PionLT (E12-19-006) Analysis Updates (Low Q^2 L/T Separated Analysis) π^+ Form Factor

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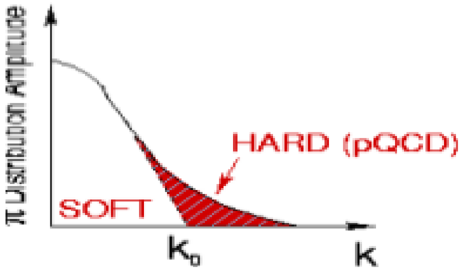
Hall C Winter 2025 Meeting

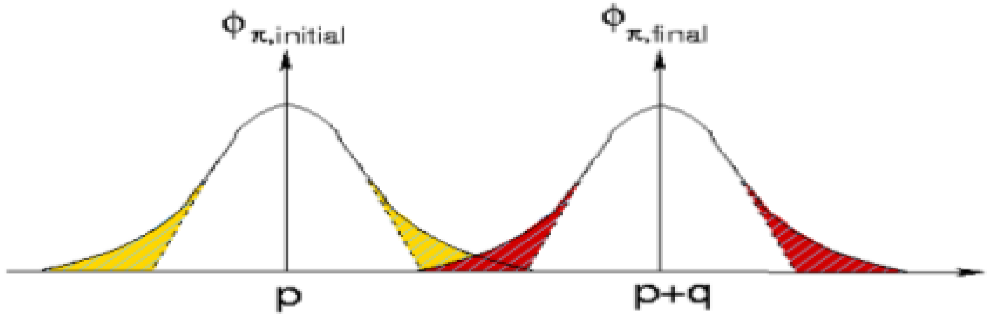
Charged Meson Form Factors

Simple $q\bar{q}$ valence structure of mesons presents the ideal testing ground for our understanding of bound quark systems.

In quantum field theory, the form factor is the overlap integral:

$$F_\pi(Q^2) = \int \phi_\pi^*(p)\phi_\pi(p+q)dp$$





The meson wave function can be separated into φ_π^{soft} with only low momentum contributions ($k<k_0$) and a hard tail φ_π^{hard}.

While φ_π^{hard} can be treated in pQCD, φ_π^{soft} cannot.

From a theoretical standpoint, the study of the Q^2–dependence of the form factor focuses on finding a description for the hard and soft contributions of the meson wave-function.

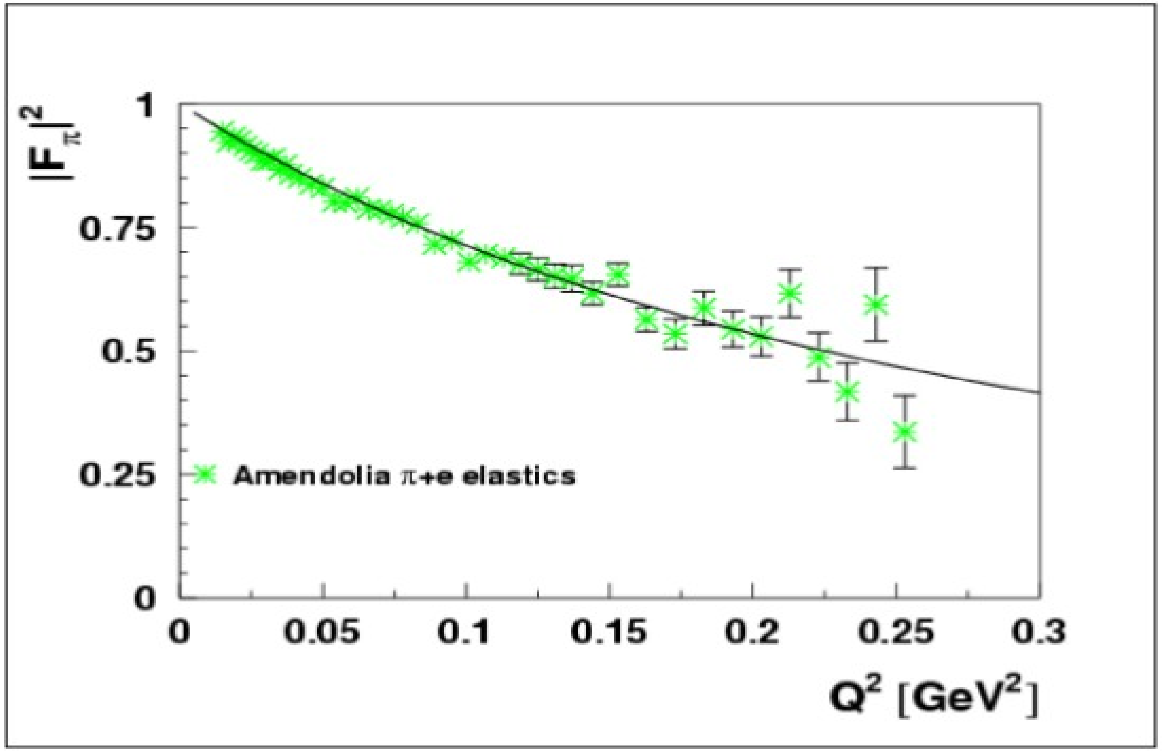
Slide credit: Dr. Garth Huber

π^+ Form Factor – Low Q^2 (Direct Measurement)

At low Q^2, F_π can be measured model–independently via high energy elastic π^- scattering from atomic electrons in Hydrogen

- CERN SPS used 300 GeV pions to measure form factor up to $Q^2 = 0.25$ GeV^2 *[Amendolia, et al., NPB* ***277****(1986)168]*

- Data used to extract pion charge radius
  $r_\pi = 0.657 \pm 0.012$ fm



Maximum accessible Q^2 roughly proportional to pion beam energy

Q^2=1 GeV^2 requires 1 TeV pion beam

Slide credit: Dr. Garth Huber

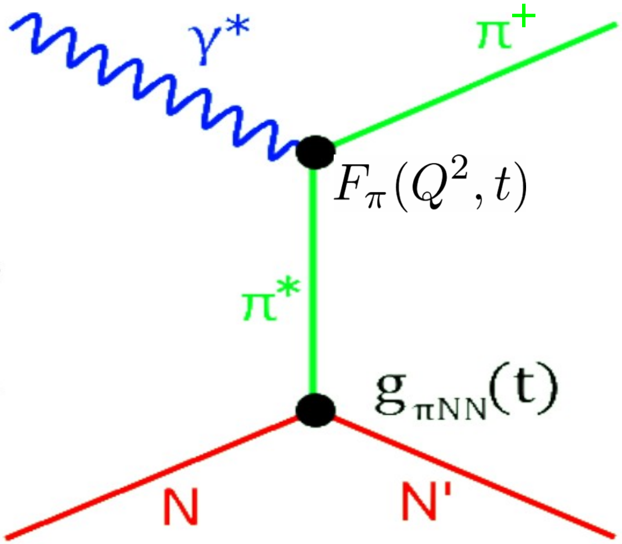
π^+ Form Factor via Electro-production (An Indirect Technique)

Above $Q^2>0.3$ GeV^2, F_π is measured indirectly using the “pion cloud” of the proton via pion electroproduction $p(e,e'\pi^+)n$

$$|p\rangle = |p\rangle_0 + |n\pi^+\rangle + \ldots$$

- At small $-t$, the pion pole process dominates the longitudinal cross section, σ_L
- In Born term model, F_π^2 appears as

$$\frac{d\sigma_L}{dt} \propto \frac{-tQ^2}{(t-m_\pi^2)}\, g_{\pi NN}^2(t)\, F_\pi^2(Q^2,t)$$



Drawbacks of this technique:

1. Isolating σ_L experimentally challenging.
2. The F_π values are in principle dependent upon the model used, but this dependence is expected to be reduced at sufficiently small $-t$.

- The main objective of my Ph.D. thesis analysis is to enhance the understanding of the **indirect technique** by analyzing data at **low Q^2** with high precision and comparing it with **direct measurements**.

Slide credit: Dr. Garth Huber

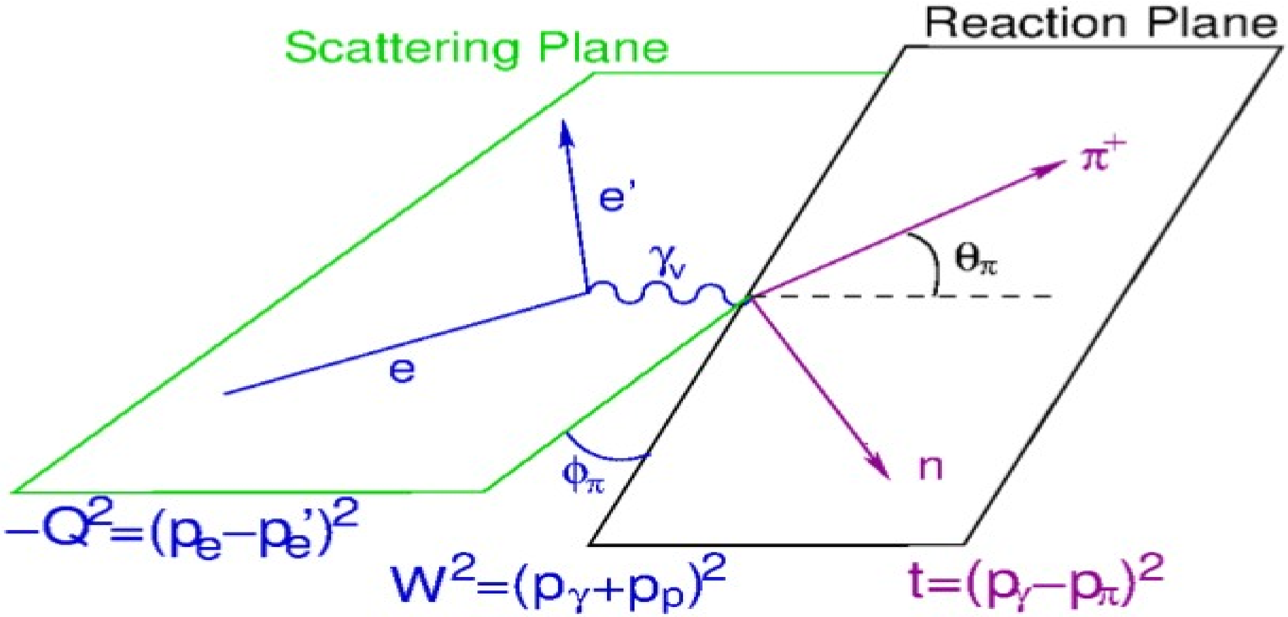
Rosenbluth (LT) Cross-Section Separation Technique

$$2\pi \frac{d^2\sigma}{dt\,d\phi} = \varepsilon \frac{d\sigma_L}{dt} + \frac{d\sigma_T}{dt} + \sqrt{2\varepsilon(\varepsilon+1)}\,\frac{d\sigma_{LT}}{dt}\cos\phi + \varepsilon\frac{d\sigma_{TT}}{dt}\cos 2\phi$$

