Three-dimensional Imaging of the Pion using Lattice QCD: GPDs

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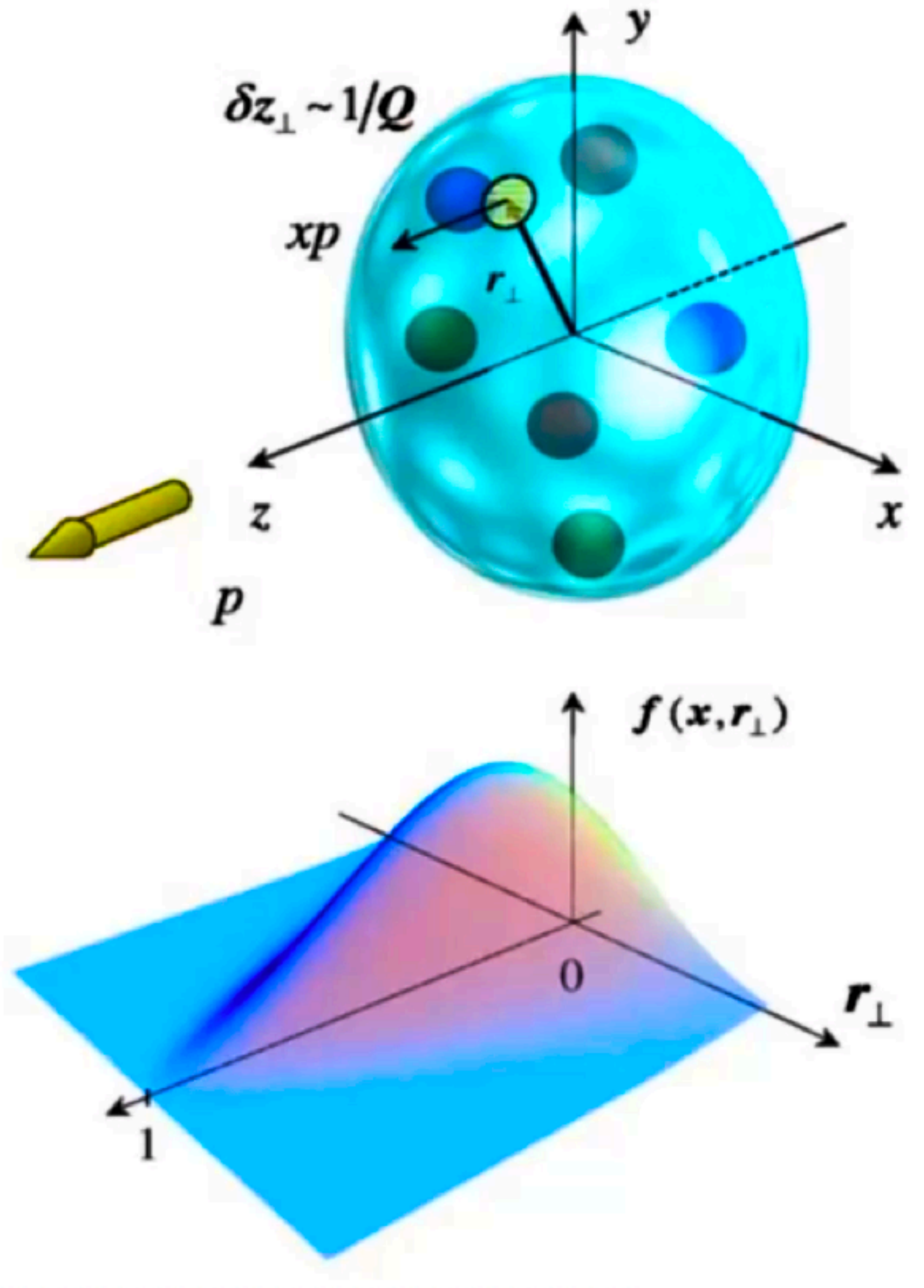
Generalized parton distributions

GPDs goes far beyond the 1D PDFs and the transverse structure encoded in the form factors.



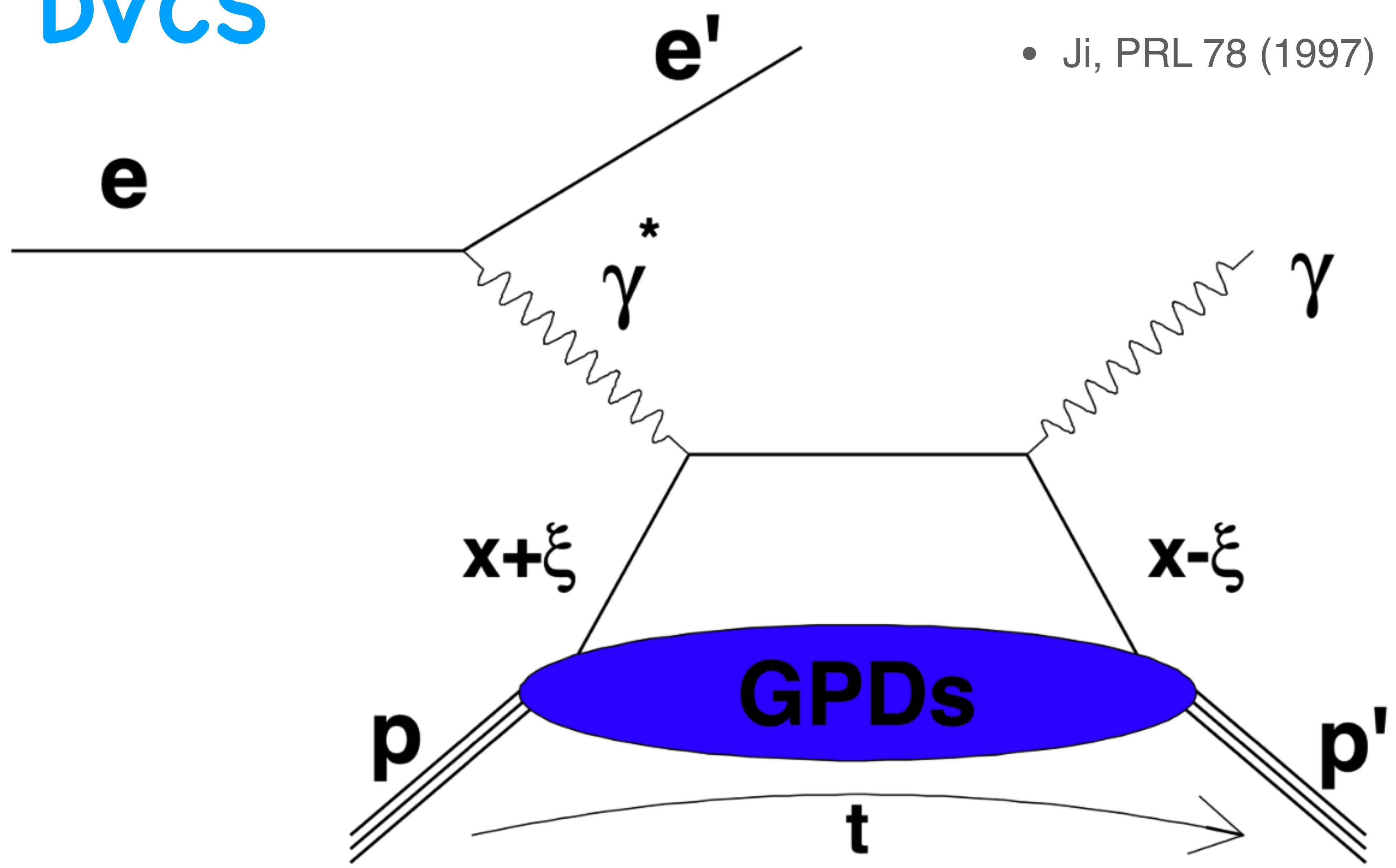
$$F_q(x, \xi, t) = \int \frac{dz^-}{4\pi} e^{-ixP^+z^-} \langle p_f | \bar{q}(-\tfrac{z}{2}) \gamma^\mu \mathscr{W}(-\tfrac{z}{2}, \tfrac{z}{2}) q(\tfrac{z}{2}) | p_i \rangle$$

- Offer insights into the 3D image of hadrons.
- Give access to the orbital motion and spin of partons.
- Have a relation to pressure and shear forces inside hadrons.

- Ji, PRL 78 (1997)
- Belitsky and Radyushkin: Phys.Rept. 418 (2005) 1-387

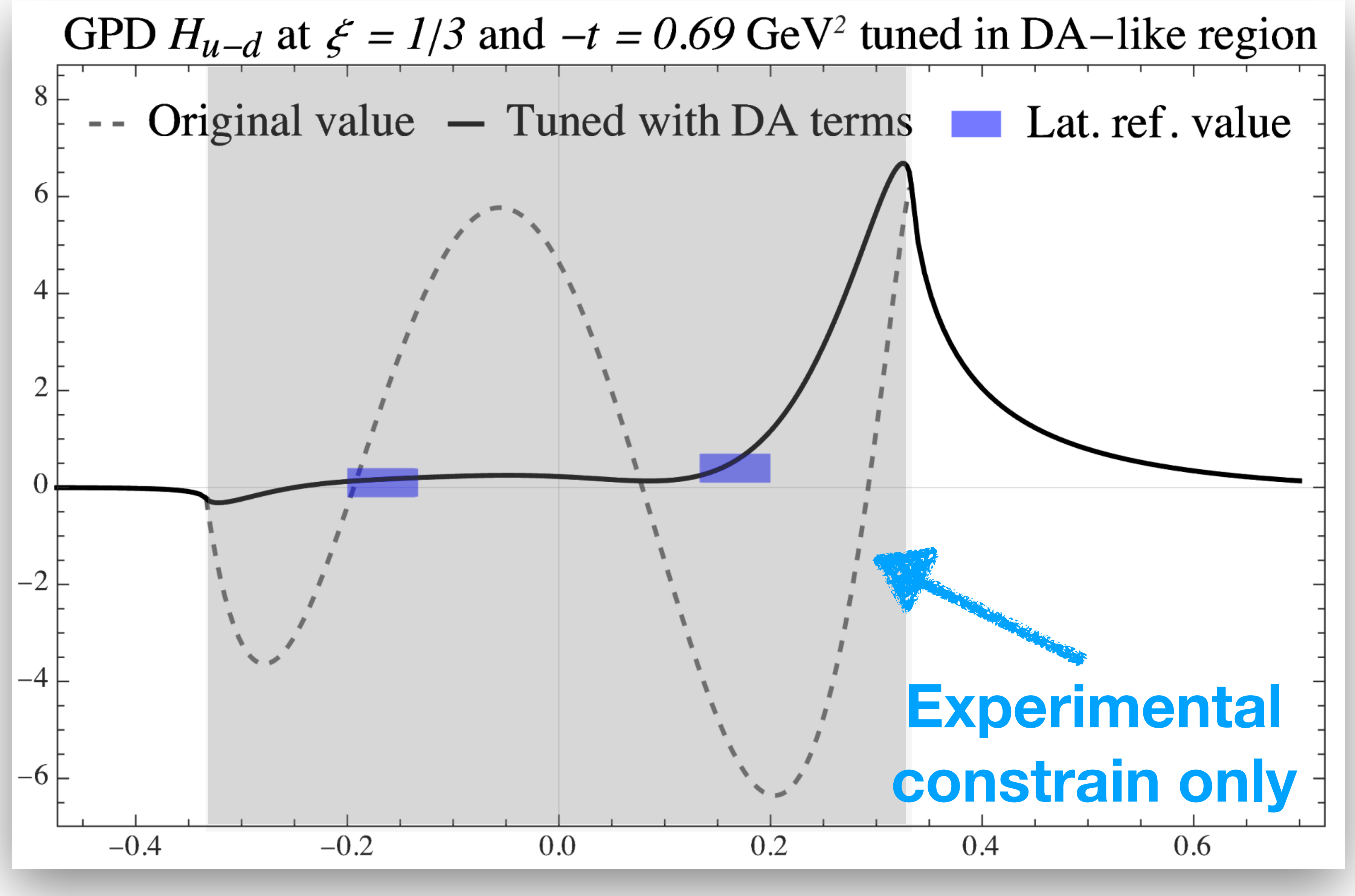
3 Generalized parton distributions

DVCS



• Ji, PRL 78 (1997)

