

# THE MECHANICAL STRUCTURE OF THE PROTON AT SOLID

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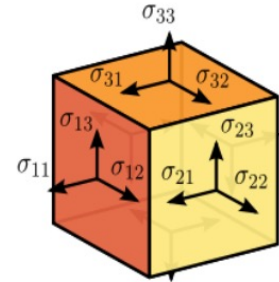
Hall A Winter Collaboration Meeting  
January 16, 2025

# PROTON MECHANICAL STRUCTURE

Proton *mechanical* structure is defined by analogy to GR via the QCD energy-momentum tensor (EMT)

$$T^{\mu\nu} = \begin{bmatrix} \text{Energy density} & \text{Momentum density} & & \\ T^{00} & T^{01} & T^{02} & T^{03} \\ T^{10} & T^{11} & T^{12} & T^{13} \\ T^{20} & T^{21} & T^{22} & T^{23} \\ T^{30} & T^{31} & T^{32} & T^{33} \\ \text{Energy flux} & \text{Momentum flux} & & \end{bmatrix}$$

Shear stress  
Normal stress (pressure)



# GRAVITATIONAL FORM FACTORS

- Proton gravitational form factors (GFFs) encode information about the matrix elements of the QCD energy-momentum tensor

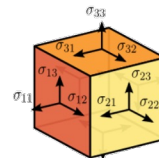
$$\langle p', s' | \hat{T}_{\mu\nu}^a(x) | p, s \rangle = \bar{u}' \left[ A^a(t) \frac{\gamma_{\{\mu} P_{\nu\}}}{2} + B^a(t) \frac{i P_{\{\mu} \sigma_{\nu\}\rho} \Delta^\rho}{4m} + D^a(t) \frac{\Delta_\mu \Delta_\nu - g_{\mu\nu} \Delta^2}{4m} + m \bar{c}^a(t) g_{\mu\nu} \right] u e^{i(p'-p)x}$$

$$P = \frac{p+p'}{2} \quad \Delta = p' - p = q - q'$$

$$t = (p-p')^2 = \Delta^2$$

## EMT Matrix Elements

Energy density	Momentum density			
T <sub>00</sub>	T <sub>01</sub>	T <sub>02</sub>	T <sub>03</sub>	
T <sub>10</sub>	T <sub>11</sub>	T <sub>12</sub>	T <sub>13</sub>	Shear stress
T <sub>20</sub>	T <sub>21</sub>	T <sub>22</sub>	T <sub>23</sub>	
T <sub>30</sub>	T <sub>31</sub>	T <sub>32</sub>	T <sub>33</sub>	Normal stress (pressure)
Energy flux	Momentum flux			



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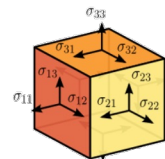
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## Form factors

Fourier transforms of spatial distributions

	Energy density	Momentum density			
	$T_{00}$	$T_{01}$	$T_{02}$	$T_{03}$	
	$T_{10}$	$T_{11}$	$T_{12}$	$T_{13}$	Shear stress
	$T_{20}$	$T_{21}$	$T_{22}$	$T_{23}$	
	$T_{30}$	$T_{31}$	$T_{32}$	$T_{33}$	Normal stress (pressure)
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## Form factors

Fourier transforms of spatial distributions

## “Gravitational”

Describing the energy-momentum tensor  
I.e. what would be seen from proton-graviton scattering

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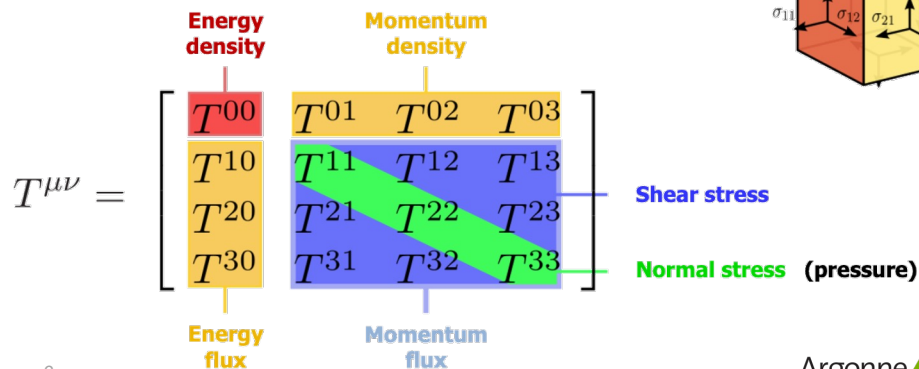
$$P = \frac{p+p'}{2} \quad \Delta = p' - p = q - q'$$

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## D-term

$D(0)$  represents a fundamental property of the proton

On par with spin, charge, mass!



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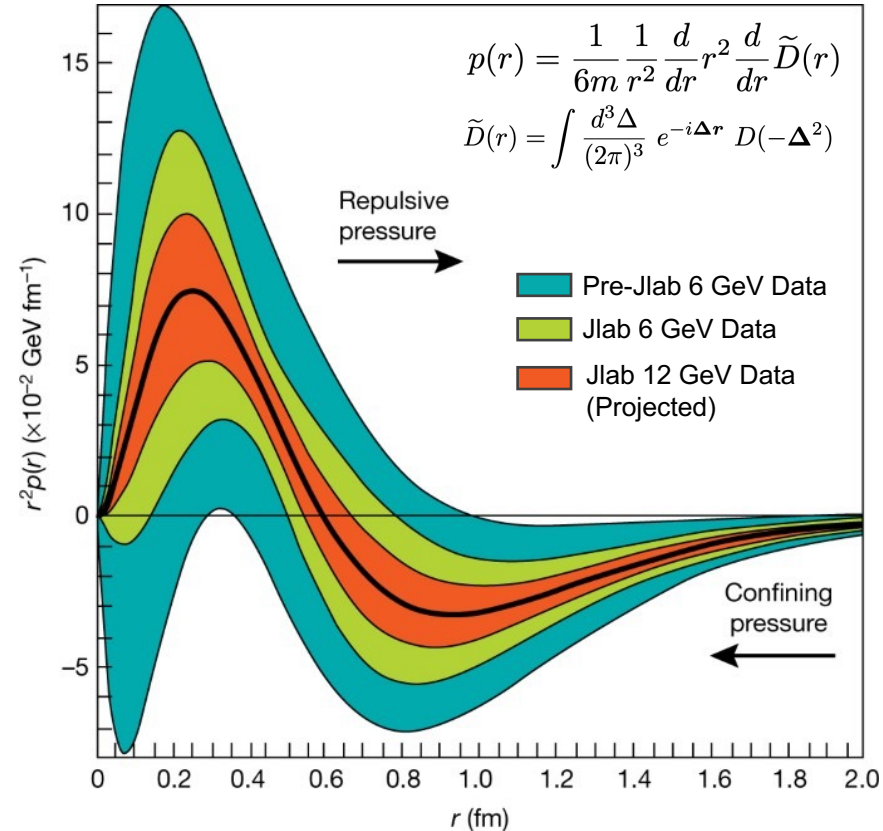
**D-term**

Often called the last global unknown property of the proton!

<b>em:</b> $\partial_\mu J_{\text{em}}^\mu = 0$	$\langle N'   J_{\text{em}}^\mu   N \rangle \rightarrow Q = 1.602176487(40) \times 10^{-19} \text{C}$
	$\mu = 2.792847356(23) \mu_N$
<b>weak:</b> PCAC	$\langle N'   J_{\text{weak}}^\mu   N \rangle \rightarrow g_A = 1.2694(28)$
	$g_p = 8.06(55)$
<b>gravity:</b> $\partial_\mu T_{\text{grav}}^{\mu\nu} = 0$	$\langle N'   T_{\text{grav}}^{\mu\nu}   N \rangle \rightarrow m = 938.272013(23) \text{ MeV}/c^2$
	$J = \frac{1}{2}$
	$D = ?$

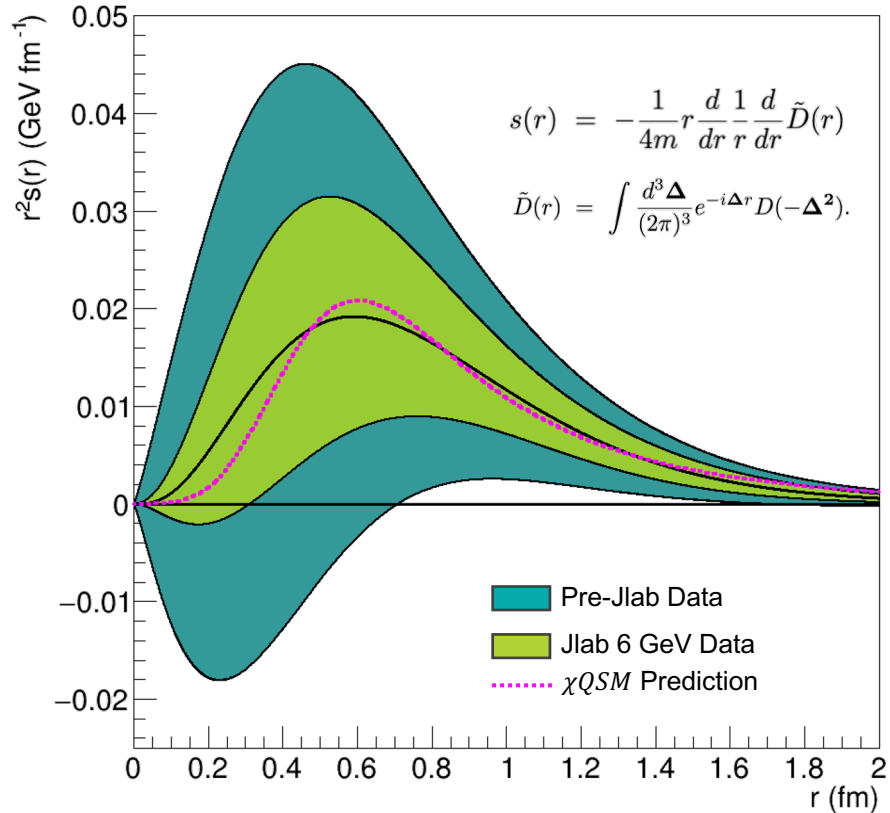
# MECHANICAL PROPERTIES

- The total  $D$ -term provides a gateway for extraction of various **mechanical** properties of the proton, including:
  - Pressure distribution
  - Shear force distribution
  - Mechanical radius
  - Tangential & normal force distributions



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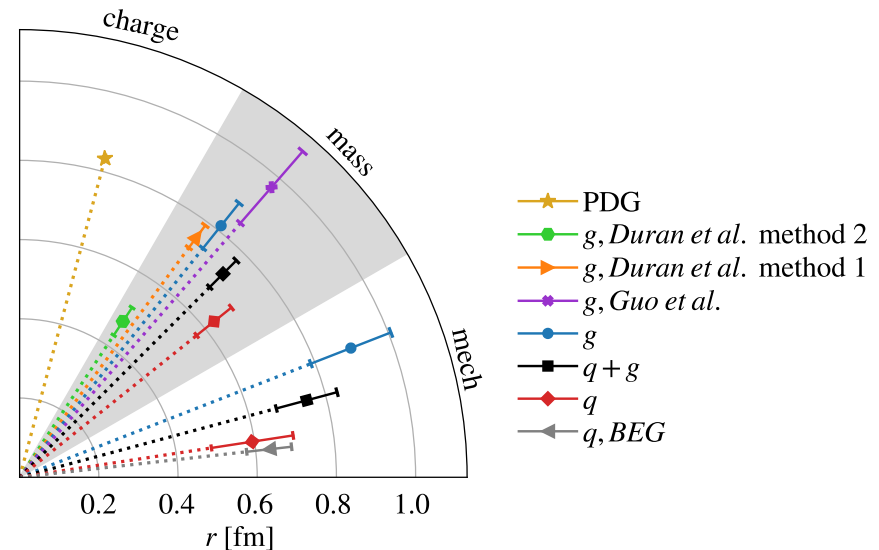


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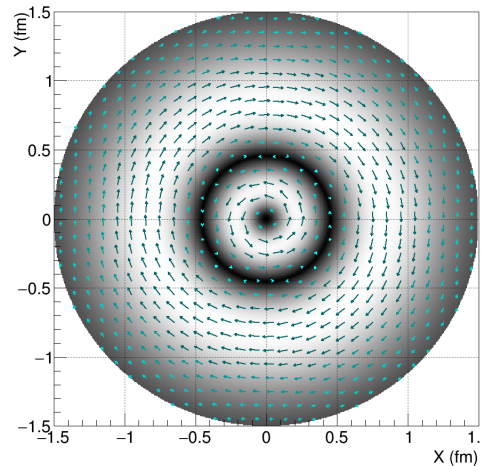
- Pressure distribution
- Shear force distribution
- **Mechanical radius**
- Tangential & normal force distributions

$$\langle r^2 \rangle_{\text{mech}} = \frac{\int d^3r r^2 \left[ \frac{2}{3}s(r) + p(r) \right]}{\int d^3r \left[ \frac{2}{3}s(r) + p(r) \right]} = \frac{6D}{\int_{-\infty}^0 dt D(t)}$$

