The Axial Form Factor Extracted from Elementary Targets

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Super Bigbite Spectrometer Collaboration Meeting

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Outline

- Neutrino Oscillation Introduction
- Deuterium Bubble Chamber Fits
- Lattice QCD Axial Form Factor Computations
- MINERνA Hydrogen/Deuterium Comparisons
- Conclusions

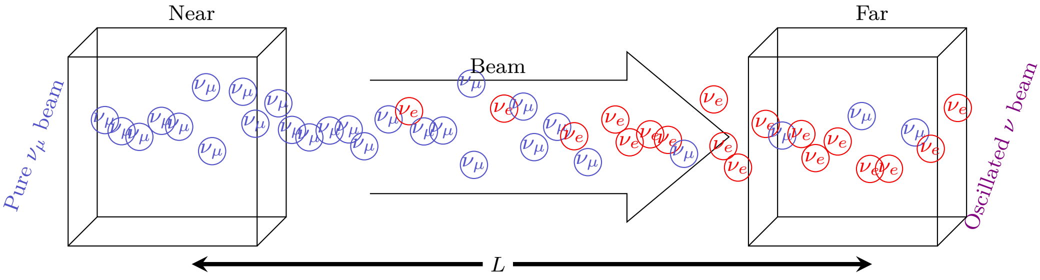
Note: all references in online slides are [hyperlinked]

From this collab meeting, see also:

- [Kordosky - Friday 10:30am]
- [Napolitano - Friday 11:10am]
- [Napolitano - Saturday 8:30am]

Introduction

Neutrino Oscillation



Neutrino oscillation: ν spontaneously change flavor

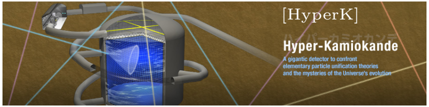
Parameters of oscillation not completely known

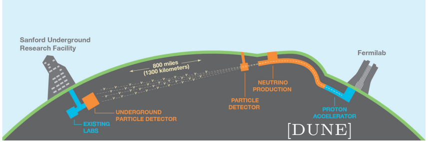
$\implies$ δ_{CP}?, $m_3 > m_{1,2}$?

Upcoming flagship accelerator experiments

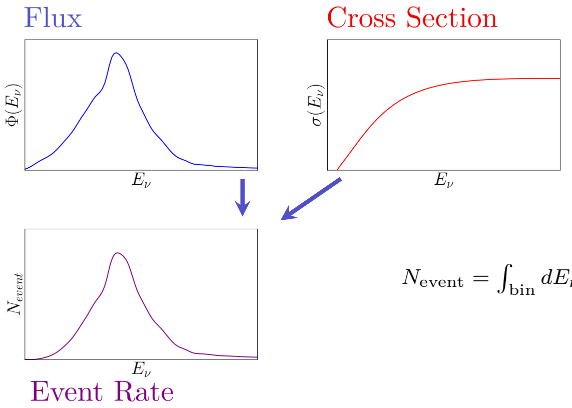
$\implies$ DUNE, HyperK

$\implies$ Measure *oscillation probability*





Measuring Oscillation Probability

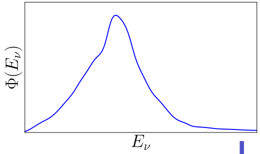


$$N_{\text{event}} = \int_{\text{bin}} dE_\nu \, \Phi(E_\nu)\sigma(E_\nu)$$

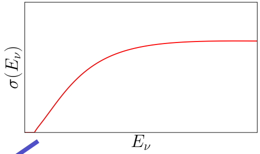
Broad flux & distribution of event E_ν

Measuring Oscillation Probability

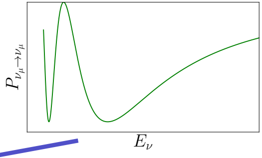
Flux

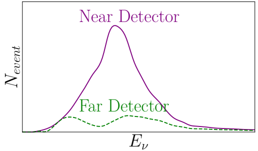


Cross Section



Oscillation Probability





Event Rate

$$N_{\text{event}} = \int_{\text{bin}} dE_\nu \, \Phi(E_\nu) \sigma(E_\nu) \underbrace{\left[\times P(L/E_\nu) \right]}_{\text{far detector only}}$$

$$\left[P(L/E_\nu) \right]_{\text{bin}} \approx \left[\frac{N_{\text{event}}[\text{FAR}]}{N_{\text{event}}[\text{NEAR}]} \right]_{\text{bin}}$$

Broad flux & distribution of event E_ν

far/near $\implies$ oscillation probability, assuming we can get E_ν dependence correct...

Neutrino Event Topologies

"Large" nucleus ($A > 10$)

$\implies$ more nucleons to interact with

$\implies$ target material = detector

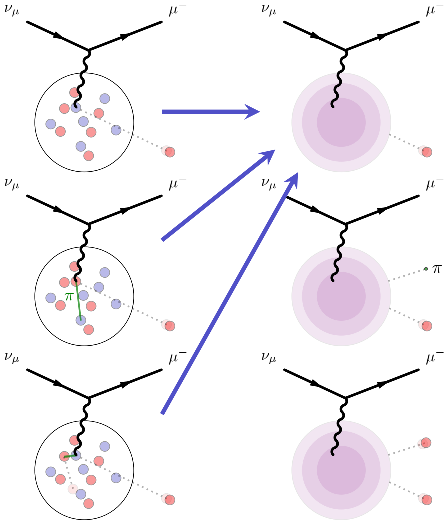
Nuclear environment complicates measurements:

- Many allowed kinematic channels
- Reinteractions within nucleus
- Only final state particles are observable

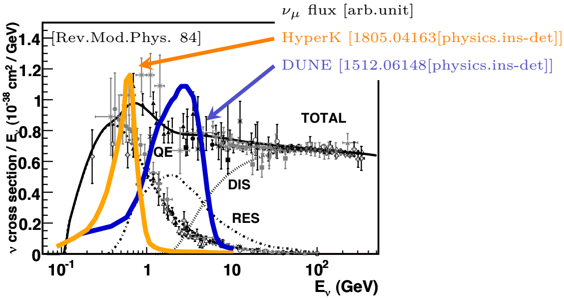
Cannot isolate E_ν event-by-event

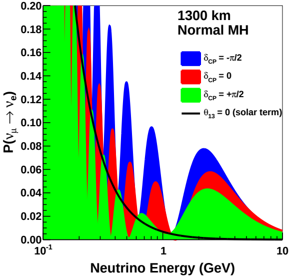
$\implies$ E_ν reconstructed from Monte Carlo distributions

$\implies$ need precise & accurate *nuclear* models built with *nucleon* amplitudes



Neutrino Cross Sections from Elementary Targets





Quasielastic is lowest E_ν, simplest $\implies$ most important

Question:

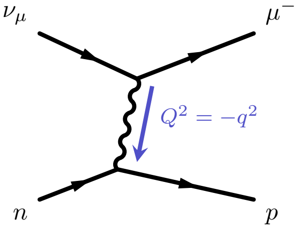
How well do we know free nucleon quasielastic cross section from elementary target sources?

Three main sources:

- Hydrogen scattering
- Deuterium scattering
- Lattice QCD

Deuterium Fits

Quasielastic Scattering



$$\mathcal{M}_{\mathrm{CCQE}} \sim \langle \ell | \mathcal{J}^\mu | \nu_\ell \rangle \langle p | \mathcal{J}_\mu | n \rangle \qquad \frac{d\sigma_{CCQE}}{dQ^2} \sim |\mathcal{M}_{\mathrm{CCQE}}|^2$$

$$\langle p_k | (\mathcal{V}_\mu - \mathcal{A}_\mu) | n_p \rangle$$
$$= \bar{u}_k^{(p)} \Big[\gamma_\mu F_1(Q^2) + \frac{i}{2M_N} \sigma_{\mu\nu} q^\nu F_2(Q^2) + \gamma_\mu \gamma_5 F_A(Q^2) + \frac{1}{2M_N} q_\mu \gamma_5 F_P(Q^2) \Big] u_p^{(n)}$$

Simplest topology, lowest E_ν – nucleon scatters elastically with neutrino

Nucleon response described by *form factors*:

- F_1, F_2: vector form factors, constrained by eN scattering
- F_P: "induced pseudoscalar" form factor, subleading in cross section related to F_A by pion pole dominance constraint
- F_A: axial form factor

Nucleon cross section uncertainty dominated by axial form factor F_A

Form Factor Parameterizations

Dipole model ansatz —

$$F_A(Q^2) = g_A \left(1 + \frac{Q^2}{m_A^2}\right)^{-2}$$

- Overconstrained by data
- Inconsistent with QCD
- Motivated by $Q^2 \to \infty$ limit, data restricted to low Q^2

z expansion [Phys.Rev.D 84 (2011)] —

$$F_A(z) = \sum_{k=0}^{\infty} a_k z^k \qquad z(Q^2; t_0, t_{\rm cut}) = \frac{\sqrt{t_{\rm cut} + Q^2} - \sqrt{t_{\rm cut} - t_0}}{\sqrt{t_{\rm cut} + Q^2} + \sqrt{t_{\rm cut} - t_0}} \qquad t_{\rm cut} \leq (3M_\pi)^2$$

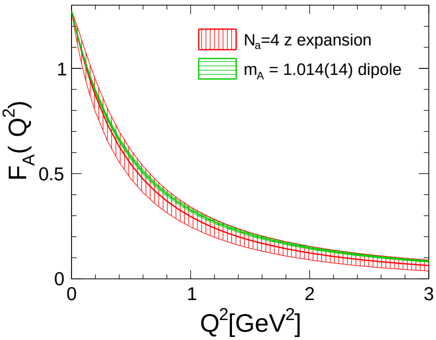
- $|z| < 1$, bounded $|a_k| \implies$ rapid convergence
- Controlled procedure for introducing new parameters
- Optional sum rule constraints regulate large-Q^2 behavior

Deuterium Constraints on F_A

with M. Betancourt, R. Gran, R. Hill

[Phys.Rev.D 93 (2016)]

- Outdated bubble chamber experiments:
  - Total $O(10^3)$ ν_μ QE events
  - Digitized event distributions only
  - Unknown corrections to data
  - Substantial uncertainty in/about flux
  - Deficient deuterium correction
- Dipole overconstrained by data,
  underestimated uncertainty $\times 10$



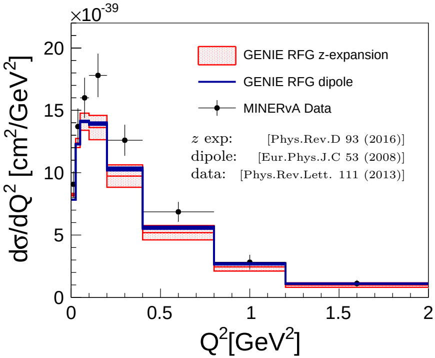
Deuterium Constraints on F_A

with M. Betancourt, R. Gran, R. Hill

[Phys.Rev.D 93 (2016)]

