Perturbative Results Related to the Axial Current and the Axial Anomaly

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Outline

- Introduction and motivation

- Perturbative calculations involving the axial current
  - parton distribution
  - local current and form factor(s)
  - generalized parton distributions

- Summary

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Introduction and Motivation

- Vector current

$$J^\mu(x) = \sum_q \bar{q}(x)\,\gamma^\mu\,q(x)$$

$$\partial_\mu J^\mu(x) = 0$$

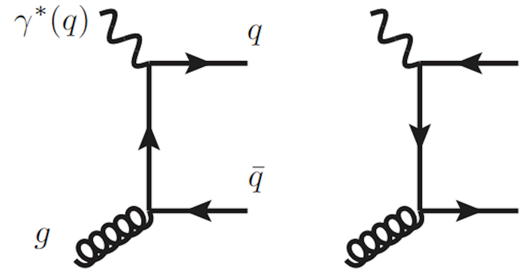
- Axial current

$$J_5^\mu(x) = \sum_q \bar{q}(x)\,\gamma^\mu\gamma_5\,q(x)$$

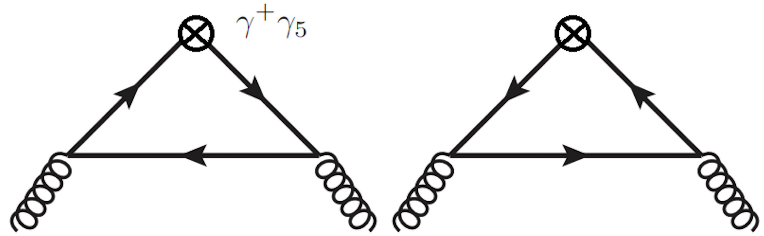
$$\partial_\mu J_5^\mu(x) = \sum_q 2im_q\,\bar{q}(x)\,\gamma_5\,q(x) - \frac{\alpha_\mathrm{s} N_f}{4\pi}\,\mathrm{Tr}\big(F^{\mu\nu}(x)\widetilde{F}_{\mu\nu}(x)\big)$$

  - axial current not conserved due to (i) nonzero fermion mass and (ii) axial anomaly (Adler, 1969 / Bell, Jackiw, 1969 / Adler, Bardeen, 1969 / ...)
  - axial anomaly can be derived, e.g., by evaluating $J_5^\mu(x)$ between gluon states
  - axial anomaly was intensively discussed in hadronic physics soon after discovery of nucleon spin crisis through DIS measurements

- Pioneering work (Altarelli, Ross, 1988 (AR) / Carlitz, Collins, Mueller, 1988 (CCM) / ...)
  - considering process $\gamma^* + g \to q + \bar{q}$



  - extracting leading power-term of $1/q^2$ expansion and integrating upon x
    $\to$ calculation of local axial current



  - overall conclusion: difference between measured $(\Delta\Sigma)$ and "intrinsic" $(\Delta\widetilde{\Sigma})$ quark-spin contributions

$$\Delta\Sigma = \Delta\widetilde{\Sigma} - \frac{\alpha_s N_f}{2\pi}\Delta G$$

    * term proportional to ΔG due to axial anomaly (?)
    * solution of nucleon spin problem ?

- Critique of pioneering papers (Jaffe, Manohar, 1989 / Bodwin, Qiu, 1989 / ...)
  - main concern: result of AR and CCM depends on infrared (IR) regulator
  - this concern, and need for very large ΔG, raised severe doubts

- Recent renewed interest in field
  (Tarasov, Venugopalan, 2021, 2022 (TV) / Bhattacharya, Hatta, Vogelsang, 2022, 2023 (BHV))
  - considered also the x-dependence as opposed to x-integrated results only
  - statements include:
    * need off-forward kinematics to capture physics of axial anomaly
    * GPDs may have more robust connection to anomaly than PDFs
    * anomaly manifests in pole contribution for $t = \Delta^2 \to 0$
    * anomaly pole could challenge factorization (not stated in all papers)
  - TV and BHV agree on certain aspects and disagree on others
  - perturbative results are important part of TV and BHV works

- Our motivations
  - revisit dependence of perturbative calculations on IR regulator
  - what role is played by quark mass ?
  - new insights on the "classic" AR and CCM papers ?