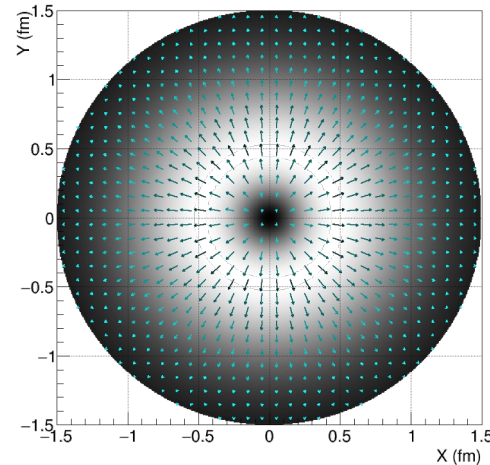


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Tangential force



Normal force

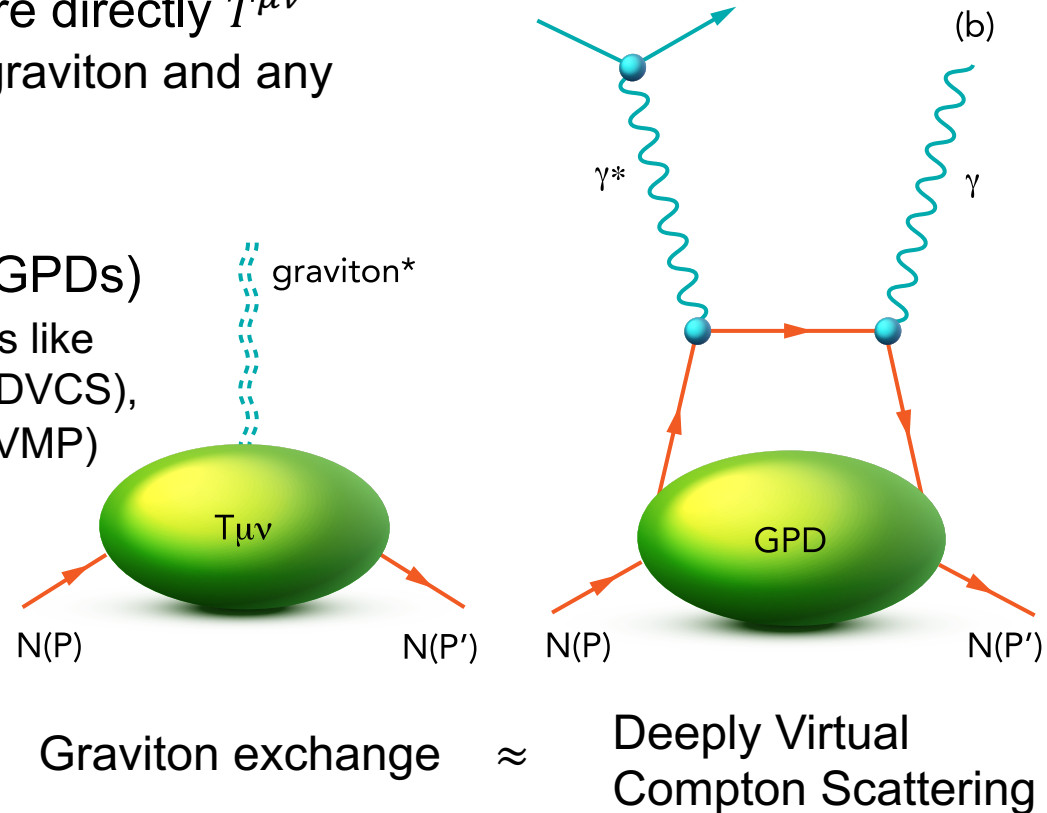
$$\frac{dF_\phi}{dS_\phi} = -\frac{1}{3}s(r) + p(r).$$

$$\frac{dF_r}{dS_r} = \frac{2}{3}s(r) + p(r)$$



# HOW DO WE MEASURE THIS STUFF?

- Graviton scattering would measure directly  $T^{\mu\nu}$ 
  - Exploit the duality between the graviton and any massless spin-2 field
- $D$ -term is a contribution to the generalized parton distributions (GPDs)
  - Measured in hard exclusive reactions like Deeply Virtual Compton Scattering (DVCS), Deeply Virtual Meson Production (DVMP)
- Extractions of  $D$ -term can go through GPDs, or use models to bypass them depending on the process



# HOW DO WE MEASURE THIS?

The total  $D$ -term is related to the partonic  $D$ -terms by a sum rule:

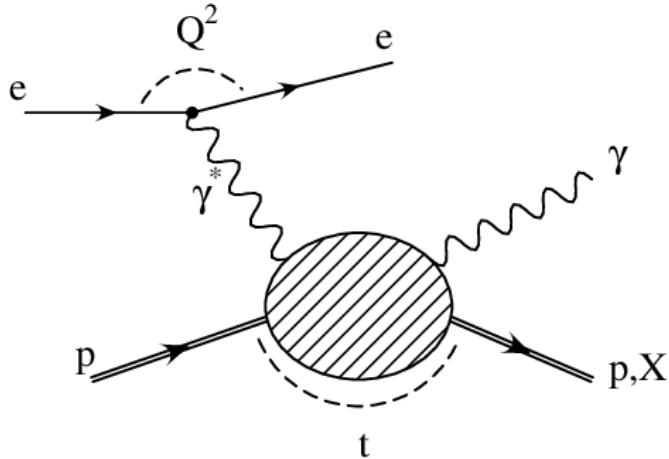
$$D(0) = D_g(0) + D_u(0) + D_d(0) + D_s(0) + \dots$$

Different exclusive processes provide access to the different partonic  $D$ -terms!

Up & Down quarks:  
Accessible via DVCS cross section &  
beam-spin asymmetries



$$D(0) = D_g(0) + D_u(0) + D_d(0) + D_s(0) + \dots$$



**The pressure distribution inside the proton**

[V. D. Burkert](#) ✉, [L. Elouadrhiri](#) & [F. X. Girod](#)

Gluons:  
 Accessible via near-threshold  
 production of  $J/\psi$  and  $\Upsilon$

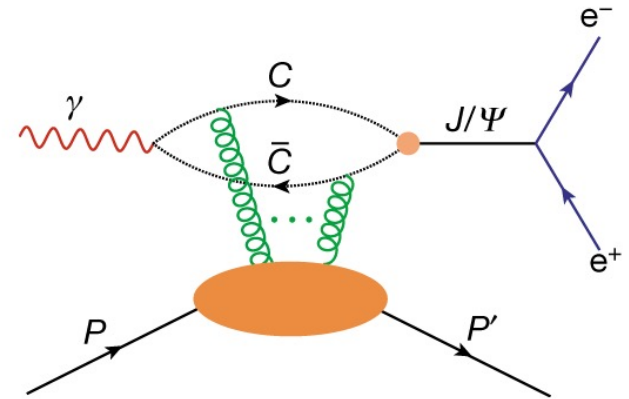
SoLID  $J/\psi$ !



$$D(0) = D_g(0) + D_u(0) + D_d(0) + D_s(0) + \dots$$

### Determining the Proton's Gluonic Gravitational Form Factors

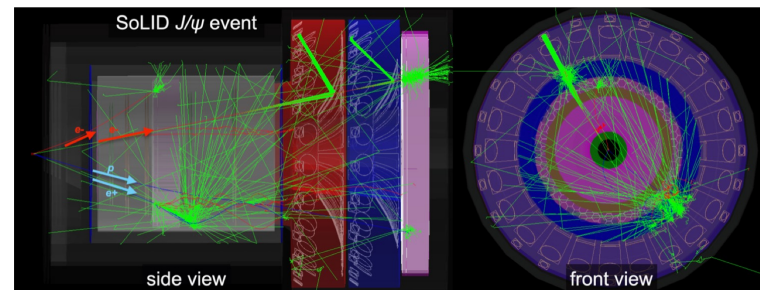
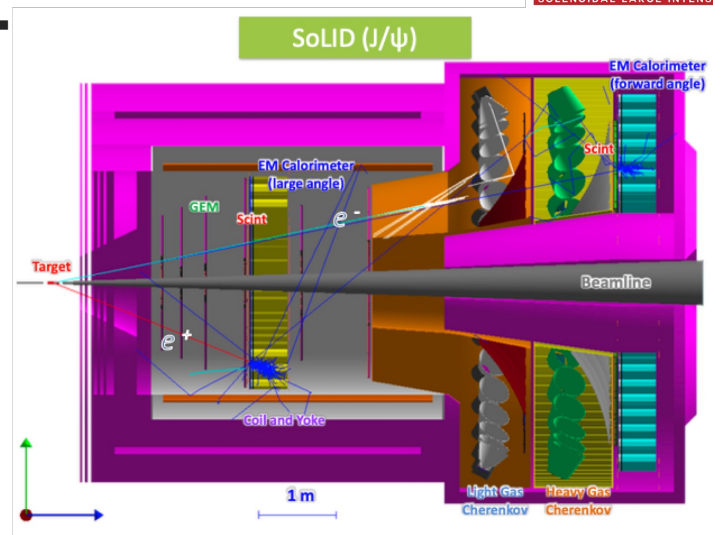
B. Duran<sup>3,1</sup>, Z.-E. Meziani<sup>1,3\*\*</sup>, S. Joosten<sup>1</sup>, M. K. Jones<sup>2</sup>, S. Prasad<sup>1</sup>, C. Peng<sup>1</sup>,  
 W. Armstrong<sup>1</sup>, H. Atac<sup>3</sup>, E. Chudakov<sup>2</sup>, H. Bhatt<sup>5</sup>, D. Bhetuwal<sup>5</sup>, M. Boer<sup>11</sup>,  
 A. Camsonne<sup>2</sup>, J.-P. Chen<sup>2</sup>, M. M. Dalton<sup>2</sup>, N. Deokar<sup>3</sup>, M. Diefenthaler<sup>2</sup>, J. Dunne<sup>5</sup>,  
 L. El Fassi<sup>5</sup>, E. Fuchey<sup>9</sup>, H. Gao<sup>4</sup>, D. Gaskell<sup>2</sup>, O. Hansen<sup>2</sup>, F. Hauenstein<sup>6</sup>,  
 D. Higinbotham<sup>2</sup>, S. Jia<sup>3</sup>, A. Karki<sup>5</sup>, C. Keppel<sup>2</sup>, P. King<sup>7</sup>, H.S. Ko<sup>10</sup>, X. Li<sup>4</sup>, R. Li<sup>3</sup>,  
 D. Mack<sup>2</sup>, S. Malace<sup>2</sup>, M. McCaughan<sup>2</sup>, R. E. McClellan<sup>8</sup>, R. Michaels<sup>2</sup>, D. Meekins<sup>2</sup>,  
 M. Paolone<sup>3</sup>, L. Pentchev<sup>2</sup>, E. Pooser<sup>2</sup>, A. Puckett<sup>9</sup>, R. Radloff<sup>7</sup>, M. Rehfuss<sup>3</sup>,  
 P. E. Reimer<sup>1</sup>, S. Riordan<sup>1</sup>, B. Sawatzky<sup>2</sup>, A. Smith<sup>4</sup>, N. Sparveris<sup>3</sup>, H. Szumila-Vance<sup>2</sup>,  
 S. Wood<sup>2</sup>, J. Xie<sup>1</sup>, Z. Ye<sup>1</sup>, C. Yero<sup>6</sup>, and Z. Zhao<sup>4</sup>

