

Application of novel computational techniques for the determination of the gravitational form factors and mechanical properties of the proton

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Co-PI: Volker Burkert, David Richard

Collaborators : Herve Dutrieux, Daniel Lersch , Pierre Chatagnon

Hall A/C

July 26th 2024

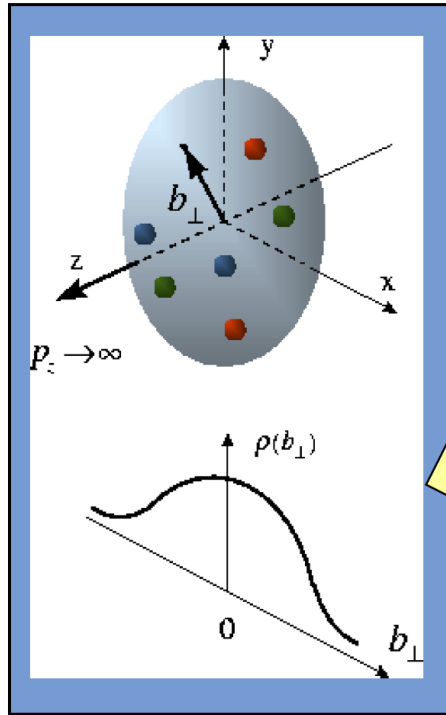
Outline

- Generalized Form Factor
- Report year 1
- Milestones for year 2
- Timeline
- Labor and expertise
- Budget request
- Conclusion

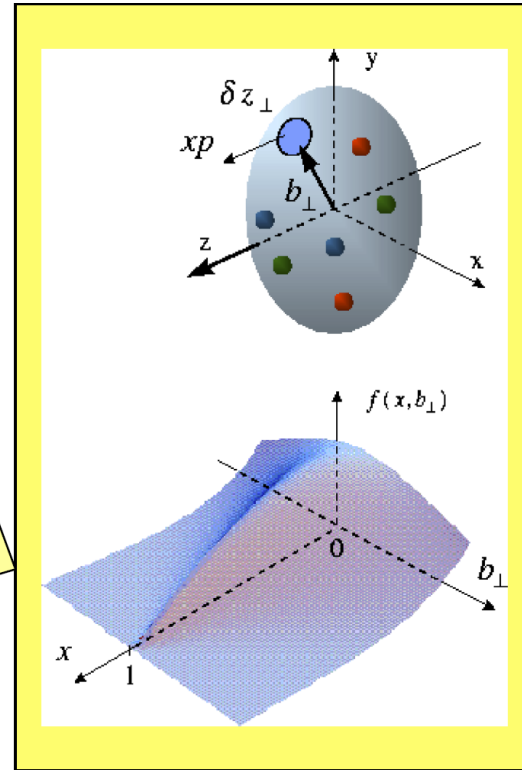
GPDs and 3D Imaging of Nucleons

X. Ji, D. Mueller, A. Radyushkin; 1994-1997

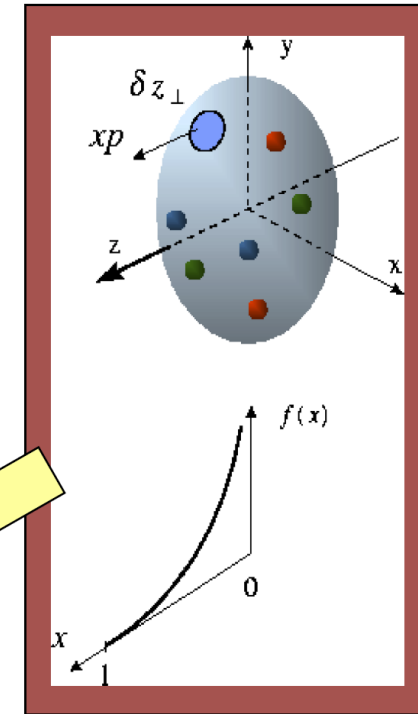
A.V. Belitsky



Elastic form factors \rightarrow
Transverse charge & current
 densities $F_1(t), F_2(t)$.



Deeply exclusive processes \rightarrow GPD's
 and **(2+1)D** images in transverse space
 and longitudinal momentum.
 4 chiral even GPDs $H, E, \tilde{H}, \tilde{E}(x, \xi, t)$



DIS structure functions
 \rightarrow **Longitudinal** parton
 momentum & helicity
 densities, $F_2(x), g_1(x)$.

Informations from GPDs

- Orbital momentum of quarks Ji's sum rule

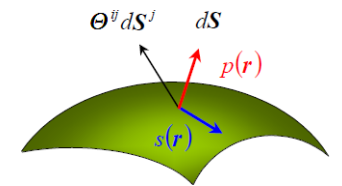
$$J^q = \frac{1}{2} [A^q(0) + B^q(0)] = \frac{1}{2} \Delta \Sigma^q + L^q$$

$$\int_{-1}^1 dx [H^q(x, \xi, t) + E^q(x, \xi, t)] = A_q(t) + B_q(t)$$

- Access to nucleon pressure

[\[hep-ph/0504030v3\]](https://arxiv.org/abs/hep-ph/0504030v3) Unraveling hadron structure with generalized parton distributions (arxiv.org) p64

$$\langle p_2 | \Theta^{a, \mu\nu} | p_1 \rangle = \frac{1}{2} \left(H^a(\Delta^2) p^{\{\mu} h^{\nu\}} + E^a(\Delta^2) p^{\{\mu} e^{\nu\}} + D^a(\Delta^2) \frac{\Delta^\mu \Delta^\nu - g^{\mu\nu} \Delta^2}{2M_N} b \right) \pm \tilde{D}(\Delta^2) M_N g^{\mu\nu} b,$$



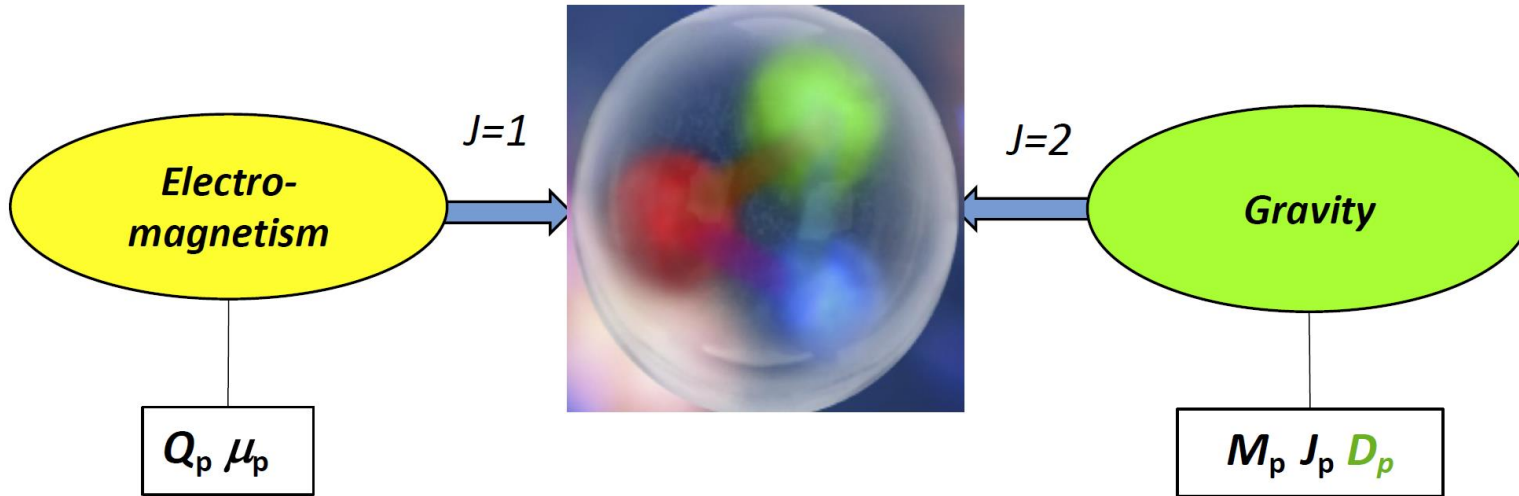
Burkert, Elouadrhiri, and Girod, "The Pressure Distribution inside the Proton." Nature 2018

Kumericki, Mueller, and Schafer, "Studying Deeply Virtual Compton Scattering with Neural Networks." arXiv:1110.3798

Dutrieux et al., "Phenomenological Assessment of Proton Mechanical Properties from Deeply Virtual Compton Scattering."

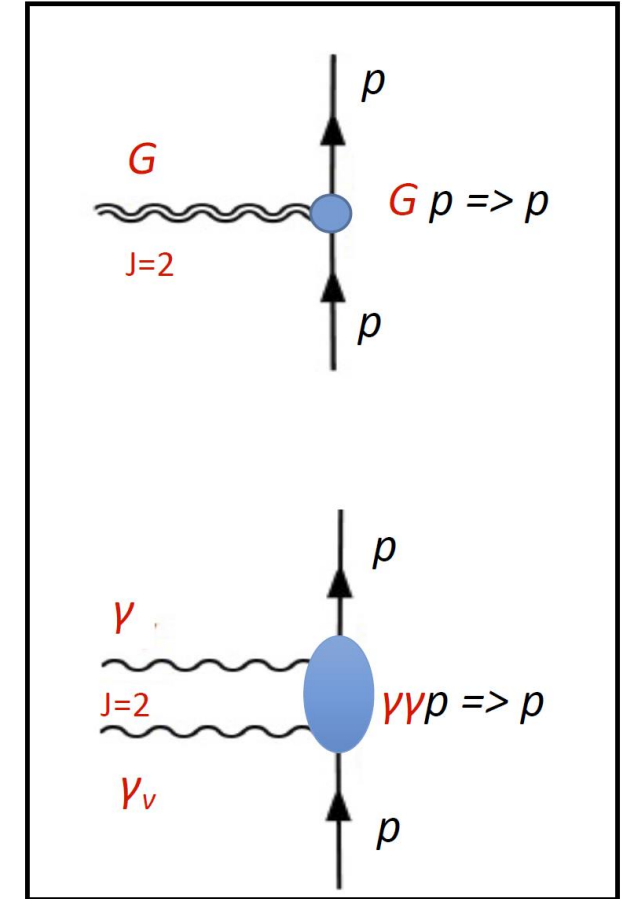
<http://arxiv.org/abs/2101.03855>

Probing structural properties of the proton



- DVCS as substitute that mimics graviton-proton interaction.

Misner, C.W.; K. S. Thorne; J. A. Wheeler (1973).
Gravitation, W. H. Freeman. ISBN 0-7167-0344-0.



The 2γ field couples to the **EMT** as gravity does with many orders of magnitude greater strength.

EMT & Gravitational Form Factors (GFF)

Gravitational form factor of the EMTs

$$\begin{aligned}
 \langle p', \vec{s}' | T_a^{\mu\nu}(0) | p, \vec{s} \rangle &= \bar{u}(p', \vec{s}') \left[\boxed{A_a(t)} \frac{P^\mu P^\nu}{M_N} \right. \\
 &+ \boxed{D_a(t)} \frac{\Delta^\mu \Delta^\nu - g^{\mu\nu} \Delta^2}{4M_N} + \bar{C}_a(t) M_N g^{\mu\nu} \quad \text{c.c.} \\
 &\left. + \boxed{J_a(t)} \frac{P^{\{\mu} i\sigma^{\nu\}\lambda} \Delta_\lambda}{M_N} - S_a(t) \frac{P^{[\mu} i\sigma^{\nu]\lambda} \Delta_\lambda}{M_N} \right] u(p, \vec{s}) \\
 &\qquad\qquad\qquad a = q, G
 \end{aligned}$$

2nd Mellin Moments of GPDs

$$\int_{-1}^1 dx x H_q(x, \xi, t) = \boxed{A_q(t)} + \xi^2 \boxed{D_q(t)}$$

$$\int_{-1}^1 dx x E_q(x, \xi, t) = B_q(t) - \xi^2 \boxed{D_q(t)}$$

$$B_q(t) = 2\boxed{J_q(t)} - A_q(t)$$

GPDs → Compton Form Factors (CFFs)

$$\text{Re}\mathcal{H}(\xi, t) + i \text{Im}\mathcal{H}(\xi, t) =$$

$$\sum_q e_q^2 \int_{-1}^1 dx \left[\frac{1}{\xi - x - i\epsilon} - \frac{1}{\xi + x - i\epsilon} \right] H_q(x, \xi, t)$$

Fixed-t subtracted dispersion relation

$$\text{Re}\mathcal{H}(\xi, t) = \boxed{C_{\mathcal{H}}(t)}$$

$$+ \frac{1}{\pi} \text{P.V.} \int_0^1 d\xi' \left[\frac{1}{\xi - \xi'} - \frac{1}{\xi + \xi'} \right] \text{Im}\mathcal{H}(\xi', t)$$

$$\boxed{C_{\mathcal{H}}(t)} = 2 \sum_q e_q^2 \int_{-1}^1 dz \frac{D_{\text{term}}^q(z, t)}{1 - z} \quad z = \xi/x$$

$$D_{\text{term}}^q(z, t) = (1 - z^2) \sum_{\text{odd } n} \boxed{d_n^q(t)} C_n^{3/2}(z) \quad \longrightarrow \quad n=1$$

Progress report year 1

- Updates
 - Postdoc hired : Melany Higuera started May
 - Implementation of Dispersion Relation with Gepard
 - First LQCD results for GFF
 - General database of existing experimental data format defined and script to convert existing data being tested
 - Start writing draft of paper
- Issues
 - Delay of start of postdoc by 2 months because of visa paperwork
 - A bit behind on labor because of availability

Team :

Volker Burkert

Pierre Chatagnon

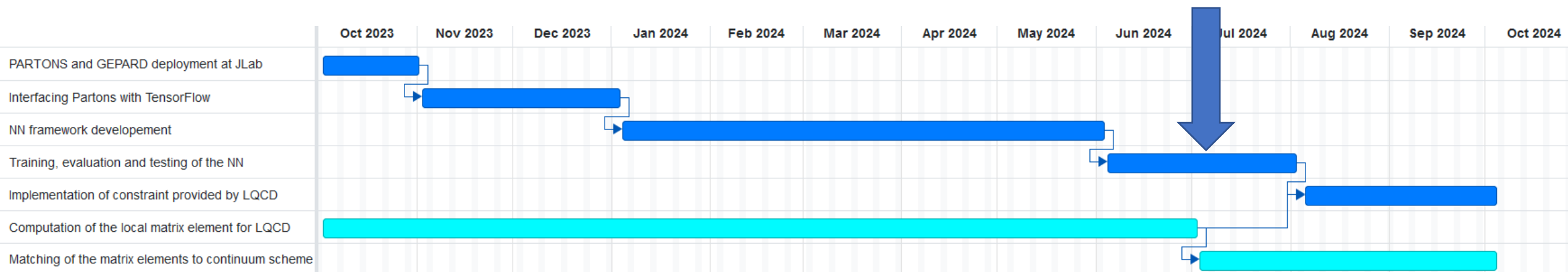
David Richards

Herve Dutrieux (WM)