Conclusions about discrepancies
depend on parameterization:

- Dipole –
  $\Longrightarrow$ form factor uncertainty small
  $\Longrightarrow$ discrepancies from *nuclear* only
- z expansion –
  $\Longrightarrow$ form factor uncertainty sizeable
  $\Longrightarrow$ discrepancies could be *nucleon form factors* and/or *nuclear modeling*



Dipole uncertainty unrealistically small – nucleon form factor not as precise as historical claims

LQCD Survey and Implications

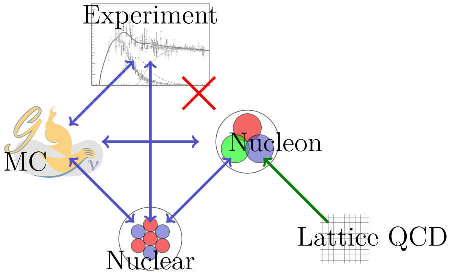
What is Lattice Quantum Chromodynamics (LQCD)?

LQCD is the **only known mathematically rigorous method**
to compute properties of hadrons in nonperturbative QCD

Constructed from **quark and gluon** degrees of freedom

After removing systematic biases, predictions of QCD **(not an approximation!)**

- ✓ Complementary to experiment
- ✓ Controlled nuclear effects
- ✓ Realistic, robust uncertainty estimates
- ✓ Systematically improvable
- ✓ Computers are (relatively) inexpensive

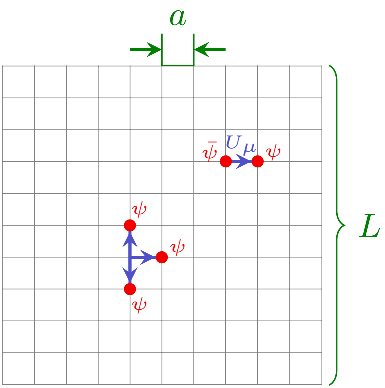


Lattice QCD Formalism

Numerical evaluation of path integral

Parameters:	$am_{(u,d),\text{bare}}$	masses
	$am_{s,\text{bare}}$	
	$\beta = 6/g^2_{\text{bare}}$	coupling
Matching:	e.g. $\frac{M_\pi}{M_\Omega}$, $\frac{M_K}{M_\Omega}$, M_Ω	experiment
	1 per parameter	

Results — first principles predictions from QCD, gluons to all orders

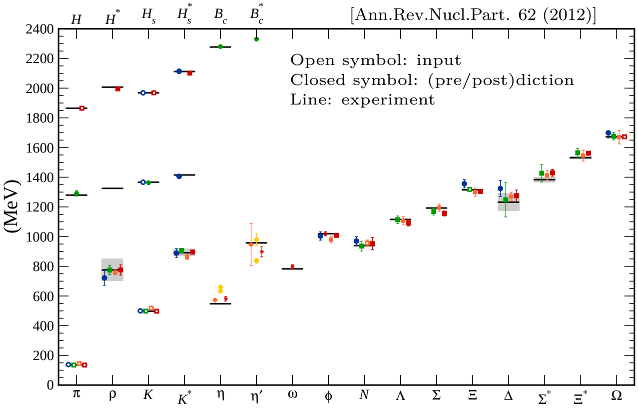


Each gauge ensemble generated with fixed a, L/a, aM_π...

"Complete" error budget $\implies$ extrapolation in a, L, M_π guided by EFT, FVχPT

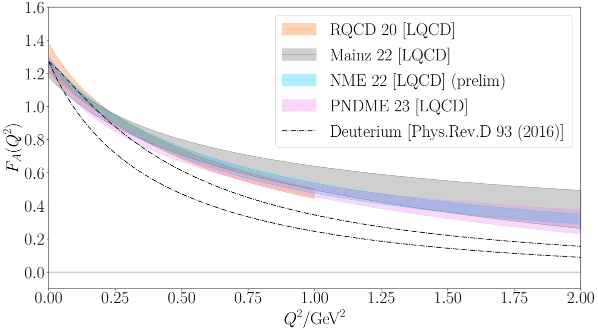
- $a \to 0$ (continuum limit)
- $L \to \infty$ (infinite volume limit)
- $M_\pi \to M_\pi^{\text{phys}}$ (chiral limit)

Successes of Lattice QCD



- $\lesssim 5$ inputs, (very) many predictions
- Heavily constrained by standard model
- Widely used in flavor physics (CKM matrix elements)

Axial Form Factor from LQCD

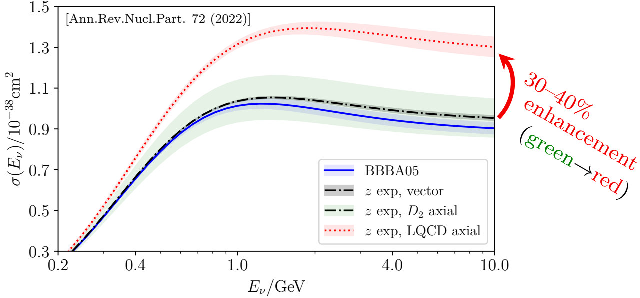


LQCD results maturing:

- Many results, all physical M_π: *independent "data" & different methods*
- Small systematic effects observed (expectation: largest at $Q^2 \to 0$)
- Subject to nontrivial consistency checks from PCAC

LQCD prediction of slow Q^2 falloff, situation unlikely to change drastically

Free Nucleon Cross Section



If LQCD form factor correct, implies big changes —

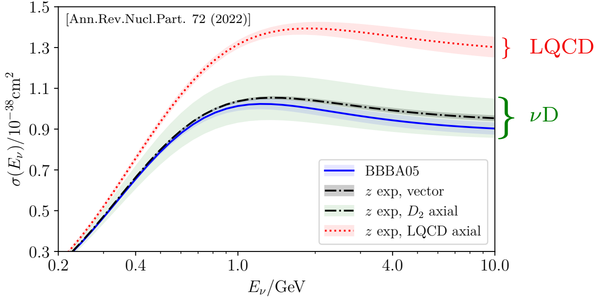
Integrate over Q^2 to get QE cross section $\sigma(E_\nu)$

$\implies$ high-Q^2 discrepancy enhances cross section 30–40%!