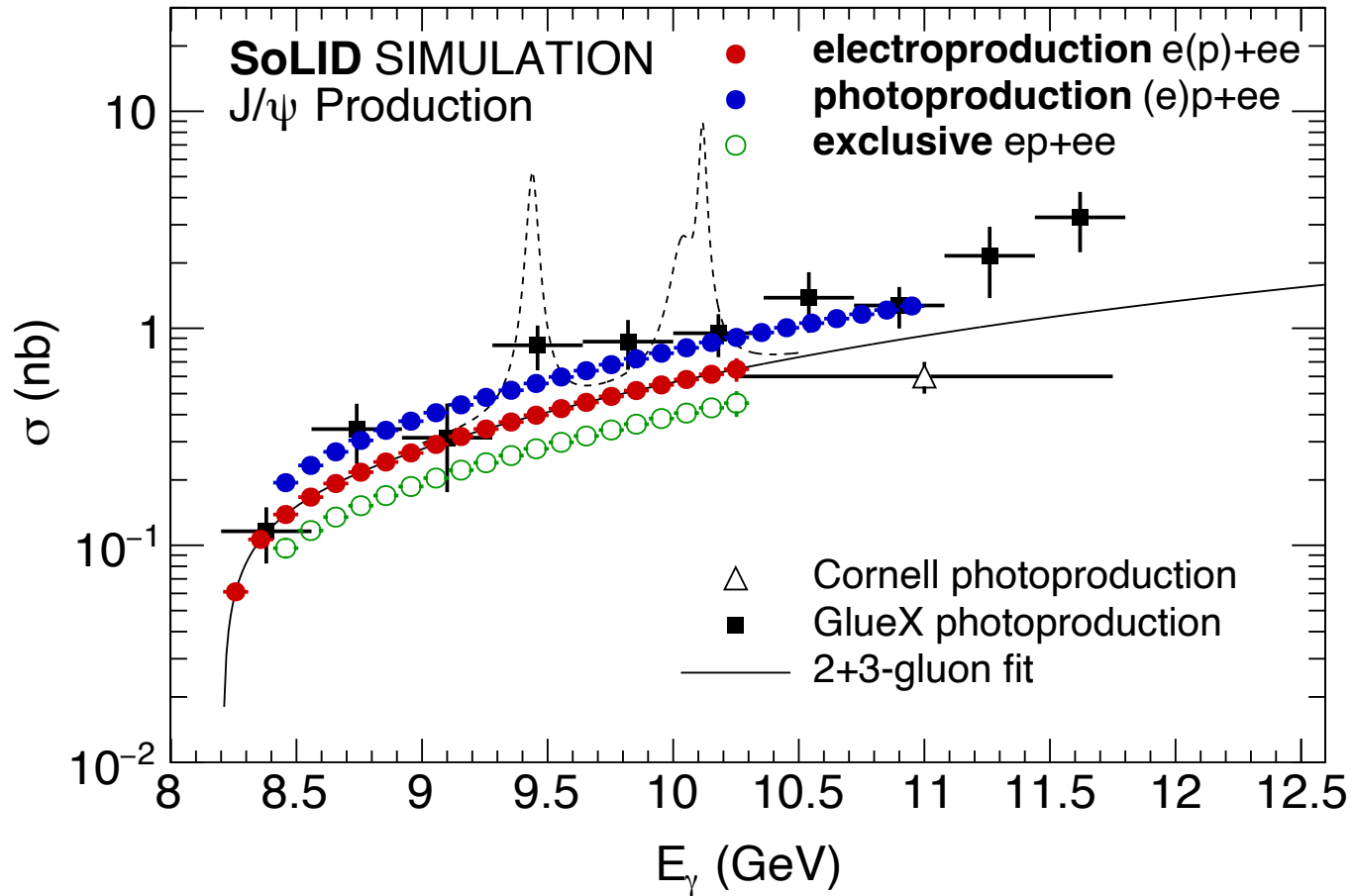


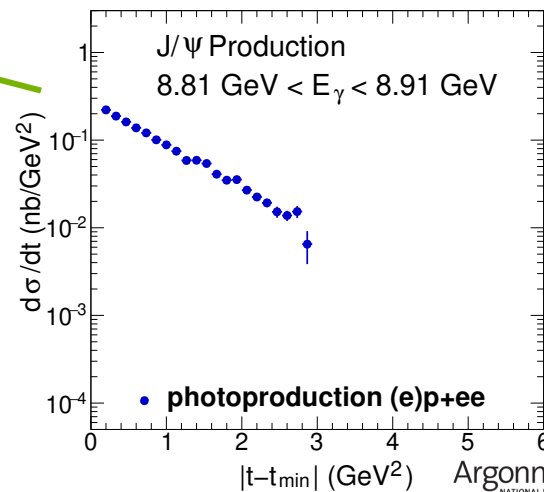
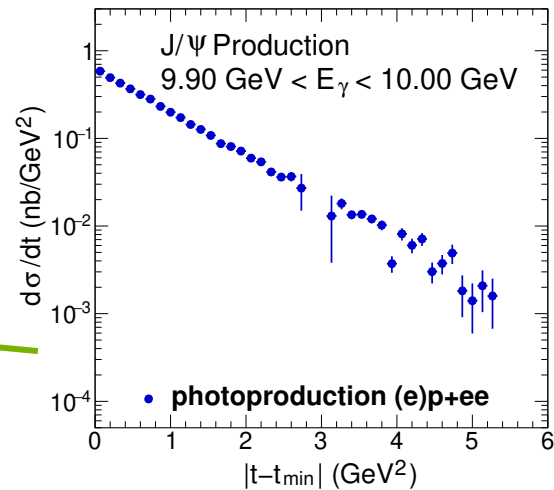
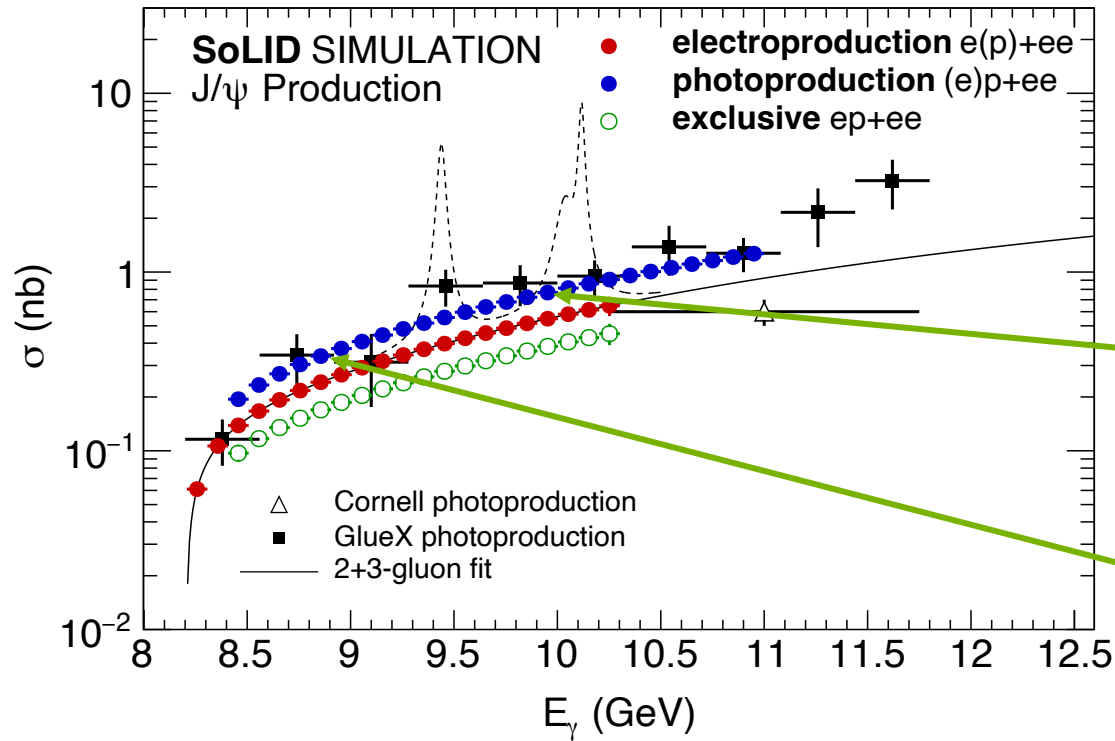
# THE SOLID-J/ $\psi$ EXPERIMENT

## Ultimate factory for near-threshold J/ $\psi$

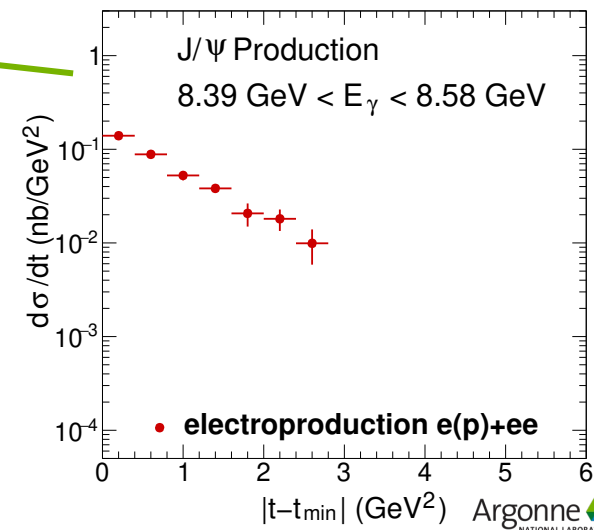
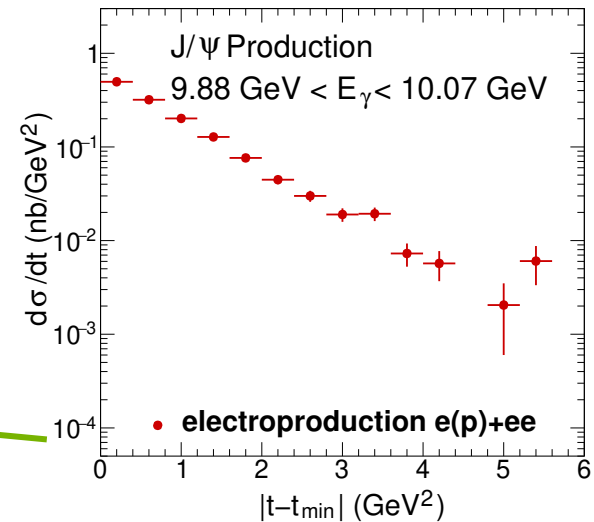
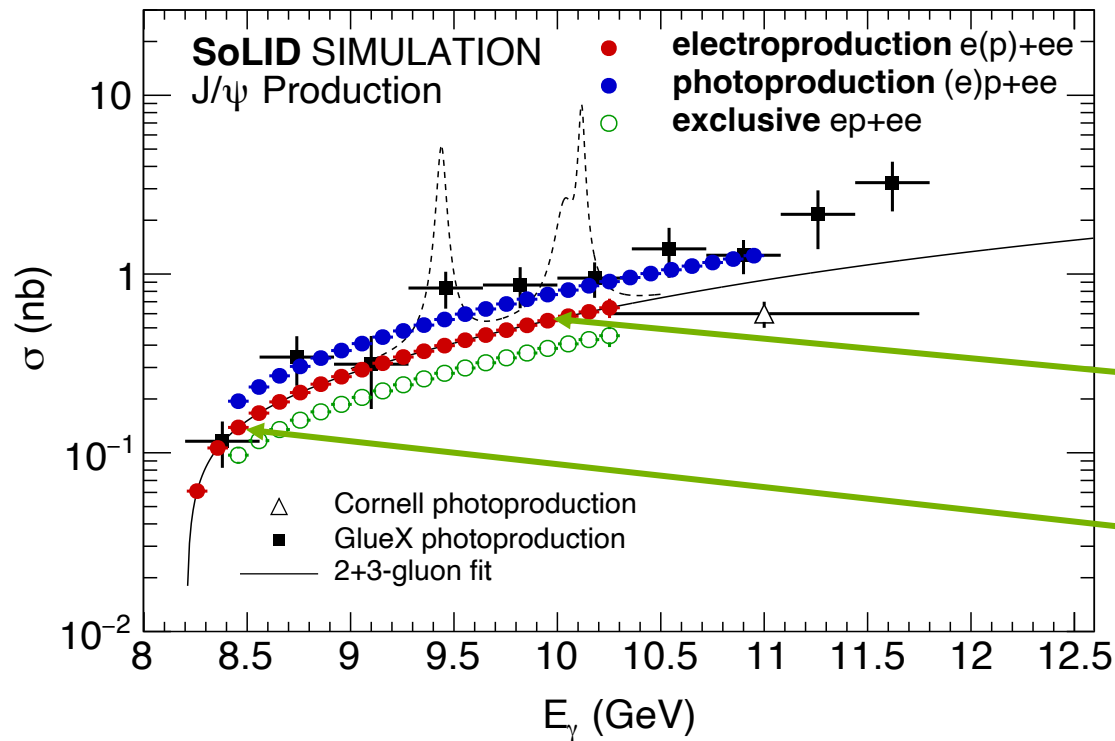
- General purpose large-acceptance spectrometer
- 50+10 days of 3 $\mu$ A beam on a 15cm long LH2 target ( $10^{37}/\text{cm}^2/\text{s}$ )
- **Ultra-high luminosity:** 43.2ab<sup>-1</sup>
- **Open 2-particle trigger**, covering J/ $\psi$  production in four channels:  
Electroproduction (e,e<sup>-</sup>e<sup>+</sup>), photoproduction (p,e<sup>-</sup>e<sup>+</sup>), inclusive (e<sup>-</sup>e<sup>+</sup>), exclusive (ep,e<sup>-</sup>e<sup>+</sup>)
- The electroproduction channel provides for a modest lever-arm in Q<sup>2</sup> near threshold

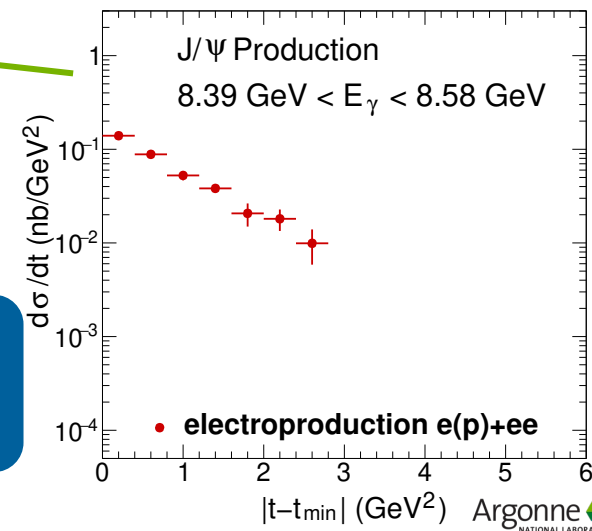
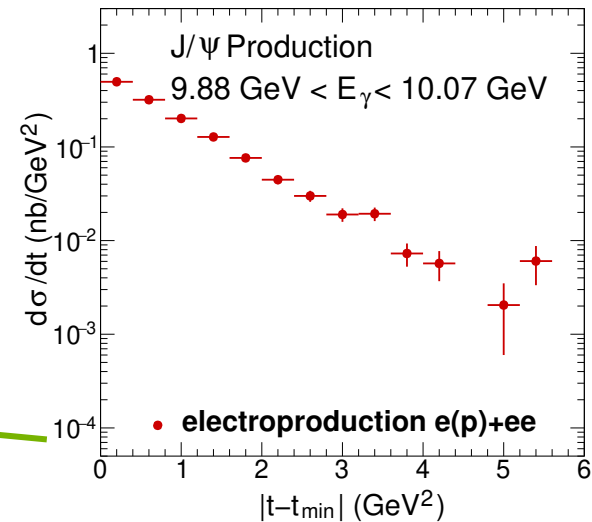
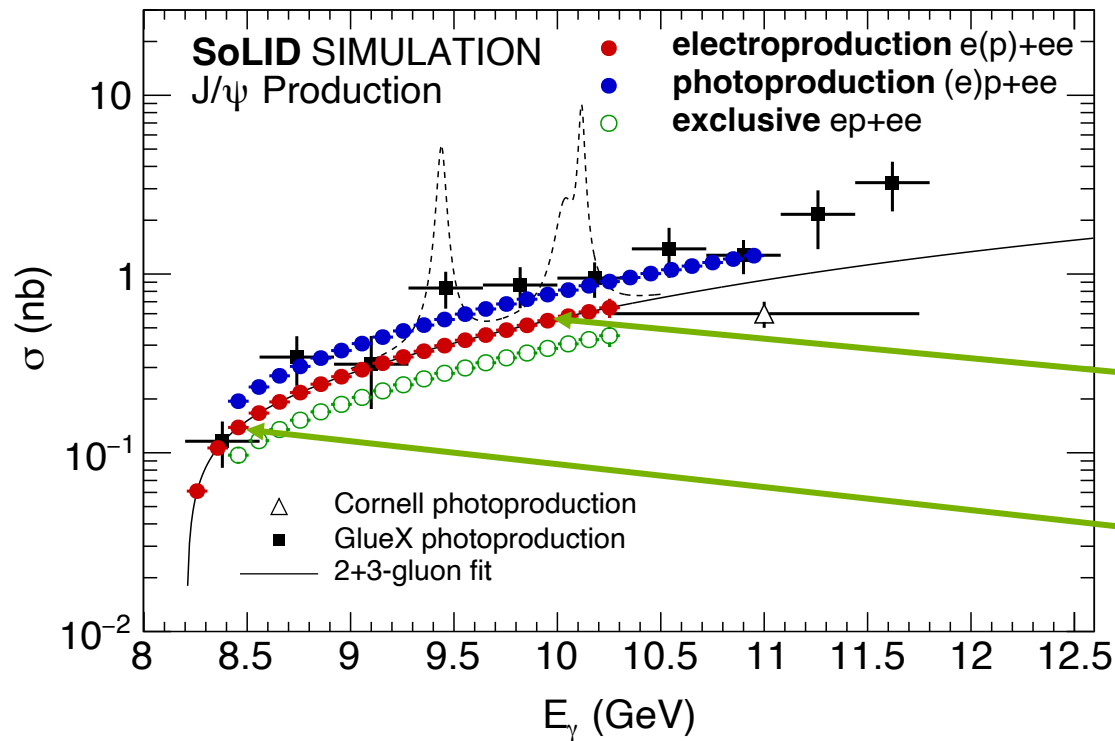








