Impact of DIS data at large x with the CJ global analysis

Shujie Li

with many thanks to

A. Alberto, M. Cerutti, C. Cocuzza, I. Fernando, X. Jing, J. Owens, S. Park, C.E. Keppel, W. Melnitchouk, P. Monaghan, N. Sato

POETIC XI Workshop @ FIU

Feb 27, 2025

BERKELEY LAB

CTEQ-Jefferson Lab Collaboration

Jefferson Lab

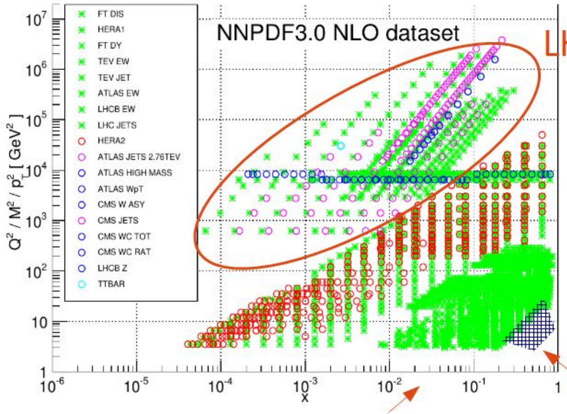
Global QCD fits

$$d\sigma_{\text{hadron}} = \sum_{f_1,f_2,i,j} \phi_{f_1} \otimes \hat{\sigma}^{f_1 f_2 \to ij}_{\text{parton}} \otimes \phi_{f_2}$$

pQCD calc.

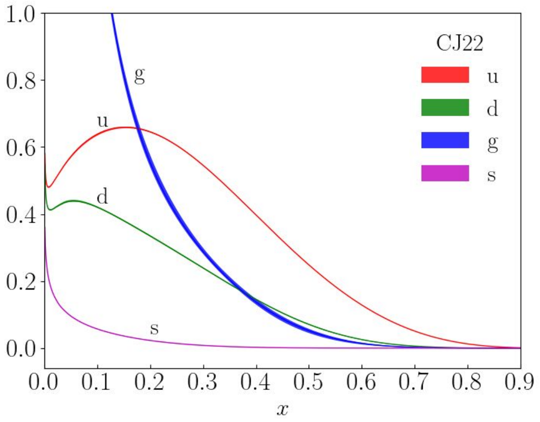
PDFs (from DIS fits)

- pQCD factorization & universality:
  can fit PDFs to a variety of hard scattering data
  - Hadron-hadron collisions
    - ⟶ Jets
    - ⟶ Electro-weak boson production
  - Electron-proton DIS
  - Electron-Deuteron DIS
- >1000 data points
- 40+ years of experience,
  - "High-energy" fitters:
    - ⟶ CTEQ-TEA, MMHT, NNPDF, HERAPDF
  - Lower-energy / nuclear focus:
    - ⟶ **CTEQ-JLab (CJ),** AKP, ABMP, JAM



CJ Overview

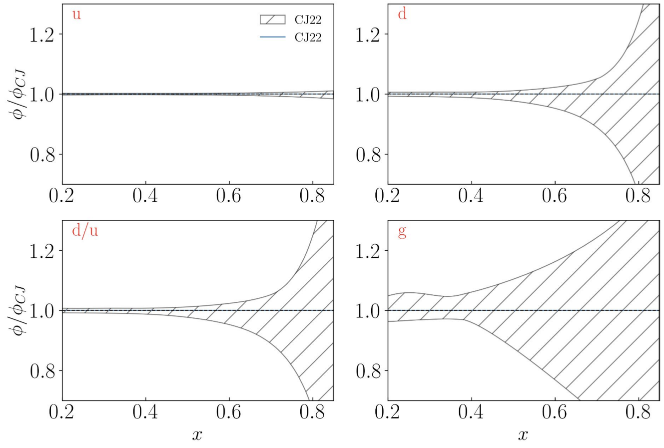
- Coordinated theory-experiment effort :
  - Alberto Accardi, **Matteo Cerutti**, Xiaoxian Jing, Ishara Fernando, Wally Melnitchouk, Jeff Owens, **Peter Risse**
  - Thia Keppel, Shujie Li, Peter Monaghan, Sanghwa Park
- Specializes in PDF fitting at JLab kinematics (low Q^2, large x)
- Latest: CJ22 A. Accardi, X. Jing, J. F. Owens, and S. Park PhysRevD.107.113005
  - 25 fitting parameters (=20 PDF + 2 off-shell + 3 higher-twist)
  - 4k+ p and d data points



Structure functions

Obs.	Experiment	Ref.	# Points	χ^2
DIS	JLab (p)	[31]	136	161.0
	JLab (d)	[31]	136	119.1
	JLab (n/d)	[32]	191	213.2
	HERMES (p)	[33]	37	29.1
	HERMES (d)	[33]	37	29.5
	SLAC (p)	[34]	564	469.8
	SLAC (d)	[34]	582	412.1
	BCDMS (p)	[35]	351	472.2
	BCDNS (d)	[36]	254	321.8
	NMC (p)	[37]	275	416.5
	NMC (d/p)	[38]	189	199.6
	HERA (NC e^-p)	[39]	159	249.7
	HERA (NC e^+p 1)	[39]	402	598.9
	HERA (NC e^+p 2)	[39]	75	98.8
	HERA (NC e^+p 3)	[39]	259	250.0
	HERA (NC e^+p 4)	[39]	209	229.1
	HERA (CC e^-p)	[39]	42	45.6
	HERA (CC e^+p)	[39]	39	52.5

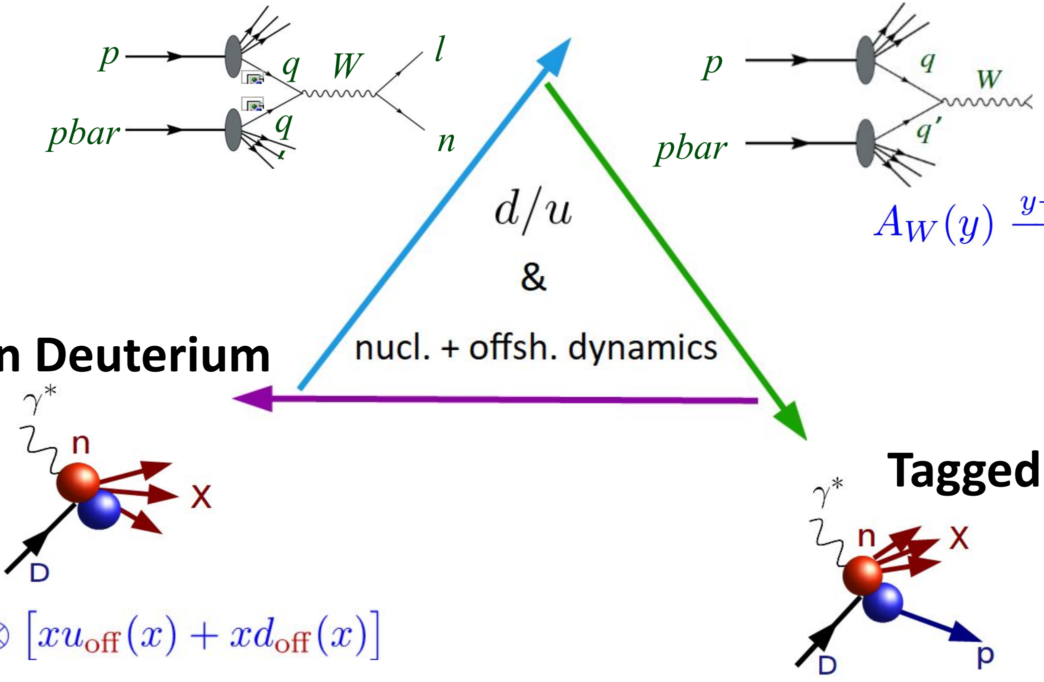
Obs.	Experiment	Ref.	# Points	χ^2
LPP	E866 (pp)	[4]	121	144.1
	E866 (pd)	[4]	129	157.4
	SeaQuest (d/p)	[5]	6	7.5
W	CDF (e)	[40]	11	12.6
	D0 (e)	[41]	13	28.8
	D0 (μ)	[42]	10	17.5
	CDF (W)	[43]	13	18.0
	D0 (W)	[44]	14	14.5
	STAR (e^+/e^-)	[6]	9	25.3
	(less η_{max} point)		(8)	(15.4)
Z	CDF	[45]	28	29.2
	D0	[46]	28	16.1
jet	CDF	[47]	72	14.0
	D0	[48, 49]	110	14.0
γ+jet	D0 1	[50]	16	8.7
	D0 2	[50]	16	19.3
	D0 3	[50]	12	25.0
	D0 4	[50]	12	12.2
	total		4557	4936.6
	total + norm		4573	4948.6



Large-x PDFs: interplay of observables

Phys. Rev. D 107, 113005

Asymmetries (check the latest **CJ22** with STAR, SeaQuest data)



$$A_W(y) \xrightarrow{y \to y_{max}} \frac{1 - d/u}{1 + d/u}$$

DIS on Deuterium

$$F_2^d \propto S_D \otimes \left[x u_{\rm off}(x) + x d_{\rm off}(x) \right]$$

Tagged DIS

$$\frac{F_2^n}{F_2^d} \propto x u(x) + 4 x d(x)$$

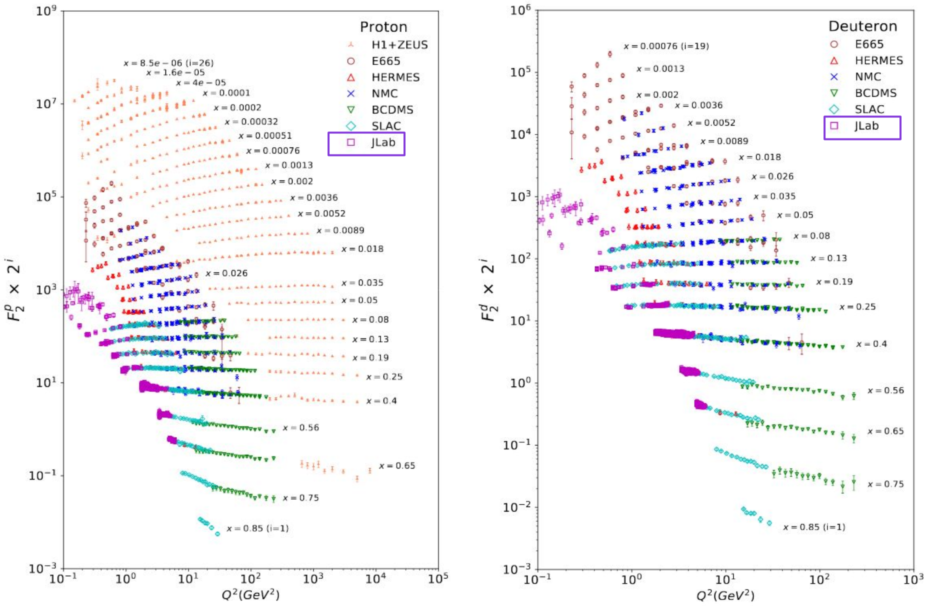
DIS data from JLab

- **CJ DIS cuts:** $Q^2 > 1.691$ GeV^2/c^2 ,$W^2 > 3.5$ GeV^2
- Fit minimization:

correlated error — normalization

$$\chi^2 = \sum_{\text{exp}} \left[\sum_{i=1}^{N_{\text{data}}} \left(\frac{D_i + \Delta_i - T_i/n}{\delta D_i} \right)^2 + \left(\lambda^{\text{norm}}\right)^2 + \sum_{k=1}^{K} \lambda_k^2 \right]_{\text{exp}}$$

uncorrelated error



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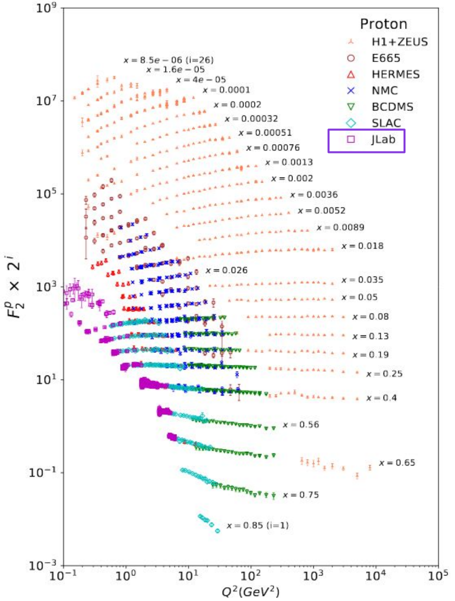
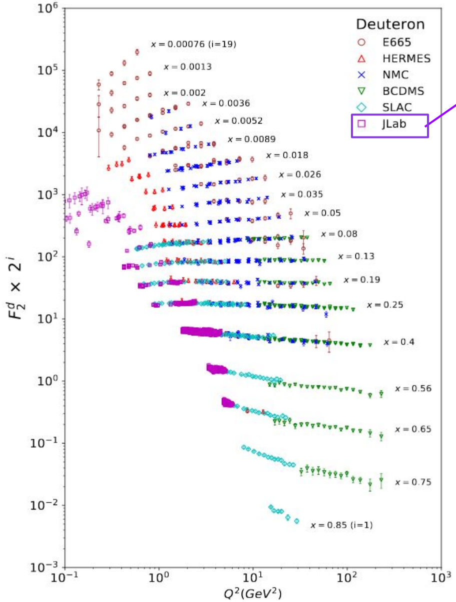
uncorrelated error





JLab 6GeV Era	# of F2p points	# of F2d points
E-00-116	136	136
E-03-103	37	69
CLAS6	609	1723
E-94-110	112	0
E-06-009	0	79
E-99-118	2	2
JLCee96	100	97
BONuS6	115 (n/d)	

More from 12 GeV:
E12-00-002: d/p
BONuS12: n/d
CLAS12: p and d
MARATHON: 3H/3He
…

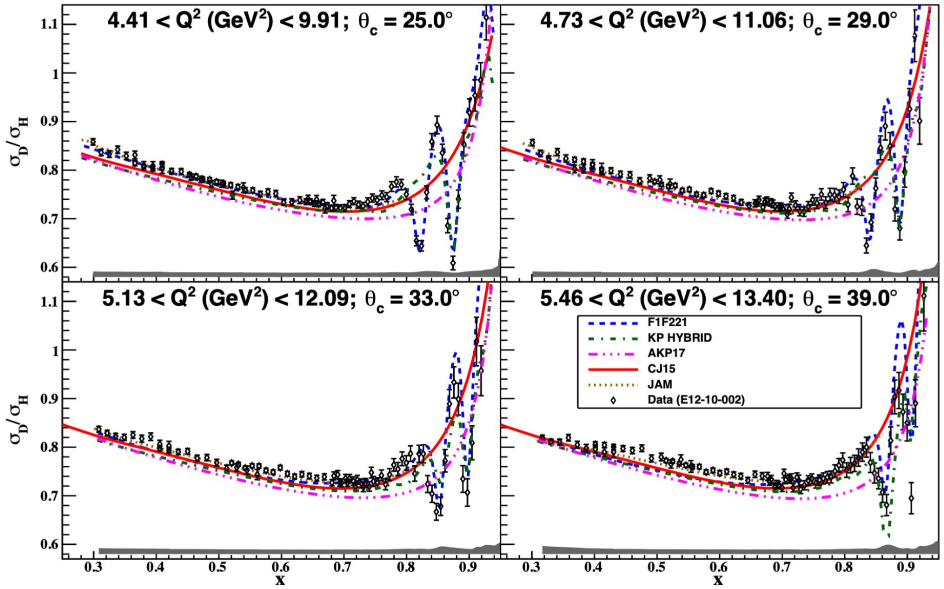
❖ Coming soon: **CJ+JAM**
- ➢ Merged DIS database and LHAPDF grids (big thanks to Christopher Cocuzza): github/CJ-JAM-database/
- ➢ unpolarized PDF benchmark
- ➢ JLab data impact study
- ➢ Impact of correlated errors

For JAM updates, see Wally's talk on Wednesday

Impact of Large-x Deuteron Data

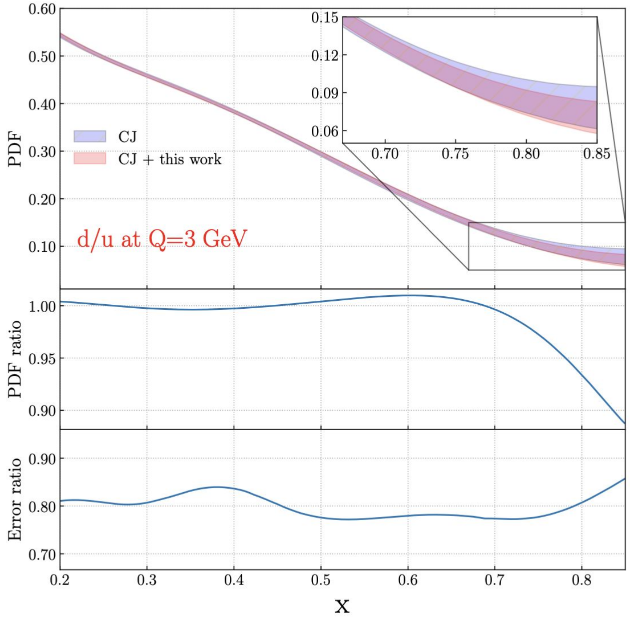
"Power of precision"

JLab E12-10-002 d/p data



arXiv:2409.15236 (submitted to PRL)

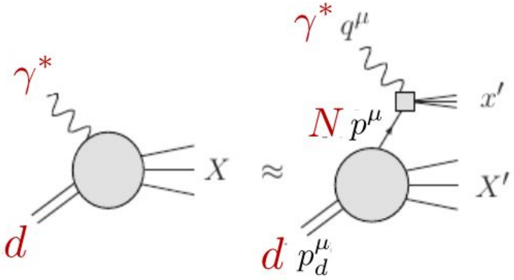
Impact on d/u



Deuteron: Fermi motion and binding

- **Weak binding approximation:**
  - Incoherent scattering from not too fast individual nucleons
  - Neglects FSI

Melnitchouk, Schreiber, Thomas, PRD 49 (1994)
Kulagin, Piller, Weise, PRC 50 (1994)
Kulagin and Petti, NPA 765 (2006)



$$F_{2d}(x,Q^2) = \int \frac{dz}{z} dp_T^2 \, \mathcal{K}(z,p^2,\gamma) \left|\psi_{N/d}(|\vec{p}\,|)\right|^2 F_{2N}(x/z,Q^2,p^2)$$

kinematic and "flux" factors

Nucleon wave function

structure function of **bound, off-shell nucleon**

→ $z = \dfrac{p \cdot q}{p_d \cdot q} \approx 1 + \dfrac{p_0 + \gamma p_z}{M} \quad \left[p_0 = M + \varepsilon, \;\; \varepsilon = \varepsilon_d - \dfrac{\vec{p}^{\,2}}{2M} \right]$

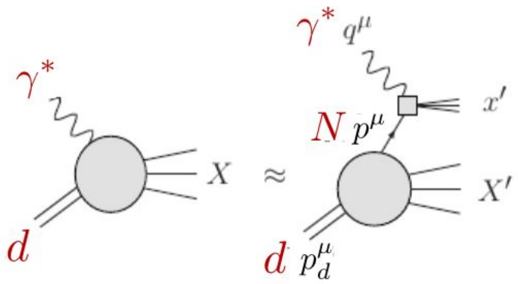
momentum fraction of d carried by N

→ at finite Q^2, $\gamma = \sqrt{1 + 4x^2p^2/Q^2}$

quantifies how far the nucleon is from the light cone (γ = 1)

Off-shell corrections in Deuteron

- **Nucleons are bound in the deuteron:**
  - $p^2 < M^2$
  - Structure functions are deformed (but not too much if x not too large)
- **Offshell expansion:**
  - parametrize first order coefficient
    - ⟶ Structure function level



$$F_{2N}(x,Q^2,p^2) = F_{2N}^{\text{free}}(x,Q^2)\left[1 + \frac{p^2 - M^2}{M^2}\delta f(x)\right]$$

Kulagin, Piller, Weise, PRC 50 (1994)
Kulagin, Melnitchouk, et al., PRC 52 (1995)
Kulagin and Petti, NPA 765 (2006)

Free proton, neutron structure function

“offshell function”

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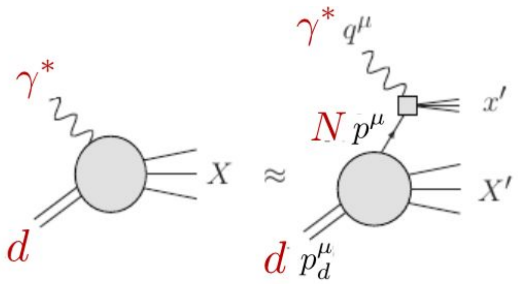
Free proton, neutron structure function

"offshell function"

Interchangeable only at LO

$\longrightarrow$ **Parton level (CJ's choice)**

$$\phi(x, Q^2, p^2) = \phi^{\text{free}}(x, Q^2) \left(1 + \frac{p^2 - M^2}{M^2} \delta f(x)\right)$$