Virtual-photon polarization:

$$\varepsilon = \left(1 + 2\frac{(E_e - E_{e'})^2 + Q^2}{Q^2}\tan^2\frac{\theta_{e'}}{2}\right)^{-1}$$

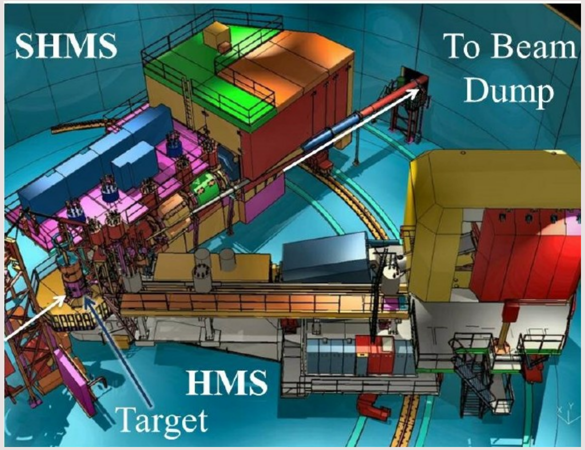
$$e + p \rightarrow e' + \pi^+ + n$$



- **L-T separation required to separate σ_L from σ_T**
- **Need to take data at smallest available $-t$, so σ_L has maximum contribution from the π^+ pole**
- **Need to measure t–dependence of σ_L at fixed Q^2,W**

Slide credit: Dr. Garth Huber

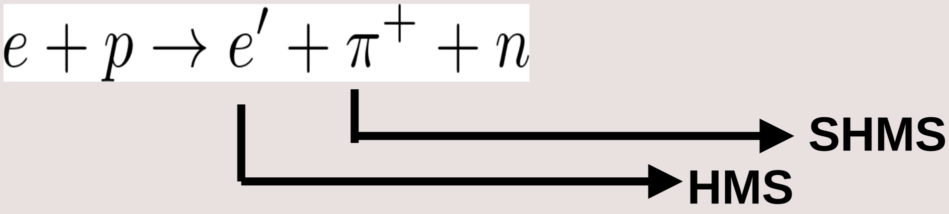
The E12-19-006 Experiment



- The experiment conducted in three phases,
  - **First run period:** ran in summer 2019 (my Ph.D. thesis data)
  - **Second run period:** ran in fall 2021
  - **Third run period:** ran in fall 2022

- Nathan Heinrich
- Muhammad Junaid

- The reaction system

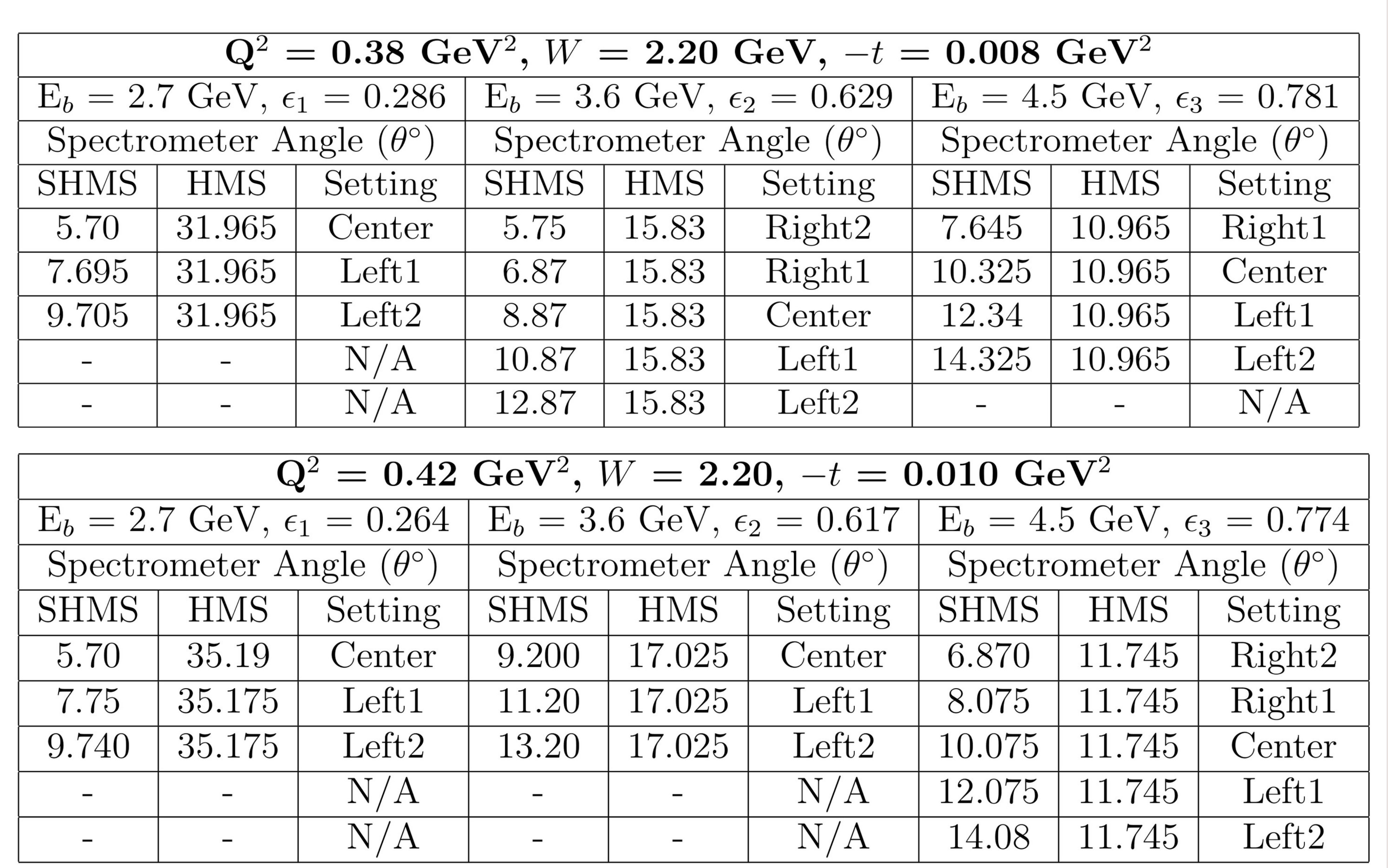


$$e + p \rightarrow e' + \pi^{+} + n$$

- Spokesperson
  - Dr. Garth Huber (UofR), Dr. Tanja Horn (CUA), and Dr. David Gaskell (JLab)

The E12-19-006 Experiment

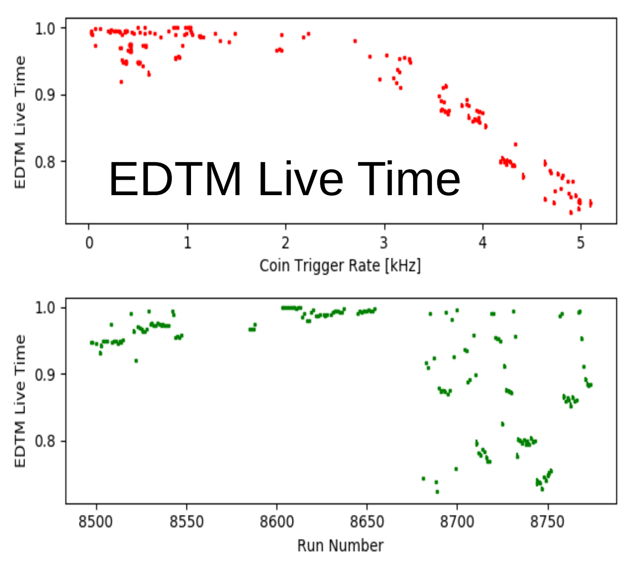
- The data acquired in the first run period,



$\mathbf{Q^2 = 0.38\ GeV^2}, W = \mathbf{2.20\ GeV}, -t = \mathbf{0.008\ GeV^2}$								
$E_b = 2.7$ GeV, $\epsilon_1 = 0.286$			$E_b = 3.6$ GeV, $\epsilon_2 = 0.629$			$E_b = 4.5$ GeV, $\epsilon_3 = 0.781$		
Spectrometer Angle ($\theta°$)			Spectrometer Angle ($\theta°$)			Spectrometer Angle ($\theta°$)		
SHMS	HMS	Setting	SHMS	HMS	Setting	SHMS	HMS	Setting
5.70	31.965	Center	5.75	15.83	Right2	7.645	10.965	Right1
7.695	31.965	Left1	6.87	15.83	Right1	10.325	10.965	Center
9.705	31.965	Left2	8.87	15.83	Center	12.34	10.965	Left1
-	-	N/A	10.87	15.83	Left1	14.325	10.965	Left2
-	-	N/A	12.87	15.83	Left2	-	-	N/A