Parton Distribution in Perturbation Theory

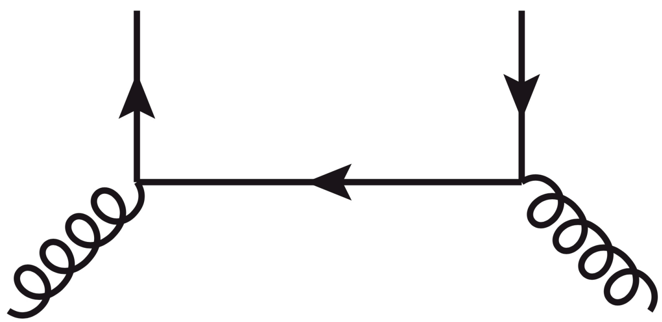
- Definition of PDF

$$\Phi^{[\gamma^+\gamma_5]}_{\lambda\lambda'}(x) = \int \frac{dz^-}{4\pi}\, e^{ik\cdot z}\, \langle g(p,\lambda')|\bar{q}(-\tfrac{z}{2})\,\gamma^+\gamma_5\, q(\tfrac{z}{2})|g(p,\lambda)\rangle\big|_{z^+=0,\vec{z}_\perp=\vec{0}_\perp}$$

$$= -\frac{i}{p^+}\,\varepsilon^{+\,\epsilon\,\epsilon'^*\,p}\; g_1(x)$$

$$g_1(x) = \frac{1}{2}\Big(\Phi^{[\gamma^+\gamma_5]}_{++}(x) - \Phi^{[\gamma^+\gamma_5]}_{--}(x)\Big) \qquad \text{gluon helicity conserved}$$

- Leading-order diagrams



plus graph with reversed arrows on quark lines

  – two diagrams contribute in different regions of x

- Result for $m \neq 0$ and off-shellness $p^2 < 0$, for $0 \leq x \leq 1$ $\quad (\bar{\mu}^2 = 4\pi e^{-\gamma_E} \mu^2)$

$$g_1(x; m, p^2) = \frac{\alpha_s}{4\pi} \left[\left(\frac{1}{\varepsilon} + \ln \frac{\bar{\mu}^2}{m^2 - p^2 x(1-x)} \right) (2x - 1) + \frac{p^2 x(1-x)}{m^2 - p^2 x(1-x)} \right]$$

  - $\int dx\, g_1(x)$ provides total spin contribution

- UV behavior
  - $g_1(x)$ UV-divergent
  - $\int dx\, g_1(x)$ UV-finite, does not depend on UV regulator

- IR behavior
  - $g_1(x)$ IR-divergent, divergence regulated using nonzero m and p^2
  - result well behaved for $m \neq 0$ and $p^2 = 0$
  - result well behaved for $m = 0$ and $p^2 \neq 0$, except for endpoints $x = 0, 1$
  - $\int dx\, g_1(x)$ IR-finite, and does depend on IR regulator

- Integral upon x
  - full result

$$\int_{-1}^{1} dx\, g_1(x;m,p^2) = \frac{\alpha_s}{2\pi}\left[-1 + \int_0^1 dx\, \frac{2m^2(1-x)}{m^2 - p^2 x(1-x)}\right]$$

$$= \frac{\alpha_s}{2\pi}\left[-1 + \frac{2}{\sqrt{\eta\,(\eta+4)}}\,\ln\frac{\sqrt{\eta+4}+\sqrt{\eta}}{\sqrt{\eta+4}-\sqrt{\eta}}\right] \qquad \eta = -\frac{p^2}{m^2} > 0$$