Off-shell corrections in Deuteron

- **Offshell expansion:**
  - parametrize first order coefficient

    ⟶ **Parton level (CJ's choice)**

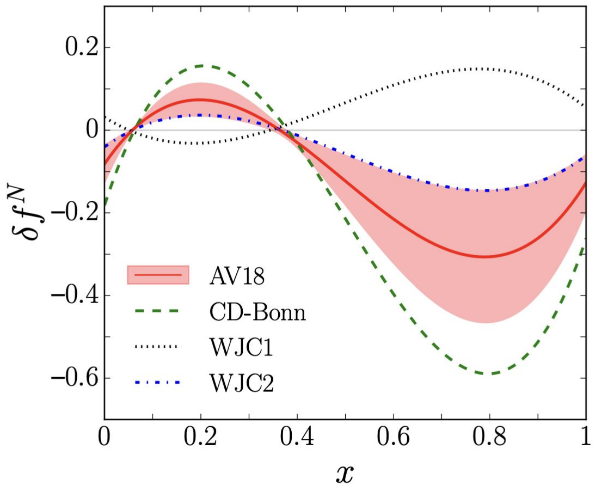
$$\phi(x,Q^2,p^2) = \phi^{\text{free}}(x,Q^2)\left(1+\frac{p^2-M^2}{M^2}\delta f(x)\right)$$

$$\delta f(x) = \left.\frac{\partial \ln \phi(x,Q^2,p^2)}{\partial \ln p^2}\right|_{p^2=M^2}$$

Assume Q^2 evolution canceled

$$\delta f(x) = C(x-x_0)(x-x_1)(1+x_0-x)$$

When fitted, this effectively becomes a phenomenological "catch-all" term (see later)



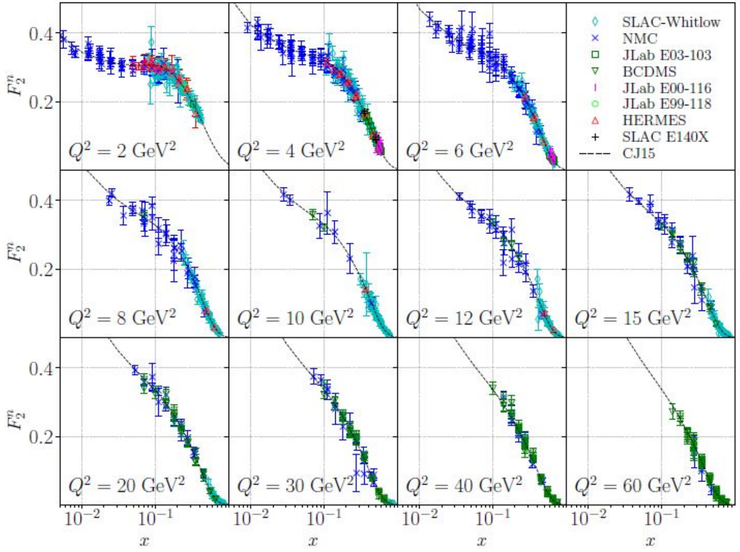
“Free” neutron

SL, Accardi, Cerruti, Fernando, et. al PhysRevD.109.074036

$$(p+n)_{data} = d_{data} * (p+n)_{cj} / d_{cj}$$

cross-normalization

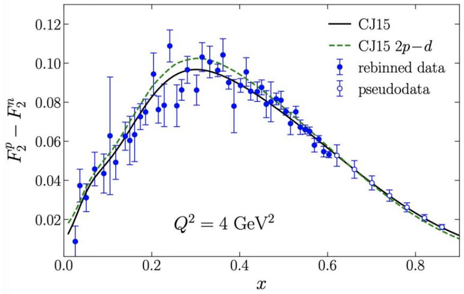
$$n_{data} = (p+n)^*_{data} - p^*_{data} = d^*_{data} * (p+n)_{cj}/d_{cj} - p^*_{data}$$



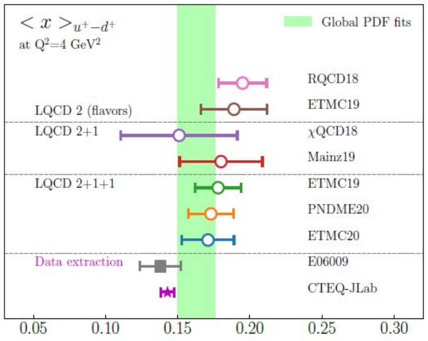
Applications:

- Isoscalar corrections for A/d
- Non-singlet moments from p-n:

$$M_2^{p-n}(Q^2) = \int_0^1 dx \frac{\xi^3}{x^3}\left[\frac{3+9r+8r^2}{20}\right] F_2^{p-n}(x, Q^2)$$



$$\frac{3}{C_2} M_2^{p-n} = \langle x \rangle_{u^+-d^+} + \text{ HT}$$

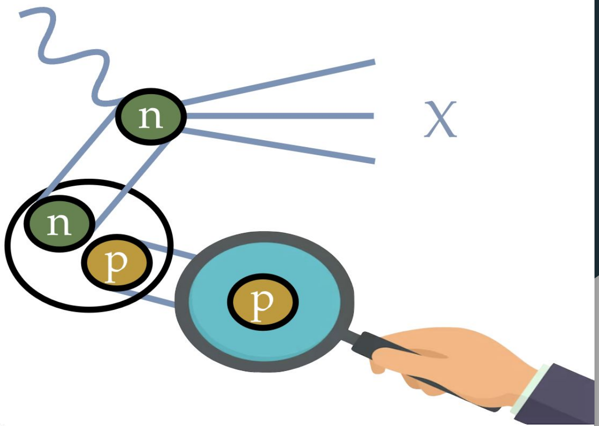


2nd order moments significantly lower than recent LQCD calculations (Rodekamp et. al, PhysRevD.109.074508), how about higher orders?

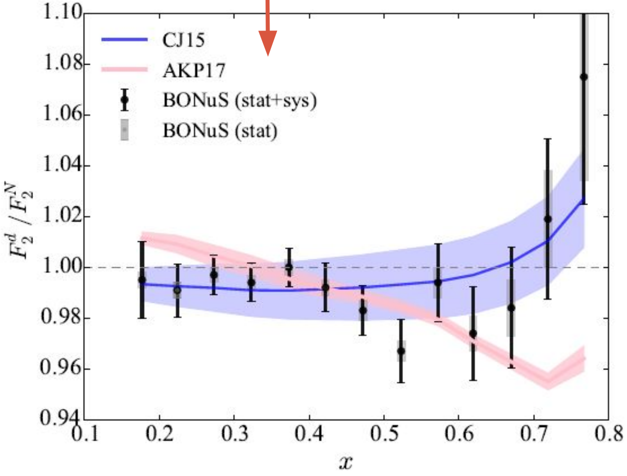
“Free” neutron

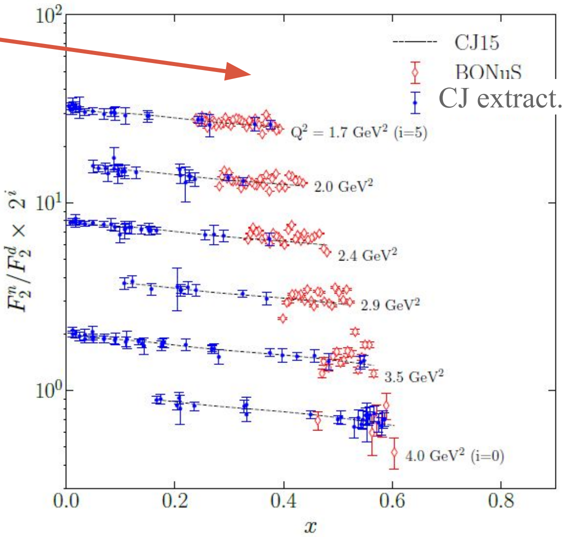
- Probe the barely off-shell neutron in deuteron by **tagging** a low-momentum recoil proton
  - Nuclear modification v.s. virtuality
  - Insensitive to isospin-dependence

$$\phi(x, Q^2, p^2) = \phi^{\text{free}}(x, Q^2)\left(1 + \boxed{\frac{p^2 - M^2}{M^2}}\delta f(x)\right)$$



6GeV BONuS: proof of principle





Coming soon:
BONuS12

Deuteron tagging at **EIC**:
- Recoil proton boosted for easier detection
- See also Dien’s talk on Tuesday

Large-x PDFs: interplay of corrections in fitting

Cerutti et. al, arXiv:2501.06849 (submitted to PRD)

Offshell:

$$F_{2N}(x,Q^2,p^2) = F_{2N}^{\text{free}}(x,Q^2)\left[1 + \frac{p^2 - M^2}{M^2}\delta f(x)\right]$$

Free proton, neutron structure function

"offshell function"

Higher-twist:

- **Multiplicative (CJ's choice)**

$$F_2(x,Q^2) = F_2^{LT}(x,Q^2)\left(1 + \frac{\boldsymbol{C}(x)}{Q^2}\right)$$

$$C(x) = a_{ht}^{(0)} x^{a_{ht}^{(1)}} (1 + a_{ht}^{(2)} x)$$

- **Additive**

$$F_2 = F_2^{LT}(x,Q^2) + \frac{\boldsymbol{H}(x)}{Q^2}$$

$$H(x) = a_{ht}^{(0)} x^{a_{ht}^{(1)}} (1-x)^{a_{ht}^{(2)}} (1 + a_{ht}^{(3)} x)$$

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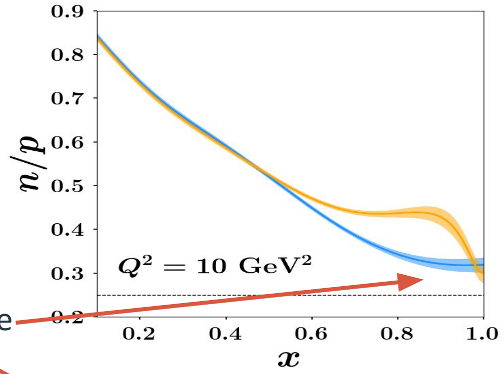

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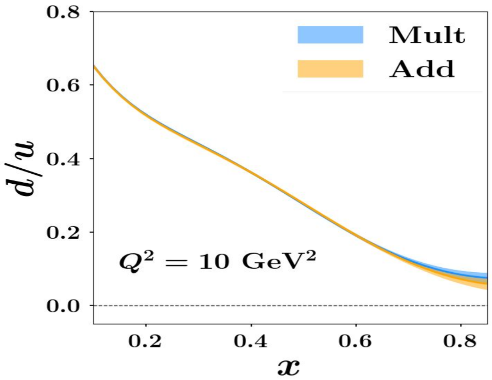
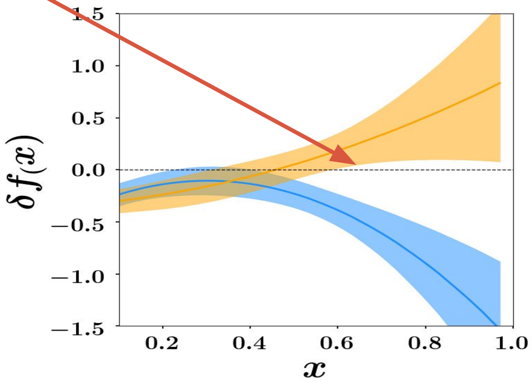
Assume **p=n** for HT and offshell:
The difference from HT choices are compensated by offshell.





