Systematic Uncertainties from Gribov Copies in CG-Fixed Correlation Functions

Based on arXiv:2408.05910

Xiang Gao, Jinchen He, Rui Zhang, and Yong Zhao

QGT Meeting 2024/09

UNIVERSITY OF MARYLAND

Argonne NATIONAL LABORATORY

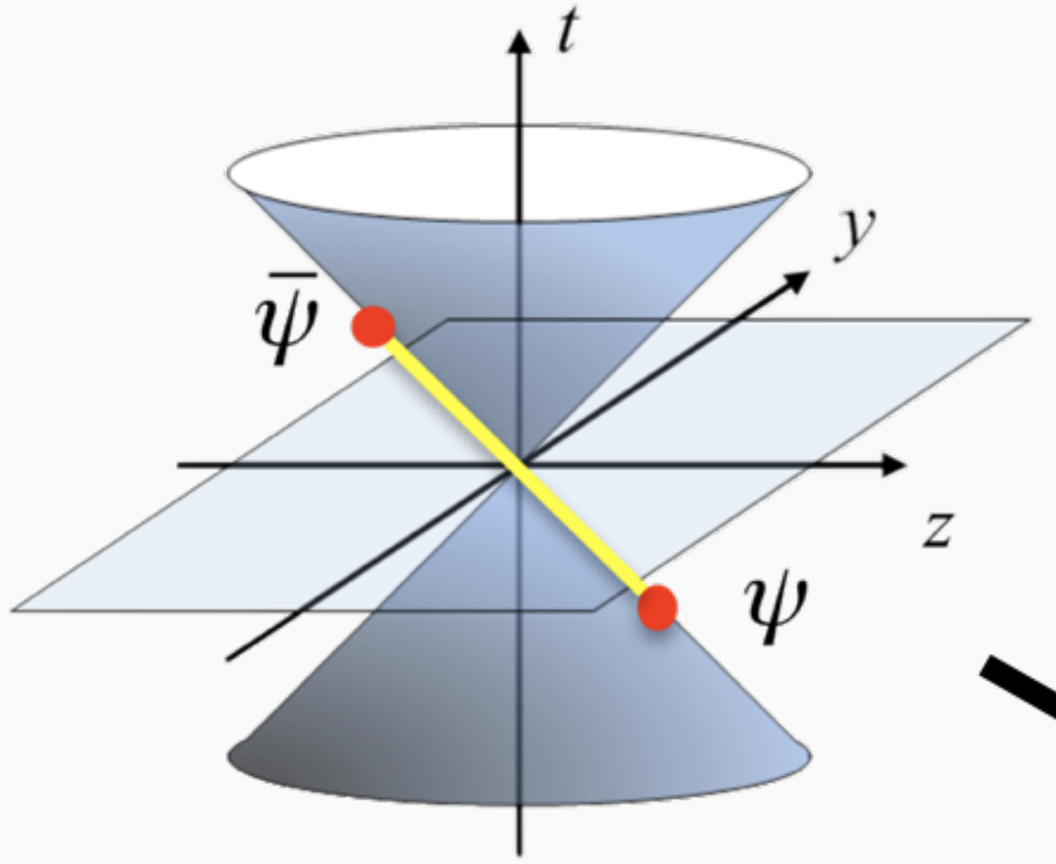
PHENOMENOLOGY LATTICE QCD THEORY

Contents

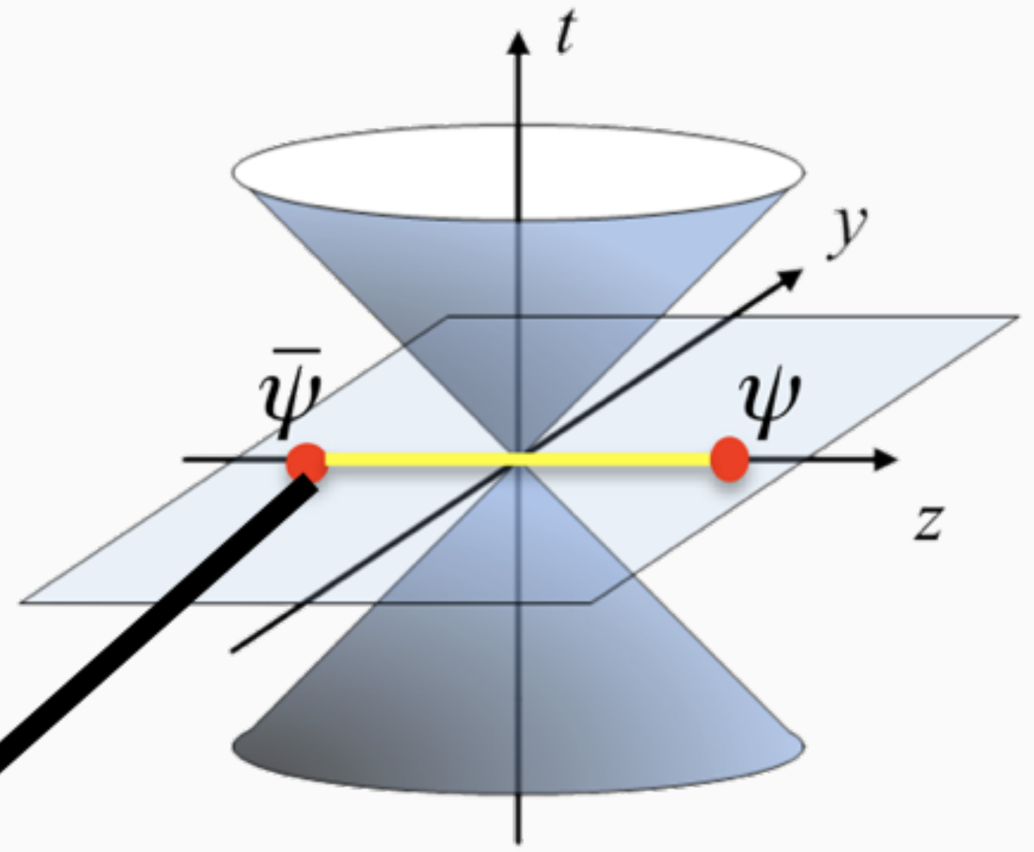
Background

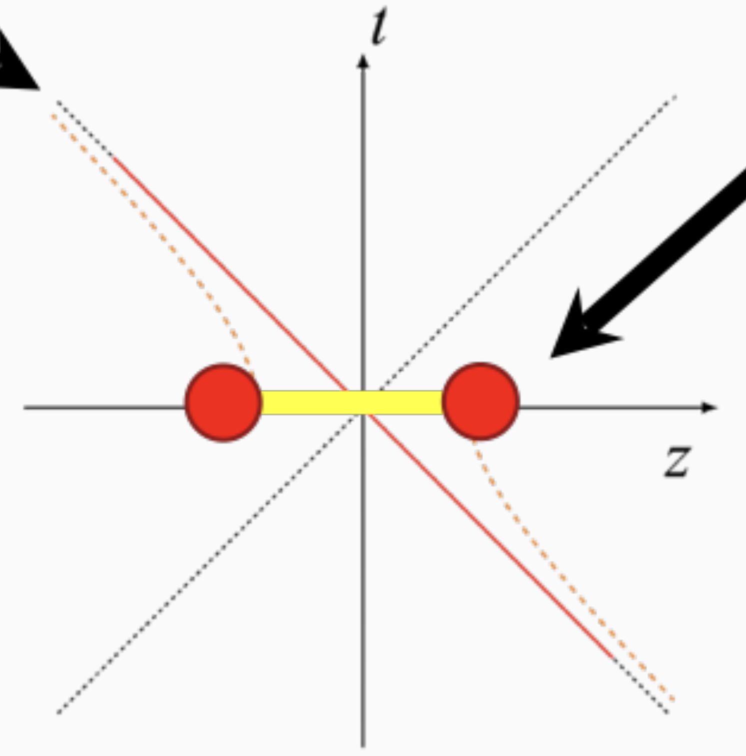
Large-Momentum Effective Theory(LaMET)

First-principle calculations of the x-dependence of parton distribution functions



X. Ji, Phys.Rev.Lett. 110, 262002 (2013)
X. Ji, et al., Rev.Mod.Phys. 93 (2021)
X. Ji, 2408.03378





Light-cone distribution:

Separated on the time axis;

