theta\Big(\frac{R_2^2}{4} - \frac{q^+}{q^-}\Big)$$

One-loop anomalus dimensions

$$\frac{d\log \tilde{S}_{\overline{R}_1 i}}{d\log\mu} = 2C_i\gamma_{\mathrm{cusp}}\ln\frac{\mu_{J_{R_1}}}{\nu\mu} - 2\gamma^{S_{\overline{R}_1 i}} \equiv \gamma_{S_{\overline{R}_1 i}}$$

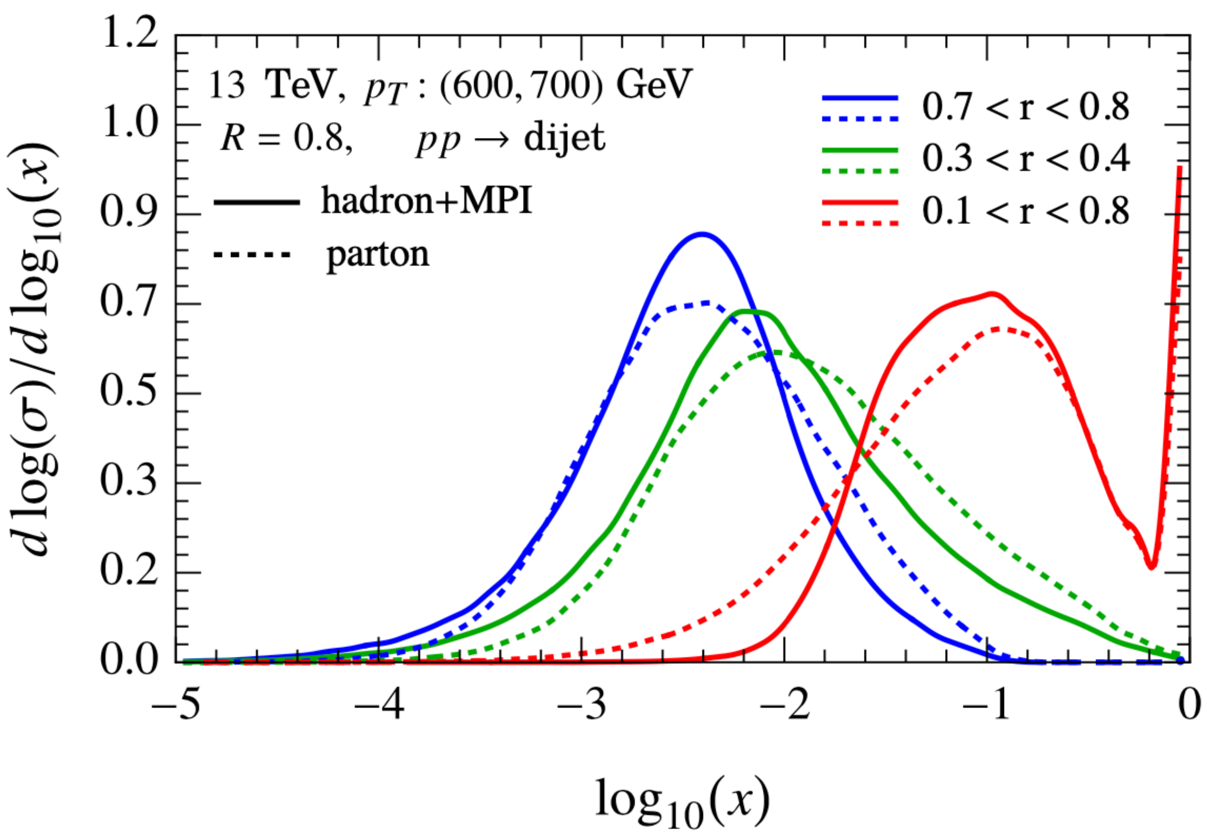
$$\frac{d\log \tilde{S}_{R_2 i}}{d\log\mu} = -2C_i\gamma_{\mathrm{cusp}}\ln\frac{\mu_{J_{R_2}}}{\nu\mu} - 2\gamma^{S_{R_2,i}} \equiv \gamma_{S_{R_2 i}}$$

