Proton GPDs from Lattice QCD at Leading Twist and Beyond

Martha Constantinou

Temple University

OUTLINE

A. Methods to access GPDs from lattice QCD

B. New results for proton GPDs
- twist-2 transversity GPFs
- twist-3 GPDs
- additional physical information

C. Synergy with phenomenology

D. Concluding remarks

OUTLINE

A. Methods to access GPDs from lattice QCD

$$f_i = f_i^{(0)} + \frac{f_i^{(1)}}{Q} + \frac{f_i^{(2)}}{Q^2} \cdots$$

Twist-2 $(f_i^{(0)})$

Quark / Nucleon	U (γ^+)	L $(\gamma^+\gamma^5)$	T (σ^{+j})
U	$H(x,\xi,t)$ $E(x,\xi,t)$ unpolarized		
L		$\widetilde{H}(x,\xi,t)$ $\widetilde{E}(x,\xi,t)$ helicity	
T			H_T, E_T $\widetilde{H}_T, \widetilde{E}_T$ transversity

B. New results for proton GPDs
- twist-2 transversity GPFs
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(Selected) **Twist-3** $(f_i^{(1)})$

$\mathcal{O}$ / Nucleon	γ^j	$\gamma^j\gamma^5$	σ^{jk}
U	G_1, G_2 G_3, G_4		
L		$\widetilde{G}_1, \widetilde{G}_2$ $\widetilde{G}_3, \widetilde{G}_4$	
T			$H_2'(x,\xi,t)$ $E_2'(x,\xi,t)$

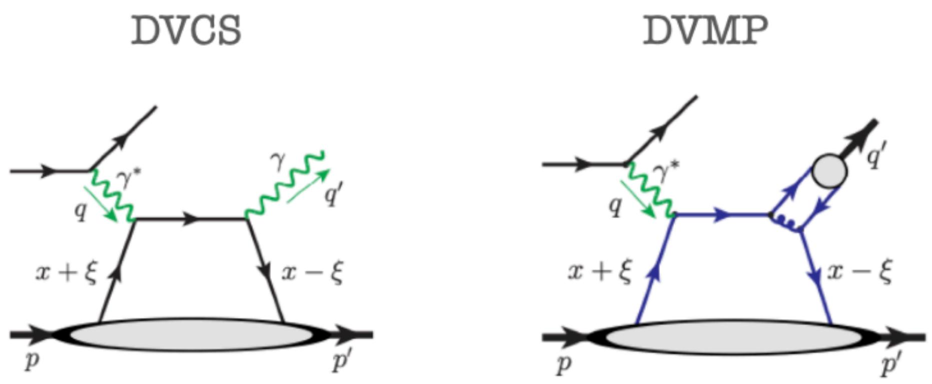
C. Synergy with phenomenology

D. Concluding remarks

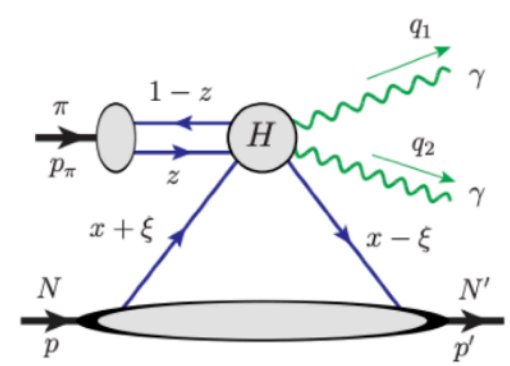
Generalized Parton Distributions

★ GPDs may be accessed via exclusive reactions (DVCS, DVMP)

★ exclusive pion-nucleon diffractive production of a γ pair of high $p_\perp$



[X.-D. Ji, PRD 55, 7114 (1997)]



[J. Qiu et al, arXiv:2205.07846]

★ GPDs are not well-constrained experimentally:

- x-dependence extraction is not direct. DVCS amplitude: $\mathscr{H} = \int_{-1}^{+1} \frac{H(x,\xi,t)}{x-\xi+i\epsilon} dx$

(SDHEP [J. Qiu et al, arXiv:2205.07846] gives access to x)

- independent measurements to disentangle GPDs
- GPDs phenomenology more complicated than PDFs (multi-dimensionality)
- and more challenges …

See Monday afternoon parallel sessions (I & II)

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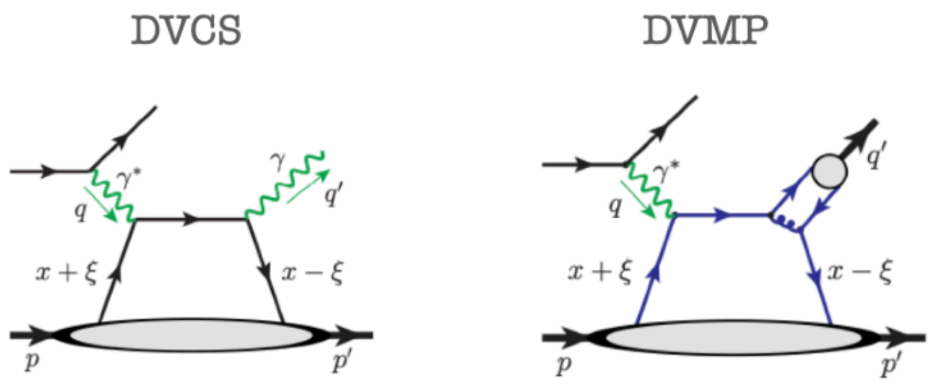

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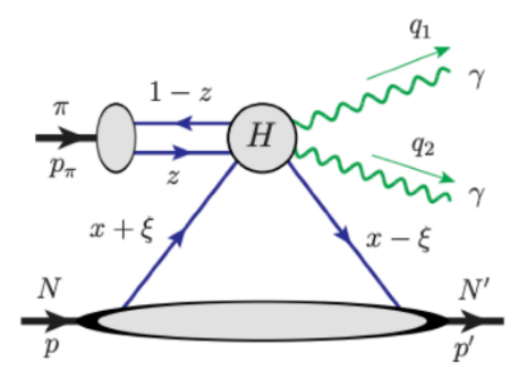

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★ Essential to complement the knowledge on GPD from lattice QCD

★ Lattice data may be incorporated in global analysis of experimental data and may influence parametrization of t and ξ dependence

Accessing information on PDFs/GPDs

★ Parton model: physical picture valid for infinite momentum frame

[R. P. Feynman, Phys. Rev. Lett. 23, 1415 (1969)]

★ PDFs via matrix elements of nonlocal light-cone operators ($-t^2 + \vec{r}^2 = 0$)

$$f(x) = \frac{1}{4\pi} \int dy^- e^{-ixP^+y^-} \langle P, S | \bar{\psi}_f \gamma^+ \mathcal{W} \psi_f | P, S \rangle$$

★ Light-cone correlations inaccessible from Euclidean lattices ($\tau^2 + \vec{r}^2 = 0$)

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A. Mellin moments (local OPE expansion)

local operators

$$\bar{q}(-\tfrac{1}{2}z)\,\gamma^\sigma W[-\tfrac{1}{2}z, \tfrac{1}{2}z]\,q(\tfrac{1}{2}z) \;=\; \sum_{n=0}^{\infty} \frac{1}{n!}\, z_{\alpha_1} \ldots z_{\alpha_n} \left[\bar{q}\gamma^\sigma \overleftrightarrow{D}^{\alpha_1} \ldots \overleftrightarrow{D}^{\alpha_n} q\right]$$

$$\langle N(P')|\mathcal{O}_V^{\mu\mu_1\cdots\mu_{n-1}}|N(P)\rangle \sim \sum_{\substack{i=0\\ \text{even}}}^{n-1} \left\{ \gamma^{\{\mu}\Delta^{\mu_1}\cdots\Delta^{\mu_i}\overline{P}^{\mu_{i+1}}\cdots\overline{P}^{\mu_{n-1}\}} A_{n,i}(t) - i\frac{\Delta_\alpha \sigma^{\alpha\{\mu}}{2m_N}\Delta^{\mu_1}\cdots\Delta^{\mu_i}\overline{P}^{\mu_{i+1}}\cdots\overline{P}^{\mu_{n-1}\}} B_{n,i}(t) \right\} + \frac{\Delta^\mu\Delta^{\mu_1}\cdots\Delta^{\mu_{n-1}}}{m_N} C_{n,0}(\Delta^2)\Big|_{n\ \text{even}} \Big) \Big\} + \frac{\Delta^\mu\Delta^{\mu_1}\cdots\Delta^{\mu_{n-1}}}{m_N} C_{n,0}(\Delta^2)\Big|_{n\ \text{even}} \Big] U(P)$$

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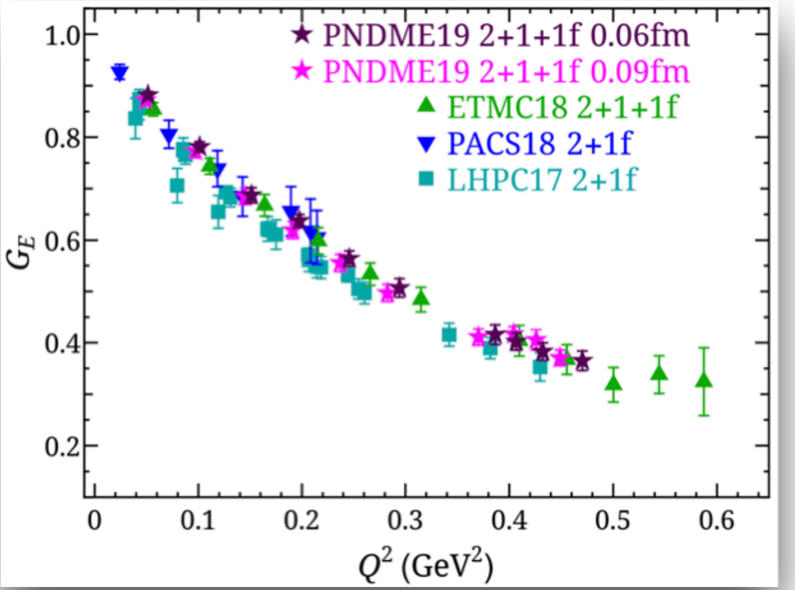
- 👍 Statistical uncertainty can be controlled
- 👍 contain physical information
- 👎 No direct access to x
- 👎 skewness independent
- 👎 Power-divergent mixing for high Mellin moments (derivatives > 3)
- 👎 Signal-to-noise ratio decays with the addition of covariant derivatives
- 👎 Number of GFFs increases with order of Mellin moment

Accessing information on PDFs/GPDs

Computationally efficient extraction of Q^2 dependence



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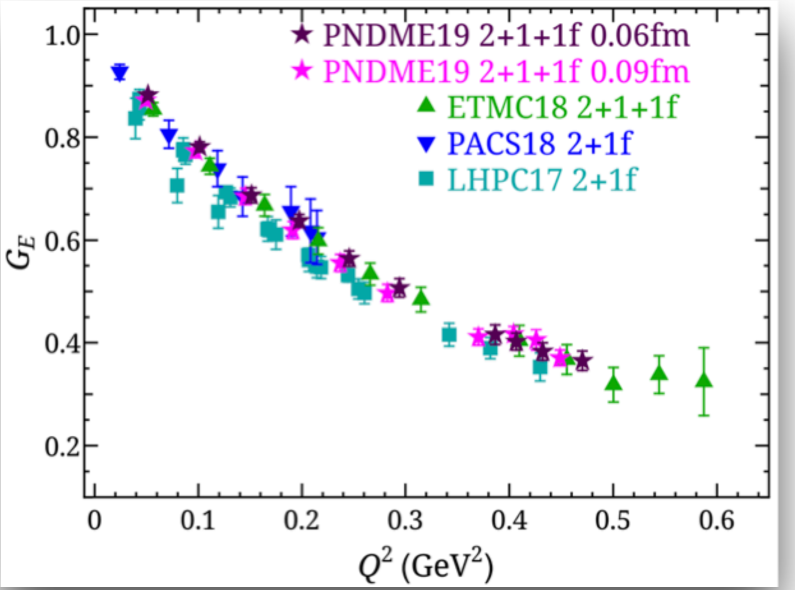

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Reconstruction of PDFs/GPDs very challenging

Accessing information on PDFs/GPDs

B. Matrix elements of nonlocal operators (quasi-GPDs, pseudo-GPDs)

$$\langle N(P_f)\,|\,\bar{\Psi}(z)\,\Gamma\,\mathcal{W}(z,0)\Psi(0)\,|\,N(P_i)\rangle_\mu$$

Nonlocal operator with Wilson line

$$\langle N(P')|O_V^\mu(x)|N(P)\rangle = \overline{U}(P')\left\{\gamma^\mu H(x,\xi,t) + \frac{i\sigma^{\mu\nu}\Delta_\nu}{2m_N}E(x,\xi,t)\right\}U(P) + \text{ht}\,,$$

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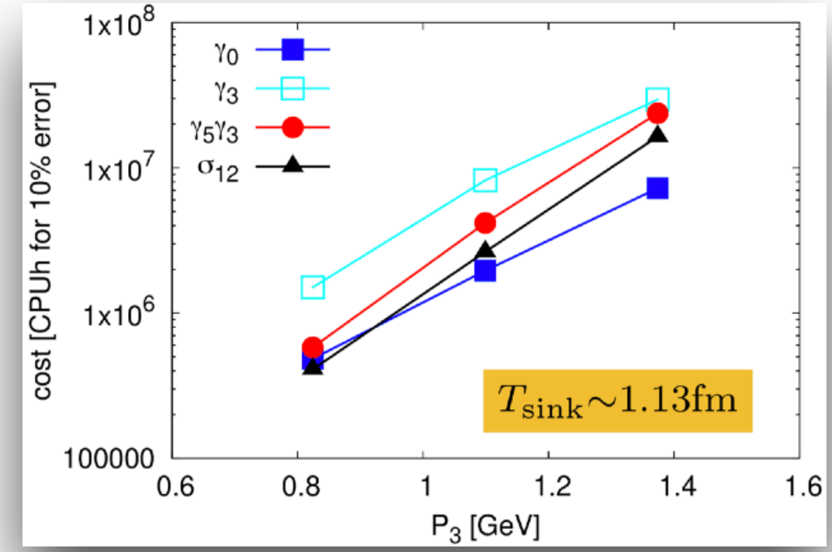
Calculation challenges

- Standard definition of GPDs in symmetric frame
  separate calculations at each t
- Statistical noise increases with P_3, t
  Projection:
  billions of core-hours for physical point at $P_3 = 3\,\text{GeV}$



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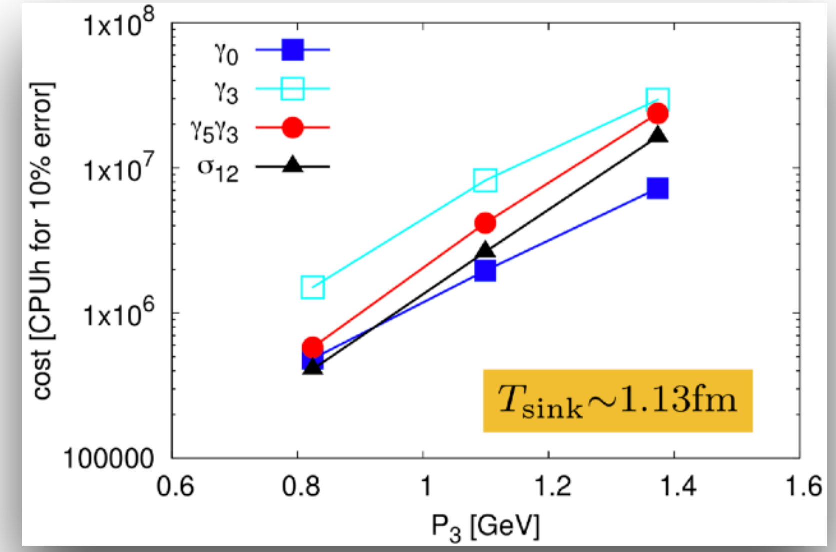
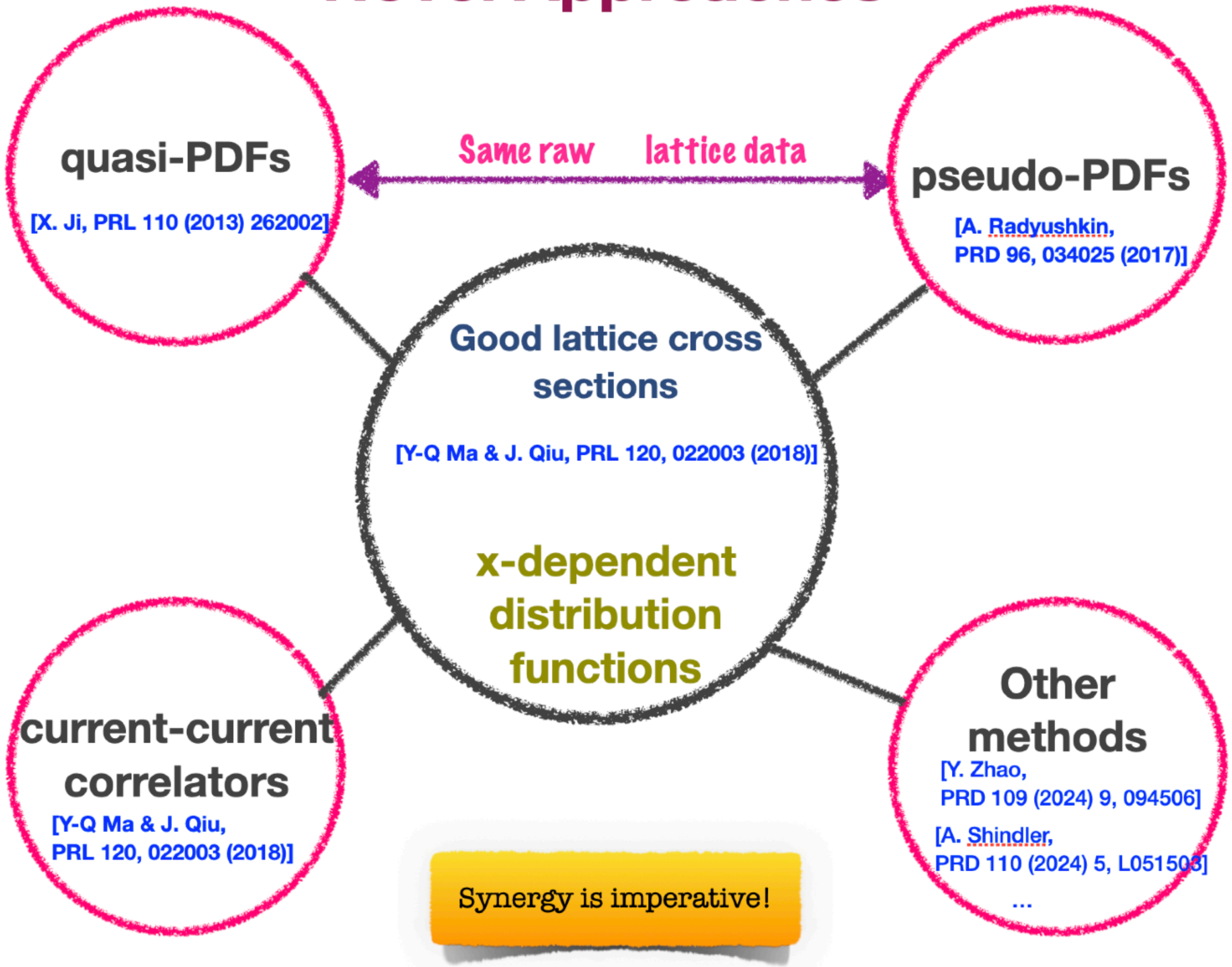


C. Other methods

See next slide

Novel Approaches



Hadronic tensor	[K.F. Liu, S.J. Dong, PRL 72 (1994) 1790, K.F. Liu, PoS(LATTICE 2015) 115]
Auxiliary scalar quark	[U. Aglietti et al., Phys. Lett. B441, 371 (1998), arXiv:hep-ph/9806277]
Fictitious heavy quark	[W. Detmold, C. J. D, Lin, Phys. Rev. D73, 014501 (2006)]
Auxiliary scalar quark	[V. Braun & D. Mueller, Eur. Phys. J. C55, 349 (2008), arXiv:0709.1348]
Higher moments	[Z. Davoudi, M. Savage, Phys. Rev. D86, 054505 (2012)]
Quasi-distributions (LaMET)	[X. Ji, PRL 110 (2013) 262002, arXiv:1305.1539; Sci. China PPMA. 57, 1407 (2014)]
Compton amplitude and OPE	[A. Chambers et al. (QCDSF), PRL 118, 242001 (2017), arXiv:1703.01153]
Pseudo-distributions	[A. Radyushkin, Phys. Rev. D 96, 034025 (2017), arXiv:1705.01488]
Good lattice cross sections	[Y-Q Ma & J. Qiu, Phys. Rev. Lett. 120, 022003 (2018), arXiv:1709.03018]
PDFs without Wilson line	[Y. Zhao Phys.Rev.D 109 (2024) 9, 094506, arXiv:2306.14960]
Moments of PDFs of any order	[A. Shindler, Phys.Rev.D 110 (2024) 5, L051503, arXiv:2311.18704]

Reviews of methods and applications

- ***A guide to light-cone PDFs from Lattice QCD: an overview of approaches, techniques and results***
  K. Cichy & M. Constantinou (invited review) Advances in HEP 2019, 3036904, arXiv:1811.07248
- ***Large Momentum Effective Theory***
  X. Ji, Y.-S. Liu, Y. Liu, J.-H. Zhang, and Y. Zhao (2020), 2004.03543
- ***The x-dependence of hadronic parton distributions: A review on the progress of lattice QCD***
  M. Constantinou (invited review) Eur. Phys. J. A 57 (2021) 2, 77, arXiv:2010.02445

Well-studied "novel" methods for PDFs/GPDs in LQCD

Matrix elements of non-local operators (space-like separated fields) with boosted hadrons

$$\mathcal{M}(P_f, P_i, z) = \langle N(P_f) \,|\, \bar{\Psi}(z)\, \Gamma\, \mathcal{W}(z,0) \Psi(0) \,|\, N(P_i) \rangle_\mu$$

Calculation very taxing!