Melany Isela Higuera

Christopher Chamness (WM)

Kishan Rajput



Milestones year 1

○ AIML

- NN-1 PARTONS and GEPARD deployment at JLab (1 month) Complete
- NN-2 Interfacing PARTONS with Tensorflow using C++ API (2 months) - Revised we decided to focus on the Python GEPARD package using the PyTorch library first after discussion with Kresimir Kumericki and Kishan Rajput (Complete)
- NN-3 Develop NN fitting framework to include arbitrary mathematical constrain function to the model (5 months) - Partially complete - We only implemented the dispersion relations and are in process of implement the custom NN.
- NN-4 Training, evaluation and testing of the NN performances on existing real and pseudo DVCS data (2 months) – Partially complete and ongoing – Fit with 6 GeV data were successfully reproduced. We skipped the production of pseudo DVCS data to focus on including new experimental CLAS and Hall A 11 GeV data in a general database for GEPARD and PARTONS
- NN-5 Implement of constraints provided by LQCD to the NN framework (2 months) (On-going)

○ LQCD

- LQCD-1 Computation of the local matrix element that give rise to the GFFs (9 months)
- LQCD-2 Commence the perturbative and non-perturbative matching of the matrix elements to a continuum scheme (3 months)

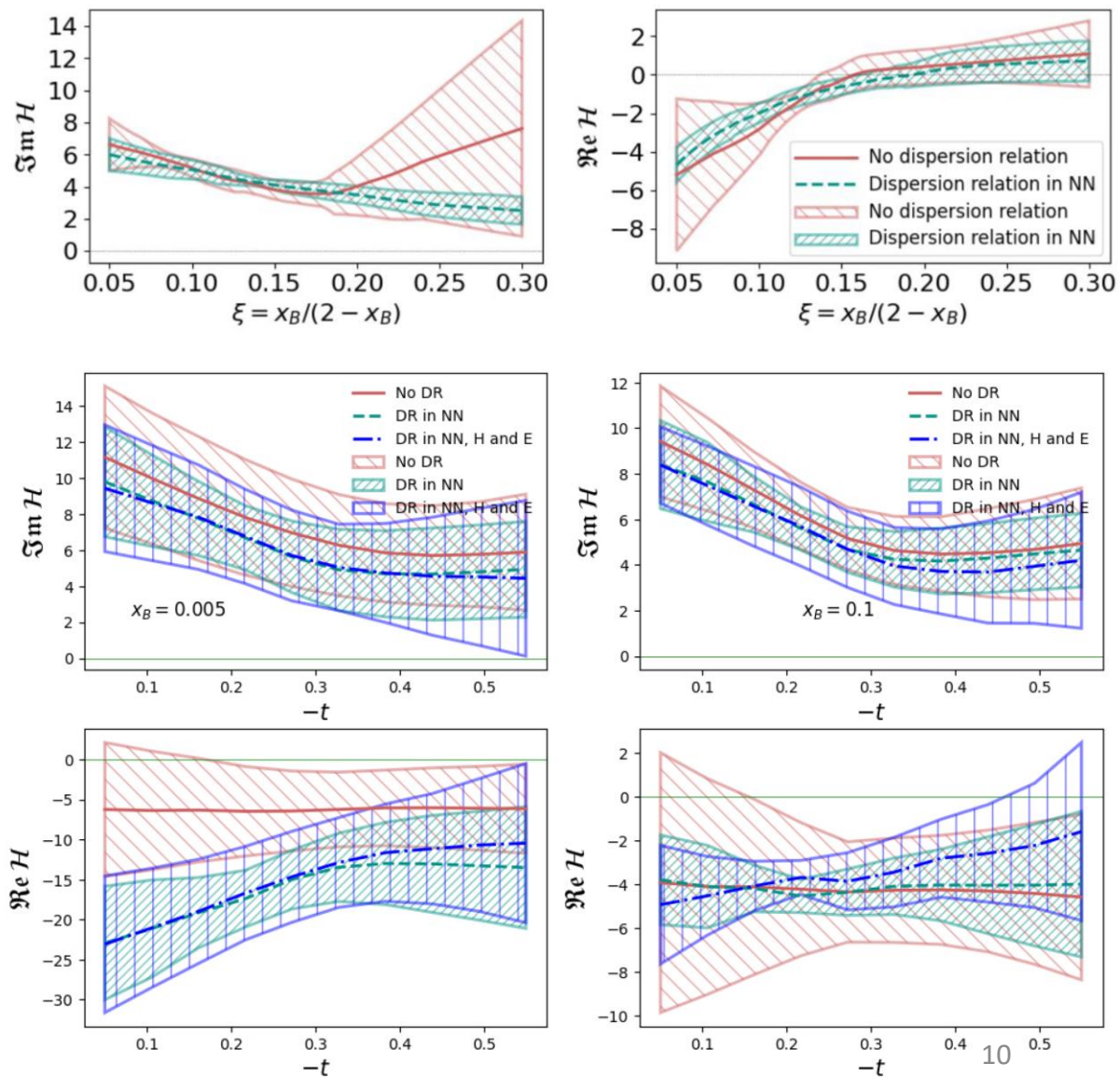
Changes compared to original proposal

- AIML
 - Focused on GEPARD with Pytorch to have results quickly for publishing
 - Format for database for experimental and LQCD data so it can be easily accessible from GEPARD and PARTONS, any other software can use
 - PARTONS implementation started later after upgrade of farm to Alma Linux 9 in June 2024
- LQCD
 - LQCD-1 Computation of the local matrix element that give rise to the GFFs (9 months) : calculation of the isovector GFF from loffe time distribution.
 - LQCD-2 Commence the perturbative and non-perturbative matching of the matrix elements to a continuum scheme (3 months) : not required with new method

Progress AIML/NN

- GEPARD with Pytorch
- CFF fit working tested on 6 GeV data
- Fit with and without Dispersion Relation
- Will redo fit with 11 GeV when data is added to database

Test with Im H and D



Database for exclusive reaction and LQCD data

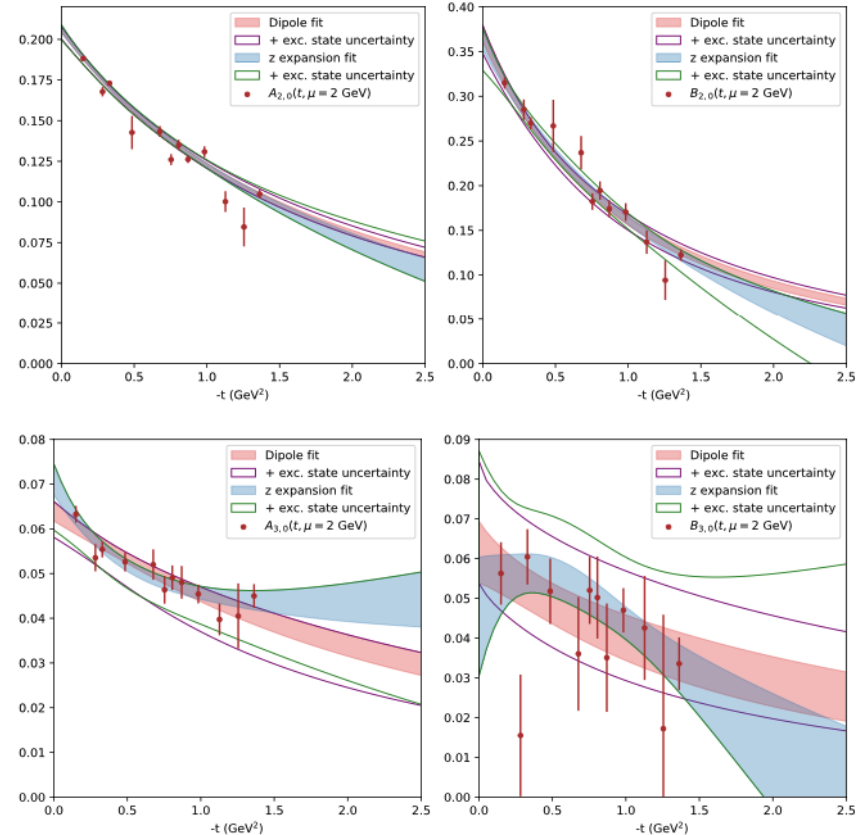
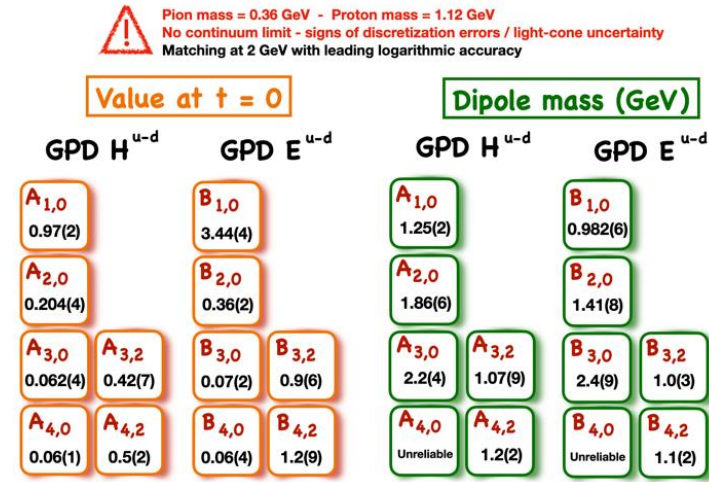
- Most data in text format
- Developed a more structured data format to hold data
- Convert all existing data in this format and save to a database accessible by anyone in the world using Github
- Interface in Python and C++ to access
- After interfacing GEPARD and PARTONS to this database, can carryout fit on exact same data sets
- Other projects can use same interface to access the same data

Database for exclusive reaction and LQCD data

- The format should store the following elements, where “ENUM” denotes predefined constants:
 - Identifier:
 - short uuid number
 - General information:
 - date
 - data type (DVCS, DVMP, latticeQCD, ...) (ENUM)
 - is pseudo-data
 - collaboration
 - reference (publication and/or arxiv number)
 - conditions (beam particle types, beam energies, $m\pi$ for latticeQCD, ...)
 - comment
 - Data group (a given file may define several data groups and it is assumed that data groups are correlated, e.g. an asymmetry extracted as a function of x_B integrating over Q^2 makes one data group; the same asymmetry extracted as a function of Q^2 integrating over x_B makes another data group, etc.):
 - kinematics
 - variableNameA value
 - variableNameB value
 - ...
 - data points
 - observable name (ENUM)
 - value
 - statistical uncertainty
 - systematic uncertainty or uncertainties (also as a covariance matrix)
 - scale uncertainty or uncertainties (also as a covariance matrix)

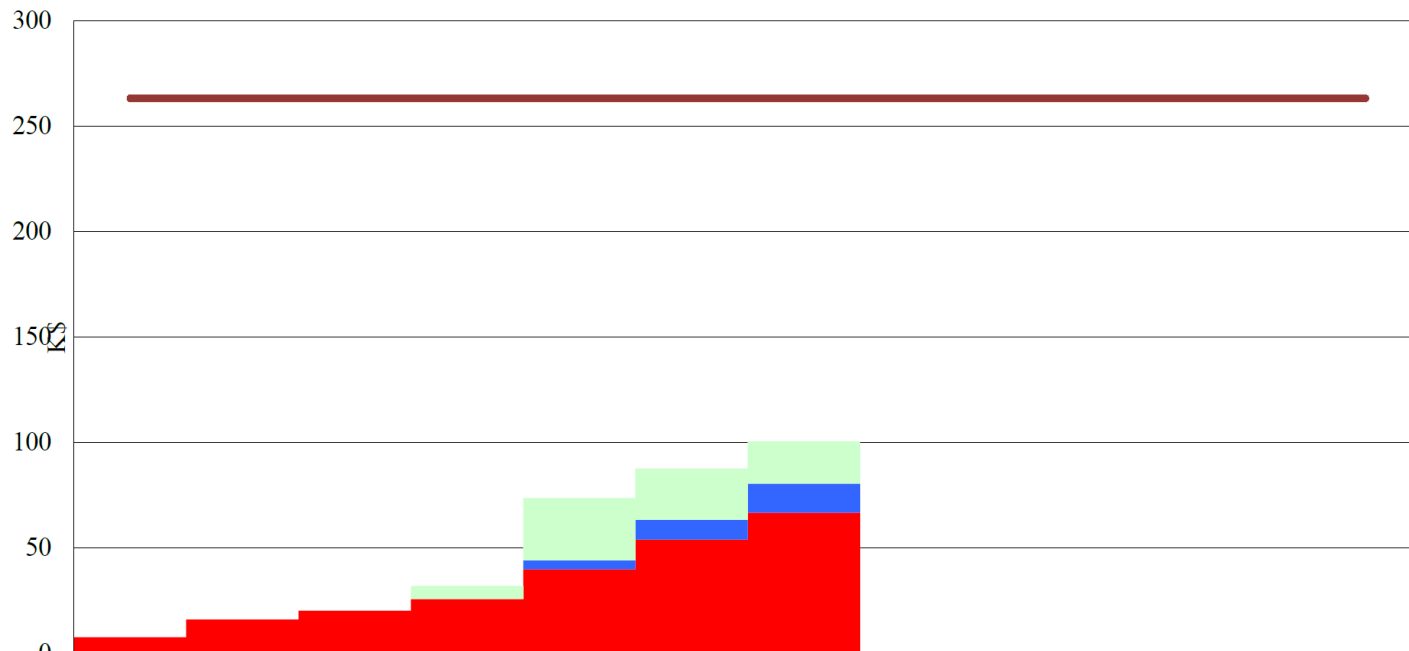
Progress LQCD

- the isovector, unpolarized Generalized Parton Distributions $H^{u-d}(x, \xi, t)$ and $E^{u-d}(x, \xi, t)$, were obtained at a pion mass of around 350 MeV
- The first and second Mellin moments of the skewness-independent GPDs. The fit of the data points, using a dipole or model-independent z expansion is shown as the shaded bands