  - after including N_f, full agreement with CCM (1988)
  - expansions for small and large η (dependence on IR regulator)

$$\int_{-1}^{1} dx\, g_1(x;m,p^2) = \frac{\alpha_s}{2\pi}\left[-\frac{\eta}{6} + \mathcal{O}\left(\eta^2\right)\right] \overset{\eta\to 0}{\longrightarrow} 0$$

$$\int_{-1}^{1} dx\, g_1(x;m,p^2) = \frac{\alpha_s}{2\pi}\left[-1 + \frac{2}{\eta}\ln\eta + \mathcal{O}\left(\frac{1}{\eta^2}\right)\right] \overset{\eta\to\infty}{\longrightarrow} -\frac{\alpha_s}{2\pi}$$

  - one can understand origin of

$$\Delta\Sigma = \Delta\widetilde{\Sigma} - \frac{\alpha_s N_f}{2\pi}\Delta G$$

Local Axial Current in Perturbation Theory

- Divergence of axial current
  - matrix element $\quad (P = \frac{1}{2}(p + p'),\ \Delta = p' - p)$

$$\langle g(p', \lambda') \,|\, \partial_\mu J_5^\mu(0) \,|\, g(p, \lambda)\rangle = -2\, \varepsilon^{\epsilon\, \epsilon'^*\, P\, \Delta}\, D(\Delta^2) \;\to \Delta \neq 0 \;\text{ needed}$$

$$D(\Delta^2; m, 0) = \frac{\alpha_s}{2\pi}\left[-1 + \frac{1}{\tau}\ln^2\frac{\sqrt{\tau+4}+\sqrt{\tau}}{\sqrt{\tau+4}-\sqrt{\tau}}\right] \quad \tau = -\frac{\Delta^2}{m^2} > 0$$

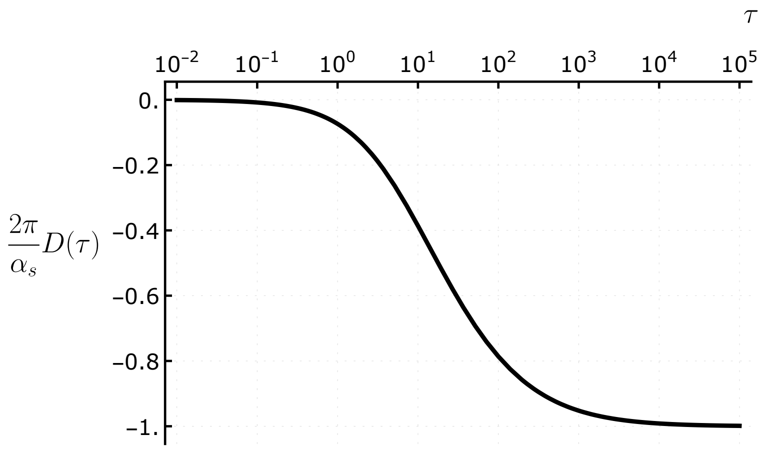
  - expansions for small and large τ

$$D(\Delta^2; m, 0) = \frac{\alpha_s}{2\pi}\left[-\frac{\tau}{12} + \mathcal{O}(\tau^2)\right] \overset{\tau \to 0}{\to} 0$$

$$D(\Delta^2; m, 0) = \frac{\alpha_s}{2\pi}\left[-1 + \frac{1}{\tau}\ln^2\tau + \mathcal{O}\left(\frac{1}{\tau^2}\right)\right] \overset{\tau \to \infty}{\to} -\frac{\alpha_s}{2\pi}$$

  * for $\Delta^2 = 0$, exact cancellation between anomaly term and fermion mass term
  * cancellation whenever m (much) larger than any other scale
  * impact of interplay between mass and anomaly terms for (non-local) axial current ?