$\implies$ recent Monte Carlo tunes prefer ~20% enhancement of QE

[Phys.Rev.D 105 (2022)] [2206.11050 [hep-ph]]

Free Nucleon Cross Section



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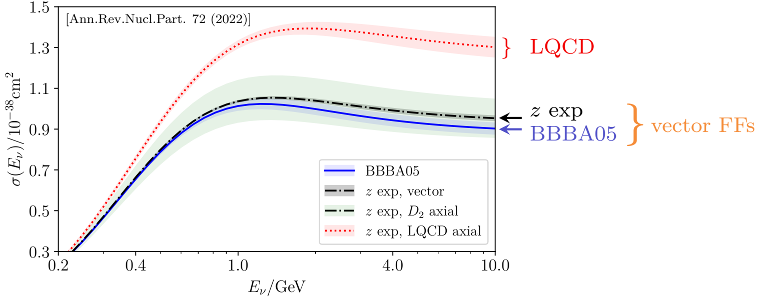
Current-generation LQCD about $\times 2$ more precise than νD scattering

Free Nucleon Cross Section



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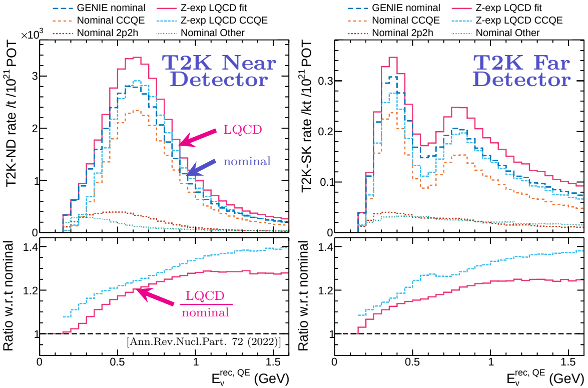
[Phys.Rev.D 105 (2022)] [2206.11050 [hep-ph]]

Current-generation LQCD about $\times 2$ more precise than νD scattering

LQCD precision small enough to be sensitive to vector form factor discrepancies

T2K Implications



Insert new form factor into Monte Carlo event generator

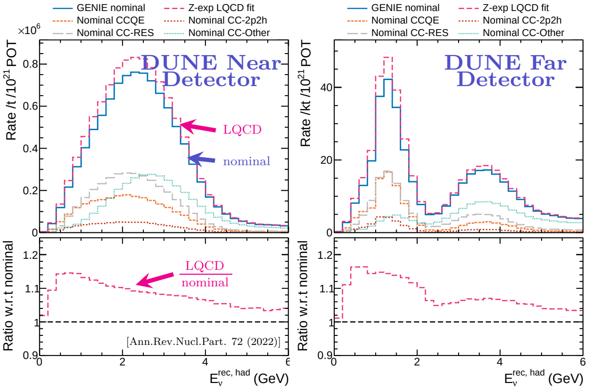
Convolve with realistic flux, nuclear model; compute neutrino event rates

Dashed dark blue (GENIE nominal) vs solid magenta (z exp LQCD fit)

E_ν-dependent event rate changes, different for near/far detectors

$\implies$ Potential source of bias – caution!

DUNE Implications



Insert new form factor into Monte Carlo event generator

Convolve with realistic flux, nuclear model; compute neutrino event rates

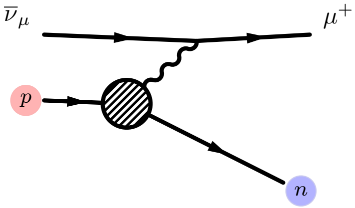
Solid dark blue (GENIE nominal) vs dashed magenta (z exp LQCD fit)

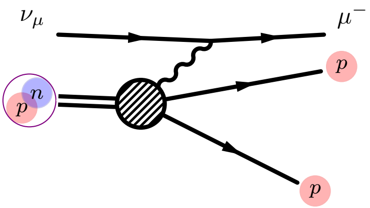
Similar story, different E_ν dependence $\implies$ different potential bias

Moving target $\implies$ other topologies adjusted to soften QE changes

Combined Hydrogen–Deuterium Fits

Hydrogen vs Deuterium



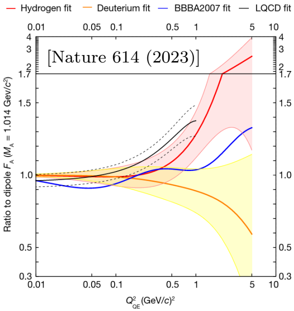


Work done with MINERνA collaboration on published data
Special thanks: Tejin Cai, Kevin McFarland, Miriam Moore