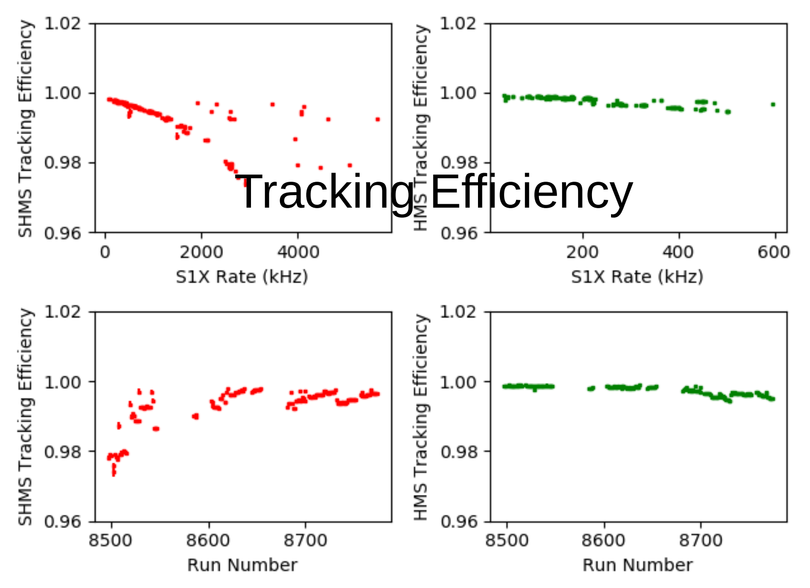
$\mathbf{Q^2 = 0.42\ GeV^2}, W = \mathbf{2.20}, -t = \mathbf{0.010\ GeV^2}$								
$E_b = 2.7$ GeV, $\epsilon_1 = 0.264$			$E_b = 3.6$ GeV, $\epsilon_2 = 0.617$			$E_b = 4.5$ GeV, $\epsilon_3 = 0.774$		
Spectrometer Angle ($\theta°$)			Spectrometer Angle ($\theta°$)			Spectrometer Angle ($\theta°$)		
SHMS	HMS	Setting	SHMS	HMS	Setting	SHMS	HMS	Setting
5.70	35.19	Center	9.200	17.025	Center	6.870	11.745	Right2
7.75	35.175	Left1	11.20	17.025	Left1	8.075	11.745	Right1
9.740	35.175	Left2	13.20	17.025	Left2	10.075	11.745	Center
-	-	N/A	-	-	N/A	12.075	11.745	Left1
-	-	N/A	-	-	N/A	14.08	11.745	Left2

Yield Correction Factors (Crucial to LT separation)



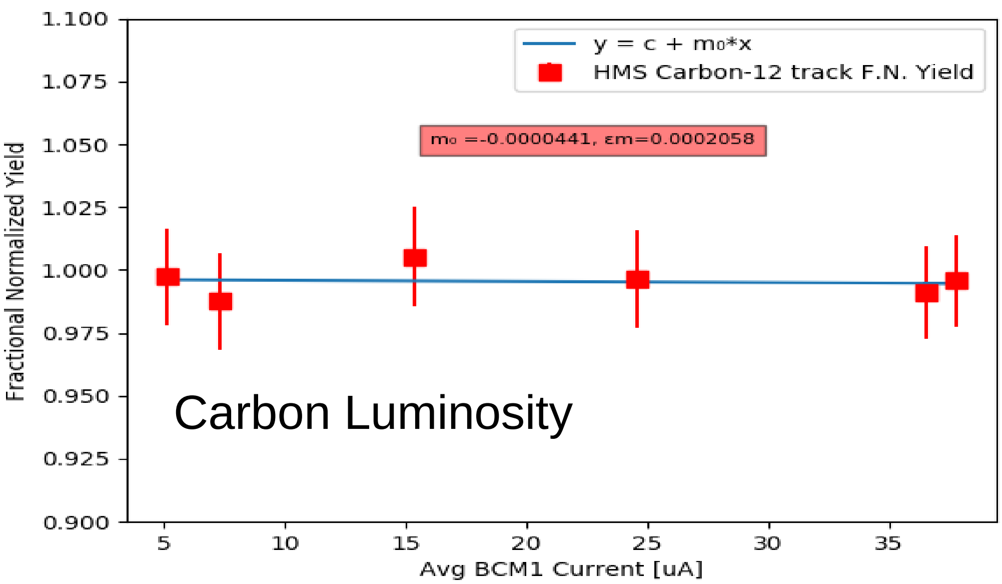
EDTM Live Time

Trigger Efficiency

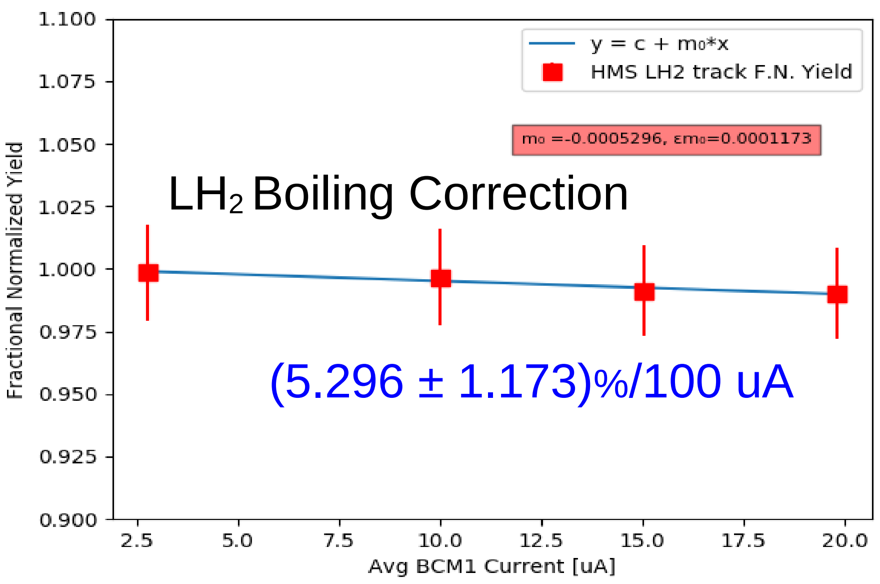


Tracking Efficiency

For $Q^2 = 0.38$ and $0.42\ \text{GeV}^2$ data analysis



Carbon Luminosity

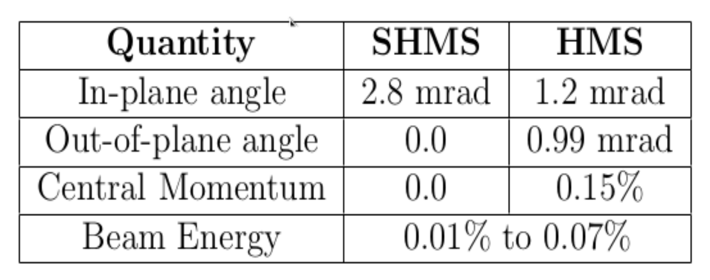


LH_2 Boiling Correction

(5.296 ± 1.173)%/100 uA

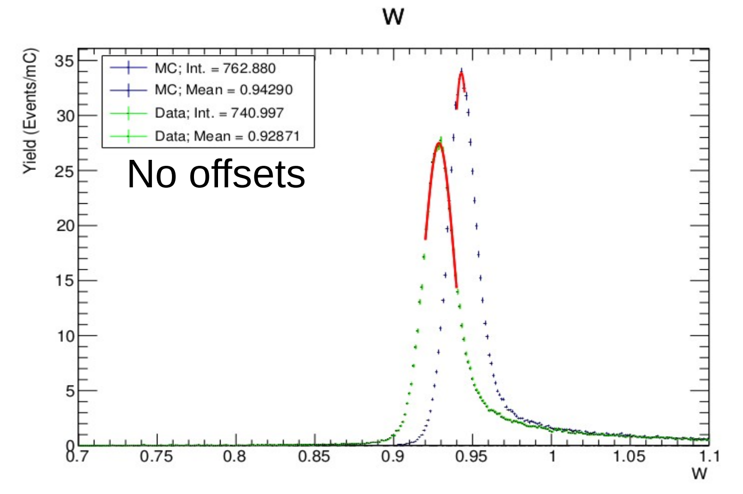
‘Heep’, $e + p \rightarrow e' + p$, Analysis (Kinematic offsets)

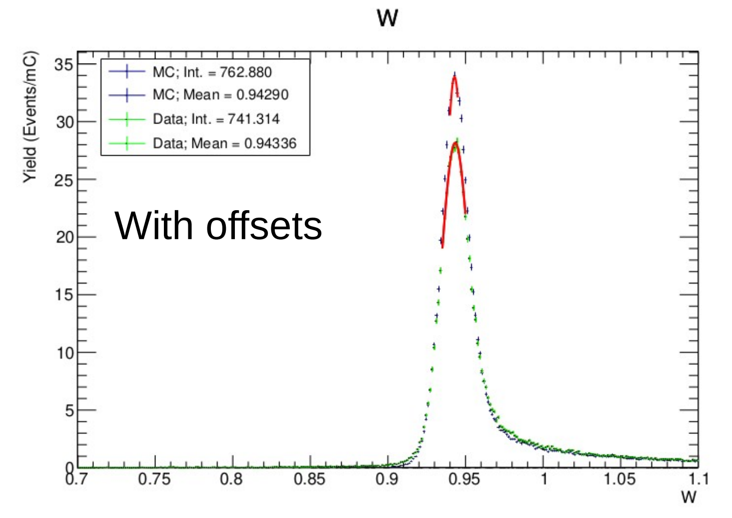
- Achieving high-quality LT separation necessitates more precise beam energy, spectrometer angles, and momenta than what is provided by power supply calibrations and floor angle markings.
  - Heep reaction kinematically over-determined (e’ & p detected).
  - We used the deviations between observed and physically required values to determine the experimental offsets.



Quantity	SHMS	HMS
In-plane angle	2.8 mrad	1.2 mrad
Out-of-plane angle	0.0	0.99 mrad
Central Momentum	0.0	0.15%
Beam Energy	0.01% to 0.07%	

For Q^2 = 0.38 and 0.42 GeV^2 (Low Q^2) LT Separation Analysis



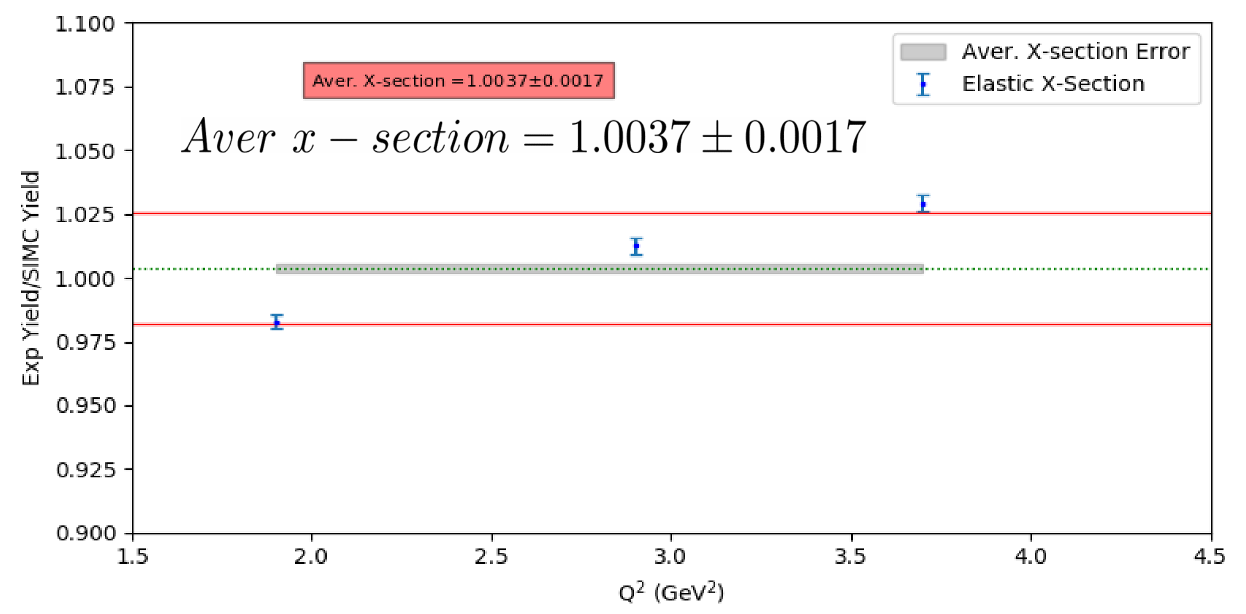


Error bar statistical only

'Heep', $e + p \rightarrow e' + p$, Analysis (Elastic X-Section)