• Y. Guo, et al., JHEP 05 (2023) 150

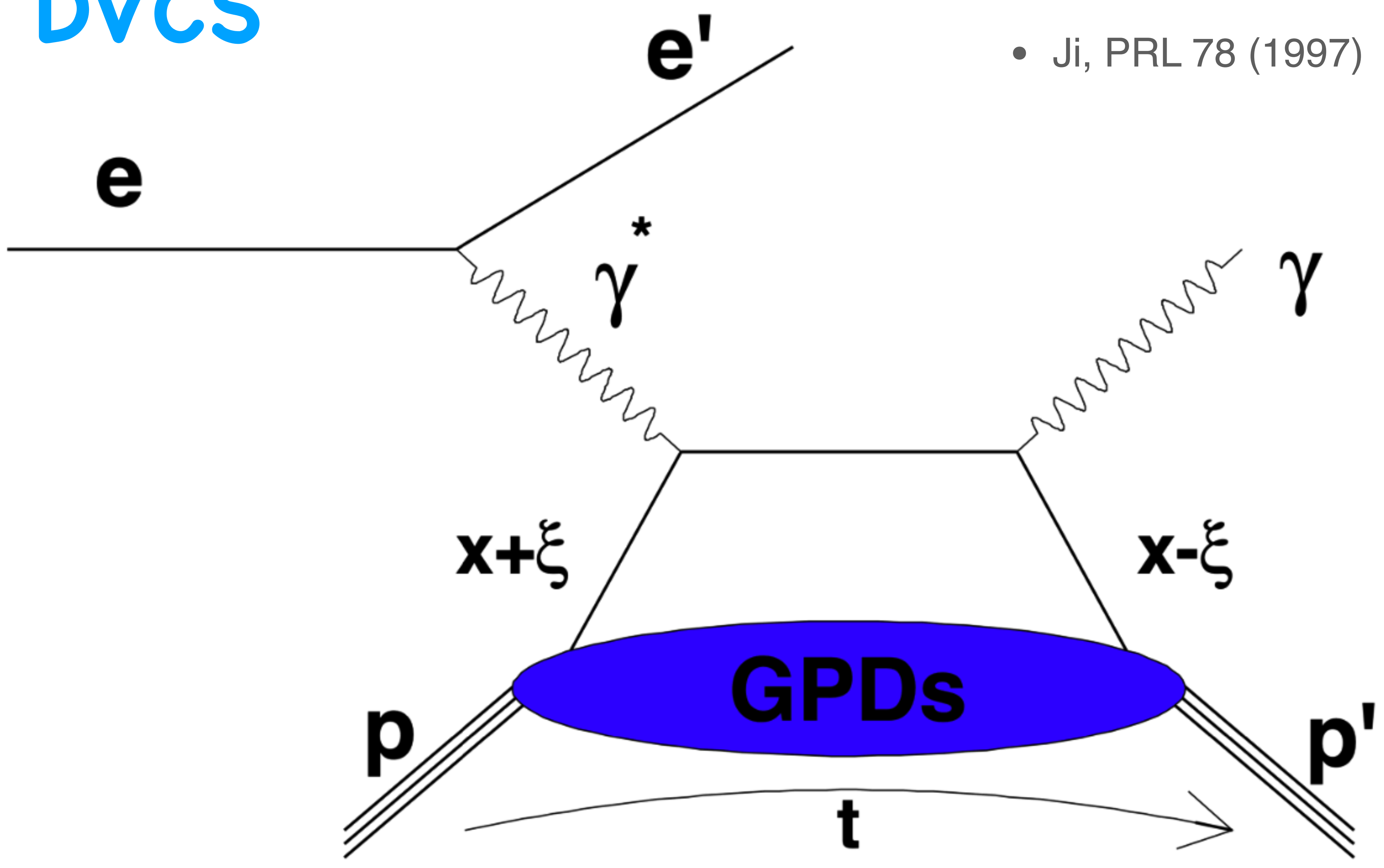


Challenging:

- Observables appear at the amplitude level.
- Multi-dimensionality $F(x, \xi, t)$.
- The momentum fraction x is integrated over (Inverse problem).

Generalized parton distributions

DVCS



- Ji, PRL 78 (1997)

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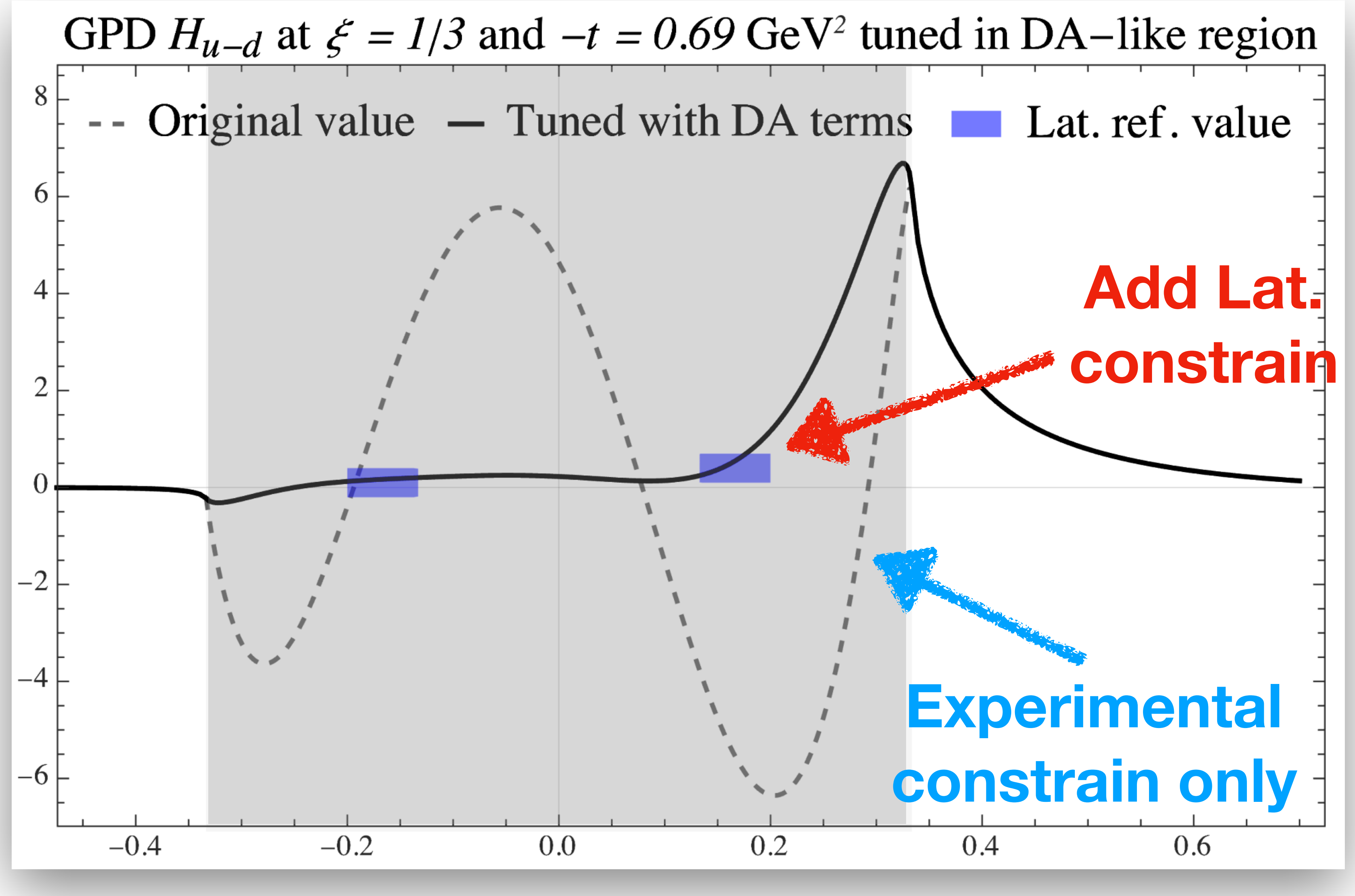
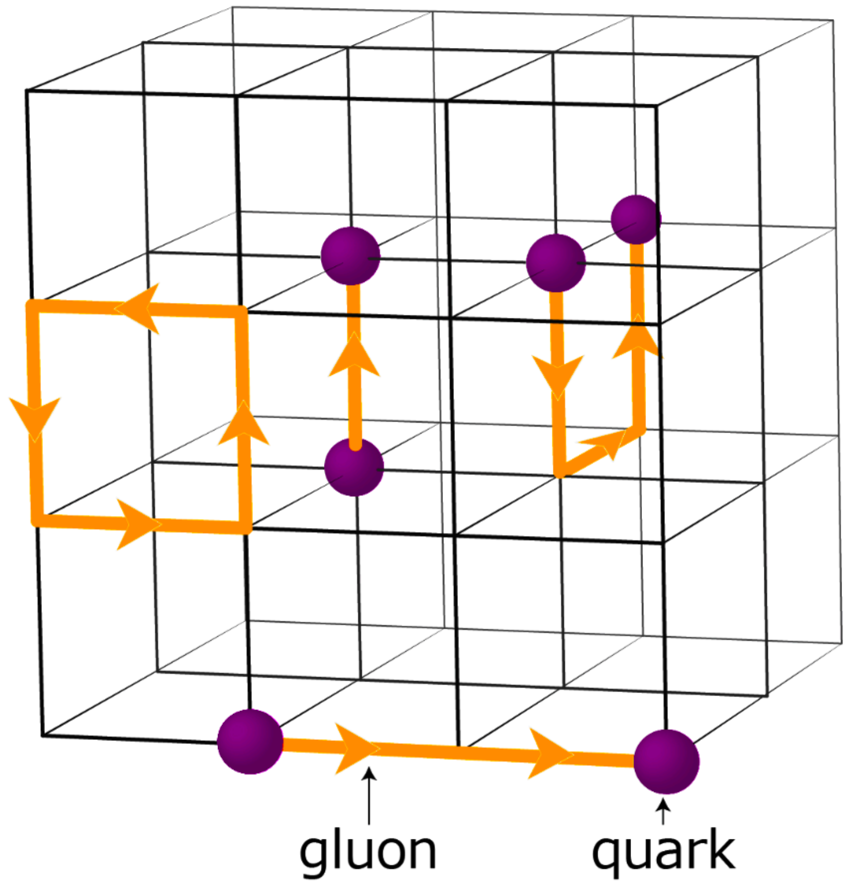
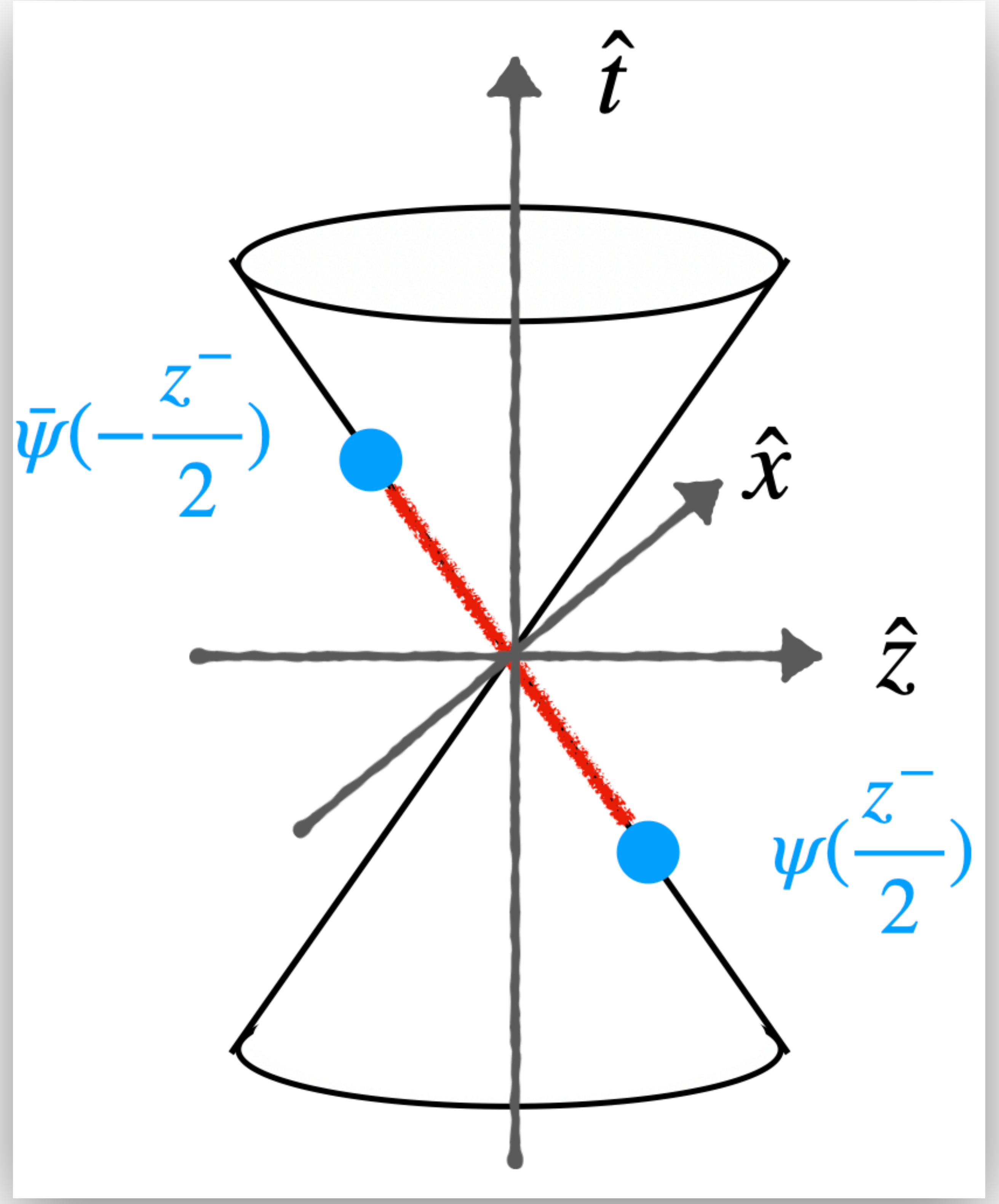