Financial report

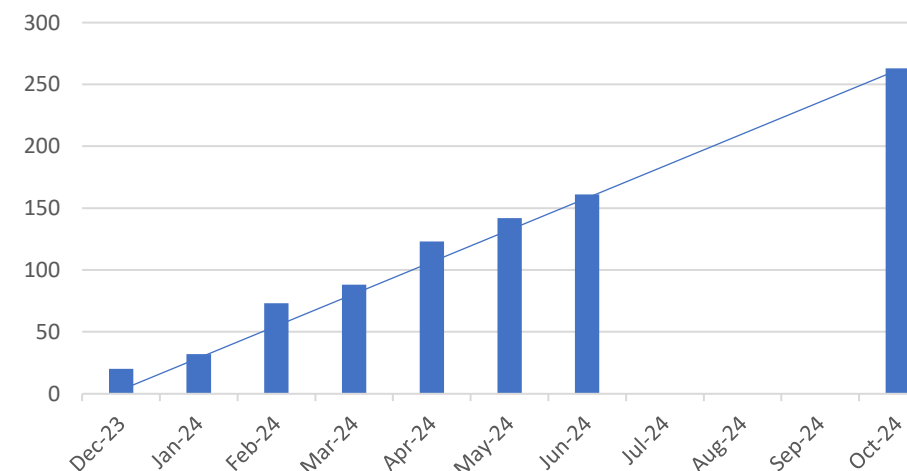
GRAVITATIONAL FORM FACTOR AND MECHANICAL PROPERTIES OF THE PROTON
A. CAMSONNE (LD2412)
WBS 1.03.LD.013 (Loaded \$k)



	Current July	Target	Remaining
Rajput, Kishan	0.1	2.2	2.1
Camsonne, Alexandre	8.1	11	2.9
Chatagnon, Pierre	2.7	6.6	3.9
Higuera Angulo, Melany	7.4	33	25.6
Burkert, Volker	3.7	6.3	2.6
Richards, David	1.1	2.2	1.1

	Oct-23	Nov-23	Dec-23	Jan-24	Feb-24	Mar-24	Apr-24	May-24	Jun-24	Jul-24	Aug-24	Sep-24
YTD Spending	7	16	20	32	73	88	100	0	0	0	0	0
Pending	0	0	0	0	0	0	0	0	0	0	0	0
Open Obligations	0	0	0	6	30	24	20	0	0	0	0	0
Expenses	0	0	0	0	4	9	14	0	0	0	0	0
Labor	7	16	20	26	40	54	67	0	0	0	0	0
Funding	263	263	263	263	263	263	263	263	263	263	263	263
3-Mo Avg Projection												

LDRD GFF



Postdoc started in May instead of January : expect ~ 37 K\$ left over
 Increased Senior staff contribution and started writing of paper

Milestones and deliverables for year 2

– Improved Neural Network based fitting procedure

- NN-25-1 Test the constrained NN fit on existing data including LQCD constraint from 1st year (1 month)
- NN-25-2 Request computing resources for the training of the NN (1 month)
- NN-25-3 Training of the NN and extraction of the GFFs (5 months)
- NN-25-4 Generation of DDVCS and TCS pseudo data with PARTONS and CLAS12 and SoLID Simulation (3 month)
- NN-25-5 Evaluation of the potential improvement on the GFF extraction when using DDVCS and TCS pseudo data at 11 and 22 GeV (1 months)
- NN-25-6 Publication of GFF fitting (2 months)

– LQCD calculation of GFFs

- LQCD-25-1: Calculation of the disconnected diagrams needed for the isosinglet proton distributions (3 months)
- LQCD-25-2: First calculation of isoscalar GFFs of the proton (9 months)

Impact studies

- Capitalize on software developed in year 1
- Procedure
 - Use PARTON framework and EPIC event generator to generate DDVCS and TCS data samples
 - Run data through CLAS12 and SoLID simulation and analysis to generate experimental pseudo data
 - Rerun fitting procedure including those pseudo-data
- First tool to quantitatively evaluate impact of DDVCS and TCS on GFF fitting accuracy. Will be useful for physics case for 12 GeV, Positron and 22 GeV program

Task and expertise

Team Member	Role	Project Contribution	Specific Aims
Alexandre Camsonne	PI	Oversee the project organization and work with the postdoc on GFF fitting method	NN-25-1 to 11
Volker Burkert	Co-PI	Oversee paper writing	NN-25-10
David Richard	Co-PI	Oversee graduate student to carry out lattice GFF computations	LQCD-25-1 and 2
Christopher Chamness	Graduate student	Carry out LQCD GFF computations	LQCD-25-1 and 2
Hervé Dutrieux	Senior personnel	Support with PARTONS framework, fitting with NN and LQCD Computation of GFF	LQCD-25-1 and 2 NN-25-1 to 11
Melany Higuera	Postdoc	Fit of GFF data with new LQCD constraints Impact studies of TCS and DDVCS on GFF fitting	NN-25-1 to 11
Daniel Lersch	Senior Scientist	Help interfacing software with standard AIML packages, add assist with NN training	NN-25-8

Kishan Rajput moving to AIML project for accelerator, Daniel Lersch is new datascience contact for NP and agreed to fill in for support of this proposal
 Pierre Chatagnon got a permanent position and will continue to contribute to the project

Budget year 2

Requested Budget for Effort by Investigator				
Name of Investigator	Role (PI, Co-I, etc.)	FY25 Budget (\$K)	FY25 Effort (% FTE)	Total Effort (%FTE)
Alexandre Camsonne	PI	50.6	20	20
David Richard	Co-PI	14.9	5	5
Volker Burkert	Co-Pi	18.1	5	5
Melany Higuera	Postdoc	103.4	100	100
Daniel Lersch	Senior personnel	9.8	5	5
Equipment				
	Non-capital			
	Capital			
Subcontracts	Hervé Dutrieux (WM)	15.9	15	15
	Christopher Chamness (WM)	24.2	50	50
Materials/ Supplies		10		
Travel		30		

Total : 277.6 K\$ mostly labor

Main change : added travel money for conference and collaboration and new computer

Potential for future funding

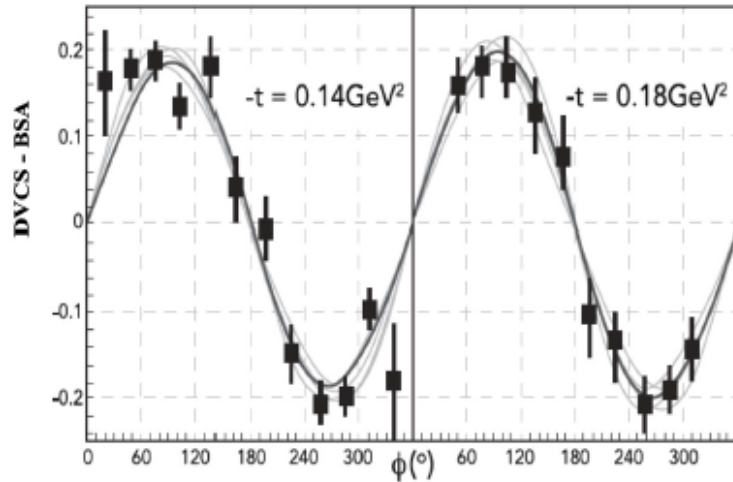
- Proposal to Polish NSF
- Collaboration with other groups
 - PARTONS
 - GEPARD
 - Contact with LQCD collaborations
 - ... other fitting collaboration

Conclusion and outlook

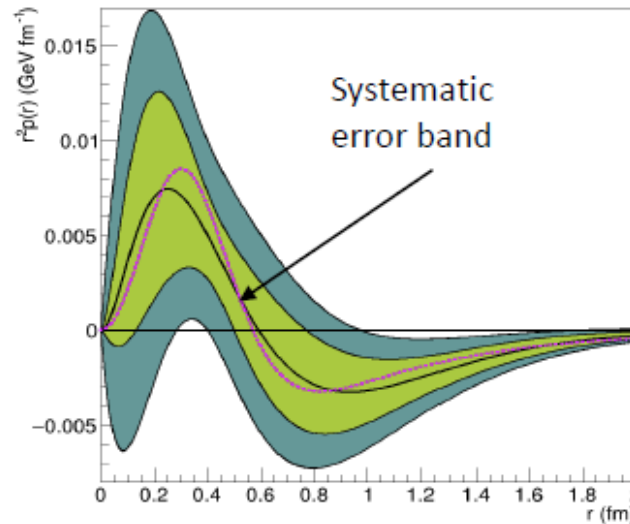
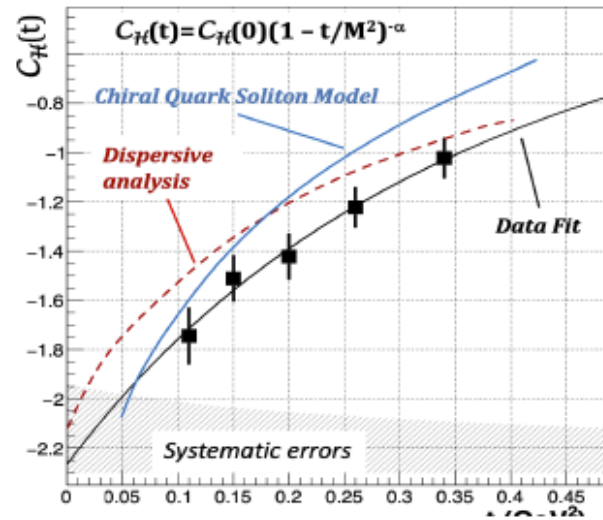
- Good progress for year 1 besides delay hiring postdoc
 - GEPARD and PARTONS software available on farm
 - Close collaboration with GEPARD and PARTONS collaboration
 - Reproduced paper result with GEPARD with Pytorch interfacing and 6 GeV data
 - Database for experimental data format defined, migrating existing data and adding 12 GeV data
 - Moments of GFF computed with lattice QCD at pion mass 350 MeV
 - Starting drafting paper
- Year 2
 - Finalize LQCD calculation
 - Include in global GFF fit
 - Impact studies of JLab 12, positron and 22 measurements for DDVCS and TCS on GFF
- Provide a database for experimental and LQCD data to be used for fitting
- Provide a GFF fitting framework for impact studies including LQCD constraints for more accurate determination of pressure distribution at surface of nucleon

Backup

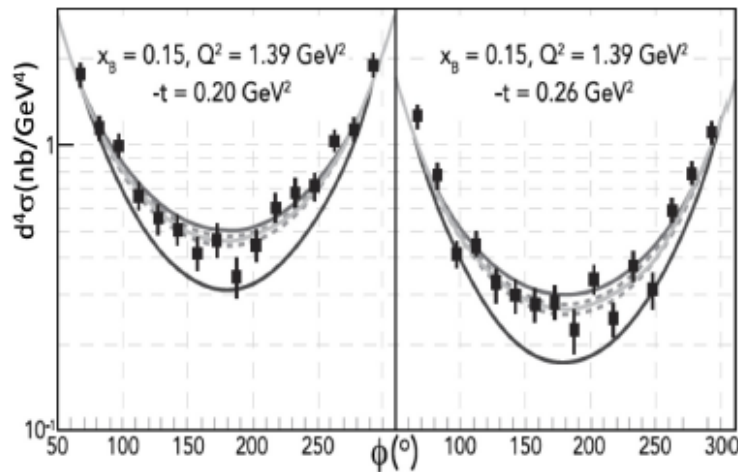
DVCS with CLAS @ 6 GeV



F.X. Girod et al., Phys.Rev.Lett. 100 (2008) 162002

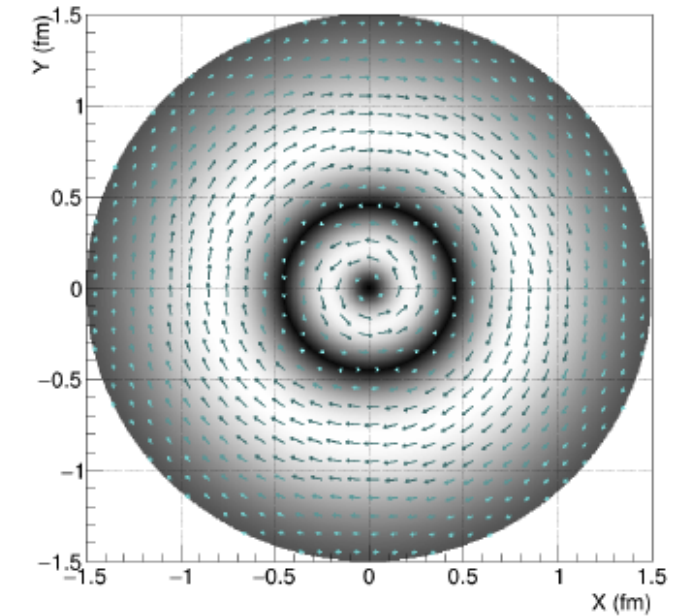


V. B., L. Elouadrhiri, F.X. Girod, Nature 557, 7705, 2018



H.S. Jo et al., Phys.Rev.Lett. 115 (2015) 21, 212003

Tangential stress in Proton

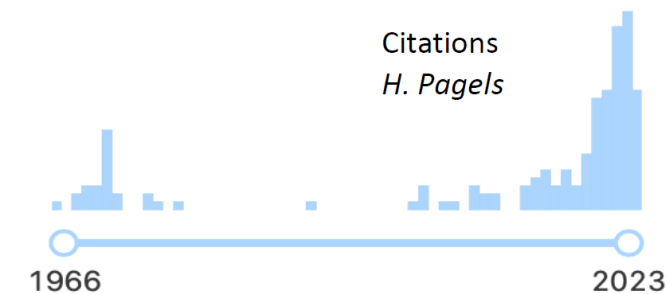


V.B., L. Elouadrhiri, F.X. Girod, C. Lorce, P. Schweitzer, P. Shanahan; to appear in: Review of Modern Physics, 2023

Mechanical properties of particles

- What are mechanical properties of nucleons?
 - The internal distribution of forces and pressure
 - The distribution of mass and energy
 - The distribution of angular momentum
 - The physical size as given by the mechanical radius
- These fundamental properties, in principle, can be probed directly in the short distance interaction of nucleons with gravitons, which is experimentally not realizable.
- Although the theoretical framework had been developed in the 1960's, there was not a known way to experimentally access any of these properties in other ways.

Yu. Kobzarev and L.B. Okun, JETP 16, 5 (1963);
H. Pagels, Phys. Rev. 144 (1966) 1250-1260;
- The development of the generalized parton distributions framework in the mid 1990's, and the discovery of Deeply Virtual Compton Scattering (DVCS) as a means of accessing them, opened the new direction of hadron 3D-imaging.
- It also led to the insight that GPDs can be related to the mechanical properties of nucleons (M. Polyakov, 2003).



Lattice QCD

We can compute moments of GPDs on the lattice thanks to well-defined matrix elements of local twist-2 operators with a various number of derivatives.