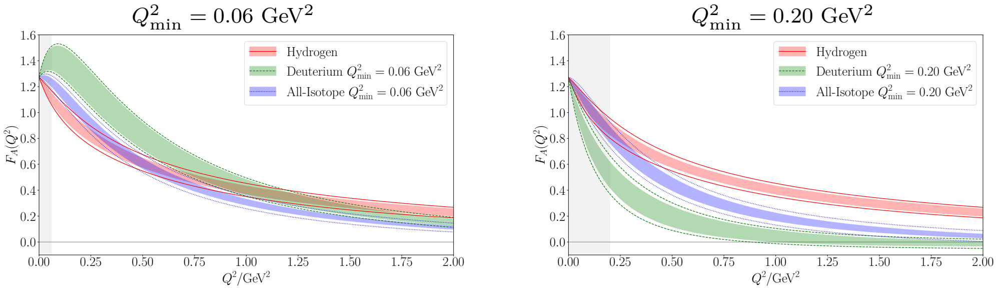
MINERνA result for $\overline{\nu}$–p scattering in plastic scintillator

Test consistency between hydrogen, deuterium fit together

Some visible disagreements between hydrogen, deuterium
$\Longrightarrow$ how does this manifest in combined fit?



Isotope Fit Comparisons

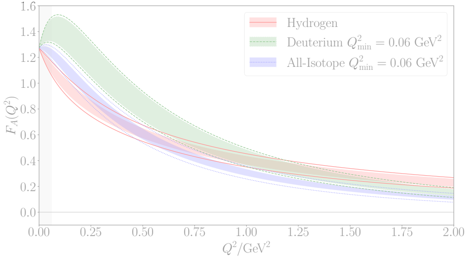


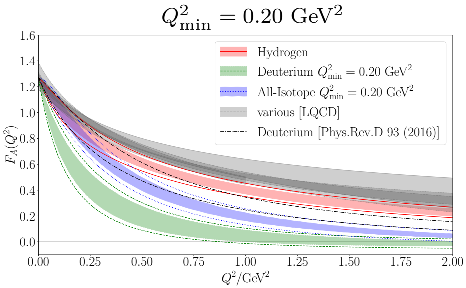
Inner band – uncertainty from axial only
Outer band – uncertainty from axial + vector [Phys.Rev.D 102 (2020)]

Cut low Q^2 in deuterium to avoid systematics (nominal $Q^2_{\text{min}} = 0.20$ GeV^2)

Degeneracy between cross section normalization and axial form factor in deuterium fits
$\Longrightarrow$ strong dependence on Q^2_{min}, suppressed by regularization in [Phys.Rev.D 93 (2016)]

Isotope Fit Comparisons





Tension in fits:

$$\Delta\chi^2 = \chi^2_{\text{H+D}} - \chi^2_{\text{D}} - \chi^2_{\text{H}} \approx 8.8 \quad \Longrightarrow \quad \Delta\chi^2 \;/\; 1 \text{ DoF yields } p\text{-Value} \approx 3.0 \times 10^{-3}$$

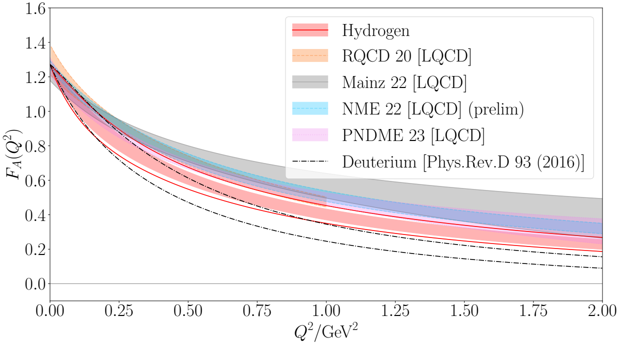
Test compatibility by fixing axial parameters (marginalize deuterium nuisance parameters):

	$\{a_k\}_{\text{D}}$	p_{D}	$\{a_k\}_{\text{H}}$	p_{H}
$\chi^2_{\text{D}}/\text{DoF}_{\text{D}}$	94.9/94	0.45	167.7/96	8.3×10^{-6}
$\chi^2_{\text{H}}/\text{DoF}_{\text{H}}$	23.3/15	0.08	10.0/13	0.69

Deuterium is incompatible with hydrogen, LQCD

Concluding Remarks

Comparison Summary



Quasielastic $F_A(Q^2)$ critical for success of accelerator neutrino oscillation experiments

Ongoing work to combine all sources of axial form factor constraint

Uncertainty historically underestimated by factor of 10 —

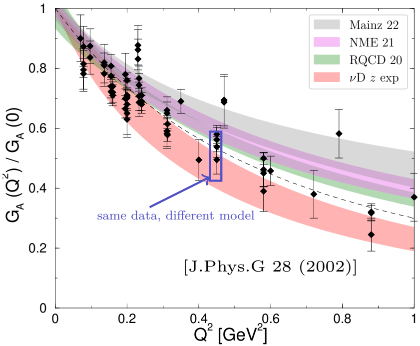
$\implies$ $F_A(Q^2)$ at $Q^2 = 1\ \mathrm{GeV}^2$ known at 20–25% level $\implies$ Tensions at $> 50\%$ level

Potential for high-impact tie-breaking result

Thank you for your attention!