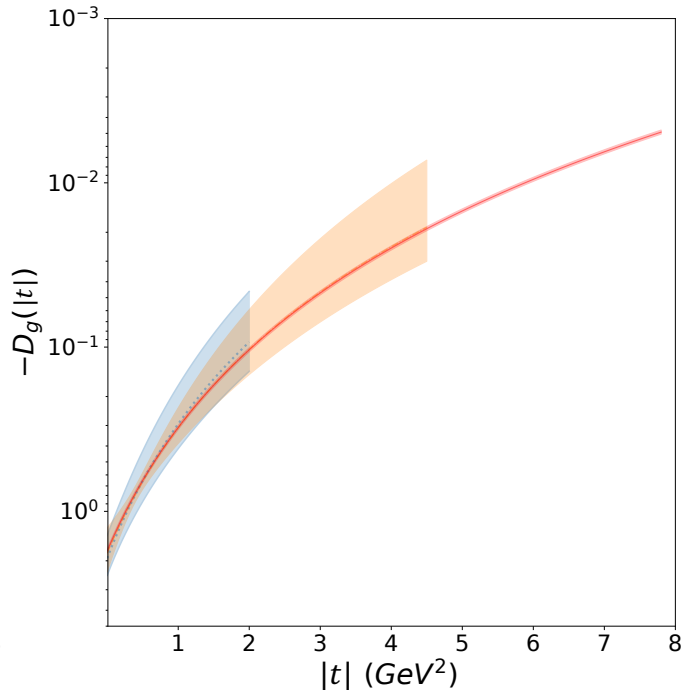
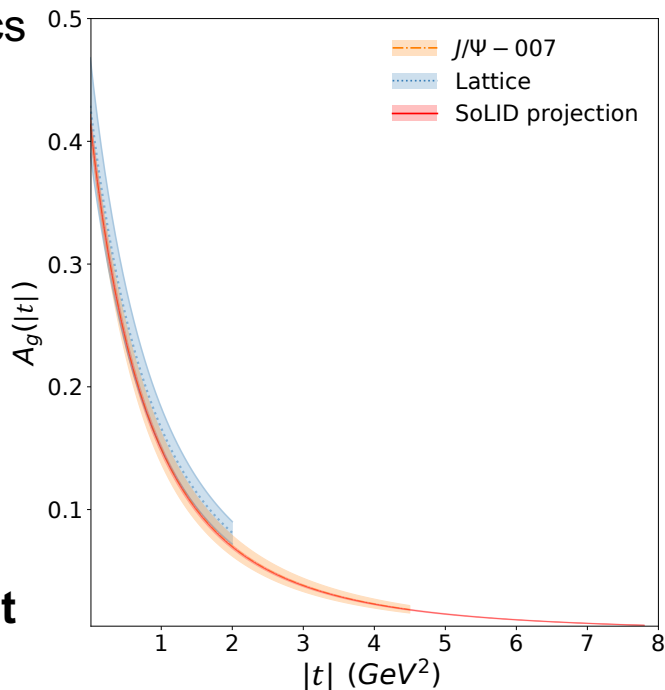




**Highly multi-dimensional measurement!**

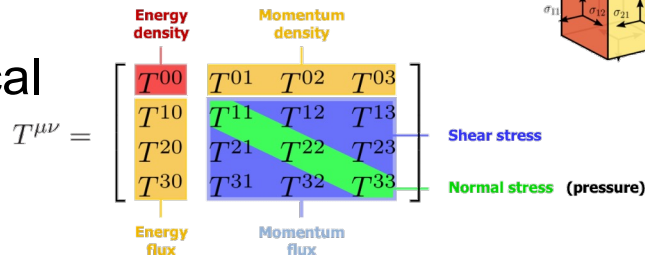
# GLUON GFFS

- Extremely high statistics enables precise extraction of  $D_g, A_g$
- No longer limited by poor experimental precision!
  - More food for thought for theorists
- **SoLID will let us take full advantage of what CEBAF can offer!**



# $\bar{c}$ CAVEAT

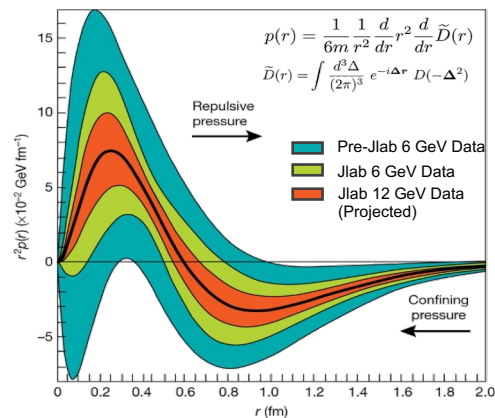
- $\bar{c}$  form factor contributes to many of the mechanical structure quantities, not only the  $D$ -term
  - $\bar{c}$  currently inaccessible by experiment



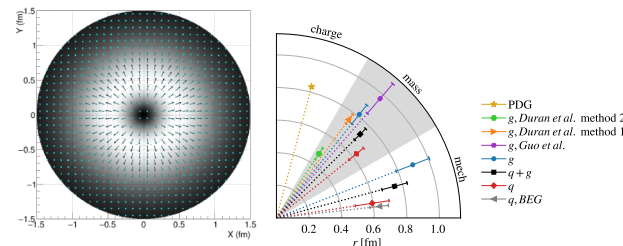
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$$t = (p-p')^2 = \Delta^2 \quad \Delta = p' - p = q - q' \quad P = \frac{p+p'}{2}$$

- However,  $\bar{c}_q = -\bar{c}_g$ ! Total  $\bar{c}$  cancels due to EMT conservation if summing over all parton species!
  - Only shear force has no contribution from  $T^{ii}$  components of the EMT, and thus no contribution from  $\bar{c}$



**This caveat means that to extract the rest of the mechanical properties rigorously, all partonic  $D$ -terms must be known!**



Since we need all terms in the sum rule to extract pressure, mechanical radius, force distributions...

$$D(0) = D_g(0) + D_u(0) + D_d(0) + \underbrace{D_s(0)} + \dots$$

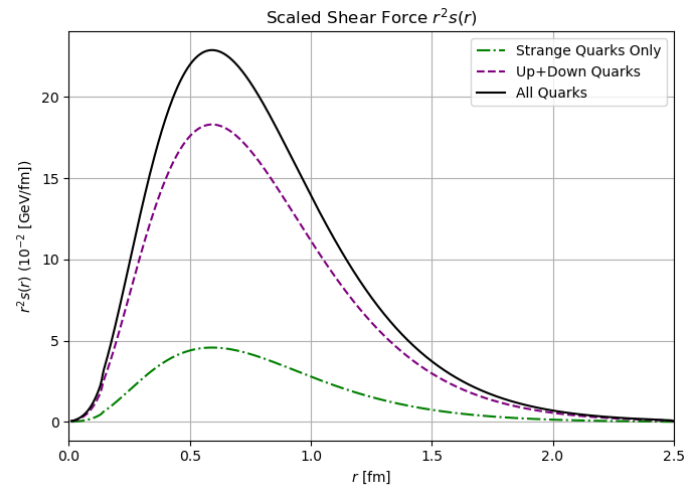
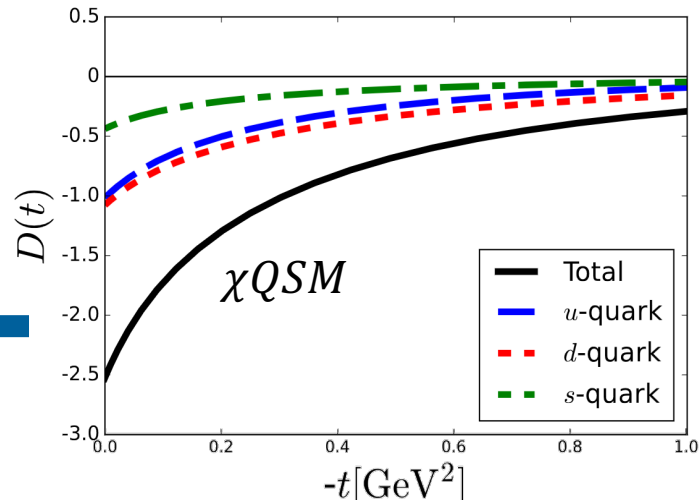
Strange quarks: Can we just neglect them...?

# THEORY PREDICTIONS

- Large- $N_c$  theory predicts that the  $D$ -term is "flavor-blind"
  - i.e.  $D_u \sim D_d$  despite their different number densities, this is supported by lattice results
- Extending this argument, could  $D_u \sim D_d \sim D_s$ ?
- Chiral quark soliton model:  $D_u \sim D_d \sim 2D_s$

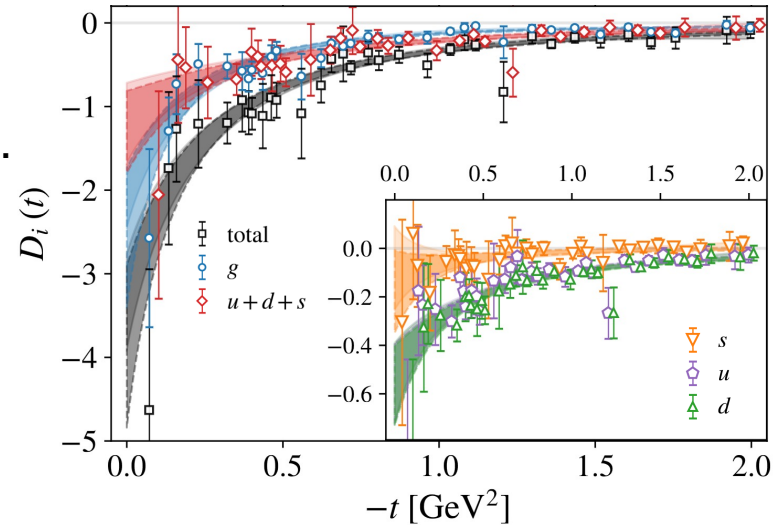
This would make  $D_s$  a non-negligible contributor to the total  $D$ -term, and thus **necessary for a full extraction of many of the mechanical properties of the proton!**

ArXiv: 2307.00740



# THEORY PREDICTIONS

- On the other hand, lattice results of Hackett et al. predict  $D_s$  consistent with zero
  - Uncertainties are still large, but the results do not exclude *positive* values of  $D_s$
- Opposite signs of sea & valence quarks is a distinct possibility, predicted by  $\chi QSM$
- $D_s > 0$  would mean that strange quarks feel forces in opposite direction to up & down quarks!
  - The pop-sci articles write themselves...



	Dipole $D_i$	$z$ -expansion $D_i$
$u$	-0.56(17)	-0.56(17)
$d$	-0.57(17)	-0.56(17)
$s$	-0.18(17)	-0.08(17)
$u + d + s$	-1.30(49)	-1.20(48)
$g$	-2.57(84)	-2.15(32)
Total	-3.87(97)	-3.35(58)



**Variety of theory predictions giving very different values for  $D_s$ , let's measure it!**

But how...?

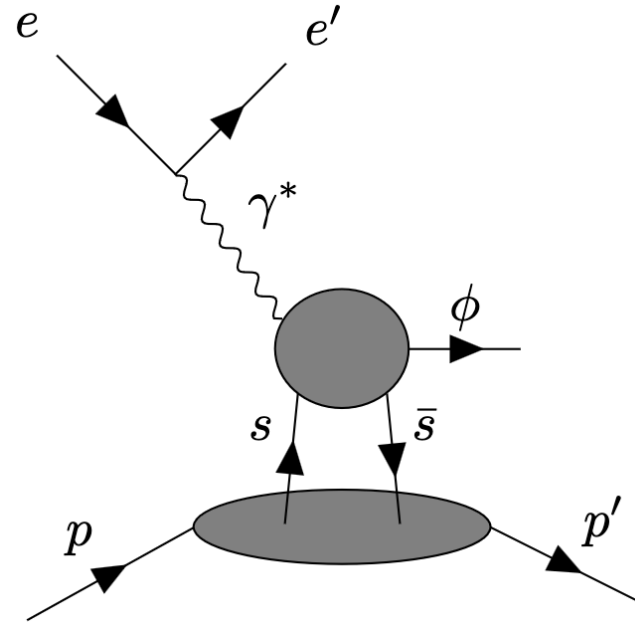
**Variety of theory predictions giving very different values for  $D_s$ , let's measure it!**

But how...?