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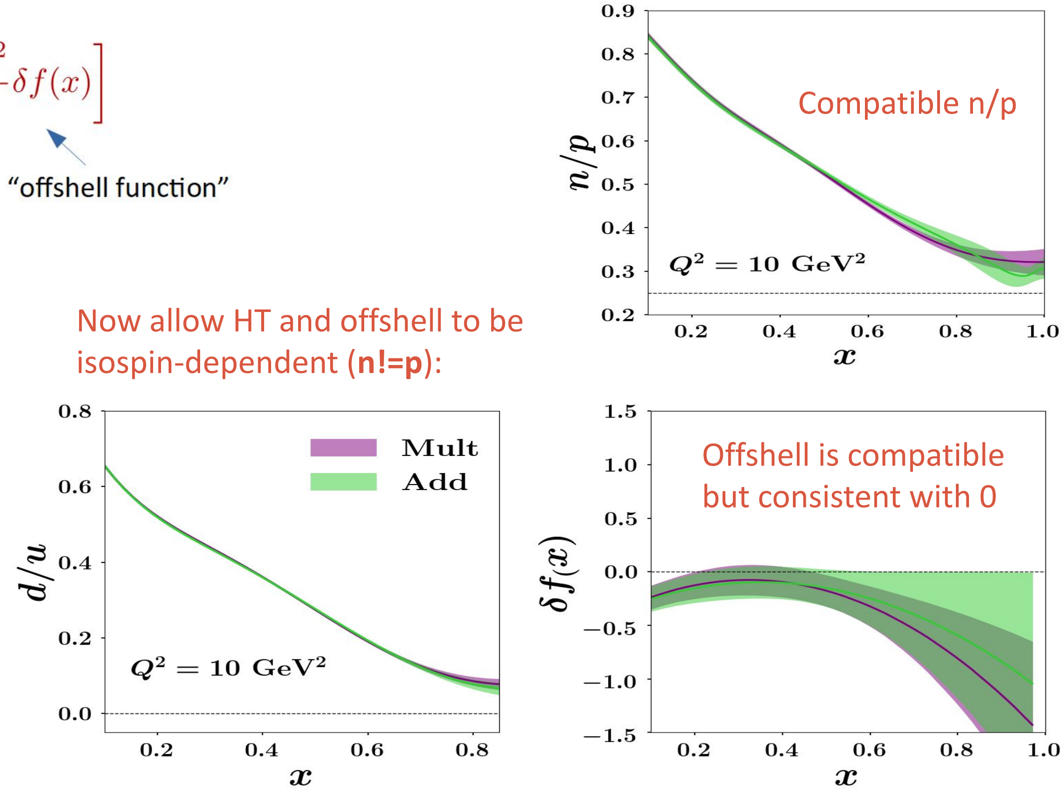
Now allow HT and offshell to be isospin-dependent (**n!=p**):



Large-x PDFs: interplay of corrections in fitting

Offshell:

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Free proton, neutron structure function

No Q^2 dependence

Higher-twist: ← **$1/Q^2$**

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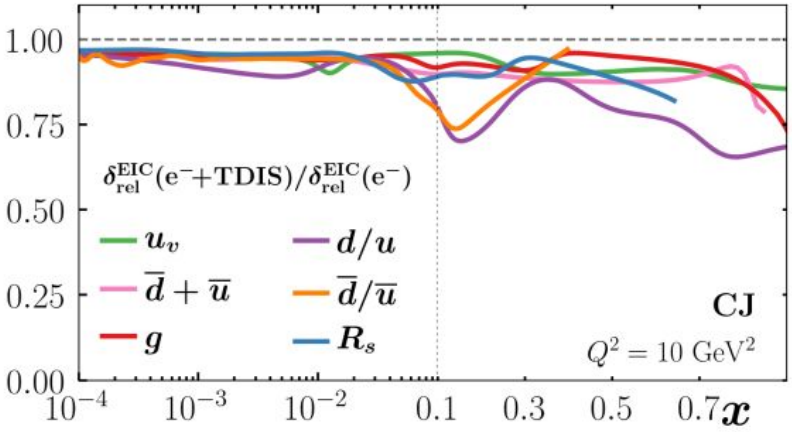
EIC:

- clean determination of offshell effects at high Q^2
- Pin down offshell effects with tagging



More thoughts on large-x PDFs at EIC

CJ+JAM discussion

"several" $\sqrt{s}$ → Direct access to gluon PDFs through F_L (hence photon PDFs!)

→ Need to revisit impact studies for the early science program (see R. Ent talk). No ep as baseline in year 1?

high Q^2 + JLab/SLAC → Fixed point for HT~1/Q^2 studies HT_2, HT_L (HT_L ~ little known)

→ Decouple HT and nuclear dynamics (offshell deformations)

Cerutti et al, arxiv:2501.06849

"easy" neutron tagging (& proton) → Get insight on medium modification v.s. virtuality

Positron beam for additional d/u determination at large x?

Backups

Impact study from yellow report

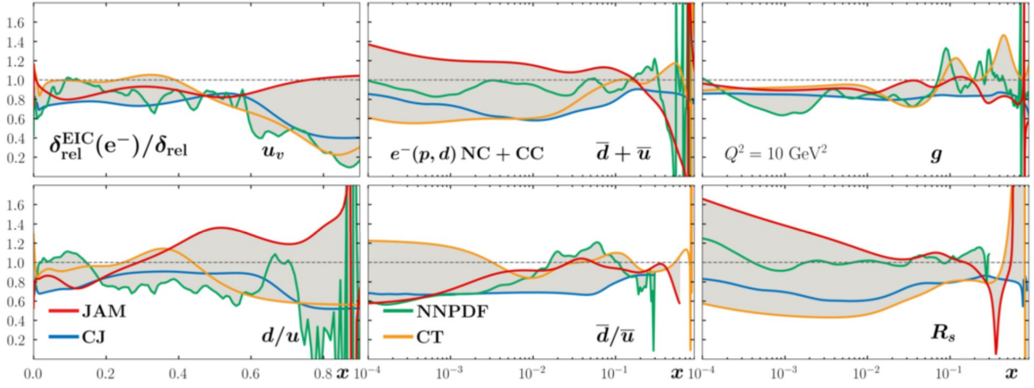


Figure 7.4: Comparison of relative uncertainties for unpolarized PDFs $xf(x)$ for different partons, before and after the inclusion of EIC data, evaluated at $Q^2 = 10$ GeV^2. We include the analysis of different collaborations, limited to e^- datasets.

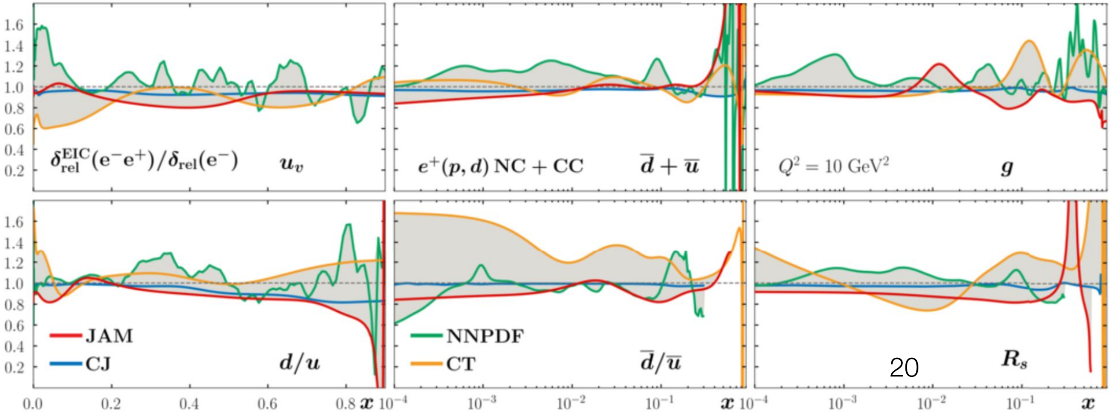
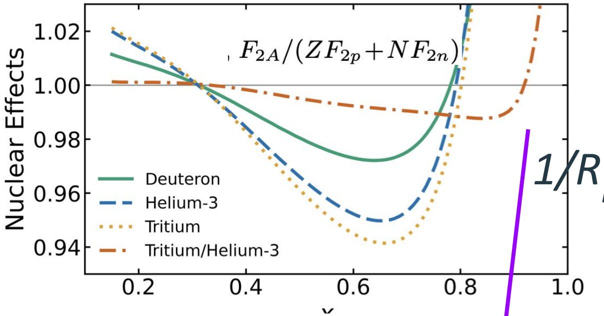


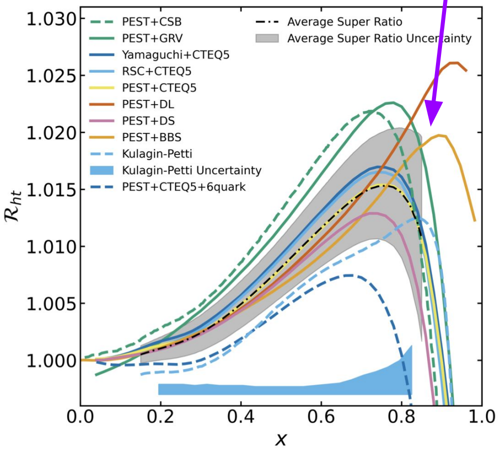
Figure 7.5: PDF relative uncertainties after inclusion of NC and CC $e^+(p,d)$ data normalized to the electron-data-only case.