– numerics for $D(\Delta^2; m, 0)$



– considering $D(0; m, p^2)$

$$D(0; m, p^2) = \frac{\alpha_s}{2\pi} \left[-1 + \frac{2}{\sqrt{\eta\,(\eta+4)}} \ln \frac{\sqrt{\eta+4}+\sqrt{\eta}}{\sqrt{\eta+4}-\sqrt{\eta}} \right]$$

$$= \int_{-1}^{1} dx\, g_1(x; m, p^2)$$

* result strongly suggests relation between $\int dx\, g_1(x)$ and matrix element of $\partial_\mu J_5^\mu$

- General structure of axial current $\Gamma_5^\mu = \langle g(p',\lambda')|\, J_5^\mu(0)\, |g(p,\lambda)\rangle$
(using symmetry between gluons, Schouten identity, and vector Ward identity)

$$\Gamma_5^\mu\Big|_{\text{real}} = G(\Delta^2; m, 0)\, A_2^\mu$$

$$\Gamma_5^\mu\Big|_{\text{virtual}} = -\frac{4p^2}{\Delta^2 - 4p^2}\, G_1(\Delta^2; m, p^2)\, A_1^\mu + \left(G_2(\Delta^2; m, p^2) + \frac{\Delta^2}{\Delta^2 - 4p^2}\, G_1(\Delta^2; m, p^2) \right) A_2^\mu$$

  - one form factor for on-shell gluons, two form factors for off-shell gluons
  - vectors A_1^μ and A_2^μ

$$A_1^\mu = -2i\, \varepsilon^{\mu\, \epsilon\, \epsilon'^*\, P} \qquad A_2^\mu = \frac{2i}{\Delta^2}\, \Delta^\mu\, \varepsilon^{\epsilon\, \epsilon'^*\, P\, \Delta}$$

    * A_2^μ structure related to axial anomaly
    * A_2^μ does not exhibit a pole for $\Delta^2 \to 0$
    * $A_1^+ \neq 0$ when gluon helicity conserved
    * $A_2^+ \neq 0$ for gluon helicity flip
    * for $\Delta_\perp = 0$, helicity flip forbidden by conservation of angular momentum

- Using (anomalous) axial Ward identity
  - relation

$$i\Delta_\mu \Gamma_5^\mu = \langle g(p', \lambda') \,|\, \partial_\mu J_5^\mu(0) \,|\, g(p, \lambda)\rangle$$

  - on-shell gluons

$$G(\Delta^2; m, 0) = D(\Delta^2; m, 0)$$

    * local current fully determined by matrix element of $\partial_\mu J_5^\mu$
    * angular momentum conservation requires $G(0; m, 0) = \int dx\, g_1(x; m, 0) = 0$
    * $D(0; m, 0) = 0$ due to cancellation between anomaly and quark mass terms
  - off-shell gluons (for $\Delta^2 \to 0$ only)

$$G_1(0; m, p^2) + G_2(0; m, p^2) = G_1(0; m, p^2) = D(0; m, p^2)$$

    * calculation provides $G_2(0; m, p^2) = 0$
    * unambiguous relation between $\int dx\, g_1(x; m, p^2)$ and matrix element of $\partial_\mu J_5^\mu$
  - overall, further insight into "classic" CCM results for on-shell and off-shell gluons

Generalized Parton Distributions in Perturbation Theory

- Definition (for on-shell gluons)

$$F^{[\gamma^+\gamma_5]}_{\lambda\lambda'}(x,\Delta) = \int \frac{dz^-}{4\pi}\, e^{ik\cdot z}\, \langle g(p',\lambda')\,|\,\bar{q}(-\tfrac{z}{2})\,\gamma^+\gamma_5\, q(\tfrac{z}{2})\,|\,g(p,\lambda)\rangle\big|_{z^+=0,\vec{z}_\perp=\vec{0}_\perp}$$
$$= \widetilde{B}_1\, H_1(x,\xi,\Delta^2) + B_2\, H_2(x,\xi,\Delta^2)$$

  - structures $\widetilde{B}_1$ and B_2

$$\widetilde{B}_1 \overset{\Delta\to 0}{\to} \frac{1}{2P^+}\, A_1^+ \qquad\qquad B_2 = \frac{1}{2P^+}\, A_2^+$$