- length of the Wilson line (z)
- nucleon momentum boost (P_3)

} PDFs, GPDs

- momentum transfer (t)
- skewness (ξ)

} GPDs

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[X. Ji, Phys. Rev. Lett. 110 (2013) 262002]
[X. Ji, Sci. China Phys. M.A. 57 (2014) 1407]

quasi-PDFs

$$\tilde{q}_\Gamma^{\rm GPD}(x, t, \xi, P_3, \mu) = \int \frac{dz}{4\pi} e^{-i\,x\,P_3\,z}\, \mathscr{M}(P_f, P_i, z)$$

pseudo-ITD [A. Radyushkin, PRD 96, 034025 (2017)]

$$\mathfrak{M}(\nu, \xi, t; z_3^2) \equiv \frac{\mathscr{M}(\nu, \xi, t; z_3^2)}{\mathscr{M}(0,0,0; z^2)} \qquad (\nu = z \cdot p)$$

Calculation very taxing!

- length of the Wilson line (z)
- nucleon momentum boost (P_3)

} PDFs, GPDs

- momentum transfer (t)
- skewness (ξ)

} GPDs

Well-studied "novel" methods for PDFs/GPDs in LQCD

Matrix elements of non-local operators (space-like separated fields) with boosted hadrons

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Matching in momentum space
(Large Momentum
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Matching in ν space

$$Q(\nu, \mu^2) = \int_{-1}^{1} dx\, e^{i\nu x} q(x, \mu^2)$$

Light-cone PDFs & GPDs

Calculation very taxing!

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quasi-PDFs

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Matching resembles factorization:

$$\sigma_{\rm DIS}(x, Q^2) = \sum_i \left[H^i_{\rm DIS} \otimes f_i\right](x, Q^2)$$

$$\tilde{q}^{\rm GPD}_{\Gamma}(x, t, \xi, P_3, \mu) = \int \frac{dz}{4\pi} e^{-i\, x\, P_3\, z}\, \mathcal{M}(P_f, P_i, z)$$

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A new approach to GPDs from lattice QCD

(leading twist)

GPDs on the lattice: the unpolarized case

★ Off-forward matrix elements of non-local light-cone operators

$$F^{[\gamma^+]}(x,\Delta;\lambda,\lambda') = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ik\cdot z} \langle p';\lambda' | \bar{\psi}(-\tfrac{z}{2})\,\gamma^+ \mathscr{W}(-\tfrac{z}{2},\tfrac{z}{2})\psi(\tfrac{z}{2})\,| p;\lambda\rangle \Big|_{z^+=0,\vec{z}_\perp=\vec{0}_\perp}$$

★ Parametrization in two leading twist GPDs

$$F^{[\gamma^+]}(x,\Delta;\lambda,\lambda') = \frac{1}{2P^+}\bar{u}(p',\lambda')\left[\gamma^+ H(x,\xi,t) + \frac{i\sigma^{+\mu}\Delta_\mu}{2M}E(x,\xi,t)\right]u(p,\lambda)$$

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γ^0 parametrization is prohibitively expensive

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Let's rethink calculation of GPDs !

Definition of GPDs on Euclidean lattice

★ Parametrization of matrix elements in Lorentz invariant amplitudes

Vector [S. Bhattacharya et al., PRD 106 (2022) 11, 114512]

Axial [S. Bhattacharya et al., PRD 109 (2024) 3, 034508]

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Tensor $F^{[i\sigma^{\mu\nu}\gamma_5]}(z,P,\Delta) \equiv \langle p_f;\lambda'|\bar{\psi}(-\tfrac{z}{2})\, i\sigma^{\mu\nu}\gamma_5\, \mathcal{W}(-\tfrac{z}{2},\tfrac{z}{2})\psi(\tfrac{z}{2})|p_i;\lambda\rangle$ [S. Bhattacharya et al., arXiv:2505.11288]

$$= \bar{u}(p_f,\lambda')\Bigg[P^{[\mu}z^{\nu]}\gamma_5 A_{T1} + \frac{P^{[\mu}\Delta^{\nu]}}{m^2}\gamma_5 A_{T2} + z^{[\mu}\Delta^{\nu]}\gamma_5 A_{T3} + \gamma^{[\mu}\left(\frac{P^{\nu]}}{m}A_{T4} + m z^{\nu]}A_{T5} + \frac{\Delta^{\nu]}}{m}A_{T6}\right)\gamma_5$$

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Advantages

- Applicable to any kinematic frame and have definite symmetries
- Lorentz invariant amplitudes A_i can be related to the standard GPDs
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Light-cone GPDs using lattice correlators in non-symmetric frames

Proof of Concept Calculation

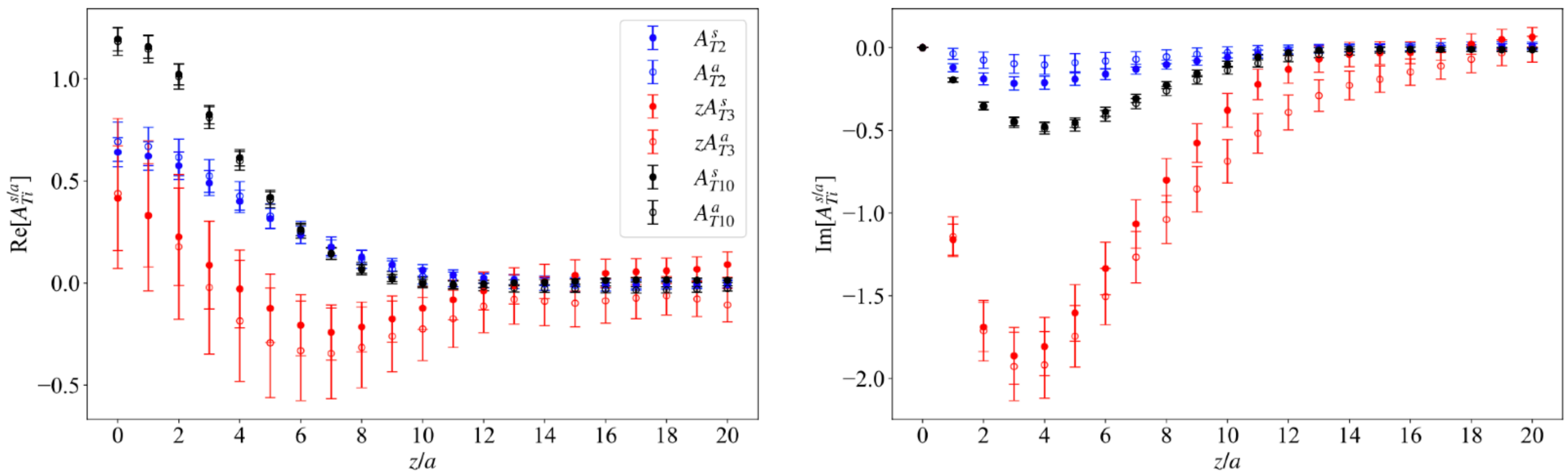
Twisted-mass fermions & clover

Name	β	N_f	$L^3 \times T$	a [fm]	M_π	$m_\pi L$
cA211.32	1.726	u, d, s, c	$32^3 \times 64$	0.093	260 MeV	4

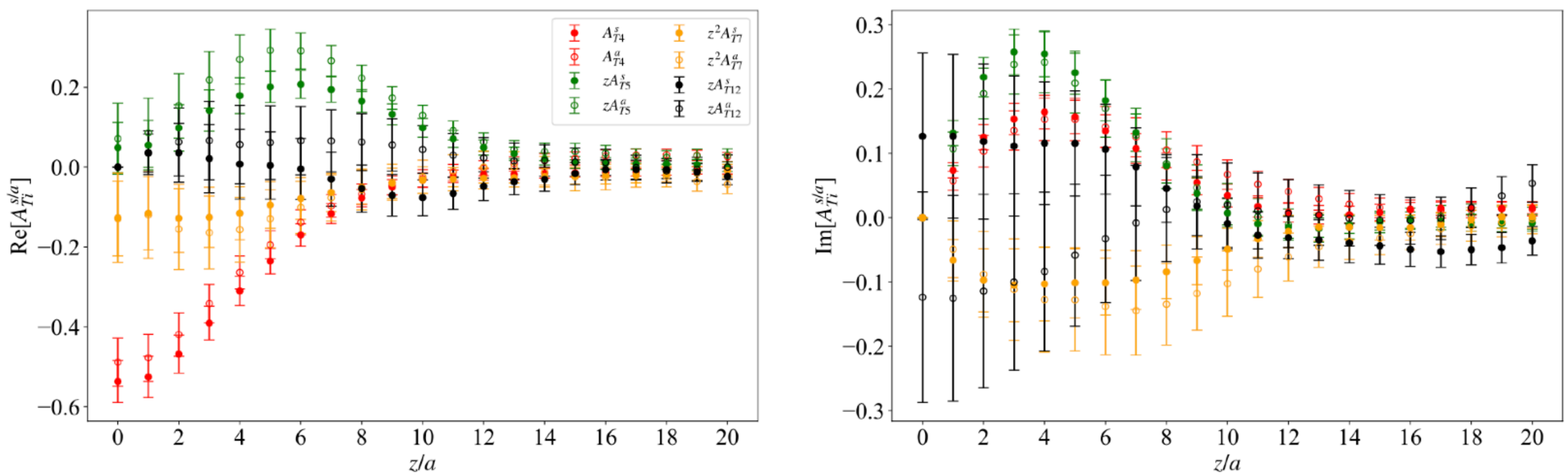
Test at zero skewness

- **symmetric frame:** $\vec{p}^{\,s}_f = \vec{P} + \vec{Q}/2\,,\quad \vec{p}^{\,s}_i = \vec{P} - \vec{Q}/2 \qquad -t^s = \vec{Q}^2 = 0.69\,GeV^2$
- **asymmetric frame:** $\vec{p}^{\,a}_f = \vec{P}\,,\quad \vec{p}^{\,a}_i = \vec{P} - \vec{Q} \qquad t^a = -\vec{Q}^2 + (E_f - E_i)^2 = 0.65\,GeV^2$

Dominant magnitude



Smaller magnitude



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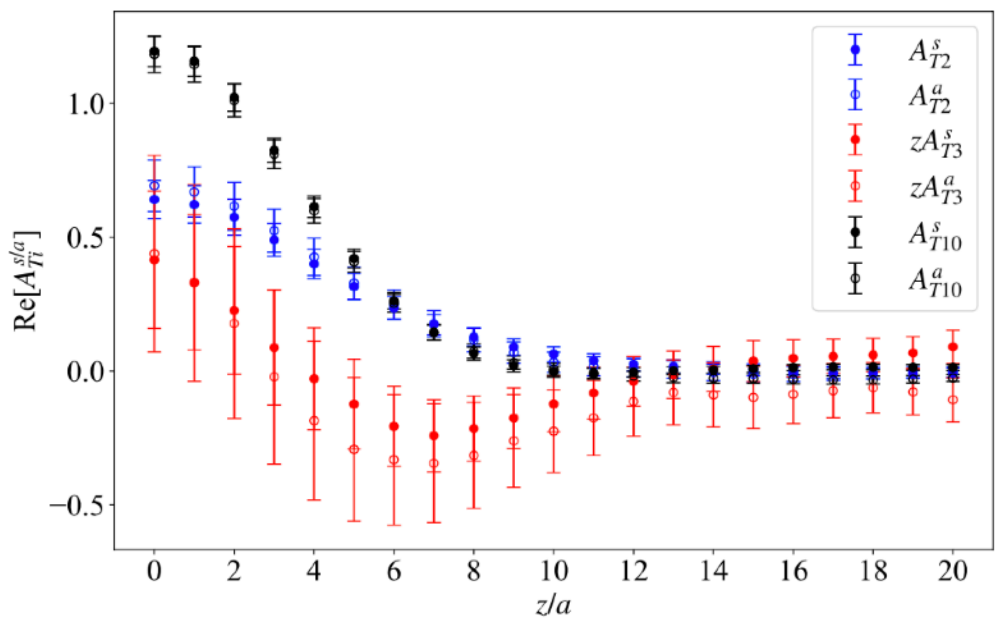
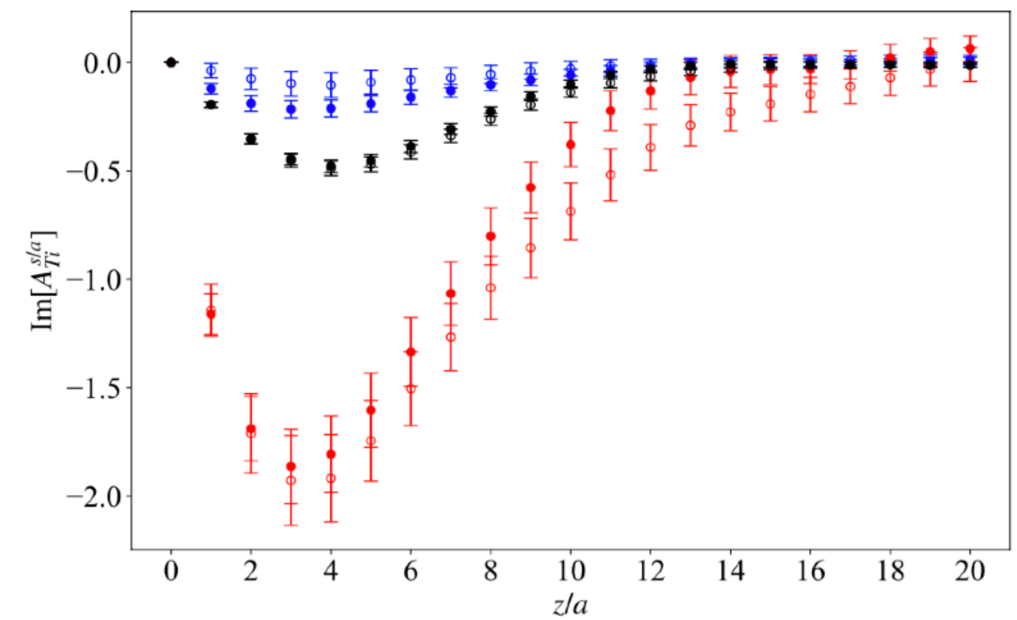
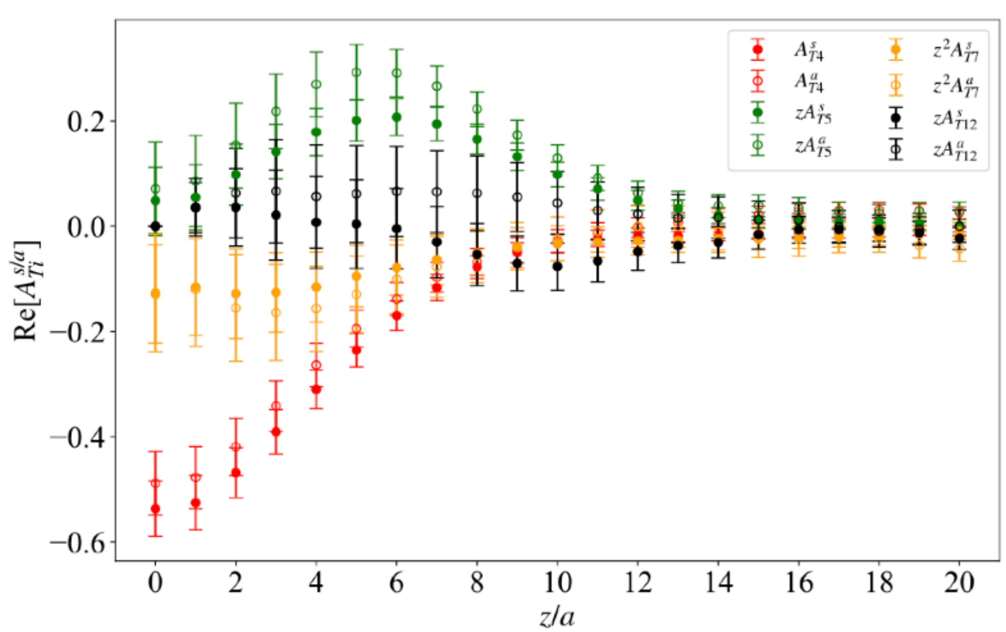
- asymmetric frame: $\vec{p}_f^{\,a} = \vec{P}\,, \quad \vec{p}_i^{\,a} = \vec{P} - \vec{Q} \quad t^a = -\vec{Q}^2 + (E_f - E_i)^2 = 0.65\,GeV^2$

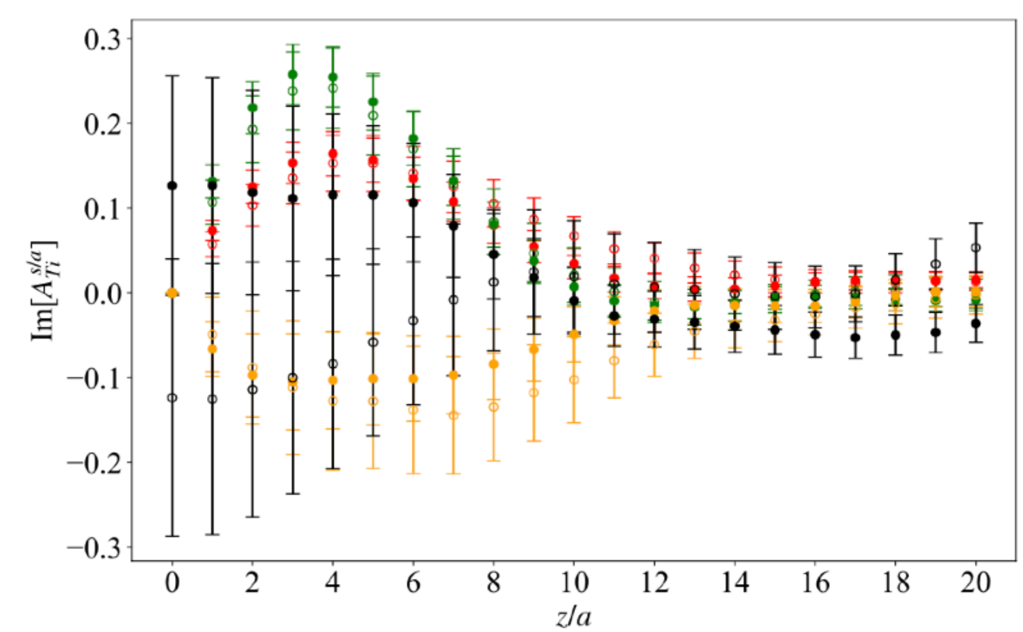
Dominant magnitude





Smaller magnitude





Indeed frame independence

Proof of Concept Calculation

Twisted-mass fermions & clover

Name	β	N_f	$L^3 \times T$	a [fm]	M_π	$m_\pi L$
cA211.32	1.726	u, d, s, c	$32^3 \times 64$	0.093	260 MeV	4

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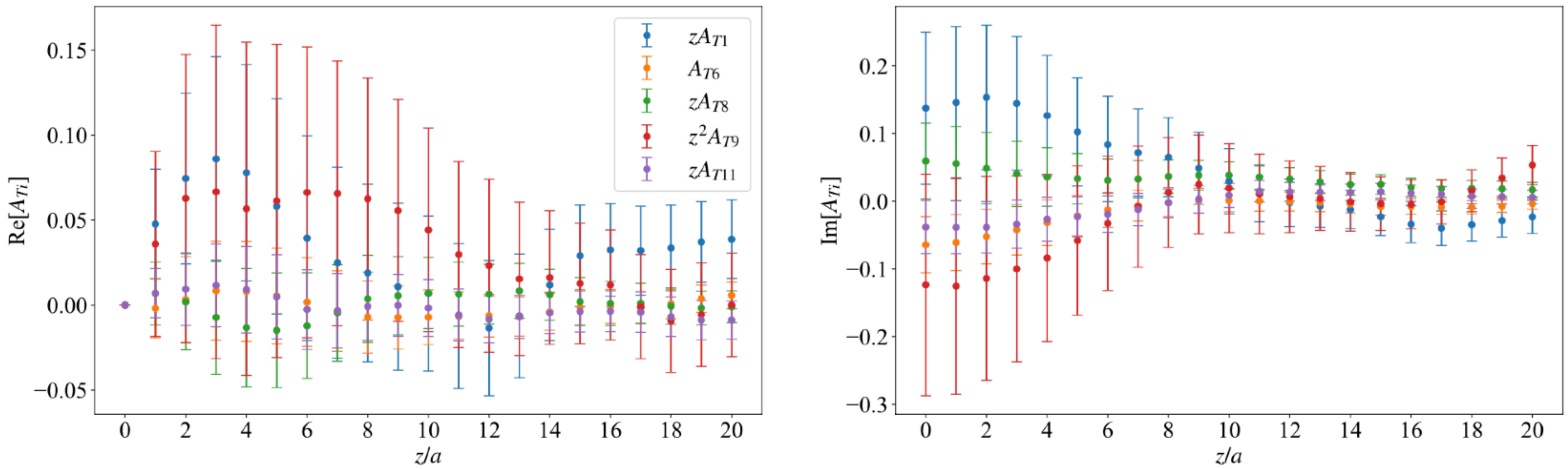
Theoretically zero at ξ=0