F2 n/p from MARATHON data impact study

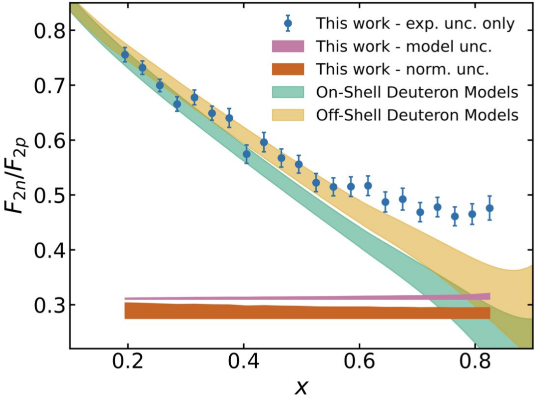
T. Hague, et. al., arxiv 2312.13499, NOT a CJ work



$1/R_{ht}$



Norm. shift: original 2.5%, here 1.3%



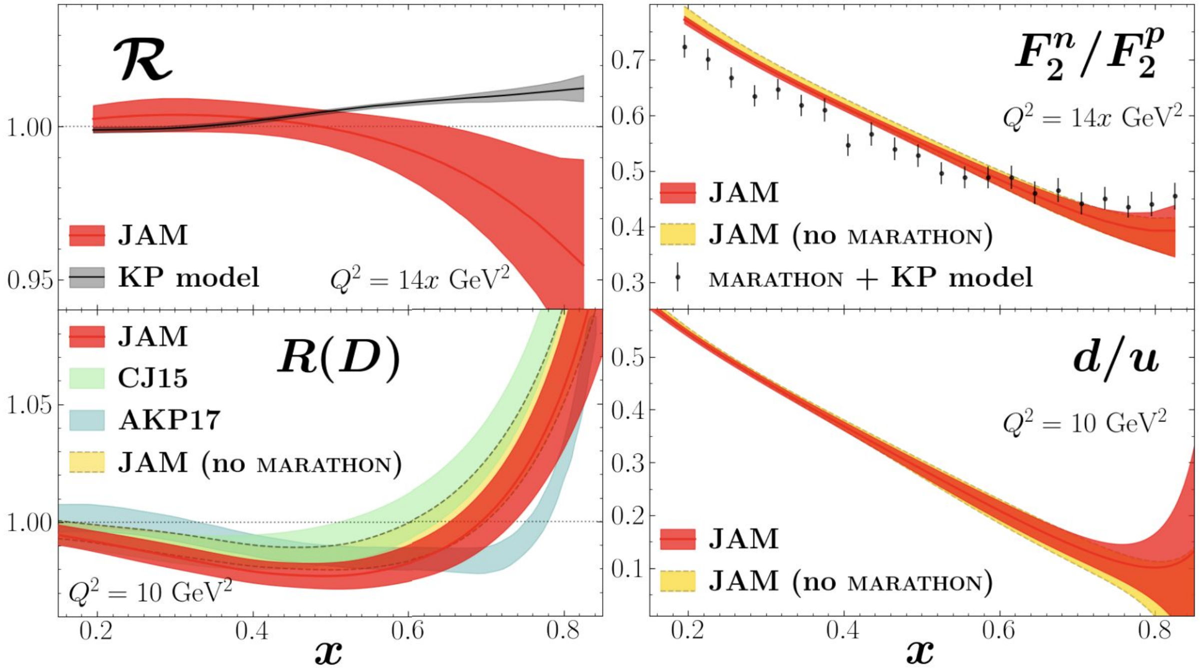
Varying R_{ht} didn't change the n/p shape at high x noticeably:

⇒ n/p is different in deuteron and A=3 nuclei?

⇒ More likely: larger than expected isospin dependence in nuclear effects

Iso-vector nuclear effect?

C. Cocuzza et. al. (JAM), arxiv: 2104.06946



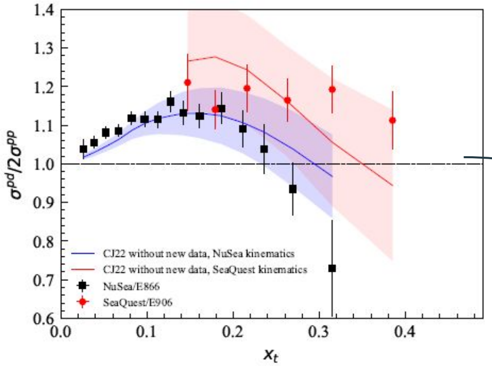
New: electroweak data

Obs.	Experiment	Ref.	# Points	χ^2
DIS	JLab (p)	[31]	136	161.0
	JLab (d)	[31]	136	119.1
	JLab (n/d)	[32]	191	213.2
	HERMES (p)	[33]	37	29.1
	HERMES (d)	[33]	37	29.5
	SLAC (p)	[34]	564	469.8
	SLAC (d)	[34]	582	412.1
	BCDMS (p)	[35]	351	472.2
	BCDNS (d)	[36]	254	321.8
	NMC (p)	[37]	275	416.5
	NMC (d/p)	[38]	189	199.6
	HERA (NC e^-p)	[39]	159	249.7
	HERA (NC e^+p 1)	[39]	402	598.9
	HERA (NC e^+p 2)	[39]	75	98.8
	HERA (NC e^+p 3)	[39]	259	250.0
	HERA (NC e^+p 4)	[39]	209	229.1
	HERA (CC e^-p)	[39]	42	45.6
	HERA (CC e^+p)	[39]	39	52.5

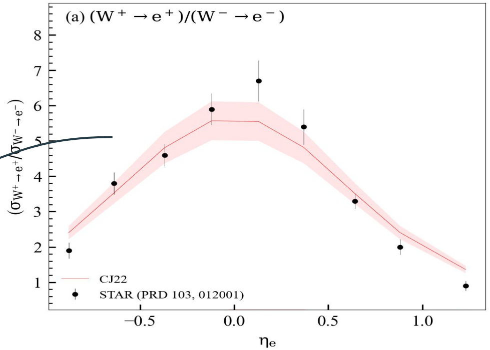
Obs.	Experiment	Ref.	# Points	χ^2
LPP	E866 (pp)	[4]	121	144.1
	E866 (pd)	[4]	129	157.4
	SeaQuest (d/p)	[5]	6	7.5
W	CDF (e)	[40]	11	12.6
	D0 (e)	[41]	13	28.8
	D0 (μ)	[42]	10	17.5
	CDF (W)	[43]	13	18.0
	D0 (W)	[44]	14	14.5
	STAR (e^+/e^-)	[6]	9	25.3
	(less $\eta_{\max}$ point)		(8)	(15.4)
Z	CDF	[45]	28	29.2
	D0	[46]	28	16.1
jet	CDF	[47]	72	14.0
	D0	[48, 49]	110	14.0
γ+jet	D0 1	[50]	16	8.7
	D0 2	[50]	16	19.3
	D0 3	[50]	12	25.0
	D0 4	[50]	12	12.2
	total		4557	4936.6
	total + norm		4573	4948.6

New: electroweak data

SeaQuest



STAR $W^+ \to e^+ / W^- \to e^-$



Fits w/o new data

$$\frac{\sigma_{pd}}{\sigma_{pp}} \approx \frac{4 + \frac{d(x_b)}{u(x_b)}}{4 + \frac{d(x_b)}{u(x_b)} \frac{\bar{d}(x_t)}{\bar{u}(x_t))}} \left(1 + \frac{\bar{d}(x_t)}{\bar{u}(x_t)}\right)$$

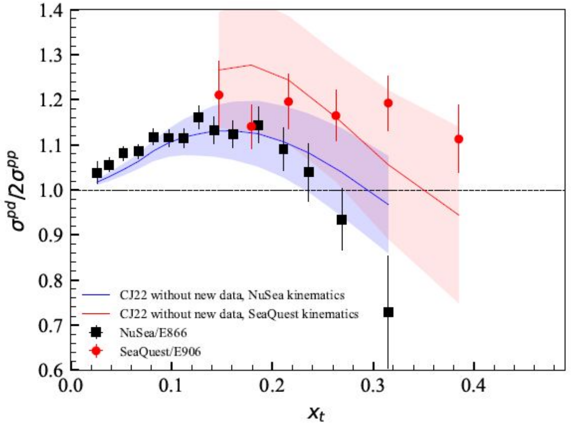
$$\frac{\sigma_{W+}}{\sigma_{W-}} \approx \frac{u(x_1)\bar{d}(x_2) + \bar{d}(x_1)u(x_2)}{d(x_1)\bar{u}(x_2) + \bar{u}(x_1)d(x_2)} \underset{y_W \approx 0}{\approx} \frac{\bar{d}/\bar{u}}{d/u}$$

Anticorrelation: db/ub ⟷ d/u
med. xt ⟷ large xb
(0.05 – 0.4) (0.3–0.7)

Correlation: db/ub ⟷ d/u
x ~ 0.16

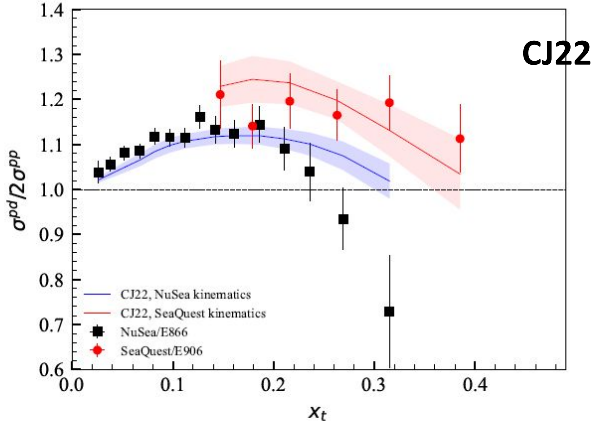
Need flexible enough parametrization

Lepton Pair Production



Fit new data

(SeaQuest & STAR)