Complementary knowledge from lattice QCD is essential.



5 Parton distributions from lattice QCD simulation



$$f(x,\mu) = \int \frac{dz^-}{4\pi} e^{-ixP^+z^-} \langle P \,|\, \bar{\psi}(0)\Gamma \mathscr{W}(0,z^-)\psi(z^-) \,|\, P' \rangle$$

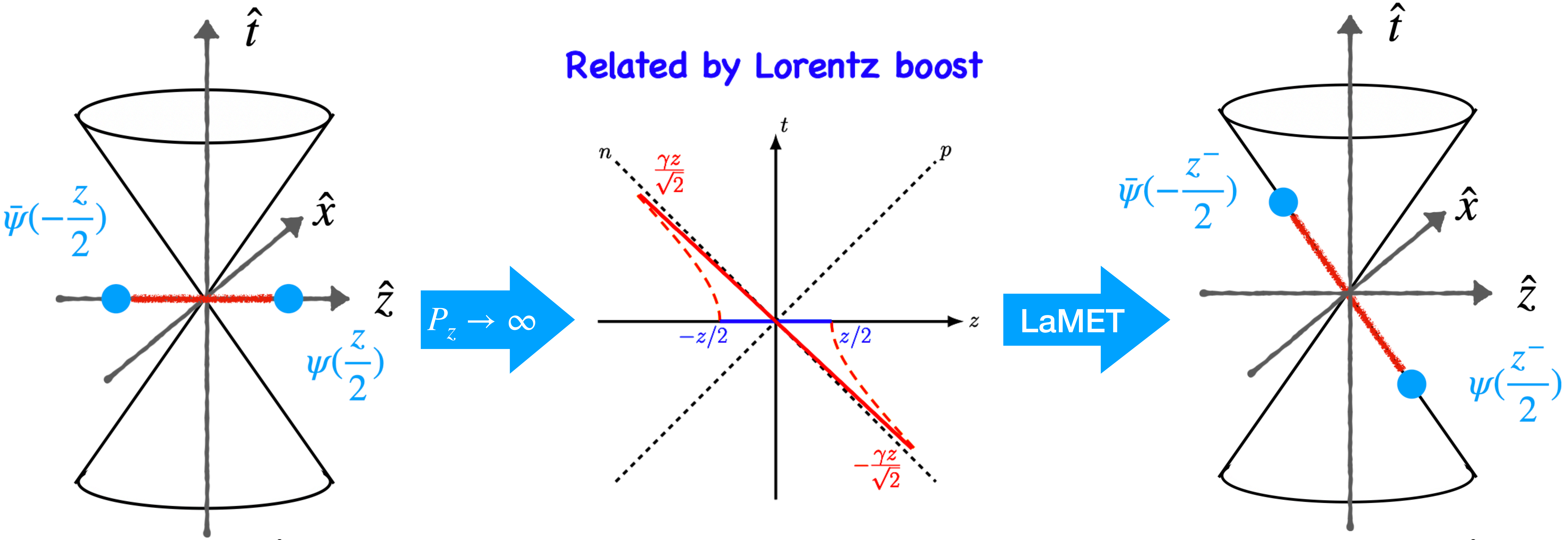
✗ Light-cone correlations: forbidden on Euclidean lattice

Large momentum effective theory

The quasi distribution from equal-time correlators,

- X. Ji, PRL 110 (2013); SCPMA57 (2014);

$$\tilde{f}(x, P_z, \mu) = \int \frac{dz}{4\pi} e^{-ixP_z z} \langle P | \bar{\psi}(0) \Gamma \mathscr{W}(0,z) \psi(z) | P \rangle$$



Quasi PDF/GPD

$$\langle P \to \infty | \bar{\psi}(0) \Gamma \mathscr{W}(0,z) \psi(z) | P \to \infty \rangle$$

Light-cone PDF/GPD

$$\langle P | \bar{\psi}(0) \Gamma \mathscr{W}(0,z^-) \psi(z^-) | P \rangle$$

Large momentum effective theory

- X. Ji, PRL 110 (2013); SCPMA57 (2014);
- X. Xiong, X. Ji, et al, 90 PRD (2014);
- Y.-Q. Ma, et al, PRD98 (2018), PRL 120 (2018);
- T. Izubuchi, X. Ji, et al PRD98 (2018).
- X. Ji, Y. Zhao, et al, RMP 93 (2021).

Large P_z expansion of quasi distribution:

Quasi PDF

$$\tilde{f}(x, P_z, \mu) = \int \frac{dy}{|y|} C(\frac{x}{y}, \frac{\mu}{yP_z}) f(y, \mu) + \mathcal{O}(\frac{\Lambda_{QCD}^2}{x^2 P_z^2}, \frac{\Lambda_{QCD}^2}{(1-x)^2 P_z^2})$$

Light-cone PDF

Power corrections

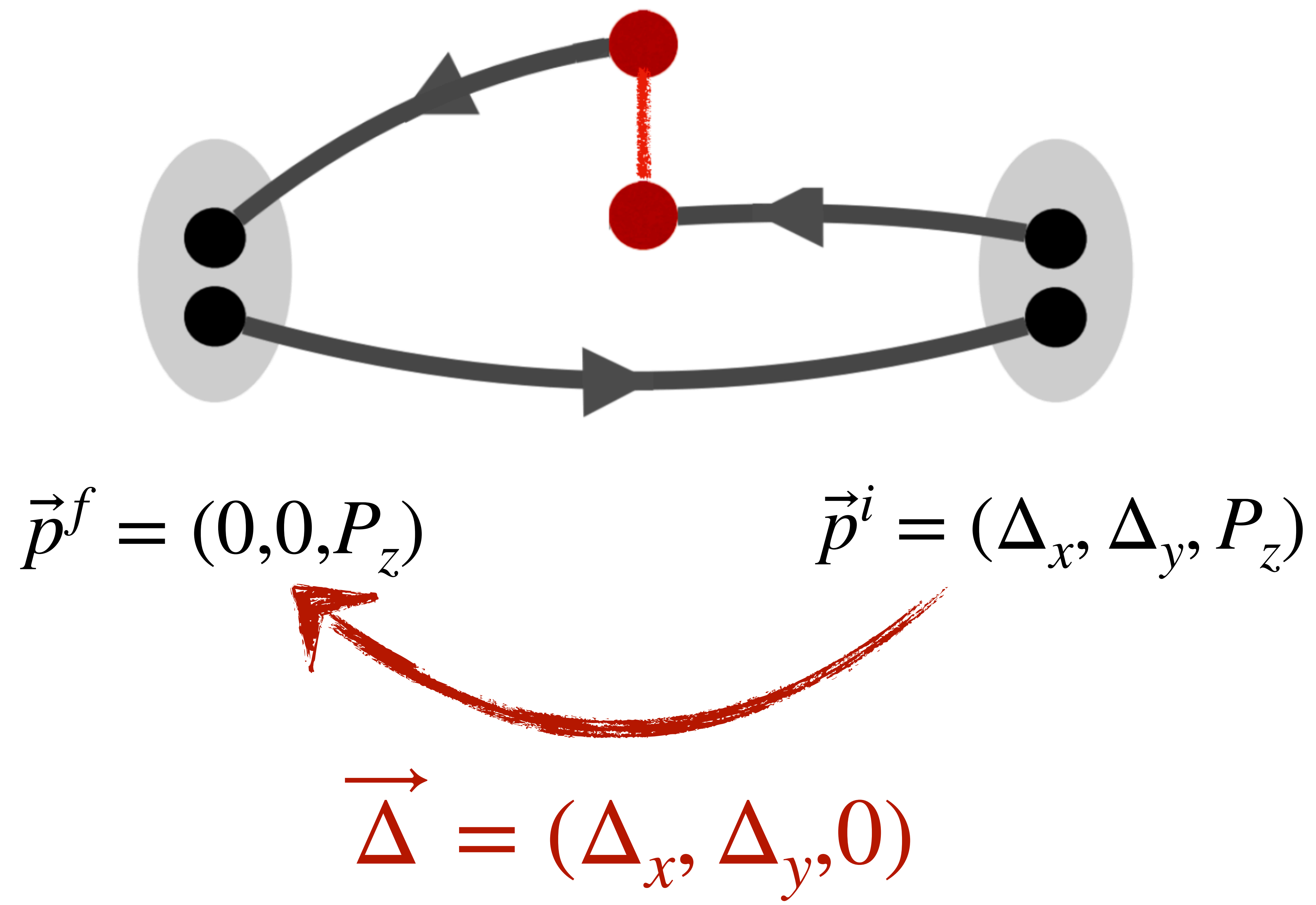
- In large P_z limit, Quasi differ from light-cone distribution by $\lim_{P\to\infty}\lim_{a\to 0}$ v.s. $\lim_{a\to 0}\lim_{P\to\infty}$, inducing a **perturbative matching** $C(\frac{x}{y}, \frac{\mu}{yP_z})$.
- **Computable** from Lattice QCD with finite $P_z < 1/a$: power corrections.