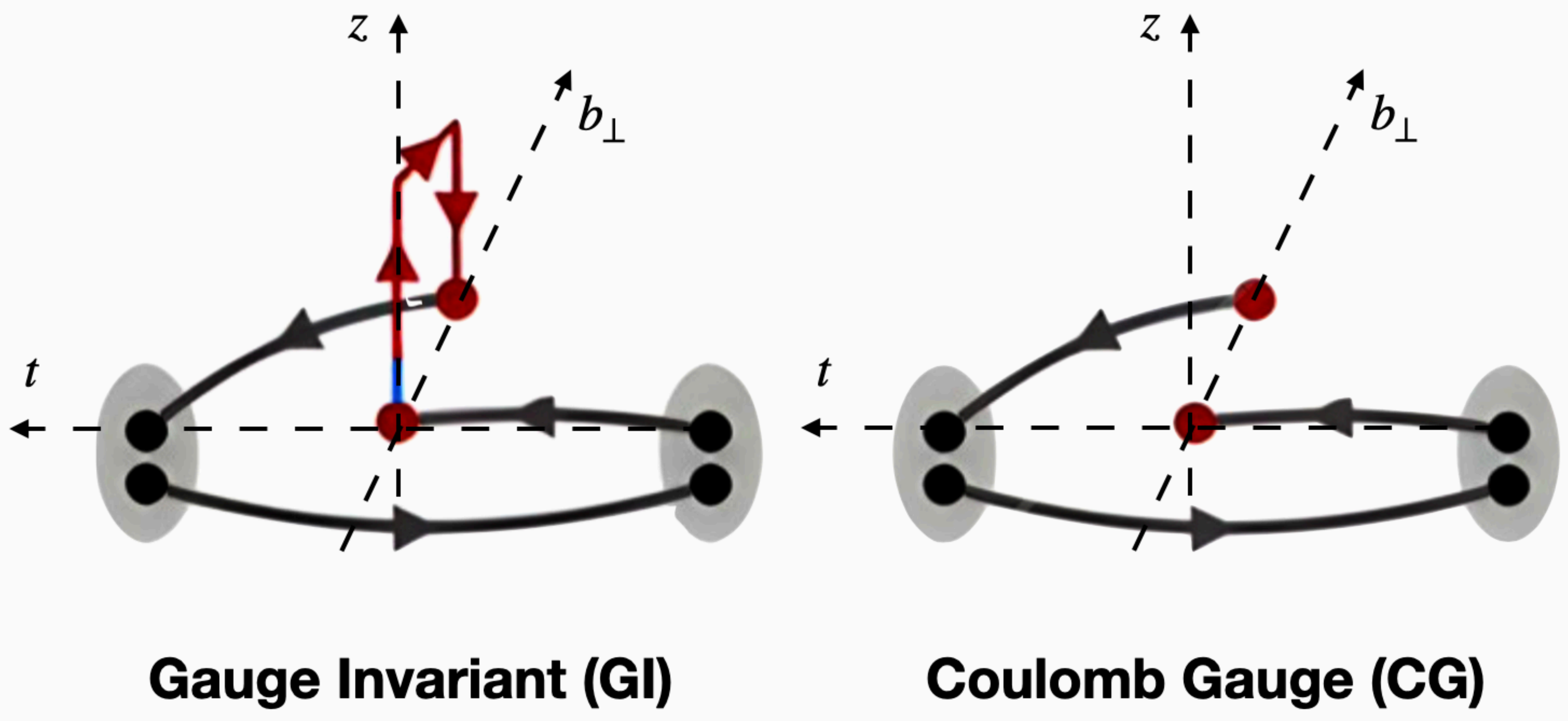
Cannot be calculated on the lattice

Quasi distribution:

Equal time in large-P hadron state;

Directly calculable on the lattice

Quasi-distributions in CG without Wilson Line



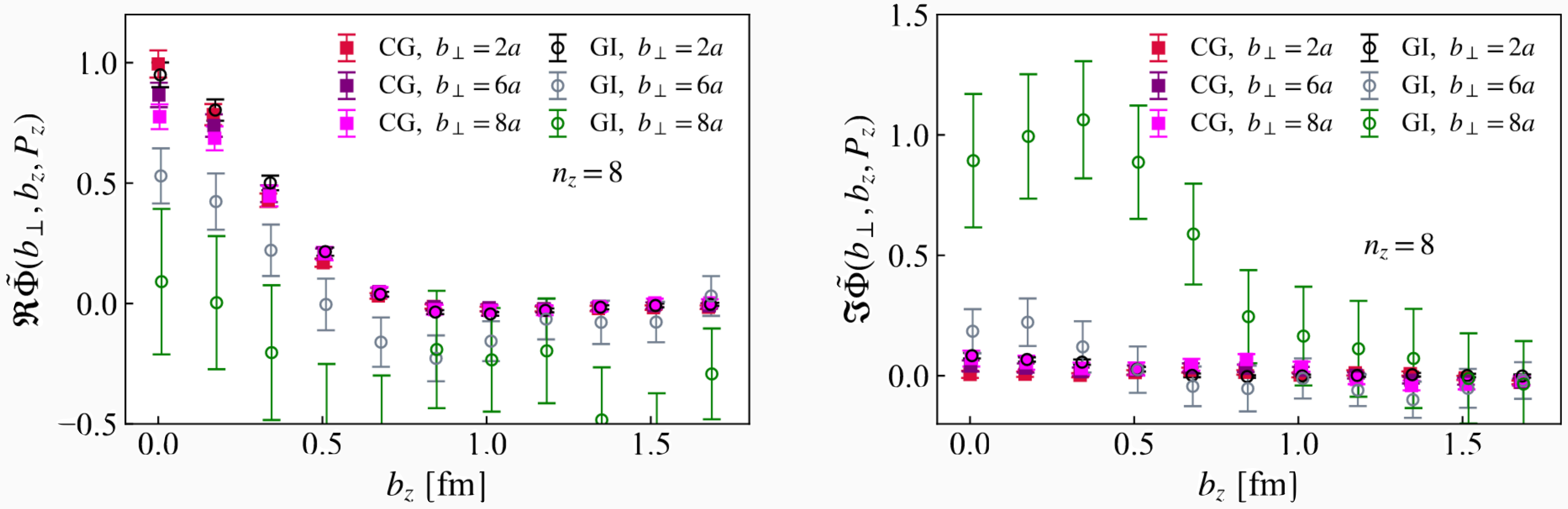
The quasi-TMD matrix elements of the pion under CG are defined as

$$\tilde{h}_{\gamma^t}(z, P^z, \mu) = \frac{1}{2P^t}\langle P \,|\, \bar{\psi}(z)\gamma^t\psi(0)\,|_{\vec{\nabla}\cdot\vec{A}=0}\,|\, P\rangle$$

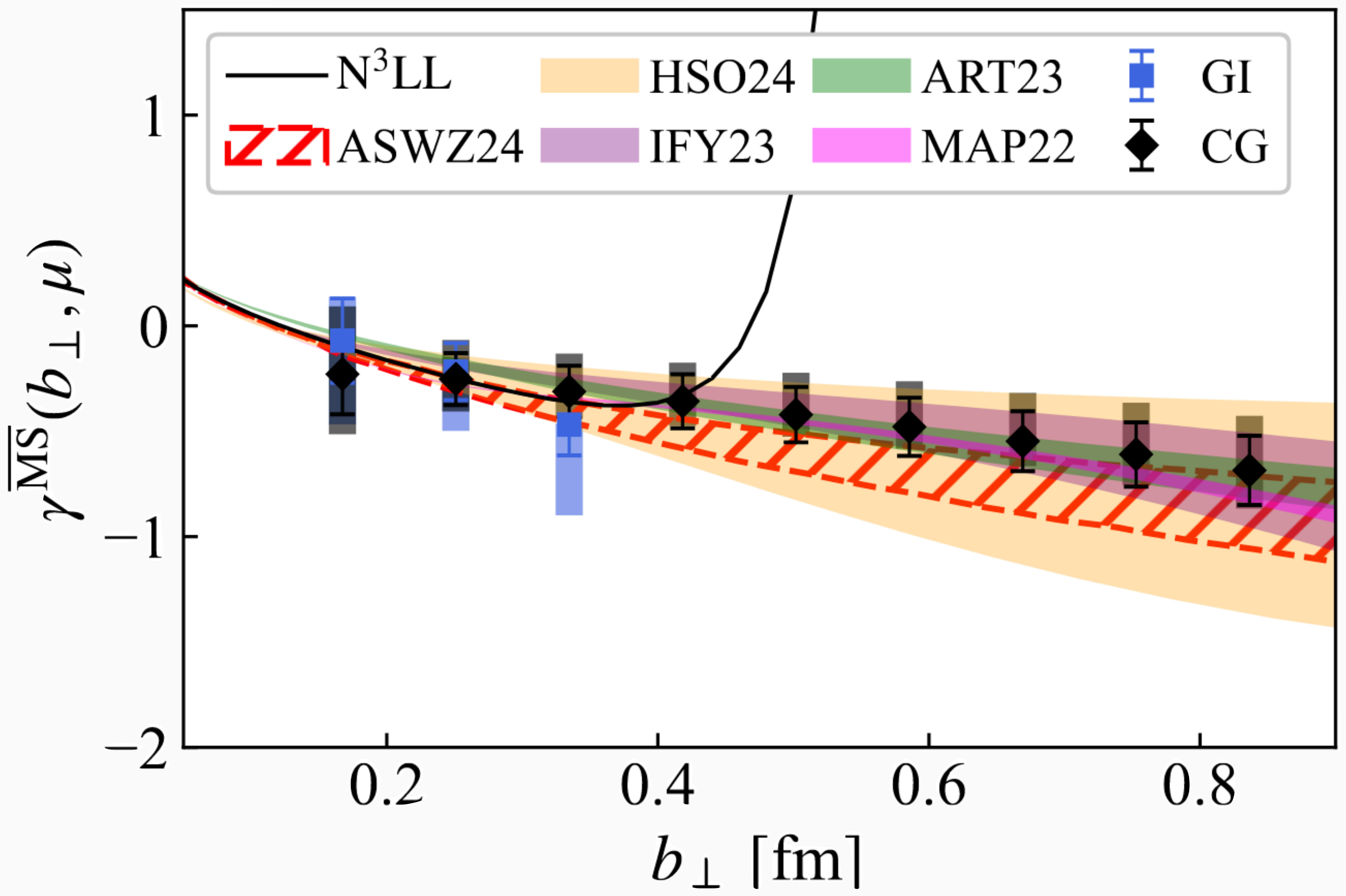
Y. Zhao, 2311.01391

Quasi-TMD in Coulomb Gauge without Wilson Line

Compared with the GI method, CG method has much better signal, especially for TMDs.