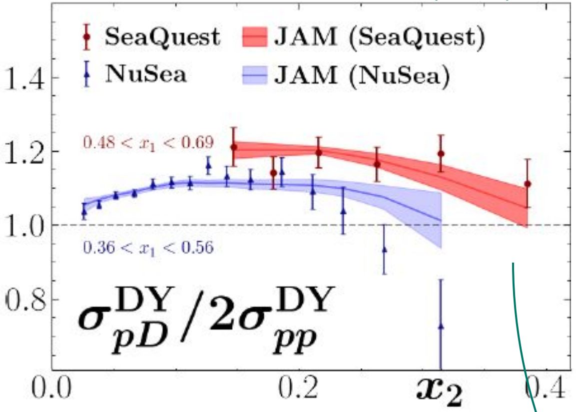
SeaQuest: χ^2/datum = 3.19 → 1.25

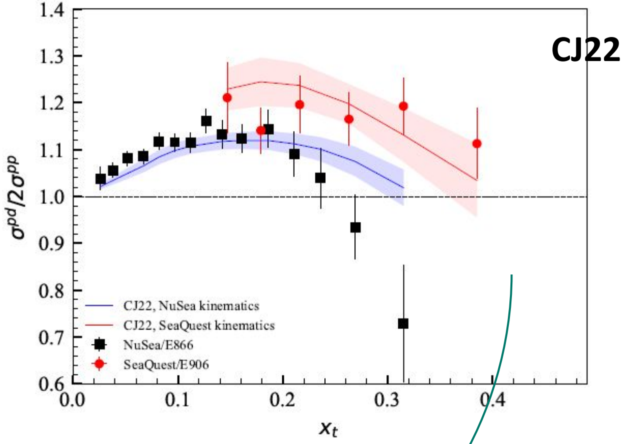
E866 : χ^2/datum = 1.63 → 1.93

Lepton Pair Production

- **Comparable results to JAM, CT:**

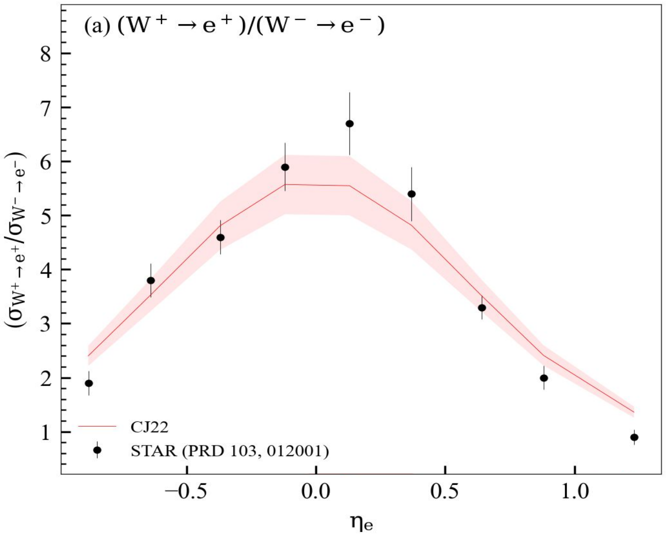
Cocuzza et al. PRD 104 (2021) 074031



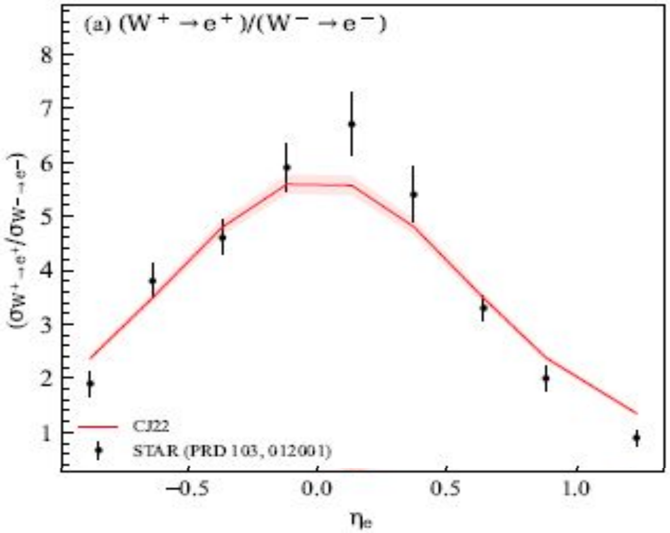


E866, SeaQuest disagree:
How to include in error bands?

Weak boson production



Fit new data
(SeaQuest & STAR)

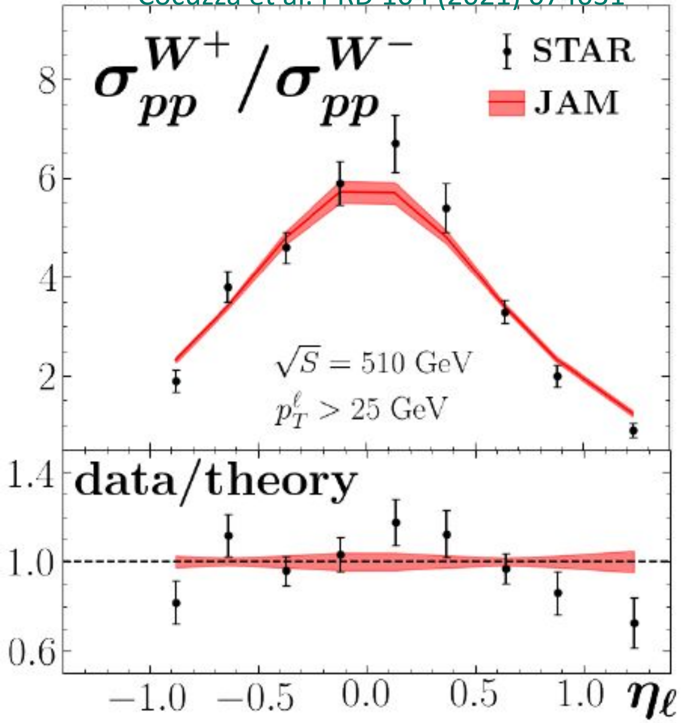


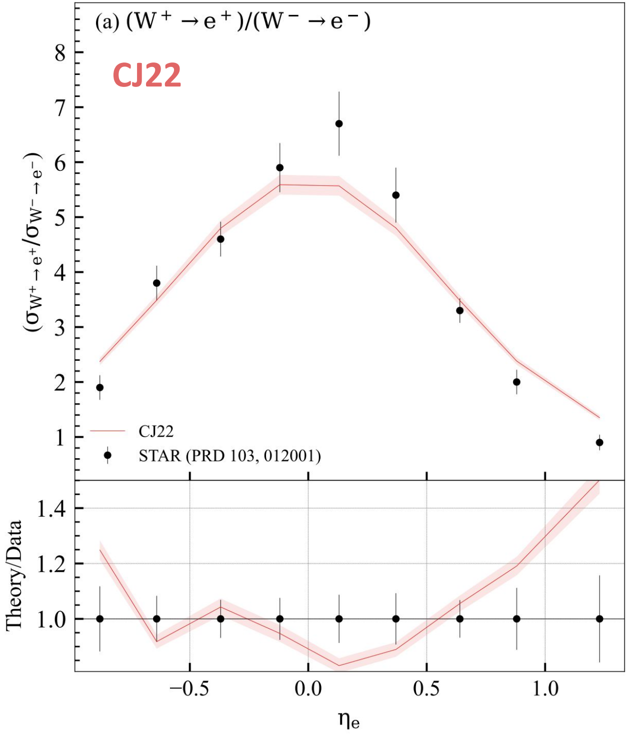
- Large reduction in uncertainty driven by SeaQuest data
- STAR contributes ~ 15% reduction around x~0.16
  - distributed between d/u (5%) and db/ub (10%) PDF ratios

Weak boson production

- Only W+/W- ratio was fitted

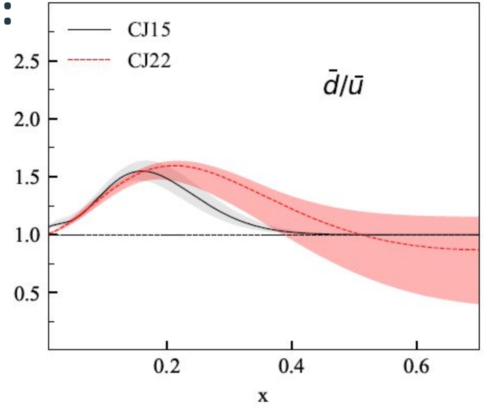
Cocuzza et al. PRD 104 (2021) 074031

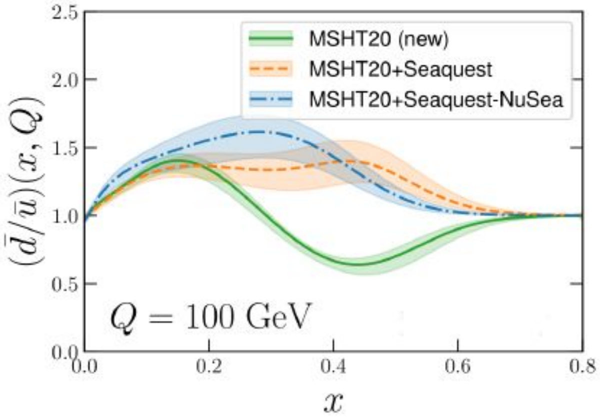




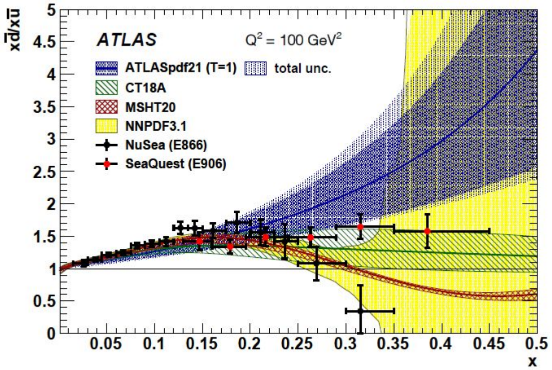
Comparison to other recent PDFs

- **SeaQuest fitted:**





- **PDFs w/o SeaQuest:**



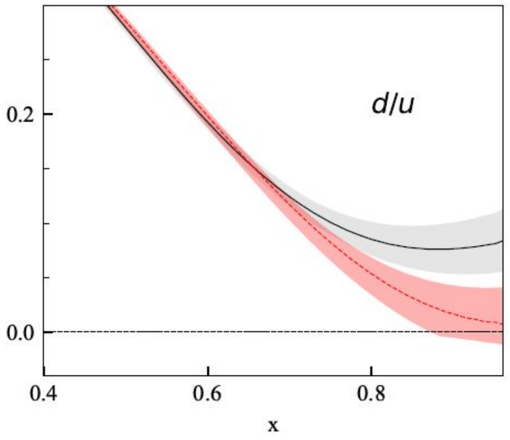
Latest results from QCD fits in CJ framework

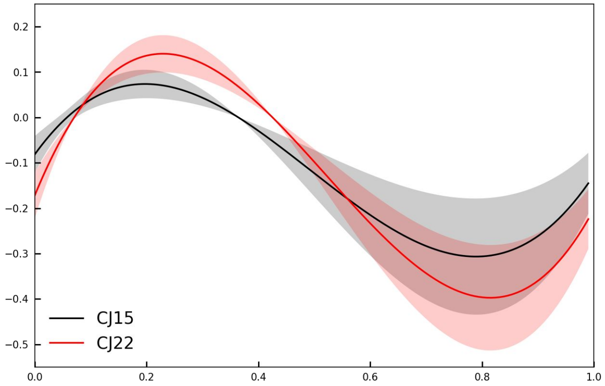
CJ22 fit

Same off-shell parameterization

More flexible parameterization of sea quarks (NuSea and SeaQuest data)

Accardi, Jing, Owens et al., PRD 107 (2023)





Different

Similar

Difference on is absorbed in something else

Higher Twist

Is the model for off-shell correction enough flexible?

