

Recent results and future prospects of Hadron Physics at J-PARC

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KEK/J-PARC

Physics Opportunities at an Electron-Ion Collider XI

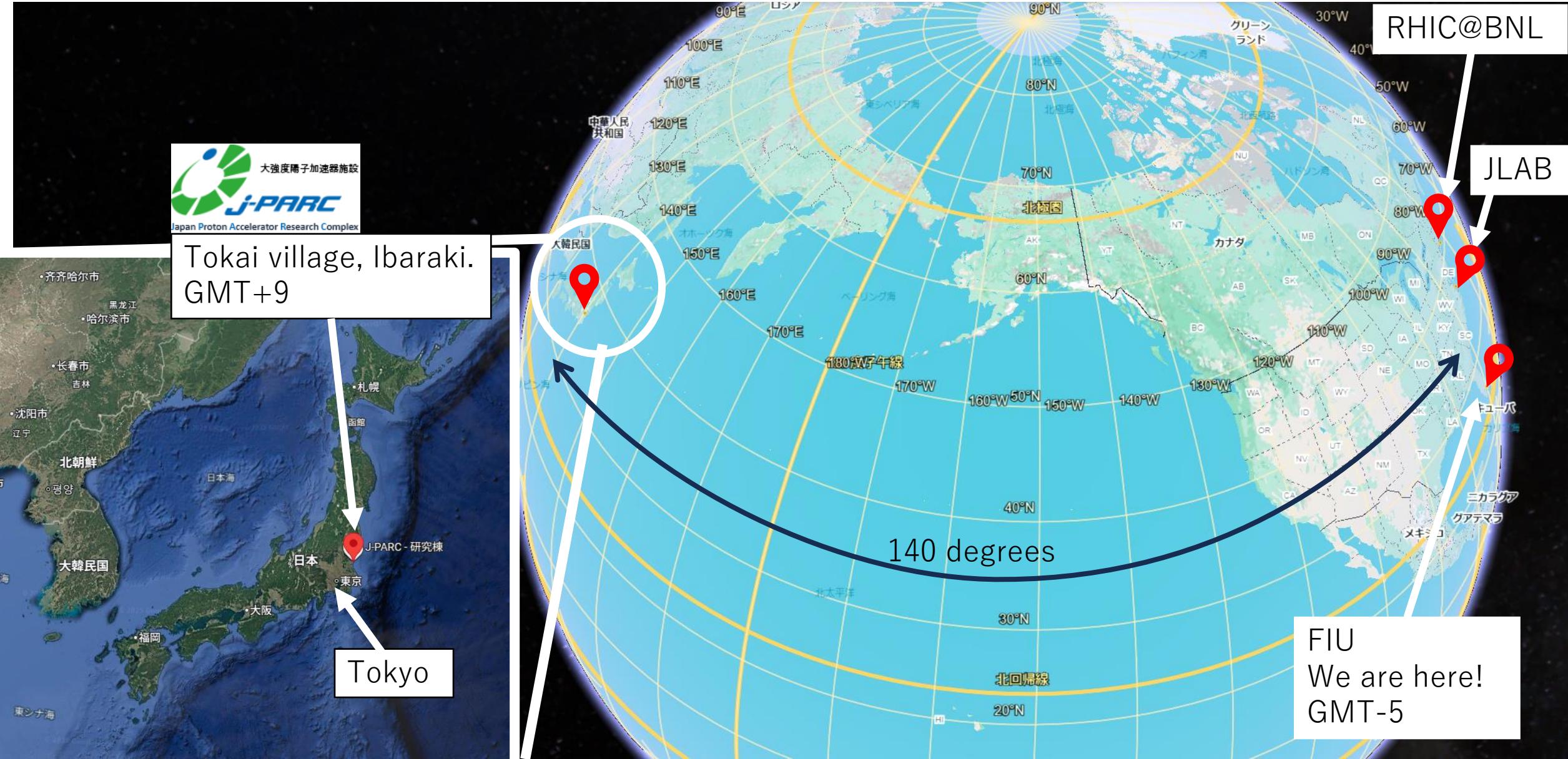
POETIC 2025

Feb24-28. 2025

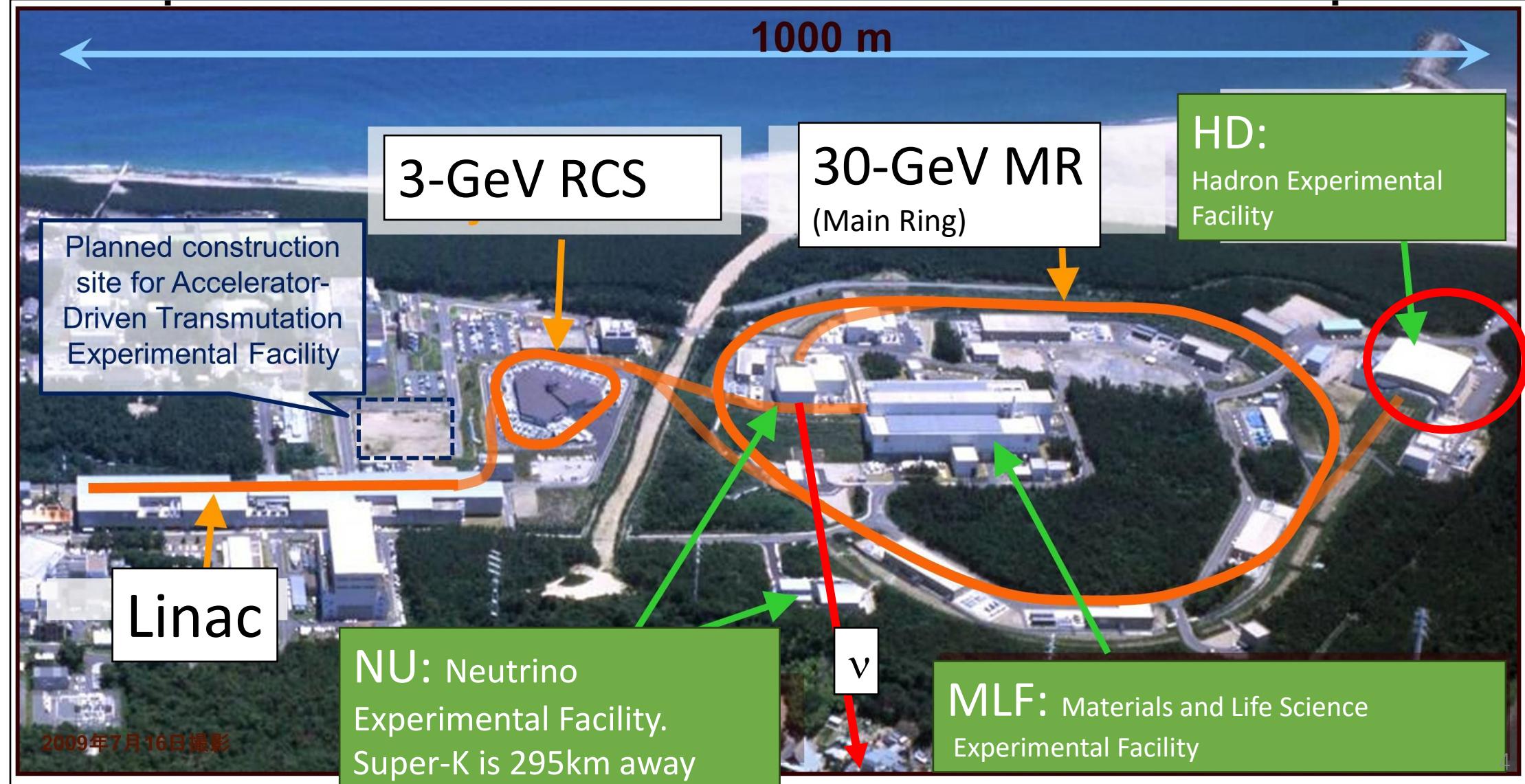
Contents

- J-PARC
- J-PARC HD (Hadron Experimental Facility) and beam lines
- Physics Motivation
- Pick up some of experiments and discuss results, along with on-going and planned experiments.

Where is J-PARC?



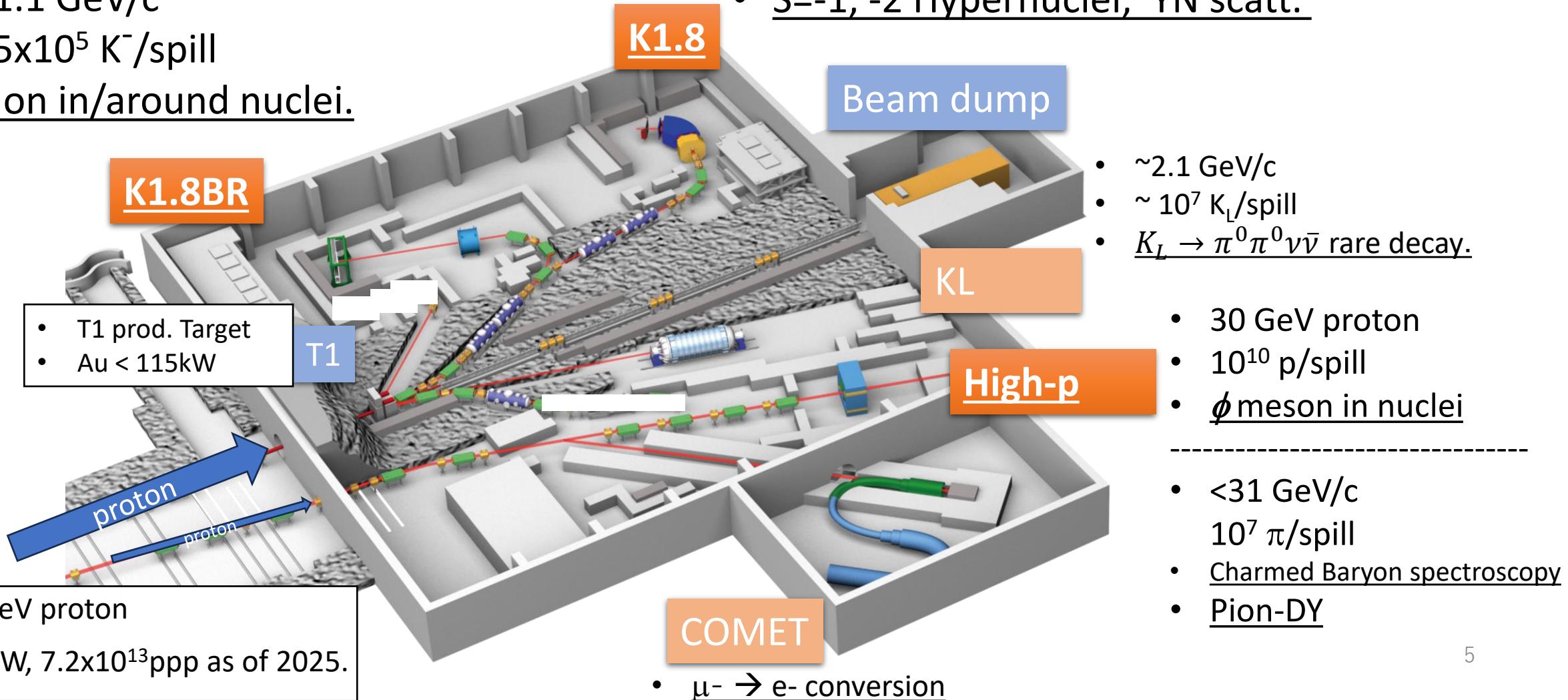
Japan Proton Accelerator Research Complex



Beam lines at J-PARC HD (Hadron Exp. Facility)

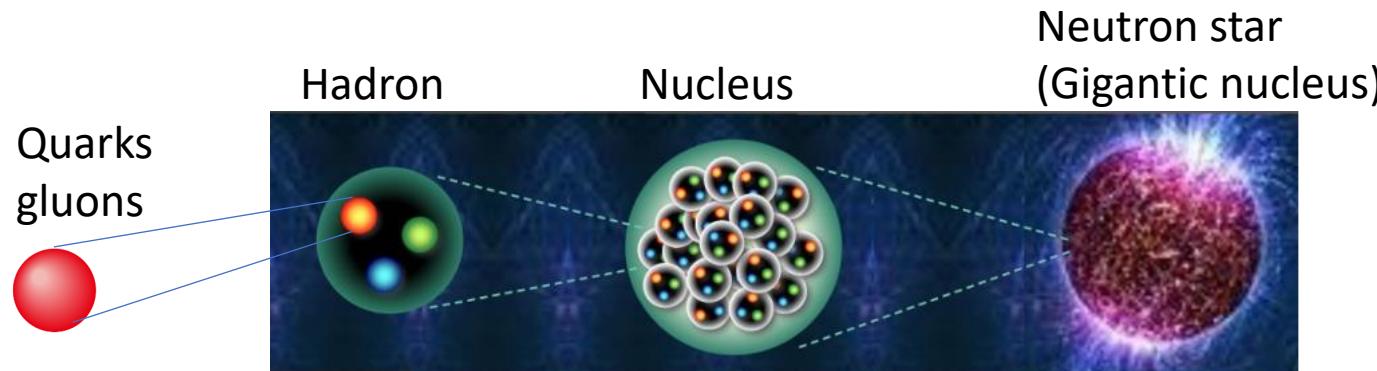
- $< 1.1 \text{ GeV}/c$
- $\sim 5 \times 10^5 K^-/\text{spill}$
- Kaon in/around nuclei.

- $< 2.0 \text{ GeV}/c$
- $\sim 10^6 K^-/\text{spill}$
- $S=-1, -2$ Hypernuclei, YN scatt.



Physics motivation in one slide:

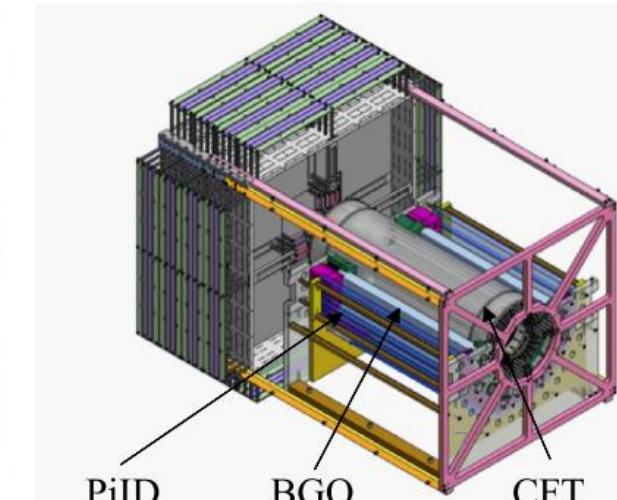
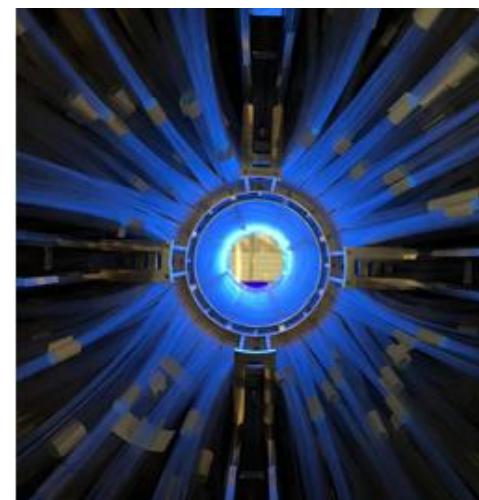
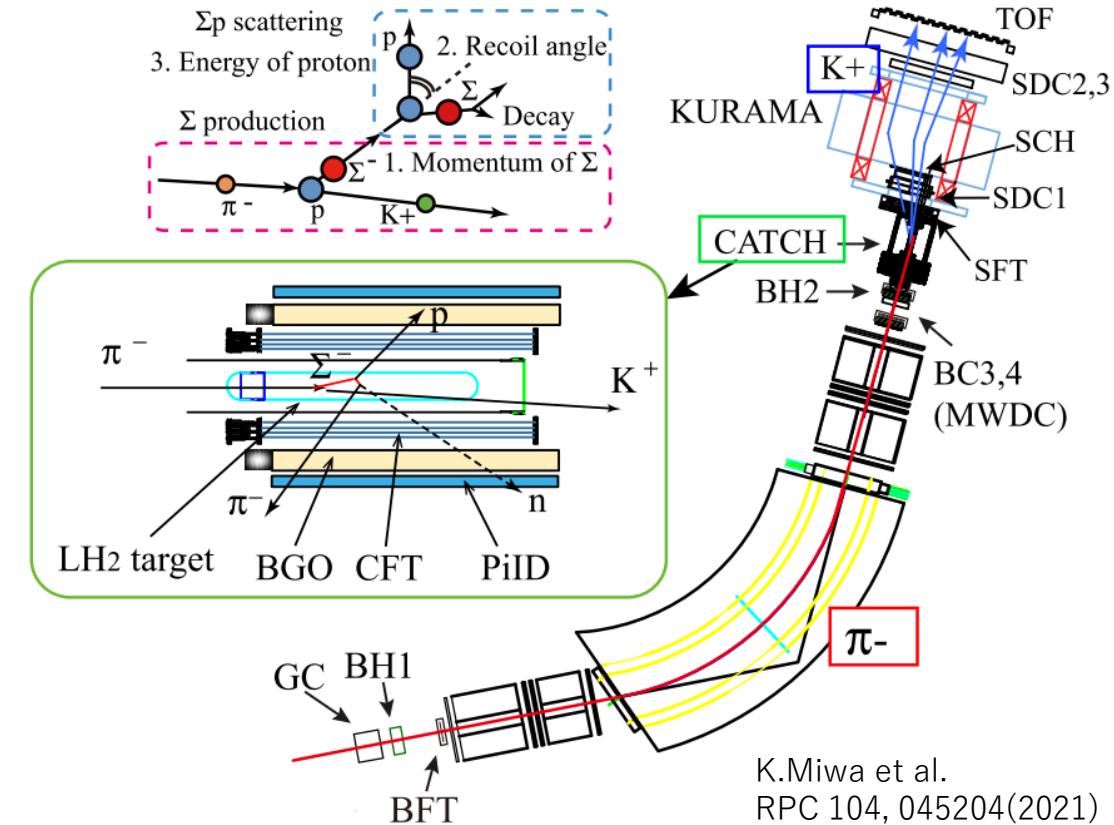
To understand strongly interacting systems from quarks to neutron stars.