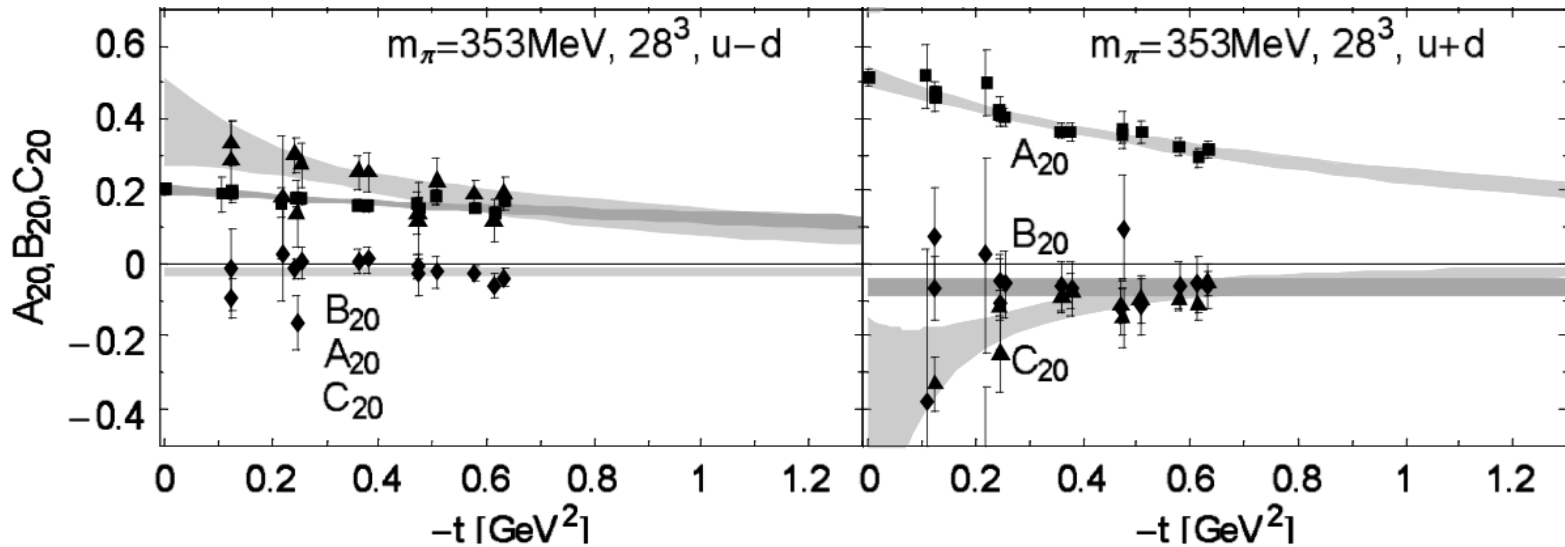
Local twist-2 operator with n-1 derivatives

$$\mathcal{O}_{[\gamma_5]}^{\{\mu_1 \dots \mu_n\}} = \bar{q}(0) \gamma^{\{\mu_1} [\gamma_5] i \overleftrightarrow{D}^{\mu_2} \dots i \overleftrightarrow{D}^{\mu_n\}} q(0)$$

Linked to moment of GPD (n=2)

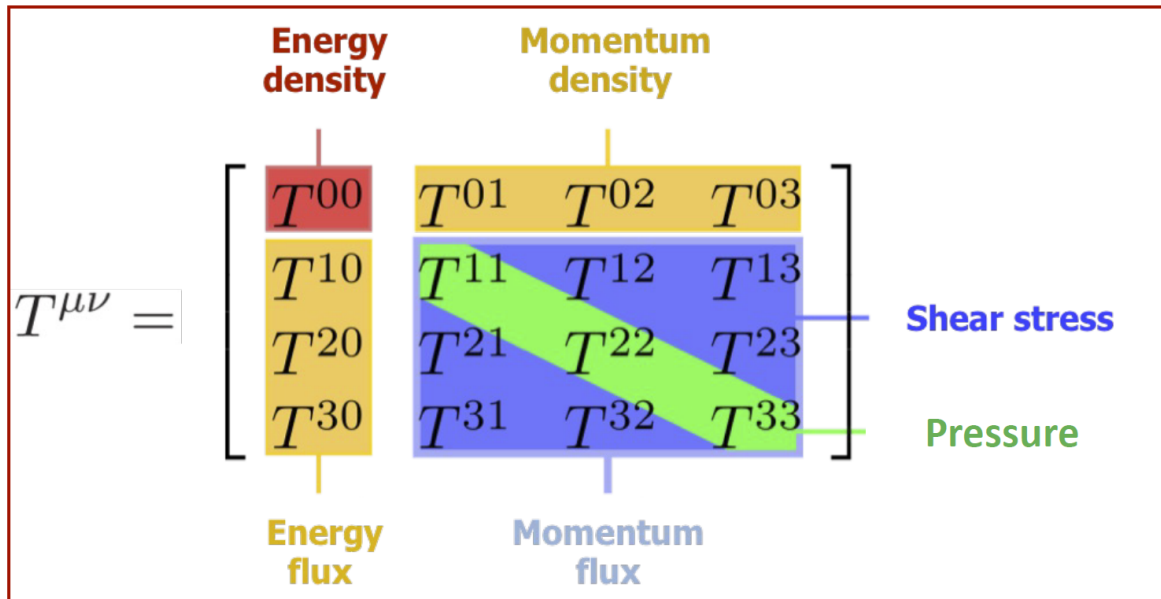
$$\int_{-1}^1 dx x^{n-1} H(x, \xi, t) \stackrel{n=2}{=} A_{20}(t) + (2\xi)^2 C_{20}(t)$$

$$\langle P' | \mathcal{O}^{\{\mu_1 \mu_2\}} | P \rangle = \bar{P}^{\{\mu_1} \langle \langle \gamma^{\mu_2} \rangle \rangle A_{20}(t) + \frac{i}{2m} \bar{P}^{\{\mu_1} \langle \langle \sigma^{\mu_2 \alpha} \rangle \rangle \Delta_\alpha B_{20}(t) + \frac{1}{m} \Delta^{\{\mu_1} \Delta^{\mu_2\}} C_{20}(t)$$



P. Hagler, DR, et al (2008)
 Opportunity to obtain a direct access to the GFFs in a first principle way.
Novel framework with better control of lattice systematics (improved sampling of the lattice + far better control of excited states)

Energy Momentum Tensor $T^{\mu\nu}$



$$T^{ij}(\mathbf{r}) = \left(\frac{r^i r^j}{r^2} - \frac{1}{3} \delta^{ij} \right) \underline{s^Q(r)} + \delta^{ij} \underline{p^Q(r)}$$

$$d_1^Q(t) = 5M_p \int d^3\mathbf{r} \frac{j_2(r\sqrt{-t})}{t} \underline{s^Q(r)}$$

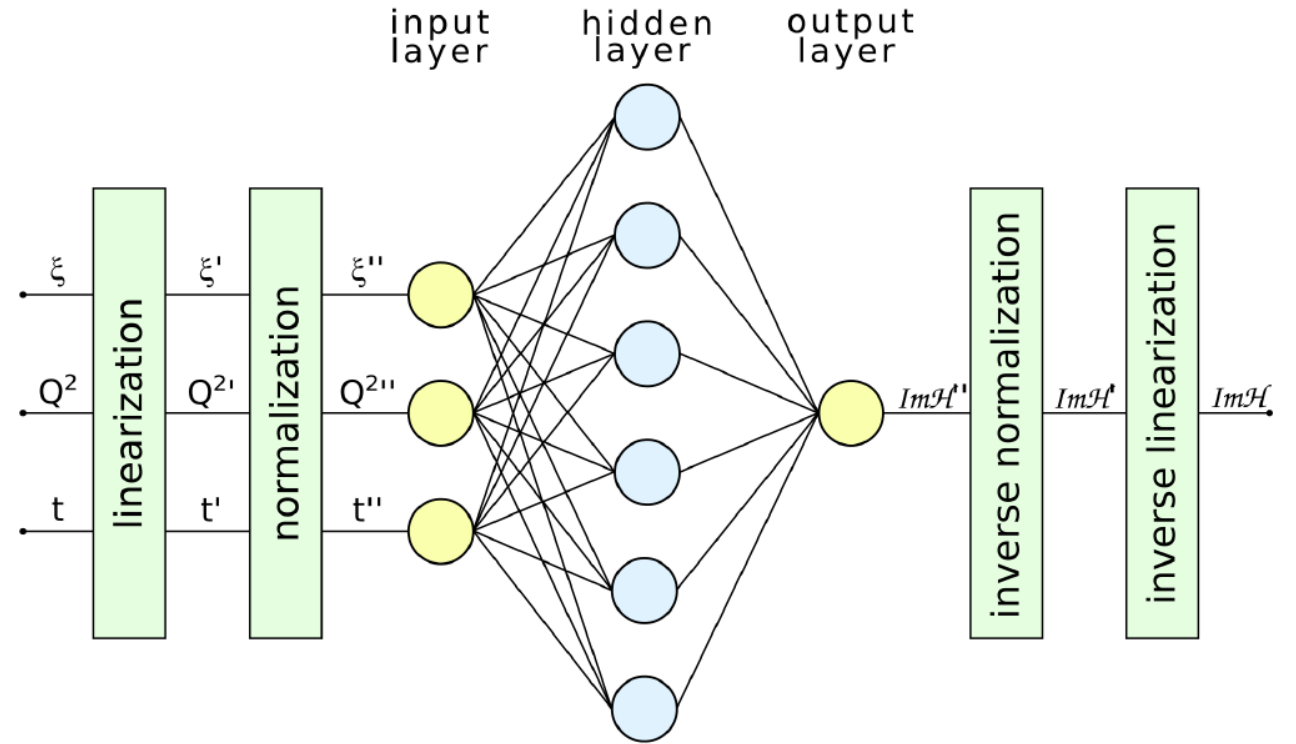
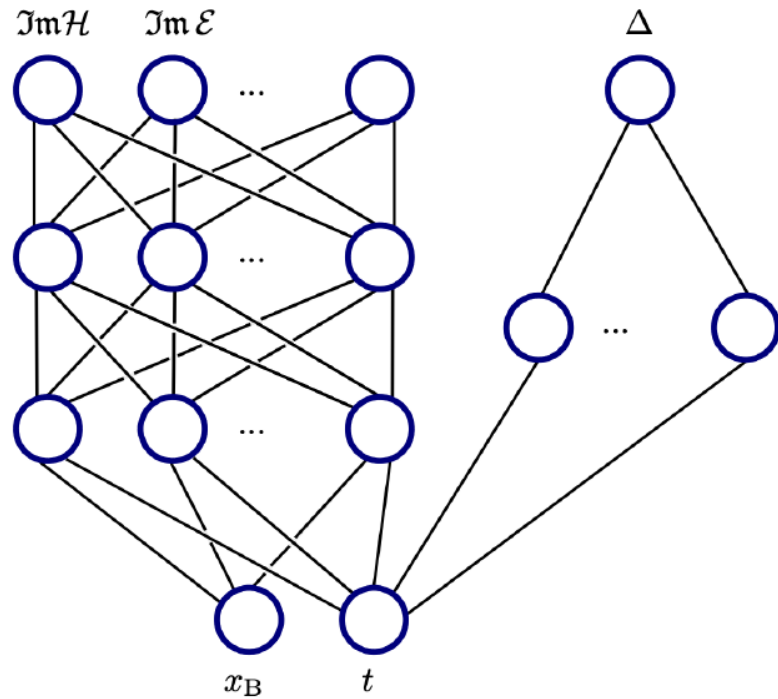
$$d_1^Q(t) = 15M_p \int d^3\mathbf{r} \frac{j_0(r\sqrt{-t})}{2t} \underline{p^Q(r)}$$

j_0, j_2 Spherical Bessel functions

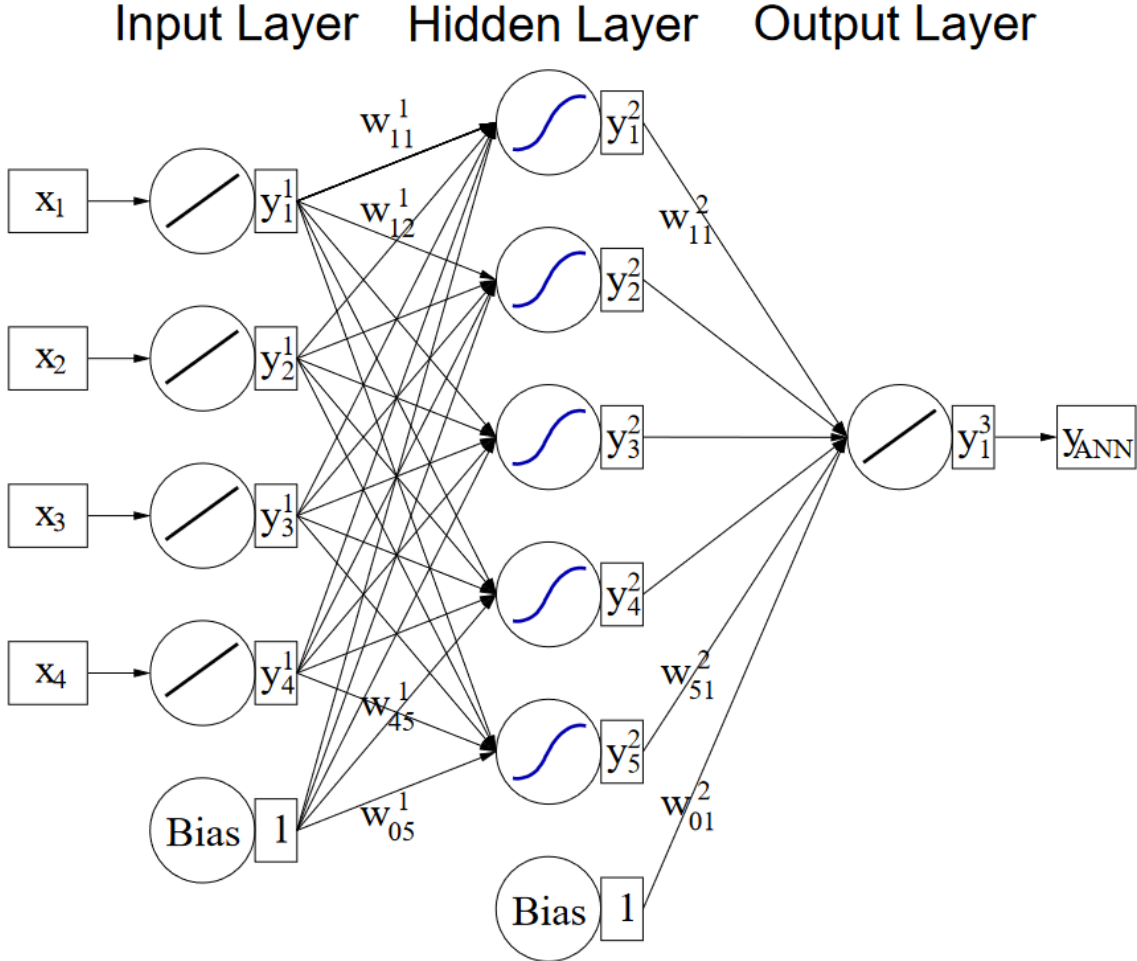
AIML fitting

- Most models do not perfectly match the data because of our lack of knowledge on modelling of GPDs
- AIML approach, model with a neural network approach and train model on existing data allow model independent extraction of parameters and prediction