Impact of HT on n/p ratio

Are experimental observables independent of the choice of the HT?

$$\xrightarrow{x \to 1} \quad \frac{4d+u}{4u+d} \simeq \frac{1}{4}$$

(extrapolation region)

Case 1: isospin-symmetric HT

Mult HT

$C_p(x) = C_n(x) = C(x)$

$$\frac{(4d+u)(1+C/Q^2)}{(4u+d)(1+C/Q^2)} \simeq \frac{1}{4}$$

No effect of HT

Add HT

$H_p(x) = H_n(x) = H(x)$

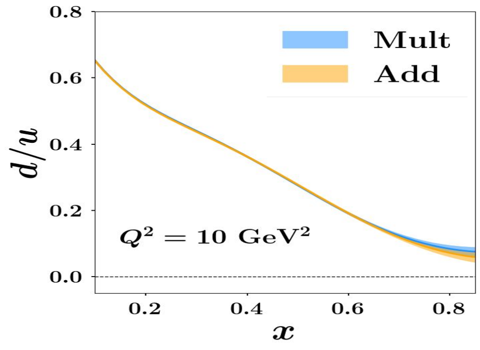
$$\frac{4d+u+H/Q^2}{4u+d+H/Q^2} \simeq \frac{1}{4} + 27\frac{H}{16uQ^2}$$

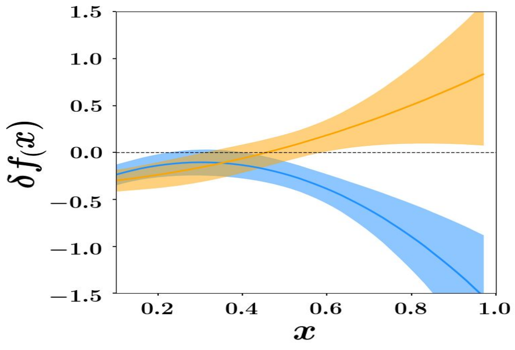
Strong effect of HT

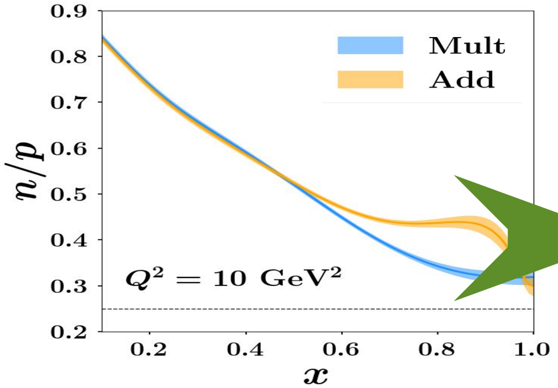
Bias identified!!

Results in the CJ fitting framework

Case 1: isospin-symmetric HT

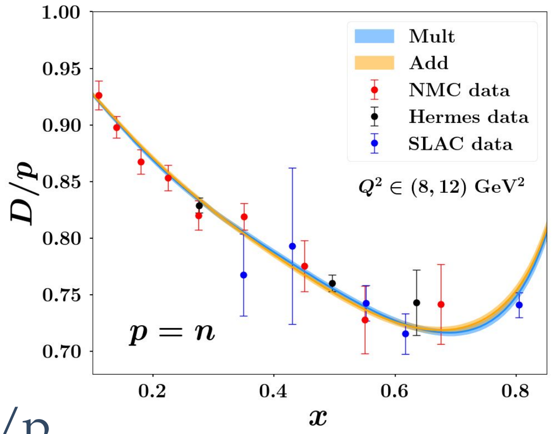






Bias identified

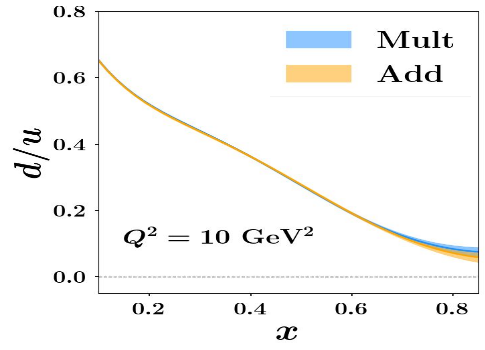
Off-shell compensates n/p

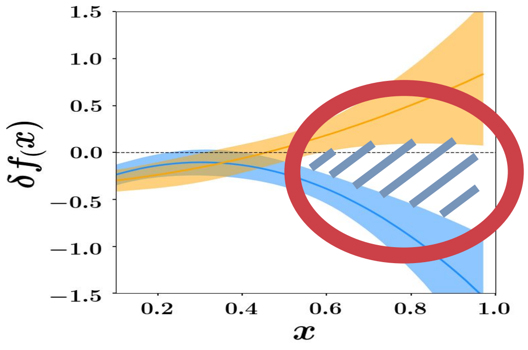


Artificially large n/p
BUT smaller d/u than Mult

Results in the CJ fitting framework

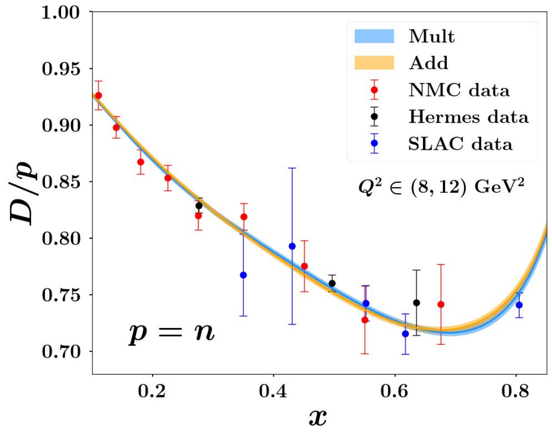
Case 1: isospin-symmetric HT

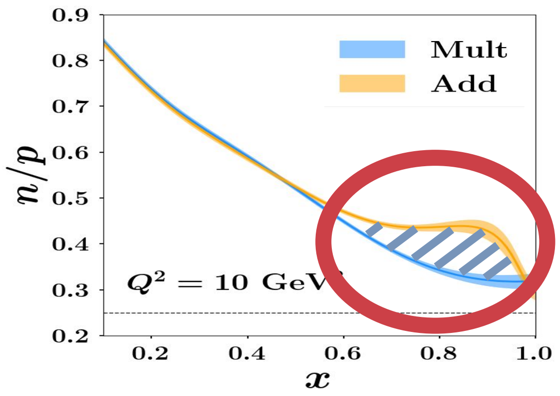




Bias identified

Off-shell compensates n/p





Systematic "implementation" uncertainty in a region of extrapolation

Impact of HT on n/p ratio

Are experimental observables independent of the choice of the HT?

$\xrightarrow{x \to 1}$ $\frac{1}{4}$ **LT** **Mult HT** $C_p(x) = C_n(x) = C(x)$

Case 2: isospin-breaking HT

Add HT

$H_p(x) \neq H_n(x)$

$$\frac{u + H_n/Q^2}{4u + H_p/Q^2}$$

$$\simeq \frac{1}{4} + 9\,\frac{4H_n - H_p}{16uQ^2}$$

$H_p(x) = H_n(x)$ → $\frac{1}{4} + 27\,\frac{H}{16uQ^2}$

$H_p(x) = 2H_n(x)$ → $\frac{1}{4} + 9\,\frac{H}{16uQ^2}$

n/p ratio is smaller

Mult HT

$C_p(x) \neq C_n(x)$

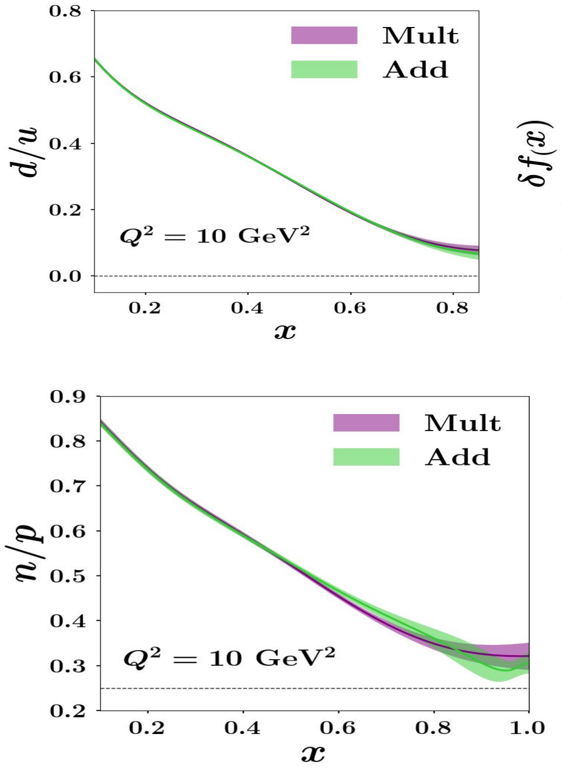
$$\frac{u + \tilde{H}_n/Q^2}{4u + \tilde{H}_p/Q^2}$$

same as Add

Bias removed!

Results in the CJ fitting framework

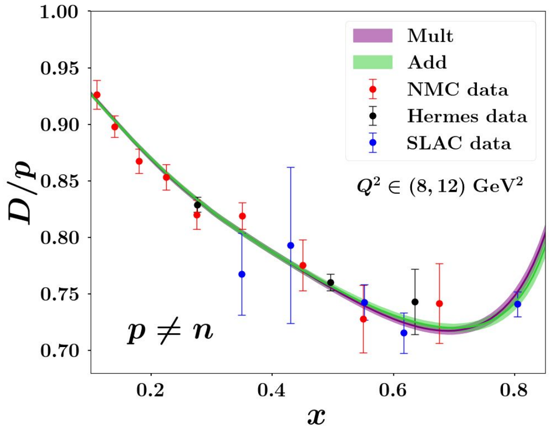
Case 2: isospin-breaking HT



Bias removed

No need of compensation by off-shell

Theory expectations confirmed!



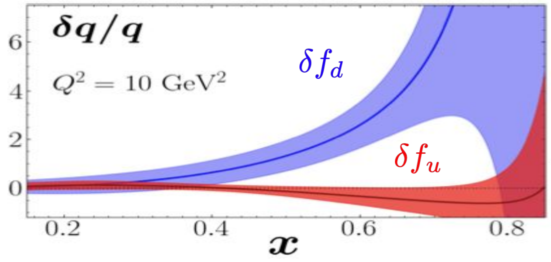
Compatible n/p

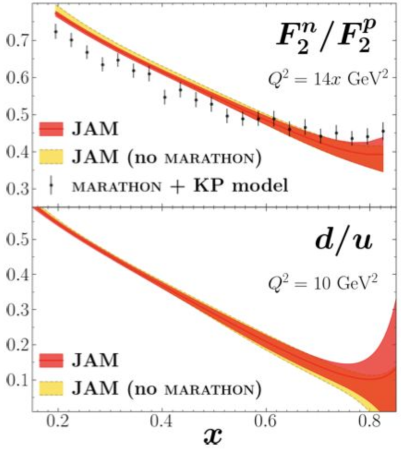
JAM results

JAM *Fit including A=3 data*

JAM Collaboration, PRL 127 (2021)