Indeed frame independence

Beyond Exploration

- ★ Symm. frame: separate calculation for each $\vec{Q}$
- ★ Asymm. frame: Two classes of $\vec{Q}$: $(Q_x,0,0), (Q_x, Q_y,0)$

frame	P_3 [GeV]	Δ [$\frac{2\pi}{L}$]	$-t$ [GeV^2]	ξ	$N_{\rm ME}$	$N_{\rm confs}$	$N_{\rm src}$	$N_{\rm tot}$
N/A	±1.25	(0,0,0)	0	0	2	731	16	23392
symm	±0.83	(±2,0,0), (0,±2,0)	0.69	0	8	67	8	4288
symm	±1.25	(±2,0,0), (0,±2,0)	0.69	0	8	249	8	15936
symm	±1.67	(±2,0,0), (0,±2,0)	0.69	0	8	294	32	75264
symm	±1.25	(±2, ±2, 0)	1.39	0	16	224	8	28672
symm	±1.25	(±4,0,0), (0,±4,0)	2.76	0	8	329	32	84224
asymm	±1.25	(±1,0,0), (0,±1,0)	0.17	0	8	429	8	27456
asymm	±1.25	(±1, ±1, 0)	0.33	0	16	194	8	12416
asymm	±1.25	(±2,0,0), (0,±2,0)	0.64	0	8	429	8	27456
asymm	±1.25	(±1,±2,0), (±2,±1,0)	0.80	0	16	194	8	12416
asymm	±1.25	(±2,±2,0)	1.16	0	16	194	8	24832
asymm	±1.25	(±3,0,0), (0,±3,0)	1.37	0	8	429	8	27456
asymm	±1.25	(±1,±3,0), (±3,±1,0)	1.50	0	16	194	8	12416
asymm	±1.25	(±4,0,0), (0,±4,0)	2.26	0	8	429	8	27456

- ★ Momentum transfer range is very optimistic
  (some values have enhanced systematic uncertainties)

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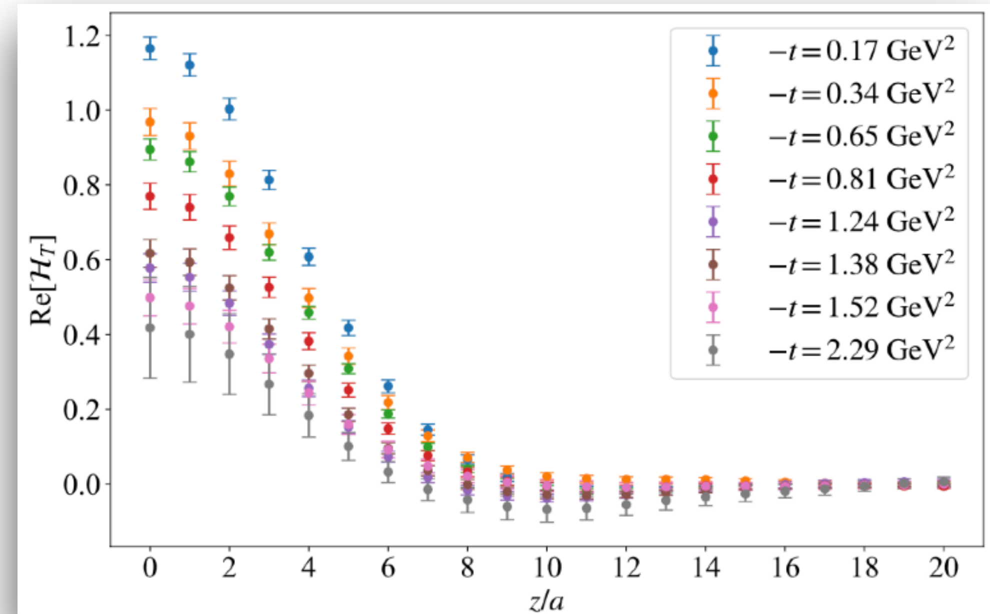
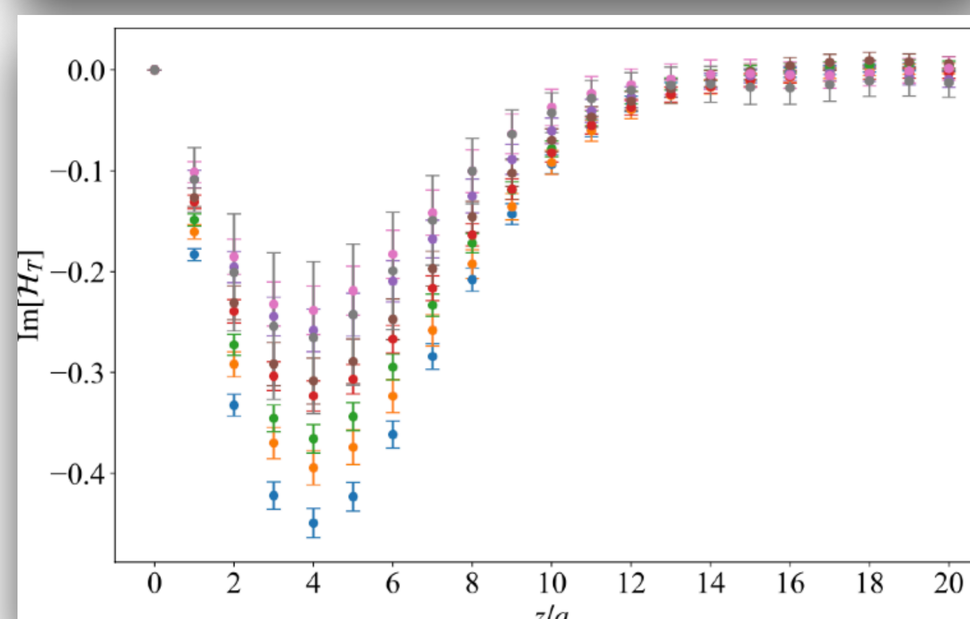
asymmetric frame





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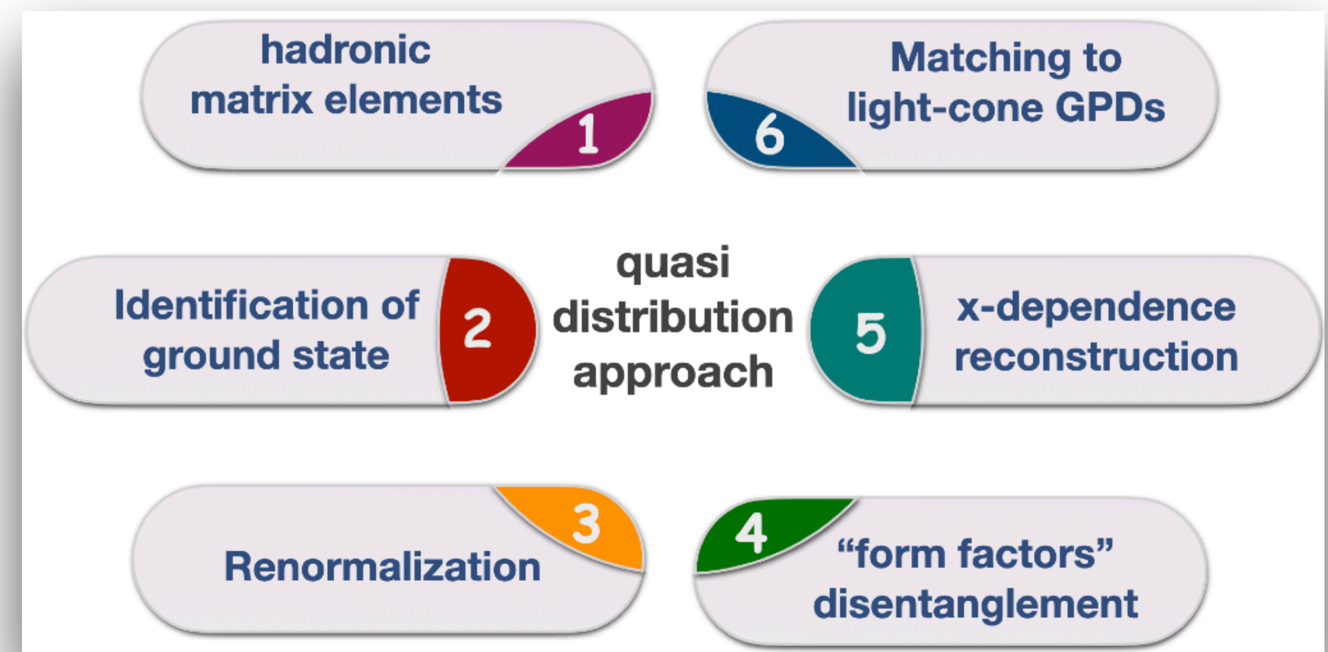
Beyond Exploration

Apply non-trivial steps



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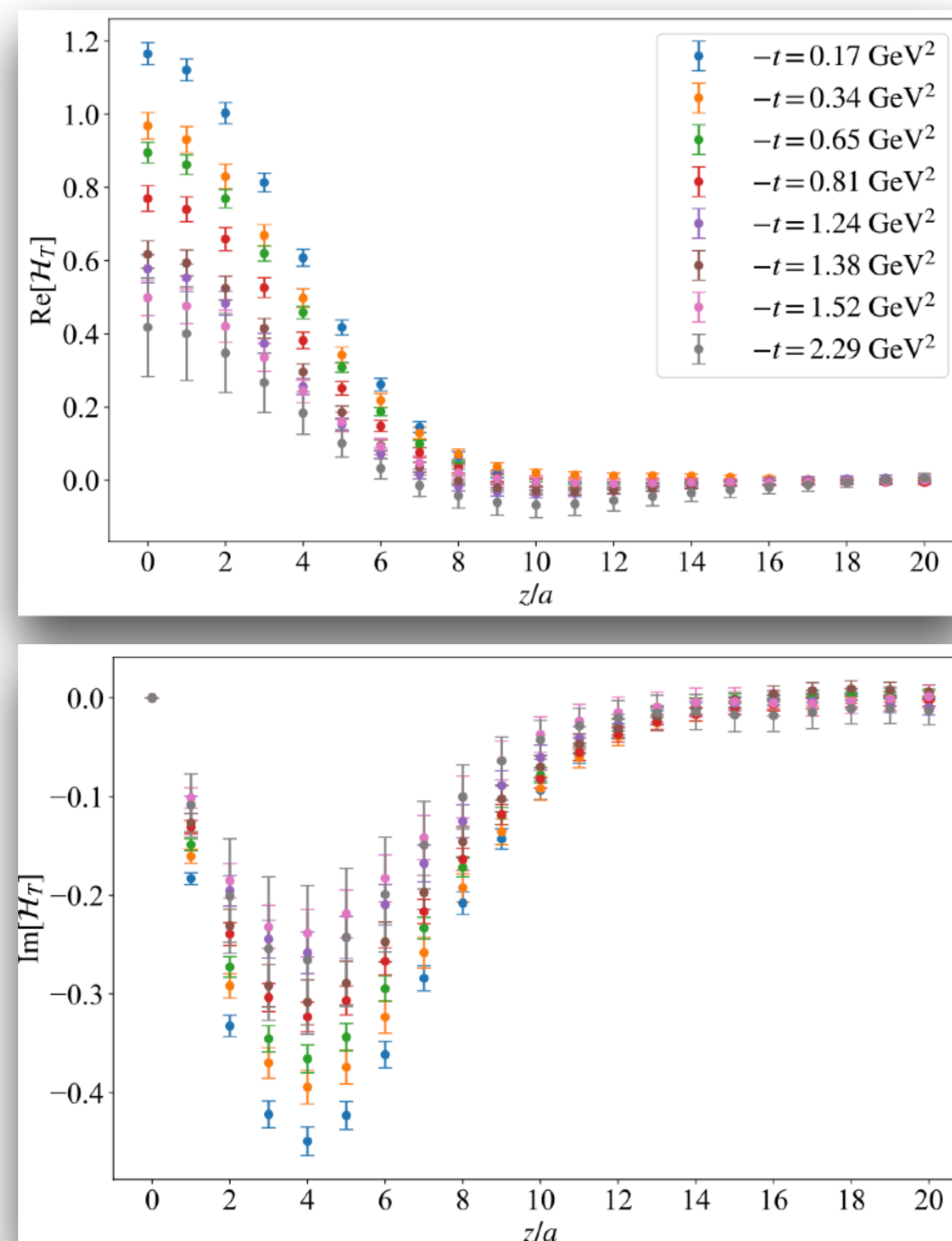

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Definition of GPDs

Lorentz invariant definition

$$\mathcal{H}_T = -2A_{T2}\left(1+\frac{P^2}{M^2}\right) + A_{T4} + A_{T10}$$

$$\mathcal{E}_T = 2A_{T2} - A_{T4}$$

$$\widetilde{\mathcal{H}}_T = -A_{T2}$$

$$\widetilde{\mathcal{E}}_T = -2A_{T6} - 2P_3\, zA_{T8}$$

Standard definition (σ^{3j})

$$\mathcal{H}_T^s = -2A_{T2}\left(1+\frac{P^2}{M^2}\right) + A_{T4} - zA_{T8}\left(\frac{E_f^2 - E_i^2}{2P_3}\right) + A_{T10}$$

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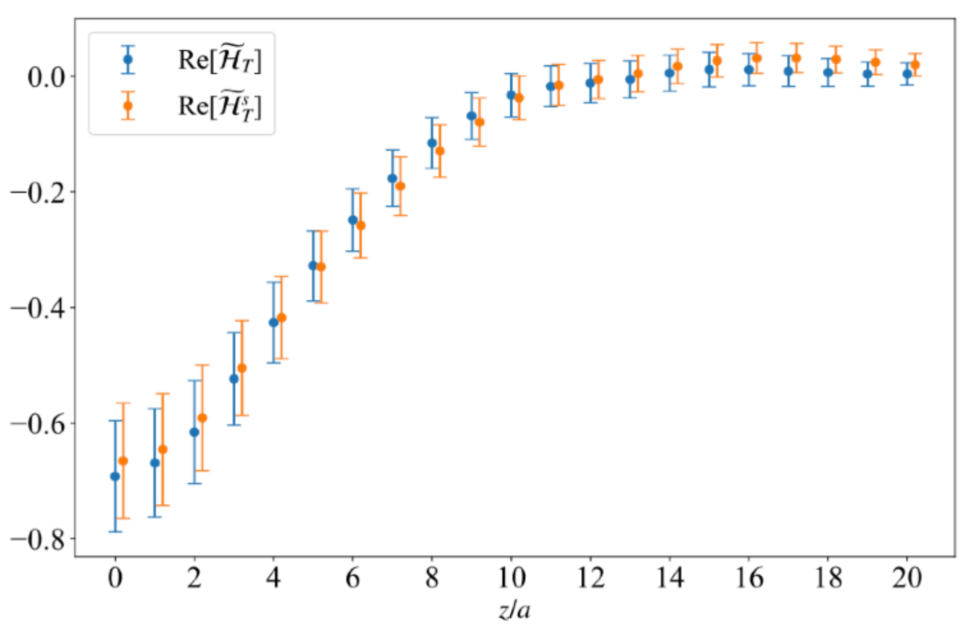
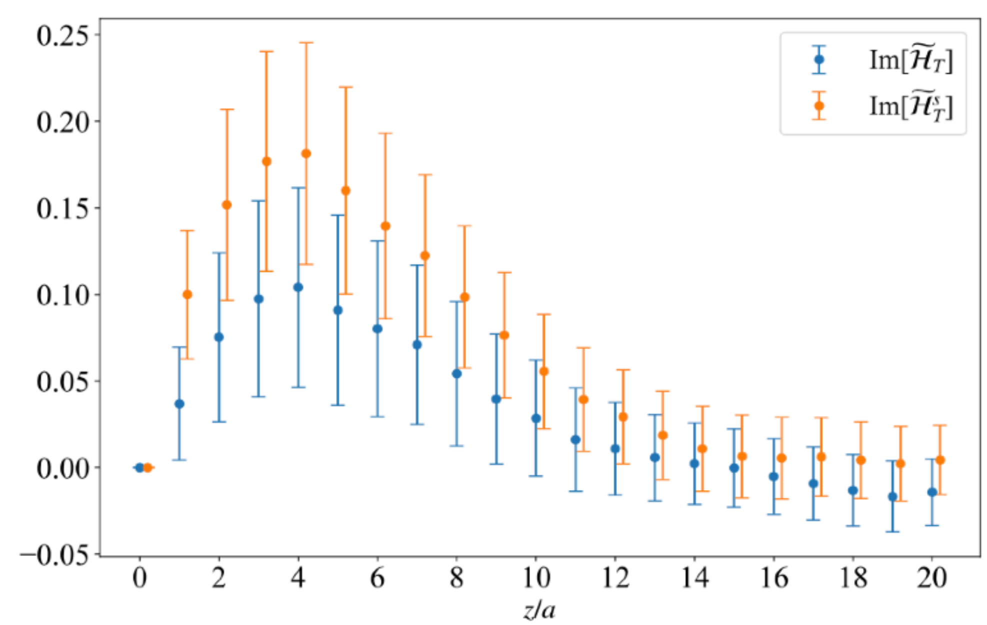
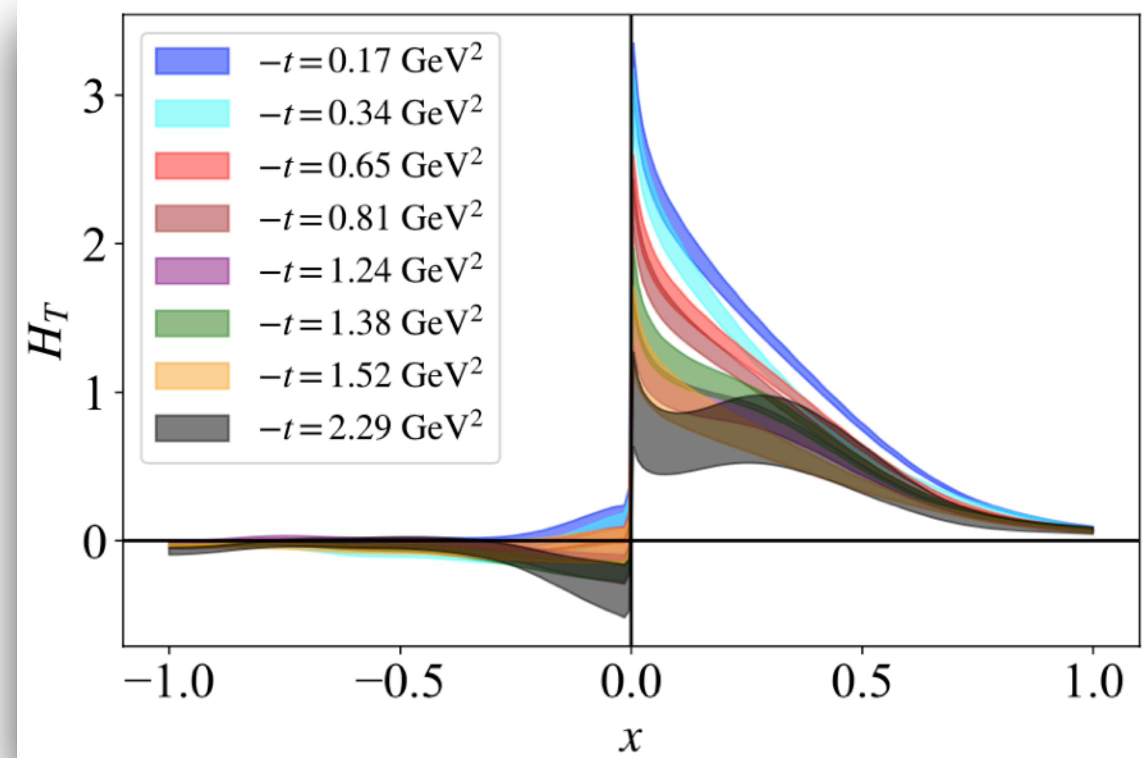