AIML fitting



Neural Network activation function



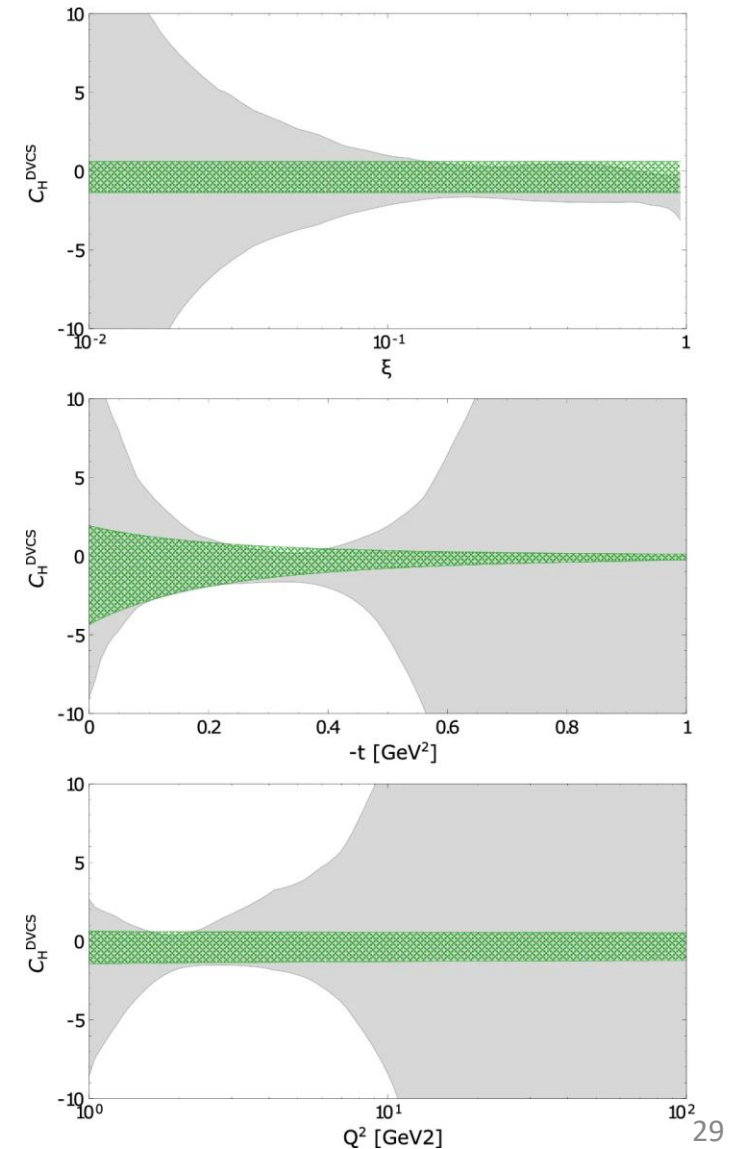
This proposal proposes to give a framework to include GPDs constraints in the neuron activation in addition to typical way in the loss function

Interface existing software with standard AIML package for best efficiency (PYTORCH or TENSORFLOW using PYTHON package or C++ API)

Example with dispersion relations

Including a mathematical constraint such as dispersion relations greatly reduce the uncertainty over the large range of variables.

Dutrieux et al., “Phenomenological Assessment of Proton Mechanical Properties from Deeply Virtual Compton Scattering.” Eur. Phys. J. C 2021



Comparison with other some experimentally driven extractions

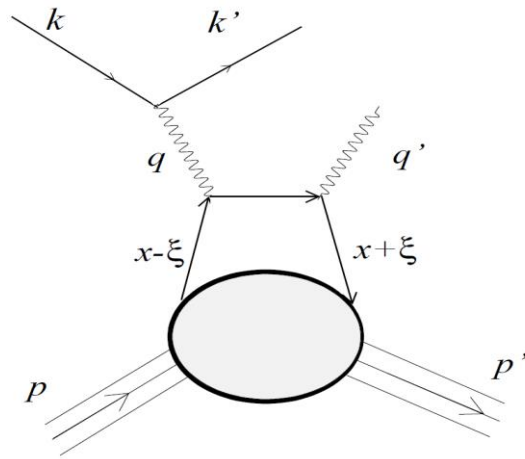
	BEG, <i>Nature</i> (2018)	Kumericki, <i>Nature</i> (2019)	DLMSTW, <i>EPJC</i> (2021)	This proposal
Dataset	JLab 6 GeV + impact study 12 GeV	JLab 6 GeV	world DVCS dataset 2019	world DVCS dataset 2024 + lattice data + impact study DDVCS
Compton Form Factors	Parametric modelling of the leading GPD diagonals and D-term (Kumericki-Muller)	One NN giving the 4 leading imaginary CFFs and a subtraction constant	8 independent NNs for the real and imaginary parts of the CFFs	NN modelling of the imaginary CFFs and subtraction constant
Dispersion relations	From the beginning	From the beginning	First, independent extraction of real and imaginary CFFs. Then dispersion relations enforced	From the beginning
D-term modelling	only first term d1 fitted + model estimate of missing higher terms	No extraction of the D-term	test of various hypotheses: d1 only, d1 and d3 jointly, free / radiative gluons,...	test of various hypotheses: do we obtain increased sensitivity to gluon, better disentanglement of d1 and d3,...
t - dependence	Factorized multipole Ansatz	Coded in the NN	First, coded in the NN. Then factorized multipole Ansatz	test of various functional forms (multipoles, z-expansion) and neural network
Evolution	not accounted for	LO evolution	First free Q ² dependence coded in the NN. Then LO evolution	LO evolution
Miscellaneous				Implementing additional constraints: positivity bounds for the GFFs A and B, end-point behavior to constraint the higher terms in the moment expansion

Example of data set

```
• uuid: 9j7gof4d
  general info:
  date: 2023-09-25
  data type: DVCS
  pseudodata: true # optional, if not set pseudodata = false
  collaboration: xsdsd # limited to 20 characters
  reference: arXiv:01/02 # optional, limited to 60 characters
  conditions:
  lepton_beam_type: e
  lepton_beam_energy: 10.
  hadron_beam_type: p
  hadron_beam_energy: 100.
  comment: "This is a comment" #optional
  data:
  - data set:
  label: "Q2_dep"
  kinematics:
  name: [xB, Q2]
  unit: [none, GeV2]
  value:
  - [0.2, 1]

• 3
  - [0.3, 2]
  - [0.4, 3]
  #or alternatively:
  #value: [[0.2, 1], [0.3, 2], [0.4, 3]]
  unc: #optional
  - [0.01, [0.01, 0.2]] #symmetric uncertainty, but can also be asymmetric
  #and given by correlation matrix
  # (see 'observable' section)
  - ["corr_matrix1", 0.01, 0.01]
  - [0.01, 0.01]
  bin: #optional #boundaries of [xB, Q2] bins:
  #[[xBmin, xBmax], [Q2min, Q2max]]
  - [[0.1, 0.3], [0, 2]]
  - [[0.2, 0.4], [1, 3]]
  - [[0.3, 0.5], [2, 4]]
  replica: #optional
  - [[0.1, 0.1, 0.2, 0.3], [1.11, 1.11, 1.21, 1.31]]
  - [[0.1, 0.1, 0.2, 0.3], [2.11, 2.11, 2.21, 2.31]]
  - [[0.1, 0.1, 0.2, 0.3], [3.11, 3.11, 3.21, 3.31]]
  observable:
  name: [ALU, ALL] #in each kinematic bin experiment measures two observables
  unit: [none, none]
  norm_unc: [0.01, 0.014] #optional
  norm_unc_contrib_label: ["target_pol", "beam_pol"] #optional
  norm_unc_contrib: [[0.01, 0.01], [0.01, 0.01]] #optional
  value:
  - [0.1, 0.2]
  - [0.11, 0.21]
  - [0.12, 0.22]
  stat_unc: #optional
  - [0.1, 0.2] #symmetric and symmetric
  - [0.1, [0.1, 0.2]] #symmetric and asymmetric
  - ["corr_matrix1", 0.1, 0.1]
  sys_unc: #optional
  - [0.1, 0.2] #cov. matrix
  - [0.1, 0.1] #symmetric and symmetric
  - [0.1, [0.1, 0.2]] #symmetric and asymmetric
  sys_unc_contrib_label: ["fit", "detector"] #optional
  sys_unc_contrib: #optional
  - [[0.06, 0.02], [0.34, 0.2]] # contribution to systematic unc.,
  #e.g. for the first point sys_fit = 0.06,
  #sys_detector = 0.02
  - [[0.06, 0.02], [0.34, 0.2]]
  - [[0.06, 0.02], [0.34, 0.2]]
  replica: #optional
  - [[0.1, 0.1, 0.2, 0.3], [0.11, 0.11, 0.21, 0.31]]
  - [[0.1, 0.1, 0.2, 0.3], [0.11, 0.11, 0.21, 0.31]]
  - [[0.1, 0.1, 0.2, 0.3], [0.11, 0.11, 0.21, 0.31]]
  correlation:
  - ["corr_matrix1", 1, 0.2, 0.2, 1]
  - ["corr_matrix2", 1, 0.4, 0.4, 1]
```

Generalized Parton Distributions



$$P = \frac{p + p'}{2}$$

$$q_M = \frac{q + q'}{2}$$

$$\Delta = p - p'$$

$$t = (p - p')^2 = \Delta^2$$

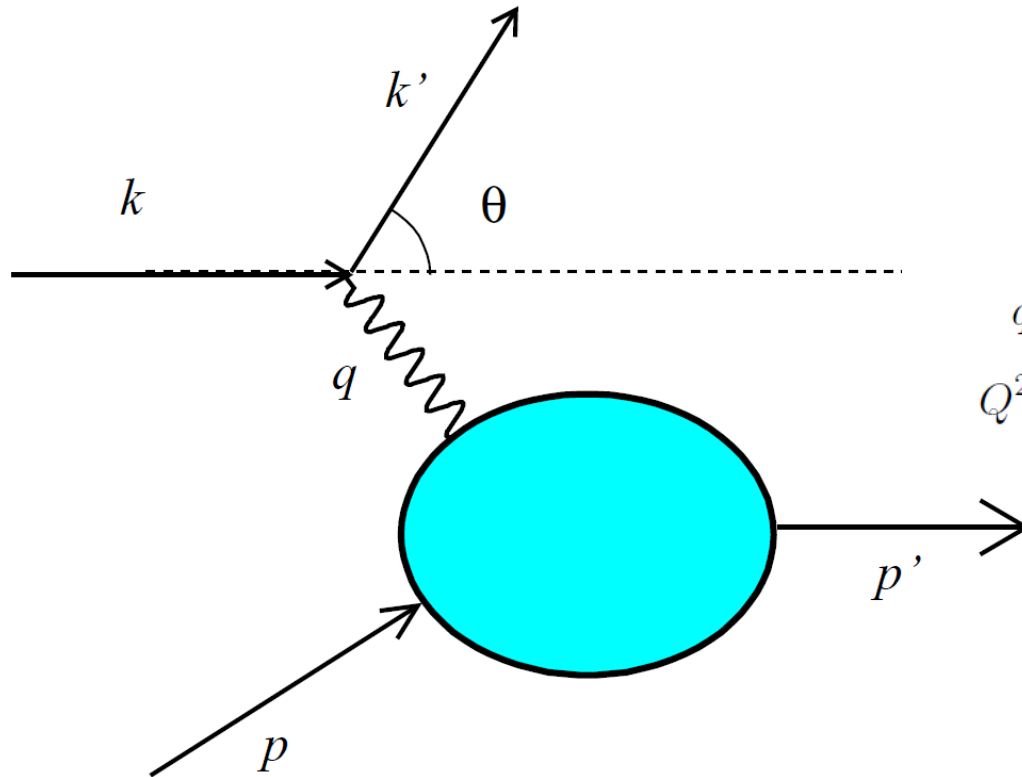
$$s = (p + q)^2 = W^2$$

$$\bar{M}^2 = P^2 = M^2 + t/4$$

$$p^\mu = \frac{\Lambda}{\sqrt{2}}(1, 0, 0, 1) \quad n^\mu = \frac{1}{\Lambda\sqrt{2}}(1, 0, 0, -1)$$

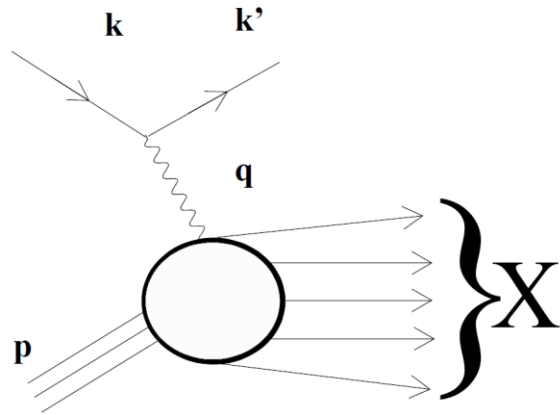
$$\xi = \frac{\Delta \cdot q_M}{P \cdot q_M}$$

Electron scattering



$$q = k - k'$$
$$Q^2 = -q^2 = -(k - k')^2 = 4EE' \sin^2\left(\frac{\theta}{2}\right)$$

Deeply Inelastic Scattering



$$W_{\mu\nu} = W_1(Q^2, \nu) \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} \right) + \frac{W_2(Q^2, \nu)}{M^2} \left(p_\mu - \frac{p \cdot q}{q^2} q_\mu \right) \left(p_\nu - \frac{p \cdot q}{q^2} q_\nu \right) \\ + G_1(Q^2, \nu) M i \epsilon_{\mu\nu\lambda\sigma} q^\lambda s_h^\sigma + \frac{G_2(Q^2, \nu)}{M} i \epsilon_{\mu\nu\lambda\sigma} q^\lambda (p \cdot q s_h^\sigma - s_h^\sigma \cdot q p^\sigma)$$

$$M W_1 = F_1(Q^2, \nu)$$

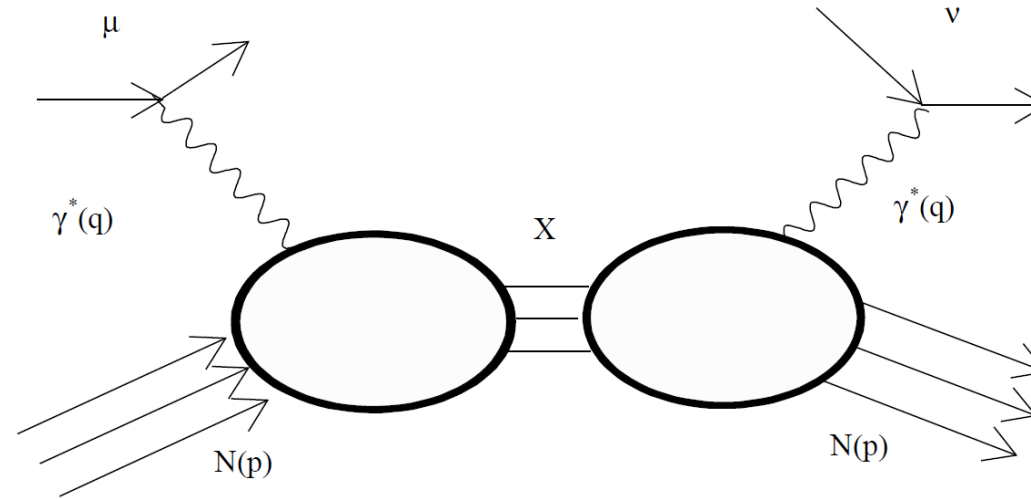
$$\nu W_2 = F_2(Q^2, \nu)$$

$$\frac{\nu}{(p \cdot q)} G_1(Q^2, \nu) = g_1(Q^2, \nu)$$

$$F_1(Q^2, \nu) = \sum_{i=1}^3 e_i q_i = \sum_{i=1}^3 e_i (q_i^\uparrow + q_i^\downarrow)$$

$$g_1(Q^2, \nu) = \sum_{i=1}^3 e_i \Delta q_i = \sum_{i=1}^3 e_i (q_i^\uparrow - q_i^\downarrow)$$