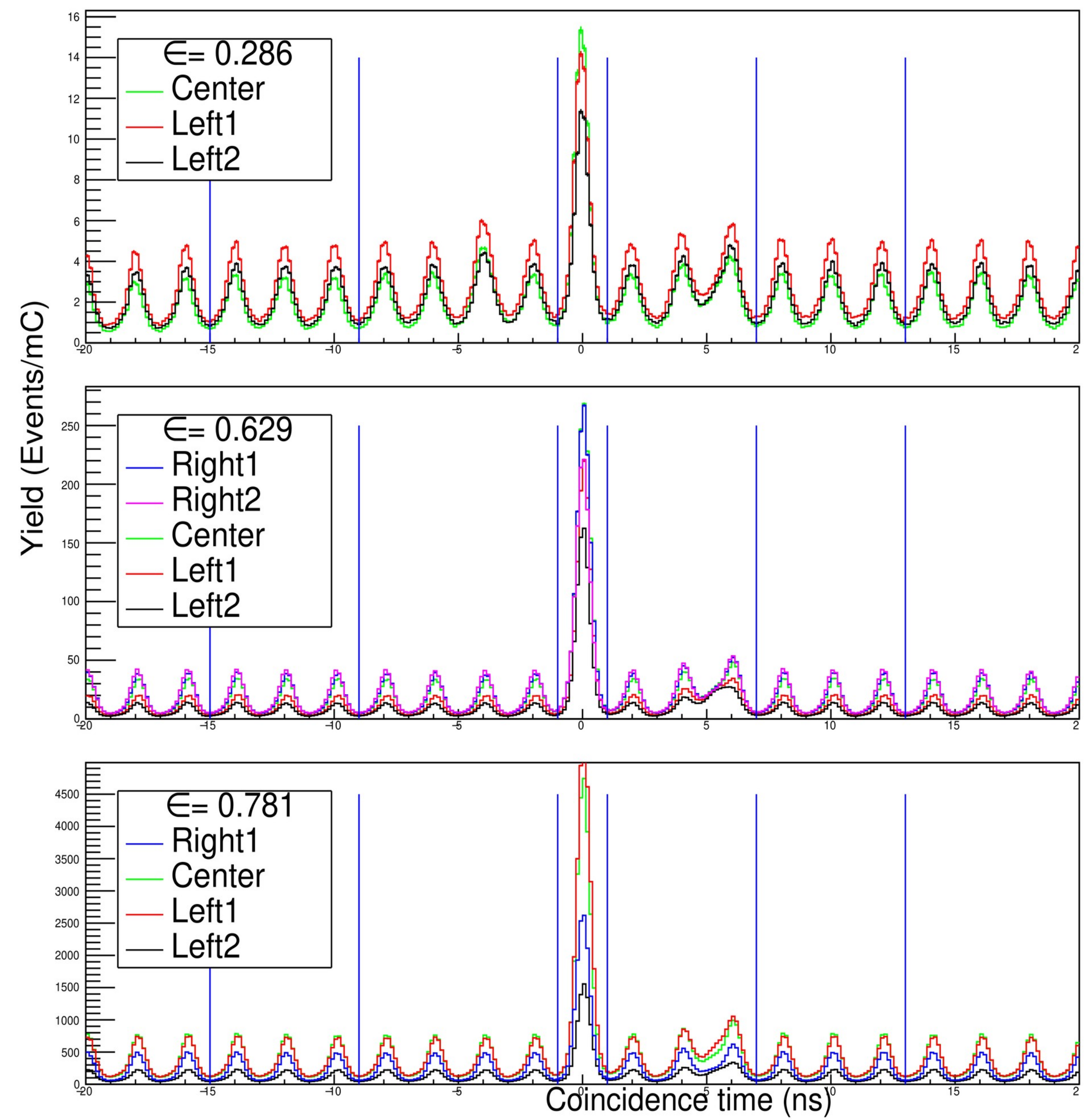
- Reproducing the known elastic cross-section before the LT separation study increases confidence in the high-quality LT separation results.
  - We studied the elastic x-section of the 1st run of the experiment

- Experimental yield $Yield = \frac{N}{Q_{tot} \times \epsilon_{tot}}$



Error bar statistical only

$e + p \rightarrow e' + \pi^{+} + n$ Event Selection (Coincidence Timing)

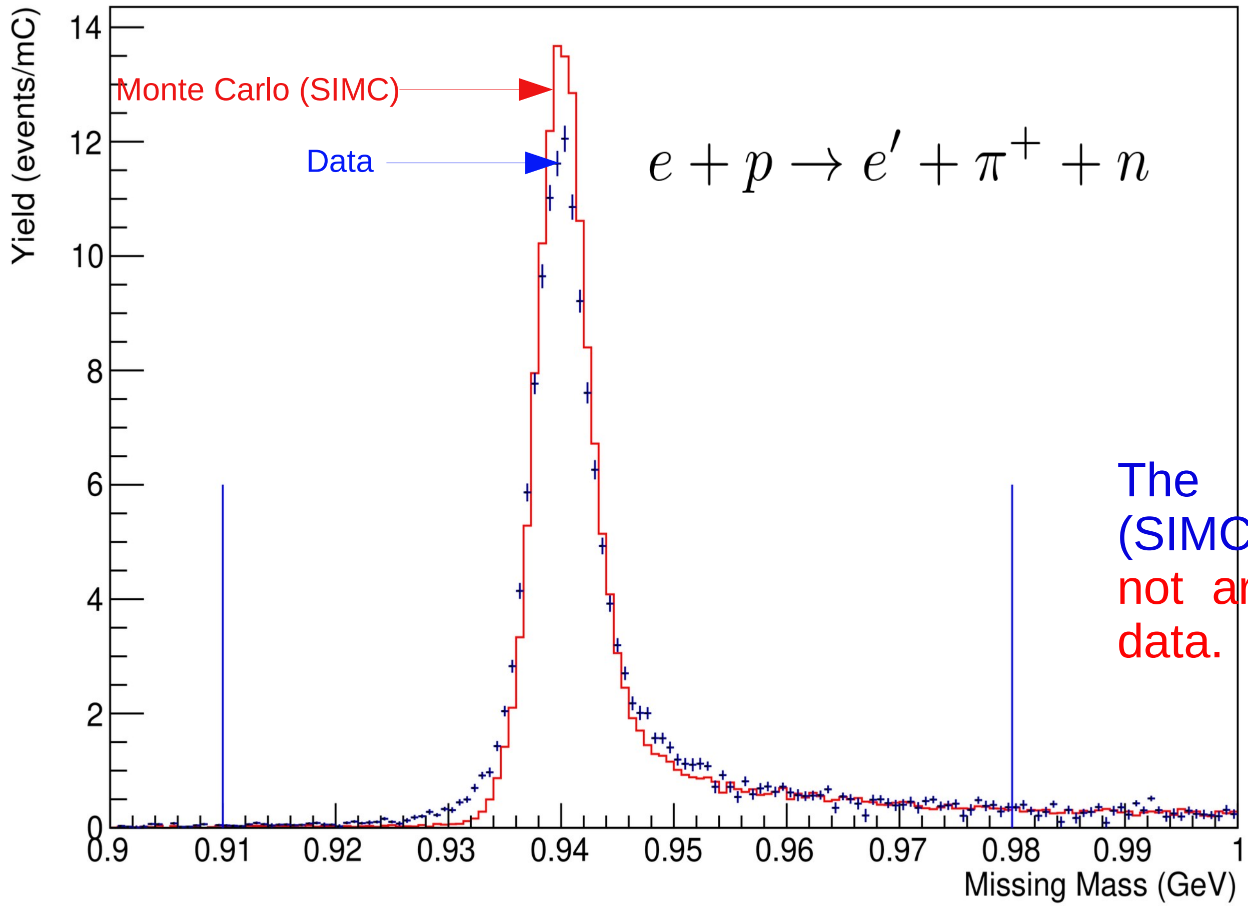


$Q^2 = 0.38\ \mathrm{GeV}^2$

$$t_{coin} = t_{HMS} - t_{SHMS}$$

$e + p \rightarrow e' + \pi^+ + n$ Event Selection (Missing Mass)

$Q^2 = 0.38$ GeV^2



We uniquely identify the exclusive final state of the produced π^+ with the missing mass technique.

The red represents the MC (SIMC) predicted cross-section, not arbitrarily normalized to the data.