- Quark → Hadron : How quarks and gluons form hadrons?
 - Quark Confinement. Spontaneous breaking of chiral symmetry.
 - Meson in nuclei → ϕ (high-p, E16), $K^{\bar{b}ar}$ (K1.8BR, E15)
 - GPD → Pion induced Drell-Yan(high-p, LOI)
- Hadron → Nucleus → Neutron Star : Property of dense matter?
 - Precise Baryon-Baryon interaction incl. hyperons and Its density dependence
 - ΣN Scattering experiment (**K1.8, E40**)
 - $S=-1, -2$ hypernuclei and γ -ray spectroscopy. (**K1.8**)

J-PARC E40 : Σp scatterings.

- Σ^\pm “beam” produced in LH2 target
 - (π, K^+) reaction creates $S=-1$.
 - $s-s^{\bar{b}a}$ pair created. K^+ takes away $s^{\bar{b}a}$. s is put in the system of interest.
 - $p(\pi^-, K^+) : \pi^- p \rightarrow K^+ \Sigma^-$
 - $p(\pi^+, K^+) : \pi^+ p \rightarrow K^+ \Sigma^+$
 - beam line spectrometer (π^\pm) + KURAMA spectrometer (K^+)
 - Missing mass (Σ^\pm)
- Σp scattering
 - $\Sigma^- p \rightarrow \Sigma^- p$ 4500 events.
 - $\Sigma^+ p \rightarrow \Sigma^+ p$ 2400 events.
 - $\Sigma^- p \rightarrow \Lambda n$ 1000 events.
 - Detected by CATCH

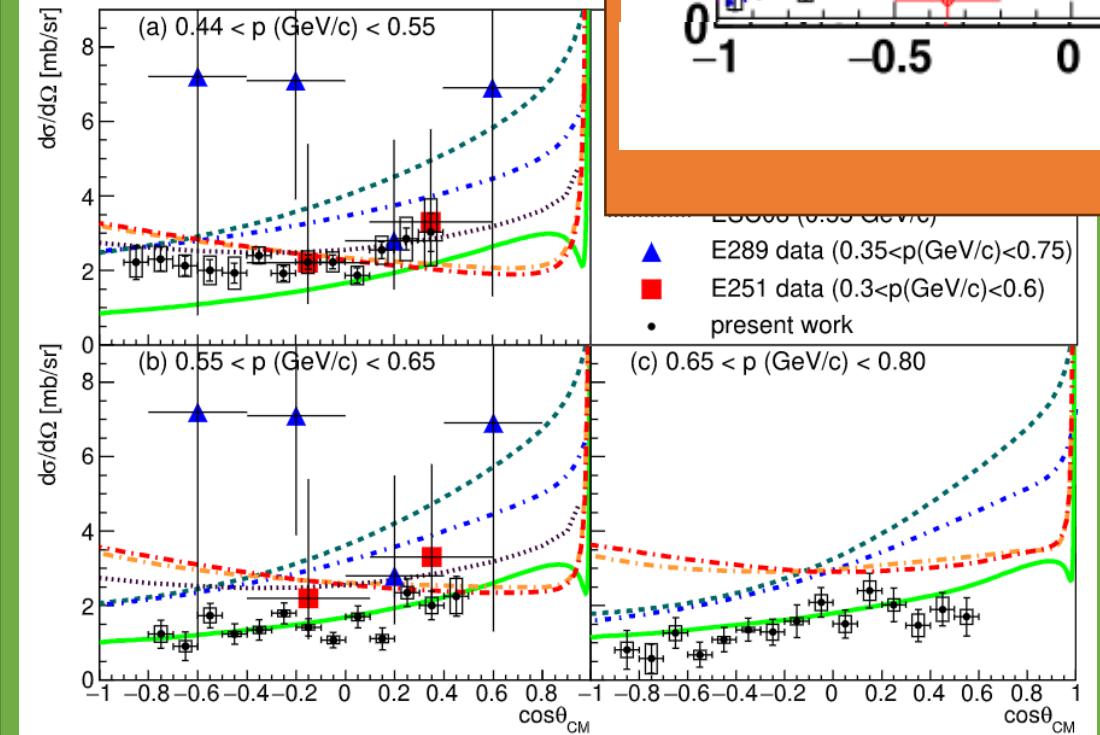


K.Miwa et al.
RPC 104, 045204(2021)

J-PARC E40 : Σ p scatterings.

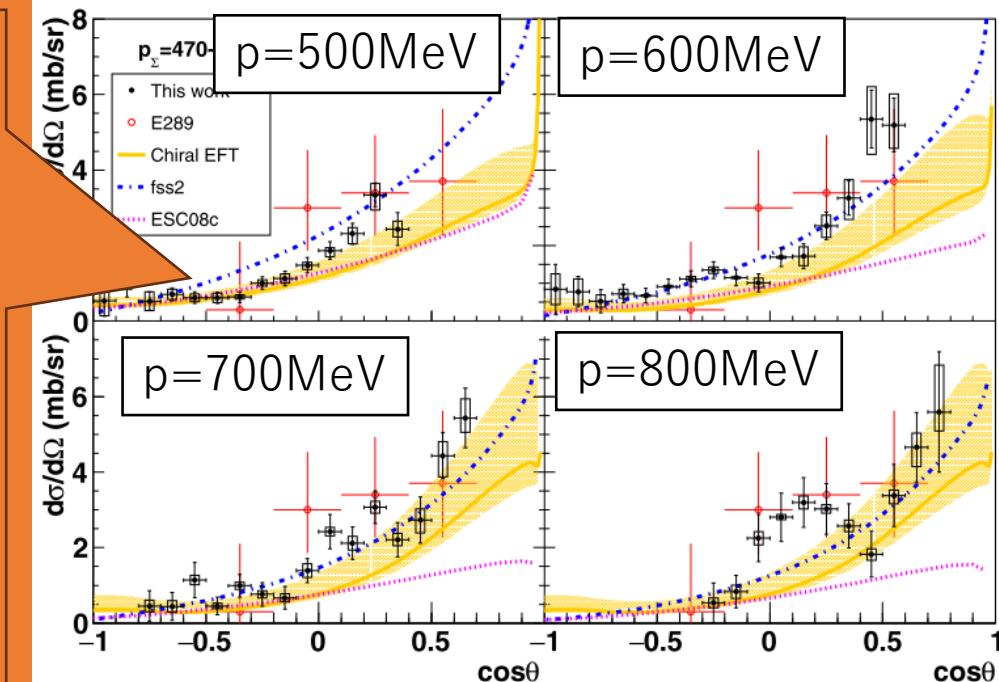
- Improved statistics compared to old measurements.

$\Sigma^+ p \rightarrow \Sigma^+ p$

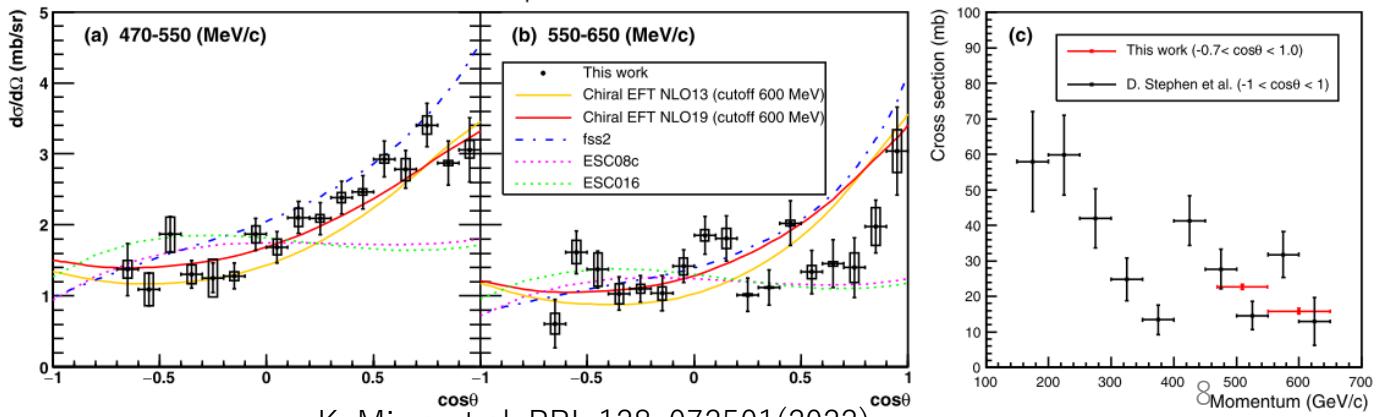


$\Sigma^- p \rightarrow \Sigma^- p$

K.Miwa et al.
RPC 104, 045204(2021)

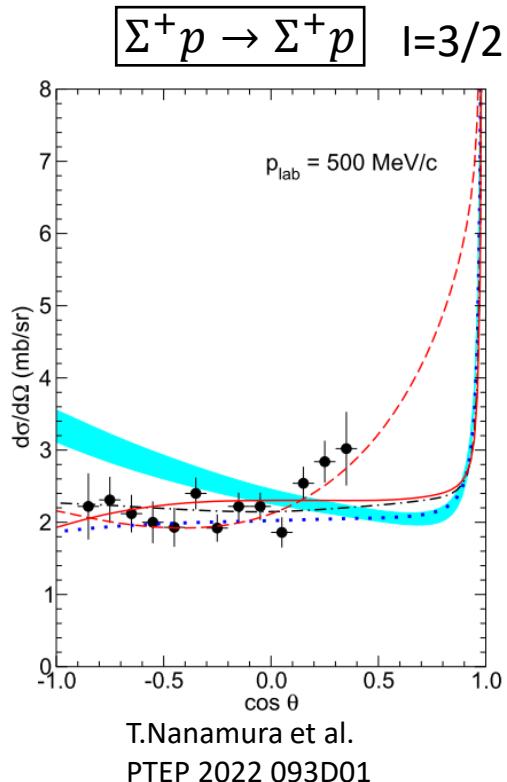
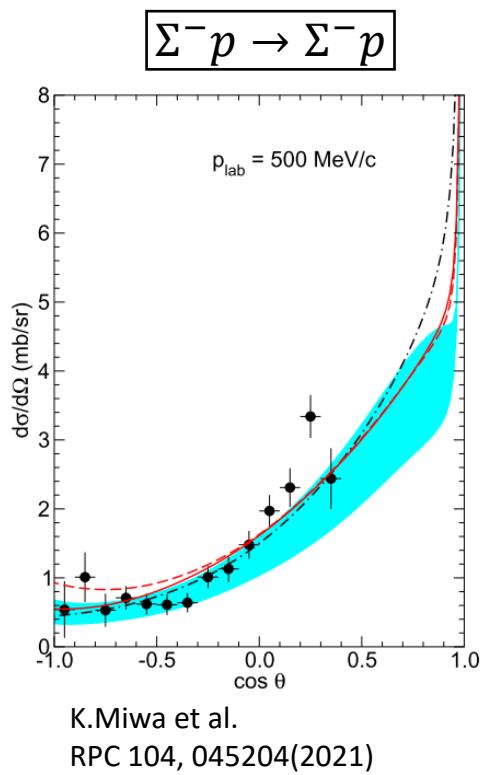
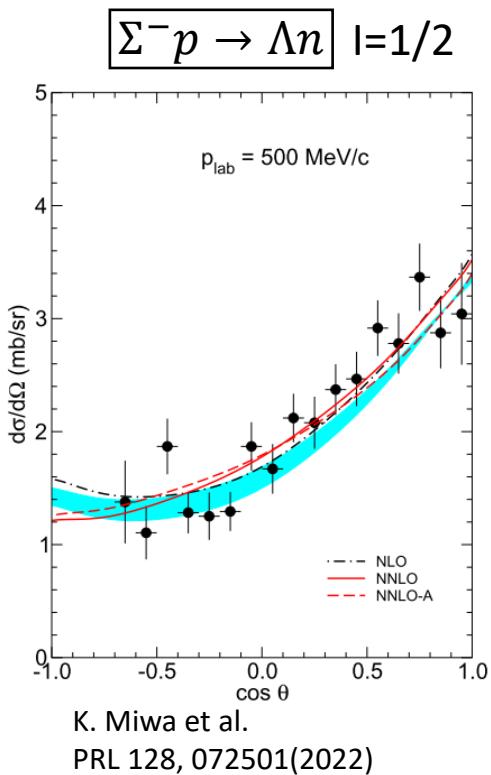


$\Sigma^- p \rightarrow \Lambda n$



J-PARC E40 : Σp scatterings.

Cyan: NLO19 (w/o E40 data) ,
 Solid: NNLO fit w/ E40 data.
 Dashed NNLO-A: fine tuned to
 500MeV/c data.



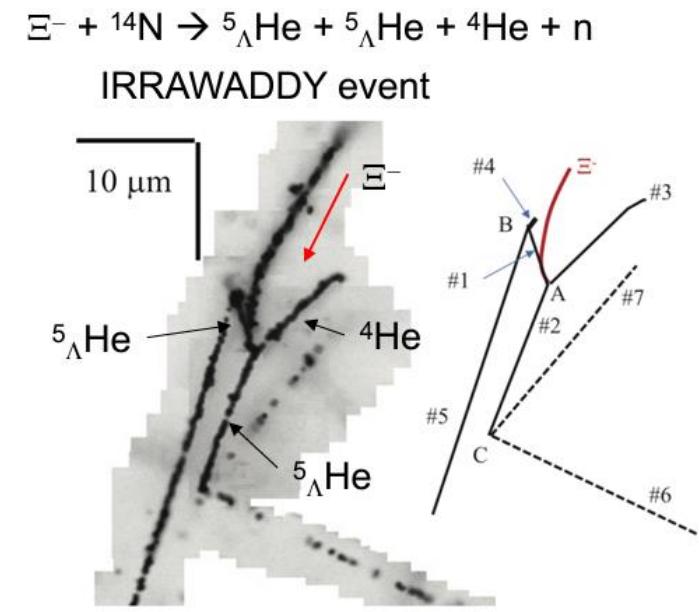
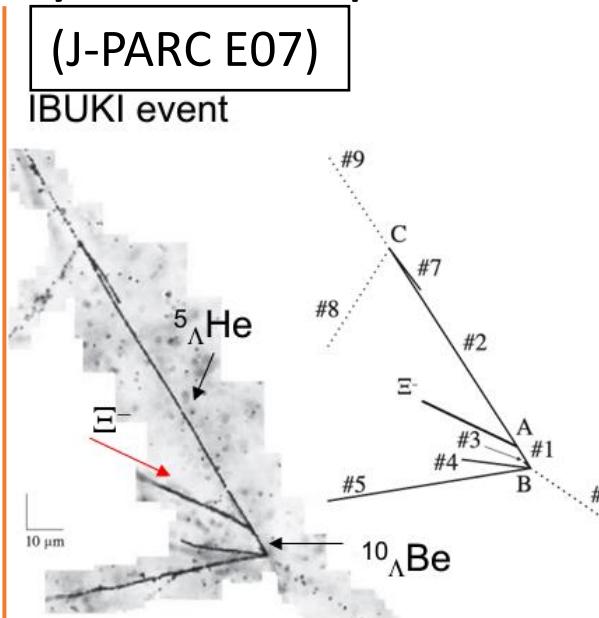
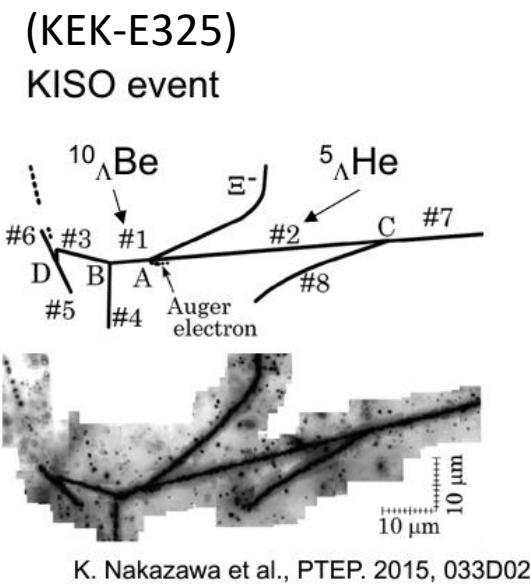
1 and 3 refer to spin singlet and triplet
 even/odd refer to OAM

BB channel (I)	${}^1\text{even or } {}^3\text{odd}$	${}^3\text{even or } {}^1\text{odd}$
$NN(I=0)$	—	(10*)
$NN(I=1)$	(27)	—
$\Lambda N(I=\frac{1}{2})$	$\frac{1}{\sqrt{10}}[(8_s) + 3(27)]$	$\frac{1}{\sqrt{2}}[-(8_a) + (10^*)]$
$\Sigma N(I=\frac{1}{2})$	$\frac{1}{\sqrt{10}}[3(8_s) - (27)]$	$\frac{1}{\sqrt{2}}[(8_a) + (10^*)]$
$\Sigma N(I=\frac{3}{2})$	(27)	(10)

- E40 data has stimulated NNLO Chiral EFT.
 - Haidenbauer et al., Eur. Phys.J.A 59, 3 (2023)
 - No additional LEC (Low Energy Constant) at this level except for 3BF. 3BF LEC not considered.
 - Cyan: NLO19 (w/o E40 data) , Solid: NNLO fit w/ E40 data.
 - NLO19 successful reproduce a) and b) but not c). Fit w/ E40 improves.
 - c) Sensitive to 10-plet in SUf(3) which does not appear NN interaction.
 - Parameters not uniquely determined. Additional information needed. Polarized ΛN scattering experiment planned. → J-PARC E86

J-PARC E07: $S=-2$ hypernuclei by Hybrid emulsion method

- (K^-, K^+) reactions : s in the beam and $s-s^{\bar{b}a}r$ created. K^+ take away $s^{\bar{b}a}r$. ss are put into the system of interest.
- Emulsions – photographic films that capture particle reactions.
- Hybrid – Ξ^- productions were recorded by counters, later search for corresponding position in emulsion by microscope.