Pion GPDs: lattice setup

- Clover-fermion on 2+1f HISQ gauge ensembles, 1-step HYP smearing.
- $64^3 \times 64$, a = 0.04 fm, $m_\pi = 300$ MeV.
- Pion P_z up to 1.94 GeV, momentum transfer up to 1.7 GeV^2.

t_s/a	$\mathbf{n}^f = (n_x^f, n_y^f, n_z^f)$	m_z	P_z[GeV]	$\mathbf{n}^\Delta = (n_x^\Delta, n_y^\Delta, n_z^\Delta)$	$-t$[GeV^2]	#cfgs	(#ex, #sl)
9,12,15,18	(0,0,0)	0	0	(0,0,0)	0	314	(3, 96)
9,12,15,18	(0,0,2)	2	0.968	(1,2,0)	0.952	314	(4, 128)
9,12,15	(0,0,3)	2	1.453	[(0,0,0), (1,0,0) (1,1,0), (2,0,0) (2,1,0), (2,2,0)]	[0, 0.229, 0.446, 0.855, 1.048, 1.589]	314	(4, 128)
9,12,15	(0,0,4)	3	1.937		[0, 0.231, 0.455, 0.887, 1.095, 1.690]	564	(4, 128)

Pion GPDs: lattice setup



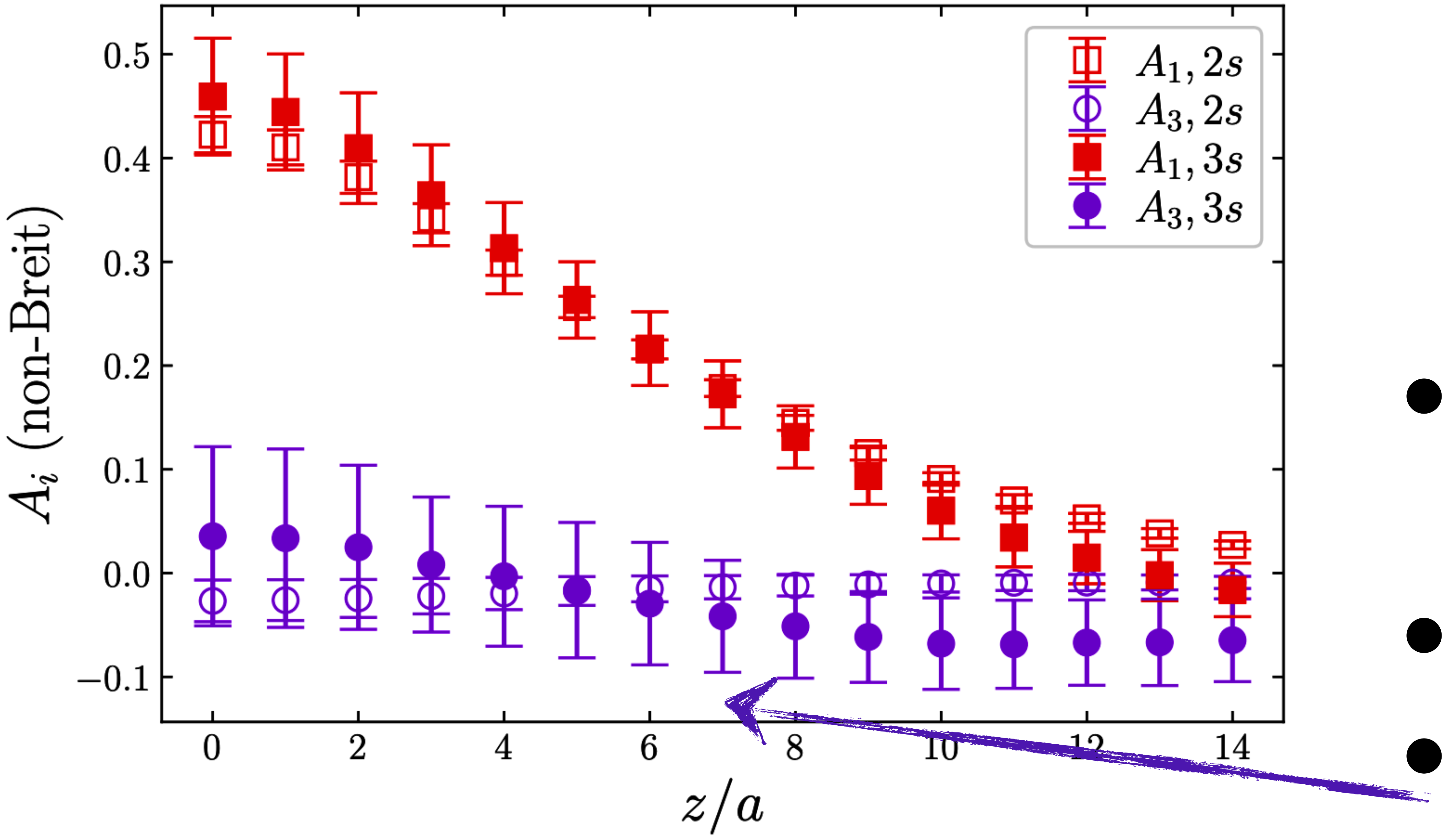
$$M^\mu(z, \bar{P}, \Delta)$$

$$= \langle \vec{p}^f | \bar{\psi}(\frac{z}{2}) \gamma^\mu W(\frac{z}{2}, -\frac{z}{2}) \psi(-\frac{z}{2}) | \vec{p}^i \rangle$$

- Zero-skewness: $\xi = -\dfrac{z\Delta_z}{2z\bar{P}_z} = 0.$
- Asymmetric frame: all Δ assigned to initial state to get multiple momentum transfer at once during contraction.

- S. Bhattacharya, XG, et al., PRD 106 (2022) 11, 114512
- S. Bhattacharya, XG, et al., PRD 108 (2023) 1, 014507

Bare matrix elements: explicit power corrections



Lorentz-covariant decomposition (spin-0)

in terms of $A_i(z \cdot P_z, z \cdot \Delta, \Delta^2, z^2)$:

$$M^\mu(z, \bar{P}, \Delta) = \bar{P}^\mu A_1 + m_\pi^2 z^\mu A_2 + \Delta^\mu A_3$$

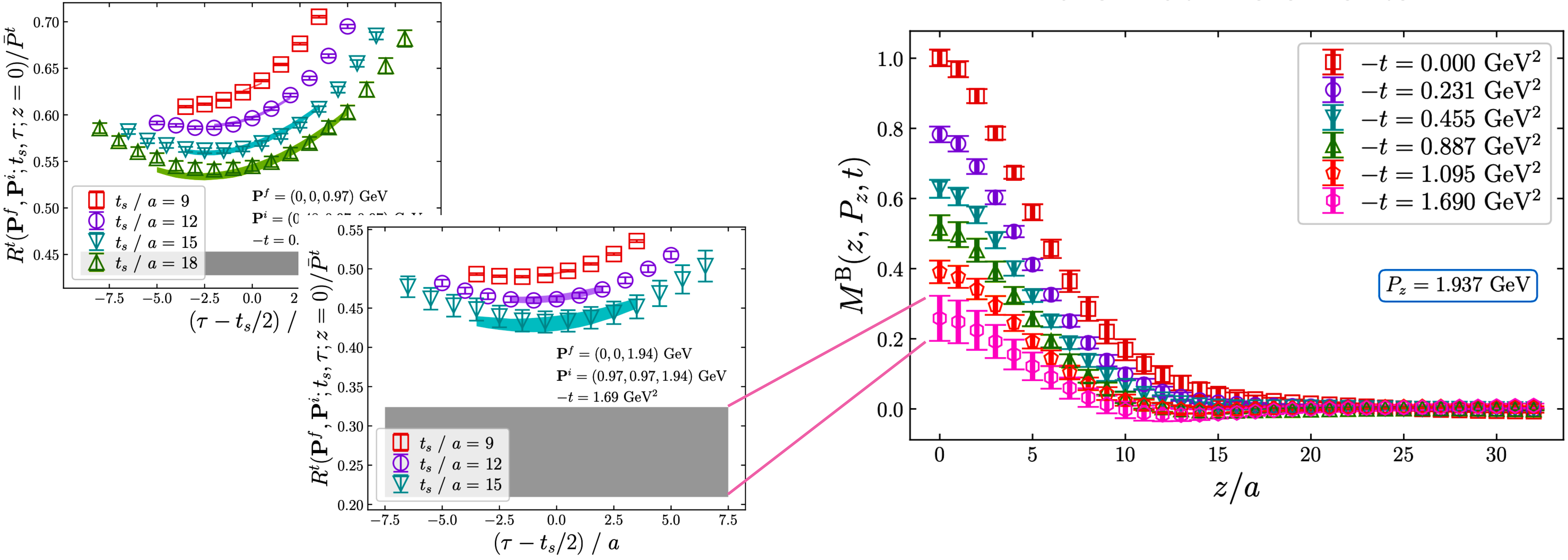
- A_2 **and** A_3 **are power corrections** existing at finite P_z but disappear in the $P_z \to \infty$ limit.
- A_2 **can be avoided** by choosing $\gamma^\mu = \gamma^t$.
- A_3 can be estimated through linear combination of $\gamma^t, \gamma^x, \gamma^y$: **consistent with zero**.