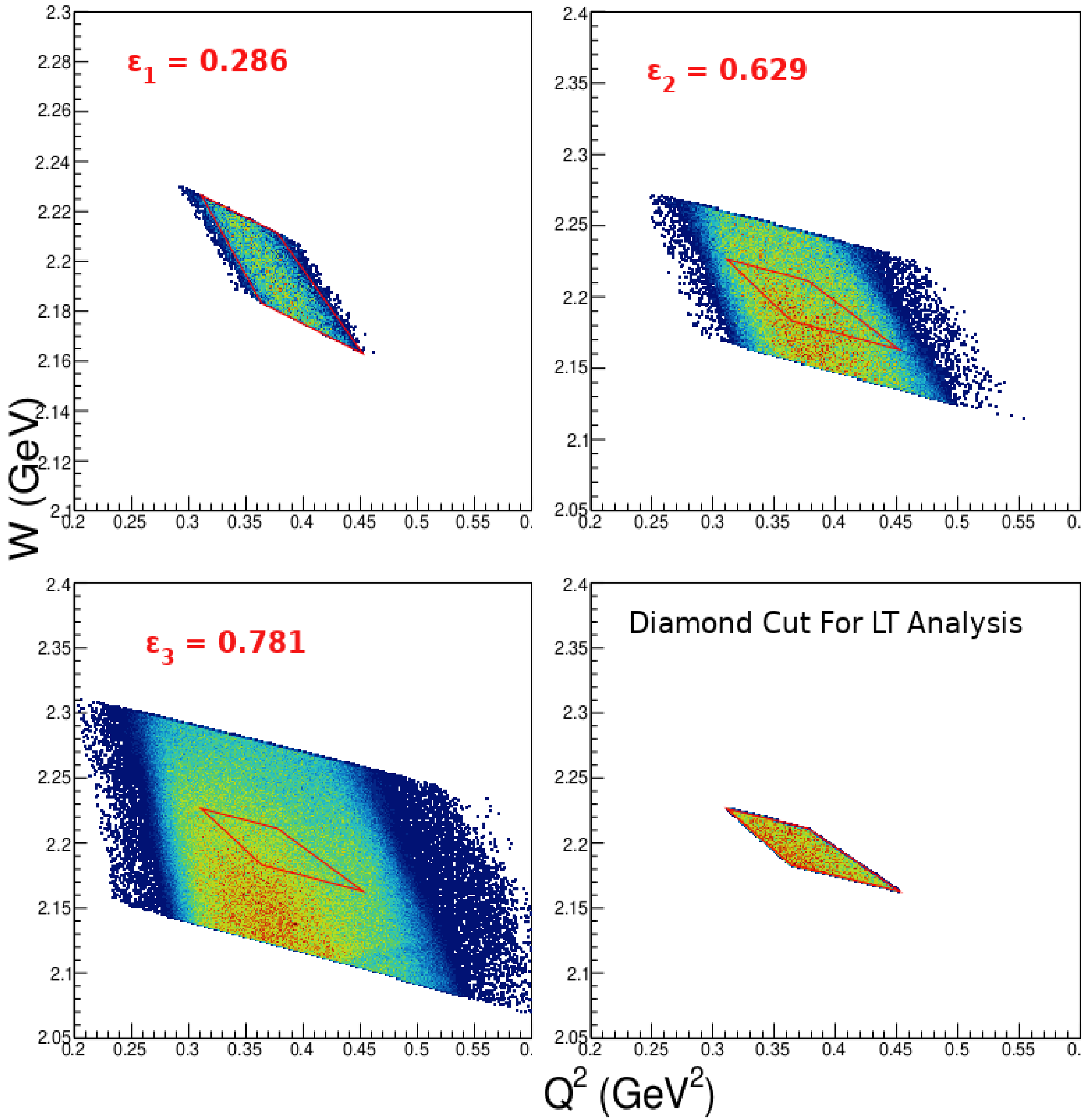
Plot: Mid ε, Center Setting of Q^2 = 0.38 GeV^2 data

$$MM = \sqrt{((E_b + m_p - E'_e - E_{\pi^+})^2 - (\vec{P_e} - \vec{P_{e'}} - \vec{P_{\pi^+}})^2)}$$

$e + p \rightarrow e' + \pi^{+} + n$ Diamond Cut

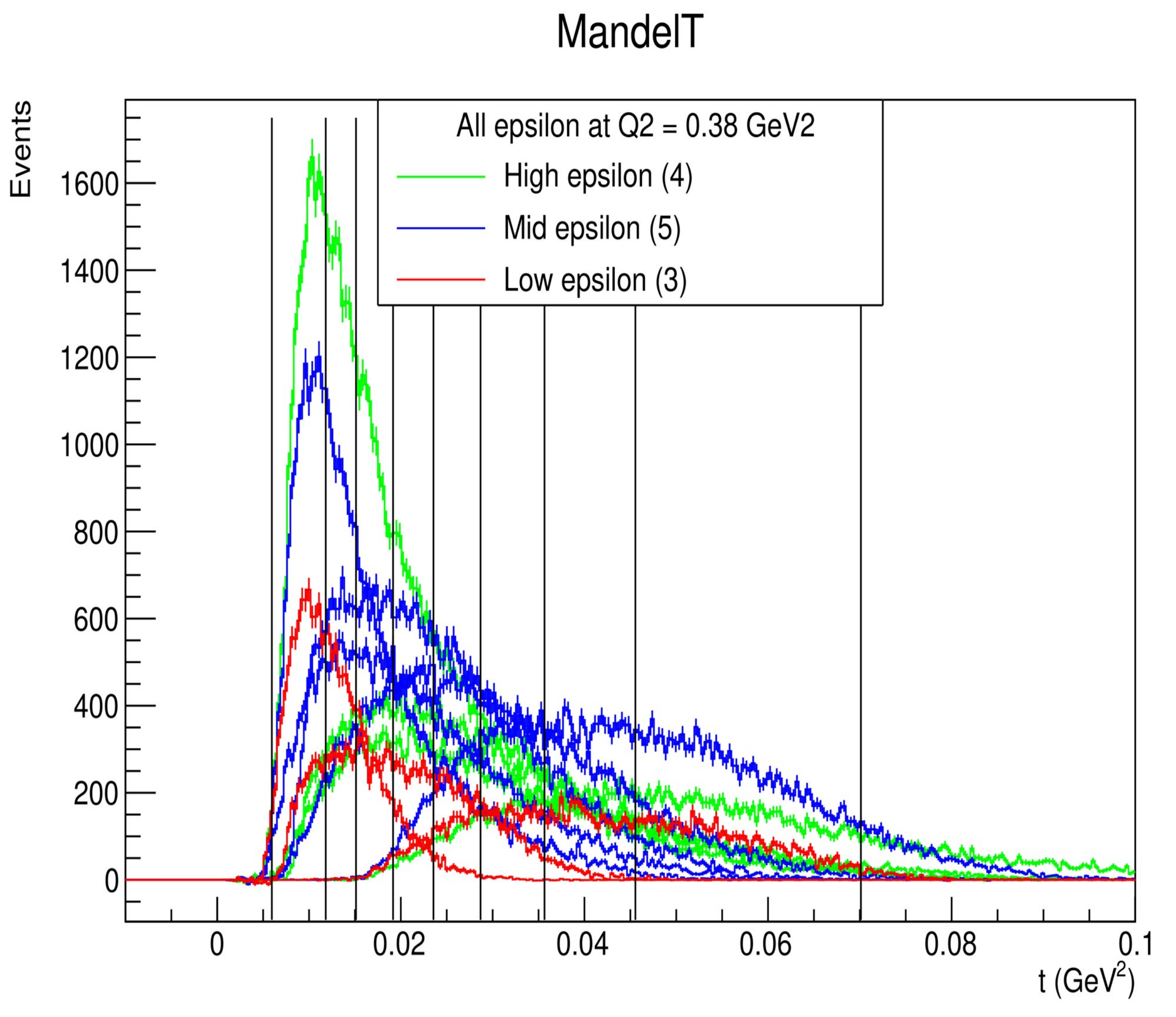
$Q^2 = 0.38\ \mathrm{GeV}^2$

- The absolute acceptance of the spectrometers in Hall C varies for different beam energies or ε settings.
- The LT separation technique requires uniform acceptance across all ε data.



$e + p \to e' + \pi^+ + n$ -t Binning & Φ Binning

- To separate the individual π^+ cross section terms as a function of -t, the data were binned into -t bins.
    - Checked the Monte Carlo simulation resolution for the fine -t binning.
    - Binned all the data into 8 t bins.
    - The general criterion that followed while binning the data was to have approximate the same statistics in each -t bin.
- Each t bin is further binned into 16 Φ bins to have the better convergence of the Rosenbluth separation technique.

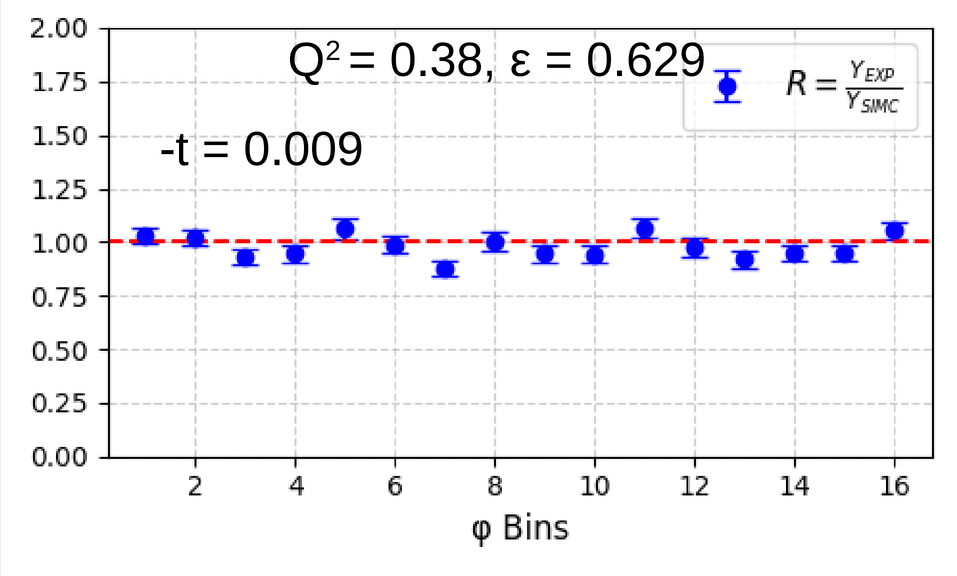


$$t = (p_\gamma - p_\pi)^2$$

$e + p \to e' + \pi^+ + n$ Experimental Cross-Section (Iterative Procedure)