$\Xi^- + {}^{14}N$
hypernuclei

Slide by
K. Miwa 2nd HEF-EX.

1st discovery of clear Ξ nuclear state

- Two possibilities of B_{Ξ^-} depending on ${}^{10}\Lambda Be$ state
- $B_{\Xi^-}({}^{10}\Lambda Be_{g.s.}) = 3.87 \pm 0.21$ MeV
 - $B_{\Xi^-}({}^{10}\Lambda Be_{1st. Ex.}) = 1.03 \pm 0.18$ MeV

1st uniquely identified Ξ nuclear state

One reaction process satisfied kinematical consistency.

- $B_{\Xi^-} = 1.27 \pm 0.21$ MeV

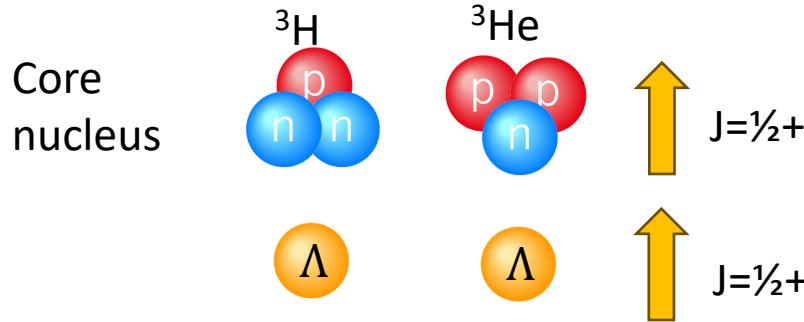
1st observation of nuclear s-state of Ξ hypernucleus

(This state maybe ${}^{14}C + \Xi^0$)
E.Friedman, A.Gal PLB837, 137640(2023)

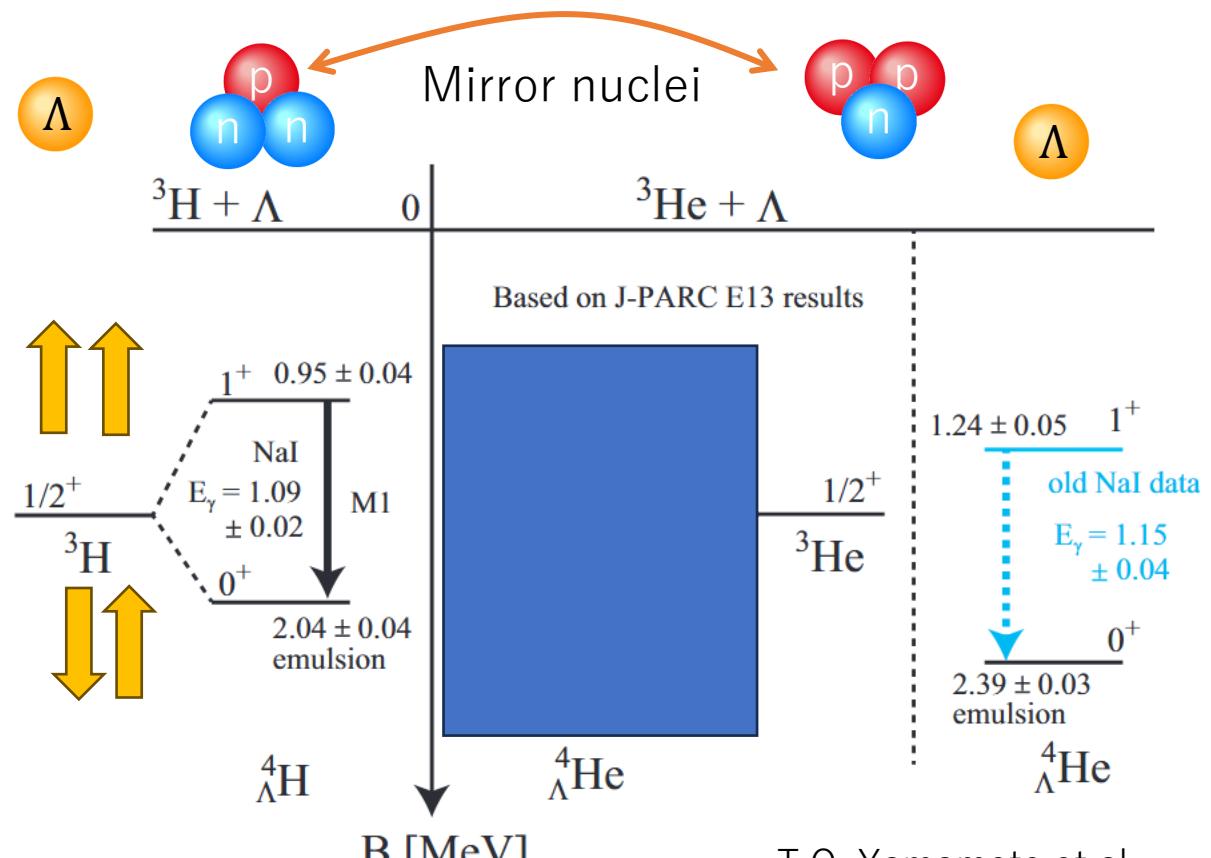
- $B_{\Xi^-} = 6.27 \pm 0.27$ MeV

J-PARC E13

$S=-1, A=4$ hypernuclei γ -ray



- Charge Symmetry Breaking (CSB) in g.s. of ${}^4_{\Lambda}\text{H}$ and ${}^4_{\Lambda}\text{He}$
 - ~350keV
 - Not theoretically understood.
- Energy splitting due to spin-spin
 - previous γ -ray measured by NaI
 - ${}^3_{\Lambda}\text{He}$: Not reliable according to experts.
- ${}^3_{\Lambda}\text{He}$ g-ray measurement needs update.
 - J-PARC E13 measured it using Germanium detector with an order of mag better resolution.



T.O. Yamamoto et al,
PRL115, 222501 (2015)

Charge symmetry holds well in NN.

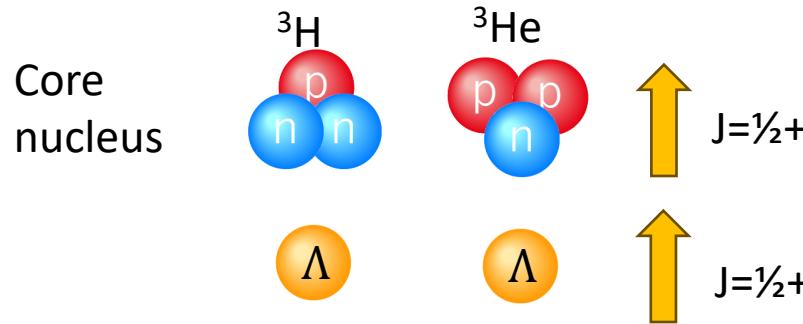
- Similarity of strong interactions btw pp and nn.

 Mirror nuclei ${}^3\text{H}$ vs ${}^3\text{He}$

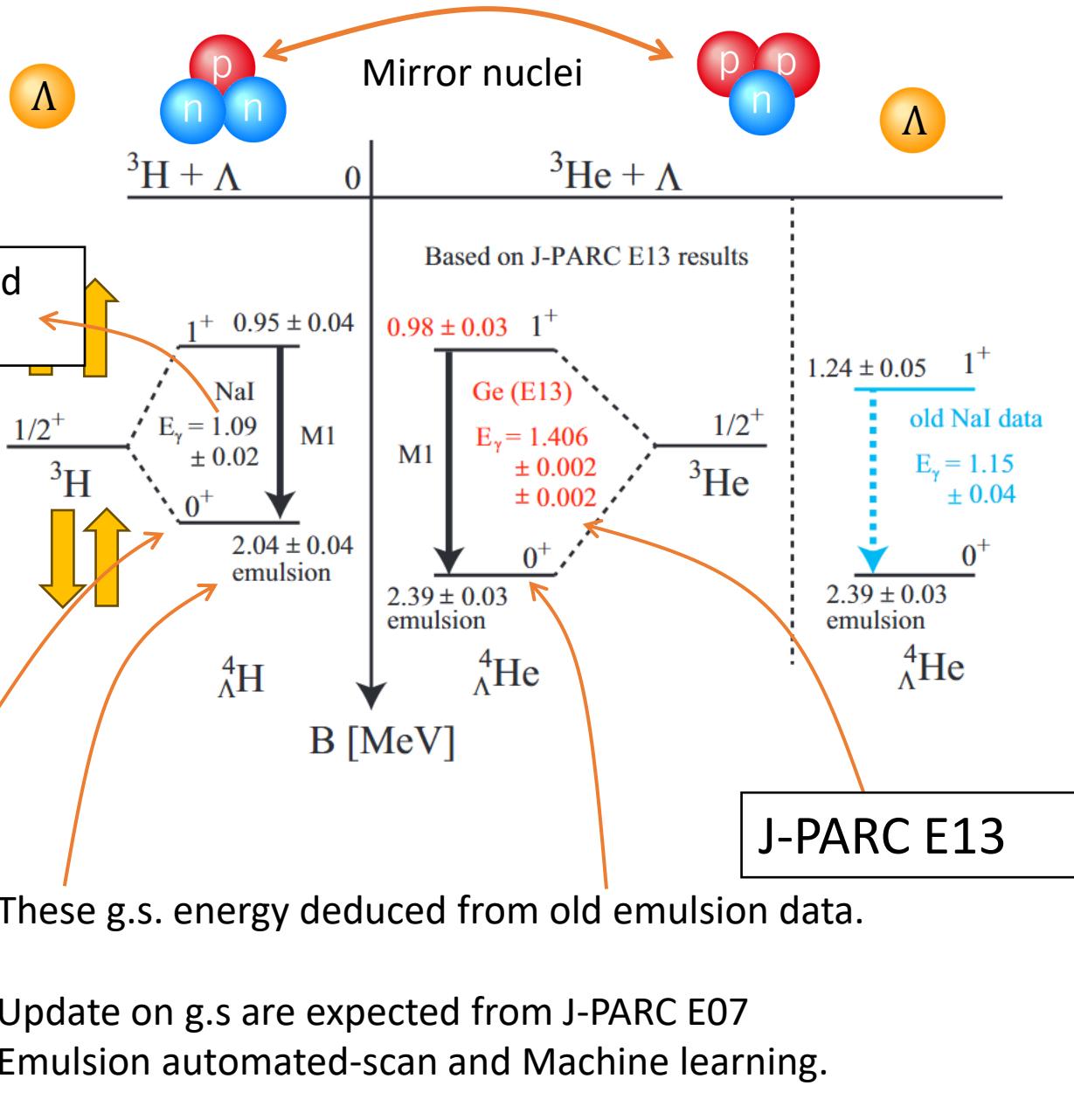
- $\Delta M = 492$ keV
- $\Delta B_{\text{total}} = 764$ keV → 70keV after Coulomb corr.
 - $\rho - \omega$ mixing : Ann. Rev. N.P. Sci. 56,253(2006)

J-PARC E13

S=-1, A=4 hypernuclei γ -ray



- J-PARC E13 confirmed that charge symmetry is broken.
 - ΛN CSB interaction has spin-dependence.
 - $\Lambda N - \Sigma N$ coupling plays an important role in CSB.



MAMI-C Measurement: $2.157 \pm 0.005 \pm 0.077$ MeV
Nucl. Phys. A 954, 149 (2016)

K1.8BR : J-PARC E15 – $\bar{K}NN$ (“K-pp”)

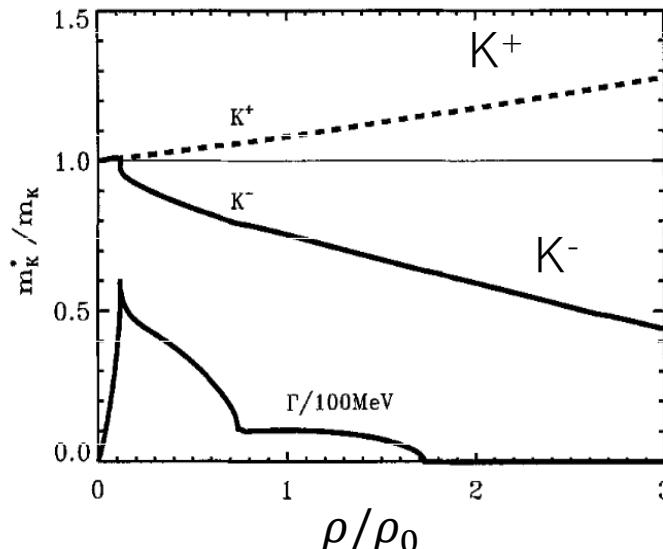
A search for deeply-bound kaonic states by in-flight ${}^3\text{He}(K^-, n)$

\bar{K} : K^- and \bar{K}^0

- $\bar{K}N$ Strongly attractive in $I=0$. $\Lambda(1405) = \bar{K}N$ molecule
- Larger Kaonic nucleus? $\bar{K}NN$, $\bar{K}NNN \dots$

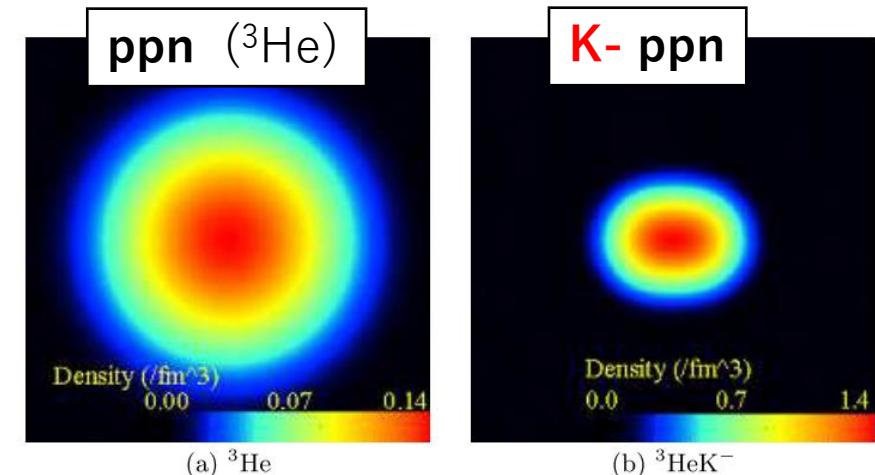
E. A. Veit et al.,
Phys. Rev. D 31, 1033 (1985)

SU(3) χ EFT that reproduce $\Lambda(1405)$ as
Bound-state of $K^{\bar{b}ar}N$,
Suggests K^- mass dropping in medium



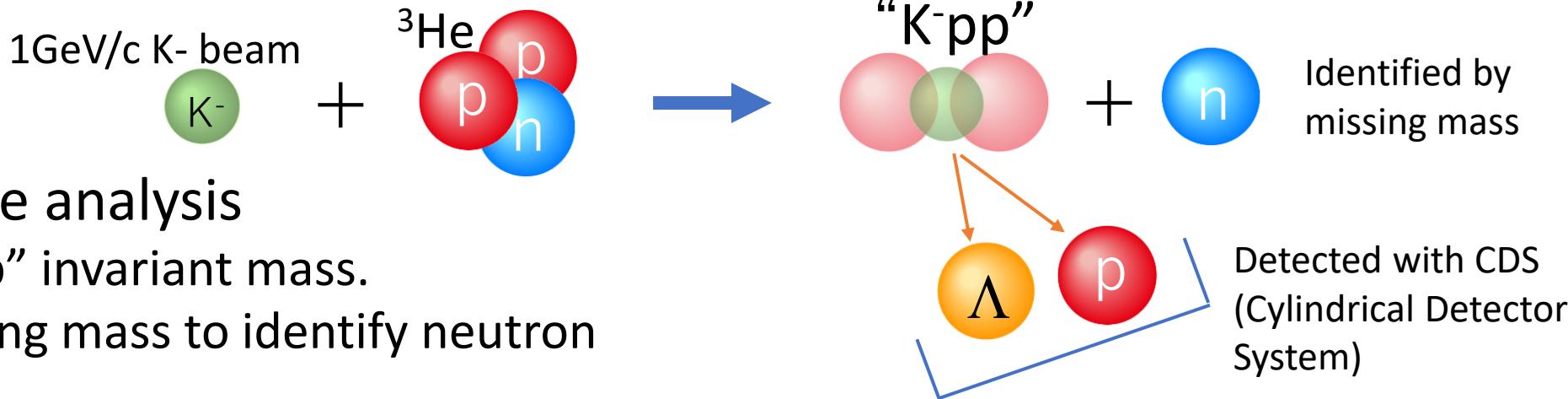
K- mass
reduction
in medium?

Theoretical calculation suggests shrinkage of
the system. Glue like role of K.
This early study may not be realistic, but interesting
thing may happen.