Optical theorem

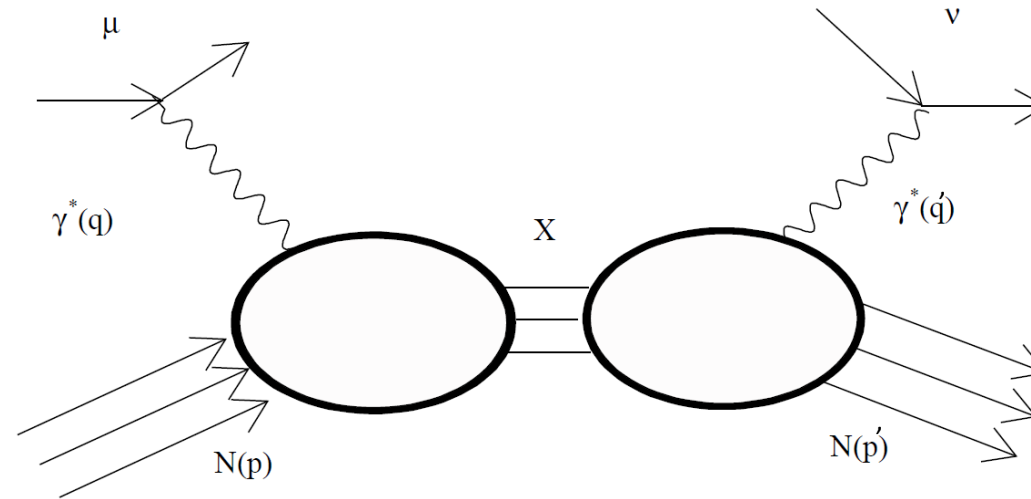


$$\begin{aligned}
 W_{\mu\nu} &= \frac{1}{4\pi} \sum_X \langle N(p) | j_\nu(0) | X \rangle \langle X | j_\mu(0) | N(p) \rangle (2\pi)^4 \delta^{(4)}(p + q - p_x) \\
 &= \frac{1}{2\pi M} \Im[T_{\mu\nu}]
 \end{aligned}$$

$$T_{\mu\nu} = i \int d^4z e^{i(q \cdot z)} \langle N(p, s) | T \{ J^\mu(-z/2), J^\nu(z/2) \} | N(p, s) \rangle$$

Description in terms of virtual photon absorbed and reemitted

Generalized Parton Distribution



$$\begin{aligned}
 W_{\mu\nu} &= \frac{1}{4\pi} \sum_X \langle N(p) | j_\nu(0) | X \rangle \langle X | j_\mu(0) | N(p') \rangle (2\pi)^4 \delta^{(4)}(p + q - p_x) \\
 &= \frac{1}{2\pi M} \Im[T_{\mu\nu}]
 \end{aligned}$$

$$T_{\mu\nu} = i \int d^4z e^{i(q \cdot z)} \langle N(p, s) | T \{ J^\mu(-z/2), J^\nu(z/2) \} | N(p', s') \rangle$$

Description in terms of virtual photon absorbed and reemitted

Properties of GPDs

- Forward limit ($\xi = 0$)

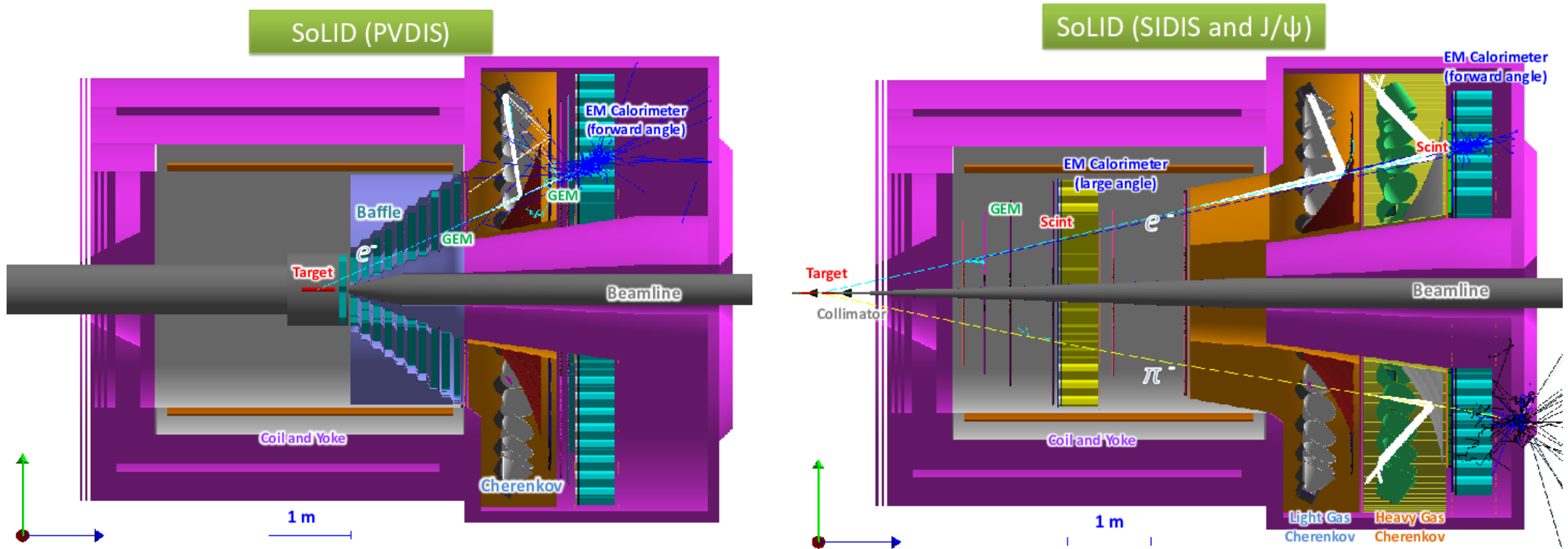
$$\begin{aligned}H^q(x, \xi = 0, t = 0) &= q(x), \\ \tilde{H}^q(x, \xi = 0, t = 0) &= \Delta q(x),\end{aligned}$$

- Integration gives back form factors

$$\begin{aligned}\int_{-1}^{+1} dx H^q(x, \xi, t) &= F_1^q(t), & \int_{-1}^{+1} dx E^q(x, \xi, t) &= F_2^q(t), \\ \int_{-1}^{+1} dx \tilde{H}^q(x, \xi, t) &= G_A^q(t), & \int_{-1}^{+1} dx \tilde{E}^q(x, \xi, t) &= G_P^q(t),\end{aligned}$$

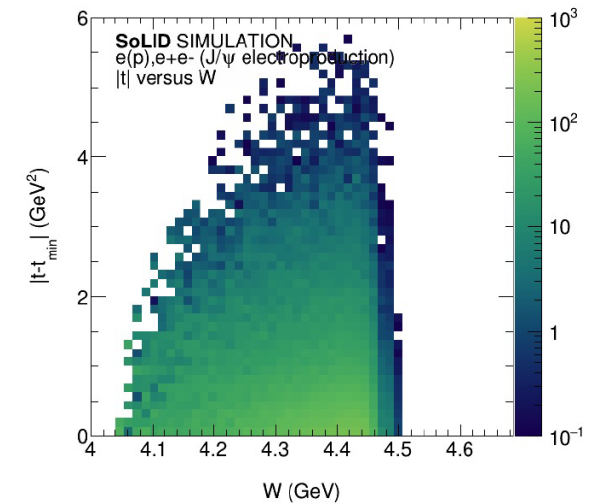
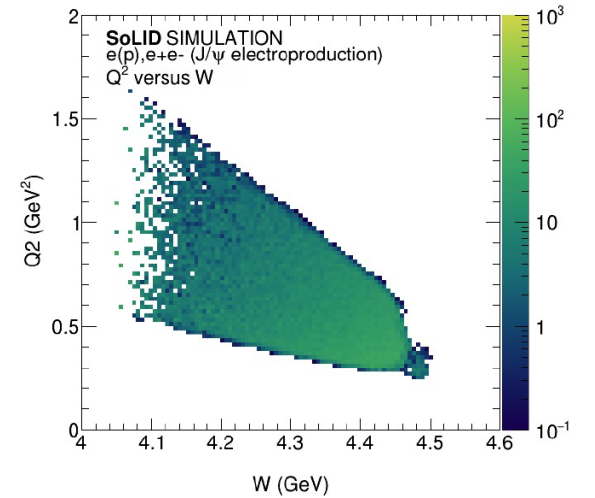
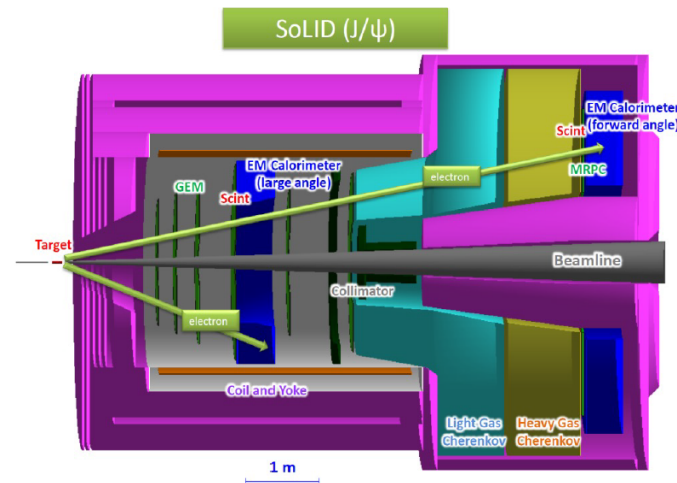
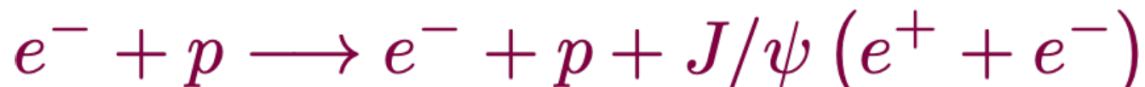
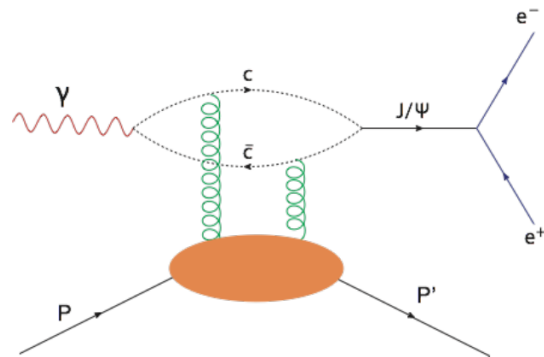
SoLID program

- SoLID detector : CLEO magnet + GEM trackers + Cerenkov + ECal
- 2 detector setup : PVDIS 60 μA , SIDIS 15 μA He3, J/Psi 3 μA 15 cm LH2 target



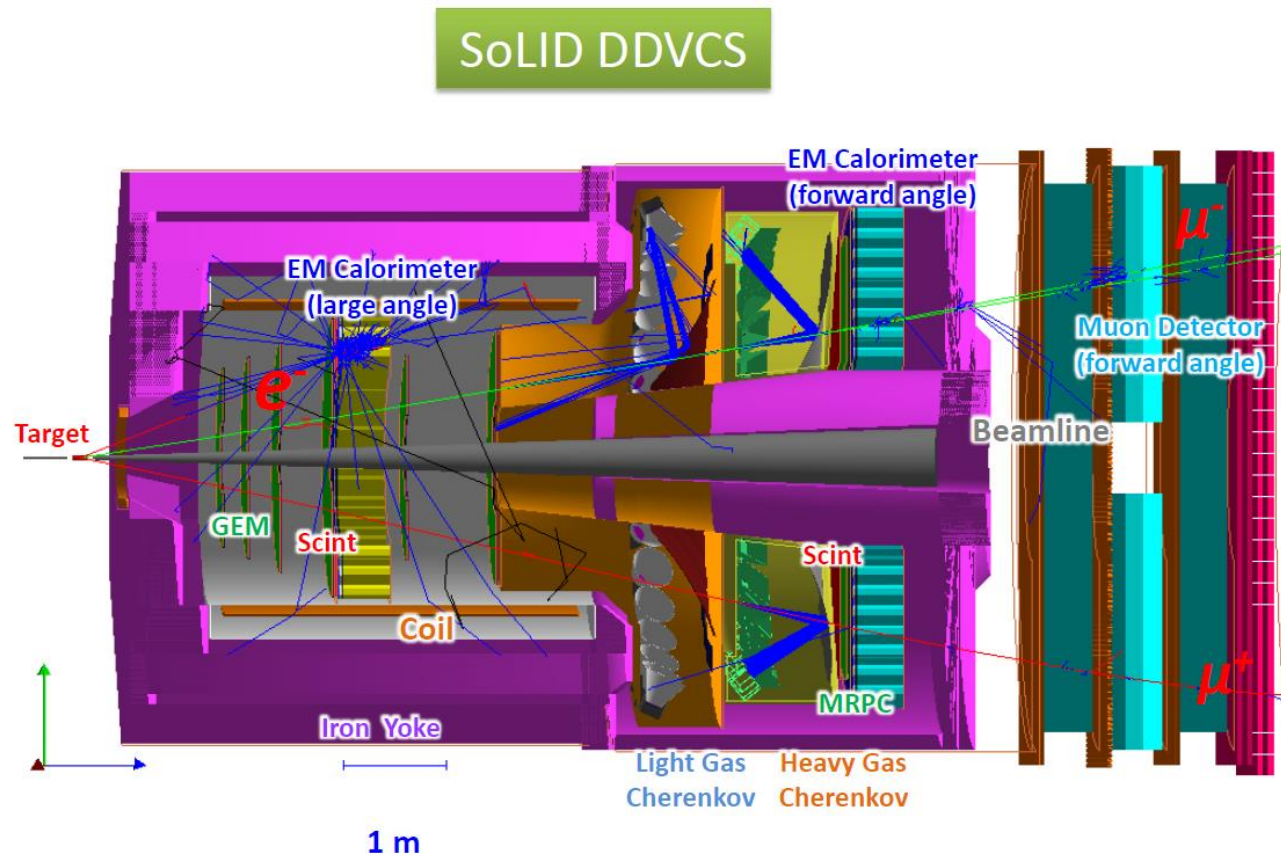
SoLID Experiment Overview

- 50 days of $3\ \mu\text{A}$ beam on a $15\ \text{cm}$ long LH_2 target at $1 \times 10^{37}\ \text{cm}^{-2}\text{s}^{-1}$
 - 10 more days include calibration/background run
- SoLID configuration overall compatible with SIDIS
 - **Electroproduction trigger:** 3-fold coincidence of e, e^-e^+
 - **Photoproduction trigger:** 3-fold coincidence of p, e^-e^+
 - **Additional trigger:** 4-fold coincidence of ep, e^-e^+
 - And (inclusive) 2-fold coincidence e^+e^-