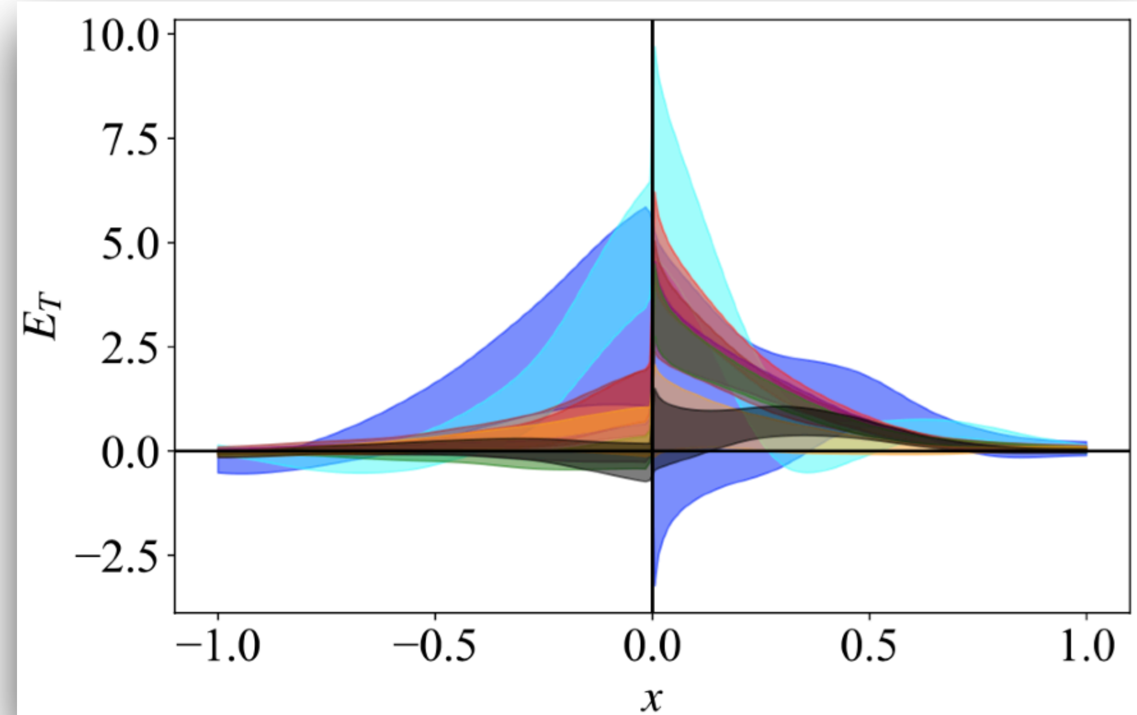


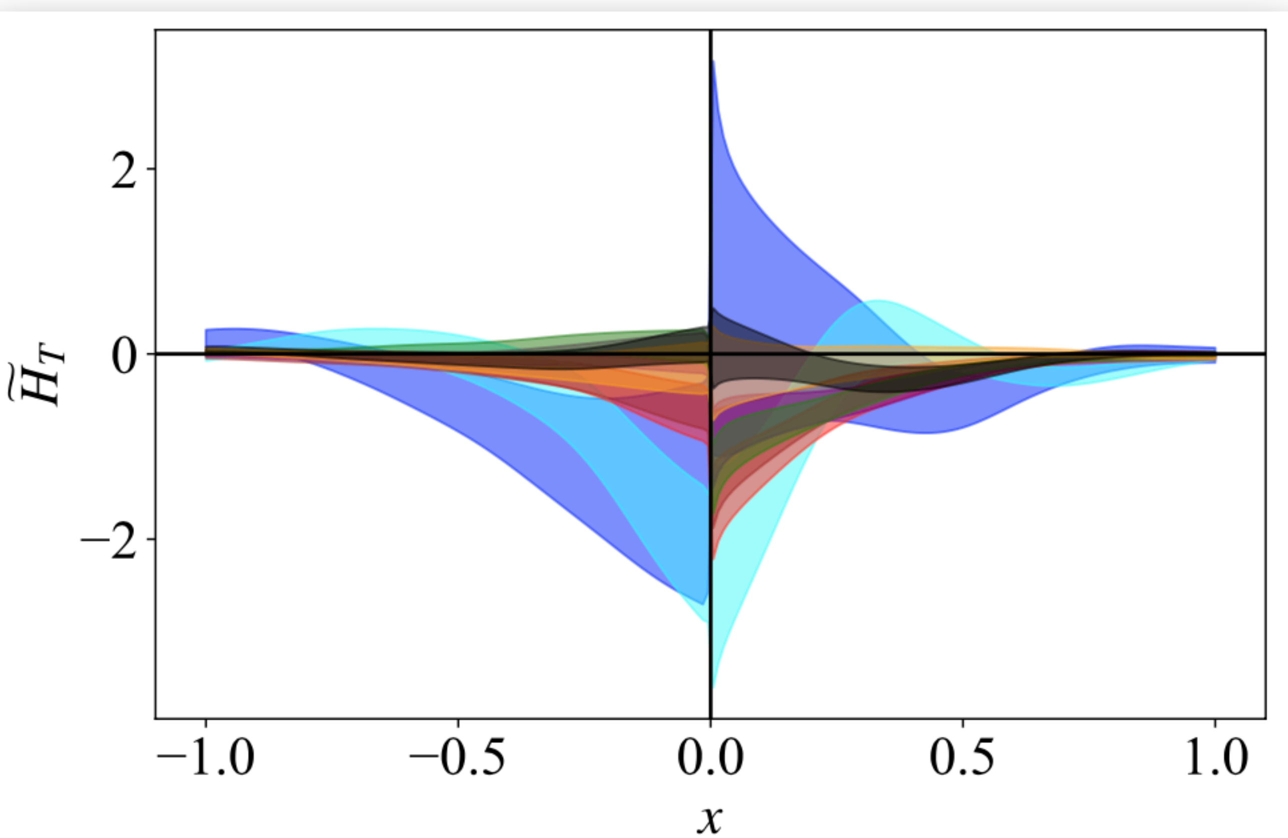
Representative example

($-t^a = 0.64$ GeV^2)

Final Results







- ★ $+x$ ($-x$) region: quarks (anti-quarks)
- ★ anti-quark region susceptible to more systematic uncertainties
- ★ small- and large-x region not reliably extracted
- ★ large -t values unreliable but free

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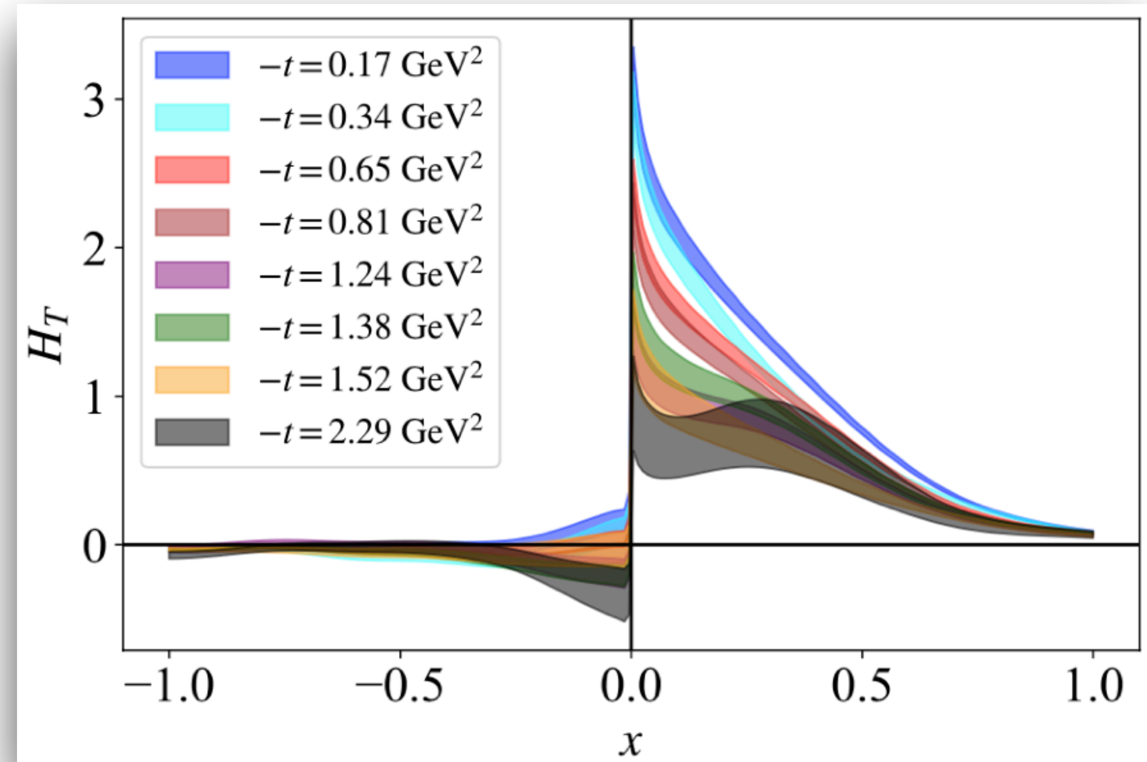

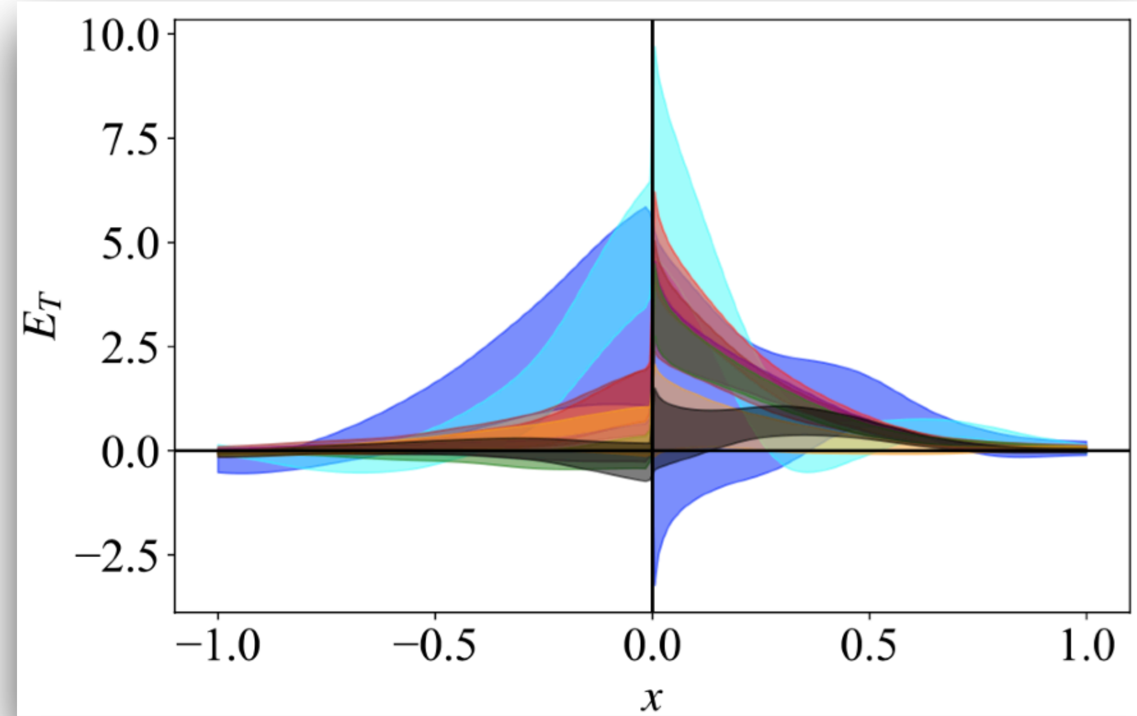

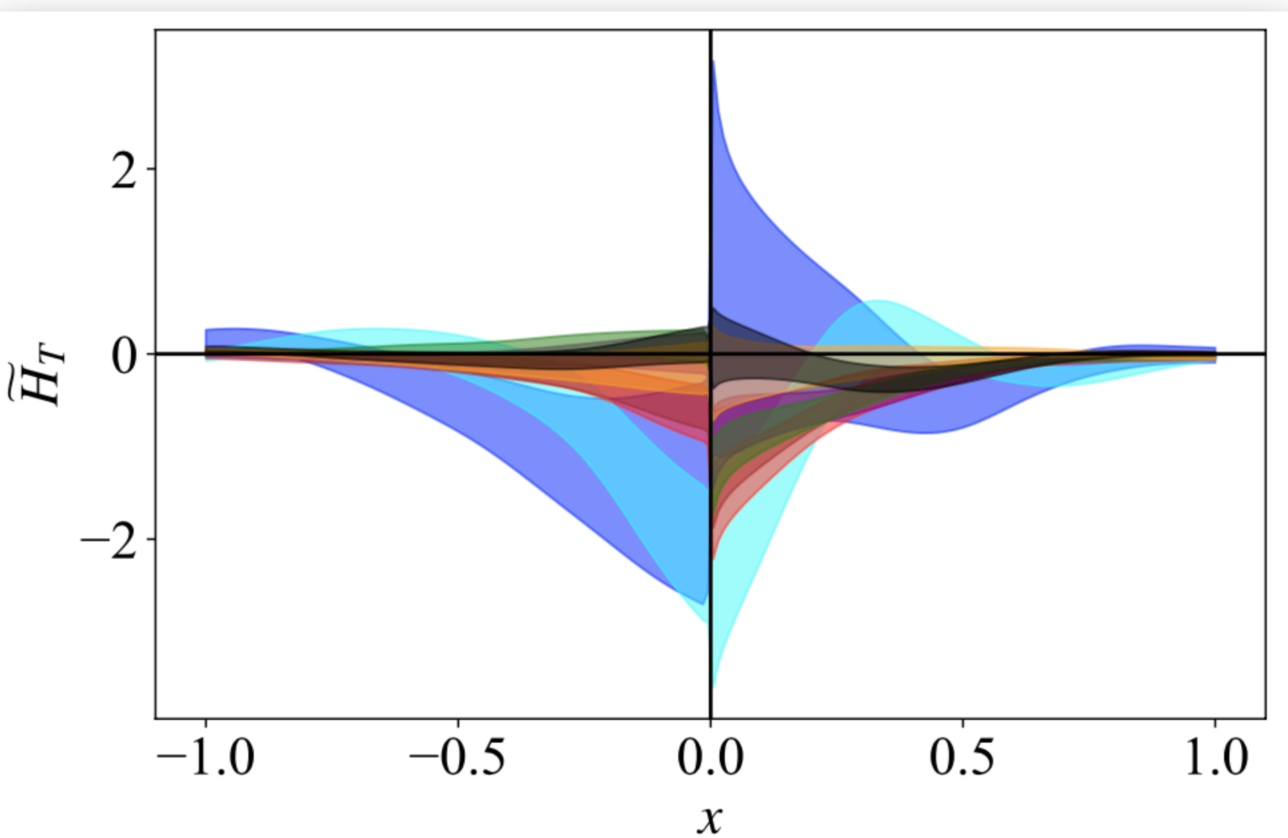

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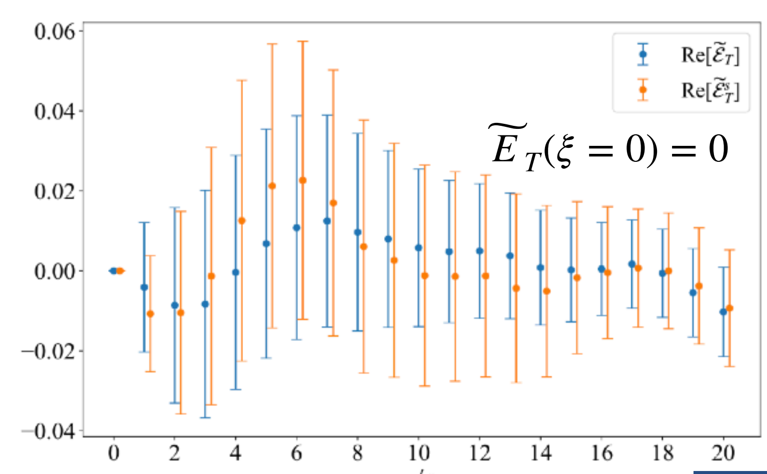
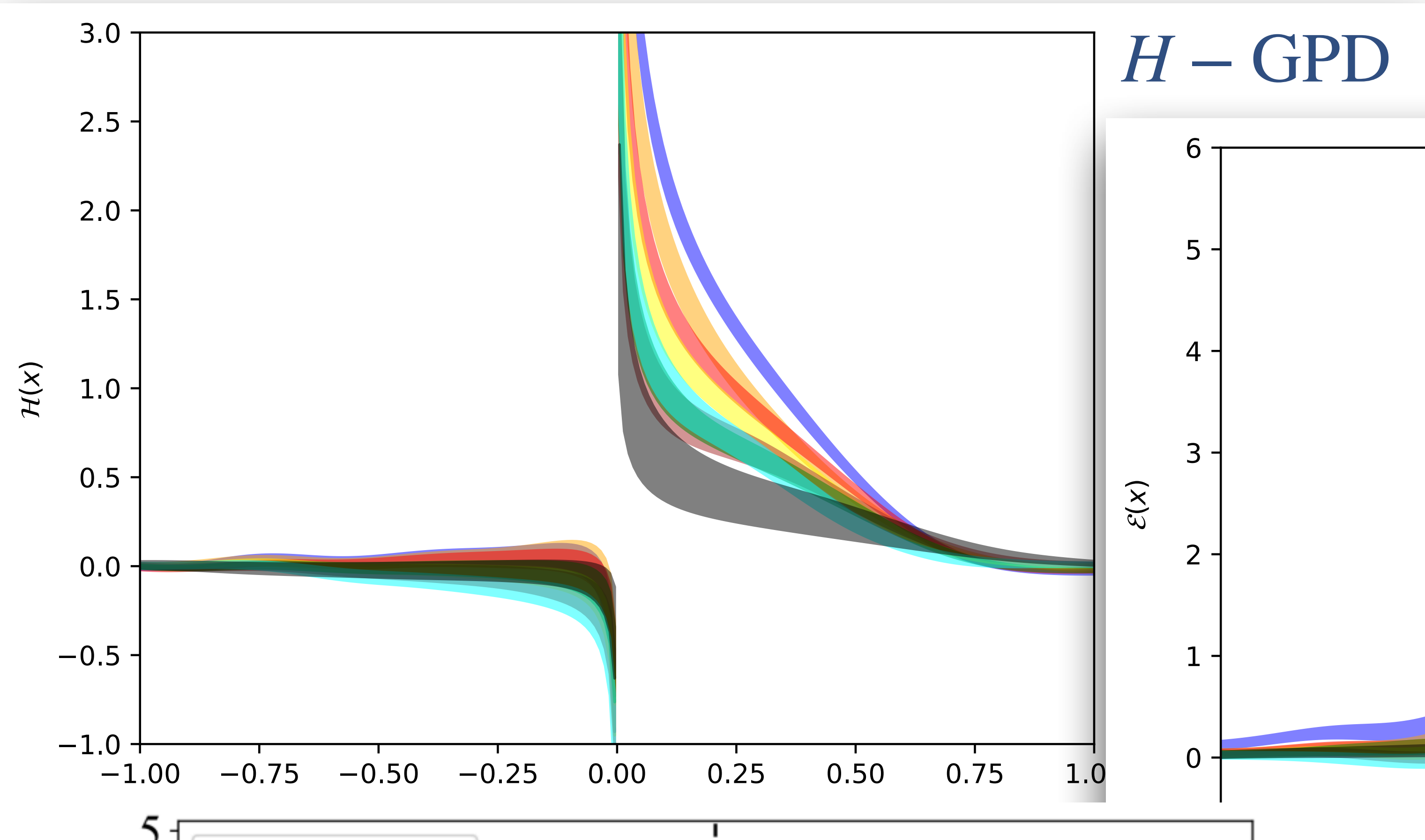
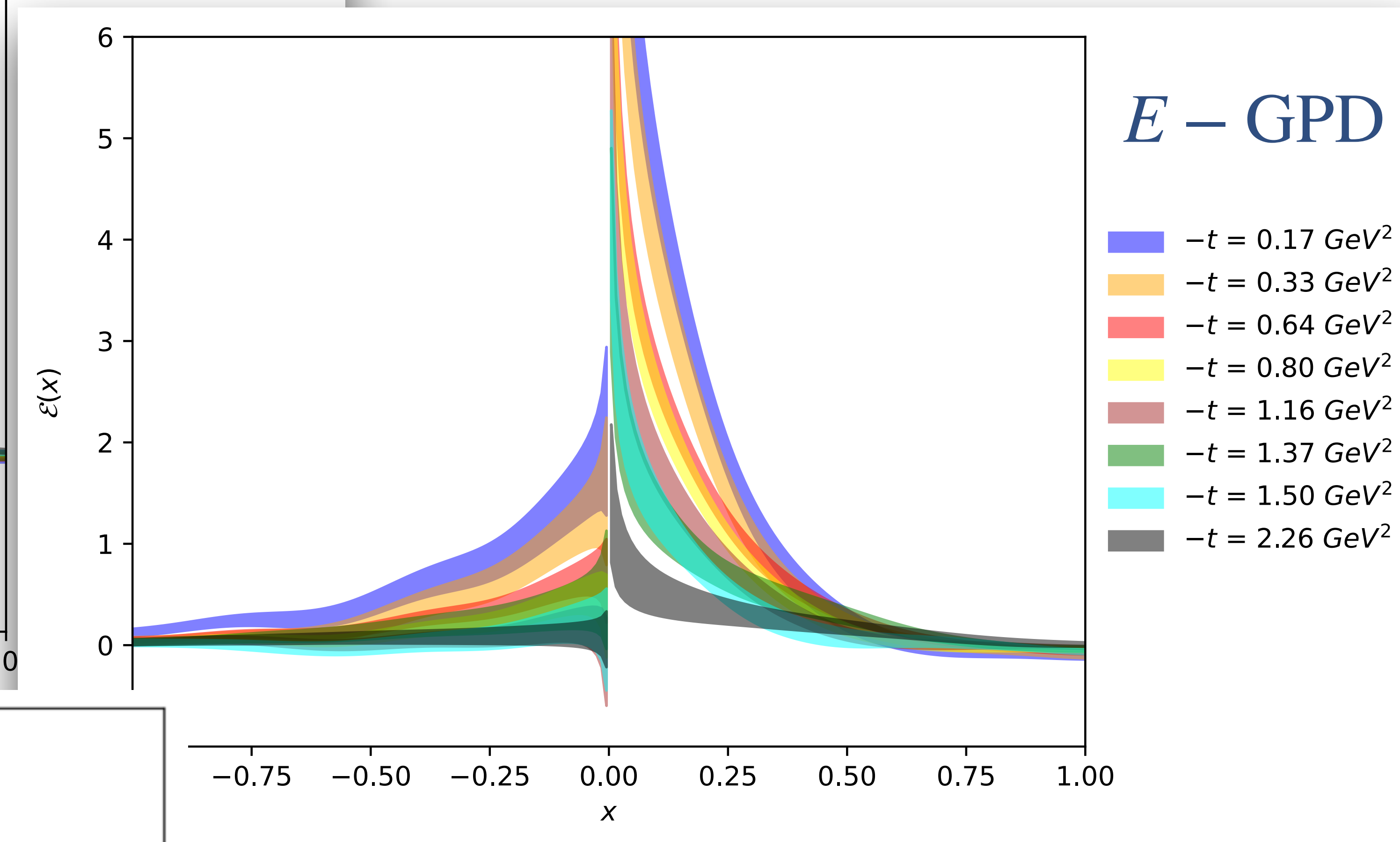
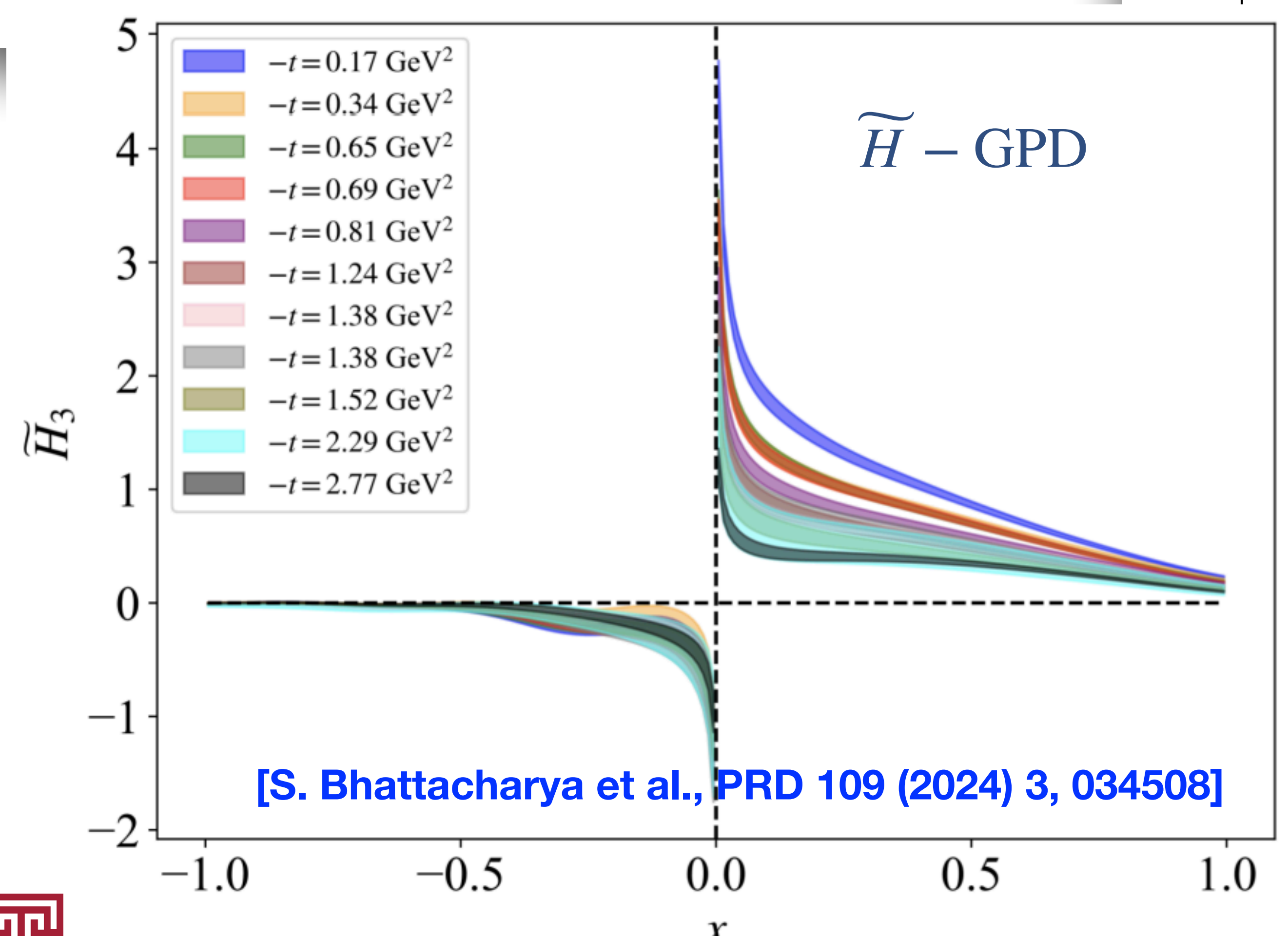