Backup

Electro Pion Production



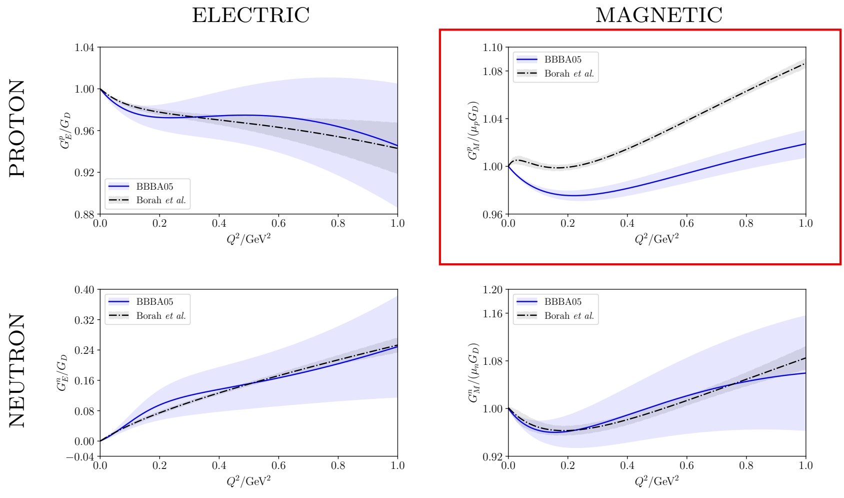
- Large model uncertainty, not included in world averages
- Valid only in $M_\pi \to 0$, $q \to 0$ limits
- Expansion to $O(M_\pi^2, Q^2)$:
  - restricted Q^2 validity
  - lacks shape freedom in Q^2
- Predates Heavy Baryon χPT, no systematic power counting

Modern experiments do not report $F_A(Q^2) \implies$ averages out of date

Possible argument for comparing to r_A^2 from low Q^2; high Q^2 untrustworthy

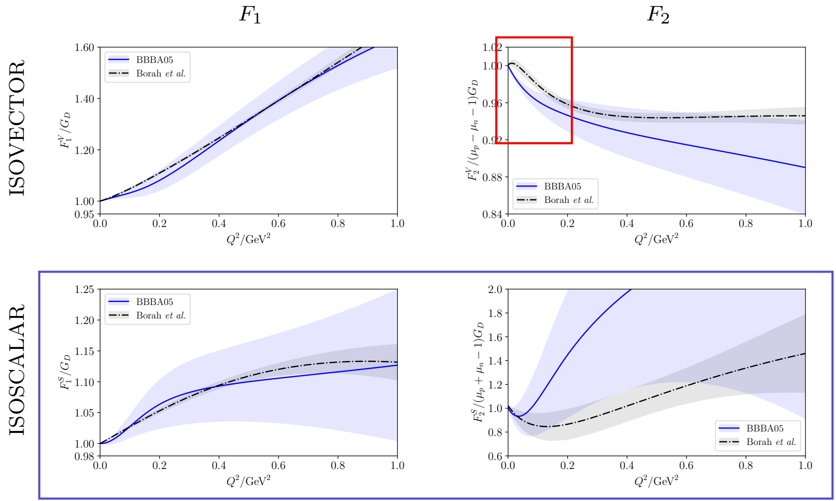
Effort needed to update prediction from photo/electro pion production

Vector Form Factors - Proton/Neutron



Large tension in proton magnetic form factor

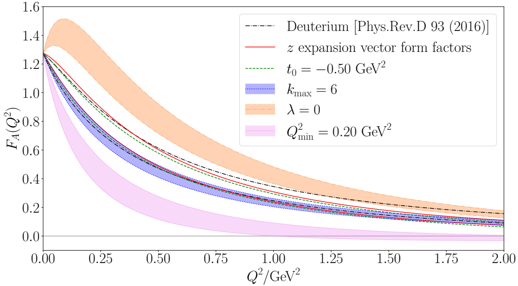
Vector Form Factors - Isospin Symmetric



Uncertain slope of F_2^V

Large uncertainty on isoscalar form factors

Cumulative Updates to Deuterium



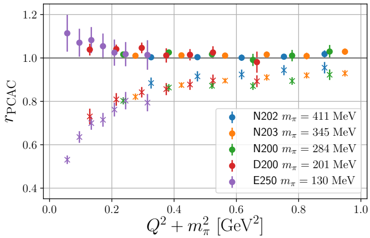
Cumulative changes between fits
$\Longrightarrow$ moving down legend labels, fits include same modifications as fits above them

Fits all 1σ consistent until regularization removed

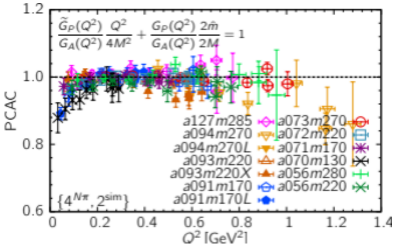
Q^2 cut emphasizes axial form factor + normalization degeneracy

PCAC Checks

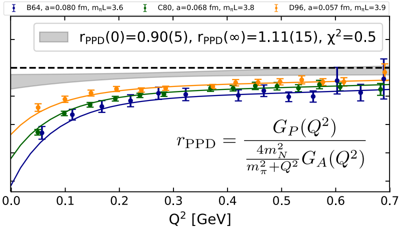
RQCD [JHEP 05 (2020)]



NME [prelim]

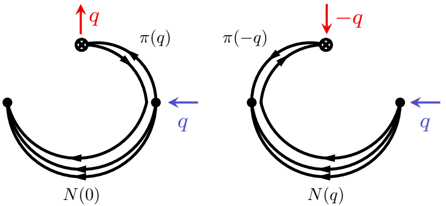


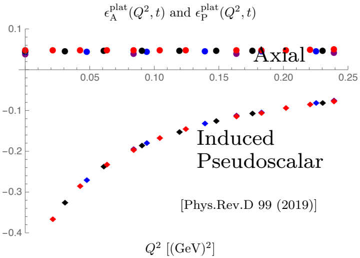
ETMC [prelim]