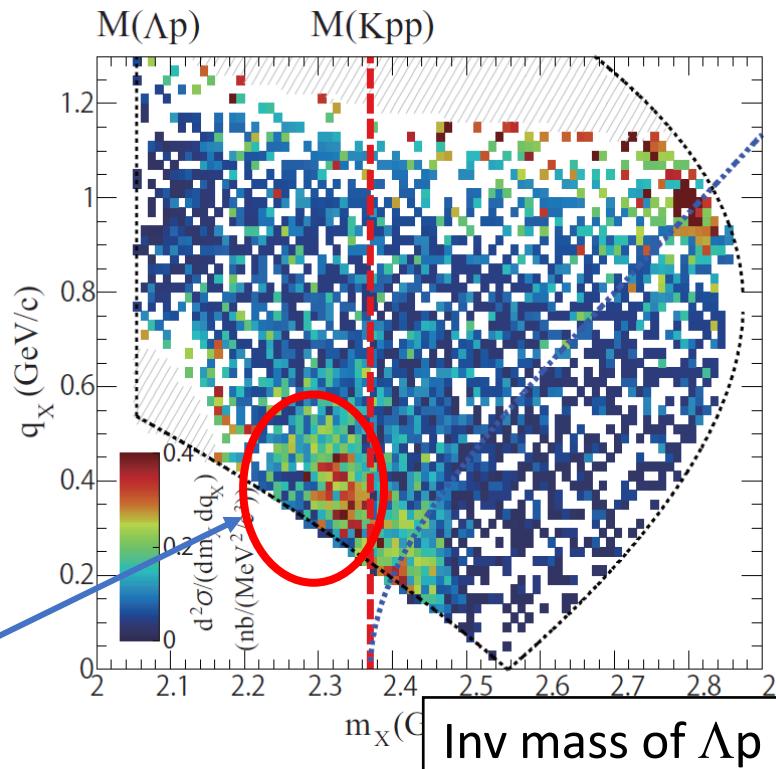
J-PARC E15 – $\bar{K}NN$ (“K-pp”)

$$q_x = |\mathbf{p}_{\bar{K}}^{\text{lab}} - \mathbf{p}_n^{\text{lab}}|,$$



- Exclusive analysis
 - “K-pp” invariant mass.
 - Missing mass to identify neutron

q_x :Momentum transfer to the Λp system



Momentum transfer independent component
“K-pp” bound state!

Quasi-free process.

$$M = \sqrt{4m_N^2 + m_K^2 + 4m_N \sqrt{m_K^2 + q_x^2}}$$

Momentum transfer = \bar{K} momentum

“ $\bar{K}NN$ ” model fitting

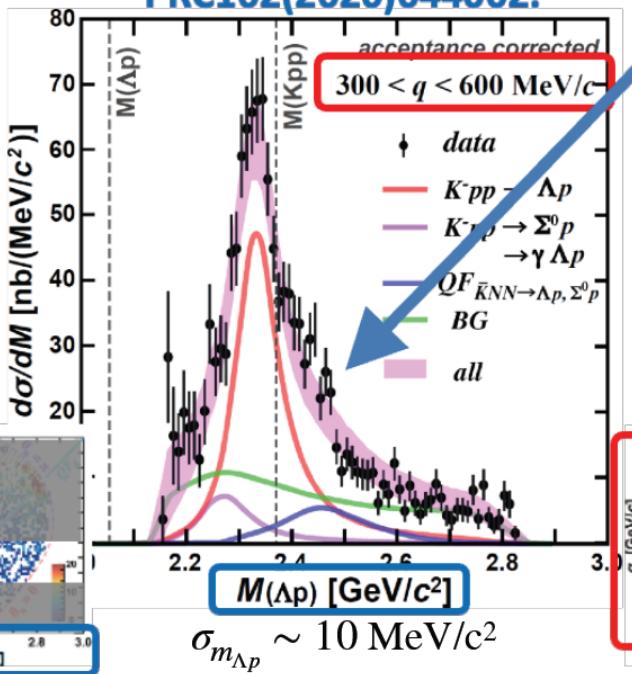
$0.3 < q_x < 0.6 \text{ GeV}/c$: Signals are well separated from other process

Fit with PWIA

$$\sigma(M, q) \propto \rho(M, q) \times \frac{\text{phase space}}{\text{Breit Wigner}} \times \text{form factor}$$

$$\frac{(\Gamma_{Kpp}/2)^2}{(M - M_{Kpp})^2 + (\Gamma_{Kpp}/2)^2} \times \exp\left(-\frac{q^2}{Q_{Kpp}^2}\right)$$

PRC102(2020)044002.



$B_{Kpp} \sim 40 \text{ MeV}$, $\Gamma_{Kpp} \sim 100 \text{ MeV}$
 \rightarrow large binding energy

$Q_{Kpp} \sim 400 \text{ MeV}$ (c.f. $Q_{QF} \sim 200 \text{ MeV}$)

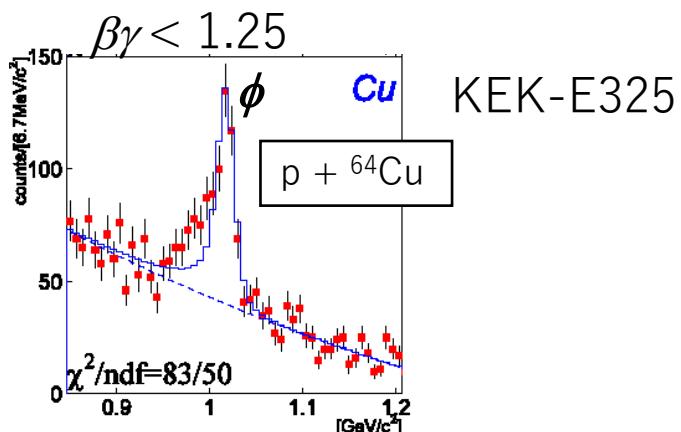
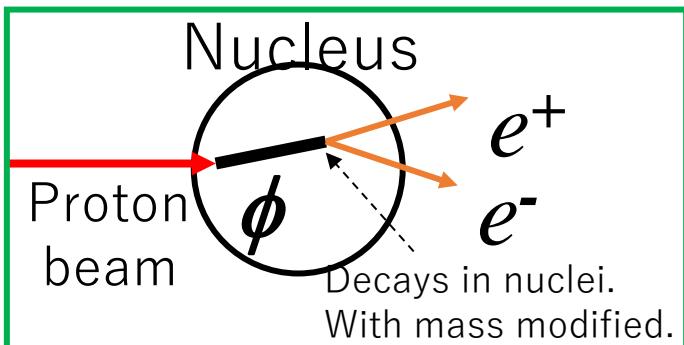
$$R = \frac{\hbar}{Q} \frac{(2m_N + m_{\bar{K}})}{2m_N} \sim 0.6 \text{ fm}$$

Indication of compact system.

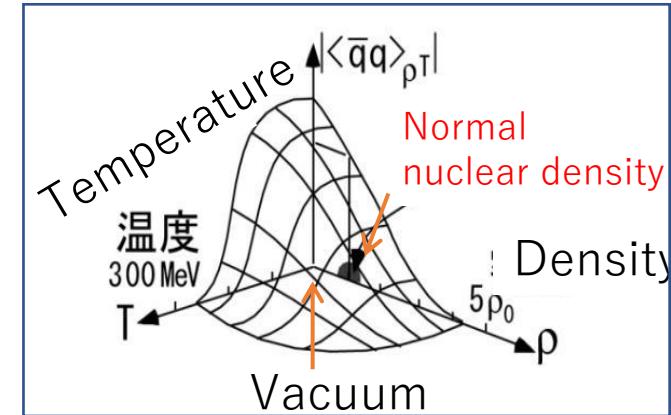
- Mesonic decay published
 - Yamaga et al. PRC110, 014002 (2024)
- Preliminary $K^{\bar{}}NNN$ results obtained. Effort continued
 \rightarrow J-PARC E80
- Upgrading detector.
 - x1.6 larger solid angle
 - x4 higher neutron detection eff.
 - Aim at data taking 2026.
- To study
 - $K^{\bar{}}NNN$ (J-PARC E80)
 - $K^{\bar{}}NN$ in detail.
 - Spin parity
 - Isospin partner $K-pn$
 - And more...

High-p : J-PARC E16 [on-going] ϕ meson in nuclei

- Spontaneous breaking of chiral symmetry plays an important role in generating hadron mass.
- Chiral symmetry breaking $\langle \bar{q}q \rangle \neq 0$ depends on density, so the hadron mass.
- KEK E325
 - 12GeV $p + A \rightarrow \rho/\omega/\phi \rightarrow e^+e^-$
 - Observed what can be interpreted as an in-medium spectral change.
 ϕ mass 3.4% reduction at $\rho_0 \rightarrow$ symmetry restored?
- **J-PARC E16 experiment:**
 - 30GeV $p + A \rightarrow \rho/\omega/\phi \rightarrow e^+e^-$, (K^+K^- E88)
 - Compared to E325, 6 times stat in 1st Run, Resolution $11 \rightarrow 6$ MeV
 - Spectrometer constructed for 1st physics run.
 - Commissioning run finished in June 2024.
 - 1st physics RUN in FY2025.



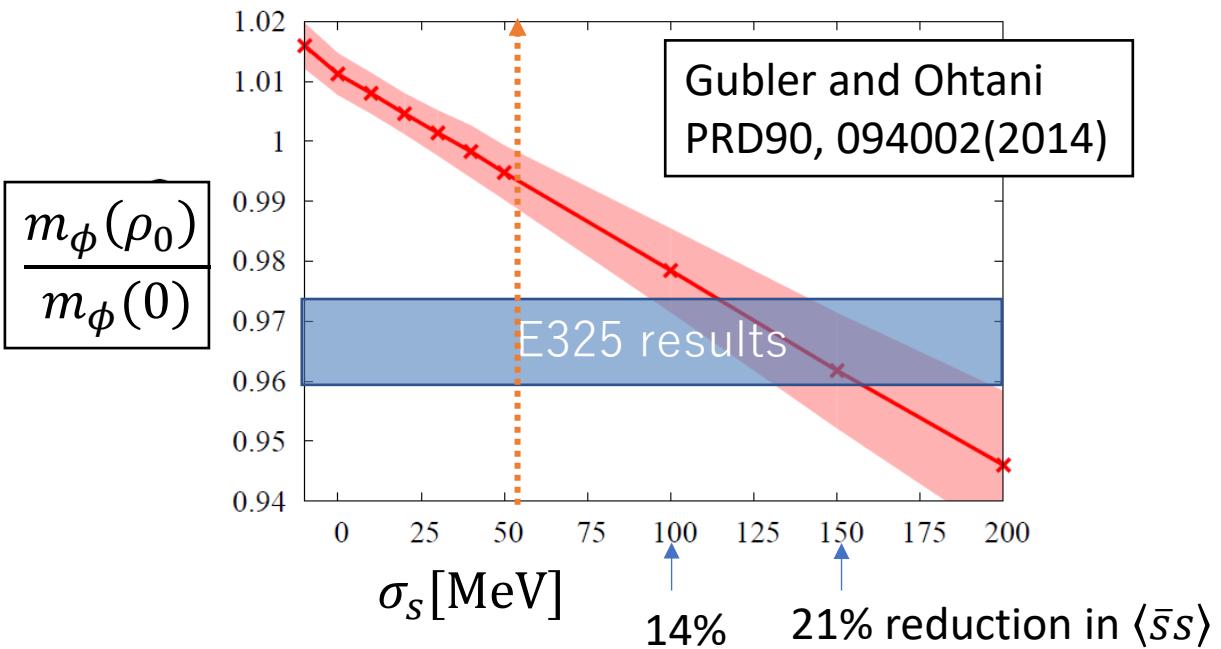
NJL model
M. Lutz et al.
Nucl. Phys. A542,52(1992)



QCD sum rule connects mass to condensates

- QCD sum rule connects:
 - [mass of ϕ meson] and $\langle \bar{s}s \rangle_\rho$
- At leading order,

$$\langle \bar{s}s \rangle_\rho = \langle \bar{s}s \rangle_0 + \langle N | \bar{s}s | N \rangle \cdot \rho$$



$\langle \bar{s}s \rangle_\rho$: s -quark condensate at density ρ

$$\sigma_s = m_s \cdot \langle N | \bar{s}s | N \rangle$$

- Chiral-odd twist-3 distribution function $e^s(x)$

Definition for parton “a” (Arxiv: hep-ph/0312044)

$$e^a(x) = M_N \int_{-\infty}^{\infty} \frac{dz_0}{2\pi} e^{ixM_N z_0} \langle N | \bar{\psi}_a(0) \psi_a(z) | N \rangle_{z_3=-z_0, z_\perp=0}$$

Integral gives σ_s

$$\int_{-1}^1 dx [e^s(x)] = \sigma_s / \hat{m}_s$$

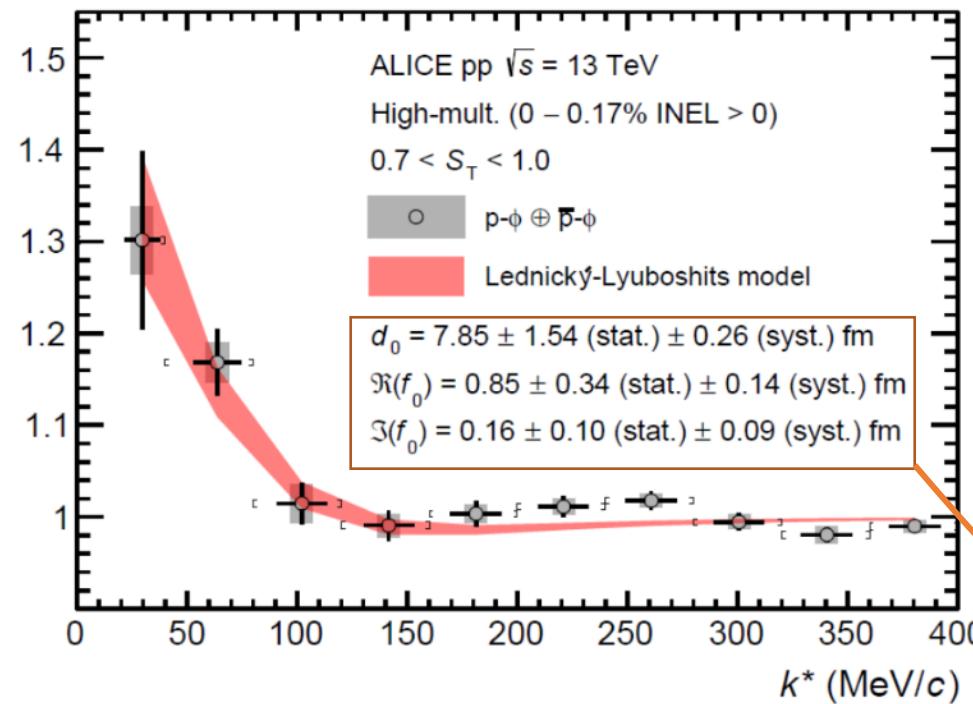
$$\int_{-1}^1 dx [e^u(x) + e^d(x)] = \sigma_{\pi N} / \hat{m}$$

Information from p- ϕ interaction

Attractive interaction \rightarrow mass reduction



ALICE two particle correlation



Phys. Rev. Lett. 127, 172301(2021)

f_0, a_0 : scattering length.
 d_0, r_0 : effective range.