Reminder: Unpolarized & Helicity GPDs



[S. Bhattacharya et al., PRD 106 (2022) 11, 114512]

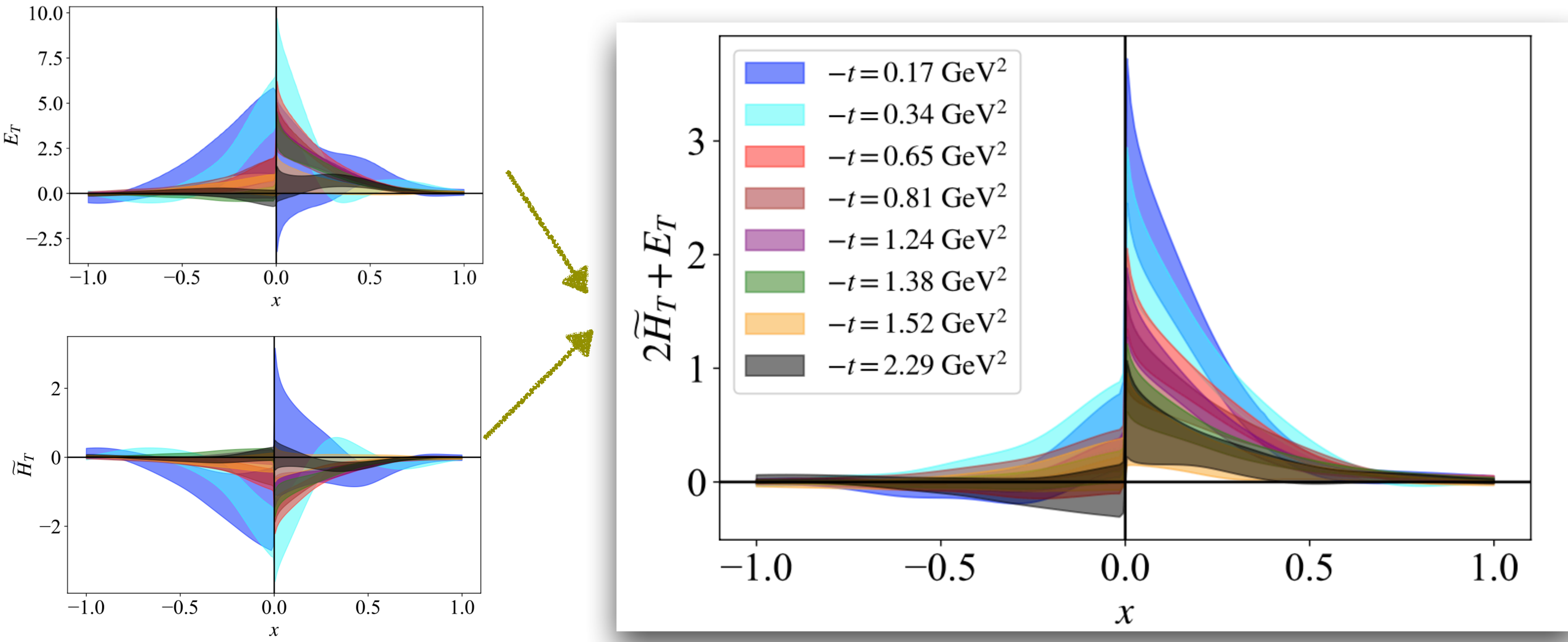




★ Signal for H_T comparable with H, $\widetilde{H}$

★ $\widetilde{E}(\xi=0)=0$

Physical Interpretation



★ $E_T + 2\widetilde{H}_T$ related to transverse spin structure of the proton

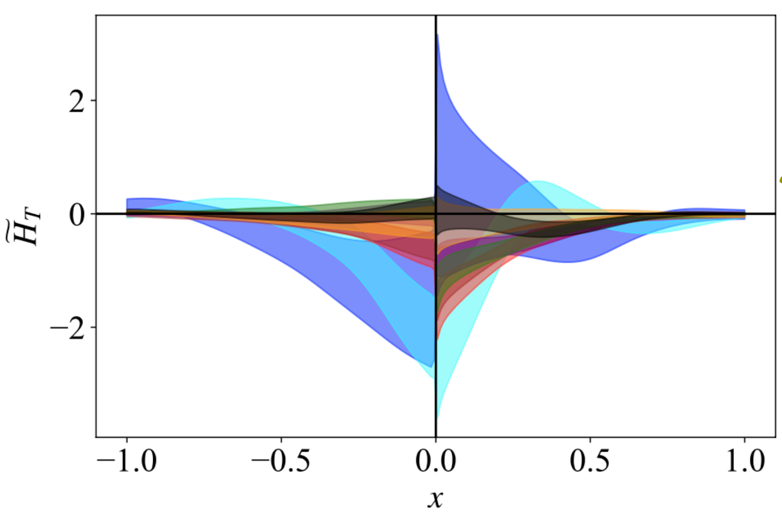
★ Impact parameter space: describes the deformation in the distribution of transversely polarized quarks within an unpolarized proton.

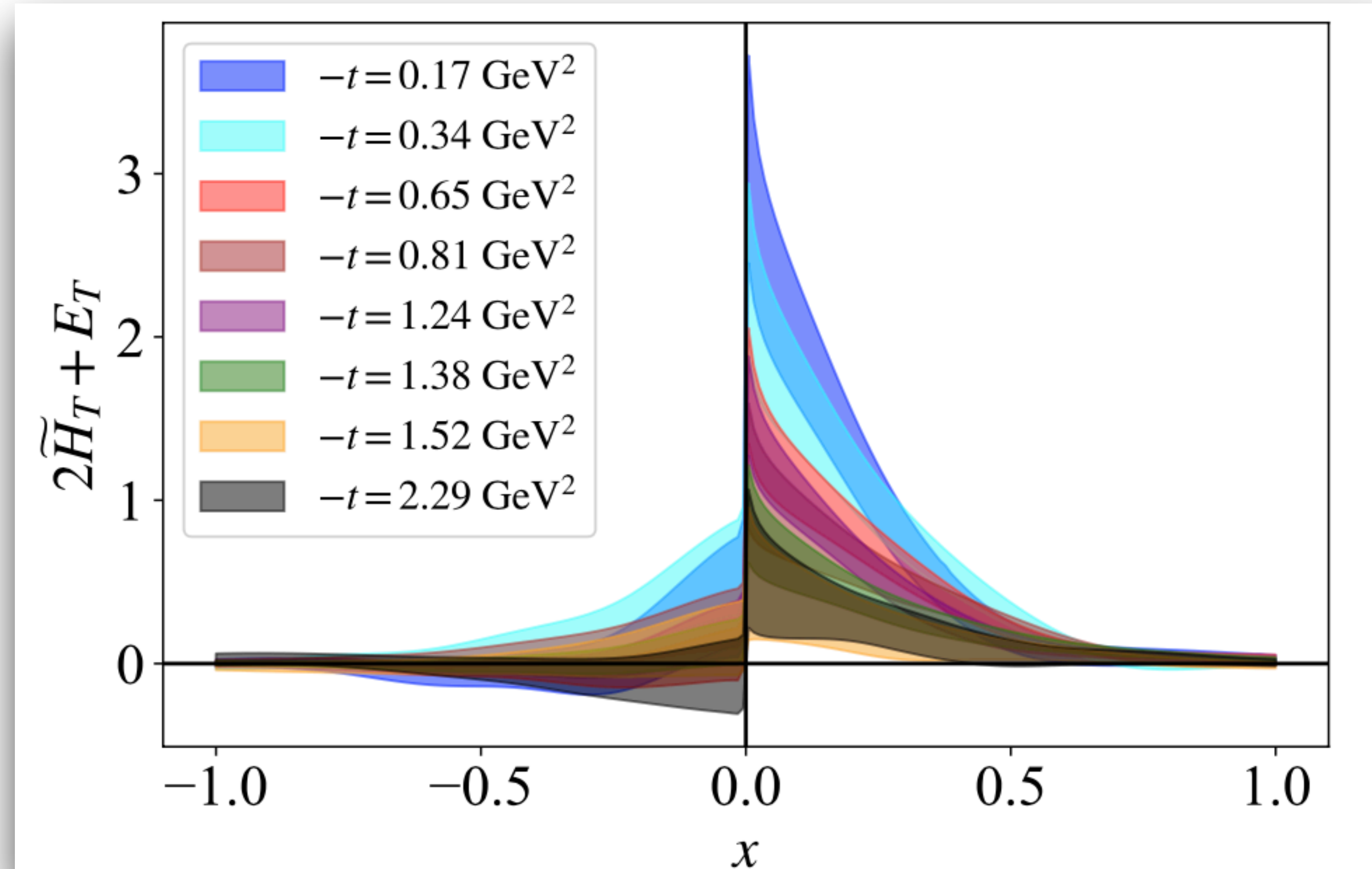
★ $k_T = \int dx \left(E_T(x,0,0) + 2\widetilde{H}_T(x,0,0) \right)$: size of dipole moment given by distribution

★ $E_T + 2\widetilde{H}_T = -A_4$ (good signal)

Physical Interpretation

E_T : No physical interpretation as a density





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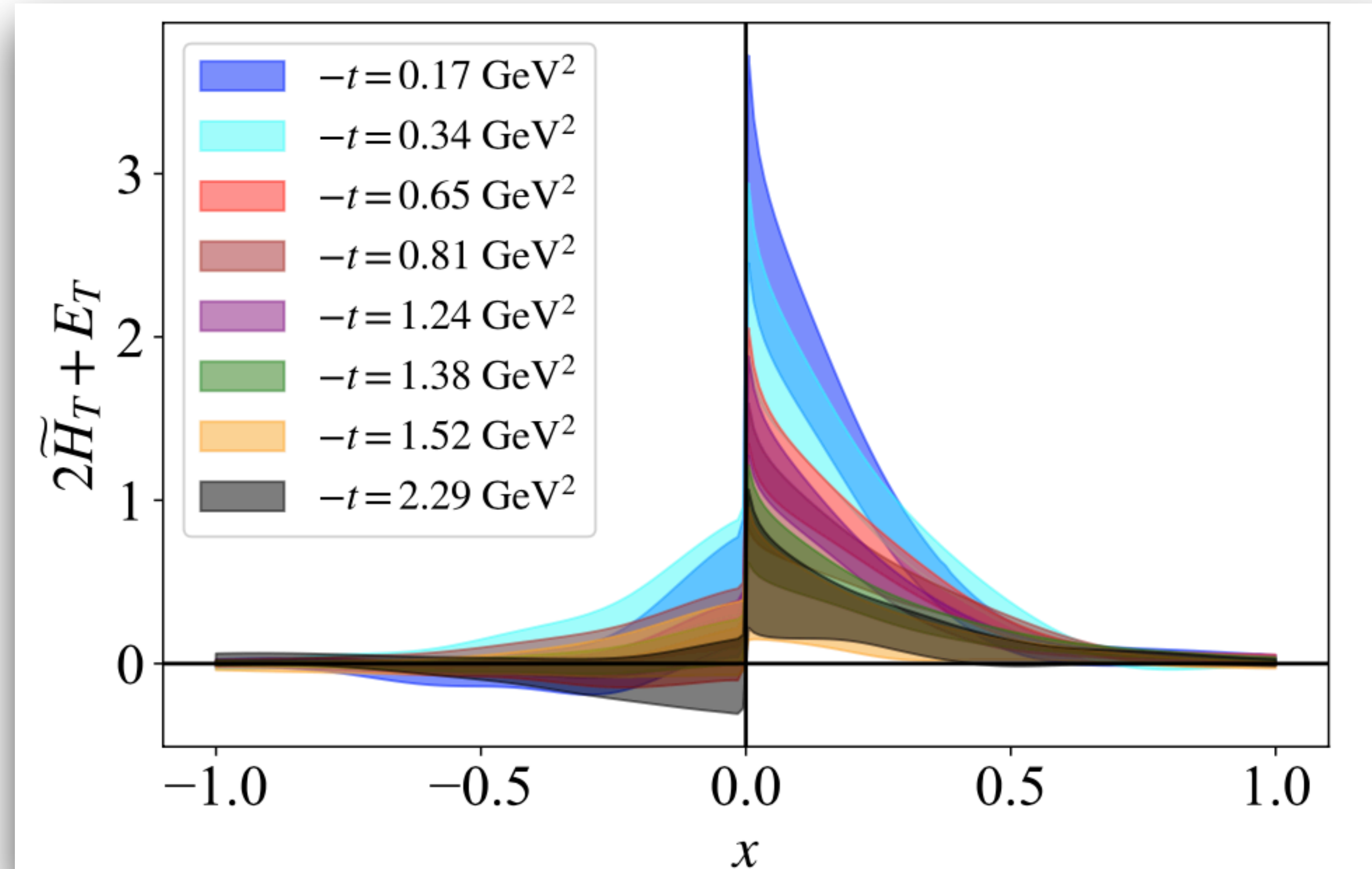
Physical Interpretation

E_T : No physical interpretation as a density

$\widetilde{H}_T$: connection with quadrupole deformation of distribution of $\perp$ pol. quarks in $\perp$ pol. proton



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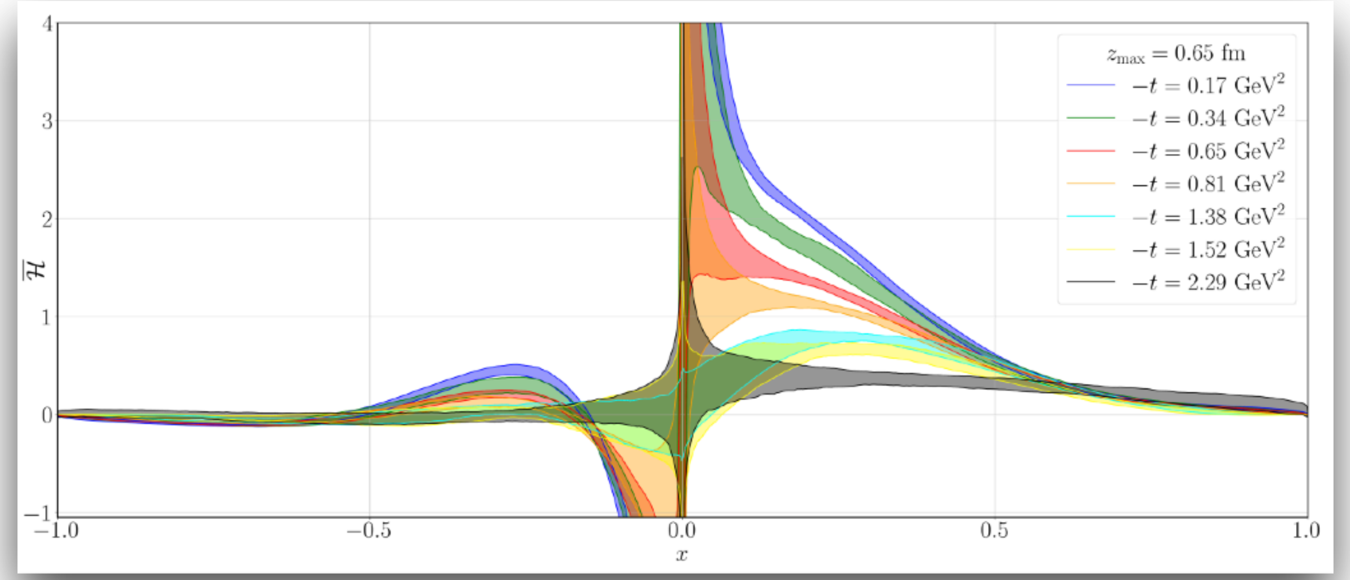
From Raw Data to

Rich Insights

Alternative approach: pseudo-ITD

Example: unpolarized GPDs case



[Battacharya et al., PRD 110 (2024) 5, 054502]

Different steps between approaches:

- renormalization

- x-dependence reconstruction

- matching formalism

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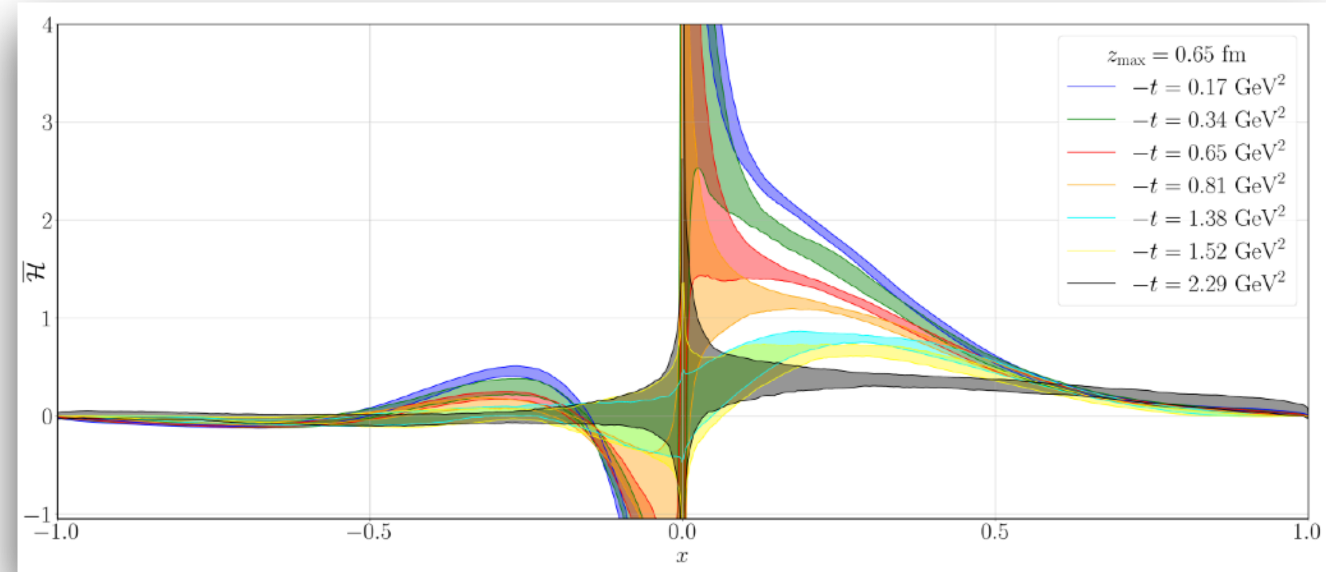
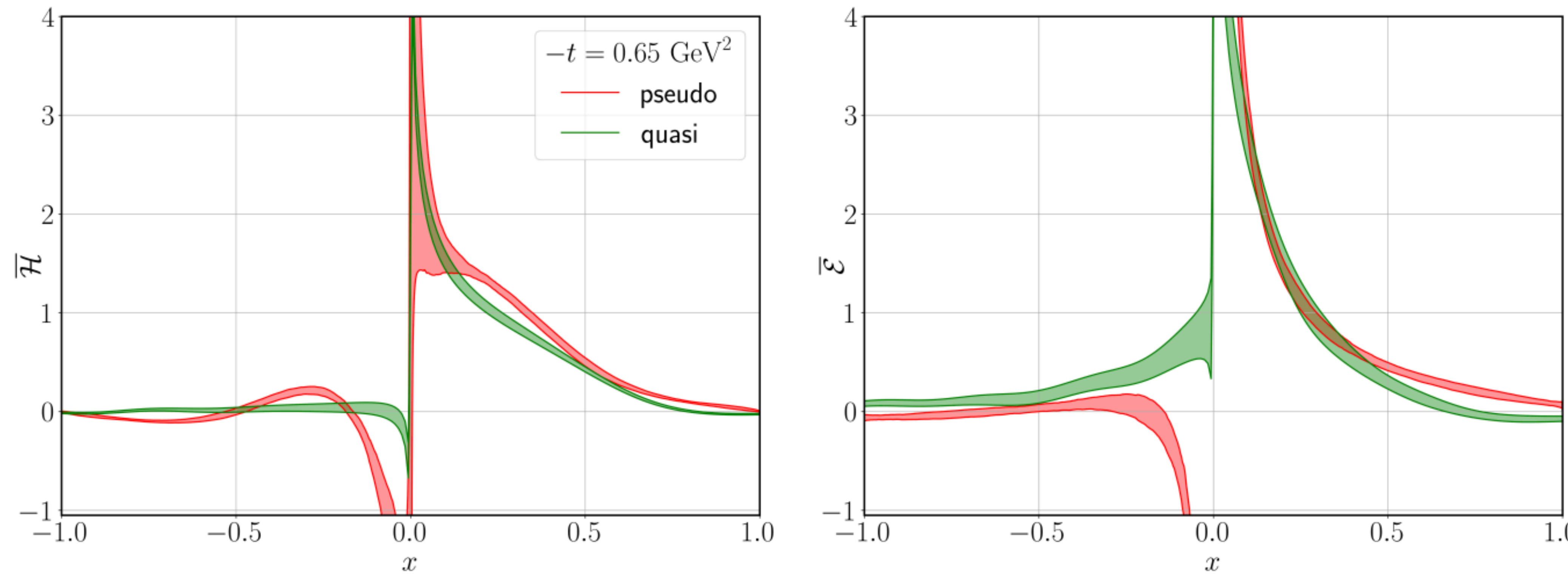


[Battacharya et al., PRD 110 (2024) 5, 054502]

Different steps between approaches:
- renormalization
- x-dependence reconstruction
- matching formalism

★ Comparison between methods helps assess systematic effects



★ x<0 and small-x regions susceptible to systematic effects

★ Comparison only includes systematic uncertainties

Mellin moments

Example: unpolarized GPDs case

★ Leading-twist factorization formula

$$\mathcal{M}(z,P,\Delta) \equiv \frac{\mathcal{F}(z,P,\Delta)}{\mathcal{F}(z,P=0,\Delta=0)} = \sum_{n=0} \frac{(-izP)^n}{n!} \frac{C_n^{\overline{\mathrm{MS}}}(\mu^2 z^2)}{C_0^{\overline{\mathrm{MS}}}(\mu^2 z^2)} \langle x^n \rangle + \mathcal{O}(\Lambda^2_{\mathrm{QCD}} z^2)$$

★ Avoid power-divergent mixing of multi-derivative operators

★ Wilson coefficients known to NLO (or NNLO)

★ Both isovector and isoscalar (ignores disconnected; found tiny) [C. Alexandrou et al., PRD 104 (2021) 5, 054503]

Mellin moments

Example: unpolarized GPDs case

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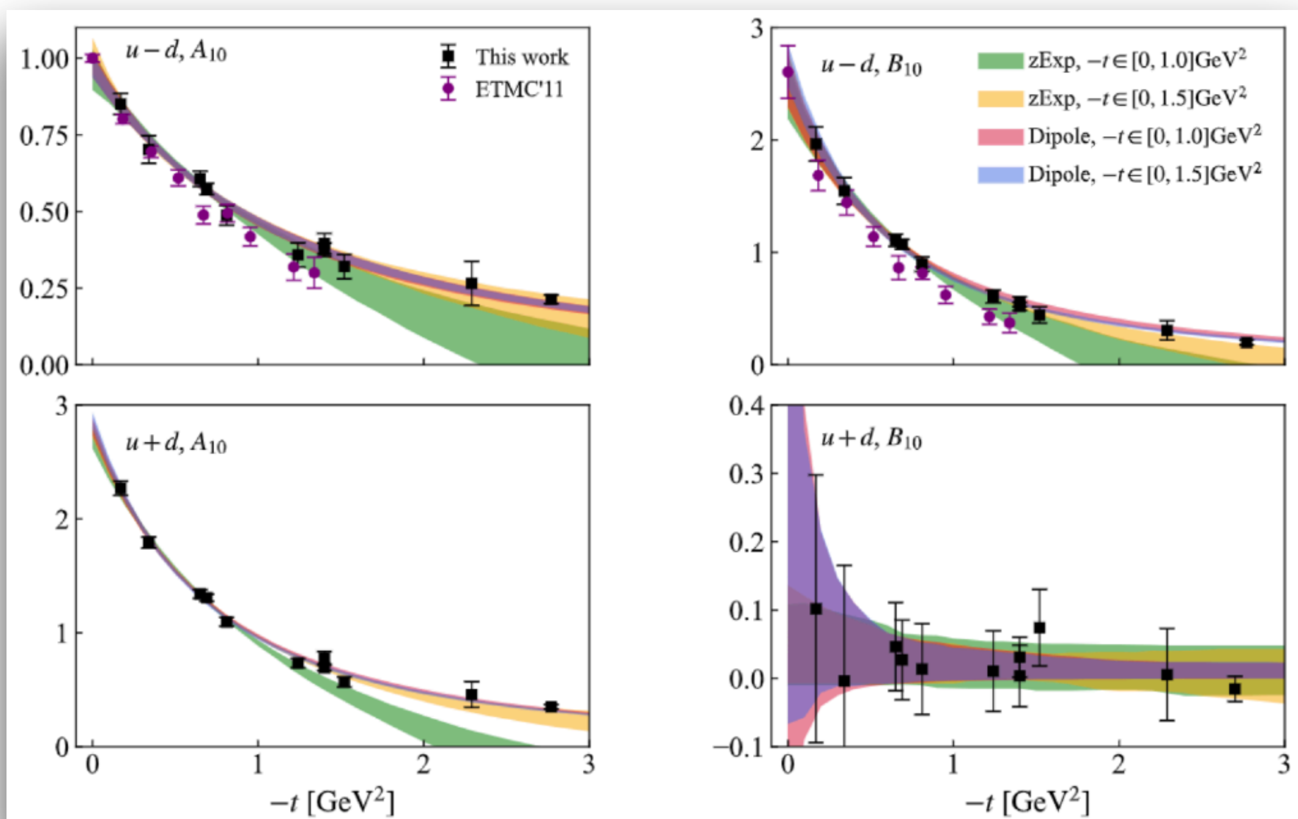
★ Avoid power-divergent mixing of multi-derivative operators

★ Wilson coefficients known to NLO (or NNLO)

★ Both isovector and isoscalar (ignores disconnected; found tiny) [C. Alexandrou et al., PRD 104 (2021) 5, 054503]

[S. Bhattacharya et al., PRD 108 (2023) 1, 014507; arXiv:2410.03539]



Mellin moments

Example: unpolarized GPDs case

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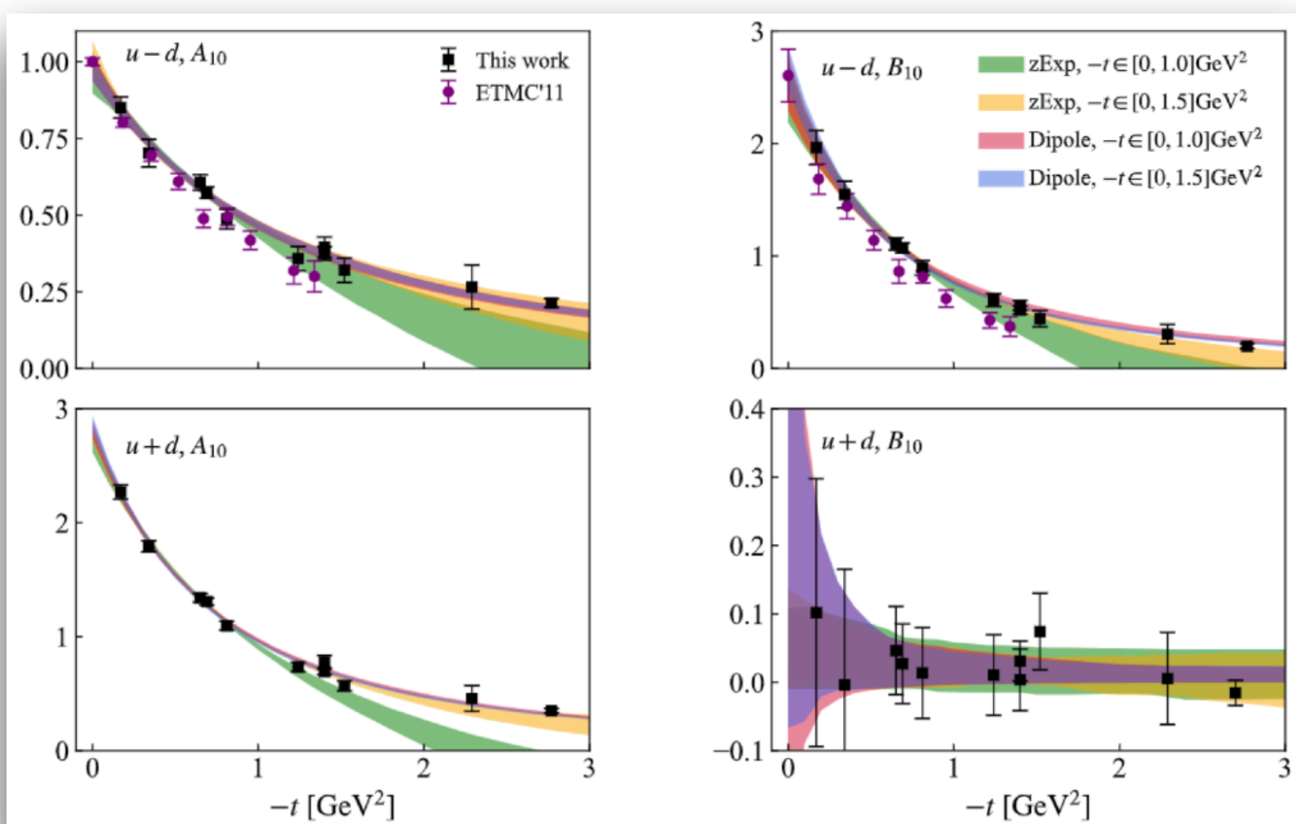

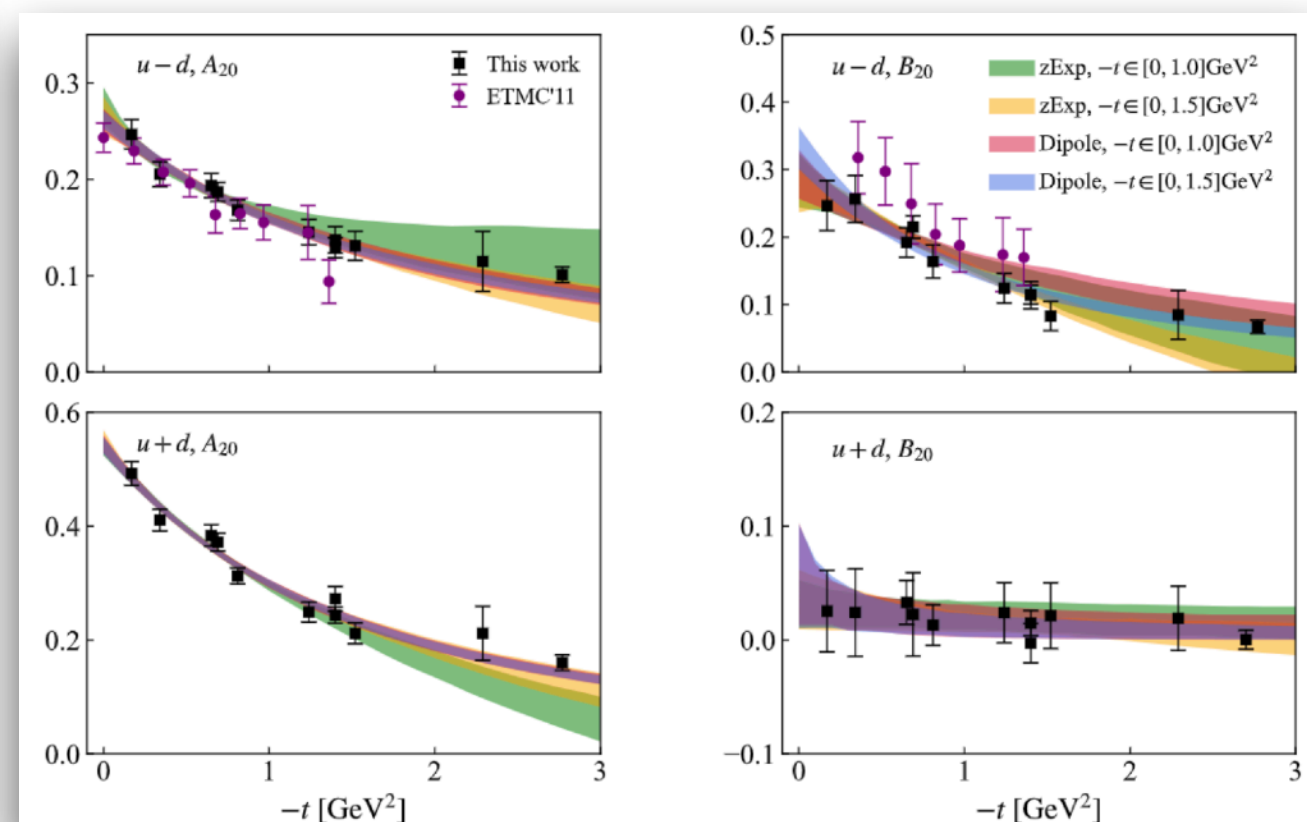

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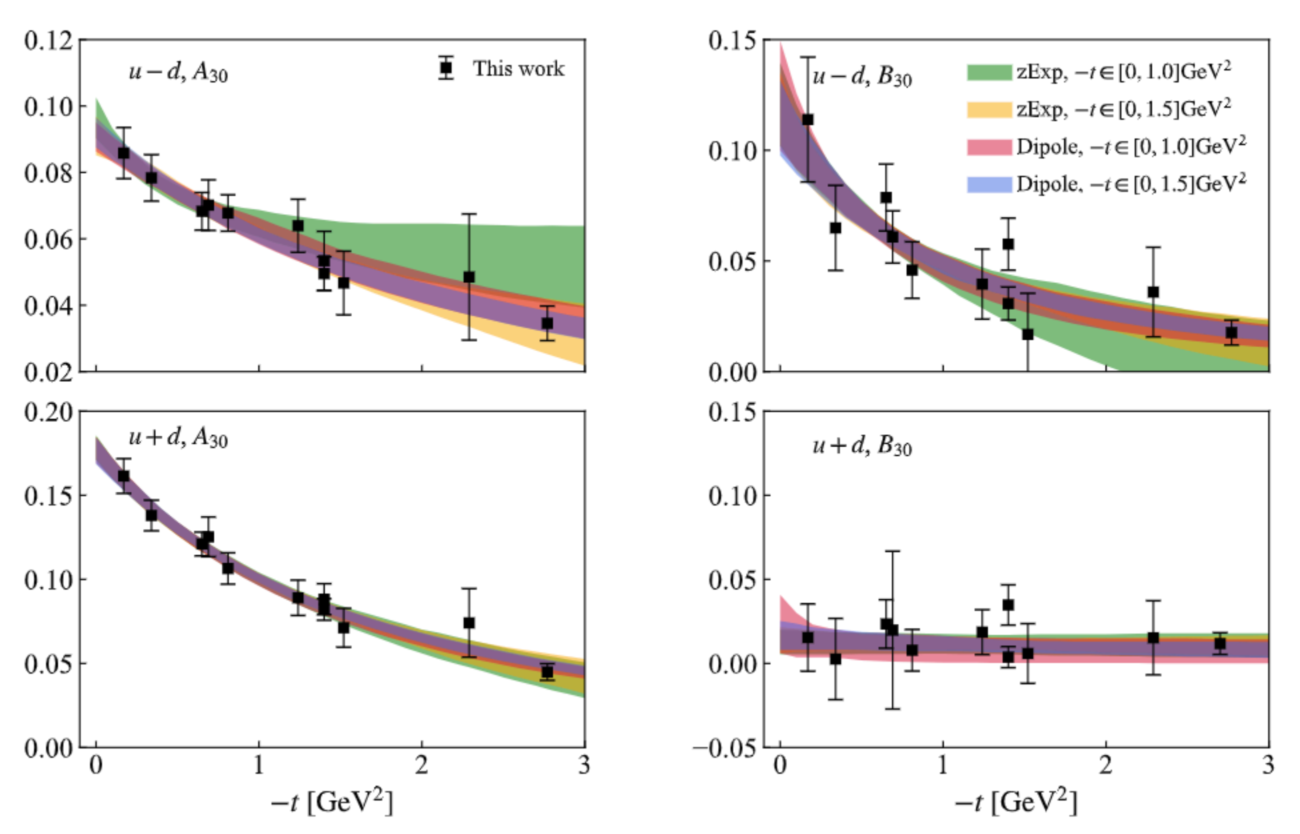