  - addressing the two GPDs

$$H_1(x,\xi,\Delta^2) = \frac{1}{2(1-\xi^2)}\Big(F^{[\gamma^+\gamma_5]}_{++}(x,\Delta) - F^{[\gamma^+\gamma_5]}_{--}(x,\Delta)\Big)$$
$$H_2(x,\xi,\Delta^2) = -\frac{1}{2\xi}\Big(F^{[\gamma^+\gamma_5]}_{+-}(x,\Delta) - F^{[\gamma^+\gamma_5]}_{-+}(x,\Delta)\Big) \;\to\; \text{helicity flip}$$

- Usage of nonzero Δ: (i) IR regulator; (ii) generates new structure
  - if no other IR regulator, one cannot recover forward limit of matrix element

- Forward limit, using (additional) IR regulator

$$\lim_{\Delta \to 0} F^{[\gamma^+ \gamma_5]}_{\lambda\lambda'}(x, \Delta) = \Phi^{[\gamma^+ \gamma_5]}_{\lambda\lambda'}(x)$$

$$H_1(x, 0, 0) = g_1(x)$$

- Comparison with local current (form factor)
  (see also TV 2021, 2022 / BHV 2022, 2023)

$$\int_{-1}^{1} dx\, H_1(x, \xi, \Delta^2) = 0$$

$$\int_{-1}^{1} dx\, H_2(x, \xi, \Delta^2) = G(\Delta^2) \to \text{relation with anomaly}$$

- Our perturbative results for the GPD satisfy required constraints

- Results for arbitrary Δ^2 and arbitrary m $(\kappa = \tau(1-x)^2/(1-\xi^2))$

$$H_1(x,\xi,\Delta^2;m) = \frac{\alpha_s}{4\pi}\begin{cases} \frac{2x-1-\xi^2}{1-\xi^2}\left[\frac{1}{\varepsilon} - \ln\frac{m^2}{\bar{\mu}^2}\right] - 1 \\ + \frac{4+(1+\xi^2)\kappa - 2x(\kappa+2)}{1-\xi^2}\frac{1}{\sqrt{\kappa(\kappa+4)}}\ln\frac{\sqrt{\kappa+4}+\sqrt{\kappa}}{\sqrt{\kappa+4}-\sqrt{\kappa}} & \xi \le x \le 1\,, \\ -\frac{(1-\xi)(\xi+x)}{2\xi(1+\xi)}\left[\frac{1}{\varepsilon} - \ln\frac{m^2}{\bar{\mu}^2}\right] - \frac{\xi+x}{2\xi} \\ -\frac{\xi^2(2-x)-x}{2\xi(1-\xi^2)}\ln\left[1+\frac{(1-\xi)(\xi+x)(\xi^2+\xi(1-x)-x)\kappa}{4\xi^2(1-x)^2}\right] \\ +\frac{4+(1+\xi^2)\kappa-2x(\kappa+2)}{2(1-\xi^2)}\frac{1}{\sqrt{\kappa(\kappa+4)}}\ln\frac{h_+}{h_-} + (x\to -x) & -\xi \le x \le \xi\,, \end{cases}$$

$$H_2(x,\xi,\Delta^2;m) = \frac{\alpha_s}{4\pi}\begin{cases} \frac{2(1-x)}{1-\xi^2}\left[-1+\frac{2}{\sqrt{\kappa(\kappa+4)}}\ln\frac{\sqrt{\kappa+4}+\sqrt{\kappa}}{\sqrt{\kappa+4}-\sqrt{\kappa}}\right] & \xi \le x \le 1\,, \\ \frac{2}{1+\xi}\left[-\frac{\xi+x}{2\xi}+\frac{1-x}{1-\xi}\frac{1}{\sqrt{\kappa(\kappa+4)}}\ln\frac{h_+}{h_-}\right] + (x\to -x) & -\xi \le x \le \xi\,. \end{cases}$$

with the auxiliary functions

$$h_\pm = 4\xi(1-x) \pm (1-\xi)(\xi+x)\,\sqrt{\kappa}\,(\sqrt{\kappa+4} \pm \sqrt{\kappa})\,.$$