Pion Quasi-TMDWF

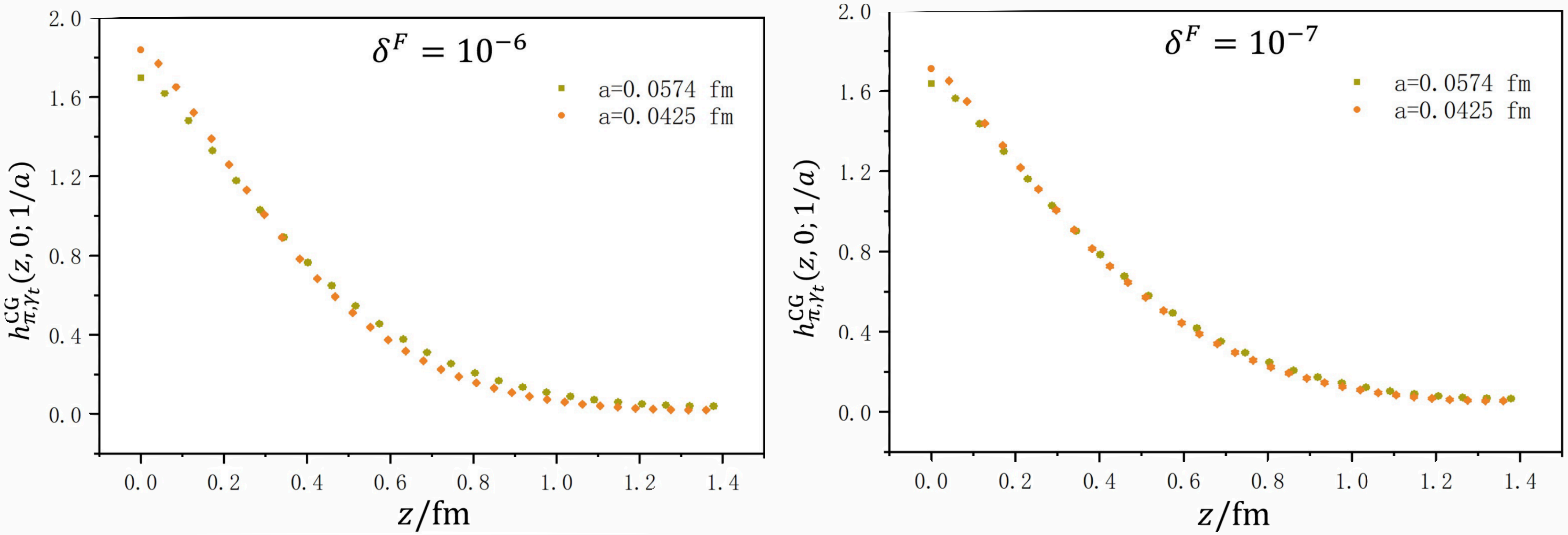


Collins-Soper kernel

D. Bollweg, et al., Phys.Lett.B 852 (2024)

Dependence on the Gauge Fixing Precision

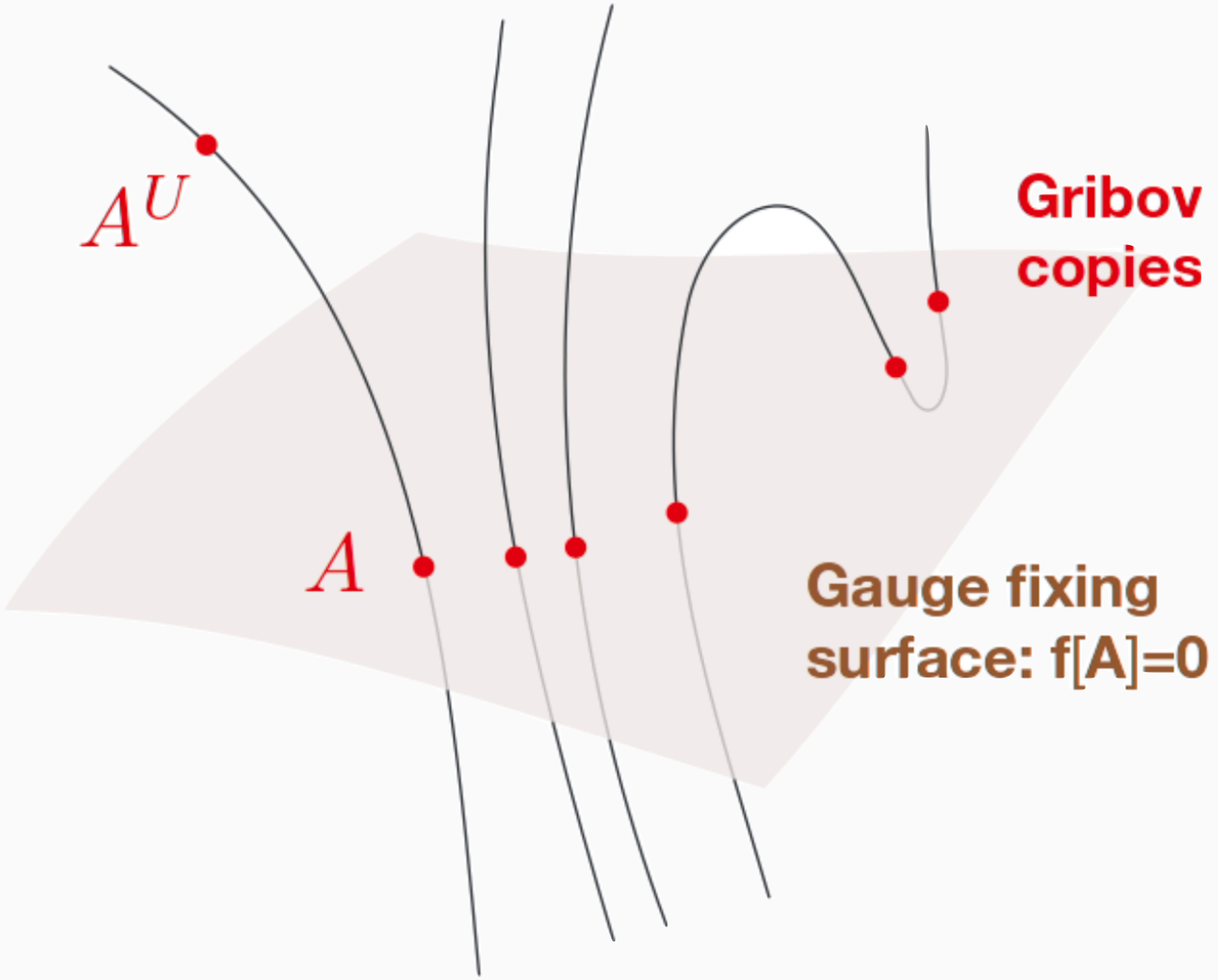
Pion PDF using Coulomb gauge method depends on the gauge fixing precision.



K. Zhang, et al.(LPC), 2405.14097

Gribov Copies

- The gauge fixing condition may have many solutions in Lattice QCD.



Ph. D. Thesis of Diego Fiorentini

Faddeev-Popov operator

- The existence of Gribov copies is related to the zero mode of the Faddeev-Popov operator

Gauge condition: $\mathscr{F} = \vec{\nabla} \cdot \vec{A} = 0$ Infinitesimal transformation: $\delta \vec{A} = -\vec{D}\omega$

Variation: $\delta\mathscr{F} = \vec{\nabla} \cdot \delta\vec{A} = -\vec{\nabla} \cdot (\vec{D}\omega)$

Faddeev-Popov: $\mathscr{M} \equiv \dfrac{\delta\mathscr{F}}{\delta\omega} = -\vec{\nabla} \cdot \vec{D}$ For QED, there is no Gribov copy in Landau gauge.

if $\mathscr{M}\theta = 0$, then $\vec{\nabla} \cdot \vec{A} = \vec{\nabla} \cdot (\vec{A} - \vec{D}\theta) \equiv \vec{\nabla} \cdot \vec{A}\,' = 0$

$* \; D_{\mu ab}\omega_b = \partial_\mu \omega_a - g f^{cab} A^c_\mu \omega_b$

Methodology

Gauge Fixing in Lattice QCD

Continuous Theory

$$F_{\mathrm{CG}}[A,\Omega] \equiv \frac{1}{2}\sum_{\mu=1}^{3}\int d^4x A^a_{\Omega\mu}(x)A^{\mu a}_{\Omega}(x)$$

$$\delta F_{\mathrm{CG}}[A,\Omega] = -\sum_{\mu=1}^{3}\int d^4x(D^{\Omega}_{\mu ab}\theta_b)A^{\mu a}_{\Omega}$$

$$= -\sum_{\mu=1}^{3}\int d^4x(\partial_\mu\theta_a - gf^{cab}A^c_{\Omega\mu}\theta_b)A^{\mu a}_{\Omega}$$

$$= \sum_{\mu=1}^{3}\int d^4x\theta_a(\partial_\mu A^{\mu a}_{\Omega})$$

$$* A_{\Omega\mu}(x) \equiv \Omega^\dagger(x)A_\mu(x)\Omega(x) + \frac{i}{g}\Omega^\dagger(x)\partial_\mu\Omega(x)$$

Lattice Theory

$$F_{\mathrm{CG}}[U,\Omega] \equiv -\Re\left[\mathrm{Tr}\sum_{x}\sum_{\mu=1}^{3}\Omega^\dagger(x+\hat{\mu})U_\mu(x)\Omega(x)\right]$$

Find ~~stationary points~~ of the functional value.

Find minimal points of the functional value, so that F-P operator (second derivative) is positive definite.

Criteria of Gauge Fixing

- Variation of the functional

$$\delta F / F < 10^{-8}$$

- Residual gradient of the functional

$$\theta^G \equiv \frac{1}{V} \sum_x \theta^G(x) \equiv \frac{1}{V} \sum_x \mathrm{Tr}\left[\Delta^G(x) \left(\Delta^G\right)^\dagger(x)\right]$$