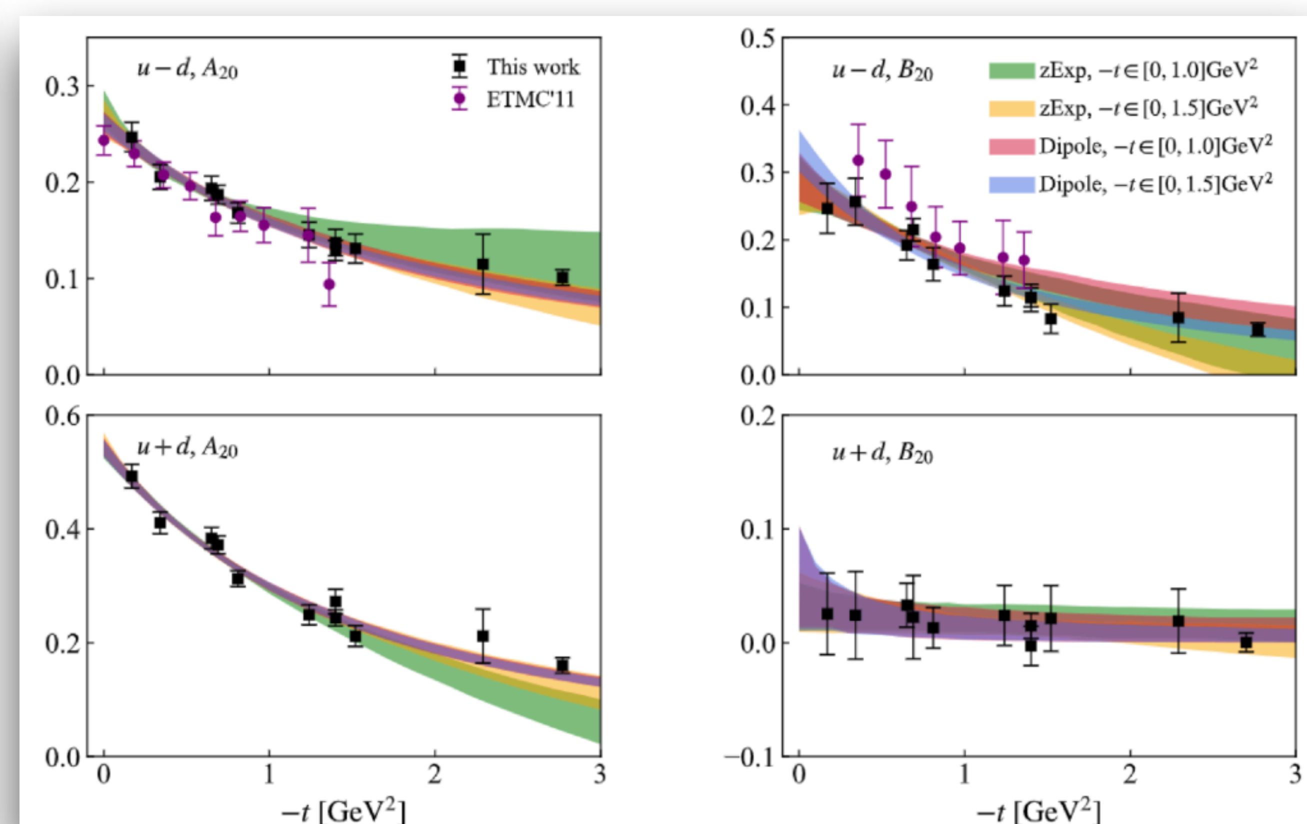

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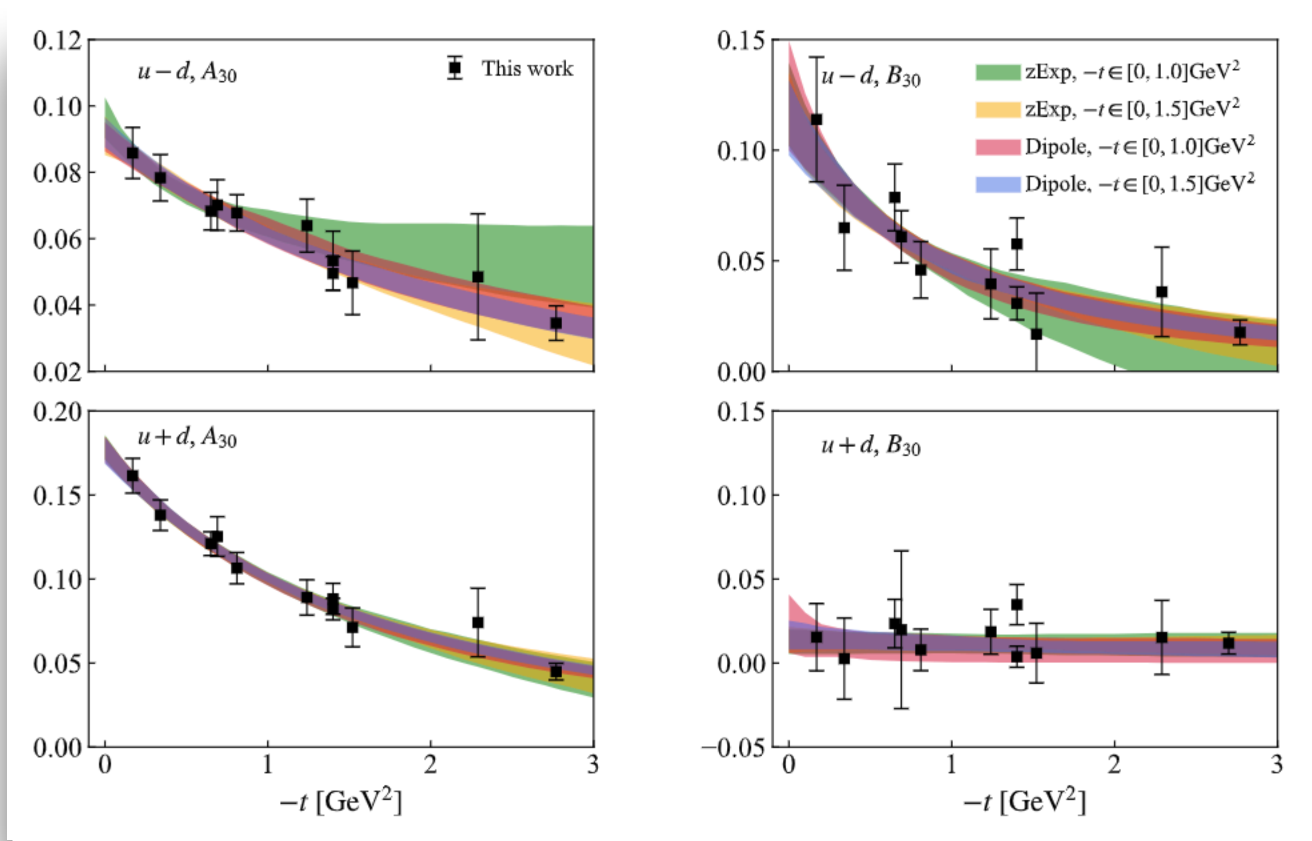

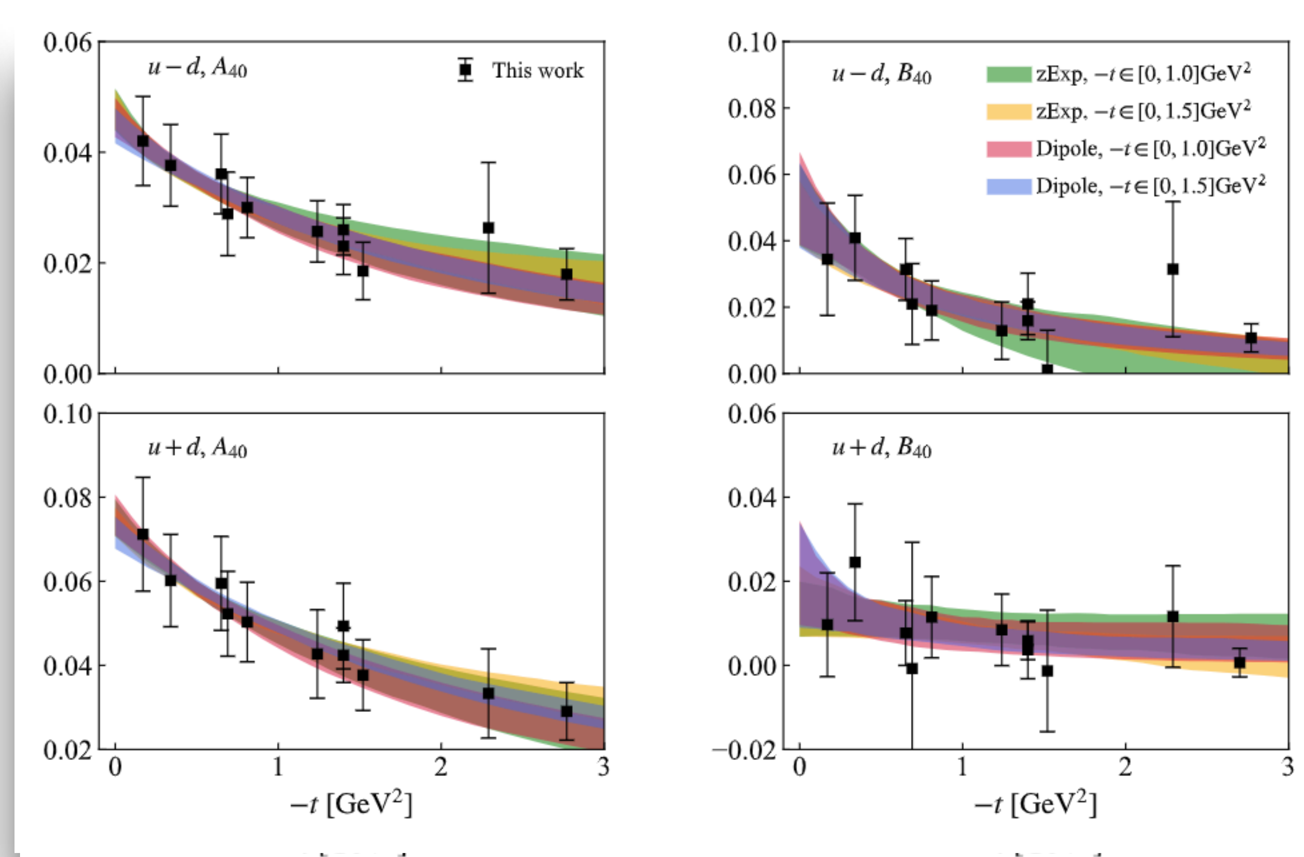

Beyond leading twist

- ★ Lack density interpretation, but have physical interpretation
- ★ Contain information about quark-gluon correlations inside hadrons
- ★ Sensitive to soft dynamics
- ★ Appear in QCD factorization theorems for various observables
- ★ Challenging to probe experimentally and isolate from leading-twist
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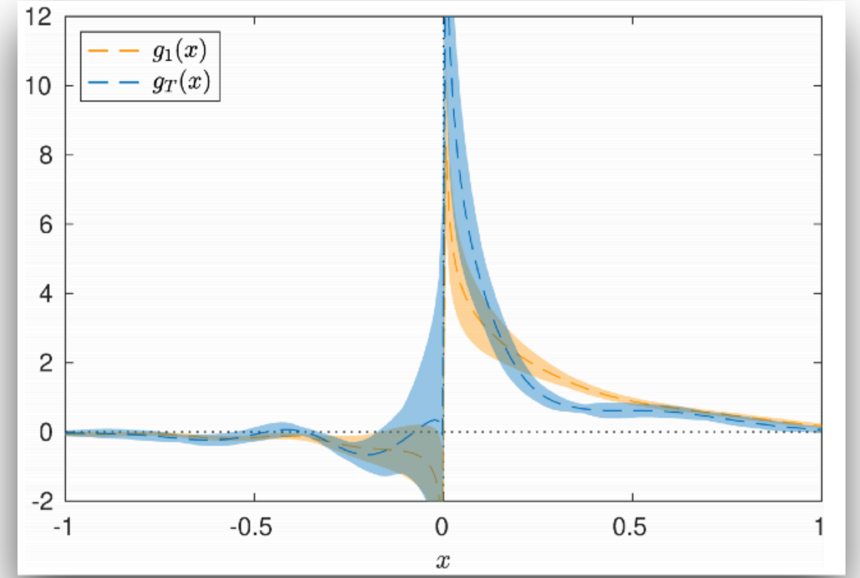

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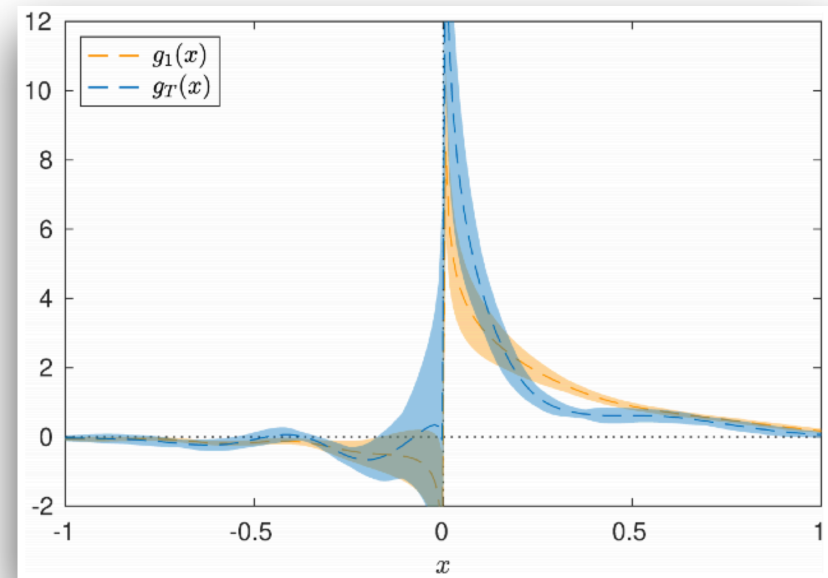
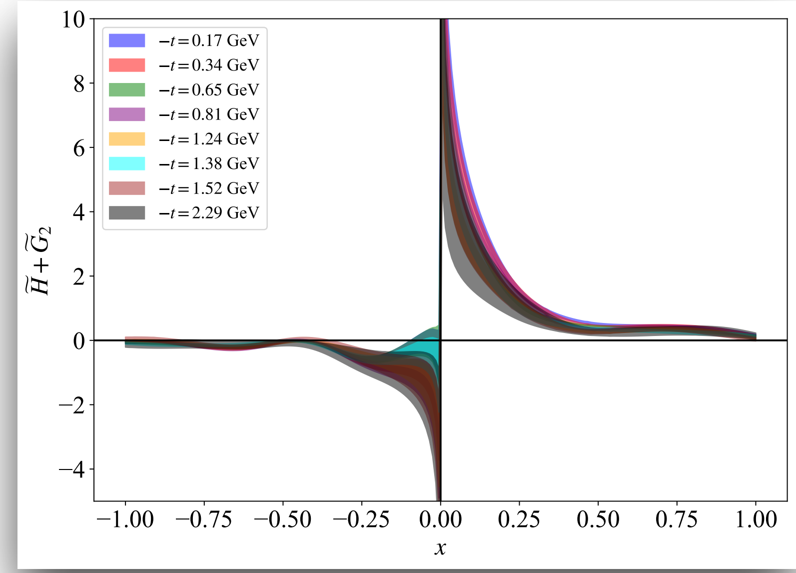
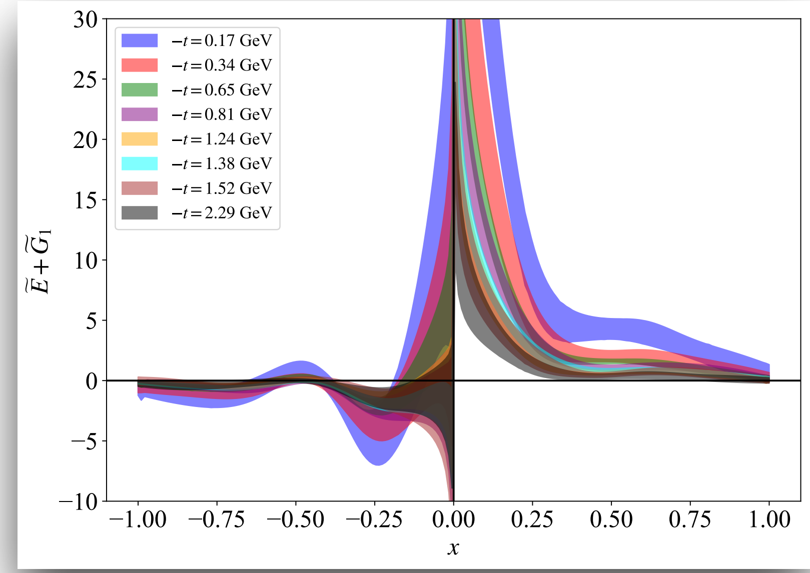


- ★ Extraction of twist-3 distributions from lattice QCD is very challenging
- ★ Mixing with q-g-q correlators; matching:
  [V. Braun et al., JHEP 05 (2021) 086; JHEP 10 (2021) 087]
- ★ Kinematic twist-3 contributions to pseudo & quasi GPDs to restore translation invariance
  [V. Braun et al., JHEP 10 (2023) 134]

Amplitude decomposition

Example: axial twist-3 GPDs





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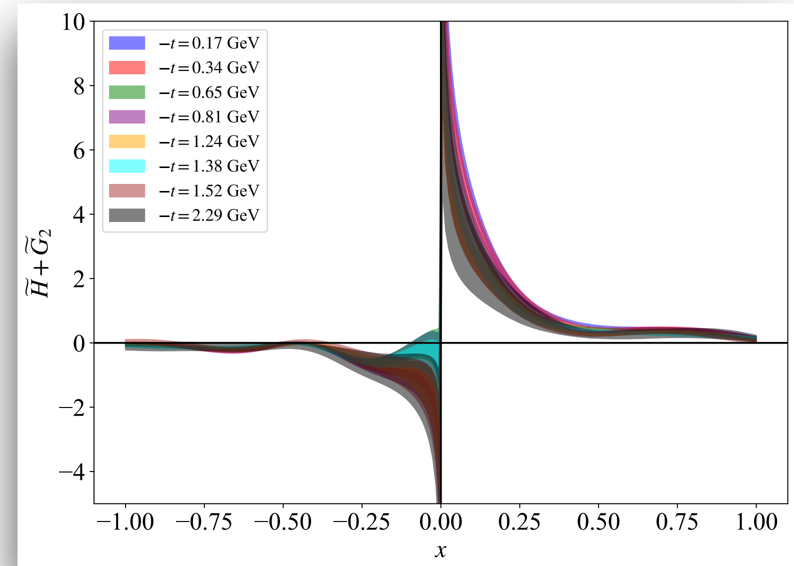
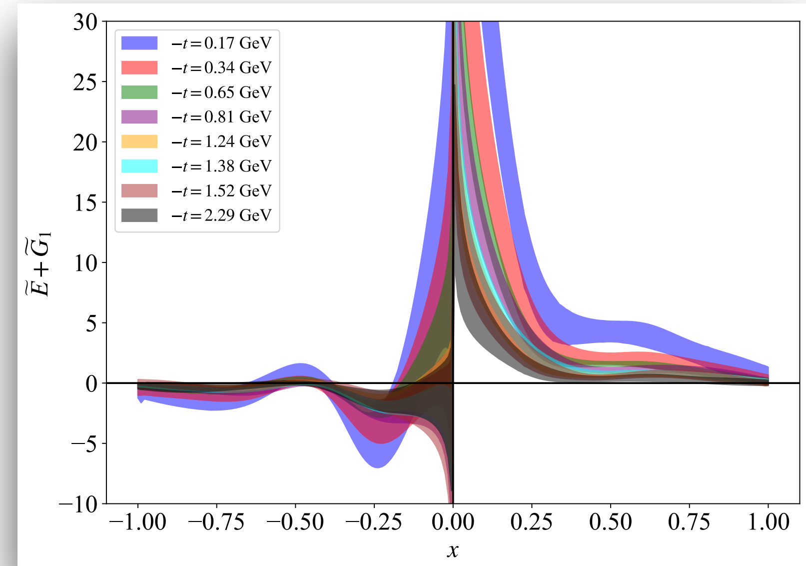
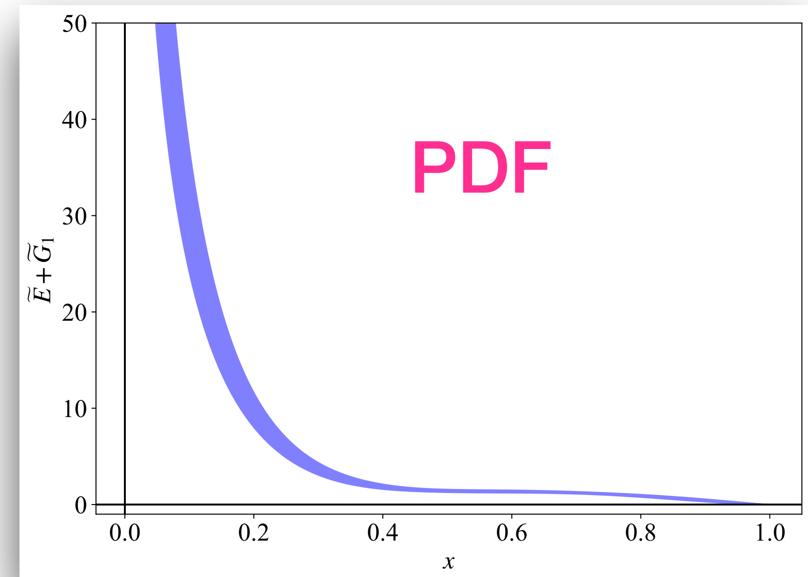




Parametrization, gives info, e.g., access to $\widetilde{E}$-GPD even at zero skewness



$$\int_{-1}^{1} dx\, \widetilde{E}(x,\xi,t) = G_P(t) \qquad \int_{-1}^{1} dx\, \widetilde{G}_i(x,\xi,t) = 0\,,$$

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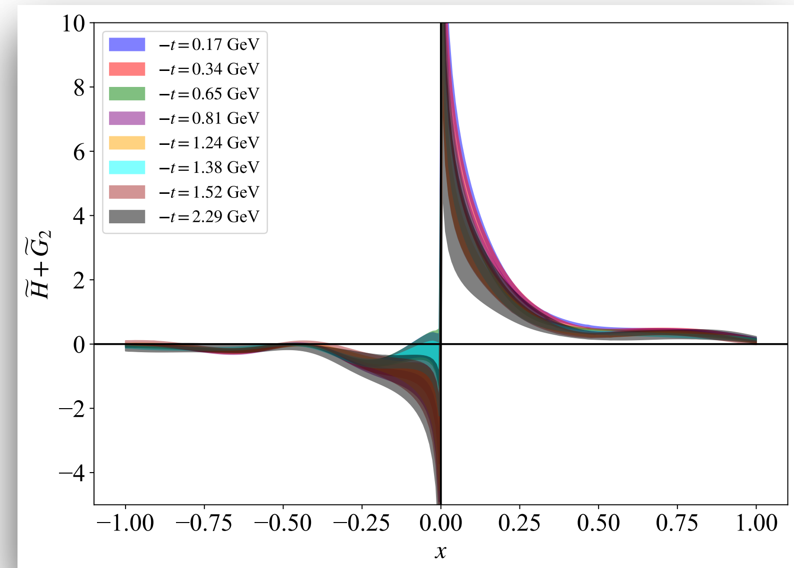