- S. Bhattacharya, XG, et al., PRD 106 (2022) 11, 114512
- S. Bhattacharya, XG, et al., PRD 108 (2023) 1, 014507

Bare matrix elements

- Three-point to two-point functions ratio

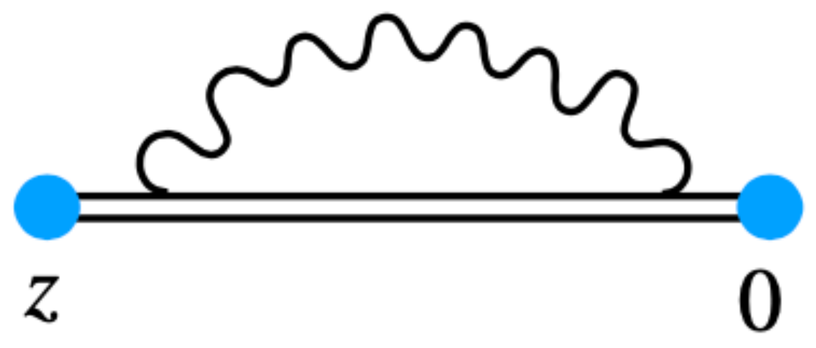
- Bare matrix elements



➡ Pion P_z up to 1.94 GeV, momentum transfer up to 1.7 GeV^2.

Renormalization

- X. Ji, J. H. Zhang and Y. Zhao, PRL120 (2018)
- J. Green, K. Jansen and F. Steffens, PRL121 (2018)
- T. Ishikawa, et al, PRD 96 (2017)



$$= \delta m(a)|z| \propto \frac{|z|}{a}$$

The operator can be multiplicatively renormalized:

$$[\bar{\psi}(0)\Gamma W_{\hat{z}}(0,z)\psi(z)]_B$$

$$= e^{-\delta m|z|} Z(a) [\bar{\psi}(0)\Gamma W_{\hat{z}}(0,z)\psi(z)]_R$$

$$\delta m(a) = m_{-1}(a)/a + \mathcal{O}(\Lambda_{\rm QCD})$$

Wilson-line self energy + renormalon ambiguity

1. **Subtract the linear divergence (Lattice scheme):**
   - $a\delta m(a) = 0.1508(12)$ from static potential.
     - **XG**, Y. Zhao et al., (BNL/ANL) PRL 128 (2022).
2. **Subtract the leading renormalon:**
   - Match the $\overline{\rm MS}$ OPE matrix element to lattice scheme.

$$Z(a) e^{\delta m(a)|z|} M^B(z, P_z = 0)$$

$$= C_0^{\overline{\rm MS},\mathrm{LRR}}(\mu, z) e^{-m_0|z|}$$

LRR: leading renormalon resummation

$$\mathcal{O}(\Lambda_{\rm QCD}/P_z) + \mathcal{O}(\Lambda_{\rm QCD}^2/P_z^2)$$

- J. Holligan, et al., (UMD) NPB 993 (2023).
- R. Zhang, et al., (UMD) PLB 844 (2023).

Renormalization

- X. Ji, J. H. Zhang and Y. Zhao, PRL120 (2018)
- J. Green, K. Jansen and F. Steffens, PRL121 (2018)
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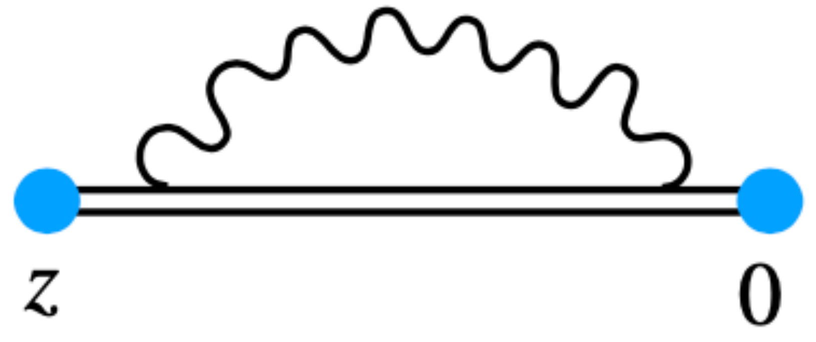

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Wilson-line self energy + renormalon ambiguity

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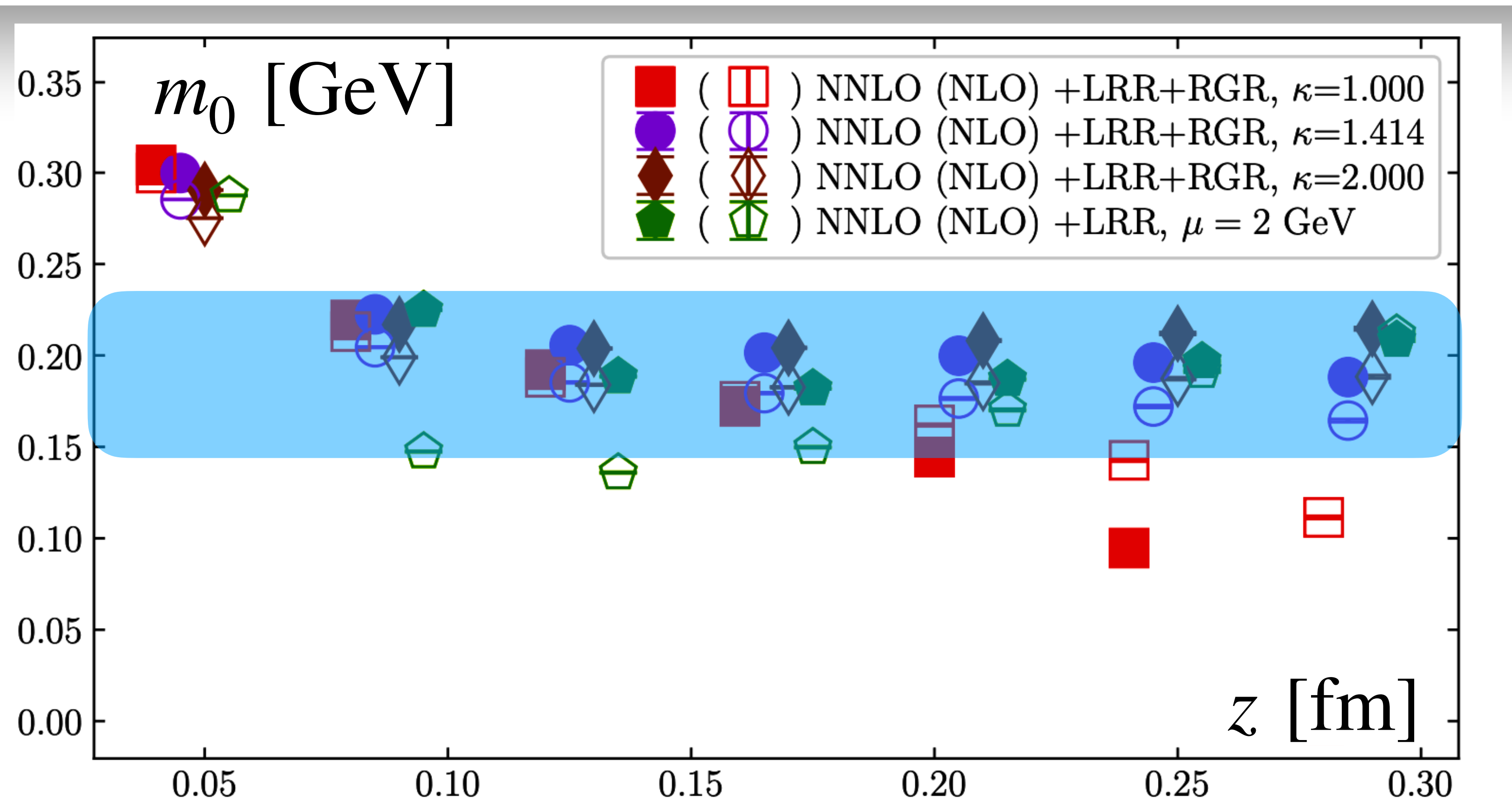
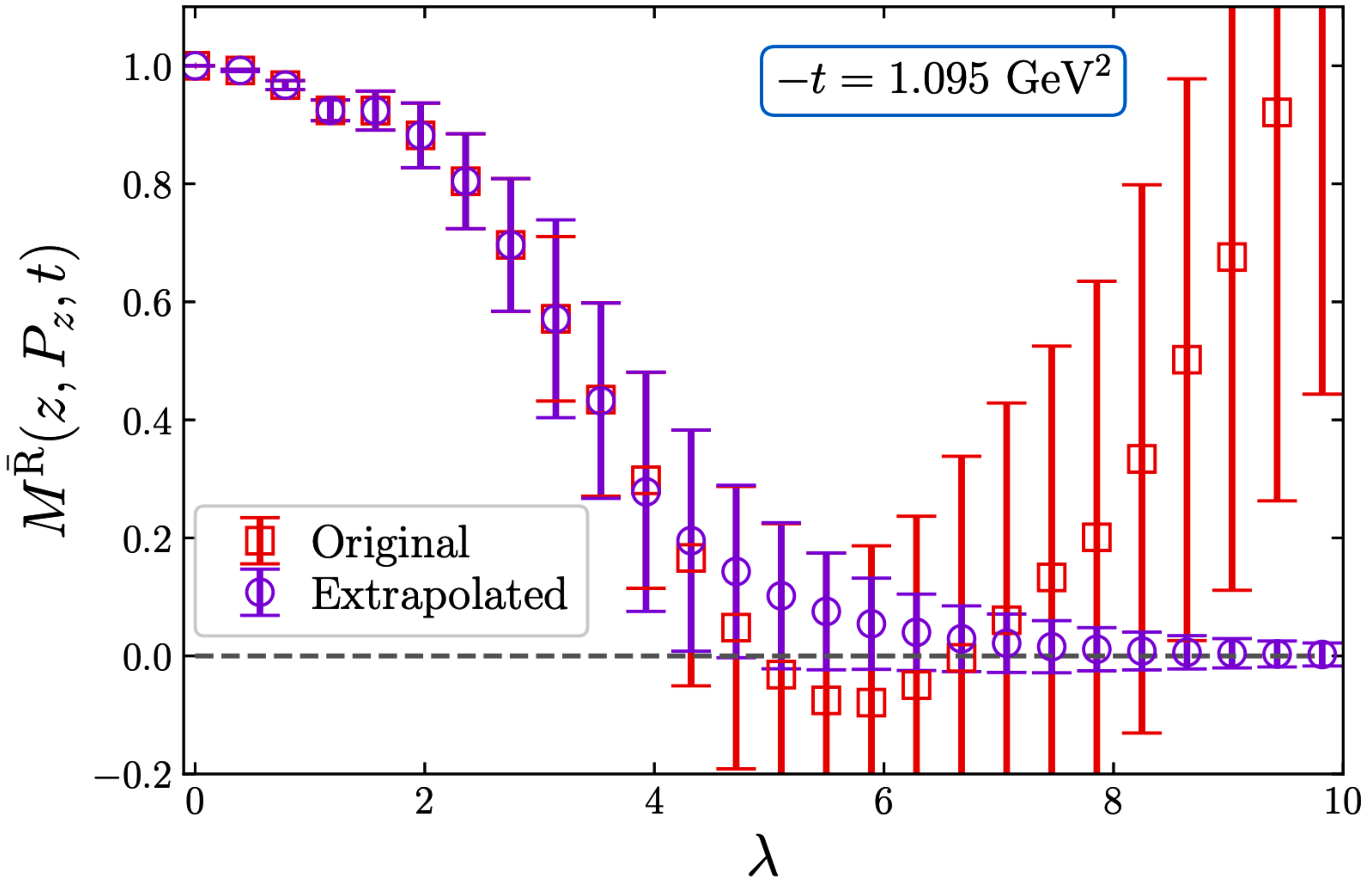