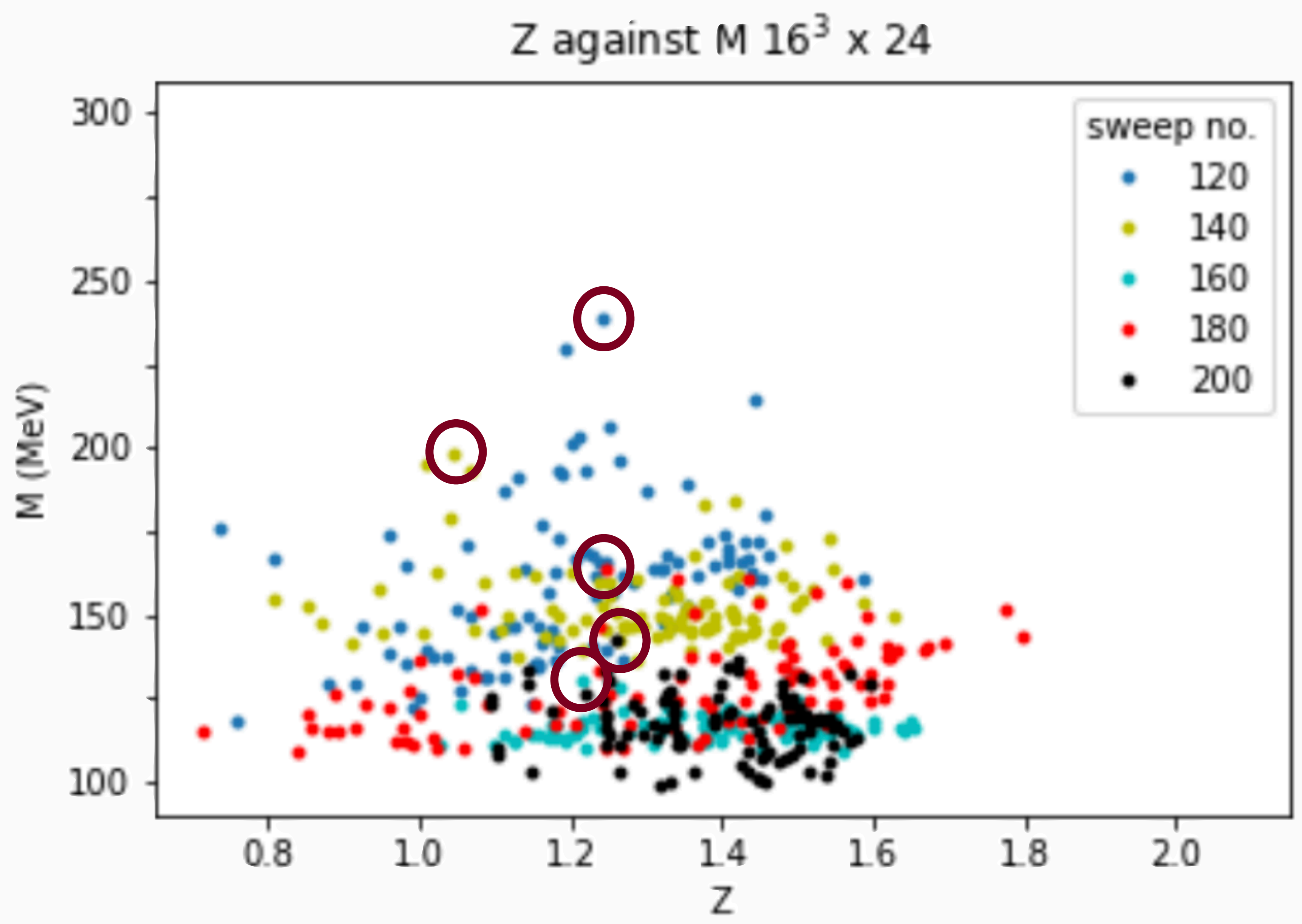
$$* \ \Delta^G(x) \equiv \sum_\mu \left(A_\mu^G(x) - A_\mu^G(x - \hat{\mu})\right)$$

Different Gribov copies can be distinguished by the difference of functional values ΔF.

Two kinds of impact from Gribov Copies

- Lattice Gribov noise: not separable from the statistical uncertainty; related to the distribution across Gribov copies.

- Measurement distortion: systematic uncertainty; related to the bias of strategies to choose representative from copies.



G. Kalusche, et al., 2405.17301

Strategies to Choose Representative in Gribov Copies

- **Mother-daughter method**: do a random gauge transformation before gauge fixing to get different copies.
- "First it": choose the first instance of gauge fixing;
- "Smallest f": choose the instance with the smallest functional value among all instances; (Fundamental Modular Region)

N. Vandersickel, et al., Phys.Rept. 520 (2012)

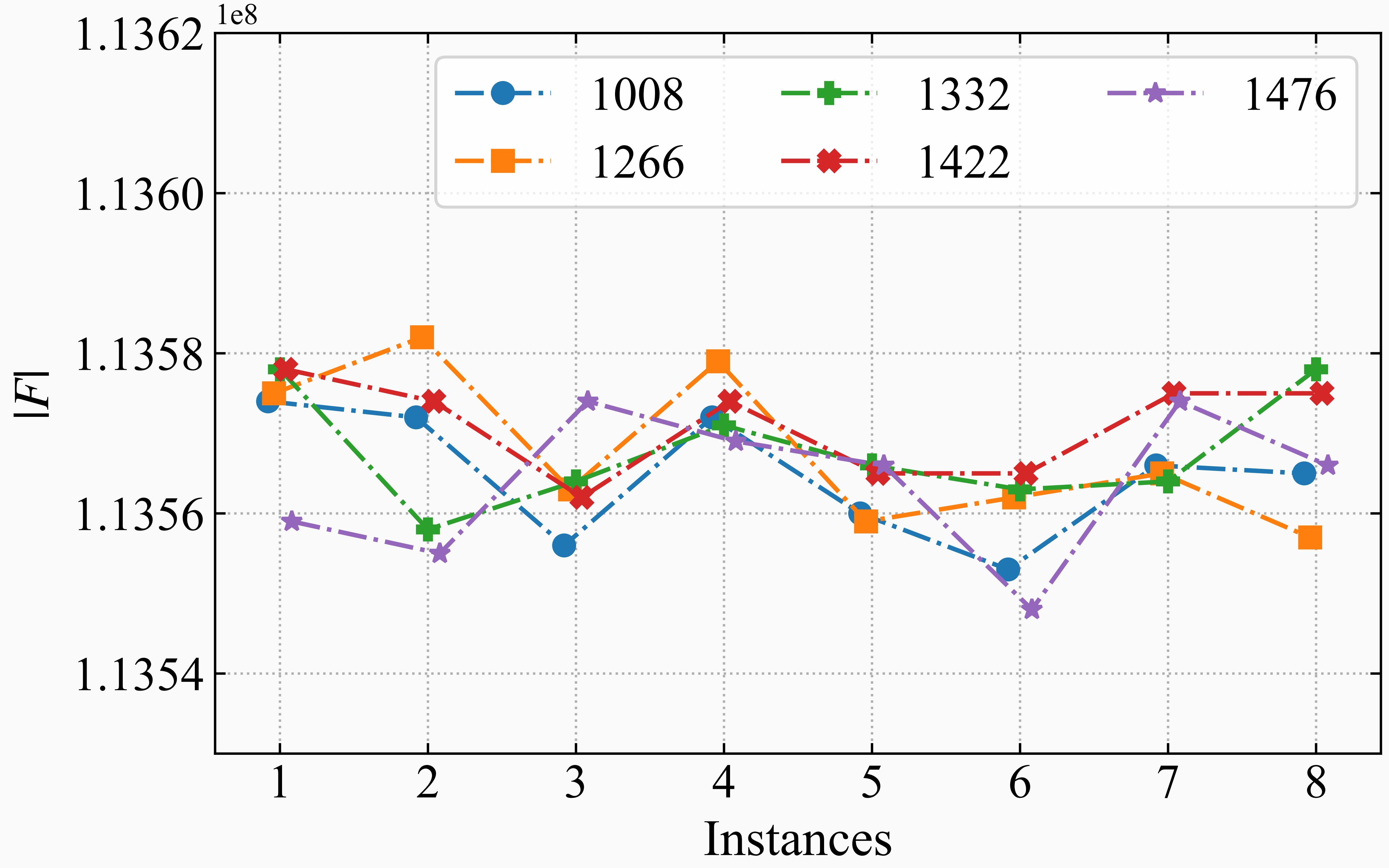
Numerical Results

Functional Values of Gribov Copies

Lattice setup: 2+1 flavor HISQ ensemble by HotQCD

a	$L_s^3 \times L_t$	$m_q a$	$m_\pi L_t$	#Cfgs	(#ex, #sl)	Inv precision
0.06 fm	$48^3 \times 64$	-0.0388	5.85	100	(1, 8)	ex: 10^{-10}; sl: 10^{-4}

We have 100 configurations, do the Coulomb gauge fixing to get 8 instances on each configuration.



Quark Propagator under the Coulomb Gauge

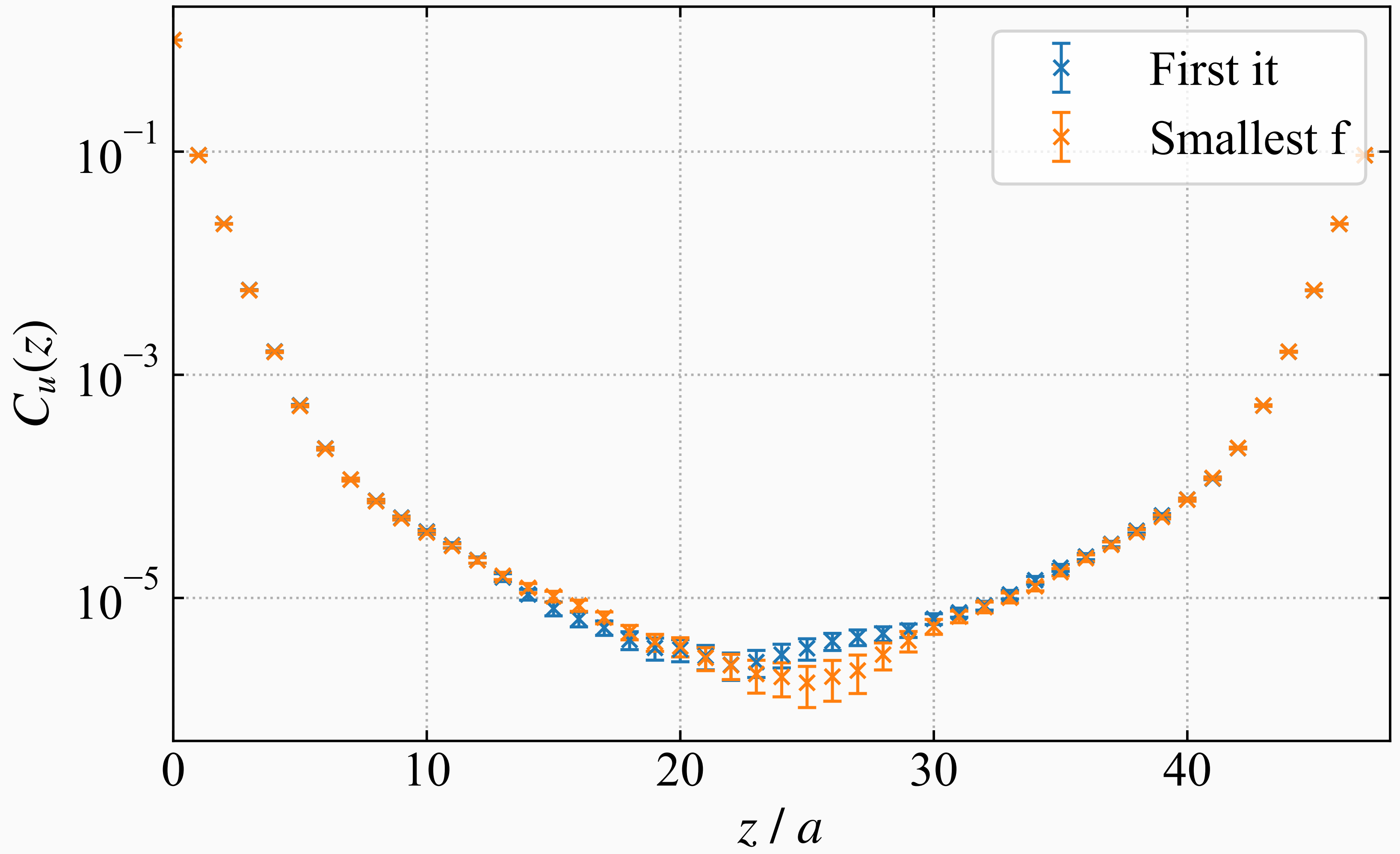
- Quark Propagator:

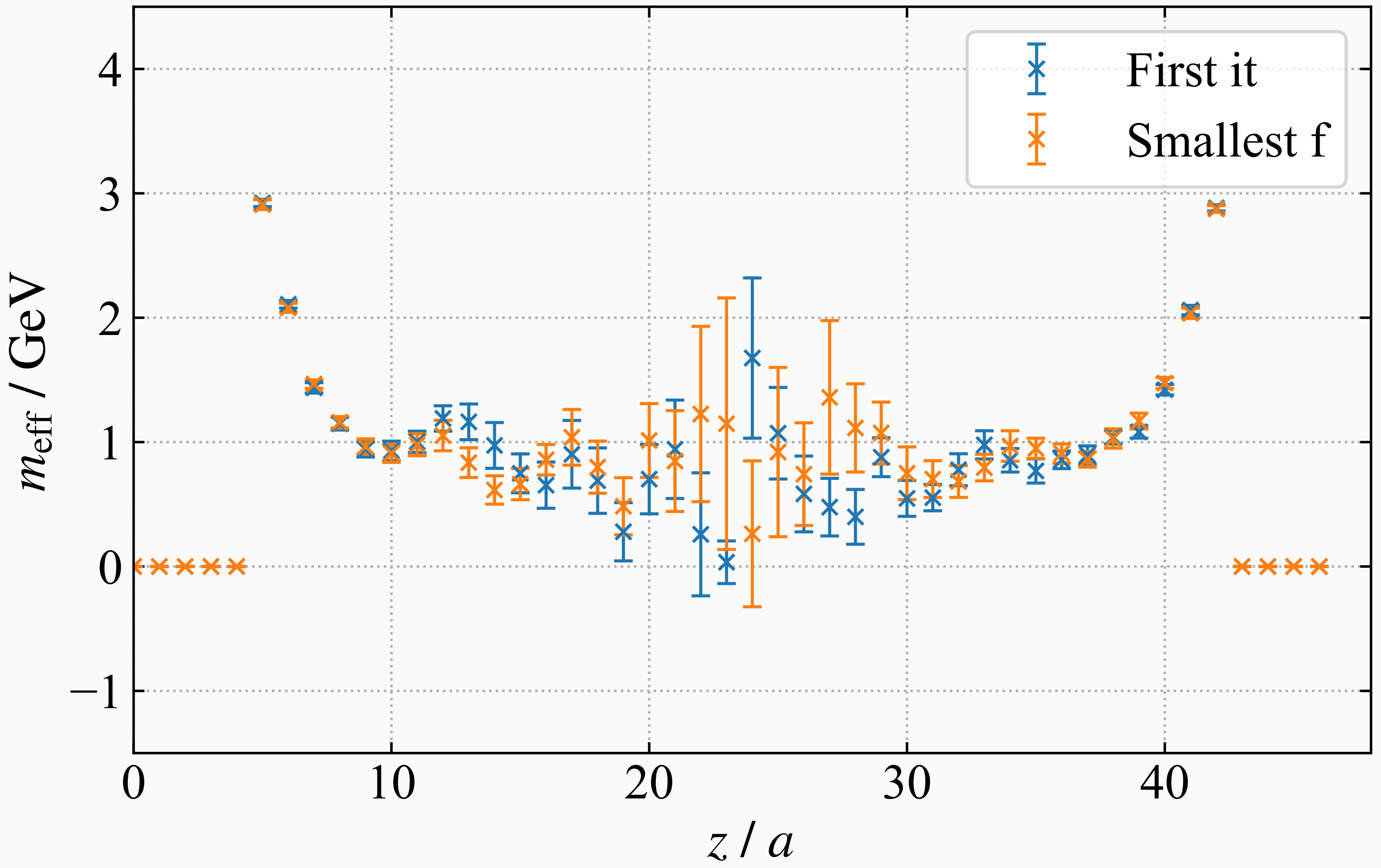
$$C_u(z) = \langle \mathrm{Tr}[u(z)\bar{u}(0)] \rangle$$

- Effective Mass:

$$\frac{C_u(z)}{C_u(z+1)} = \frac{\cosh(m_{\mathrm{eff}} \cdot (z - L_s/2))}{\cosh(m_{\mathrm{eff}} \cdot (z + 1 - L_s/2))}$$

Both 2pt and meff show a good consistency between two strategies.





Quark Propagator under the Coulomb Gauge

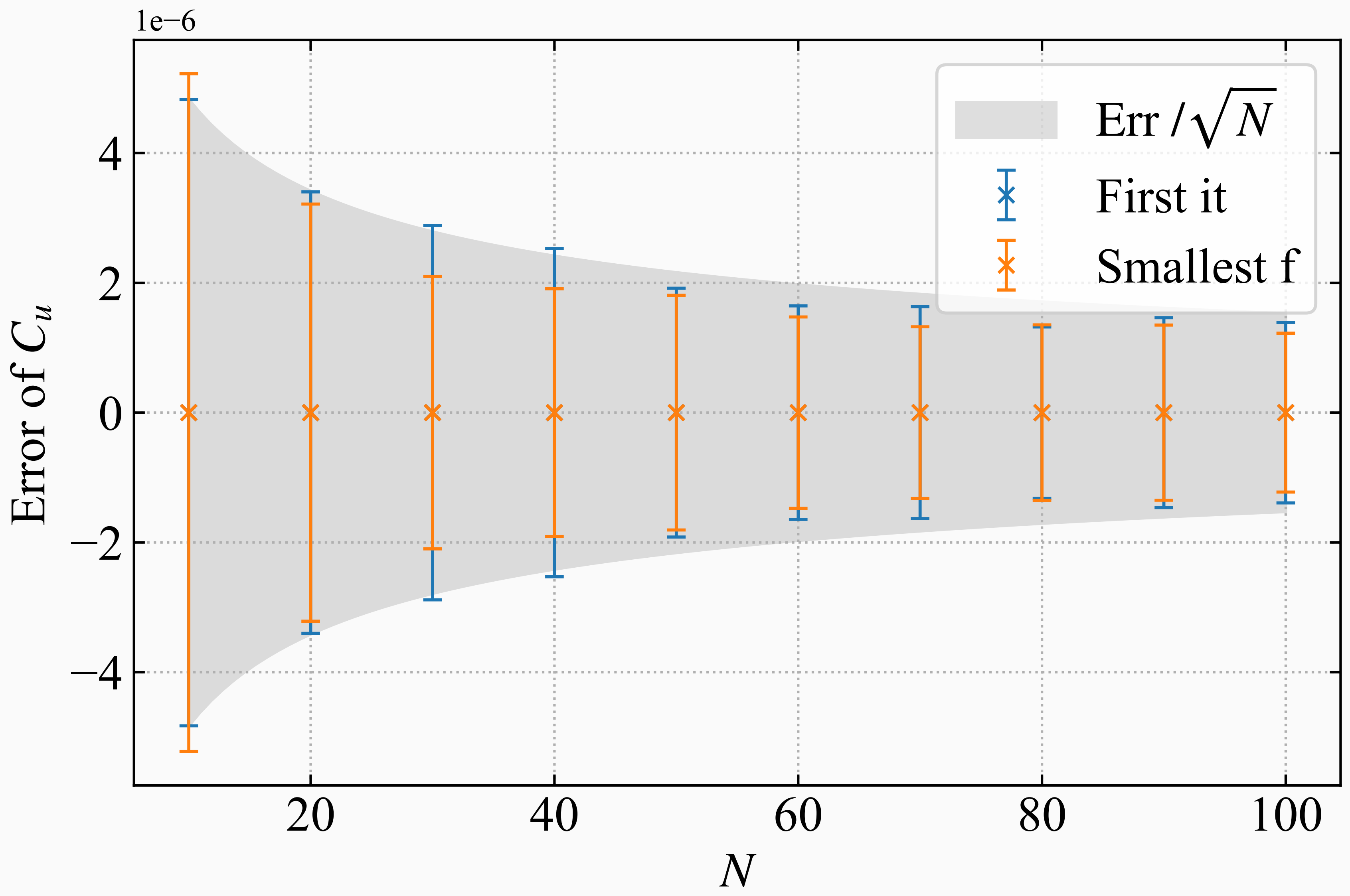
- Quark Propagator:

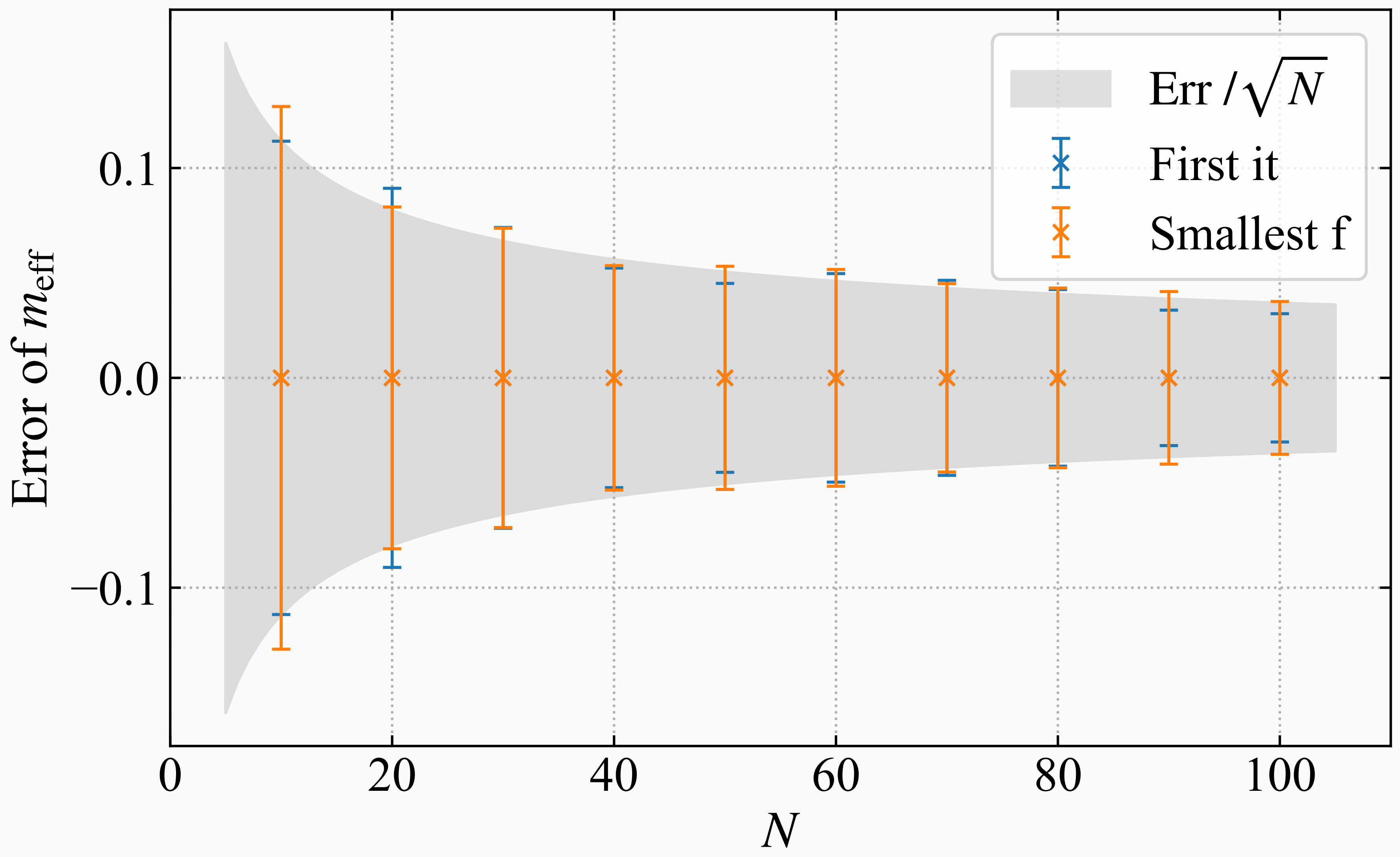
$$C_u(z) = \langle \mathrm{Tr}[u(z)\bar{u}(0)] \rangle$$

- Effective Mass:

$$\frac{C_u(z)}{C_u(z+1)} = \frac{\cosh(m_{\mathrm{eff}} \cdot (z - L_s/2))}{\cosh(m_{\mathrm{eff}} \cdot (z + 1 - L_s/2))}$$

The behavior of error is consistent with $1/\sqrt{N}$ when varying the number of configurations N.





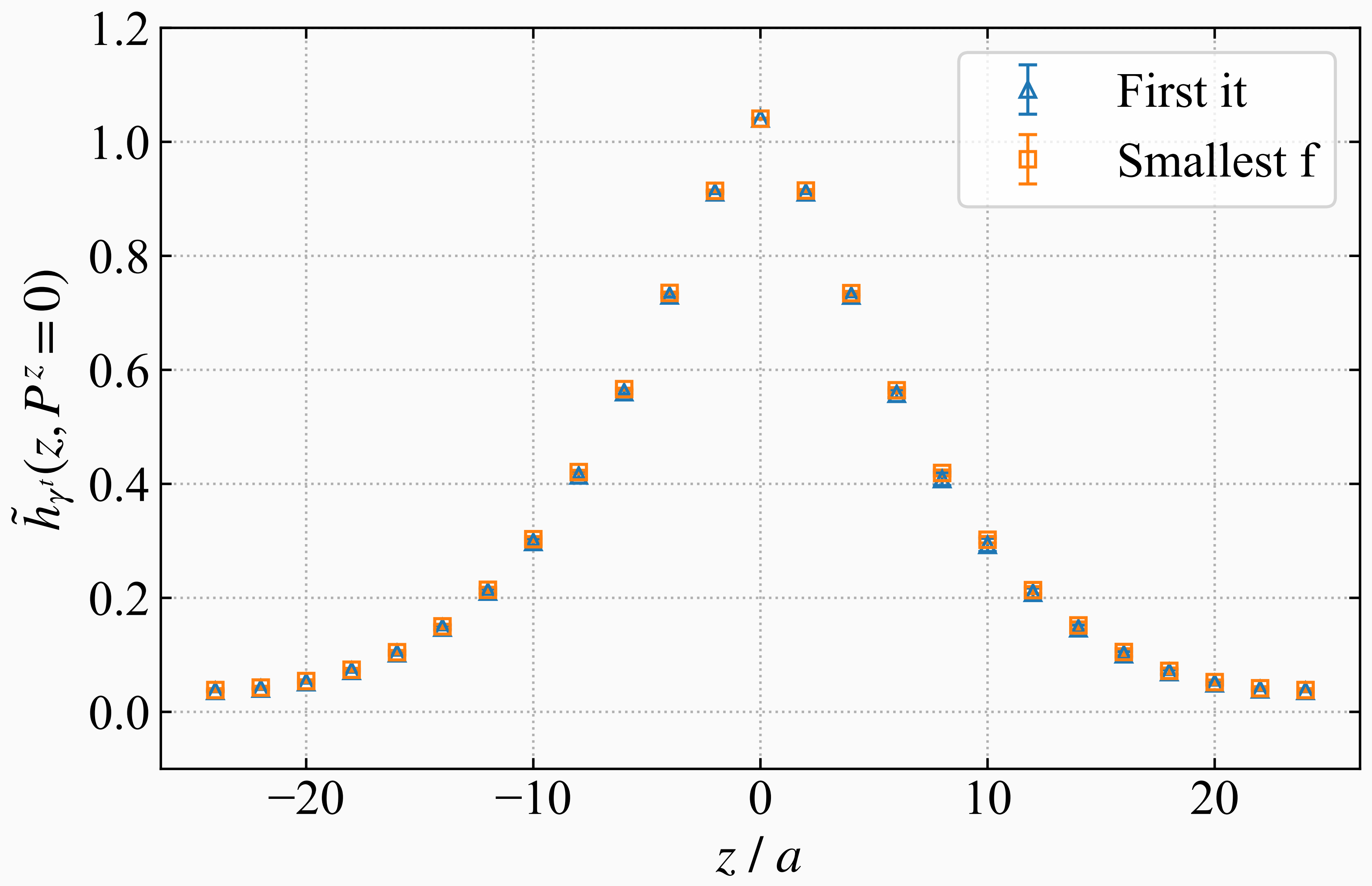
$z = 10\ a$

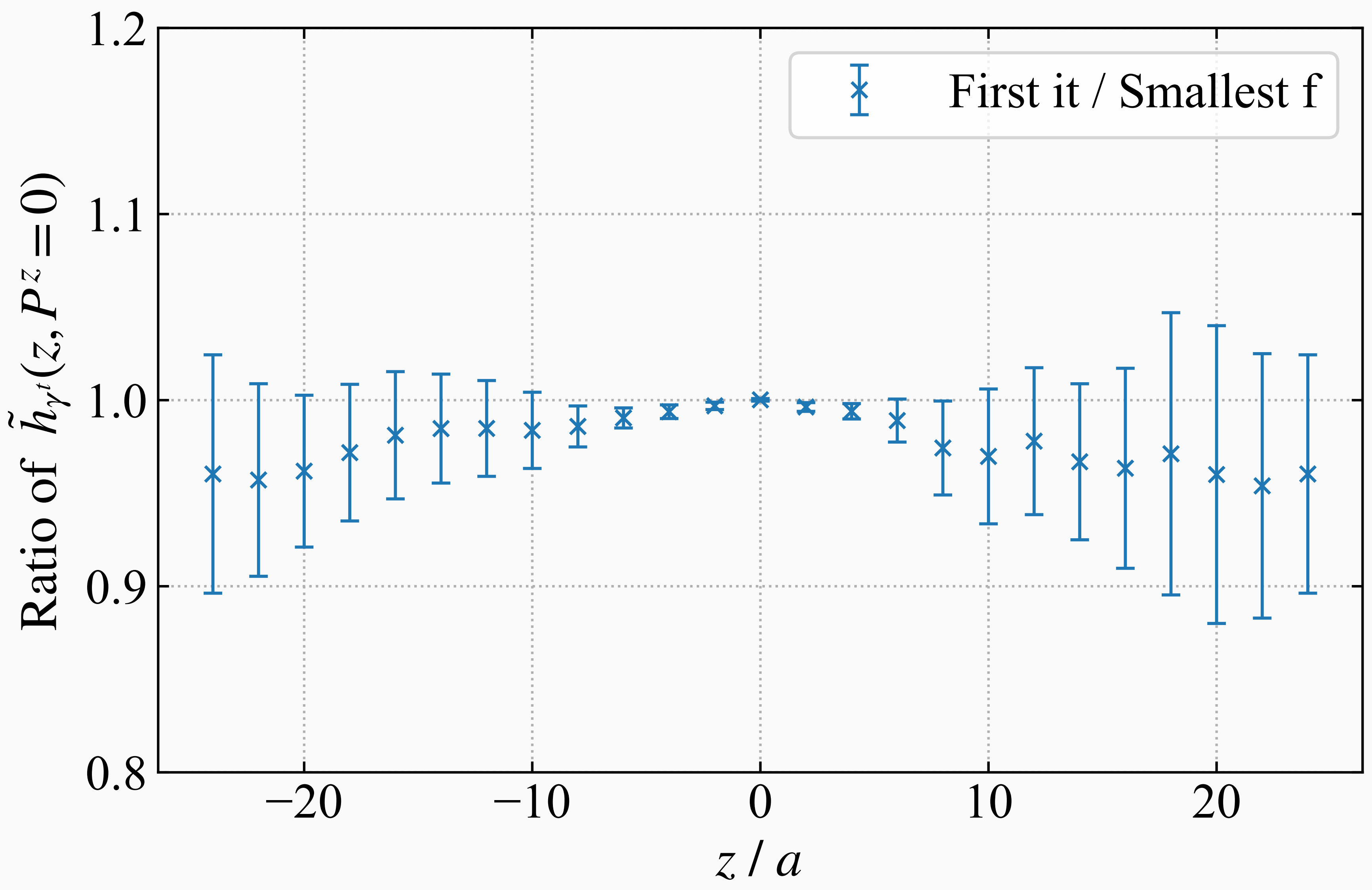
Quasi-distribution under the Coulomb Gauge