- Relation btw F_A, F_P, $\tilde{F}_P$ via PCAC
- Contamination in F_A and $\tilde{F}_P$, F_P very different
  $\Longrightarrow$ nontrivial consistency check
  [Phys.Rev.D 99 (2019)]

LQCD Excited States — χPT and $N\pi$





Contamination in $g_A(Q^2)$ primarily from enhanced $N\pi$, mostly from induced pseudoscalar

Correlator fits without axial current not sensitive to $N\pi$ [Phys.Rev.C 105 (2022)] [Phys.Rev.D 105 (2022)]

Alternate fit strategies:

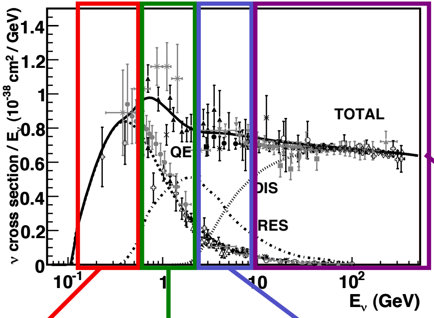
- explicit $N\pi$ operators
- include $\mathcal{A}_4$ (strong $N\pi$ coupling)
- Kinematic constraints ($F_P = 0$)

Prediction from χPT: [Phys.Rev.D 99 (2019)]

First demonstration of $N\pi$: [Phys.Rev.Lett. 124 (2020)]

χPT-inspired fit methods for fitting form factor data
[Phys.Rev.D 105 (2022)] [JHEP 05 (2020) 126]

Energy Regimes



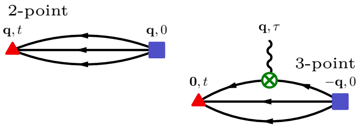
Deep Inelastic Scattering
–Axial quasi/pseudo PDF

Quasielastic
–Nucleon Form Factors
–Full Error Budgets

$N \to \Delta, N \to N^*$
–Transition Matrix Elements
–Multiparticle Operators

"Shallow Inelastic Scattering" (SIS)
–Hadronic Tensor
–Four Point Functions

LQCD Computation Anatomy



Correlation functions in euclidean time:

$\implies e^{-E_n t}$ decay of excited state contribs

2-point function

$\langle \blacktriangle(t) \blacksquare(0) \rangle = \sum_n \langle 0|\blacktriangle|n\rangle\langle n|\blacksquare|0\rangle e^{-E_n t}$

3-point function

$\langle \blacktriangle(t) \otimes(\tau) \blacksquare(0) \rangle = \sum_{mn} \langle 0|\blacktriangle|n\rangle\langle n|\otimes|m\rangle\langle m|\blacksquare|0\rangle e^{-E_n(t-\tau)-E_m\tau}$

Extract masses from 2-point, matrix elements from 3-point

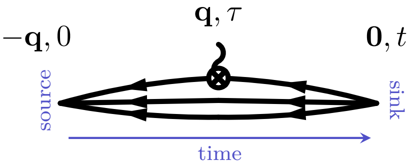
Complications:

- exponentially degrading signal/noise with t
- $n > 0$ contaminations from excited states

Use many source/sink operators ($\blacksquare$, $\blacktriangle$) to suppress excited states:

$$C_{ij}(t) = \sum_n z_{i,n} z_{j,n}^\dagger e^{-E_n t} \quad \implies \quad v^T C(t) v \approx e^{-E_0 t} \quad \text{when} \quad \sum_i v_i^T z_{i,n} \approx \delta_{0,n}$$

Fit Setup



Fit exponential dependence of axial “3-point” functions:

$$C^{3\text{pt}}_{\mathcal{A}_z}(t,\tau,\mathbf{q}) = \langle \mathcal{N}(\mathbf{0},t) \mathcal{A}_z(\mathbf{q},\tau) \overline{\mathcal{N}}(-\mathbf{q},0)\rangle$$

$$\sim \sum_{mn} z_n^{\mathbf{0}}\, A^{\mathbf{q}}_{nm}\, z_m^{\mathbf{q}\dagger}\, e^{-E_n^{\mathbf{0}}(t-\tau)} e^{-E_m^{\mathbf{q}}\tau}$$

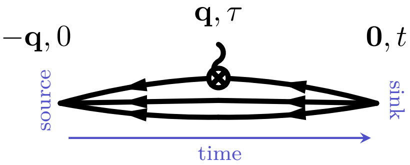
Towers of excited states m, n depend on momenta injected

Current $\mathcal{A}_z$ couples to axial, induced pseudoscalar form factors

Overlaps, energies constrained by “2-point” functions

$$C^{2\text{pt}}(t,\mathbf{q}) = \langle \mathcal{N}(\mathbf{q},t) \overline{\mathcal{N}}(-\mathbf{q},0)\rangle \sim \sum_{m} z_m^{\mathbf{q}}\, z_m^{\mathbf{q}\dagger}\, e^{-E_m^{\mathbf{q}} t}$$