- GPD results for $m = 0$ and $\Delta_\perp \neq 0$ $\;(\tau = -\Delta^2/m^2 \to \infty)$
  - we confirm results of BHV (2023) for H_1 and H_2
  - H_1 has logarithmic divergence for $\Delta_\perp \to 0$
  - result for (anomaly-related) H_2

$$H_2(x, \xi, \Delta^2; m) \overset{\tau \to \infty}{\to} \frac{\alpha_s}{4\pi} \begin{cases} -\dfrac{2(1-x)}{1-\xi^2} & \xi \leq x \leq 1 \\[2ex] -\dfrac{2}{1+\xi} & -\xi \leq x \leq \xi \end{cases}$$

    * result independent of Δ^2
    * H_2 for $\Delta^2 \to 0$ not defined
    * result cannot be used to draw conclusion about forward limit
    * angular momentum conservation requires H_2 to vanish in forward limit
    * no anomaly-related "pole" when approaching forward limit

- GPD results for $m \neq 0$ and $\Delta_\perp = 0$ $\;(\tau = -\Delta^2/m^2 \to 0)$

$$H_1(x,\xi,\Delta^2;m) \overset{\tau\to 0}{\to} \frac{\alpha_s}{4\pi}\begin{cases} \dfrac{2x-1-\xi^2}{1-\xi^2}\left[\dfrac{1}{\varepsilon} - \ln\dfrac{m^2}{\bar{\mu}^2} - 1\right] & \xi \leq x \leq 1 \\ -\dfrac{1-\xi}{1+\xi}\left[\dfrac{1}{\varepsilon} - \ln\dfrac{m^2}{\bar{\mu}^2} - 1\right] & -\xi \leq x \leq \xi \end{cases}$$

$$H_2(x,\xi,\Delta^2;m) \overset{\tau\to 0}{\to} \frac{\alpha_s}{4\pi}\begin{cases} -\dfrac{(1-x)^3}{3(1-\xi^2)^2}\,\tau & \xi \leq x \leq 1 \\ -\dfrac{(\xi+x)^2\,(\xi^2+2\xi(1-x)-x)}{12\xi^3(1+\xi)^2}\,\tau + \ldots & -\xi \leq x \leq \xi \end{cases}$$

  - H_1 well behaved in forward limit (quark mass acts as IR regulator)
  - H_2 vanishes in forward limit, which is required by angular momentum conservation
  - for $m \neq 0$, meaningful results in forward limit
  - vanishing result for H_2 can be considered the non-local generalization of

$$D(\Delta^2;m,0) = G(\Delta^2;m,0) = \frac{\alpha_s}{2\pi}\left[-\frac{\tau}{12} + \mathcal{O}(\tau^2)\right] \overset{\tau\to 0}{\to} 0$$

Summary

- Potential imprints of chiral anomaly in polarized DIS and DVCS have been discussed in literature
- We confirm "classic" result by CCM (1988) for DIS
- Perturbative results (for PDF, FF, GPDs) depend on IR scheme
- Going from $m = 0$ to $m \neq 0$ qualitatively changes results
- Additional (anomaly-related) contribution arises for $\Delta \neq 0$
  (Tarasov, Venugopalan, 2021, 2022 / Bhattacharya, Hatta, Vogelsang, 2022, 2023)
- Perturbative calculations show that imprints of anomaly can be seen by (i) using off-shell photons and/or (ii) going to off-forward kinematics
- Anomaly-related contribution ($\sim H_2$) has no pole for $\Delta \to 0$ (no challenge for factorization)
- In forward limit, H_2 must vanish due to angular momentum conservation
- For $m \neq 0$, H_2 does vanish (cancellation between anomaly and quark mass terms)