- Quasi-distribution:

$$\tilde{h}_{\gamma^t}(z, P^z, \mu) = \frac{1}{2P^t}\langle \vec{P} = \vec{0}\,|\,\bar{\psi}(z)\gamma^t\psi(0)\,|_{\vec{\nabla}\cdot\vec{A}=0}\,|\,\vec{P} = \vec{0}\rangle$$

The conclusion holds for both collinear and TMD case because of 3D rotational symmetry.

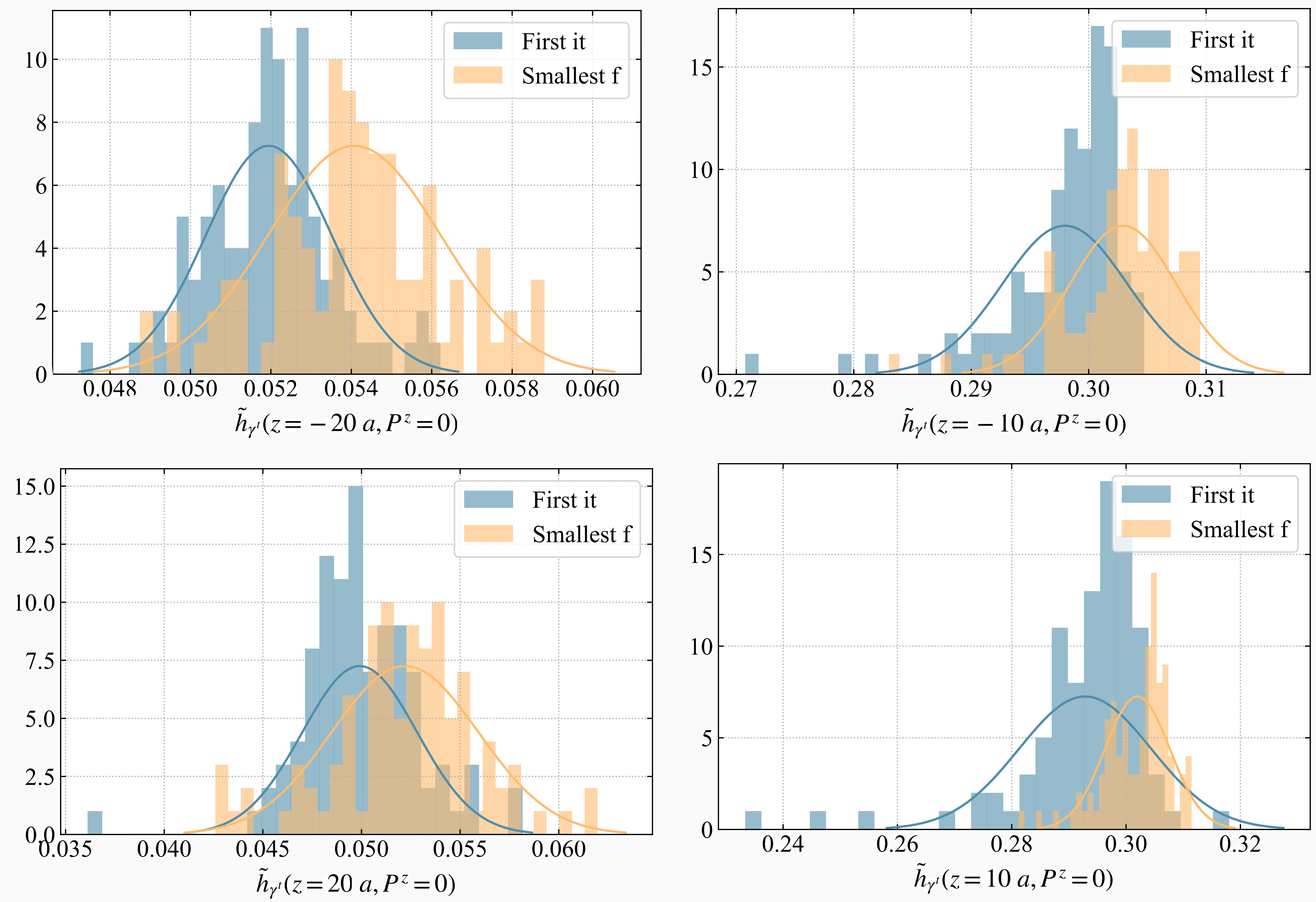




Quasi-distribution under the Coulomb Gauge

- Quasi-distribution:

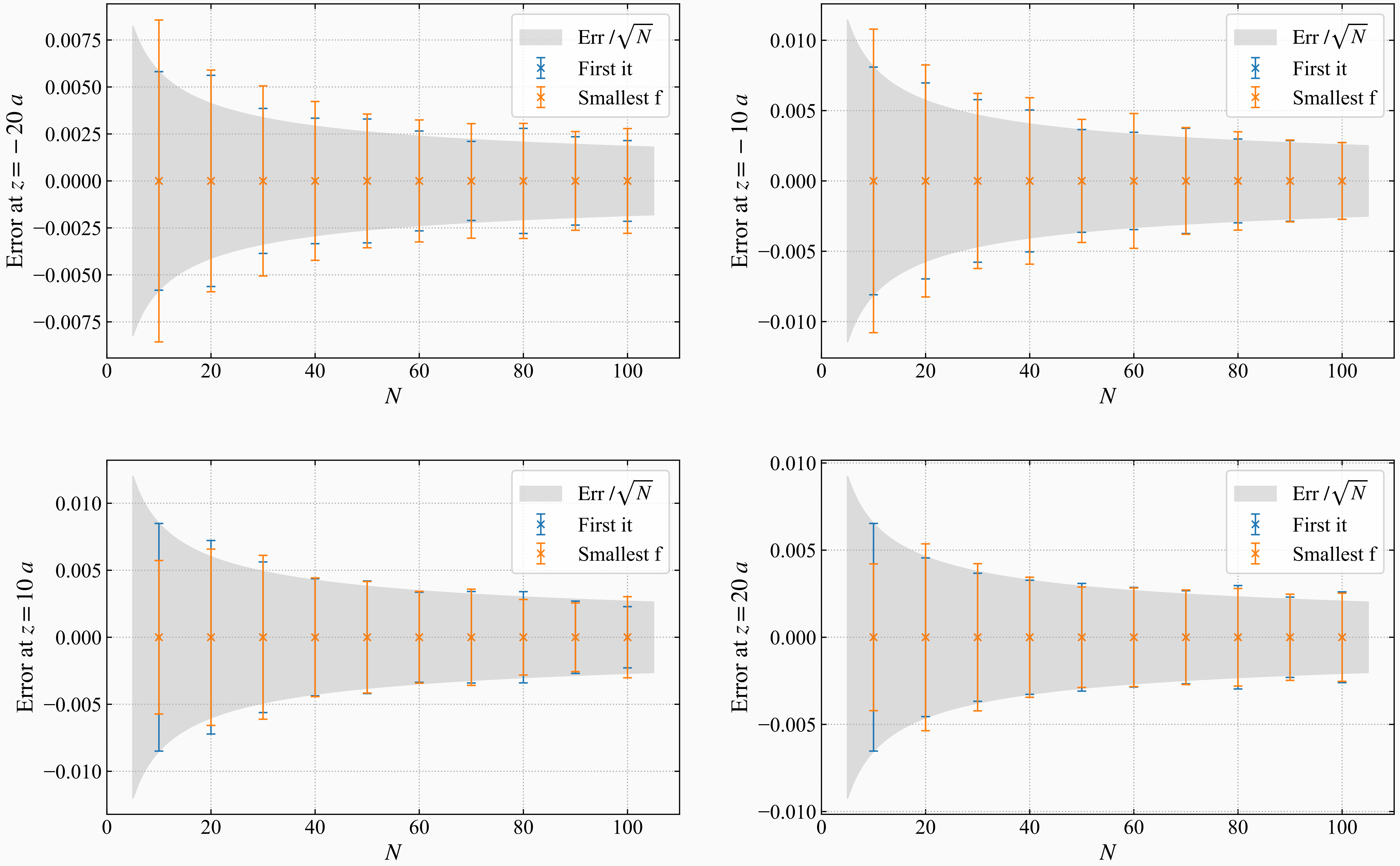
$$\tilde{h}_{\gamma^t}(z, P^z, \mu) = \frac{1}{2P^t} \langle \vec{P} = \vec{0} \,|\, \bar{\psi}(z)\gamma^t\psi(0) \,|_{\vec{\nabla}\cdot\vec{A}=0} \,|\, \vec{P} = \vec{0} \rangle$$



Quasi-distribution under the Coulomb Gauge

- Quasi-distribution:

$$\tilde{h}_{\gamma^t}(z, P^z, \mu) = \frac{1}{2P^t} \langle \vec{P} = \vec{0} \,|\, \bar{\psi}(z)\gamma^t \psi(0) \,|_{\vec{\nabla}\cdot\vec{A}=0} \,|\, \vec{P} = \vec{0} \rangle$$