HAL QCD method

arXiv:2205.10544 (2022)

Scattering length and effective ranges are deduced for spin 3/2 combination.

$$a_0^{(3/2)} = -1.43(23) \text{ fm}$$

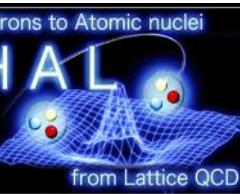
$$r_0^{(3/2)} = 2.36(10) \text{ fm}$$

• Mass reduction

- HAL QCD: $5.3\% \pm 0.4\%$
- ALICE: $5.8\% \pm 1.8\%$
- E325 : $3.4\% {}^{+0.6}_{-0.7}$

$f_0, d_0 \rightarrow$ First order optical potential

$$V(r) \sim \frac{1}{2m_\phi} 4\pi \rho(r) \frac{b}{1 + \frac{b}{d_0}}, b = f_0 \left(1 + \frac{m_\phi}{m_{proton}} \right) = \Delta M + i \Gamma/2$$

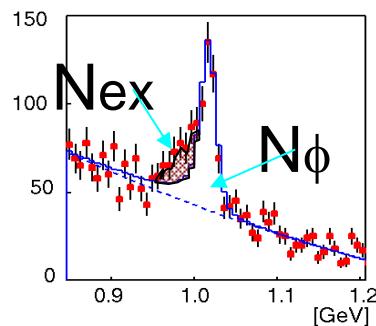
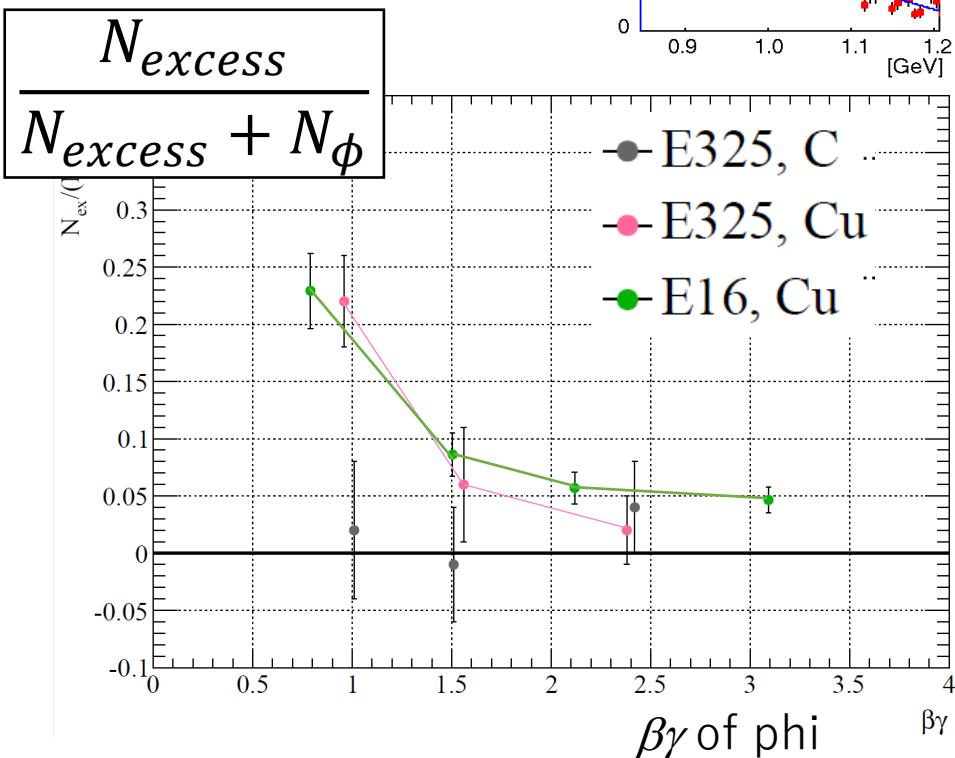


nucl-ex/0306011(2003)
And
E. Chizzali, R. Del Grande, L. Fabbietti

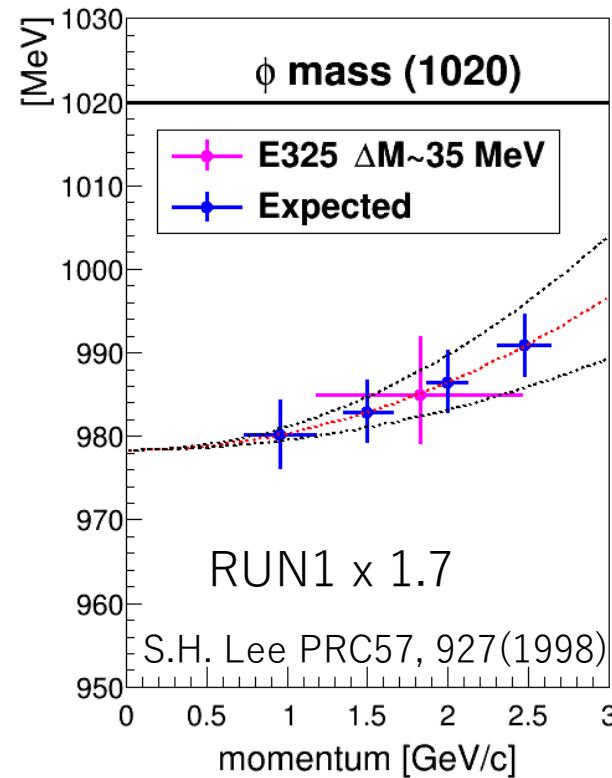
J-PARC E16

- 1st RUN expectation and beyond.

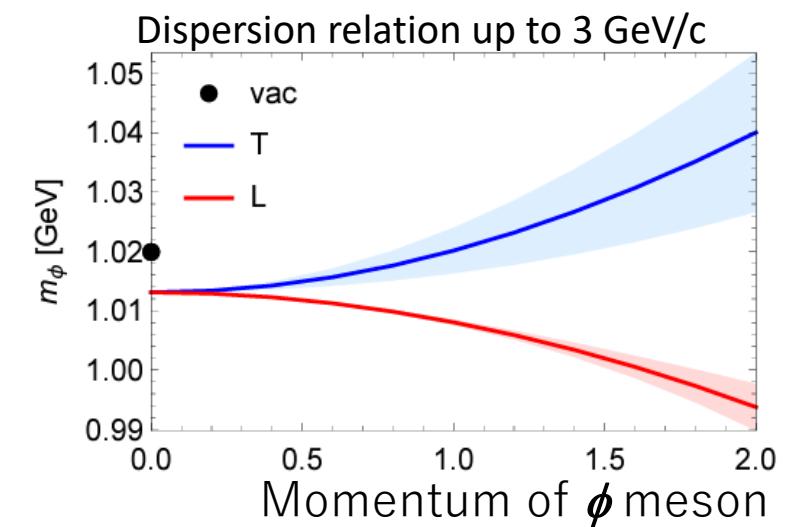
1st RUN expectation.
Clear tendency



Dispersion relation
can be obtained.



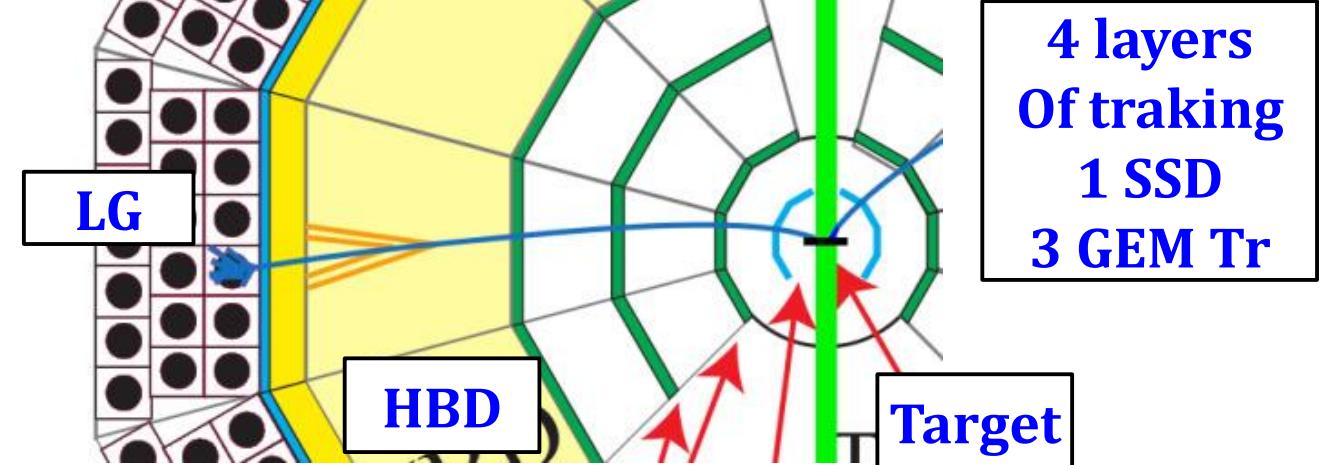
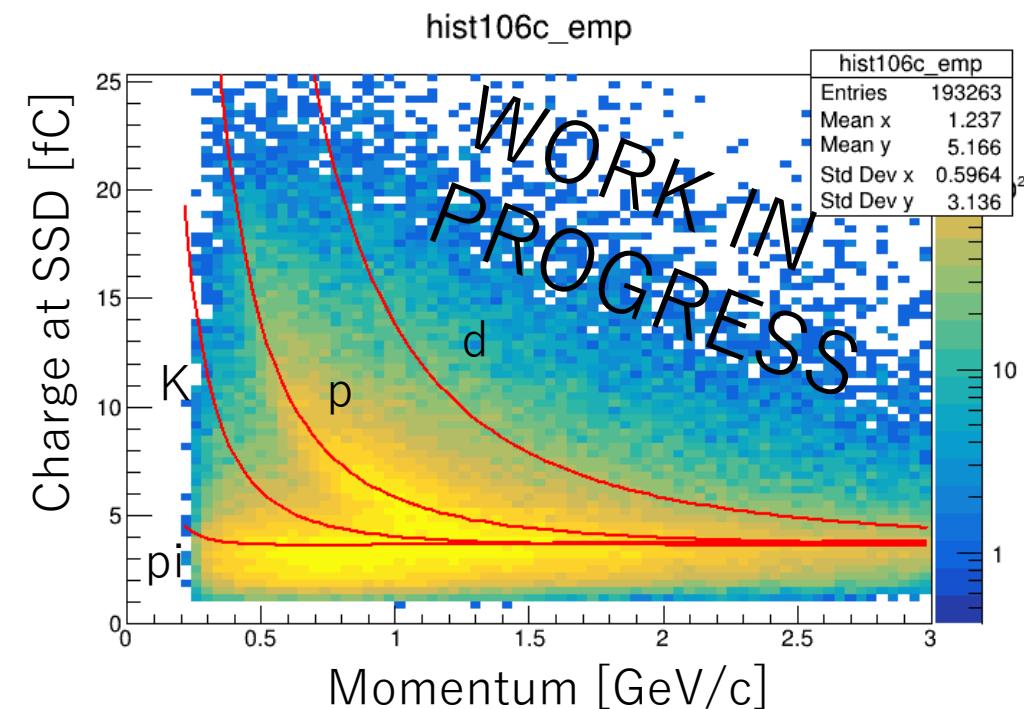
Polarization dependence
can be accessed.



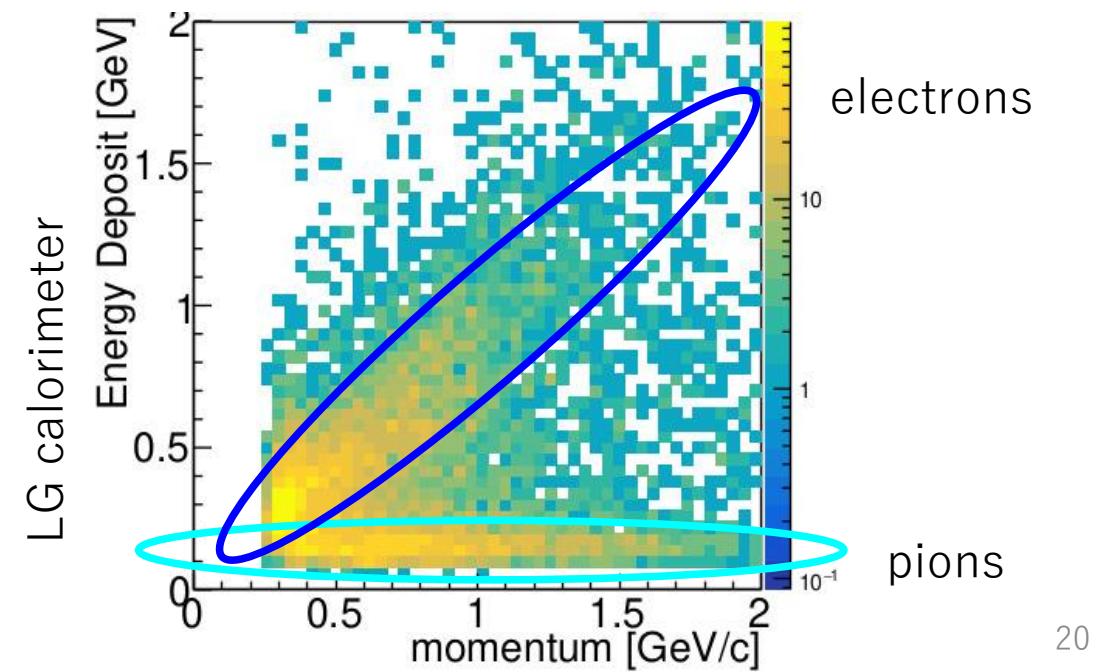
H.J. Kim, P.Gubler, PLB805, 135412 (2020)
I.W. Park et al, PRD 107, 074033 (2023)

J-PARC E16 from commissioning run.

- Tracking: 1-layer of Silicon detector and 3-layers of GEM tracker.
- Silicon detector was developed in collaboration with GSI-CBM.
- Clear proton locus. SSD worked.



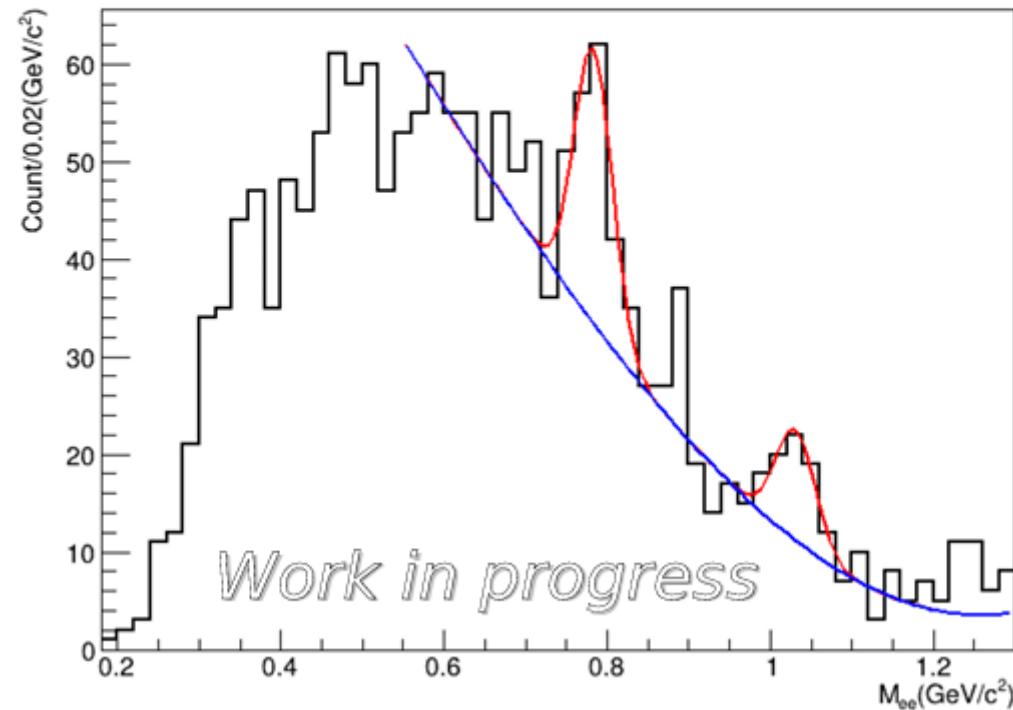
- Hadron Blind Detector (Cherenkov detector) to enhance electrons.
- Clear Momentum vs Energy (deposit on LG calorimeter) correlation of electron seen.



J-PARC E16 : ω and ϕ meson peaks.

- Calibration ongoing to get better resolution and for better reconstruction efficiency.

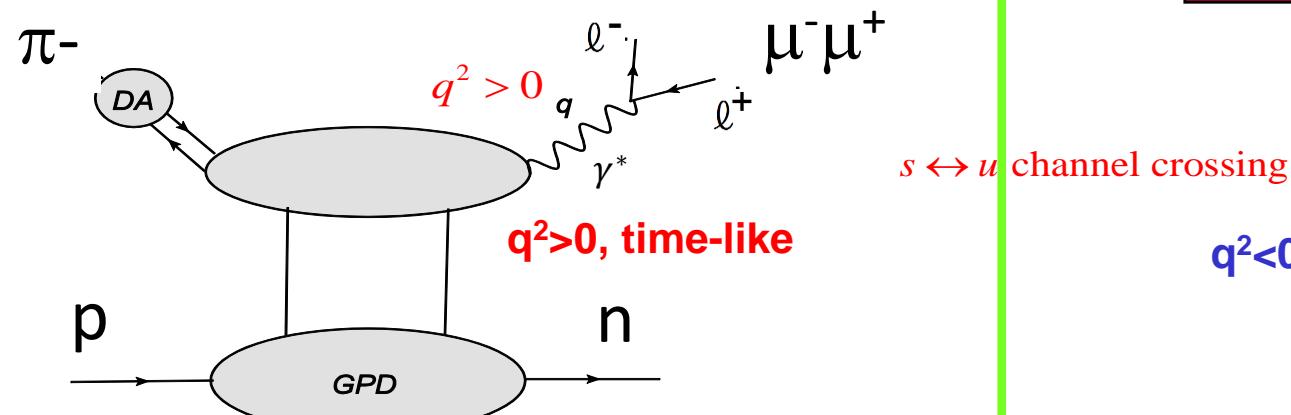
e^+e^- invariant mass
(red: Gauss fit,
blue: polynomial bkg)



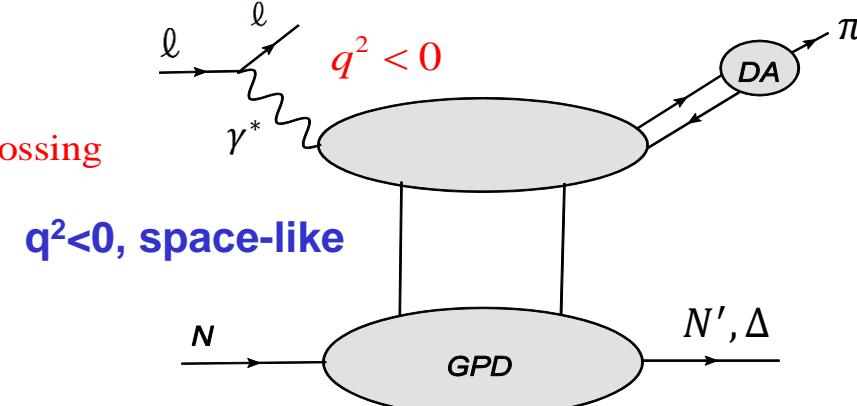
Exclusive Drell-Yan [high-p, LOI]

Muller et al., PRD 86 031502(R) (2012)

Exclusive meson-induced DY



Deeply Virtual Meson Production (DVMP)