**SoLID  $\phi$ !**

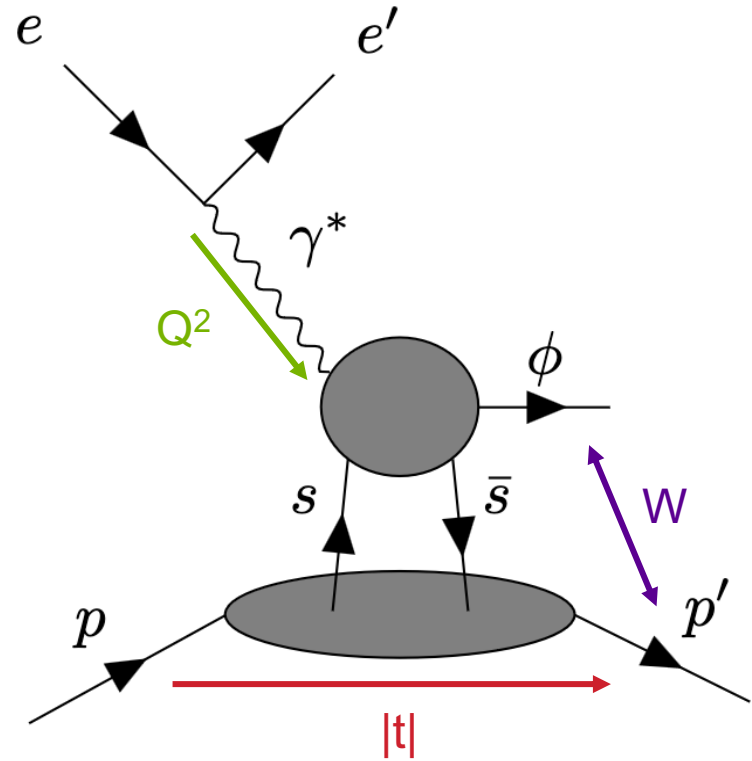
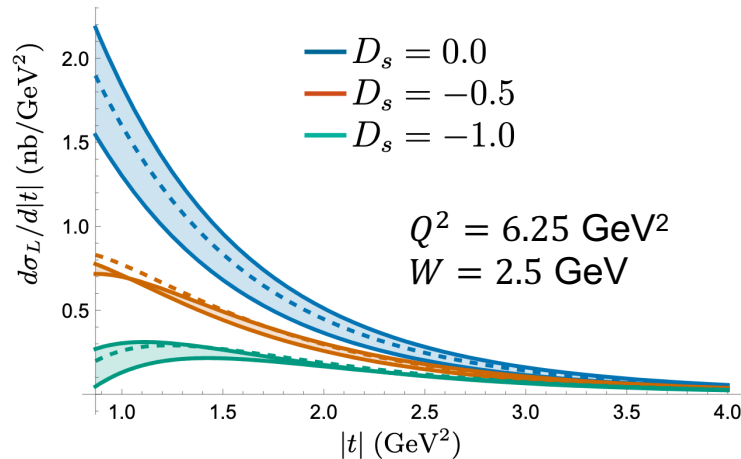
# ACCESSING THE STRANGENESS D-TERM

- Information on strangeness in the valence region of the proton is limited
  - Disentangling it from up & down requires use of specialized processes
  - e.g. W/Z exchange or kaon production in SIDIS
- Recently, it was proposed that *near-threshold electroproduction of  $\phi$  mesons* could provide sensitivity to the strangeness *D*-term
  - $\phi$  meson is very nearly a pure  $s\bar{s}$  state
  - Expected to couple strongly to strangeness in the proton
- Never measured in the required kinematic region!



# DEEP NEAR-THRESHOLD $\phi$ KINEMATICS

- Deep = high momentum transfer = **high  $Q^2$**
- Near-threshold = invariant mass of final-state hadrons  **$W \sim M_\phi + M_p \sim 1.96$  GeV**
- Small momentum transfer to proton = **Low- $|t|$** 
  - Strong sensitivity to strangeness  $D$ -term!



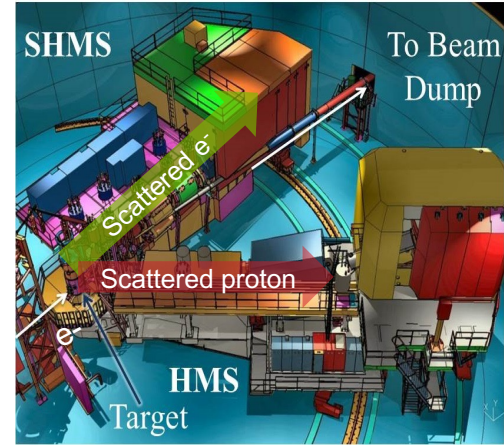
$\phi$ -meson lepto-production near threshold and the strangeness  $D$ -term

# THE STRANGENESS D-TERM IN HALL C

2501.01582

- **Proposed Measurement:** Exclusive  $\phi$  meson electroproduction near threshold in Hall C at Jefferson Lab (2024 LOI)

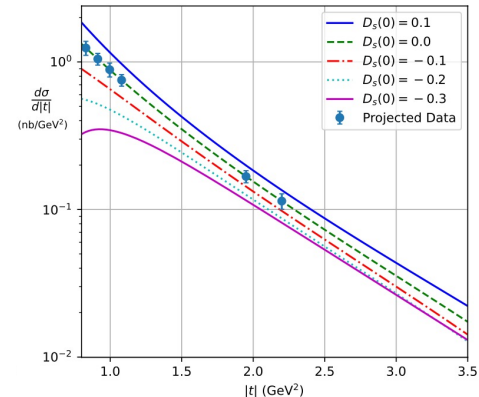
- Measure the  $|t|$ -dependence of the electroproduction cross-section using the reaction  $H(e, e'p)\phi$  at  $Q^2 \sim 3.5$  and  $W \sim 2.2$
- Uses the missing mass technique with standard Hall C spectrometers to identify exclusive events
  - No hit from  $\phi \rightarrow KK$  BR, but large DIS background!



- **Theoretical Challenges:**

Two points highlighted by the PAC:

- Model Dependence: Extracting  $D_s$  requires understanding the dynamics of  $\phi$  meson production and **final-state interactions**
- Separating Quark and Gluon Contributions: **Need to distinguish between strange quark and gluonic effects**

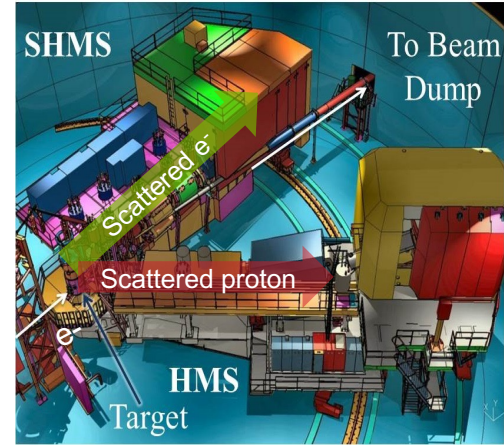


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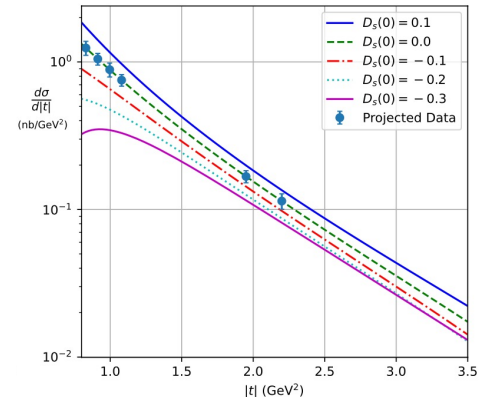


- **Theoretical Challenges:**

Two points highlighted by the PAC:

Jobs for theorists...

- Model Dependence: Extracting  $D_s$  requires understanding the dynamics of  $\phi$  meson production and **final-state interactions**
- Separating Quark and Gluon Contributions: **Need to distinguish between strange quark and gluonic effects**



# THEORY PREDICTIONS

- New predictions available from Hatta et al. using GPD framework in the near-threshold region
  - Typical issue for GPDs near-threshold is final-state interactions
  - FSI calculated to be 2-3 orders of magnitude smaller than production cross section for  $\phi + p$  in photoproduction ([S. H. Kim et al.](#))
- Theoretical uncertainty on cross section from this approximation is  $\sim 10\%$  or less for  $\xi > 0.3$ !
  - Focus on high  $\xi$

$$\frac{d\sigma_L}{dt} = \frac{2\pi^2 \alpha_{em}}{(W^2 - M^2)W p_{cm}} \left( (1 - \xi^2) |\mathcal{H}|^2 - \left( \frac{t}{4M^2} + \xi^2 \right) |\mathcal{E}|^2 - 2\xi^2 \text{Re}(\mathcal{H}\mathcal{E}^*) \right)$$

$$\begin{pmatrix} \mathcal{H}(\xi, t) \\ \mathcal{E}(\xi, t) \end{pmatrix} = \kappa \sum_{j=1}^{\text{odd}} \sum_{k=0}^{\text{even}} \sum_a \frac{2}{\xi^{j+1}} \begin{pmatrix} H_j^a(\xi, t) \\ E_j^a(\xi, t) \end{pmatrix} T_{jk}^a(\xi) \varphi_k, \quad \kappa \equiv e_s \frac{C_F f_\phi}{N_c Q}.$$



“Threshold Approximation” –  
Keep only  $j = 1$

$$\begin{pmatrix} \mathcal{H}(\xi, t) \\ \mathcal{E}(\xi, t) \end{pmatrix} \approx \frac{2\kappa}{\xi^2} \sum_a \begin{pmatrix} H_1^a(\xi, t) \\ E_1^a(\xi, t) \end{pmatrix} T_{10}^a(\xi, \mu^2).$$

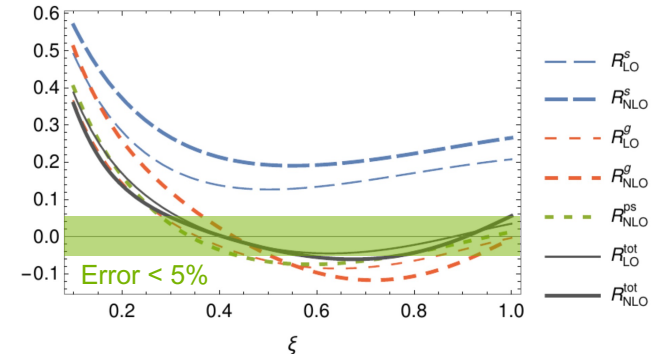
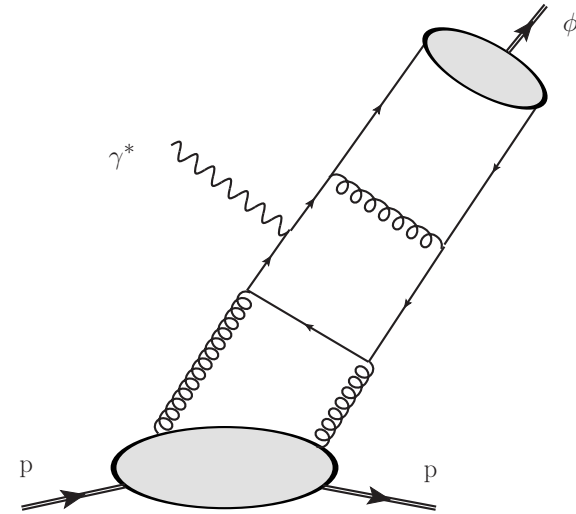
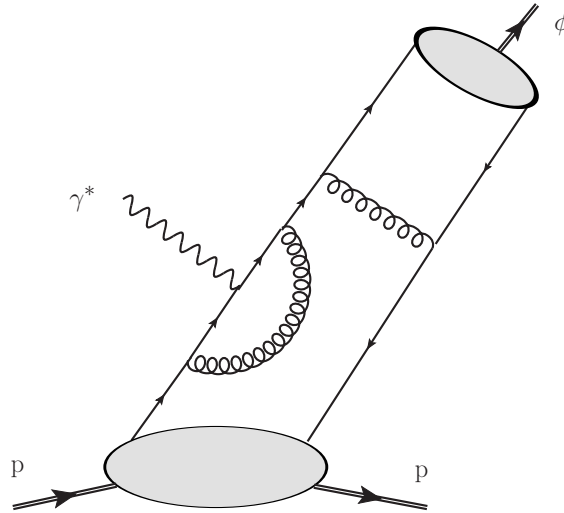
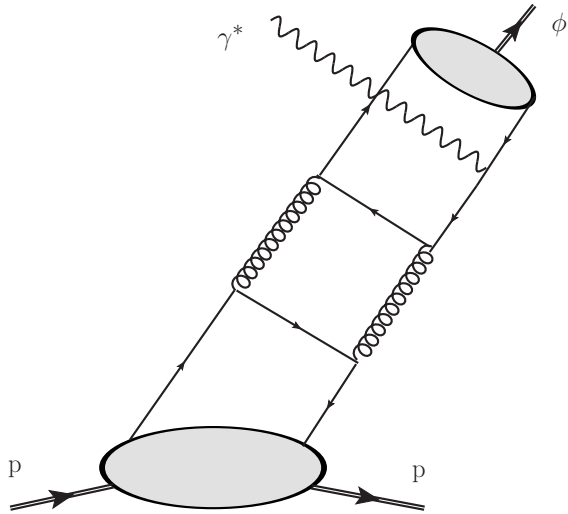


FIG. 4: Relative error for the amplitude  $\mathcal{H}$  from truncating the conformal partial wave expansion after the first term. Plotted quantities are defined in (40). The subscript denotes whether the leading order (LO) or next-to-leading order (NLO) coefficient function has been used. In this and the next plots, we have set  $t = t_{\min}(\xi)$ ,  $\alpha_s = 0.3$  and  $\kappa = 1$ .