$t_{\text{sep}} = 8\ a,\ \tau = 4\ a$

Summary

Summary

- Gribov copies stem from multiple solutions of gauge condition;
- Two impacts of Gribov copies: noise and distortion;
- Gribov noise is undistinguishable from the statistical noise;
- No significant distortion on quark propagator & quasi-distribution;

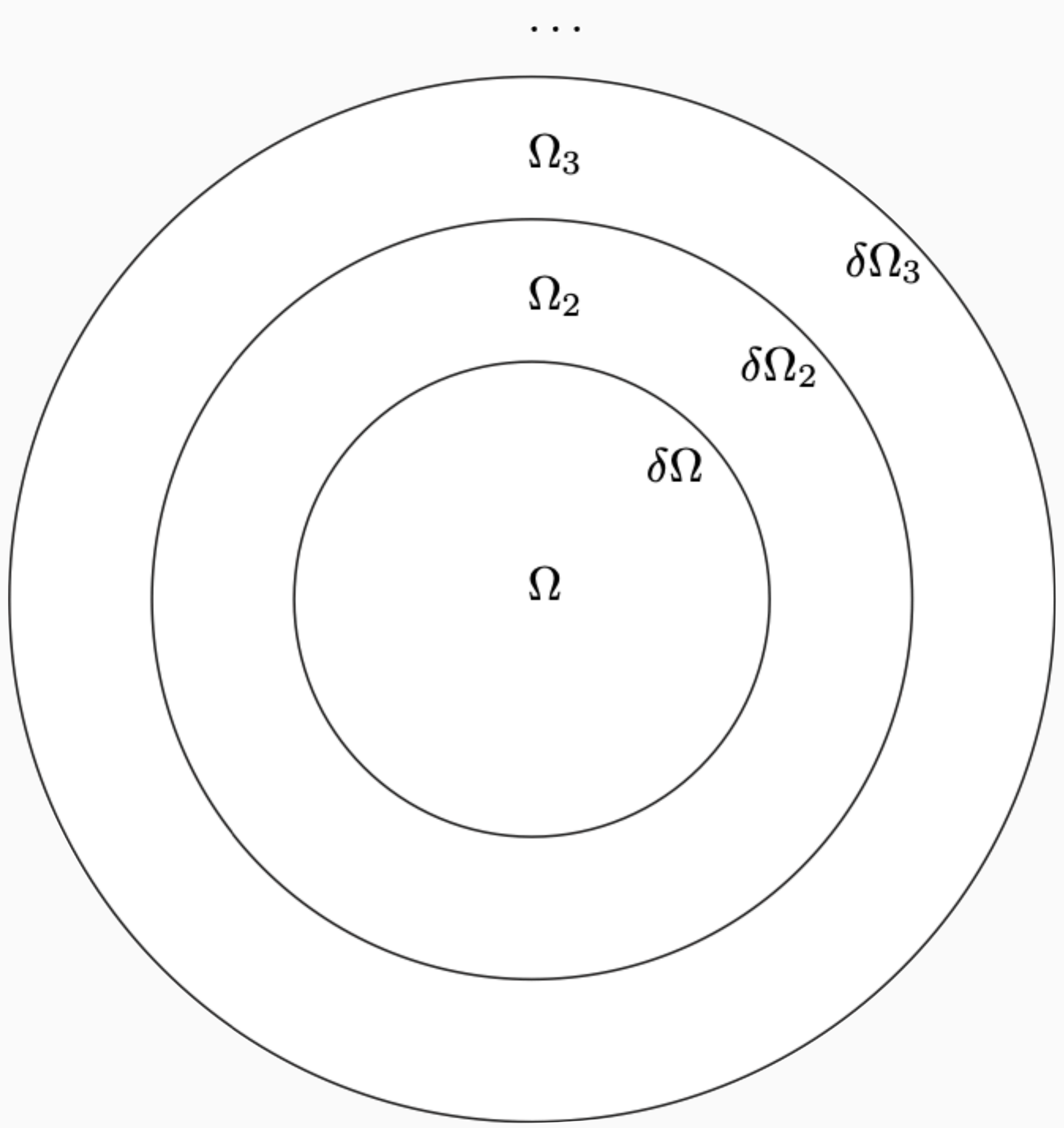
Prospect

- Including more strategies;
- Including more instances;
- Study the gluon propagator for the gluon parton distribution;

Back Up

First Gribov Region

- First Gribov Region: Faddeev-Popov operator is positive definite;
- Gribov Horizon: Faddeev-Popov determinant is zero.



N. Vandersickel, et al., Phys.Rept. 520 (2012)

First Gribov Region

Take minimum point of the functional

$$F_{\mathrm{CG}}[A,\Omega] \equiv \frac{1}{2}\sum_{\mu=1}^{3}\int d^4x A^a_{\Omega\mu}(x)A^{\mu a}_{\Omega}(x)$$

$$\delta F_{\mathrm{CG}}[A,\Omega] = -\sum_{\mu=1}^{3}\int d^4x (D^{\Omega}_{\mu ab}\theta_b)A^{\mu a}_{\Omega}$$

$$= -\sum_{\mu=1}^{3}\int d^4x (\partial_\mu\theta_a - gf^{cab}A^c_{\Omega\mu}\theta_b)A^{\mu a}_{\Omega}$$

$$= \sum_{\mu=1}^{3}\int d^4x \theta_a(\partial_\mu A^{\mu a}_{\Omega})$$

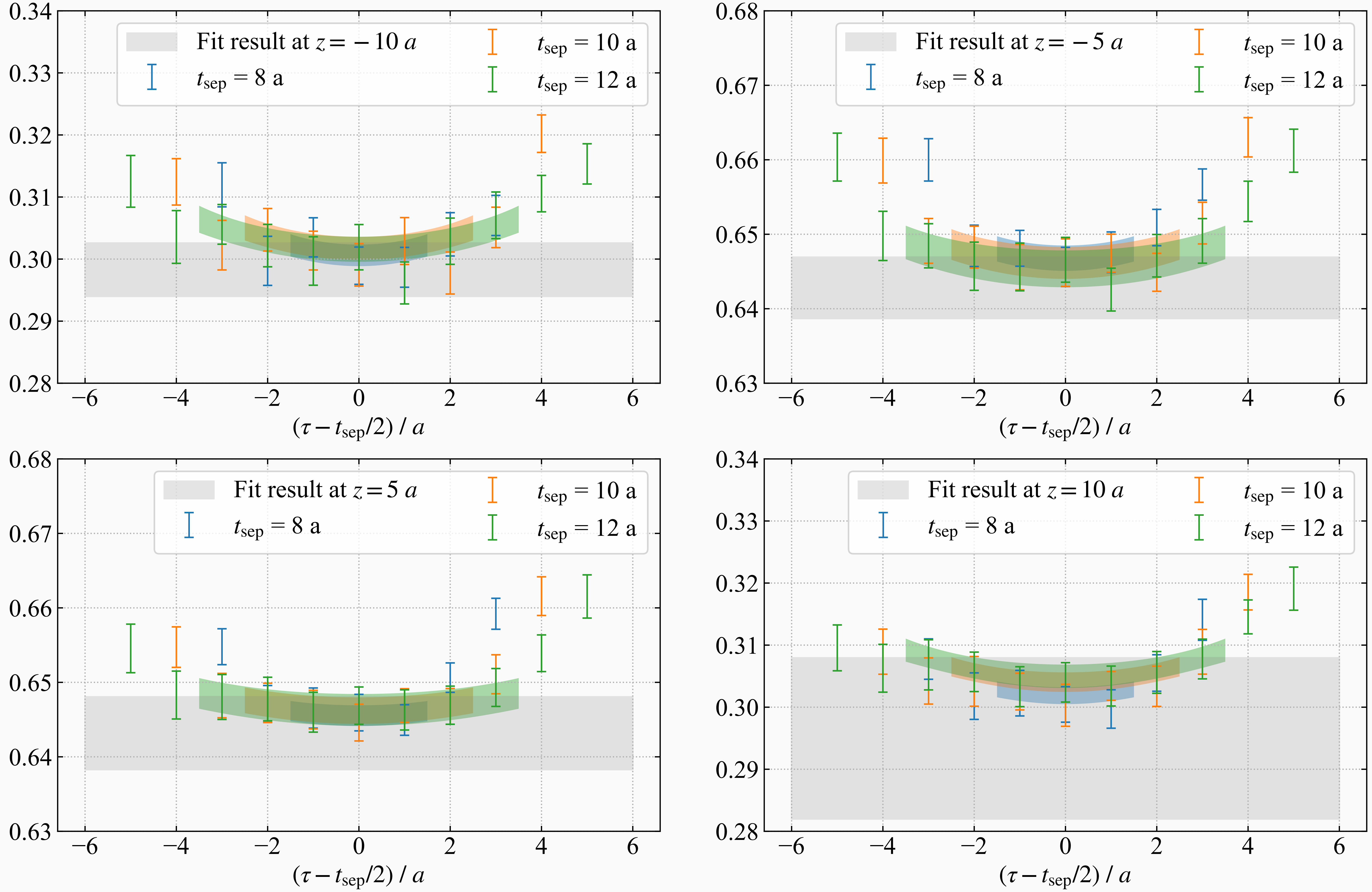
$$\delta^2 F_{\mathrm{CG}}[A,\Omega] = -\sum_{\mu=1}^{3}\int d^4x \partial_\mu\theta_a\delta A^{\mu a}_{\Omega} = -\sum_{\mu=1}^{3}\int d^4x \theta_a(\partial_\mu D^{ab}_{\mu})\theta^b$$

$\delta^2 F_{\mathrm{CG}}[A,\Omega] \geq 0$ for $\forall\theta \Longrightarrow \mathscr{M} = -\partial_\mu D^{ab}_{\mu}$ is positive definite

In the first Gribov region

$* \; A_{\Omega\mu}(x) \equiv \Omega^\dagger(x)A_\mu(x)\Omega(x) + \frac{i}{g}\Omega^\dagger(x)\partial_\mu\Omega(x)$

Ground State Fit of First it



Ground State Fit of Smallest f