SoLID DDVCS Setup

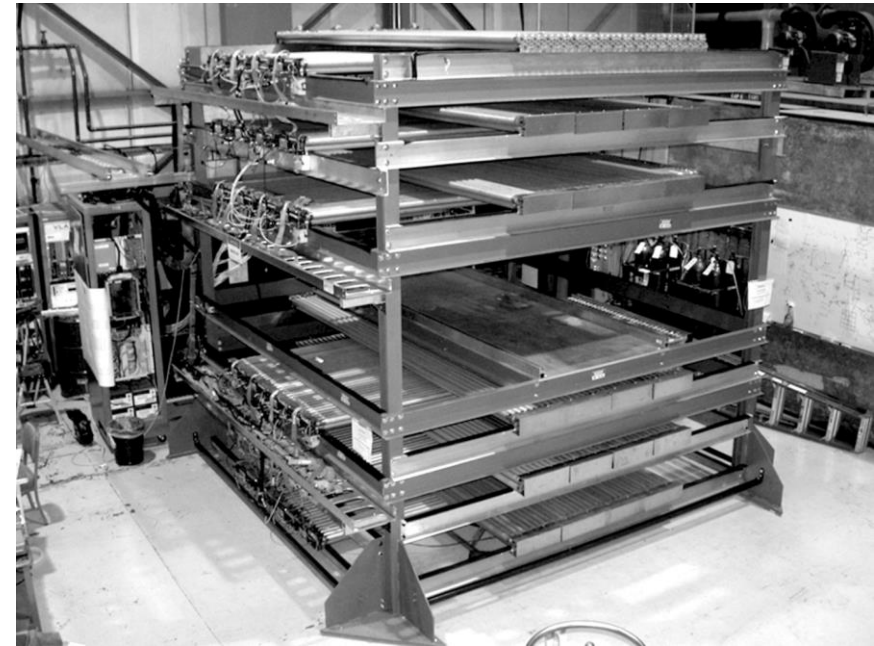
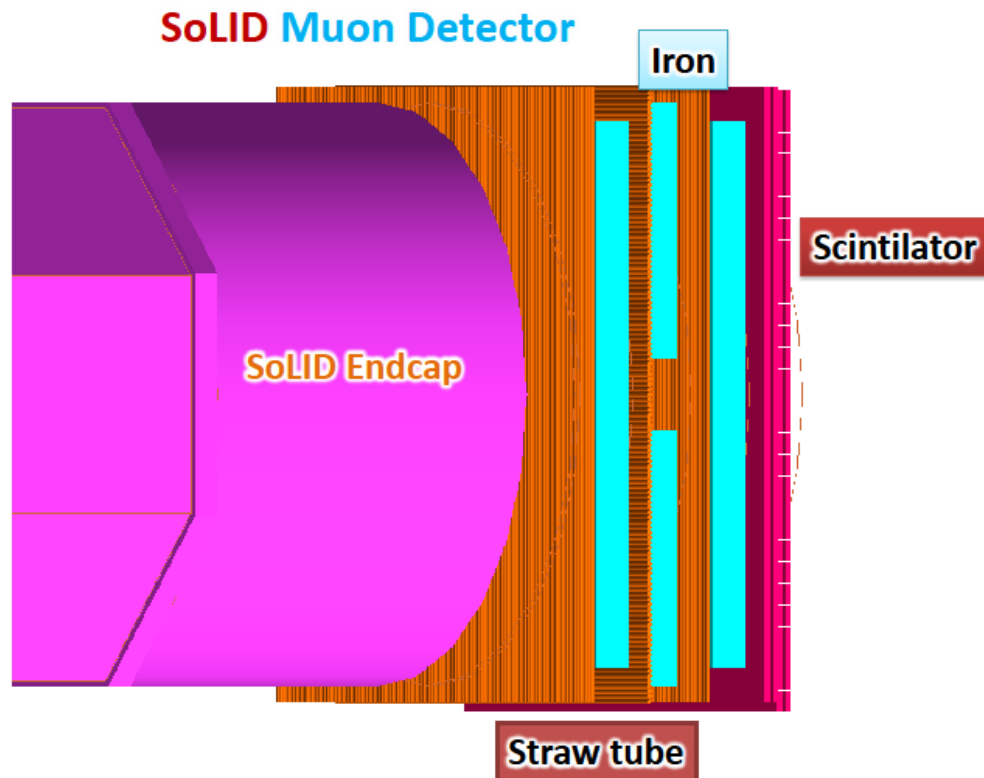
- Based on SoLID JPsi setup with forward muon detector added



- Muon detector outside from SoLID so lower background than e^+e^-
- Muons detection remove ambiguity with scattered electron compared to e^+e^- channel
- Main background behind calorimeter are pions
- Pions can be ranged out with iron plates while muon go through all layers

forward angle muon detector

- 3 layers iron to block pion, 3 layers straw tube for tracking, 2 layer2 scintillator for trigger

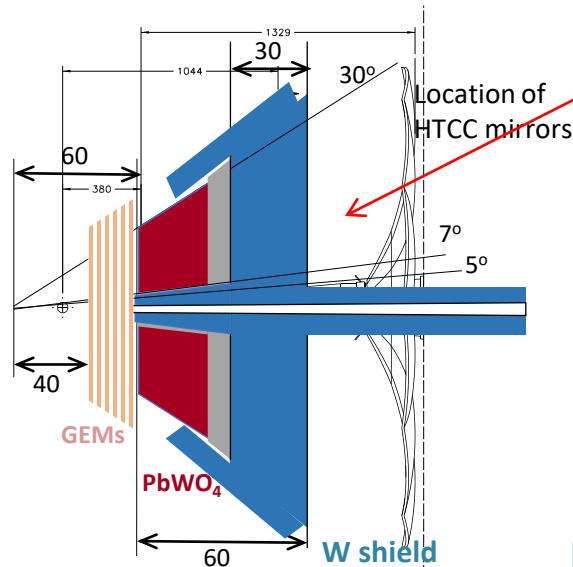
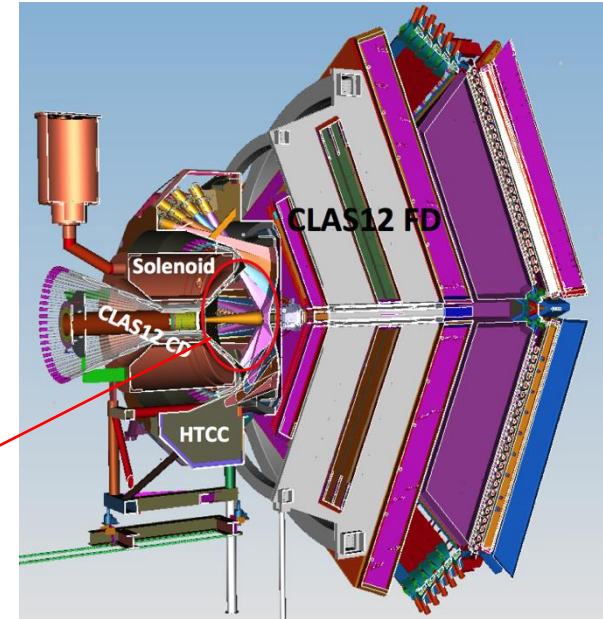


Example of straw tube chambers similar to Seaquest experiment

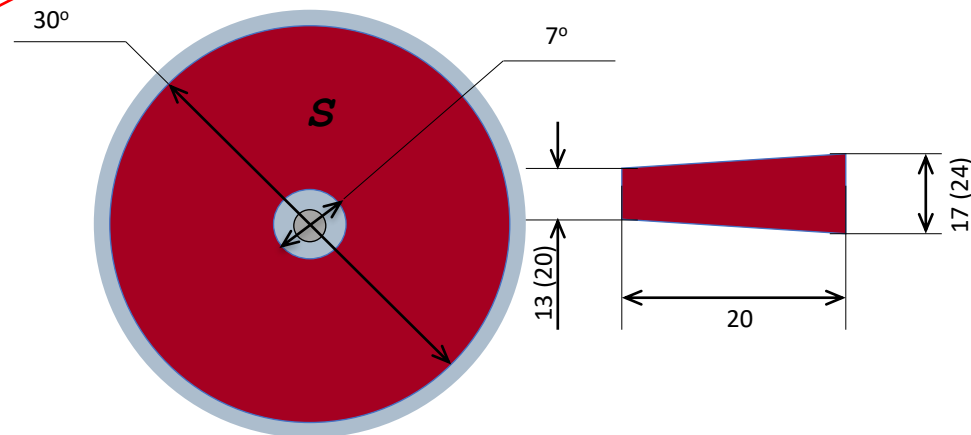
CLAS12 modifications for

$$ep \rightarrow e'p'm^+m^- @ 10^{37} \text{ cm}^{-2} \text{ s}^{-1}$$

- Remove HTCC and install in the region of active volume of HTCC
 - a new Moller cone that extends up to 7°
 - a new PbWO_4 calorimeter that covers 7° to 30° polar angular range with 2π azimuthal coverage.
- Behind the calorimeter, a 30 cm thick tungsten shield covers the whole acceptance of the CLAS12 FD
- GEM tracker in front of the calorimeter for vertexing



$$S = \pi l^2 [(tg^2(30^\circ) - tg^2(7^\circ))] = 3600 \text{ cm}^2; l = 60 \text{ cm}$$



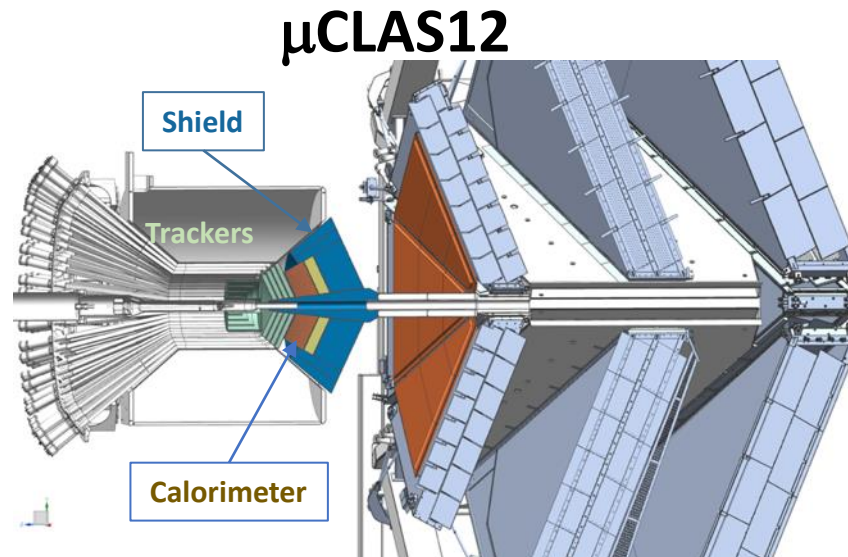
PbWO₄ modules with APD readout - ~ 1200 modules

High luminosity CLAS12 for DDVCS

Two main challenges in DDVCS measurements:

- Cross section is three orders of magnitude smaller than the DVCS cross section;
- Ambiguities and anti-symmetrization issues with the decay leptons of the outgoing virtual photon and the incoming-scattered lepton.

Di-muon electroproduction using upgraded CLAS12 will overcome these challenges.