# THEORY PREDICTIONS

- Predictions available at NLO for  $\frac{d\sigma_L}{d|t|}$ 
  - Requires our experiment to have an L/T separation (or modelling of  $R$ ) for comparison



These predictions are valid for  $\xi \gtrsim 0.4$  and  $Q^2 \gtrsim 3|t|$

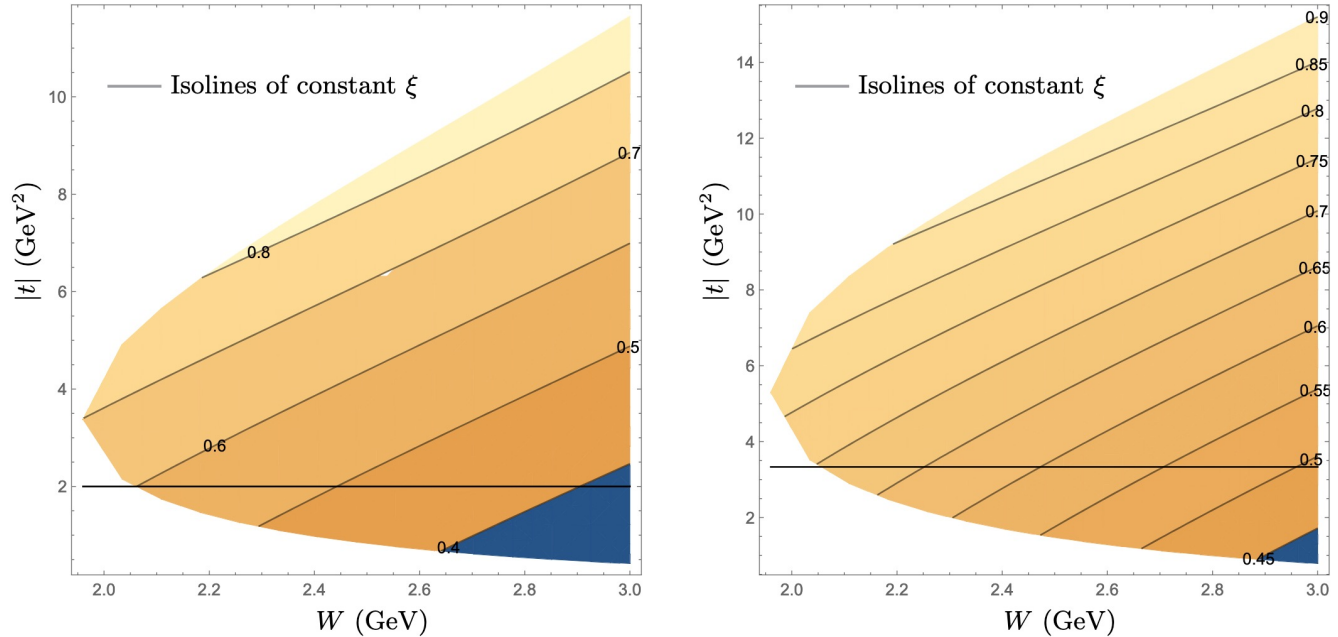
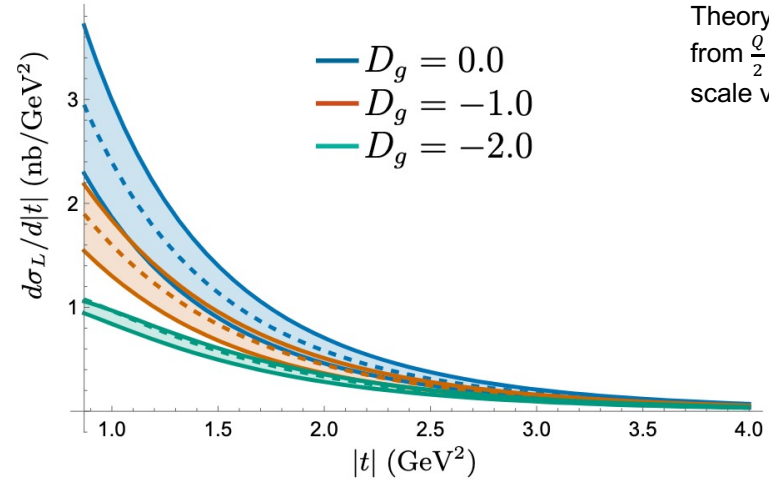
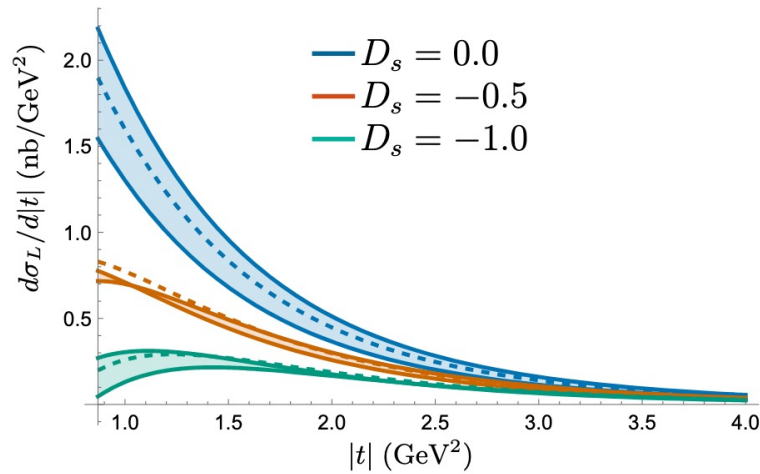


FIG. 1: Contour plots of  $\xi$  in the  $(W, |t|)$  plane at  $Q^2 = 6$  GeV<sup>2</sup> (left) and  $Q^2 = 10$  GeV<sup>2</sup> (right). The horizontal line is at  $|t| = \frac{Q^2}{3}$ .

Challenging to satisfy! Need high  $Q^2$  &  $W < 3$

# THEORY PREDICTIONS



Theory uncertainty  
from  $\frac{Q}{2} < \mu < 2Q$   
scale variation

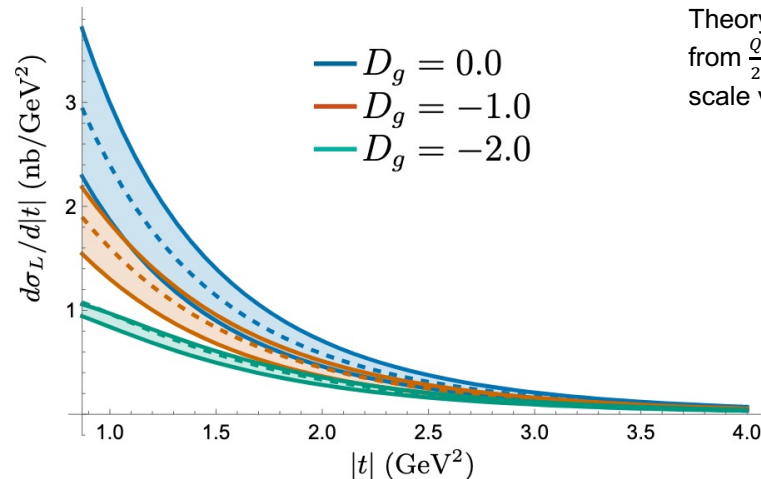
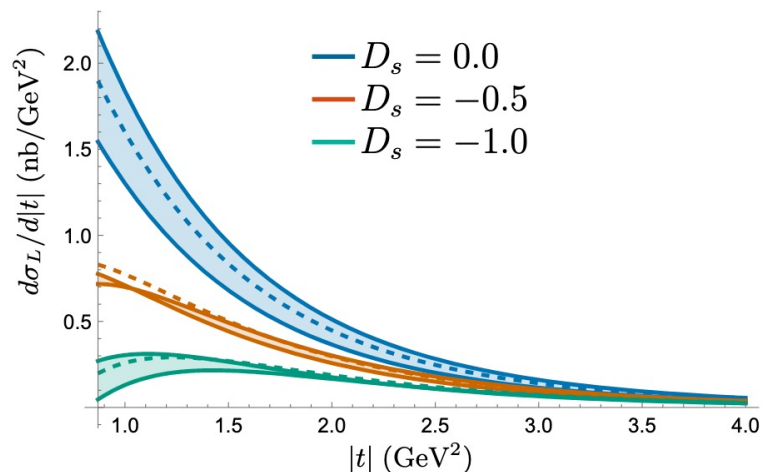
FIG. 7: NLO longitudinal cross section at  $W = Q = 2.5$  GeV as a function of  $|t|$ . Left:  $D_s = 0, -0.5, -1$  from top to bottom at fixed  $D_g = -1$ . Right:  $D_g = 0, -1, -2$  from top to bottom at fixed  $D_s = 0$ .

**Near-threshold  $\phi$  exhibits  
factor  $\sim 4$  greater sensitivity to  $D_s$  compared to  $D_g$ !**

# THEORY PREDICTIONS

$$\mathcal{H}(\xi, t) \approx \frac{2\kappa}{\xi^2} \frac{15}{2} \left[ \left\{ \alpha_s(\mu) + \frac{\alpha_s^2(\mu)}{2\pi} \left( 25.7309 - 2n_f + \left( -\frac{131}{18} + \frac{n_f}{3} \right) \ln \frac{Q^2}{\mu^2} \right) \right\} (A_s(t, \mu) + \xi^2 D_s(t, \mu)) \right. \\ \left. + \frac{\alpha_s^2}{2\pi} \left( -2.3889 + \frac{2}{3} \ln \frac{Q^2}{\mu^2} \right) \sum_q (A_q + \xi^2 D_q) + \frac{3}{8} \left\{ \alpha_s + \frac{\alpha_s^2}{2\pi} \left( 13.8682 - \frac{83}{18} \ln \frac{Q^2}{\mu^2} \right) \right\} (A_g + \xi^2 D_g) \right],$$

$\xi \sim 0.5$   
 $A_g \sim A_{u,d} \gg A_s$   
 $D_g \sim D_{u,d} \sim D_s?$

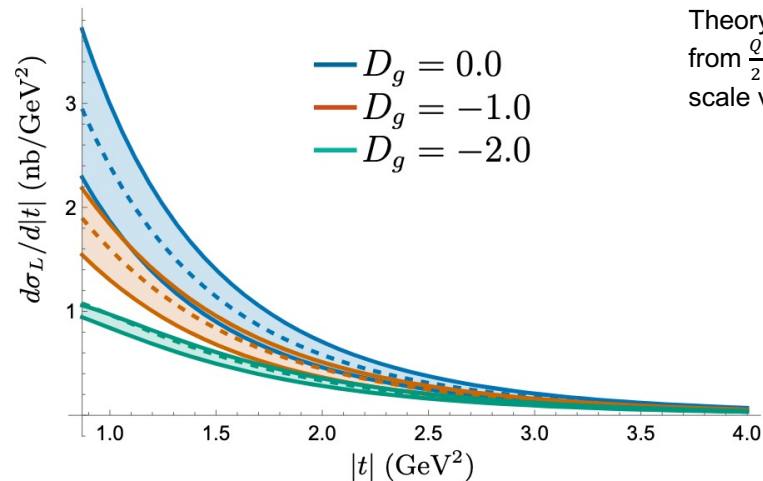
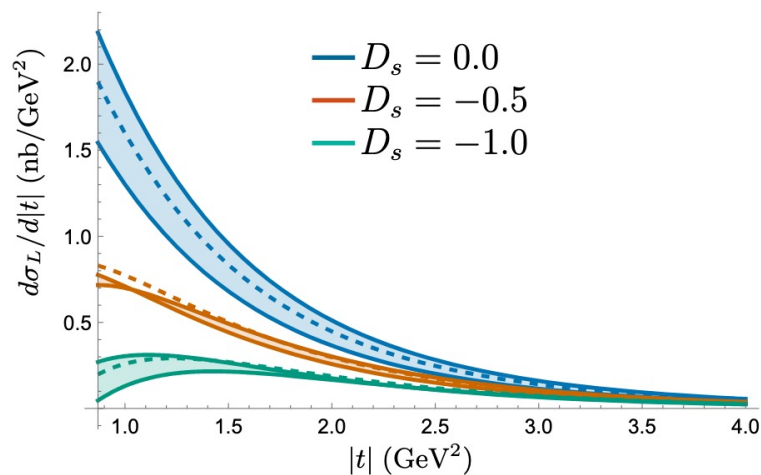


Theory uncertainty  
 from  $\frac{Q}{2} < \mu < 2Q$   
 scale variation

FIG. 7: NLO longitudinal cross section at  $W = Q = 2.5$  GeV as a function of  $|t|$ . Left:  $D_s = 0, -0.5, -1$  from top to bottom at fixed  $D_g = -1$ . Right:  $D_g = 0, -1, -2$  from top to bottom at fixed  $D_s = 0$ .

Near-threshold  $\phi$  exhibits  
factor  $\sim 4$  greater sensitivity to  $D_s$  compared to  $D_g$ !

This is the green light for our experiments to measure  $D_s$ , so let's go!