Renormalized matrix elements and F. T. to quasi-GPDs

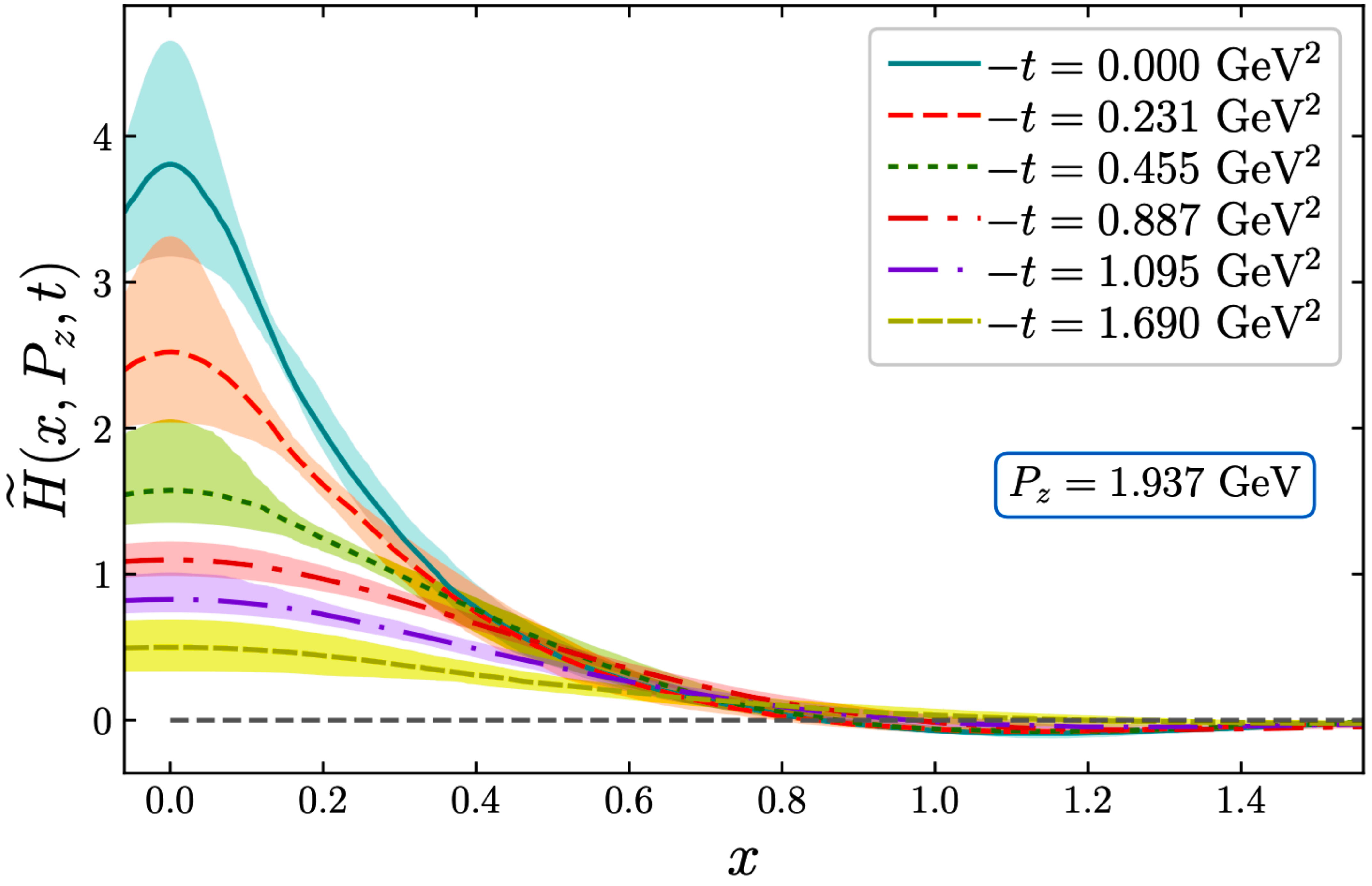
●Renormalized matrix elements



- Extrapolation of spacial correlation at long distance:

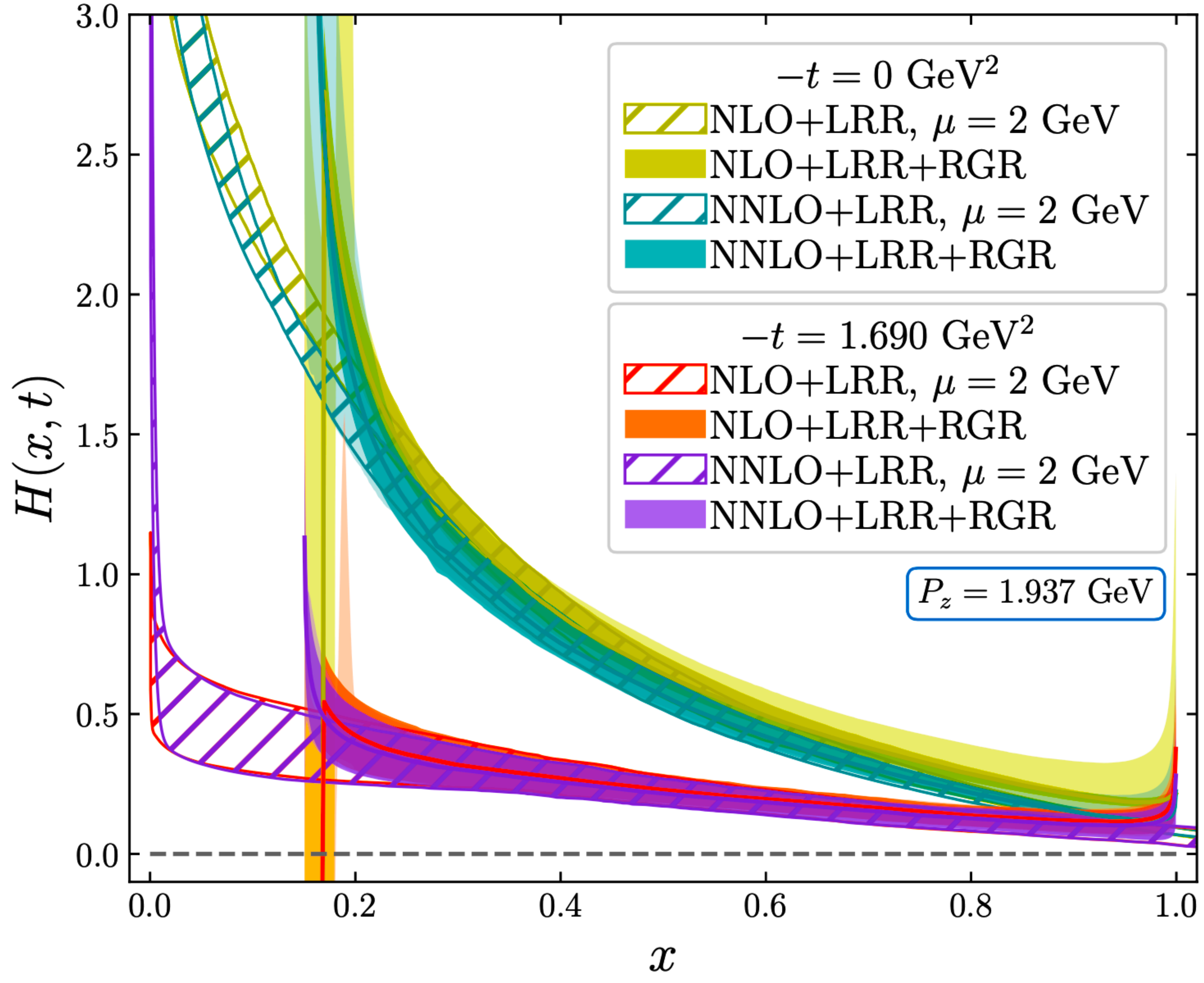
$$\sim \frac{e^{-m\cdot z}}{\lambda^d}$$

●Quasi-GPDs



- Quasi-GPDs decay as momentum transfer $-t$ increases.

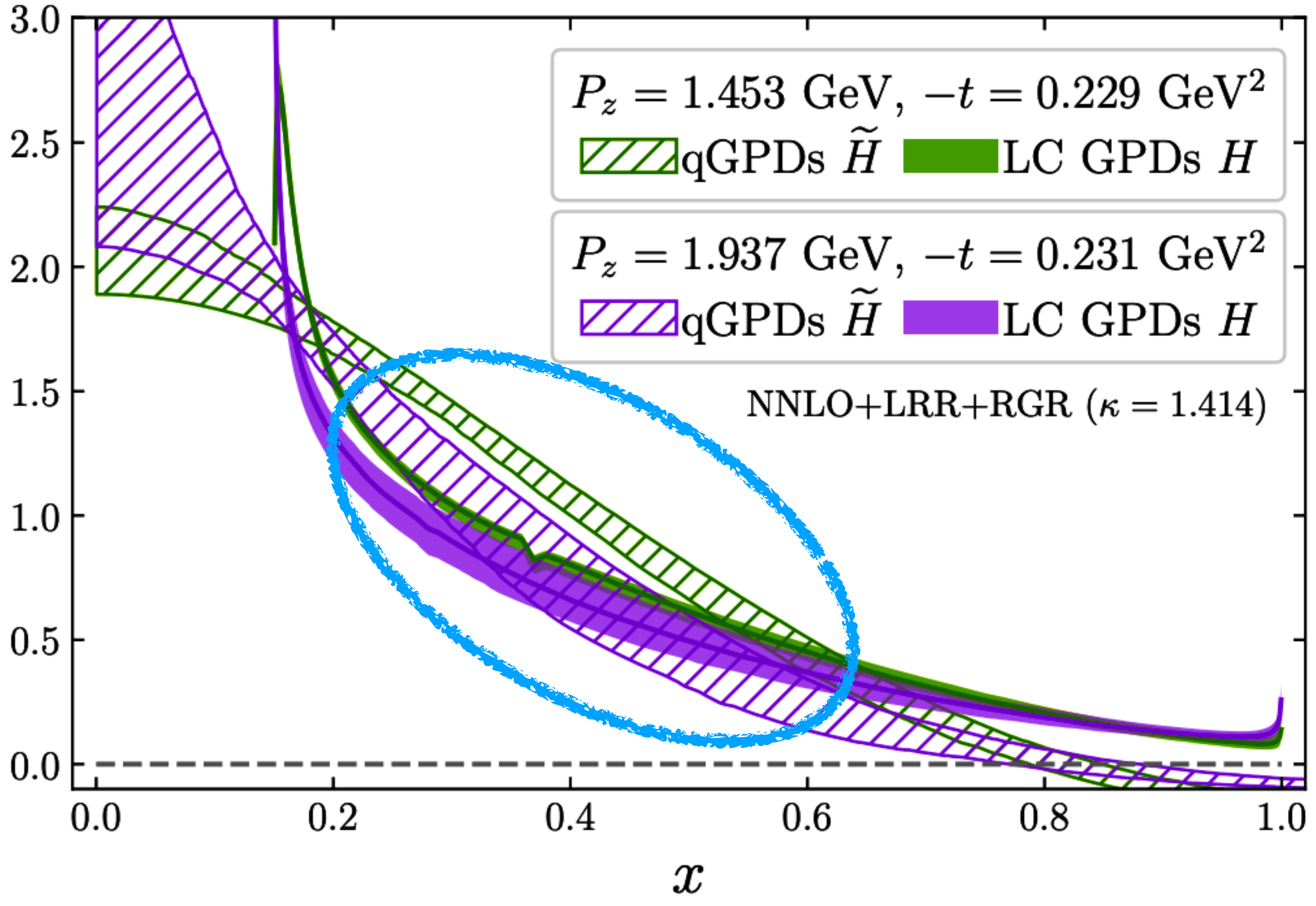
Perturbative matching up to NNLO



- NLO → NNLO: excellent perturbative convergence can be observed except small-x region, where LaMET factorization breaks down.