Fit Setup



Plot ratio correlator:

$$\mathcal{R}_{\mathcal{A}_z}(t,\tau,\mathbf{q}) = \frac{C^{3\text{pt}}_{\mathcal{A}_z}(t,\tau,\mathbf{q})}{\sqrt{C^{2\text{pt}}(t-\tau,\mathbf{0})C^{2\text{pt}}(\tau,\mathbf{q})}} \sqrt{\frac{C^{2\text{pt}}(\tau,\mathbf{0})}{C^{2\text{pt}}(t,\mathbf{0})} \frac{C^{2\text{pt}}(t-\tau,\mathbf{q})}{C^{2\text{pt}}(t,\mathbf{q})}}$$

$$\xrightarrow[t-\tau,\tau\to\infty]{} \frac{1}{\sqrt{2E_0^{\mathbf{q}}(E_0^{\mathbf{q}}+M)}} \left[-\frac{q_z^2}{2M} \mathring{F}_P(Q^2) + (E_0^{\mathbf{q}}+M)\mathring{F}_A(Q^2) \right]$$

$Q^2 = |\mathbf{q}|^2 - (E_0^{\mathbf{q}} - M)^2$

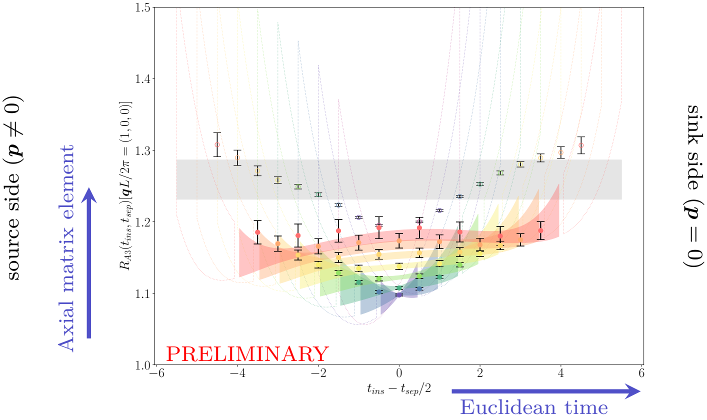
$\mathcal{A}_z$ with $q_z = 0 \quad \Longrightarrow \quad \mathcal{R}_{\mathcal{A}_z}(t,\tau,\mathbf{q}) \to \sqrt{\frac{E_0^{\mathbf{q}}+M}{2E_0^{\mathbf{q}}}} \mathring{g}_A(Q^2)$

$\Longrightarrow$ No induced pseudoscalar

$\Longrightarrow$ Simplified analysis of $\mathring{F}_A(Q^2) = \mathring{g}_A(Q^2)$

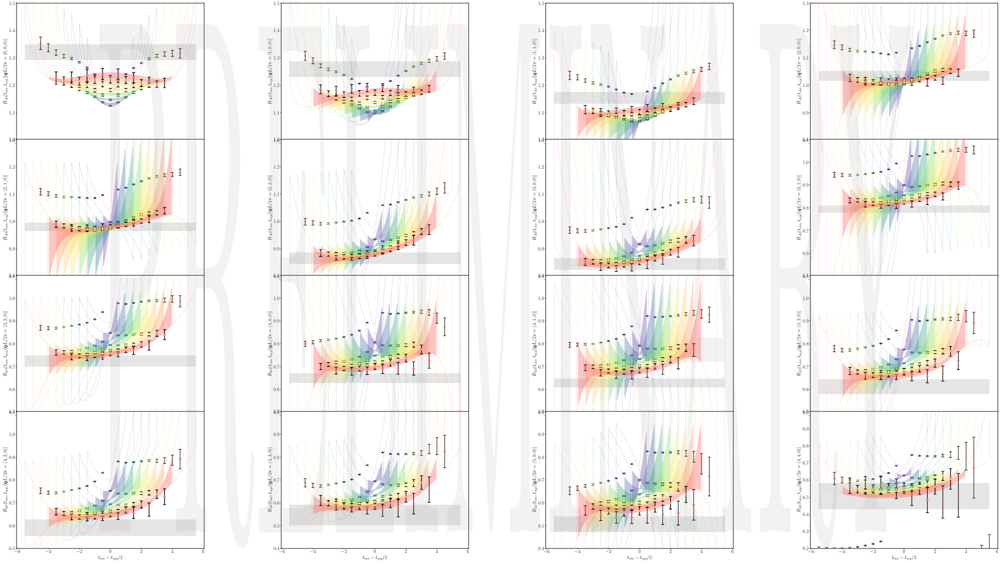
$\Longrightarrow$ `a12m130` ensemble only, $N_{state} = 3$ only

Correlation Function Ratio

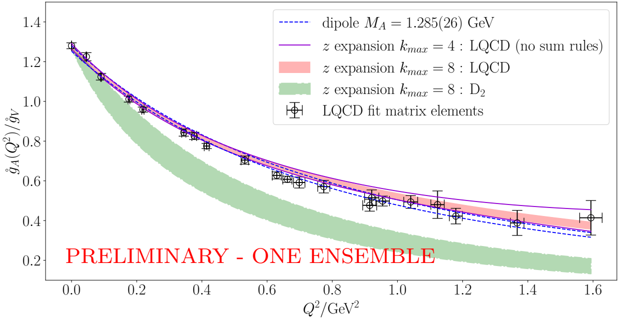


- Color: source-sink separation time
- Colored bands: fit
- Gray band: $\mathring{g}_A$ posterior value
- Curvature: excited state contamination

$\mathring{g}_A(Q^2)$ Correlators



Axial Form Factor Fit



Trend of high-Q^2 enhancement seen in other LQCD results

2–4% LQCD uncertainty vs 10% uncertainty on D_2 result

TODO list:

- $\boldsymbol{q}L/2\pi = (1, 0, 0)$ matrix element larger than expectation
- Deep dive into excited states systematics, prior dependence
- More momenta, $q_z \neq 0$, full set of ensembles