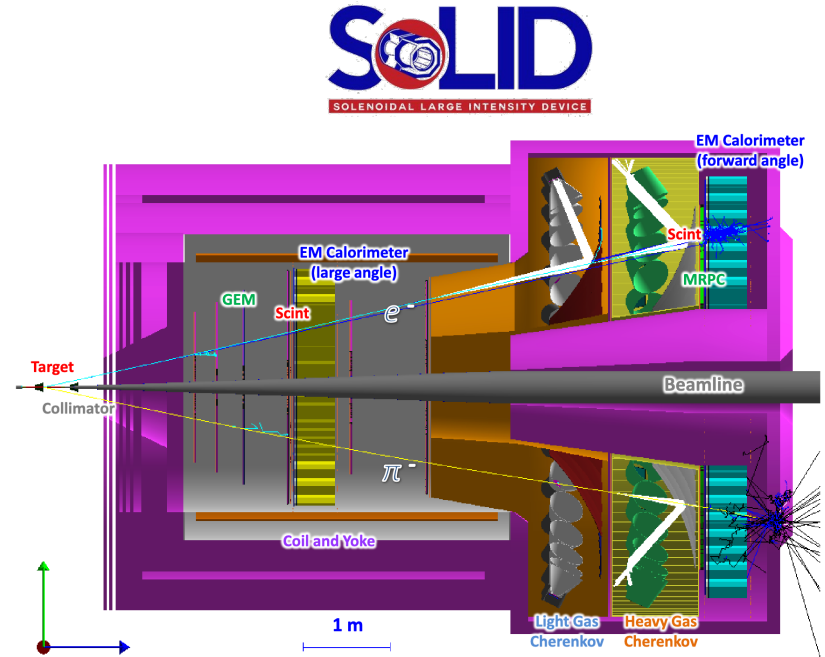


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# EXCLUSIVE $\phi$ IN SOLID

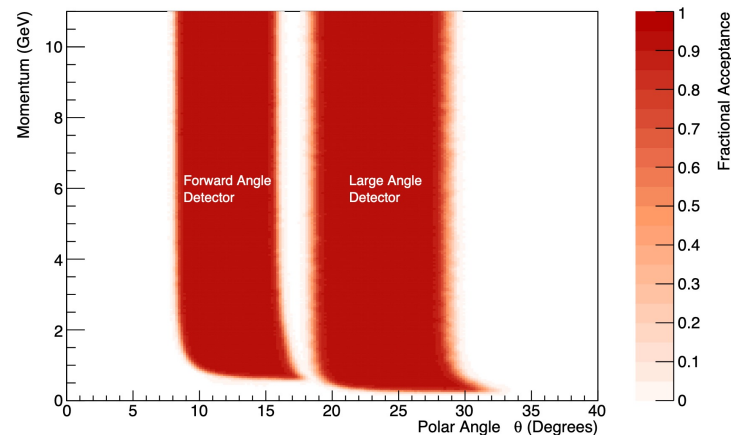
- Large acceptance & luminosity!
  - $\phi$  decay products can be measured directly
    - Fully exclusive, low background
    - Measure  $R$  to extract  $\sigma_L$
  - PID from ToF, Cherenkovs
  - High statistics & continuous kinematic coverage for multidimensional measurement



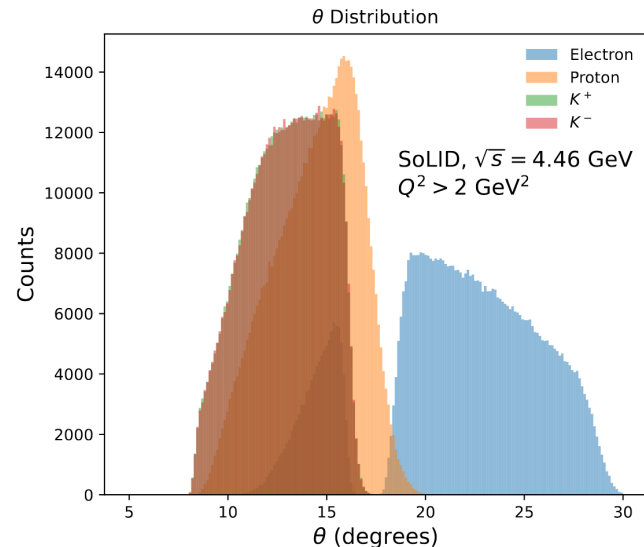
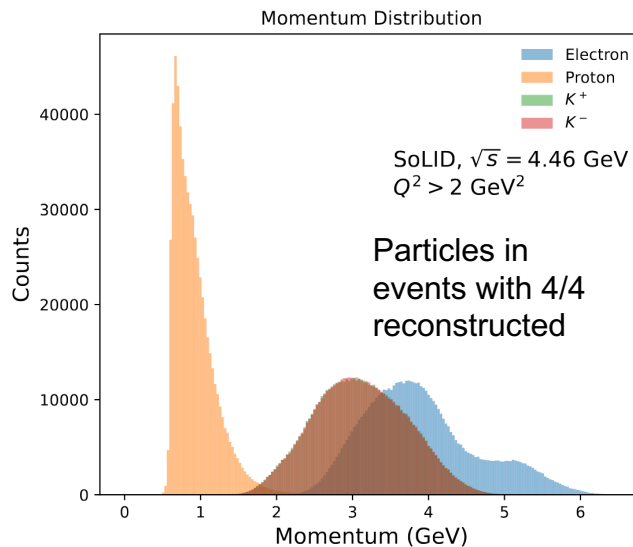
# PROJECTIONS



- First look at projections for SoLID
  - $43.2 \text{ ab}^{-1}$  (existing  $J/\psi$  proposal)
  - **Fully exclusive!** - Detect all four final state particles:  $e'$ ,  $p'$ ,  $K^+$ ,  $K^-$
- Require kaons, proton to be detected in forward angle detector
  - Better TOF PID



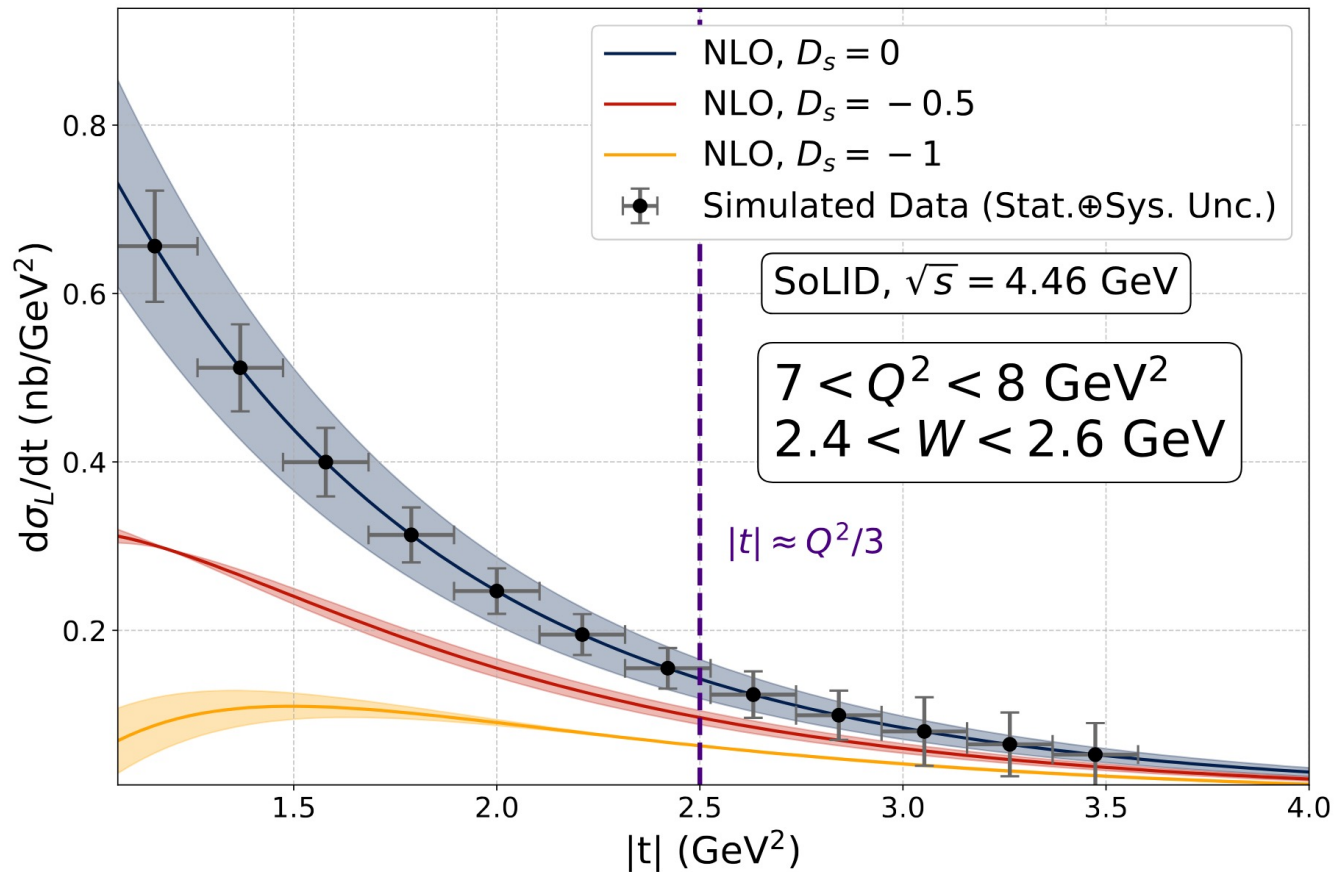
Particle momenta nicely suited for the SoLID PID systems!



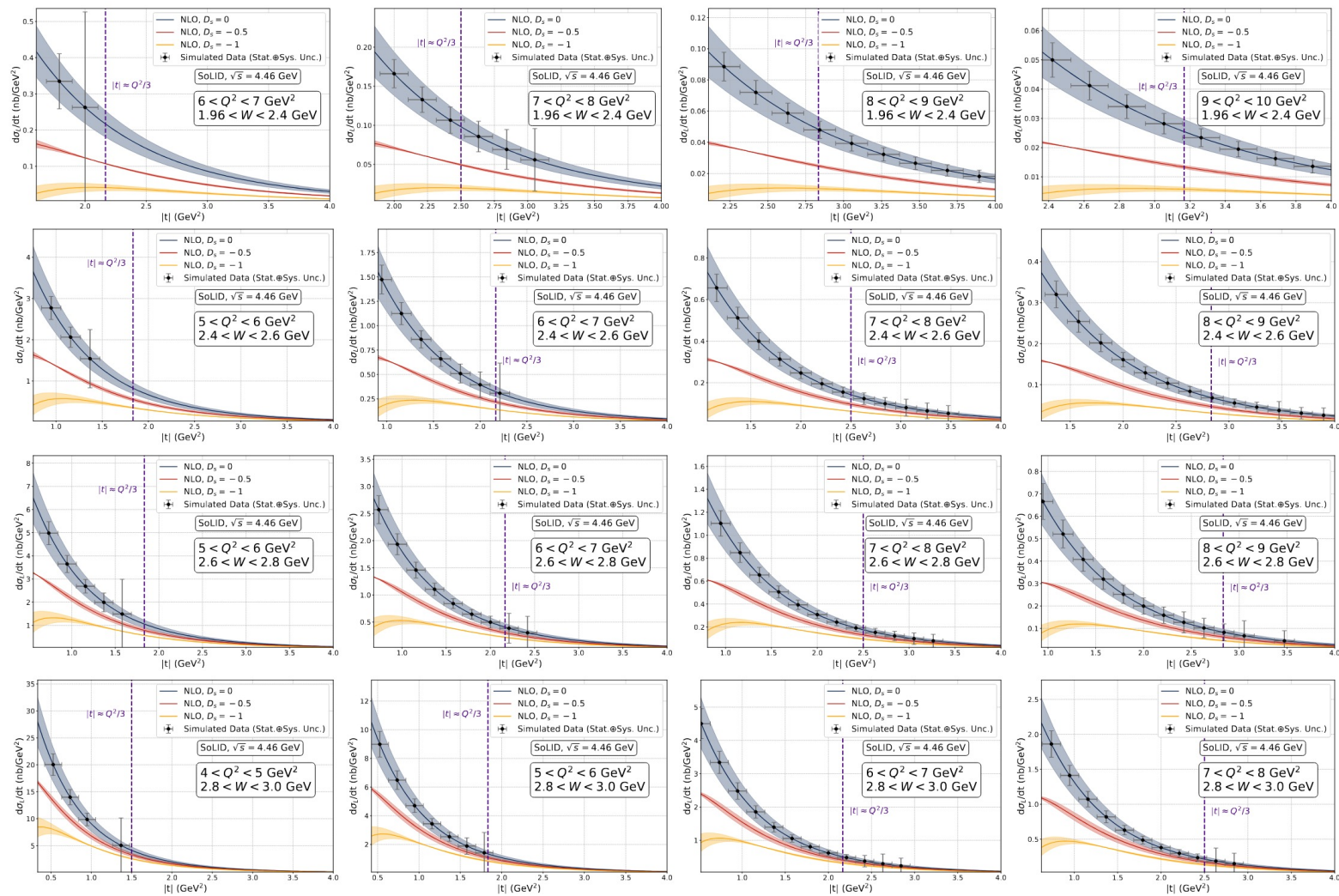
# PROJECTIONS



Assumption of 10% systematic uncertainty still **exhibits good sensitivity to  $D_s$ !**