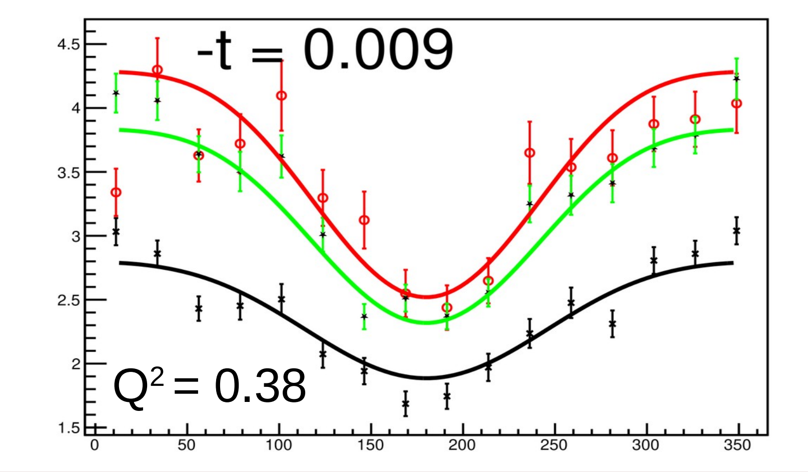
For each ε, SHMS setting analyzed and form the Yield ratio. Each ε is then combined. Error propagated accordingly.

$$R = \frac{Yield_{\exp}}{Yield_{MC}}$$



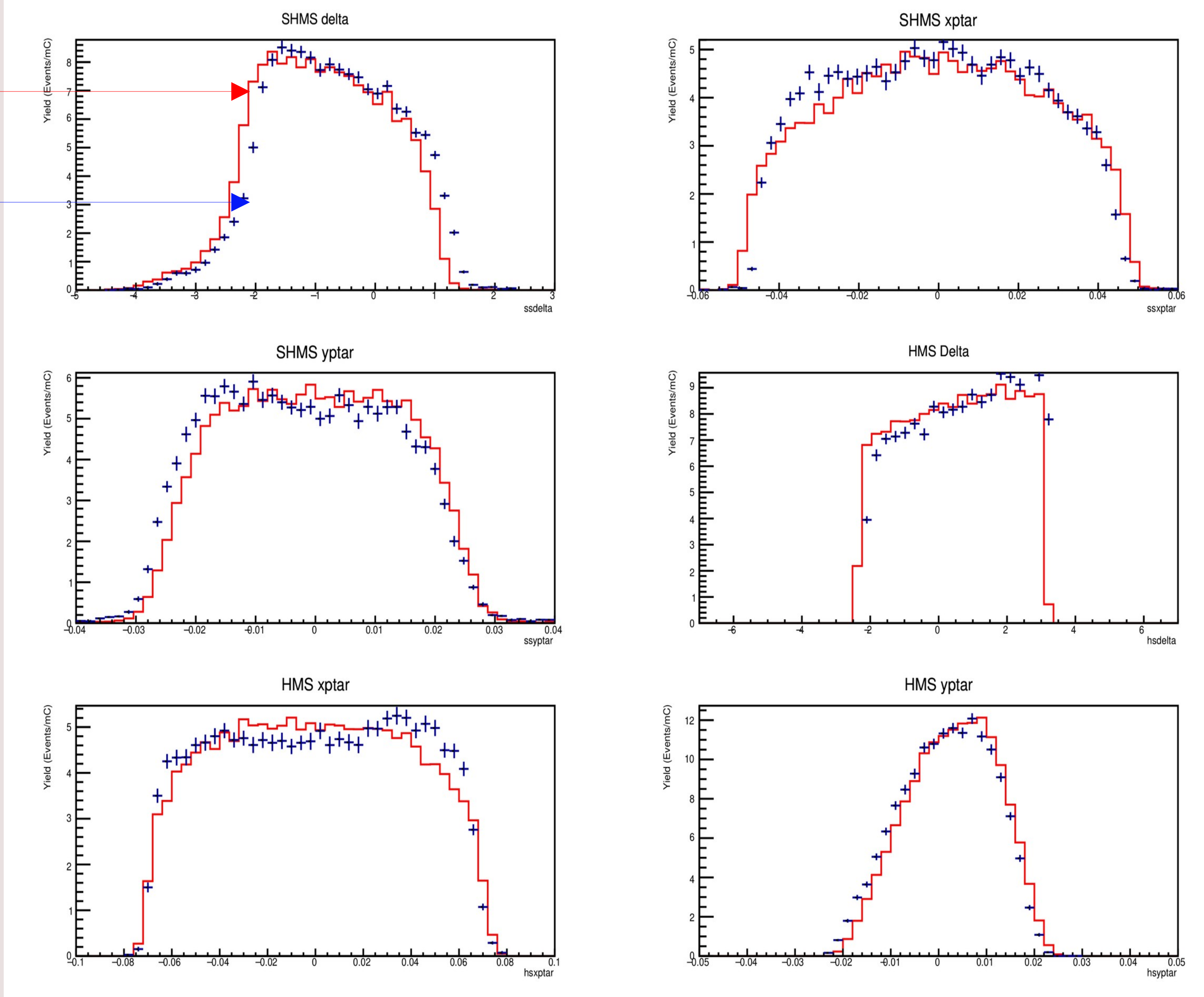
$$\sigma_{exp}(\bar{W}, \bar{Q^2}, t, \phi; \bar{\theta}, \bar{\epsilon}) = \frac{Y_{exp}}{Y_{MC}} \sigma_{MC}(\bar{W}, \bar{Q^2}, t, \phi; \bar{\theta}, \bar{\epsilon}).$$

Fit the models to σ_L, σ_T, σ_{LT} & σ_{TT} to determine a new set of parameters.



$$2\pi \frac{d^2\sigma}{dt d\phi} = \epsilon \frac{d\sigma_L}{dt} + \frac{d\sigma_T}{dt} + \sqrt{2\epsilon(\epsilon+1)} \frac{d\sigma_{LT}}{dt} cos\phi + \epsilon \frac{d\sigma_{TT}}{dt} cos2\phi$$

$e + p \rightarrow e' + \pi^{+} + n$ Yield Comparison (Acceptance Variables)



SIMC

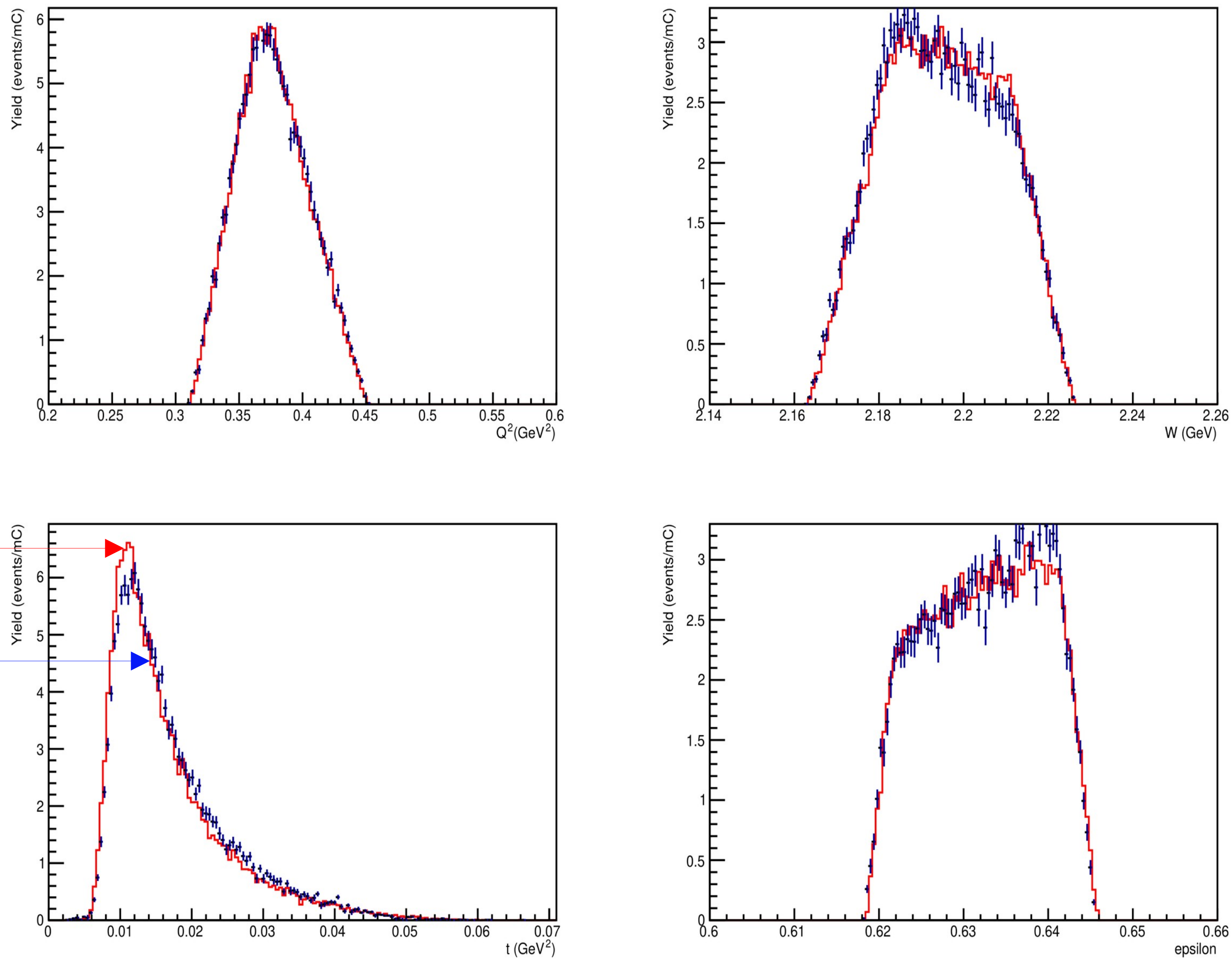
Data

- The red represents the MC (SIMC) predicted cross-section after the iteration procedure, not arbitrarily normalized to the data.

Plot: Mid ε, Center Setting of Q^2 = 0.38 GeV^2 data

$e + p \rightarrow e' + \pi^+ + n$ Yield Comparison (Kinematic Variables)

- The red represents the MC (SIMC) predicted cross-section after the iteration procedure, not arbitrarily normalized to the data.