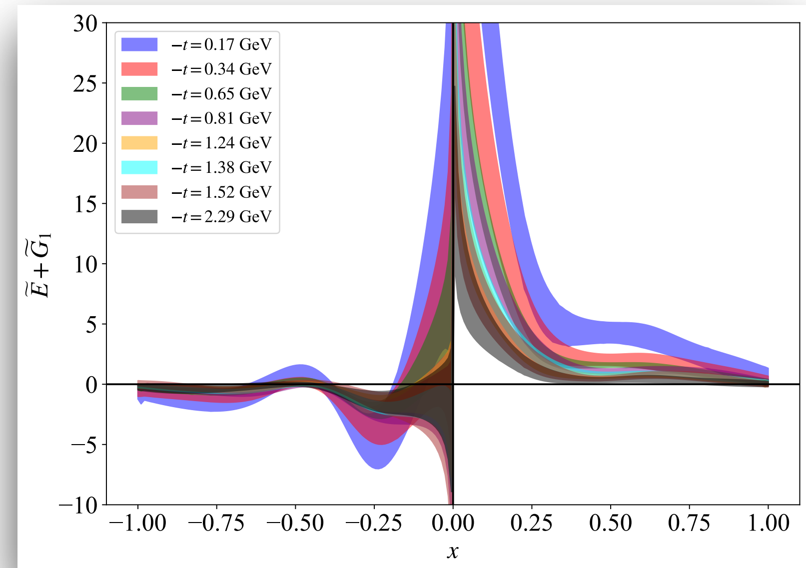

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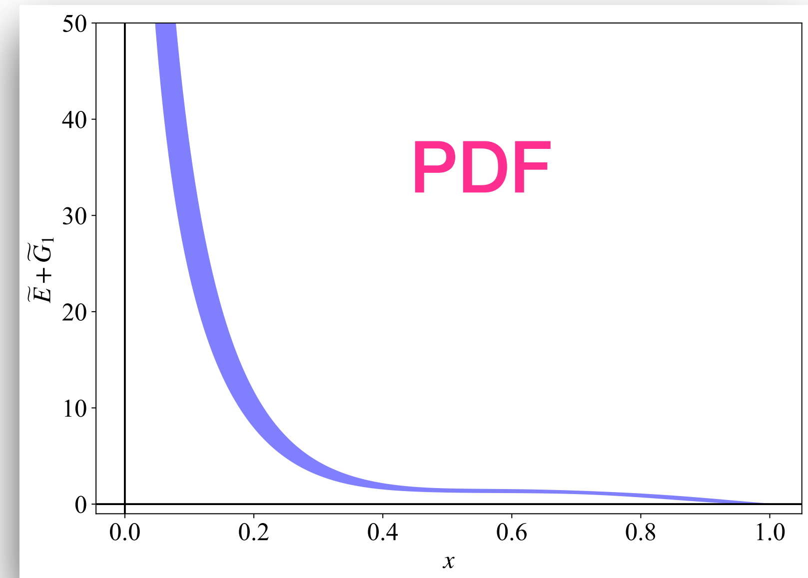

Similar methodology for tensor twist-3 GPDs (chiral odd)

$$F^{[\sigma^{+-}\gamma_5]} = \bar{u}(p')\left(\gamma^+\gamma_5\,\widetilde{H}_2' + \frac{P^+\gamma_5}{M}\,\widetilde{E}_2'\right)u(p)$$

[Meissner et al., *JHEP* 08 (2009) 056]

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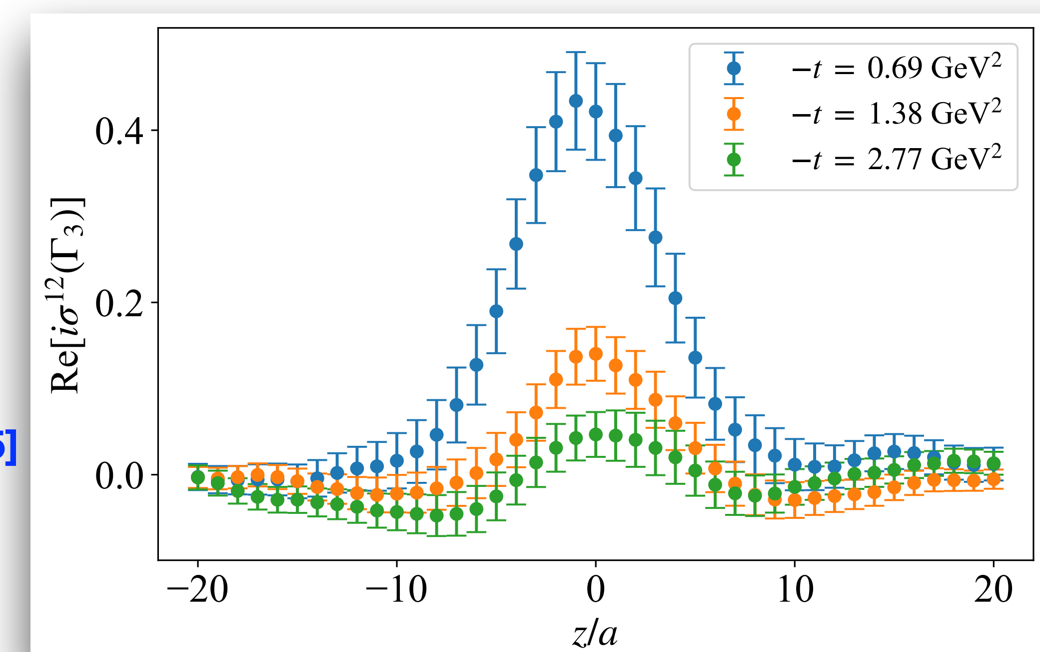
★ Fwd limit (h_L) may be accessed via:

- double-polarized Drell-Yan process
[R. Jaffe, PRL 67 (1991) 552-555; Y. Koike et al., PLB 668 (2008) 286]

- di-hadron single spin asymmetries (CLAS)
[Gliske et al., PRD 90 (2014) 11, 114027; A. Vossen, CIPANP2018, arXiv: 1810.02435]

- single-inclusive particle production in proton-proton collisions [Y. Koike et al., PLB 759 (2016) 75]

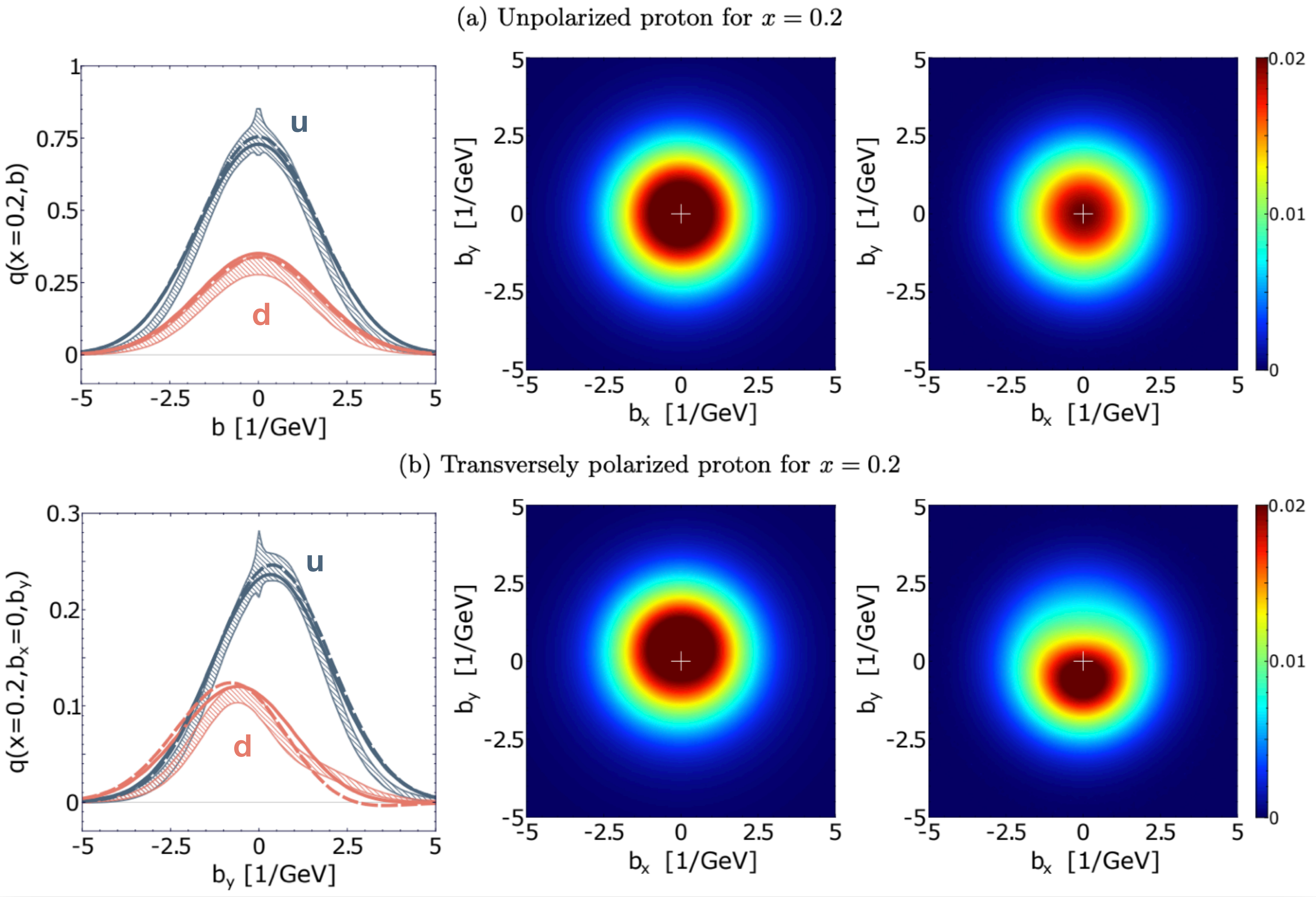


Synergy/Complementarity of lattice and phenomenology

Toward synergy for GPDs

[K. Cichy et al., PRD 110 (2024) 11, 114025]

Example: unpolarized GPDs



- GK (solid line),
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- Further study needed on how to combine lattice results with data

How to lattice QCD data fit into the overall effort for hadron tomography

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PHENOMENOLOGY
LATTICE QCD
THEORY

QUARK-GLUON TOMOGRAPHY COLLABORATION

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Award Number: DE-SC0023646

1. Theoretical studies of high-momentum transfer processes using perturbative QCD methods and study of GPDs properties
2. Lattice QCD calculations of GPDs and related structures
3. Global analysis of GPDs based on experimental data using modern data analysis techniques for inference and uncertainty quantification

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Other GPD global analysis efforts:

- Gepard [https://gepard.phy.hr/]
- PARTONS [https://partons.cea.fr]
- EXCLAIM [https://exclaimcollab.github.io/web.github.io/#/]

Concluding Remarks

- ★ New developments in several promising directions
- ★ Extensive program in extracting GPDs from lattice QCD
- ★ New methods can optimize computational resources
- ★ Access to higher-twist GPDs feasible from lattice QCD
- ★ Synergy with phenomenology has the potential to enhance the impact of lattice QCD data and complement data sets

DOE Early Career Award
Grant No. DE-SC0020405 &
Grant No. DE-SC0025218

Award Number:
DE-SC0023646

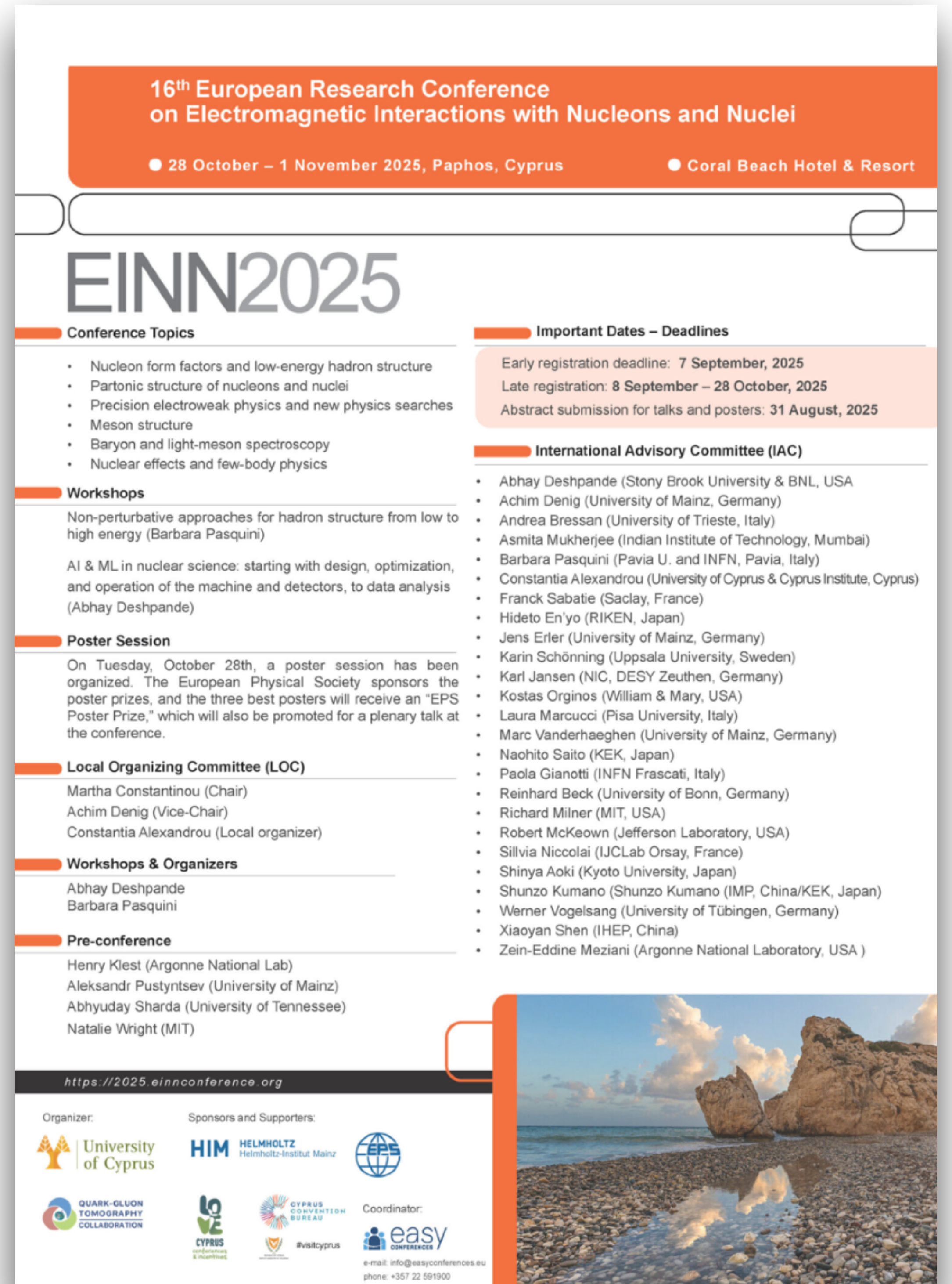
Join us at EINN 2025

https://2025.einnconference.org/

28 October – 01 November, 2025

Frontiers and Careers Workshops: 26 - 27 October, 2025

Organizers:
M. Constantinou (Chair)
A. Denig (Vice-Chair)
C. Alexandrou
A. Deshpande
B. Pasquini



16th European Research Conference on Electromagnetic Interactions with Nucleons and Nuclei

28 October – 1 November 2025, Paphos, Cyprus — Coral Beach Hotel & Resort

EINN2025

Conference Topics
- Nucleon form factors and low-energy hadron structure
- Partonic structure of nucleons and nuclei
- Precision electroweak physics and new physics searches
- Meson structure
- Baryon and light-meson spectroscopy
- Nuclear effects and few-body physics

Workshops

Non-perturbative approaches for hadron structure from low to high energy (Barbara Pasquini)

AI & ML in nuclear science: starting with design, optimization, and operation of the machine and detectors, to data analysis (Abhay Deshpande)

Poster Session

On Tuesday, October 28th, a poster session has been organized. The European Physical Society sponsors the poster prizes, and the three best posters will receive an "EPS Poster Prize," which will also be promoted for a plenary talk at the conference.

Local Organizing Committee (LOC)

Martha Constantinou (Chair)
Achim Denig (Vice-Chair)
Constantia Alexandrou (Local organizer)

Workshops & Organizers

Abhay Deshpande
Barbara Pasquini

Pre-conference

Henry Klest (Argonne National Lab)
Aleksandr Pustyntsev (University of Mainz)
Abhyuday Sharda (University of Tennessee)
Natalie Wright (MIT)

Important Dates – Deadlines

Early registration deadline: 7 September, 2025
Late registration: 8 September – 28 October, 2025
Abstract submission for talks and posters: 31 August, 2025

International Advisory Committee (IAC)
- Abhay Deshpande (Stony Brook University & BNL, USA
- Achim Denig (University of Mainz, Germany)
- Andrea Bressan (University of Trieste, Italy)
- Asmita Mukherjee (Indian Institute of Technology, Mumbai)
- Barbara Pasquini (Pavia U. and INFN, Pavia, Italy)
- Constantia Alexandrou (University of Cyprus & Cyprus Institute, Cyprus)
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- Karl Jansen (NIC, DESY Zeuthen, Germany)
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- Sillvia Niccolai (IJCLab Orsay, France)
- Shinya Aoki (Kyoto University, Japan)
- Shunzo Kumano (Shunzo Kumano (IMP, China/KEK, Japan)
- Werner Vogelsang (University of Tübingen, Germany)
- Xiaoyan Shen (IHEP, China)
- Zein-Eddine Meziani (Argonne National Laboratory, USA)

https://2025.einnconference.org

Organizer: University of Cyprus

Sponsors and Supporters: HIM Helmholtz-Institut Mainz; Quark-Gluon Tomography Collaboration; Cyprus Convention Bureau

Coordinator: easy conferences

Abstract submission is Open!

Other topics relevant to EINN
Poster
Talk in workshop 1 "Non-perturbative approaches for hadron struct
Talk in workshop 2: "AI & ML in nuclear science: starting with design,

Thank you

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PHENOMENOLOGY · LATTICE QCD · THEORY

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INCITE
LEADERSHIP COMPUTING

NERSC

ASCR Leadership Computing Challenge
ALCC

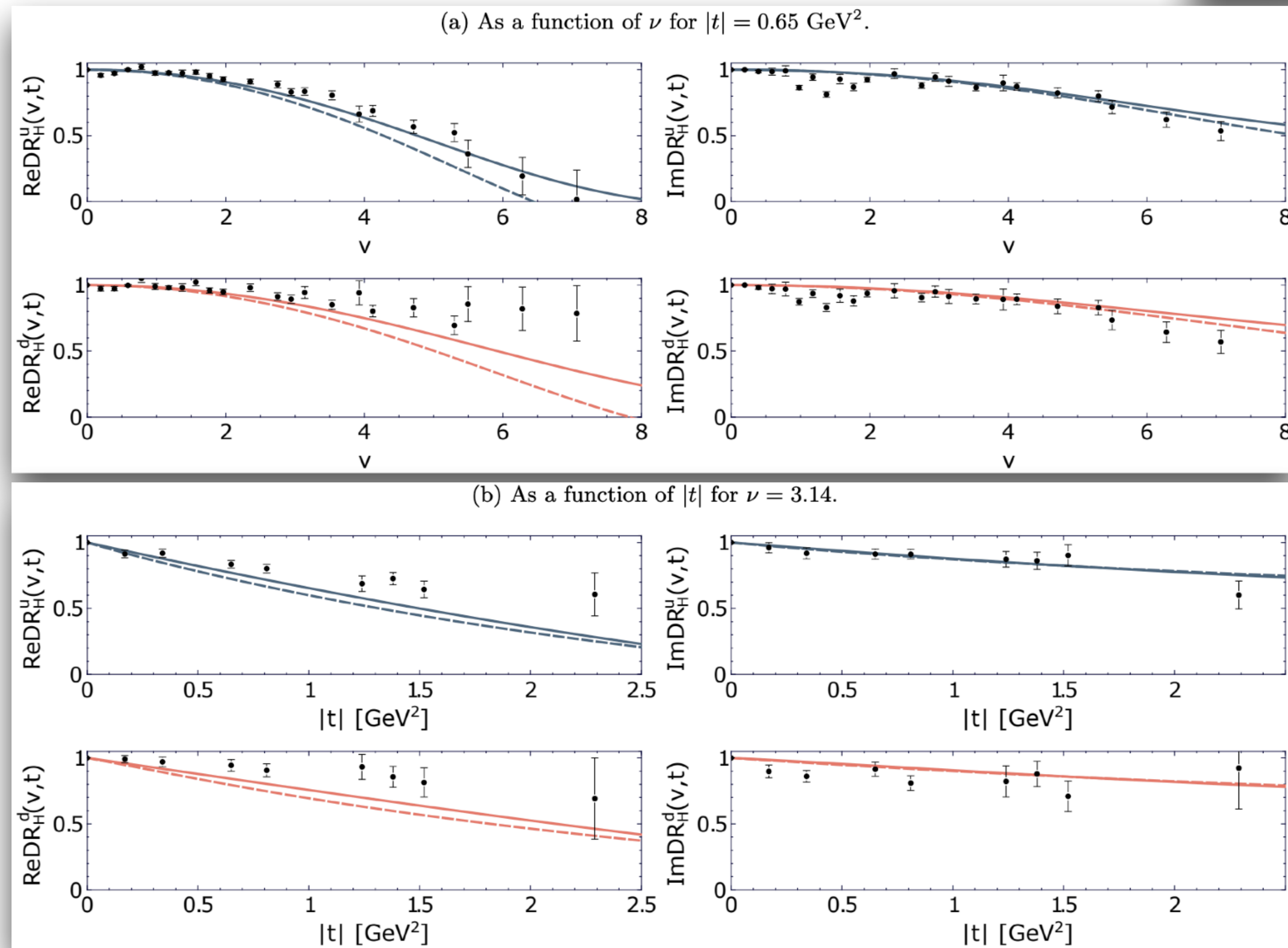
Toward synergy for GPDs

★ Forming ratios of GPDs seems to suppress systematic uncertainties

[K. Cichy et al., PRD 110 (2024) 11, 114025]

$$\mathrm{DR}_{\mathrm{Re}}^{\hat{H}^q}(\nu,t) = \frac{\mathrm{Re}\hat{H}^q(\nu,t)}{\mathrm{Re}\hat{H}^q(\nu,0)}\frac{\mathrm{Re}\hat{H}^q(0,0)}{\mathrm{Re}\hat{H}^q(0,t)},$$

$$\mathrm{DR}_{\mathrm{Im}}^{\hat{H}^q}(\nu,t) = \lim_{\nu'\to 0}\frac{\mathrm{Im}\hat{H}^q(\nu,t)}{\mathrm{Im}\hat{H}^q(\nu,0)}\frac{\mathrm{Im}\hat{H}^q(\nu',0)}{\mathrm{Im}\hat{H}^q(\nu',t)}$$



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