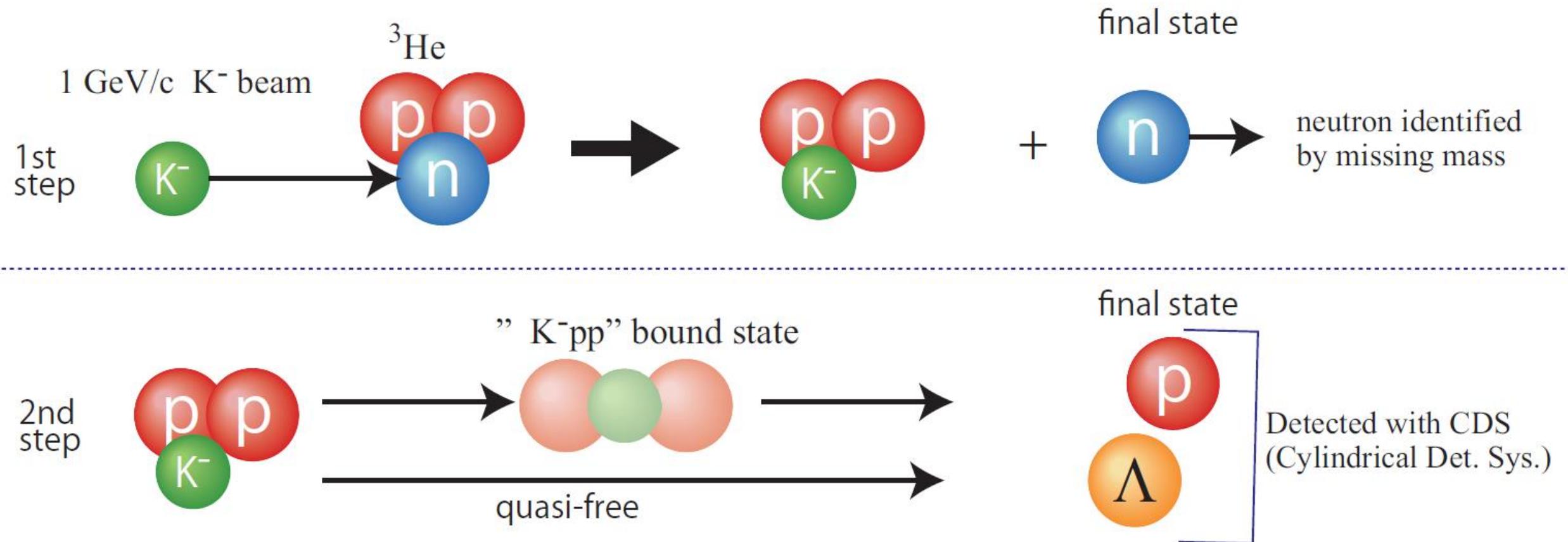


- Upgraded high-p beamline can provide high momentum secondary particles.
 - $\pi^- p \rightarrow n \gamma^* \rightarrow n \mu^+ \mu^-$
 - $\pi^- = 10\text{--}20 \text{ GeV}/c$, $\sim 10^7$ / spill, $d\mu/p \sim 0.1\%$
 - $\mu^+ \mu^-$ measured by spectrometer. $dM/M \sim 1\%$
 - Neutron identified by missing mass—ensuring exclusive proc.
- Sensitive to pion-DA and, nucleon GPDs (in ERBL region)
 $\tilde{H}^d - \tilde{H}^u$ and $\tilde{E}^d - \tilde{E}^u$.

$$\tilde{H}^u(x, \xi, t) \text{ etc..}$$

Summary

- J-PARC hosts various physics programs to understand strongly interacting system from quarks to neutron stars.
- Some of experimental results, on-going and planned experiments are discussed
 - S=-1, -2 hyper nucleus spectroscopy
 - Kbar NN
 - Phi in nucleus
 - Pion-DY.



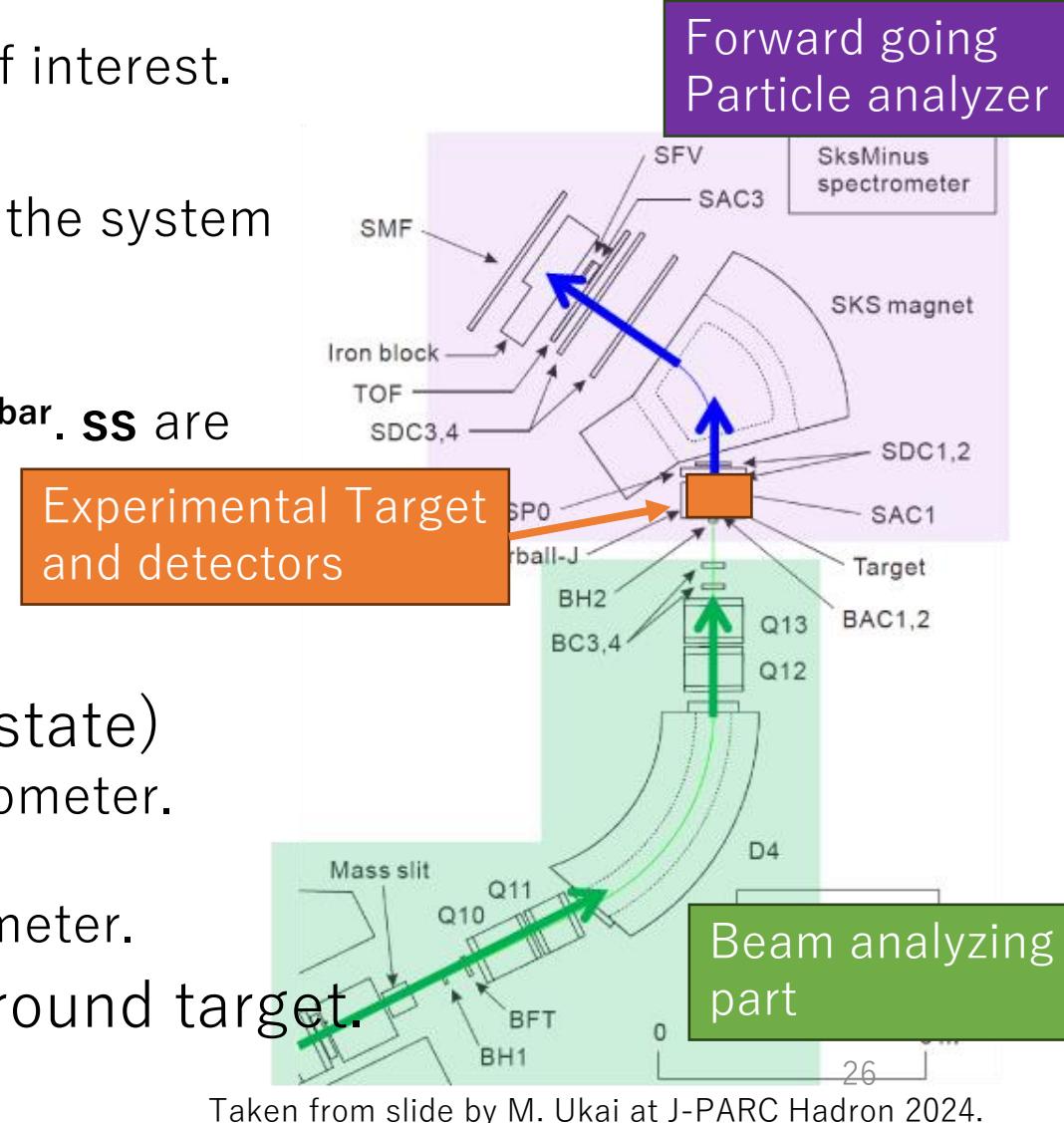
K1.8 beam line : S=-1,-2 hypernucleus, YN scattering experiment.

Hyperon, hypernuclei production scheme

- (beam , out-going particle)
- (K^-, π^-) -- S=-1 system
 - \mathbf{s} in the beam and the \mathbf{s} is put into the system of interest.
- (π, K^+) -- S=-1 system
 - $\mathbf{ss}^{\bar{\text{bar}}}$ pair created. K^+ take away $\mathbf{s}^{\bar{\text{bar}}}$. \mathbf{s} is put in the system of interest.
- (K^-, K^+) -- S=-2 system
 - \mathbf{s} in the beam and $\mathbf{ss}^{\bar{\text{bar}}}$ created. K^+ take away $\mathbf{s}^{\bar{\text{bar}}}$. \mathbf{ss} are put into the system of interest.

K1.8 common concept

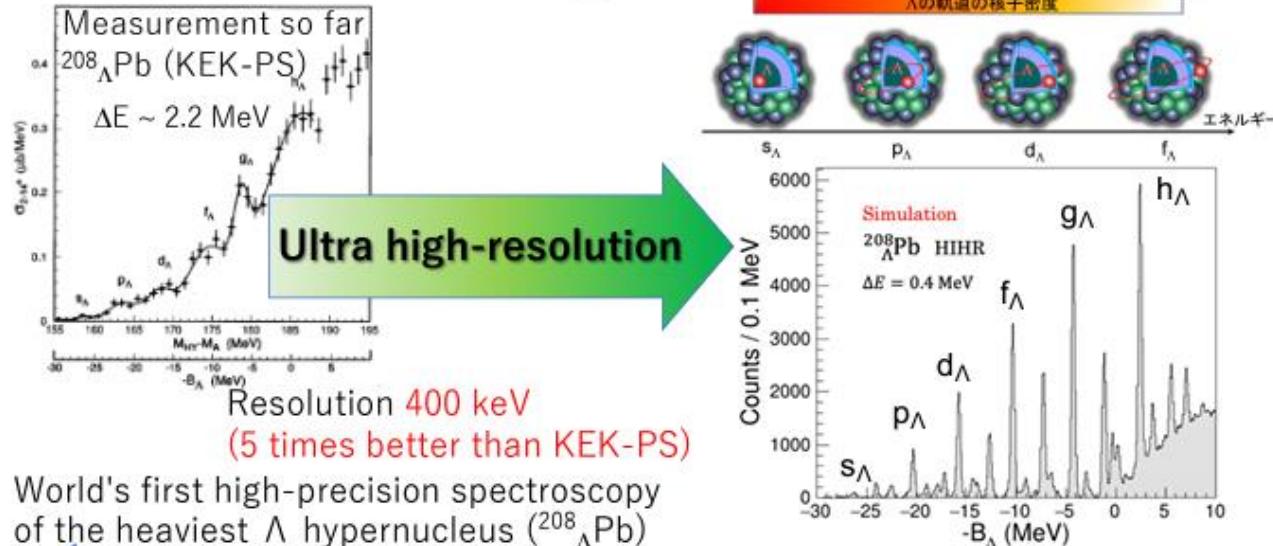
- Missing mass technique. (Initial state)
 \longleftrightarrow invariant Mass (Final state)
 - Incoming beam is analyzed by beam line spectrometer.
 - Experimental target. (Seed of hypernuclei)
 - Out-going particle analyzed by forward spectrometer.
- Experiment specific detectors are placed around target.
 - γ -ray detectors / nuclear emulsion etc..



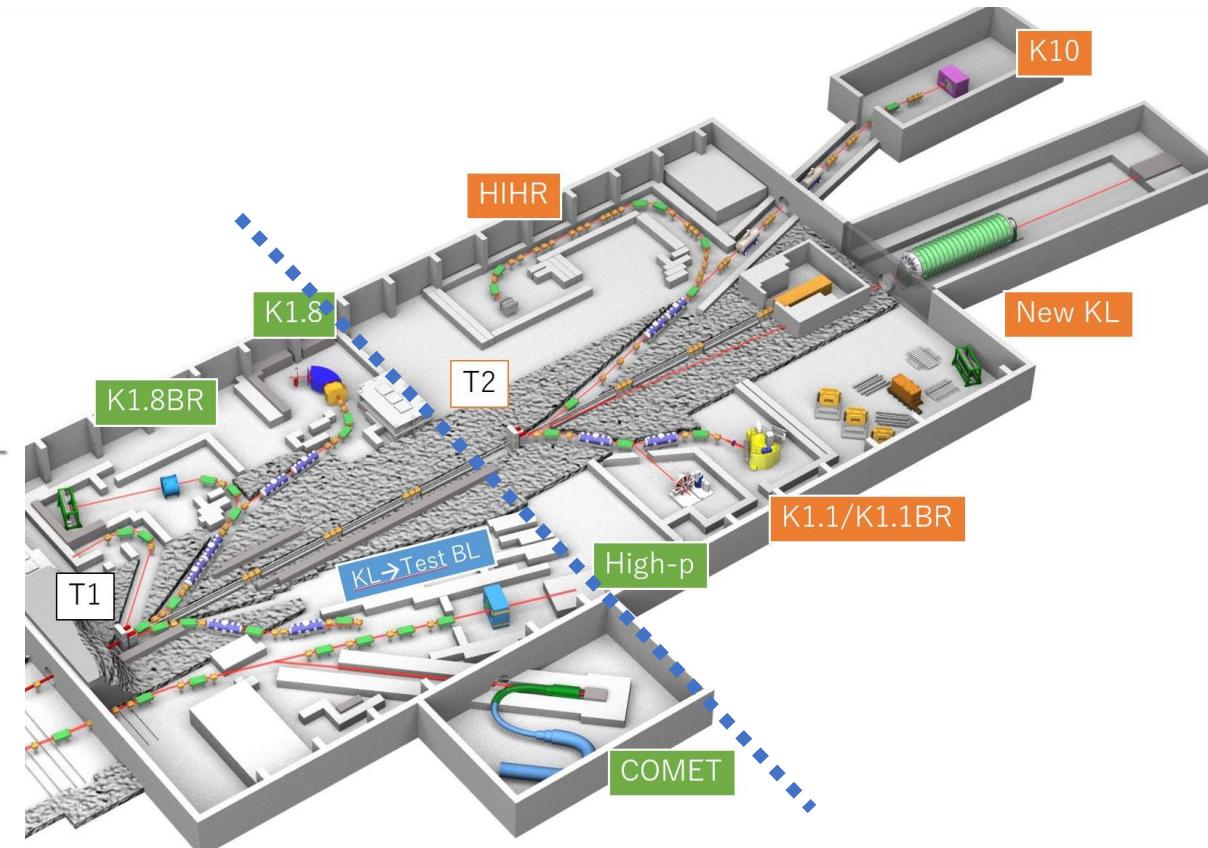
Taken from slide by M. Ukai at J-PARC Hadron 2024.

Hadron Hall Extension

Elucidation of YN interaction in nuclear matter
First high-resolution spectroscopy of the heaviest Λ hypernucleus



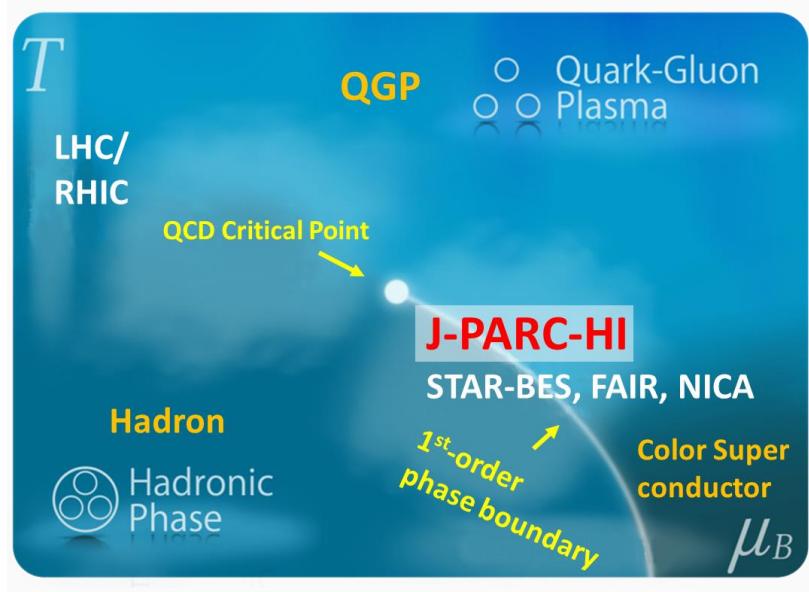
World's first high-precision spectroscopy
of the heaviest Λ hypernucleus ($^{208}\Lambda\text{Pb}$)



- Selected 1st priority in KEK PIP2022 (Project Implementation Plan)
- More beam lines with additional functionality
 - HIHR : High Intensity High Resolution beam by dispersion matching.
 - High precision systematic Λ hypernuclear spectroscopy up to Pb. Reveal 3BF.
 - K10 $\Omega \Xi^*$ spectroscopy, K1.1 polarized Λp scattering, New KL.