$$\tilde{f}(x, P_z, \mu) = \int \frac{dy}{|y|} C\left(\frac{x}{y}, \frac{\mu}{yP_z}\right) f(y, \mu) + \mathcal{O}\left(\frac{\Lambda_{QCD}^2}{x^2 P_z^2}, \frac{\Lambda_{QCD}^2}{(1-x)^2 P_z^2}\right)$$

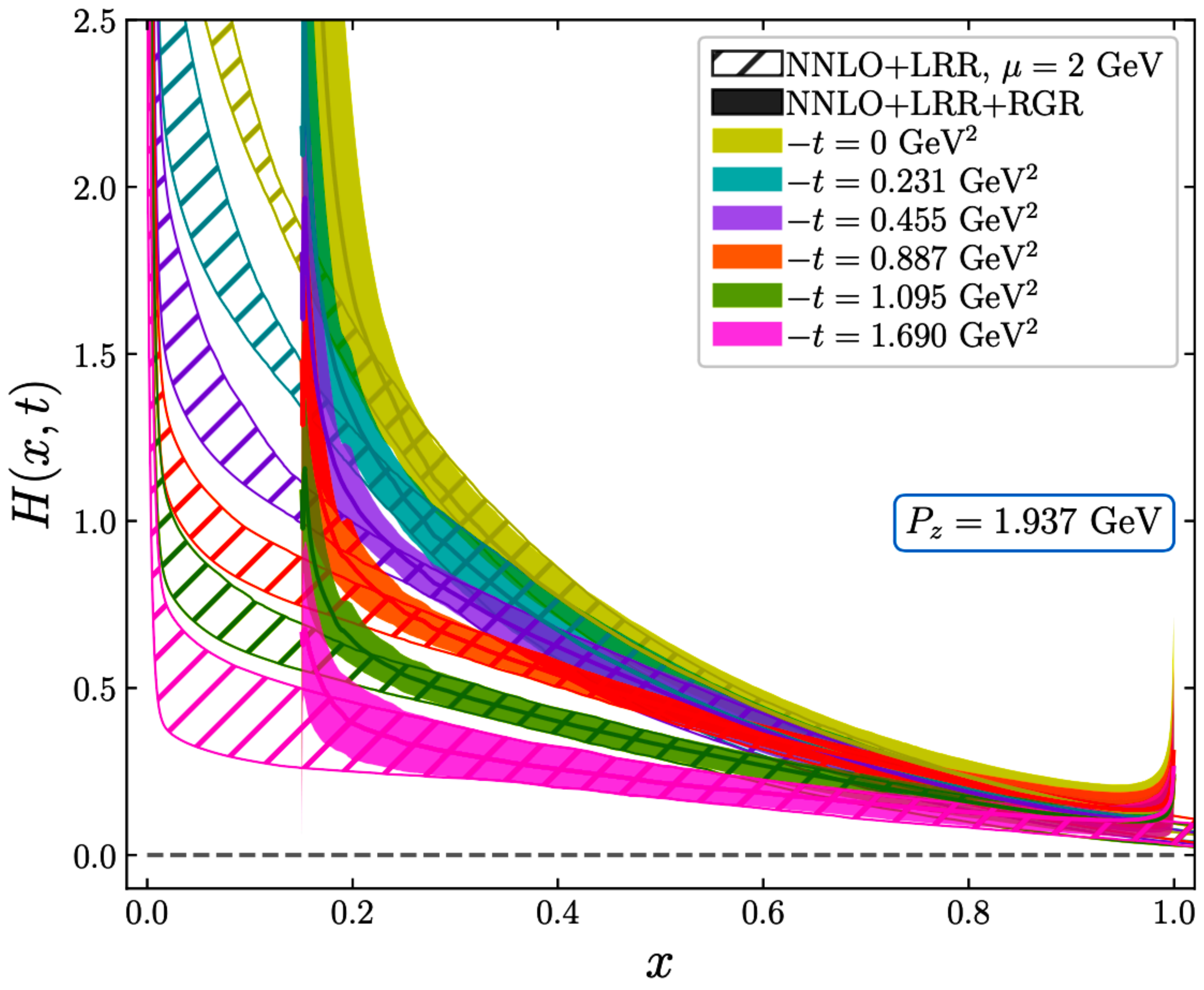
Pion GPDs from different momentum boost



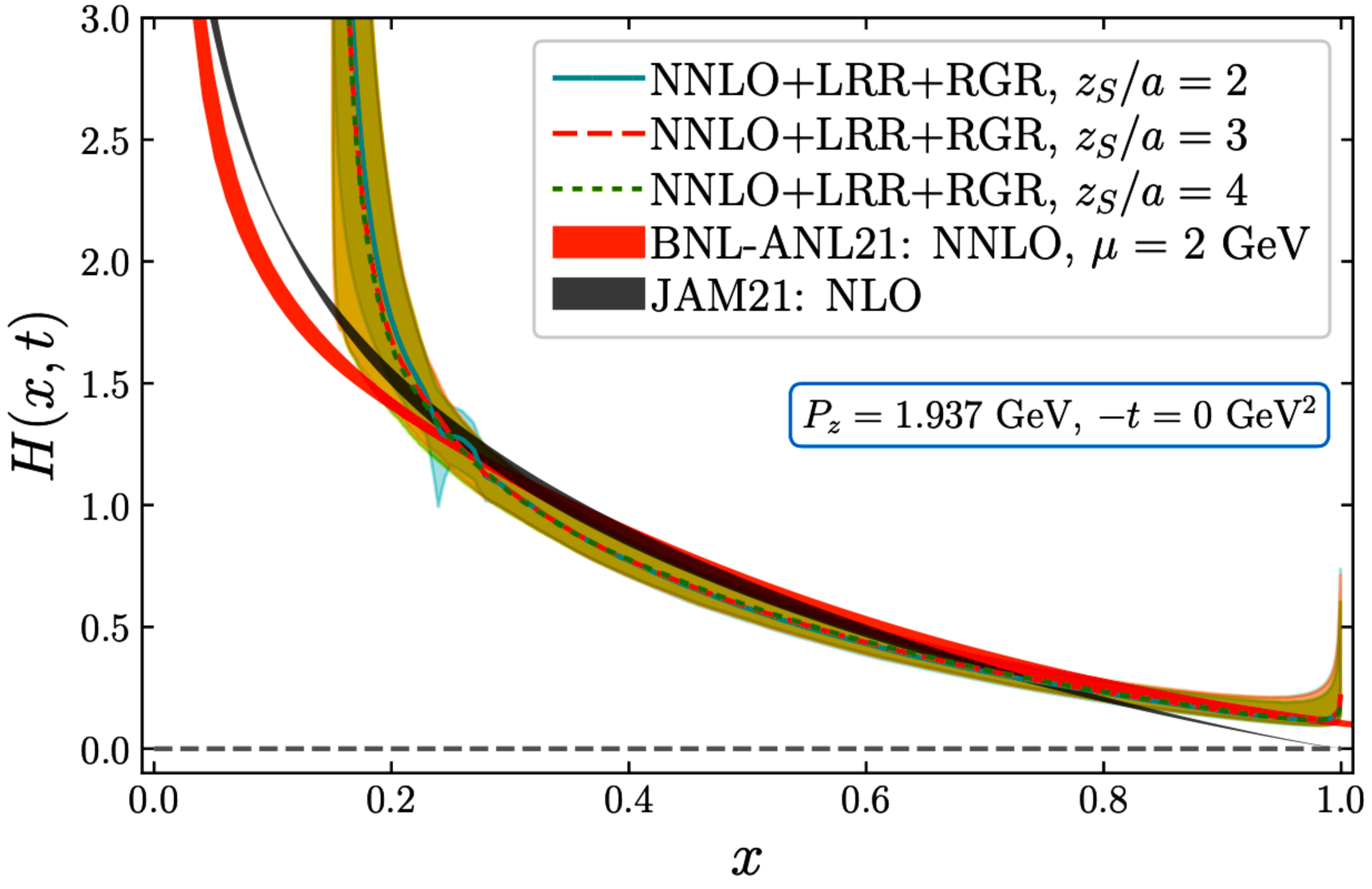
- Good momenta convergence can be observed in the moderate region of x: power corrections under control.

$$\tilde{f}(x, P_z, \mu) = \int \frac{dy}{|y|} C\left(\frac{x}{y}, \frac{\mu}{yP_z}\right) f(y, \mu) + \mathcal{O}\left(\frac{\Lambda_{QCD}^2}{x^2 P_z^2}, \frac{\Lambda_{QCD}^2}{(1-x)^2 P_z^2}\right)$$