SIMC

Data

Plot: Mid ε, Center Setting of Q^2 = 0.38 GeV^2 data

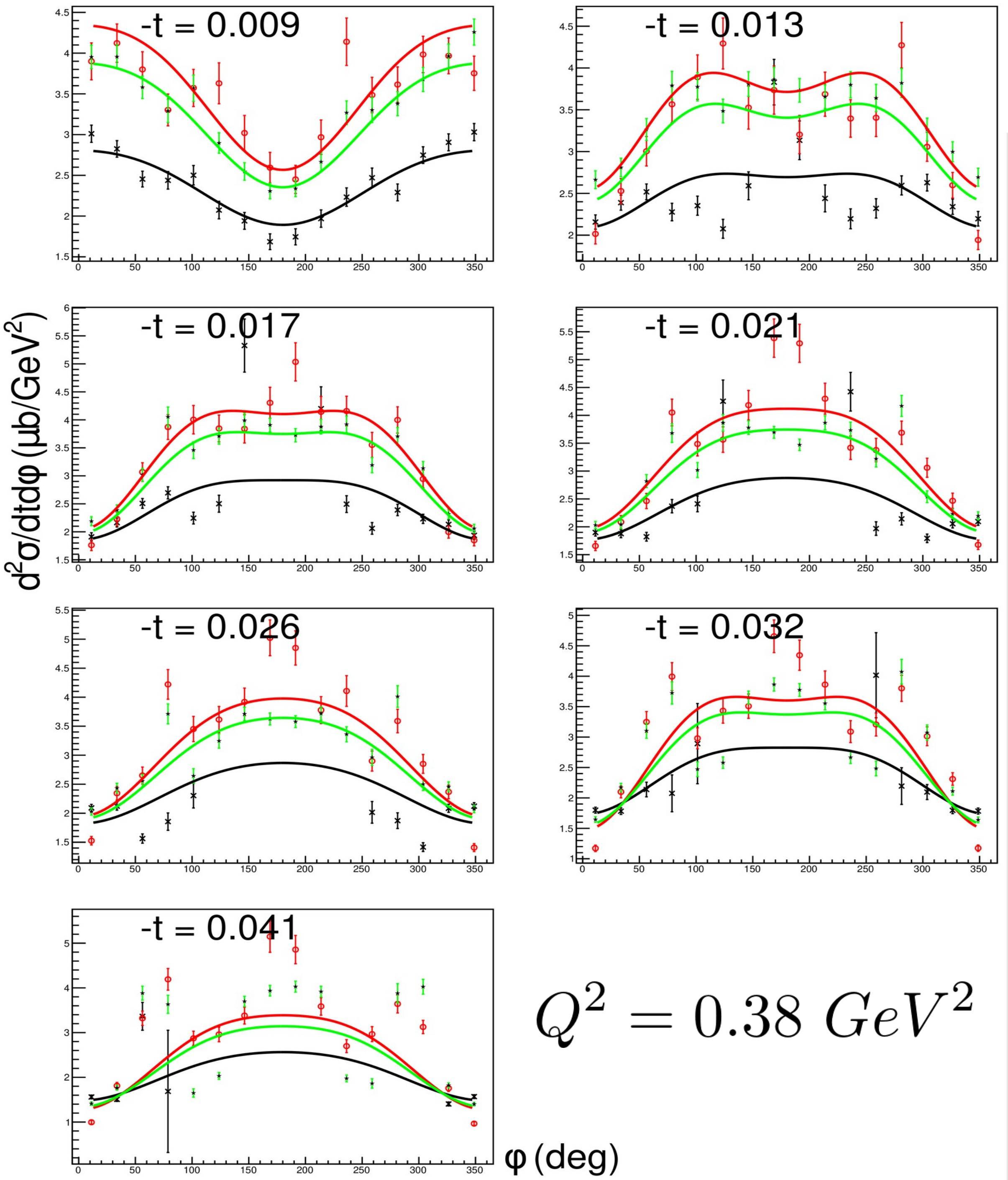
The Unseparated Cross-Section

$$e + p \rightarrow e' + \pi^+ + n$$

- **Low ε = 0.286**
- **Mid ε = 0.629**
- **High ε = 0.781**.

FINAL PRELIMINARY RESULTS
(PENDING A FEW FINAL
CHECKS)

$$2\pi\frac{d^2\sigma}{dtd\phi} = \epsilon\frac{d\sigma_L}{dt} + \frac{d\sigma_T}{dt} + \sqrt{2\epsilon(\epsilon+1)}\frac{d\sigma_{LT}}{dt}cos\phi + \epsilon\frac{d\sigma_{TT}}{dt}cos2\phi$$