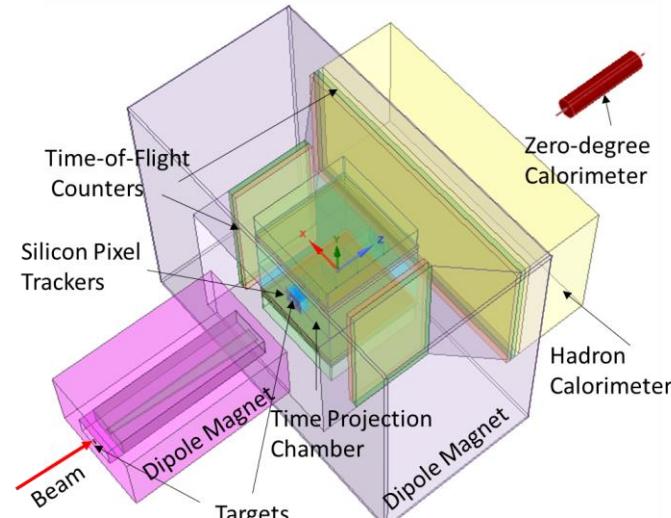
Future J-PARC Heavy Ion program

Explore the QCD phase diagram



EOS of Neutron Star
New state of the matter
Quark Phase
Color Super conductivity
Hadron physics in finite density

- Facility Upgrades Plan
 - New accelerator injector
 - New spectrometer
- Staging approach
 - Phase1:
 - Beam Intensity: 10^8 Hz for Au
 - Upgrade of the current E16 spectrometer
 - New LINAC and reuse of KEK-PS booster
 - Phase 2
 - Beam Intensity: 10^{11} Hz
 - New booster and new spectrometer

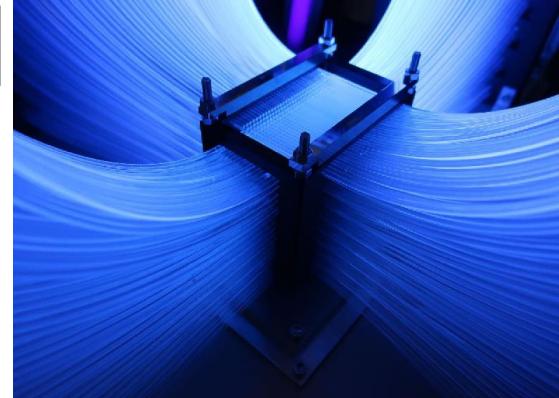


Schematic view of final spectrometer

$$\text{Elab(Au)} = 1\text{-}12 \text{AGeV}$$
$$\sqrt{s_{NN}} = 1.9\text{-}4.9 \text{GeV}$$

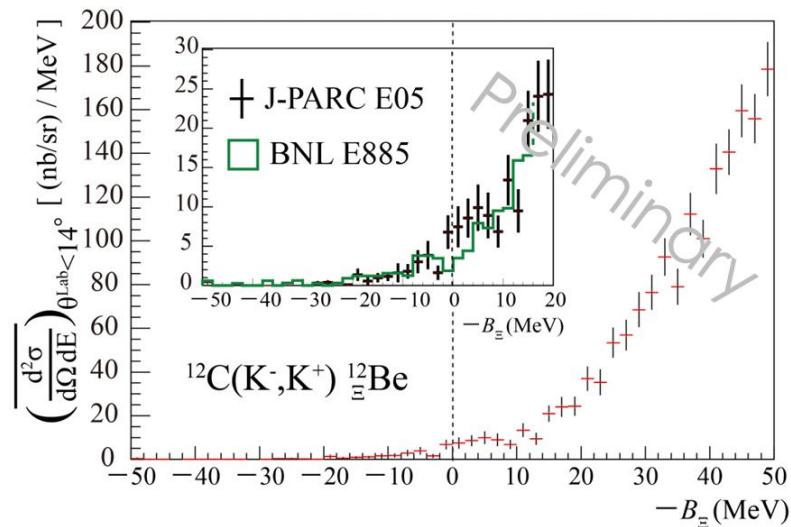
J-PARC E05(done) and E70(data taking)

AFT



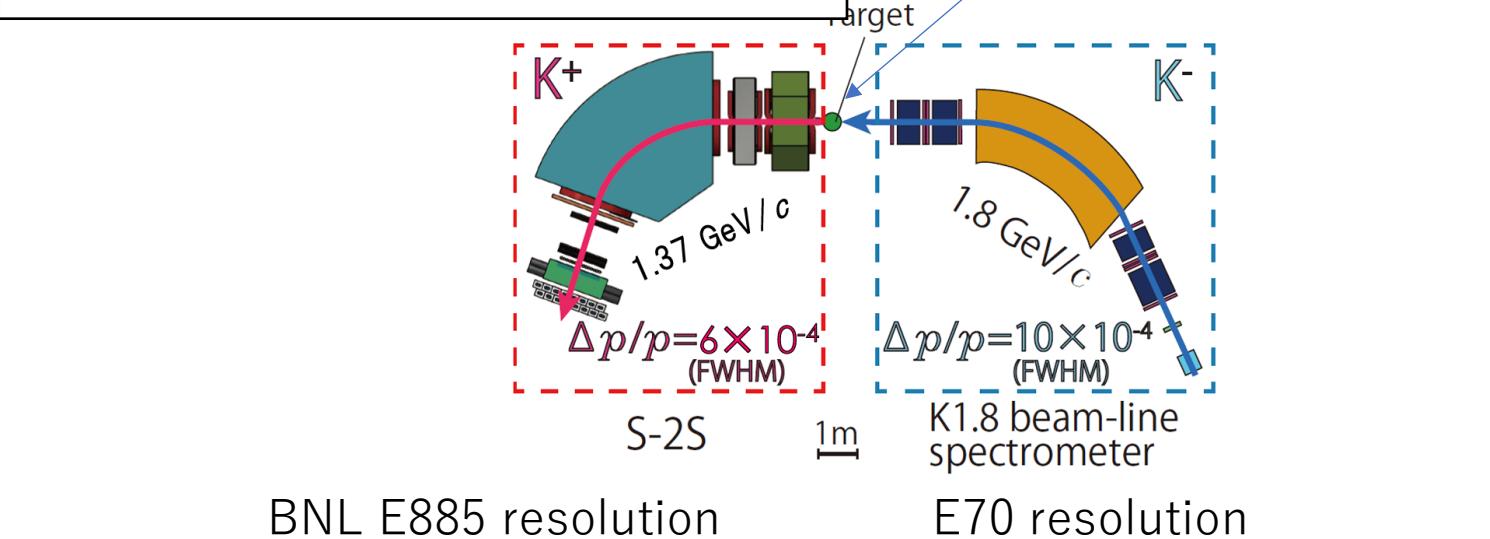
- ${}^{12}_{\Xi}\text{Be}$ via ${}^{12}\text{C}(\text{K}^-, \text{K}^+)$ reaction.
- Ξ^- nucleus search by
 - Missing mass.

J-PARC E05 results [submitted]
Events in bound region. Not clear.

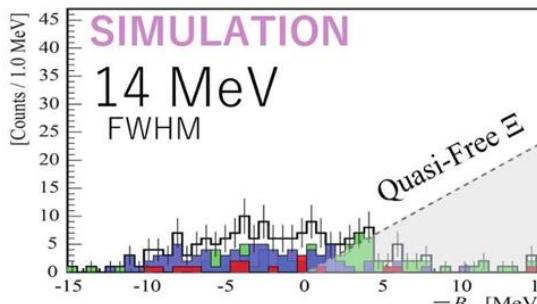


Resolution : 8 MeV (FWHM)

J-PARC E70: data taking.
+Spectrometer upgrade
+Active Fiber Target : Measure energy loss in target. Correction.



BNL E885 resolution

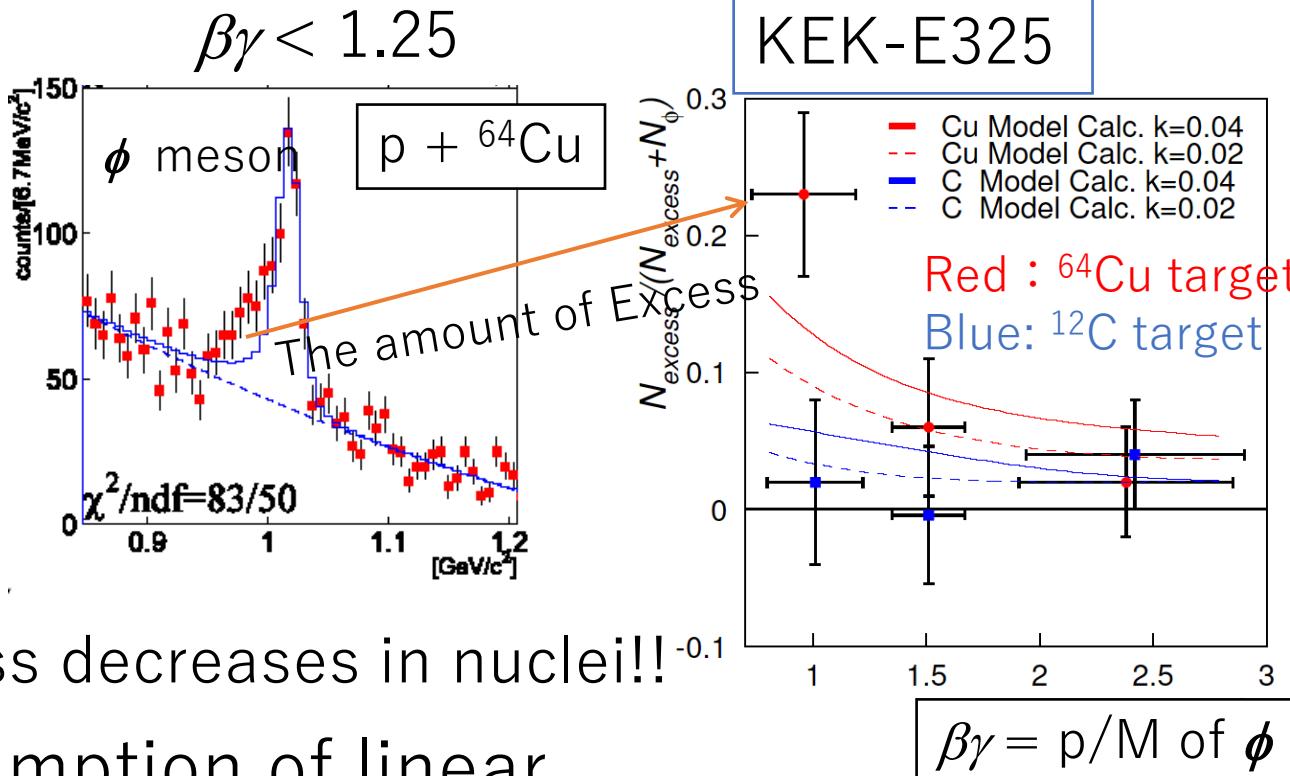
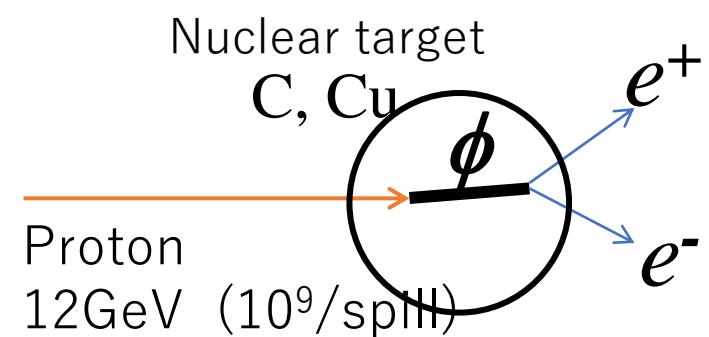


T. Gogami et al, EPJ web conf 271, 11002

($J^\pi = 1_1^-, 1_2^-$ and 1_3^-)

KEK-E325 results of ϕ meson

- The world's first results of ϕ modification.



- Conclusion: Mass decreases in nuclei!!
 - Under the assumption of linear dependence of mass and width on density.
 - Mass: $-3.4^{+0.6}_{-0.7}\%$ ↓ At normal nuclear density
 - Width: $\times 3.6^{+1.8}_{-1.2}$

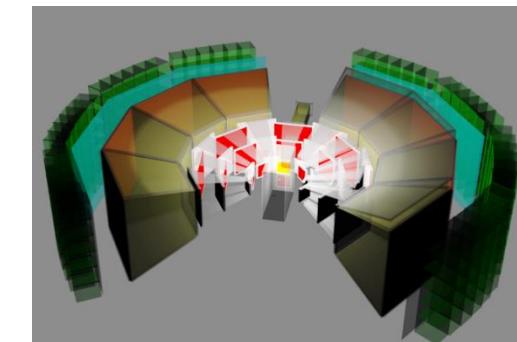
Assumption
In analysis

$$\frac{m(\rho)}{m(0)} = 1 - k_1 \left(\frac{\rho}{\rho_0} \right)$$
$$\frac{\Gamma(\rho)}{\Gamma(0)} = 1 + k_2 \left(\frac{\rho}{\rho_0} \right)$$

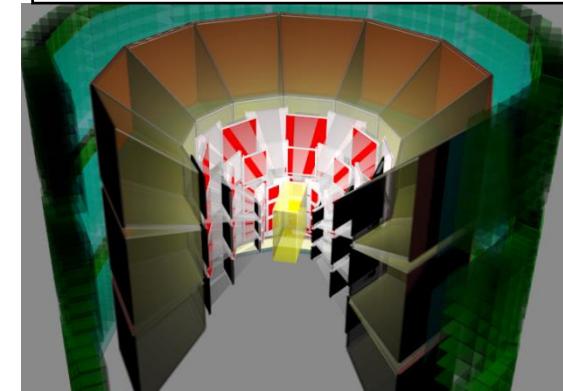
Staging approach

- **RUN 0a/b/c/d - 2020,2021,2023** – 413hrs.
 - **10 (SSD) + 8 (GTR) + 8 (HBD) + 8 (LG)** at last
 - Gradually increased acceptance and reached interm. Goal.
 - C+Cu targets
 - Beam / Detector commissioning
- **RUN 0e - 2024 (Apr.19-Jun.3) -- 206 hours.**
 - **8(SSD) + 10 (GTR) + 8 (HBD) + 8 (LG)**
 - Beam / Detector comm. + yield.
 - Upgraded Accelerator / DAQ. / Detectors.
- **RUN 1 2025(?)** -- 1280hrs (~53days)
 - **10 (SSD) + 10 (GTR) + 8 (HBD) + 8(LG)**
 - Physics data taking. ϕ : 15k for Cu.
 - Needs PAC approval based on comm. Runs.
- **RUN 2** -- 2560 hrs (~107 days)
 - **26 (SSD) + 26 (GTR) + 26 (HBD) + 26 (LG)**
 - + Pb/CH₂ target
 - Needs additional budget.

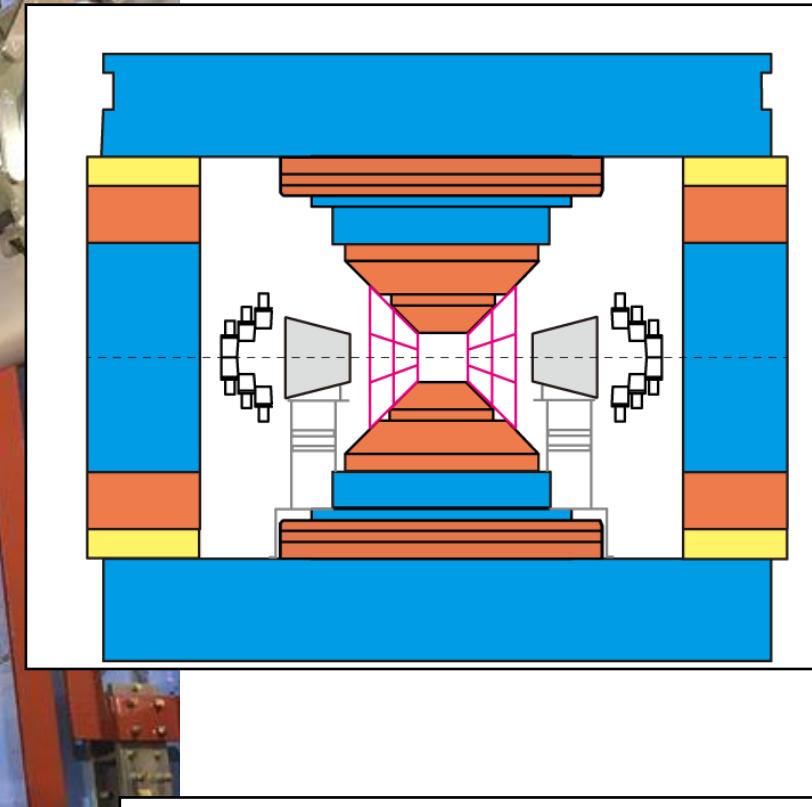
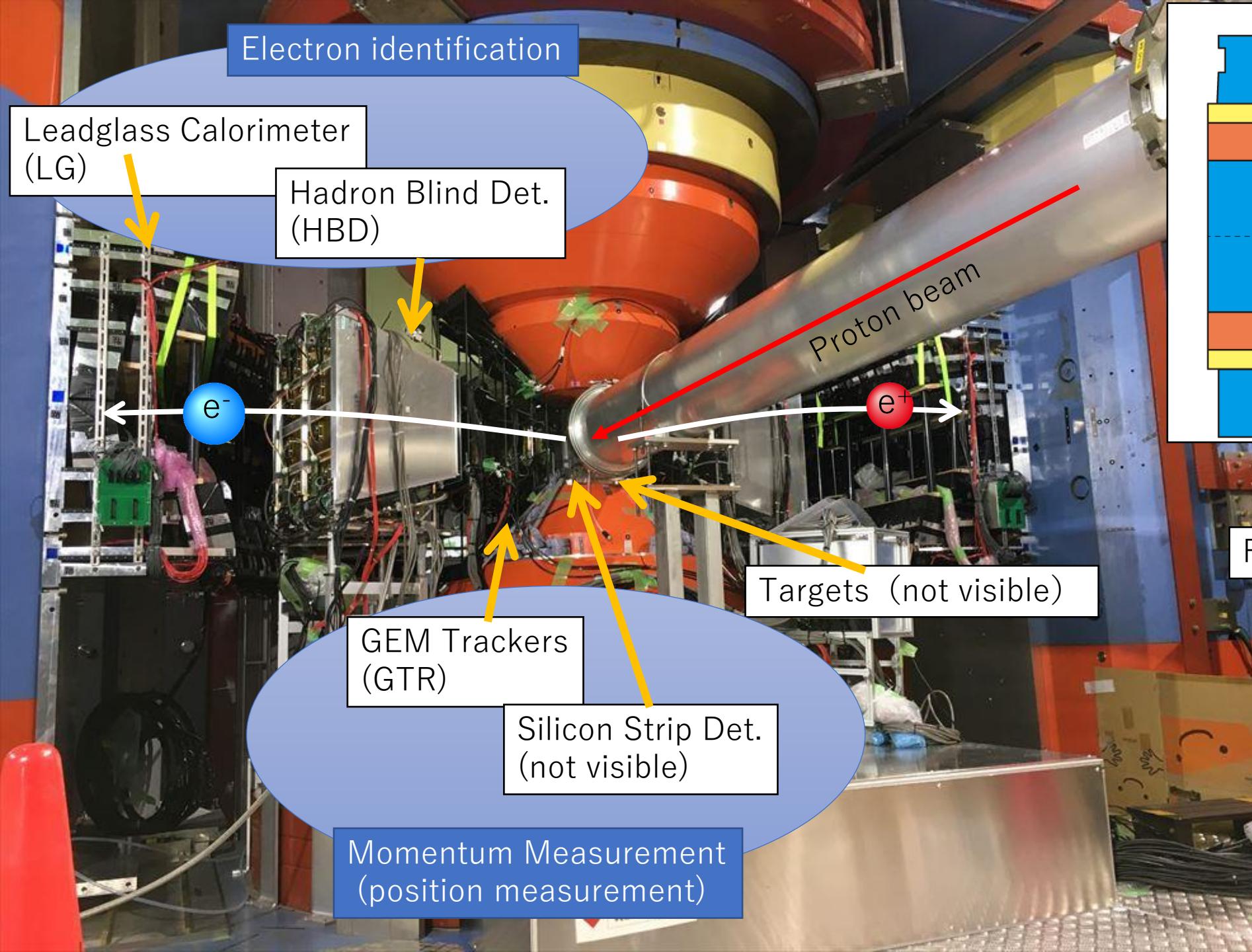
RUN 1 (8 modules)