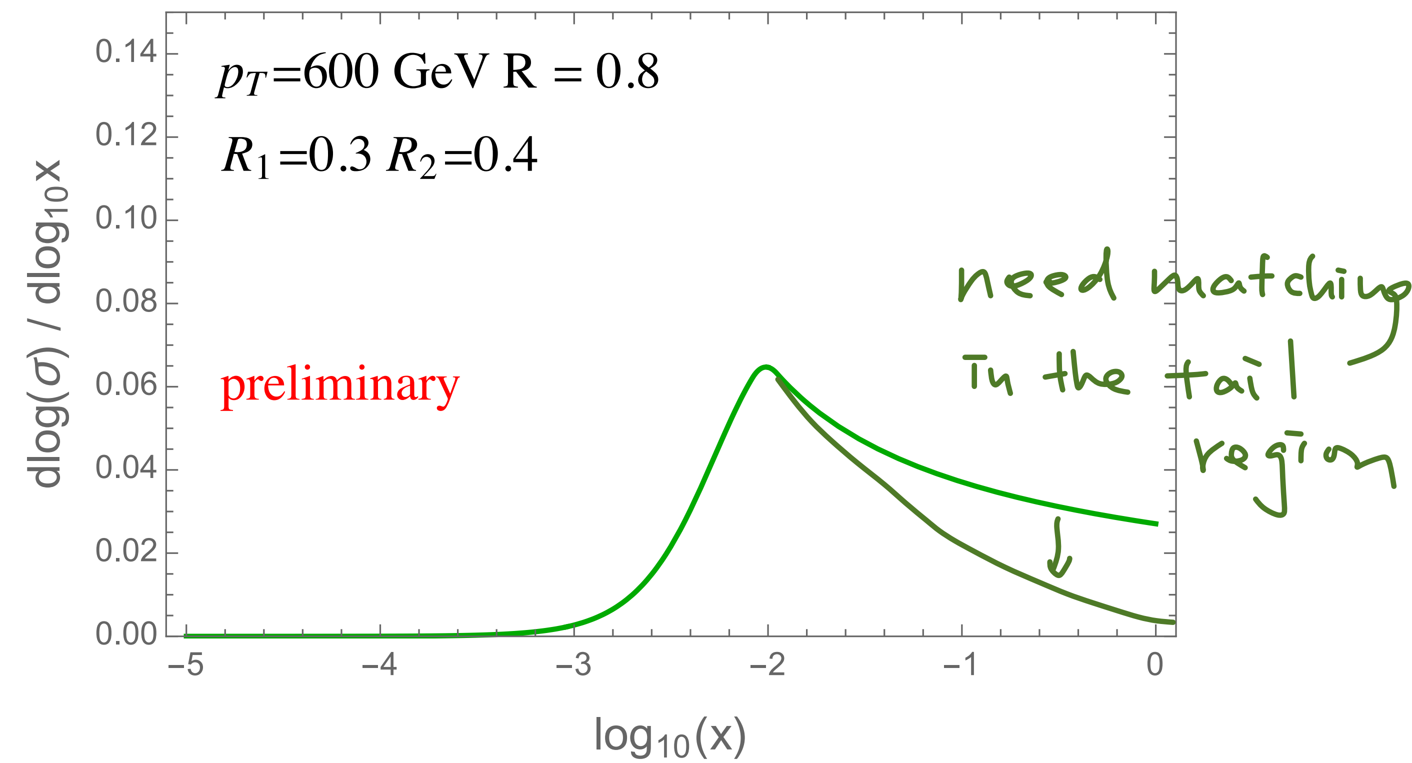
$$\gamma_{P_i^{\mathrm{FA}}} = \gamma_{S_{\overline{R}_1 i}} + \gamma_{S_{R_2 i}} = -2C_i\gamma_{\mathrm{cusp}}\log\frac{R_2}{R_1} = \gamma_{J_{R_2 i}} - \gamma_{J_{R_1 i}}$$



# Monte Carlo and analytic studies (preliminary)

Chien, Stewart, JHEP06(2020)064




- Peripheral rings ensure dropping collinear radiation and probes soft radiation
- Hadronization corrections shift to **smaller** values
- Native scale choices: $\mu_{\bar{R}_1} = E_J x R_1$, $\mu_{R_2} = E_J x R_2$ breaks down at large values of $x$ and scale merging/fixed order matching needs to be done. Constraints from recoil need to be considered.
- Sudakov peak position predicted and sensitive to the running of strong coupling constant

# Conclusions

- Explore a new jet substructure observable called flattened jet angularity
  - a jet-shape based generalization of the classic jet angularity
  - worked out the factorization expressions, still need to complete EFT scale merging and matching to fixed-order calculation
  - phenomenological applications is on the way
- Such observables do not rely on grooming nor clustering algorithms
- Their hadronization corrections can give interesting information about nonperturbative QCD effects within jets

Stay tuned !