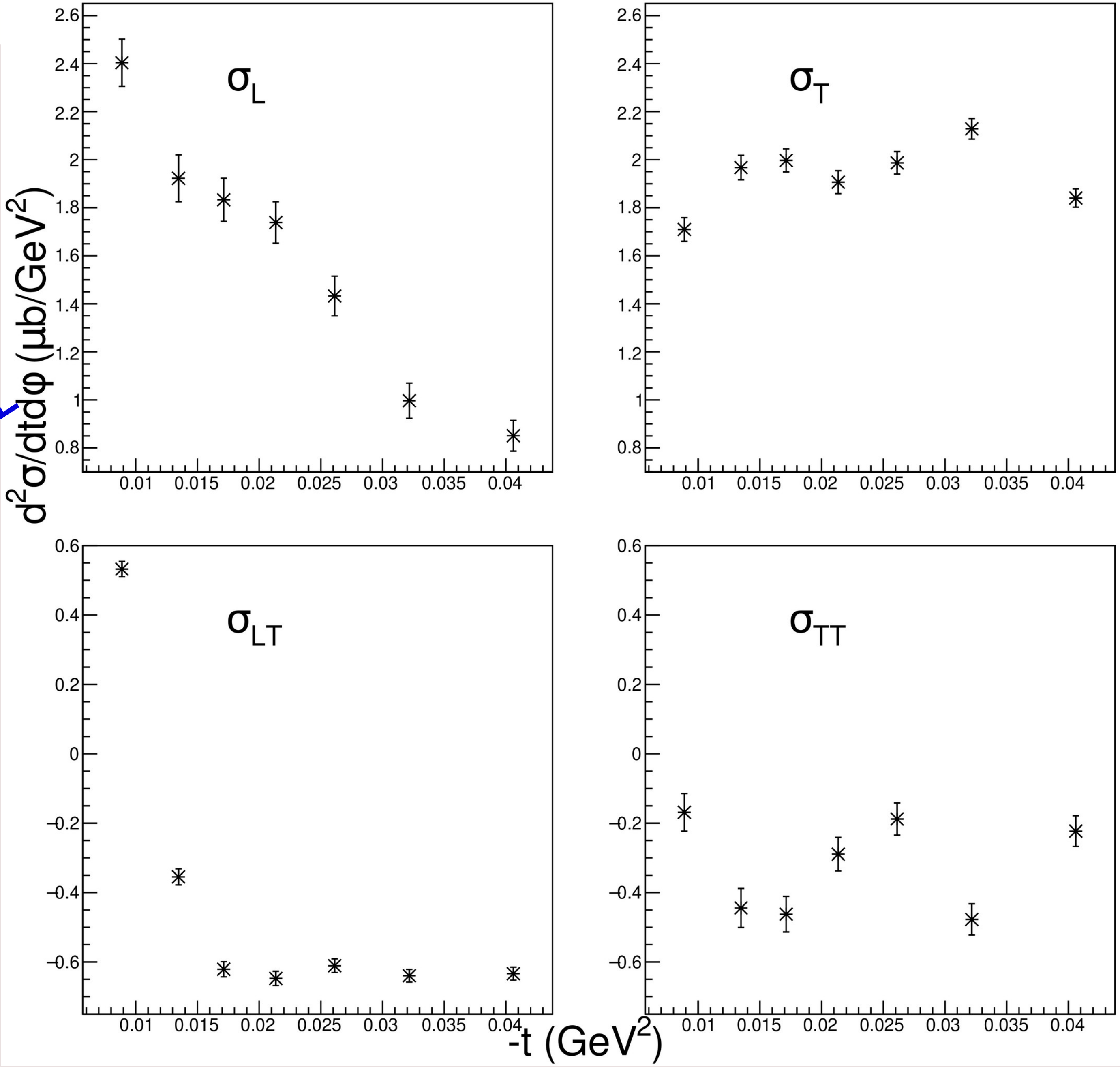
$$Q^2 = 0.38\ GeV^2$$

The Separated Cross-Section

$Q^2 = 0.38\ GeV^2$

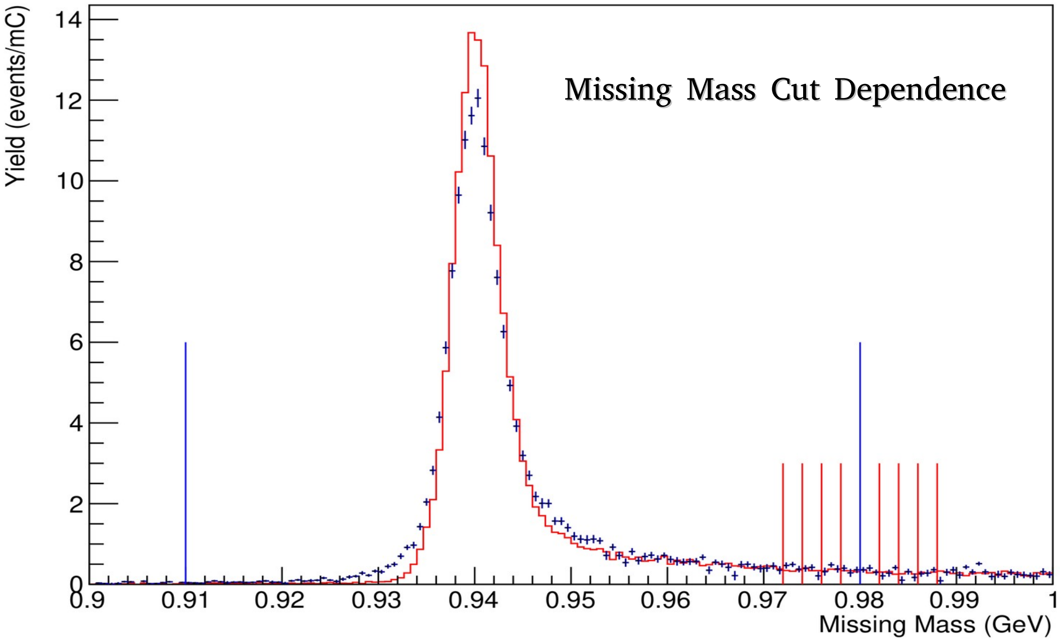
$$e + p \rightarrow e' + \pi^+ + n$$

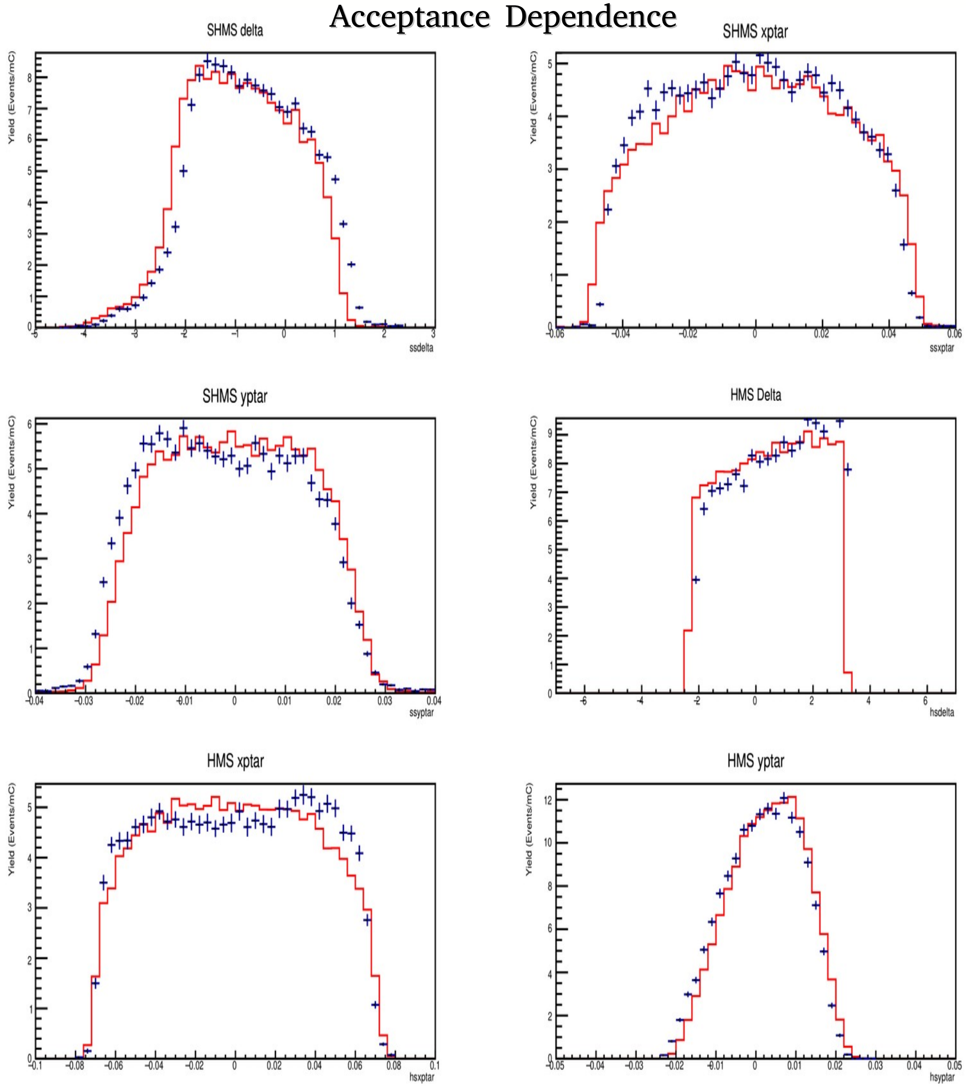
FINAL PRELIMINARY RESULTS (PENDING A FEW FINAL CHECKS)

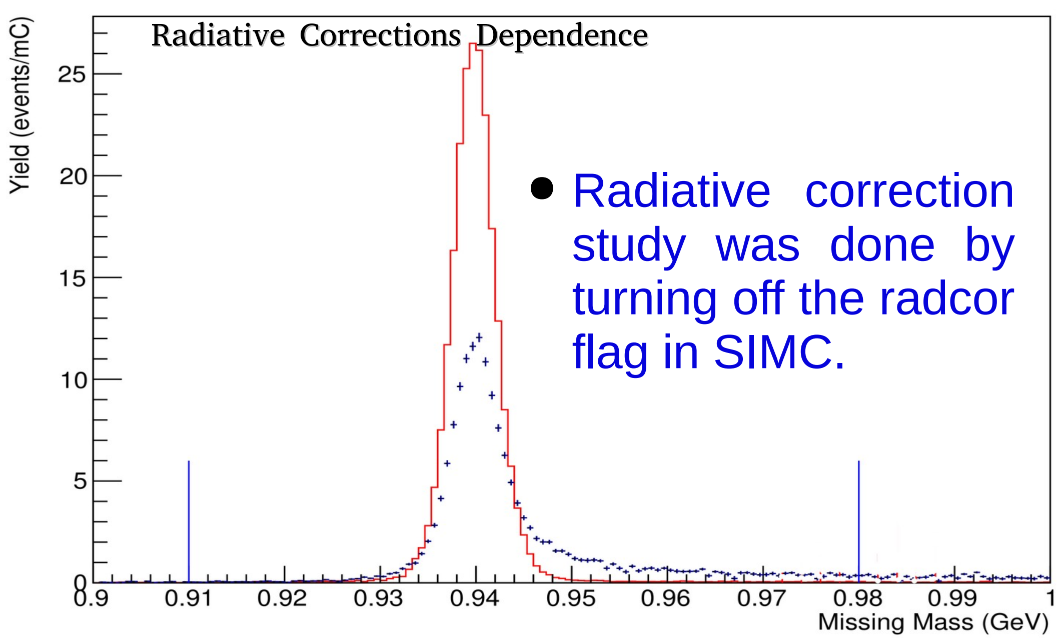


The Systematic Uncertainty

$$e + p \rightarrow e' + \pi^{+} + n$$


Missing Mass Cut Dependence


Acceptance Dependence


Radiative Corrections Dependence

- Radiative correction study was done by turning off the radcor flag in SIMC.

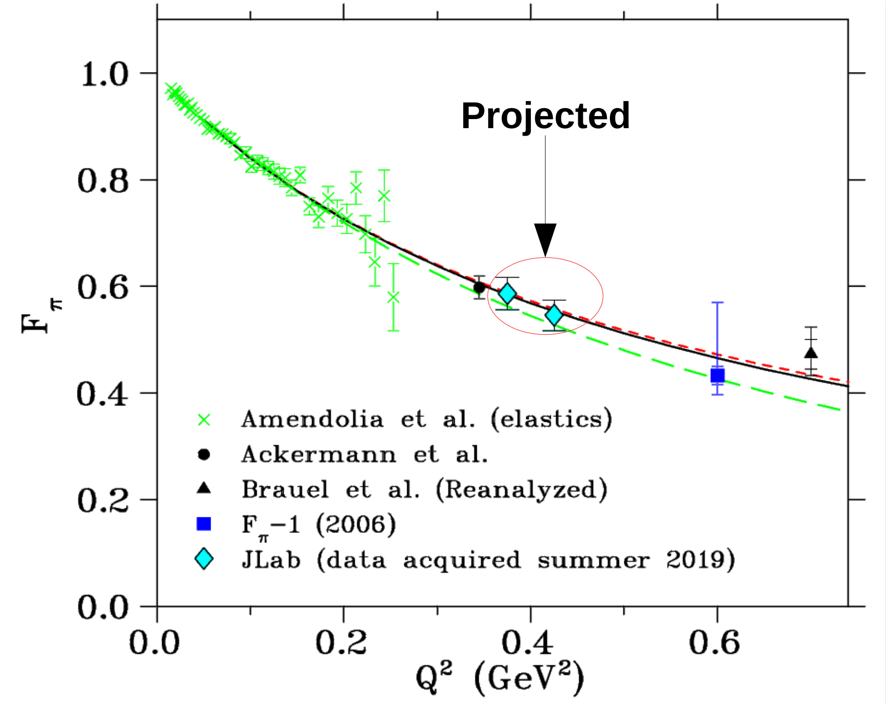
Plot: Mid ε, Center Setting of Q^2 = 0.38 GeV^2 data

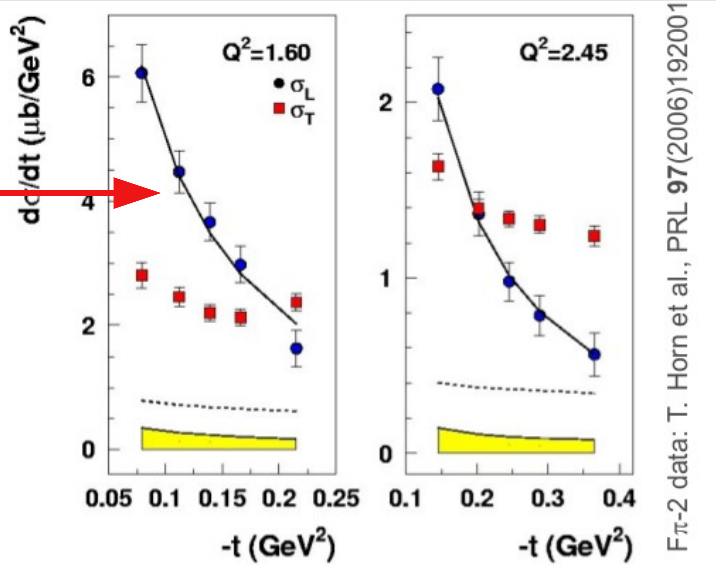
Extract $F_\pi(Q^2)$ From Separated σ_L (Not Yet Done)

- VGL Regge Model:
  - Feynman propagator, $\frac{1}{t-m^2}$ replaced by π and ρ Regge propagators.
  - Represents the exchange of a series of particles, compared to a single particle.
  - At small -t, σ_L only sensitive to $F_\pi(Q^2)$.
  - $F_\pi(Q^2)$ extract as

$$F_\pi(Q^2) = \frac{1}{1+\frac{Q^2}{\Lambda_\pi^2}}$$

- Fit to σ_L to the model gives the free parameter, Λ_π^2 (trajectory cutoff) for each Q^2.
- CKY and Perry & Thomas models are also available now, we will use to compare the results.





Fπ-2 data: T. Horn et al., PRL **97**(2006)192001

Outlook/Future Work

- Completing the LT separation analysis for $Q^2 = 0.42$ GeV^2 data.
- Final checks are in progress for $Q^2 = 0.38$ GeV^2 data.
- The form factor will be extracted for both Q^2 (0.38 and 0.42 GeV^2) and compared with the elastic measurements.
- Preparing to begin work on the final Physical Review Letters (PRL) publication.

SAPIN-2021-00026

Thank You!

Group Members:

Garth Huber, Tanja Horn, David Gaskell, Pete Markowitz, Richard Trotta, Ali Usman, Nathan Heinrich, Julie Roche, Muhammad Junaid, Alicia Postuma, Konrad Aniol, Abdennacer Hamdi and Casey Morean.