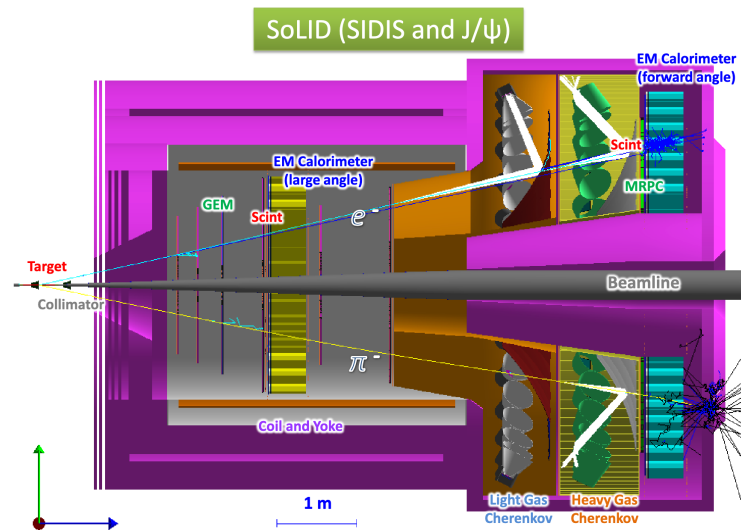
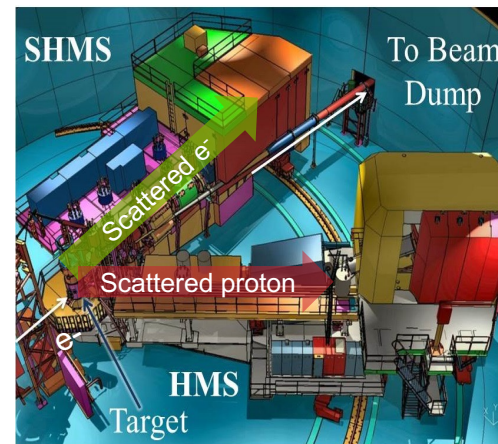




# CONCLUSION

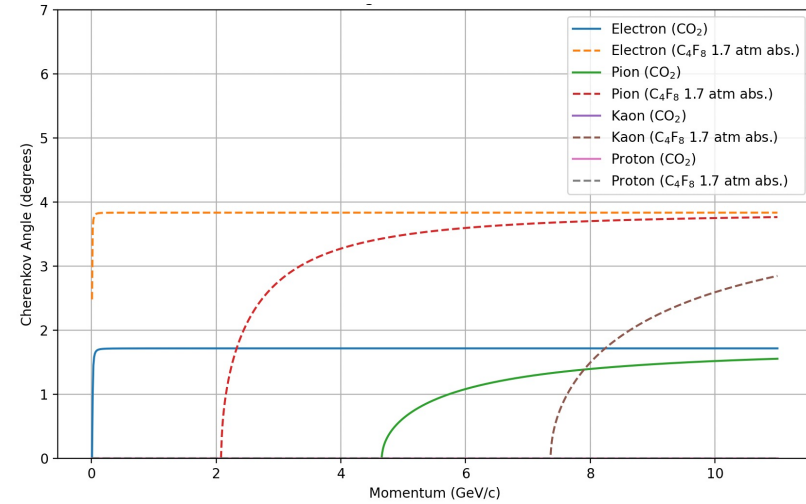
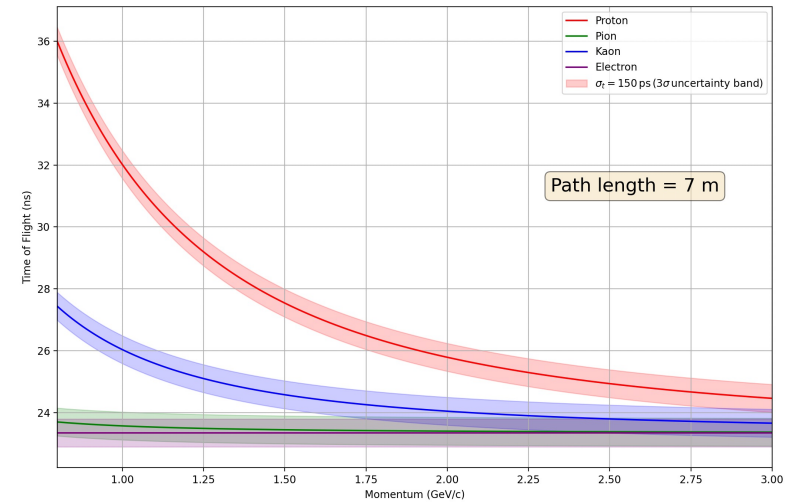
- QCD EMT framework provides new insights into the structure of hadrons
  - In particular,  $D$ -term accesses a new & exciting set of measurable quantities
- If we ever want a complete experimental measurement of the total  $D$ -term of the proton, **will need to measure the strangeness  $D$ -term**
  - Can be done at CEBAF, with the right tools

SoLID provides the opportunity to measure  $D_g$  and eventually  $D_s$ , bringing us into the **precision era of proton mechanical structure!**



# PID

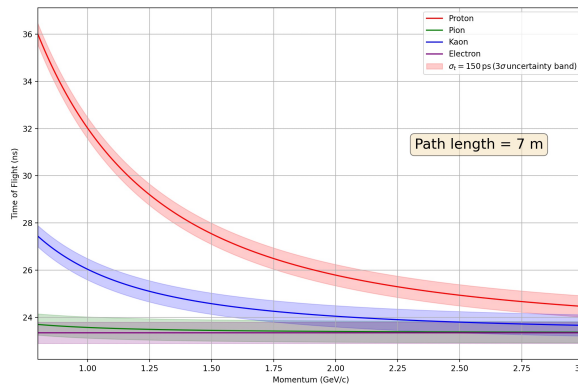
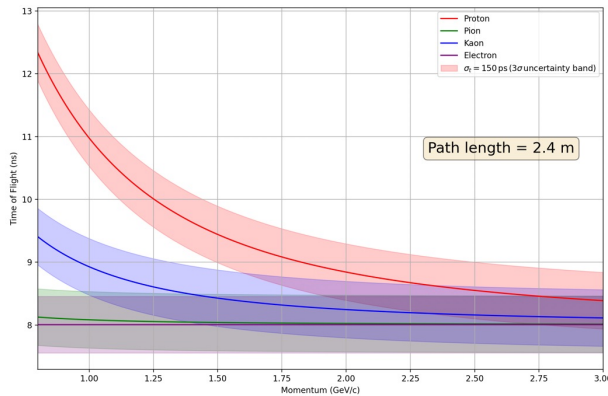
- Kaons from decay of  $\phi$  should be PID'd to reduce background
- Range of  $K^{+,-}$  momentum from  $\sim 1$ -4 GeV in forward detector
- HGC will provide  $\pi$  rejection above 2.5 GeV
- 150 ps TOF covers  $3\sigma$   $\pi/K$  up to  $\sim 2.5$  GeV
  - MRPC would handle this better, reduce the reliance on HGC near its threshold
- Scattered proton is low momentum, typically 1-2 GeV
  - TOF should be able to handle it



# ANALYSIS STRATEGY

## ▪ Kaons:

- Forward detector has superior PID
- Longer TOF baseline + Cherenkovs to reject fast pions
- **MRPC would handle PID over whole momentum range**
- SPD TOF could handle it up to where the HGC turns on
- Require kaons to be in forward detector

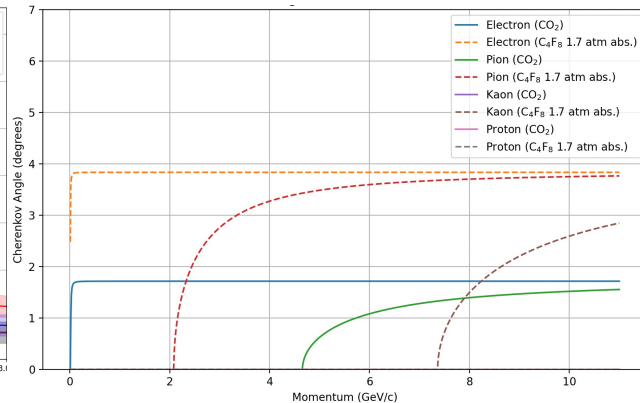


## ▪ Protons:

- Large-angle detector can PID protons up to  $\sim 2$  GeV with SPD TOF
- Allow protons in forward or large angle detectors

## ▪ Electrons:

- Acceptance for fully exclusive reconstruction is best when electron is at large angle
- Require electron in large angle



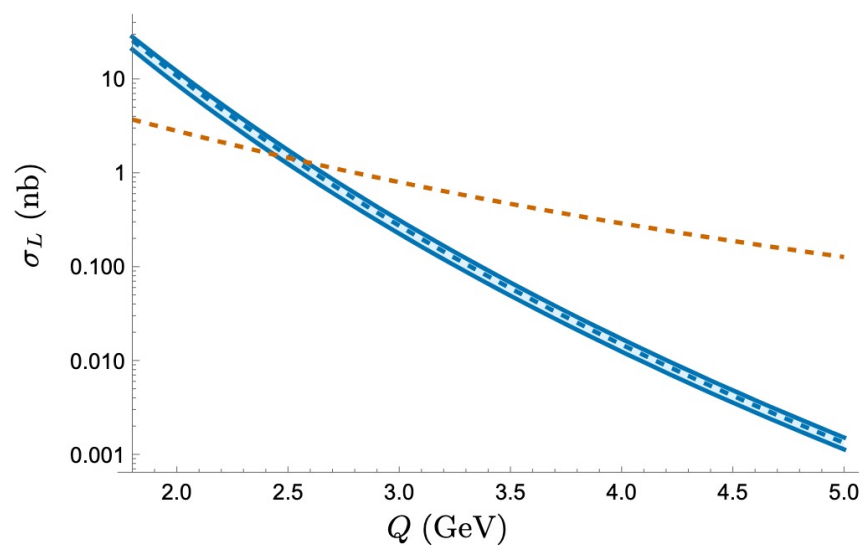
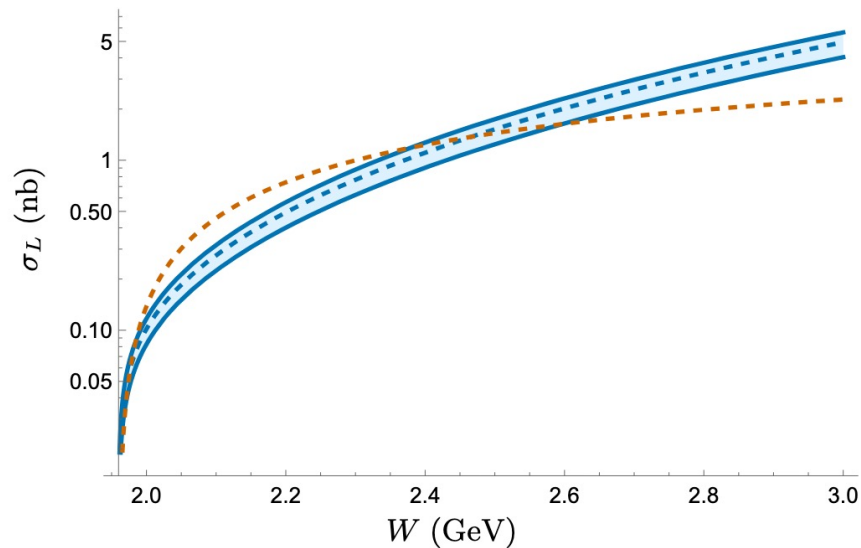


FIG. 8: NLO longitudinal cross section integrated over  $t$  with  $D_s = 0$  and  $D_g = -1$  as a function of  $W$  at fixed  $Q = 2.5$  GeV (left), and as a function of  $Q$  at fixed  $W = 2.5$  GeV (right). The red dashed curve is the CLAS parametrization (56).

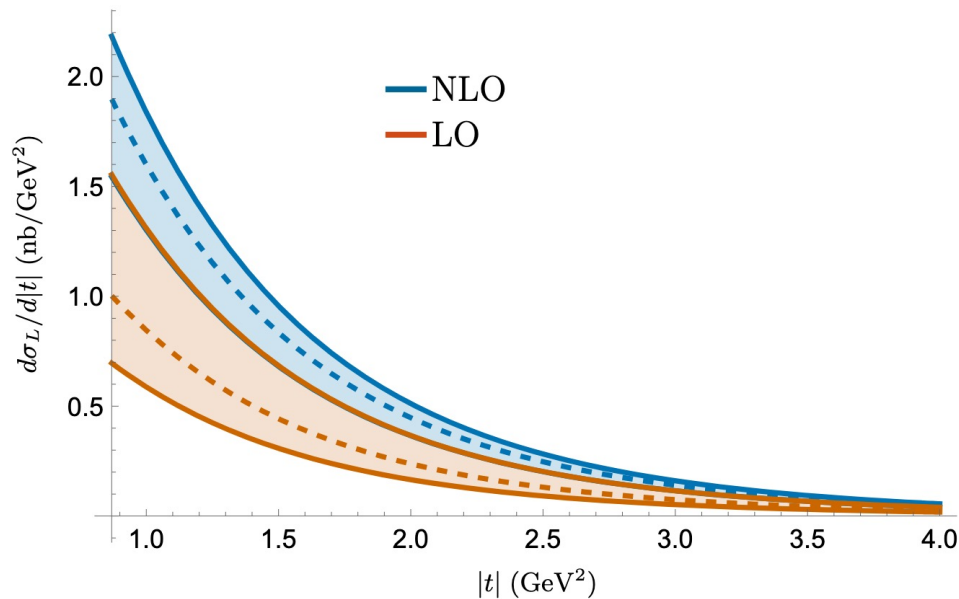


FIG. 6: Differential  $\gamma^*p$  cross section in units of  $\text{nb}/\text{GeV}^2$  at  $W = Q = 2.5$  GeV,  $D_g = -1$ , and  $D_s = 0$  as a function of  $|t|$ . The orange and blue bands represent the LO and NLO cross sections, respectively, with the renormalization scale varied in the range  $Q/2 < \mu < 2Q$ .