RUN 2 (26 modules)

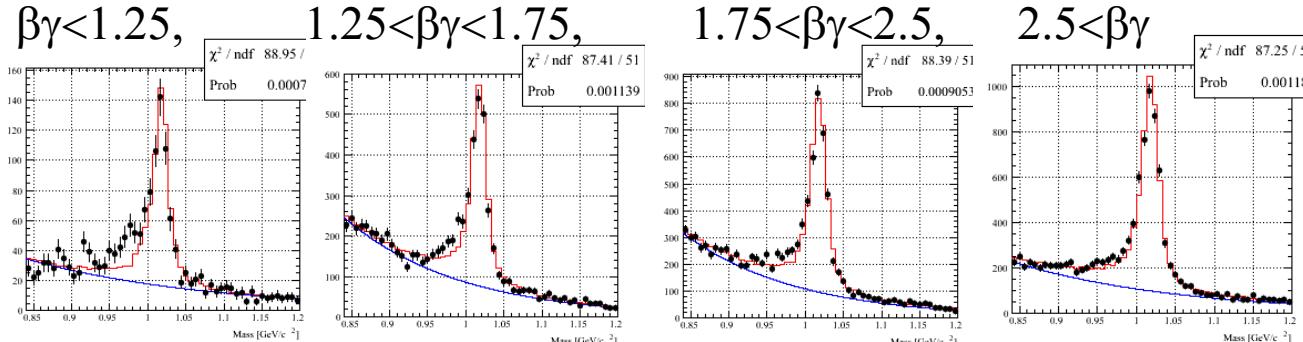


Electron identification



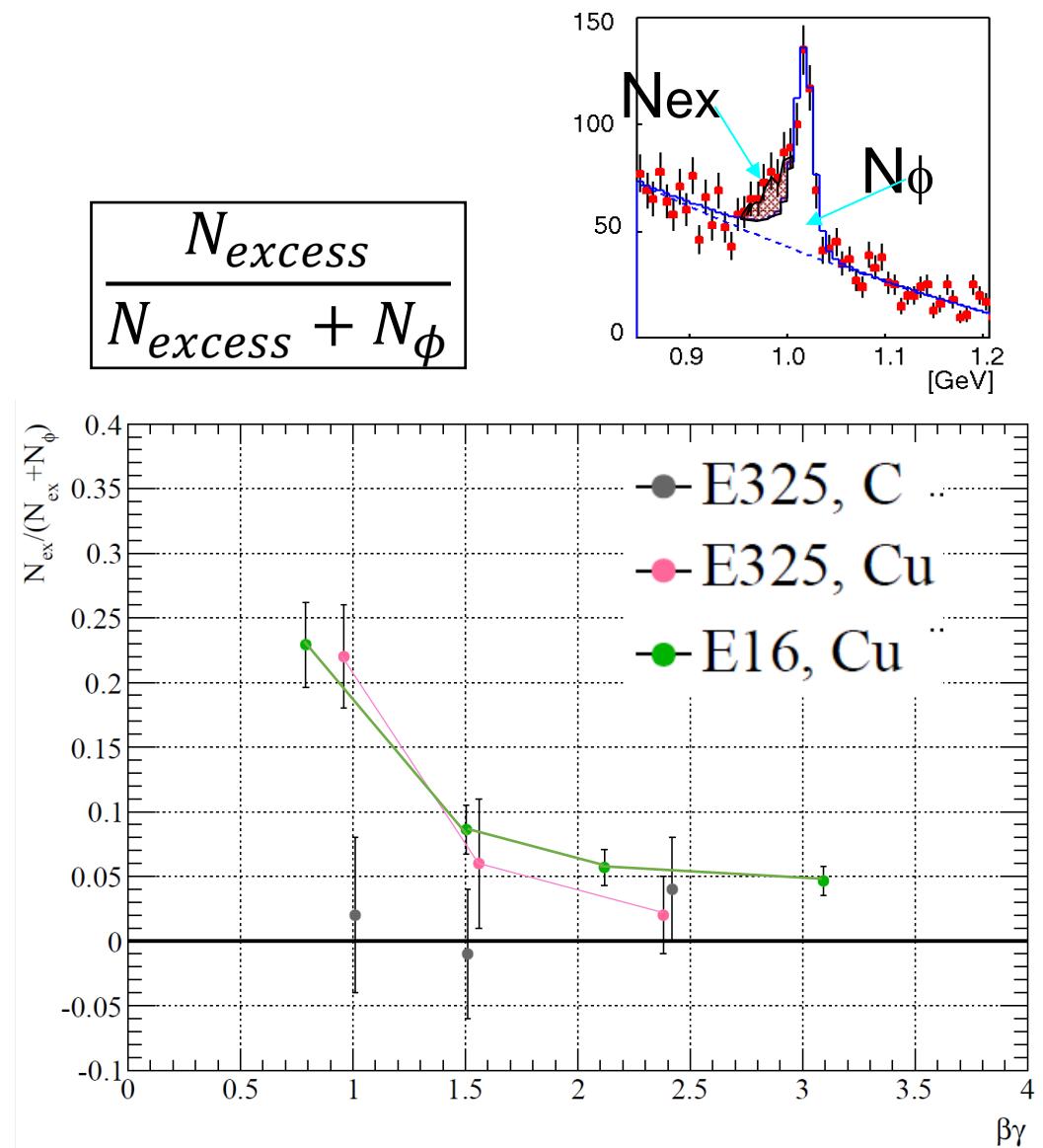
RUN1, Cu (INPUT:E325-BW)

Excess ratio vs $\beta\gamma$



(Fit fails when vacuum shapes are used.)

- ~15k ϕ for Cu target expected in RUN1
- All $\beta\gamma$ bins for Cu are significant in E16
- (cf) E325 only fastest $\beta\gamma$ bin is significant.
- Larger excess in lower $\beta\gamma$ bin.
- The tendency becomes clearer and more significant compared to E325.



- PRD93, 114034

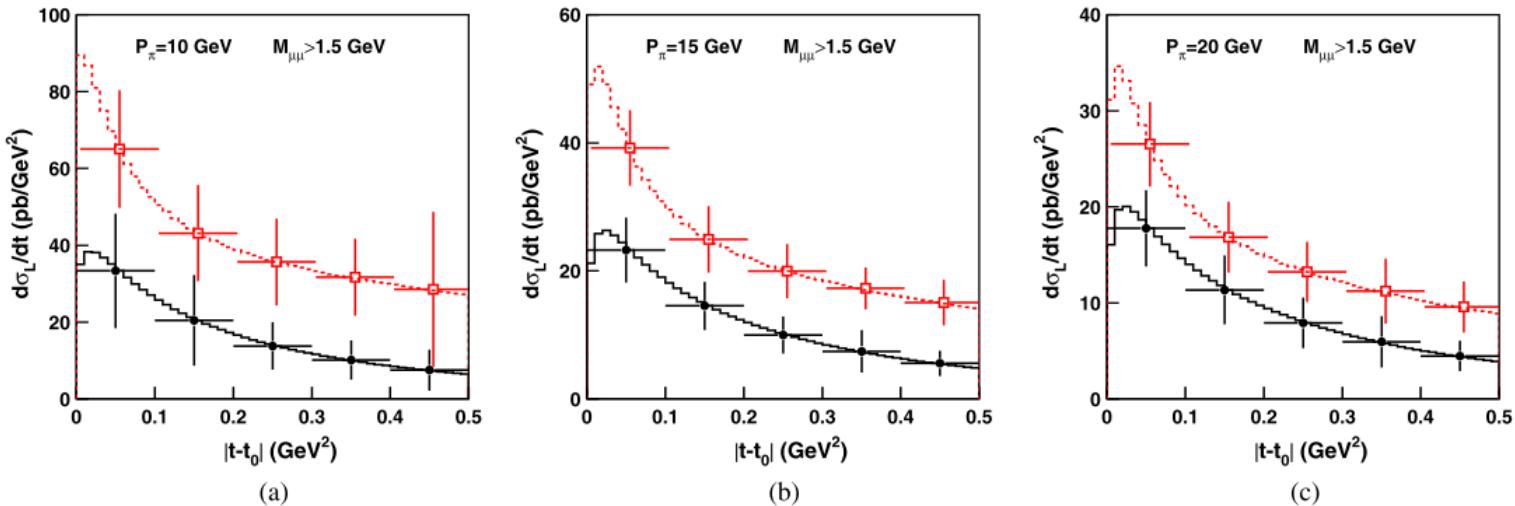
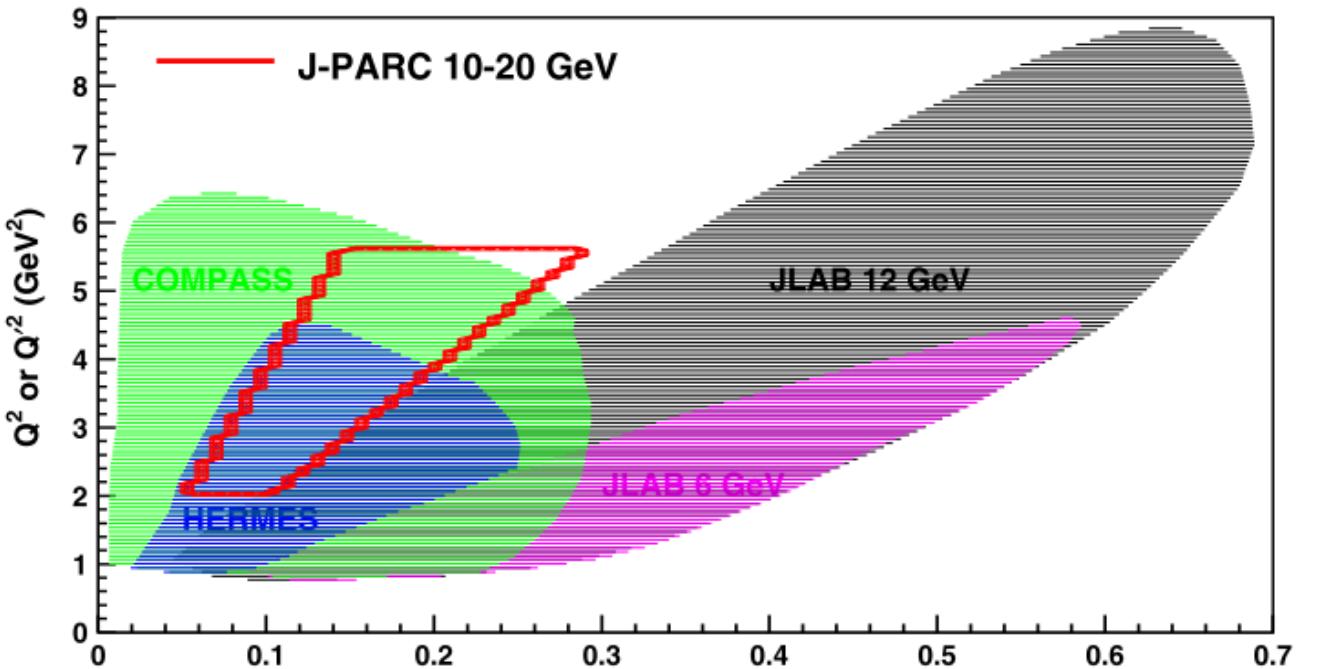
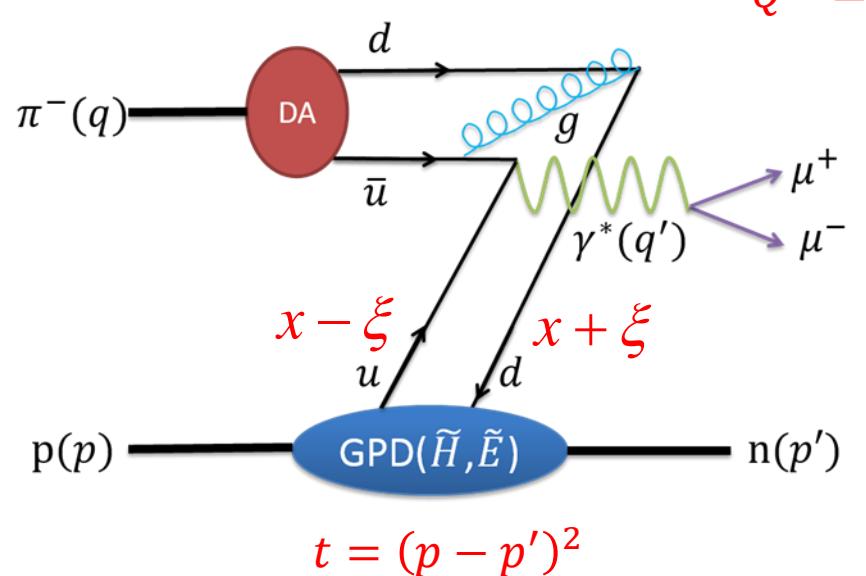


FIG. 15. The expected statistical errors of the exclusive Drell-Yan measurement for two GPDs inputs, BMP2001 (black) and GK2013 (red), as a function of $|t - t_0|$ in the dimuon mass region of $M_{\mu^+\mu^-} > 1.5$ GeV for 10 (a), 15 (b), and 20 (c) GeV beam momentum.

$\pi N \rightarrow l^+ l^- N$ (handbag diagram)

E.R. Berger, M. Diehl, B. Pire, PLB 523 (2001) 265



$$Q'^2 = q'^2 > 0$$

$$\tau = \frac{Q'^2}{2pq} \approx \frac{Q'^2}{s - M_N^2} \quad \xi = \frac{(p - p')^+}{(p + p')^+} = \frac{\tau}{2 - \tau}$$

$$\tilde{x} = -\frac{(q + q')^2}{2(p + p') \cdot (q + q')} \approx -\frac{Q'^2}{2s - Q'^2} = -\xi$$

$$\boxed{\frac{d\sigma}{dQ'^2 dt d(\cos\theta) d\varphi}} \\ = \frac{\alpha_{\text{em}}}{256\pi^3} \frac{\tau^2}{Q'^6} \sum_{\lambda', \lambda} |M^{0\lambda', \lambda}|^2 \sin^2 \theta,$$

$$\boxed{\left. \frac{d\sigma_L}{dt dQ'^2} \right|_\tau = \frac{4\pi\alpha_{\text{em}}^2}{27} \frac{\tau^2}{Q'^8} f_\pi^2 \left[(1 - \xi^2) |\tilde{\mathcal{H}}^{du}(\tilde{x}, \xi, t)|^2 \right.} \\ \left. - 2\xi^2 \text{Re} (\tilde{\mathcal{H}}^{du}(\tilde{x}, \xi, t)^* \tilde{\mathcal{E}}^{du}(\tilde{x}, \xi, t)) - \xi^2 \frac{t}{4m_N^2} |\tilde{\mathcal{E}}^{du}(\tilde{x}, \xi, t)|^2 \right],$$

$$\begin{aligned}\tilde{\mathcal{H}}^{du}(\tilde{x}, \xi, t) &= \frac{8}{3}\alpha_s \int_{-1}^1 dz \frac{\phi_\pi(z)}{1-z^2} \\ &\quad \times \int_{-1}^1 dx \left(\frac{e_d}{\tilde{x}-x-i\epsilon} - \frac{e_u}{\tilde{x}+x-i\epsilon} \right) \left(\tilde{H}^d(x, \xi, t) - \tilde{H}^u(x, \xi, t) \right),\end{aligned}$$

$$\xi \approx \frac{Q'^2}{2s-Q'^2} = \frac{\tau}{2-\tau}.$$

$$\tau = \frac{Q'^2}{2p \cdot q} \approx \frac{Q'^2}{s-m_N^2} \approx \frac{Q'^2}{s},$$

$$\tilde{x} = -\frac{(q+q')^2}{2(p+p')\cdot(q+q')} \approx -\frac{Q'^2}{2s-Q'^2} = -\xi$$