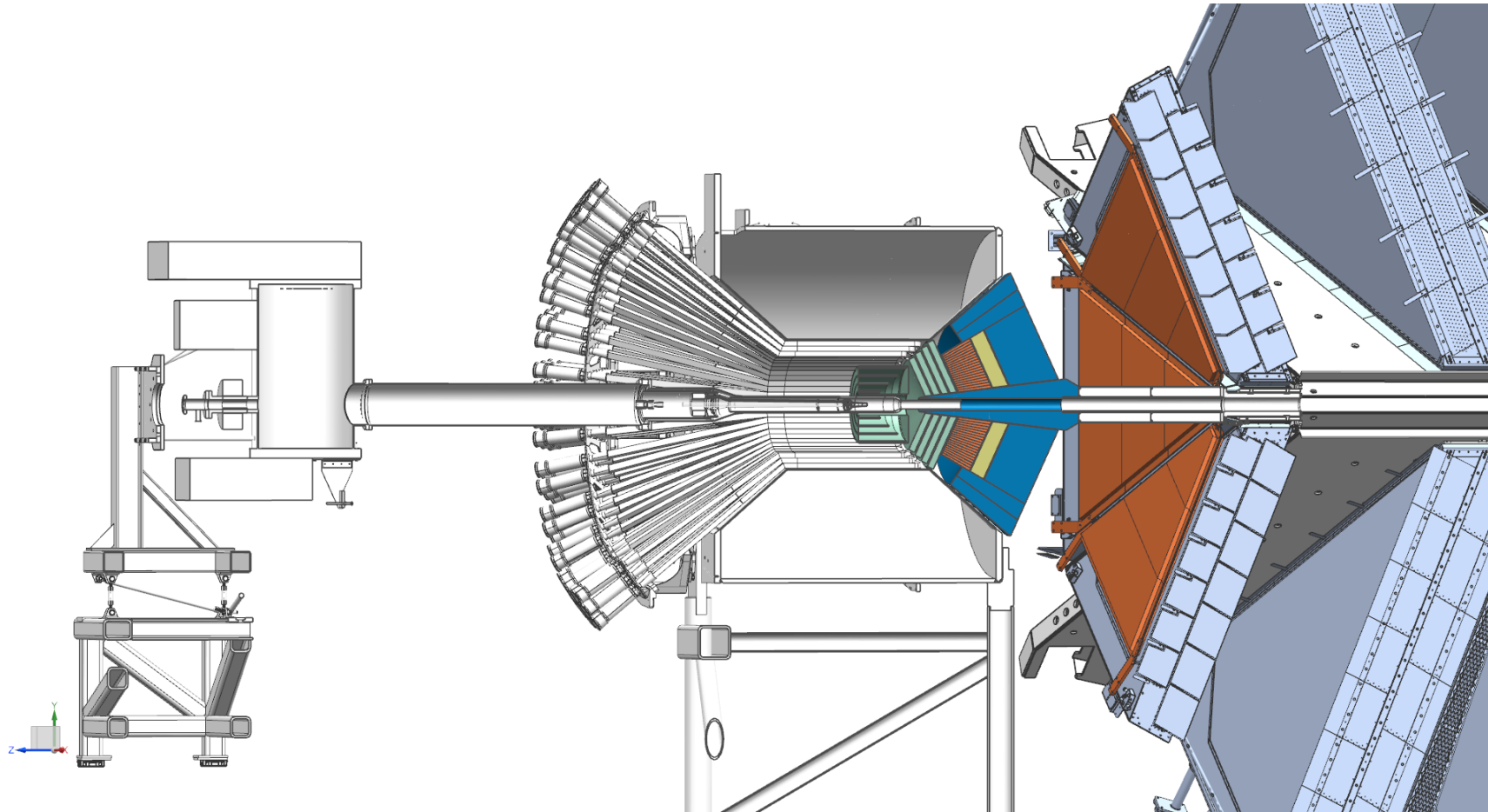


A simple concept:

- Remove HTCC and block the CLAS12 forward with a W-shield and PbWO_4 calorimeter to prevent flooding of DC by EM background;
- Shield will work as pion filter, as most of charged pions will shower;
- Remove CVT, instead use a high rate MPGDs for central and forward (in front of the calorimeter) tracking

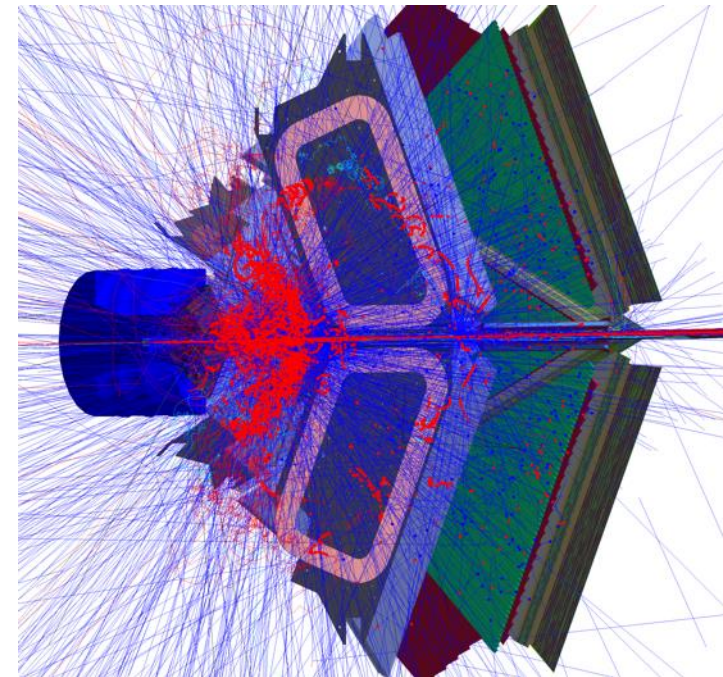
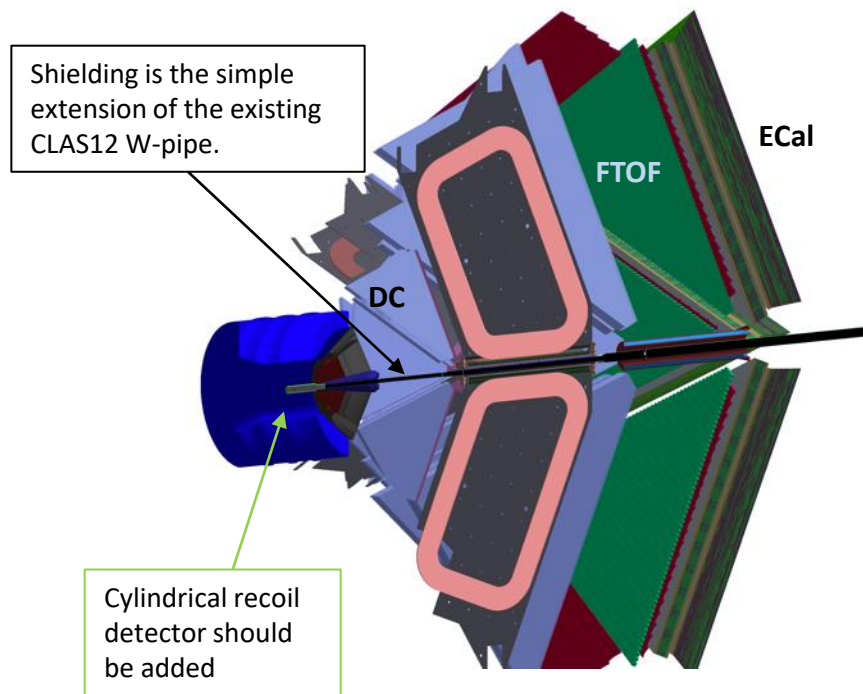
Detector capable of measuring
 $ep \rightarrow e'p'\mu^+\mu^- @ L > 10^{37} \text{ cm}^{-2} \text{ sec}^{-1}$

Starting Flow AS12LAS 12 with μ sPAC Etometer



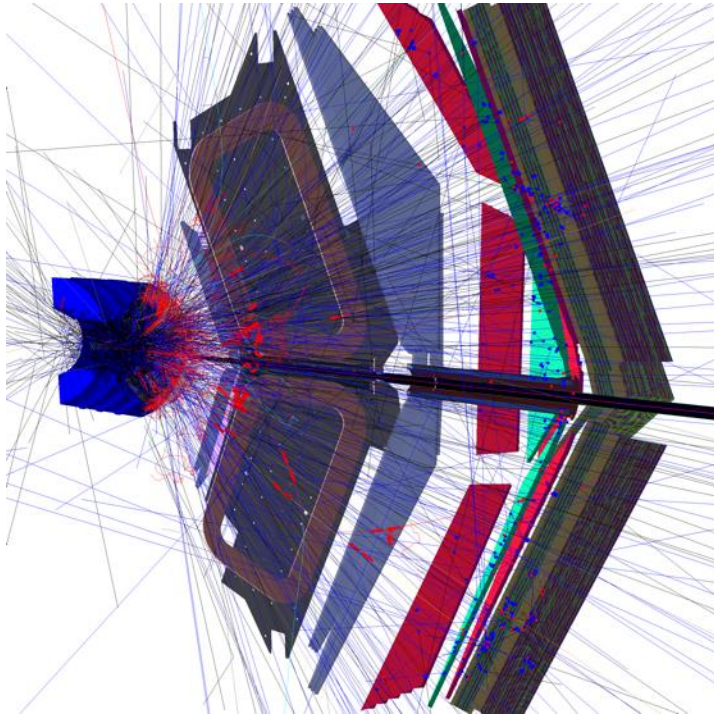
GEANT4 model

- The forward part of the proposed upgrade (calorimeter and the shielding) is in the CLAS12 MC, GEMC (M. Ungaro).
- Simulations are underway to understand backgrounds in detectors, optimize shielding and determine luminosity limitations. (*Earlier studies for LOI12-16-004 validated the concept for $L = 10^{37} \text{ cm}^{-2} \text{ sec}^{-1}$*)



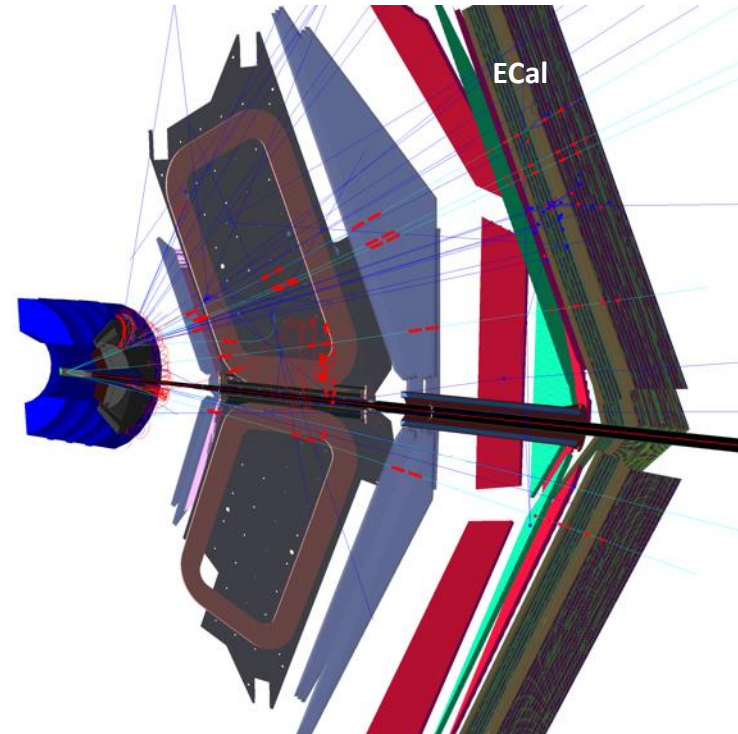
μ/π – separation

6 GeV π^+



Most pions will shower in the calorimeter/shielding and will not reach drift chambers, much less the ECal.

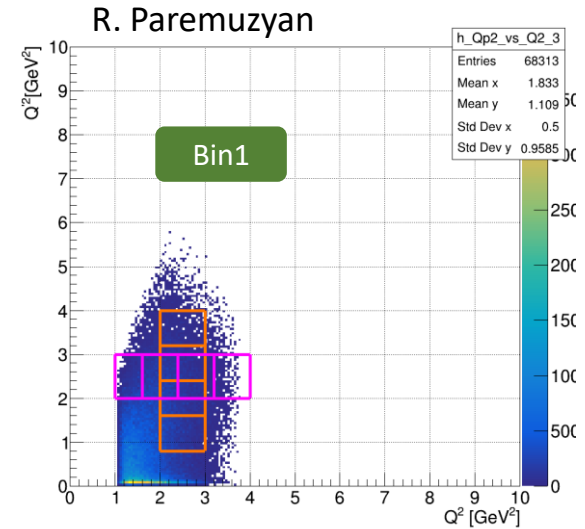
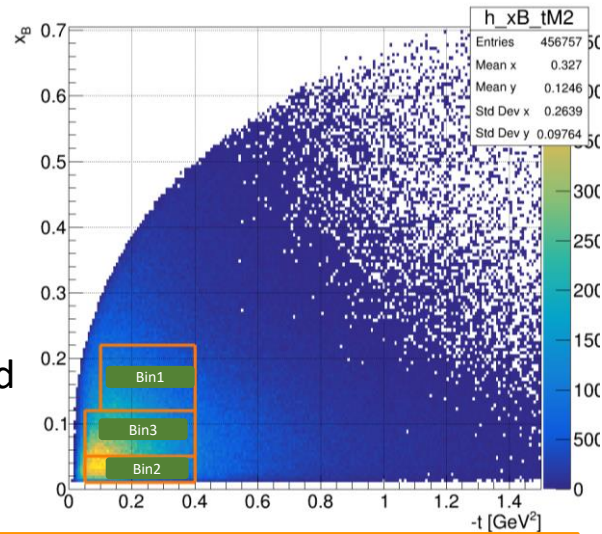
6 GeV μ^+



Conversely, muons will lose some energy in the calorimeter/shielding but will reach drift chambers and Ecal. Ecal is where muons are IDed.

Kinematical coverage for DDVCS

- GRAPE event generator, BH only.
- Shown one t-bin, but measurements can be extended to $-t \sim 1 \text{ GeV}^2$.
- The whole region is measured simultaneously.



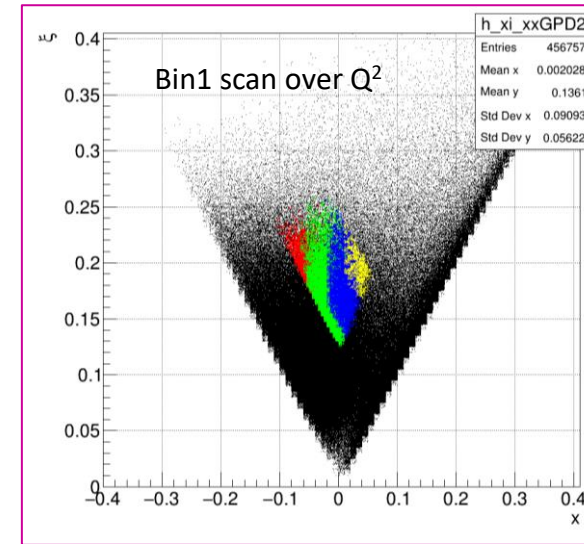
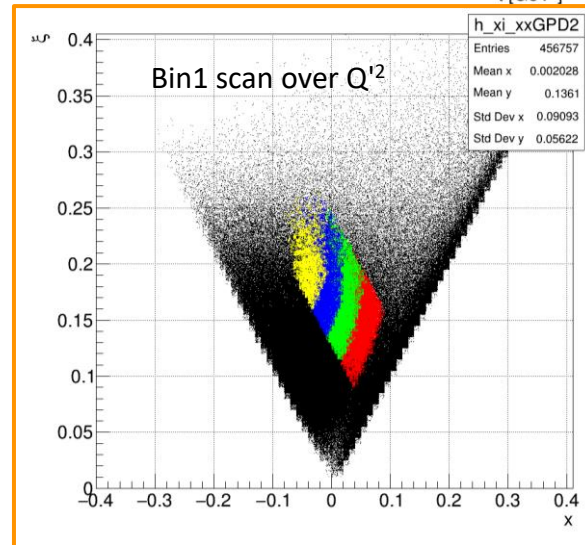
$$Q^2 = -q^2 = (e - e')^2$$

$$x_B = \frac{Q^2}{2pq}$$

$$Q'^2 \equiv M^2 = (l^+ - l^-)^2$$

$$\xi' = \frac{Q^2 + Q'^2}{2Q^2/x_B - Q^2 - Q'^2 + t}$$

$$\xi = \xi' \frac{Q^2 - Q'^2 + t/2}{Q^2 + Q'^2}$$



Final state particles

- Electrons and muons are confined within the calorimeter and FD.
- Recoil proton detection will be limited to $\vartheta > 40^\circ$, not crucial for DDVCS.