Zero-skewness pion quark GPD

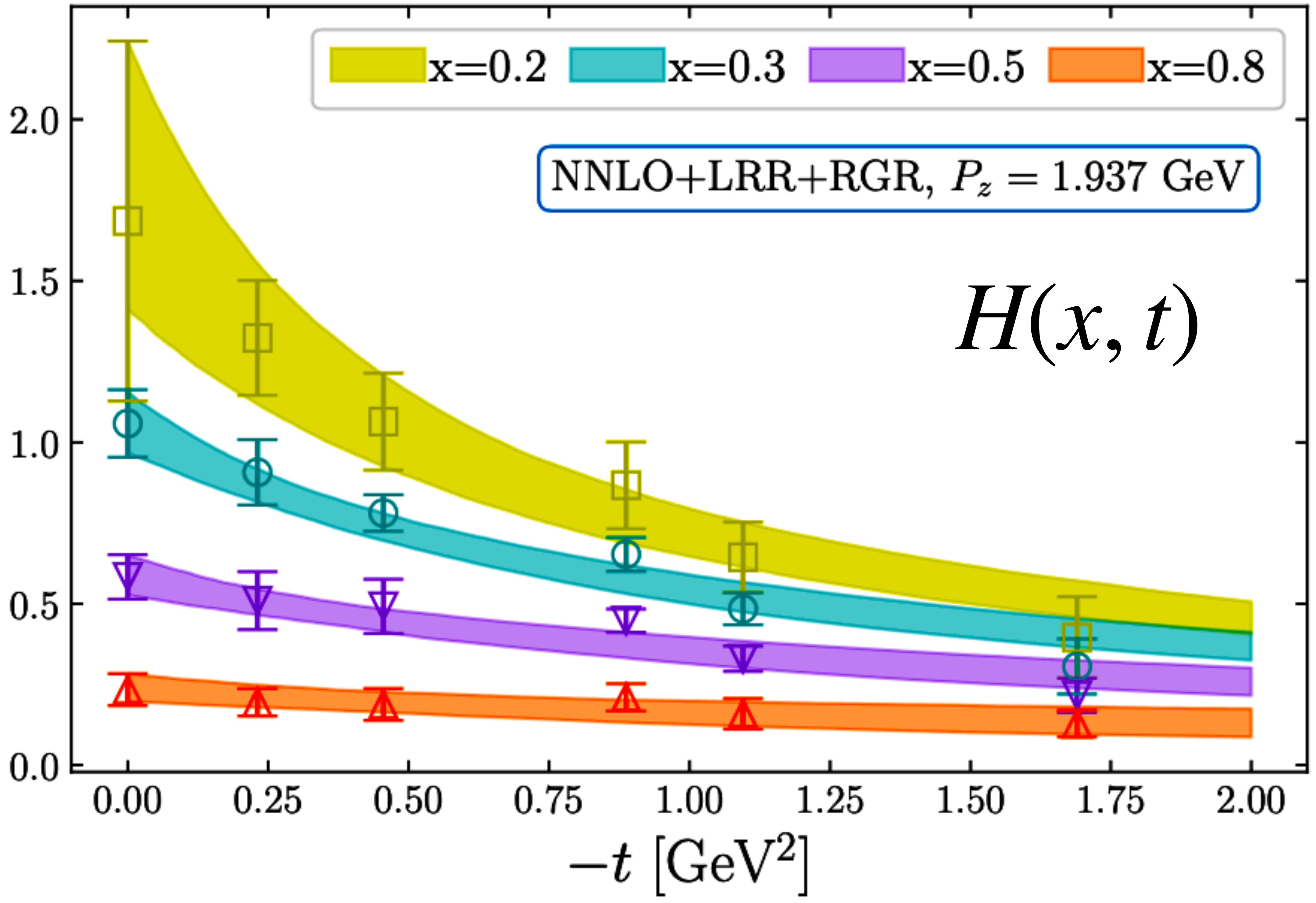


$-t = 0$: pion valence quark PDF



- PDF at $-t = 0$ agree with previous determination from fixed-order analysis and recent global analysis.

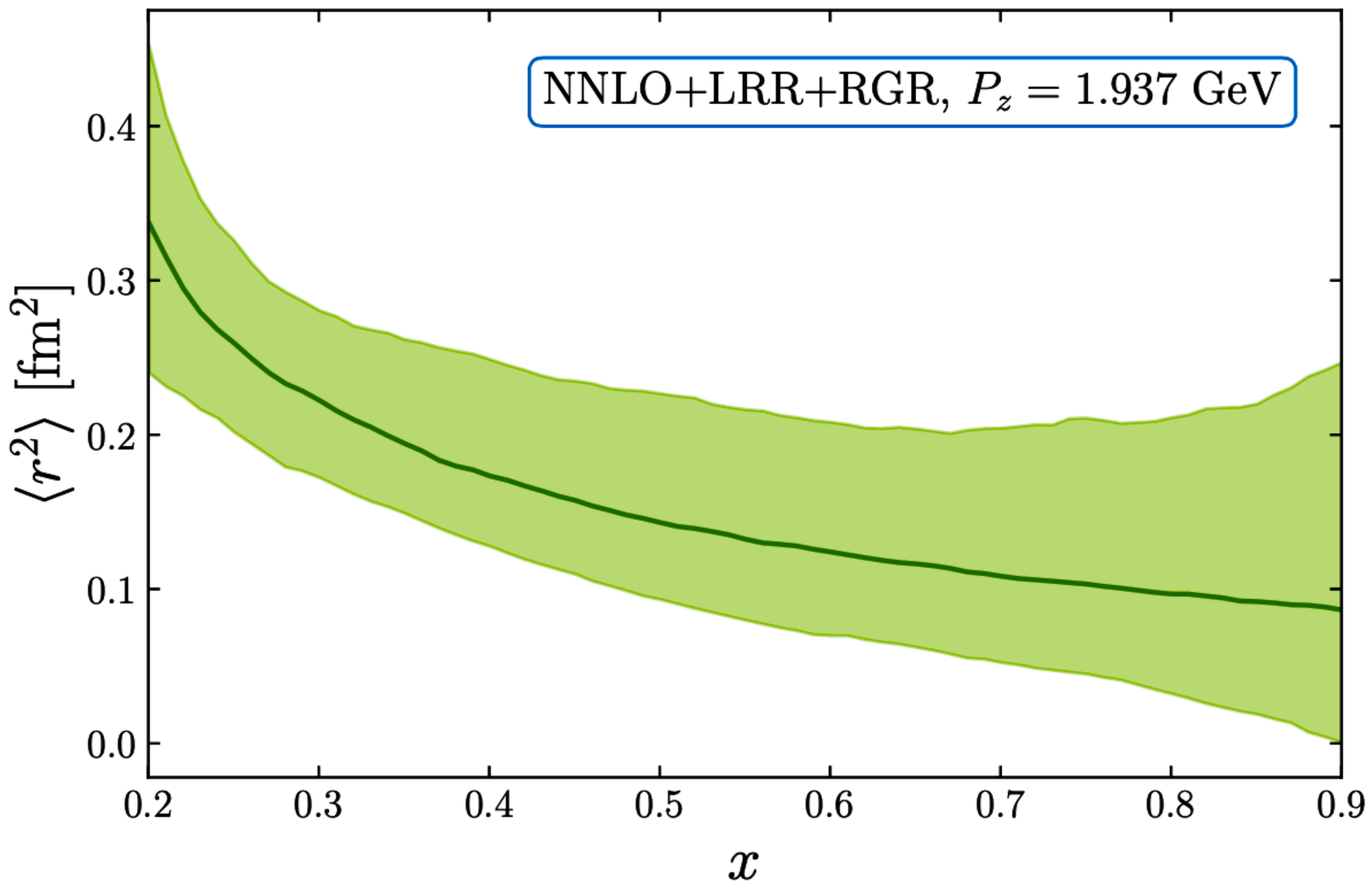
Zero-skewness pion quark GPD

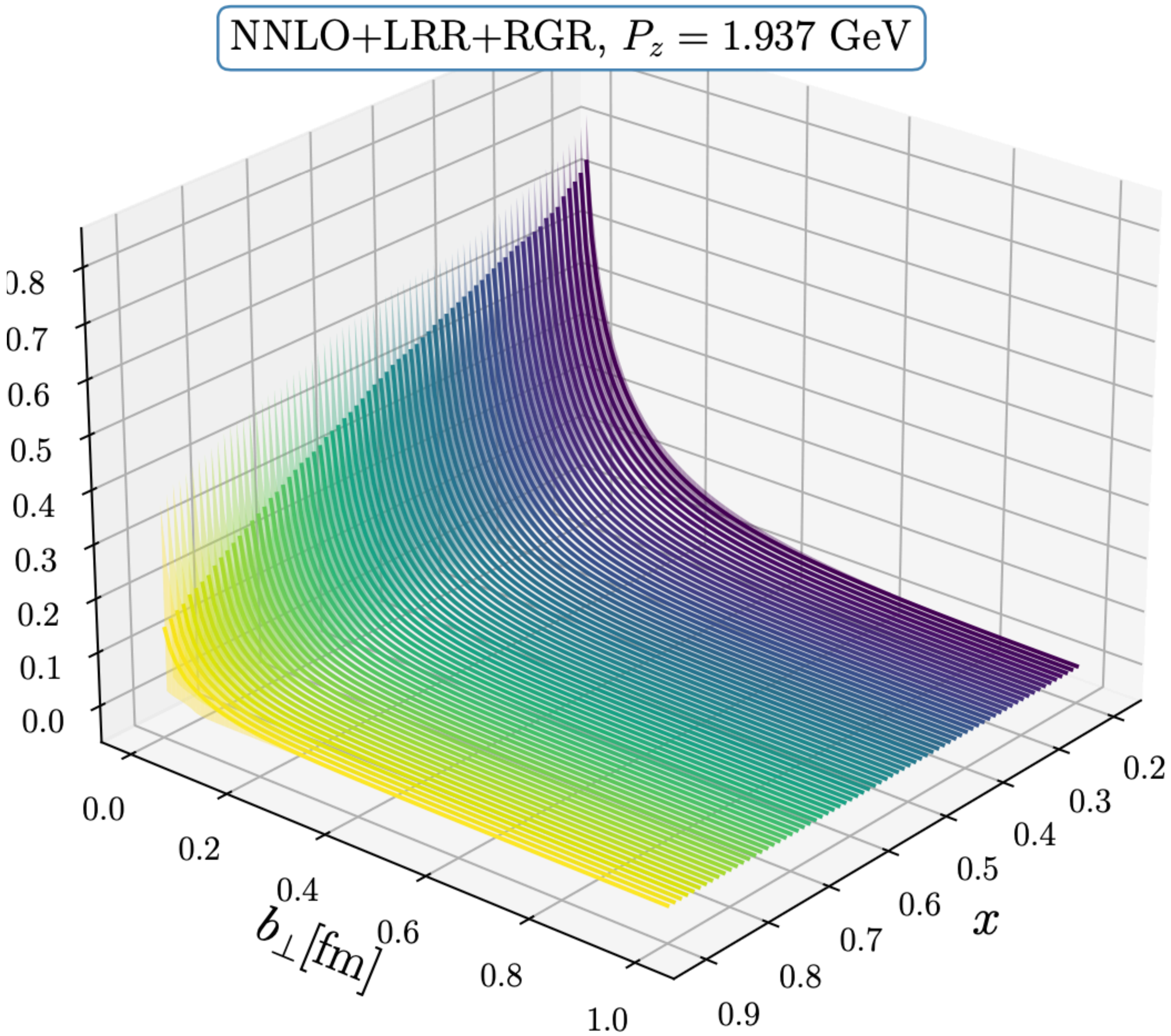


- The t dependence can be parametrized through:

$$H(x,t) = \frac{H(x,0)}{1 - t/M^2(x)}$$

3D imaging of pion from GPDs





$$\langle r^2 \rangle(x) = 6\frac{dH(x,t)}{dt}\Big|_{-t=0}$$

$$q(x, \mathbf{b}_\perp) = \int \frac{d^2\boldsymbol{\Delta}_\perp}{(2\pi)^2} H(x, \xi = 0, \boldsymbol{\Delta}_\perp^2) e^{i\mathbf{b}_\perp \cdot \boldsymbol{\Delta}_\perp}$$

Summary

- We study the pion valence quark GPDs at zero skewness under the framework of LaMET.
- We have utilized improved analysis to control the systematics including hybrid renormalization scheme, NNLO matching, leading-renormalon resummation (LRR) and renormalization group resummation (RGR). The power corrections and perturbative convergence are investigated.
- We are able to derive pion GPDs with multiple values of momentum transfer from a calculation in an asymmetric frame. From there we parametrize the t dependence, and provide a 3D imaging of pion.

Thanks for your attention!