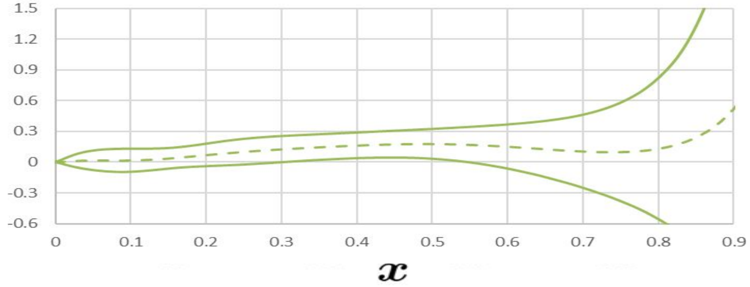
Mult HT (p=n) as default choice





$$\delta f(x)|_{\text{CJ-like}} = \frac{u\delta f_u + d\delta f_d}{u+d}$$

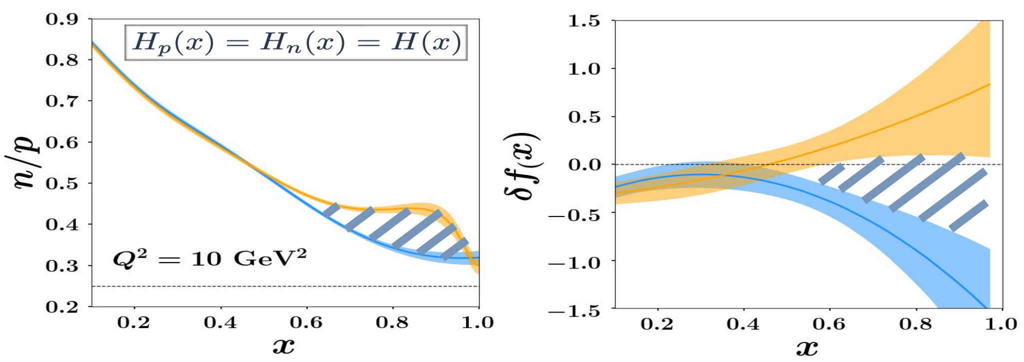
Compatible with CJ results

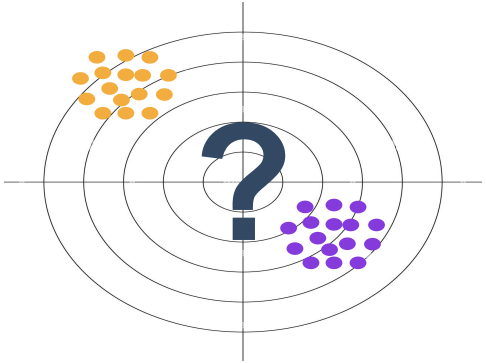


Courtesy of C. Cocuzza

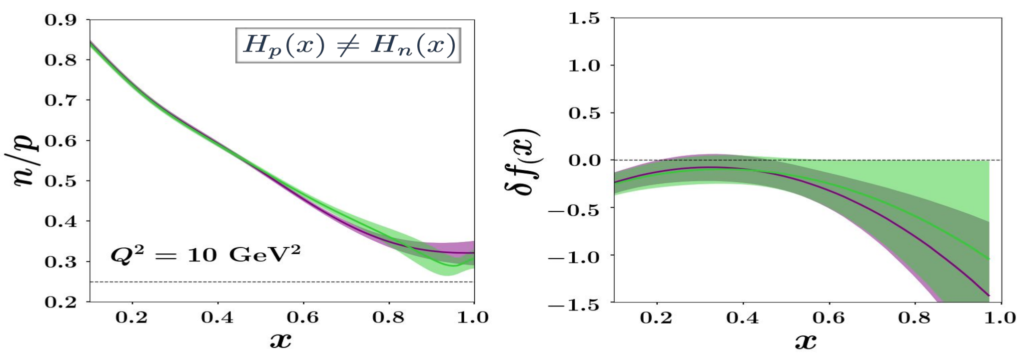
TAKE-HOME message

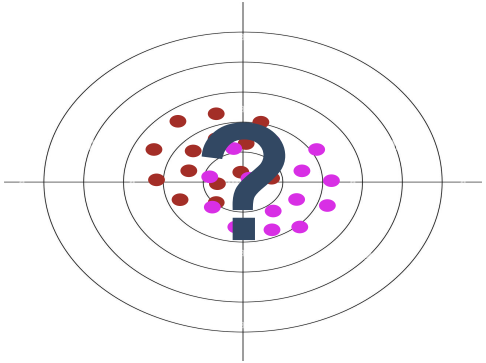
Case 1: isospin-symmetric HT





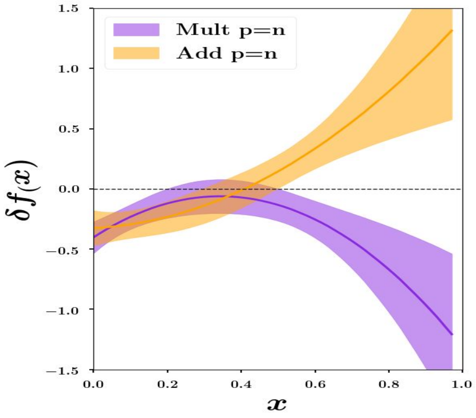
Case 2: isospin-breaking HT

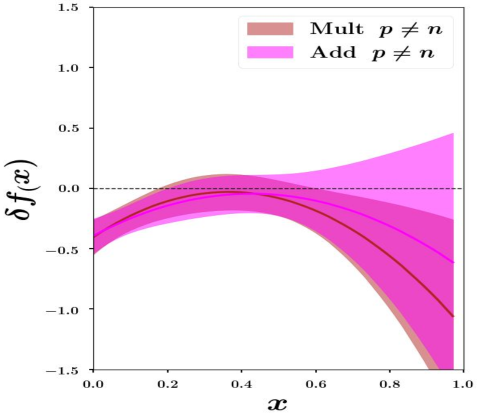




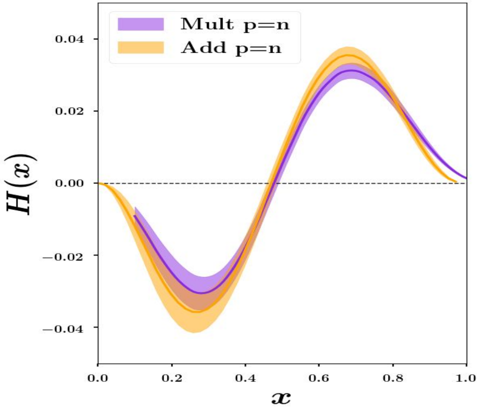
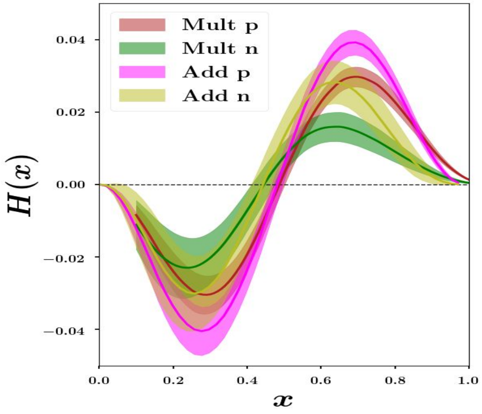
Off-shell table

higher-twist \ isospin	Symmetric	Non symmetric
Multiplicative		
Additive		





Higher-Twist table

higher-twist \ isospin	Symmetric	Non symmetric
Multiplicative	$\tilde{H} = F_{2,N}(x, Q^2) H_{\text{Mult}}(x)$	$\delta\tilde{H} = F_{2,N}(x, Q^2) \delta H_{\text{Mult}}(x)$
Additive		

Some implementation differences

Theoretical choices →

Corrections (increasing-x) ↓

	KP	AKP	CJ15	AKP-like
shadowing	yes	yes (which one?)	MST x<0.1	(same)
smearing	Paris	AV18	AV18 x>0.1	(same)
pi-cloud	yes	yes	----	----
TMC	GP O(Q4)?	GP O(Q4)??	GP approx.	(same)
HT	H (p=n ??)	H (p=n)	C (p=n)	H & C, p=n & p!=n
HT(x)	??	5 pt. spline	parametrized	parametrized
off-shell	O(p2-M2)	O(p2-M2)	O(p2-M2)	(same)
df(x)	factorized	polyn. 2nd/3rd	factorized + sum rule	polyn. 2nd/3rd
pi thresh.	yes	yes	----	----

Data - Fit Residual After Cross-normalization

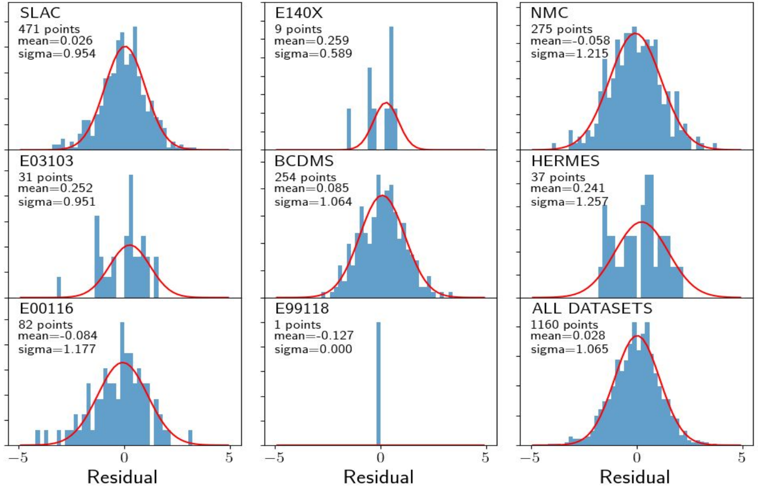
Methods: 10.1103/RevModPhys.92.045003

Correlated uncertainty

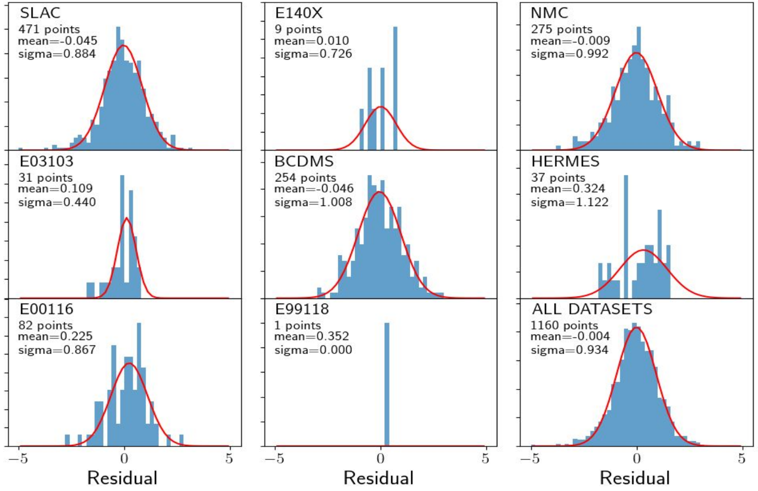
$$r_k(a_{\text{fit}}, \lambda_{\text{fit}}) = \frac{D_k - T_k(a_{\text{fit}})}{\sigma_k} - \sum_I \beta_{kI} \lambda_I^{\text{fit}}$$

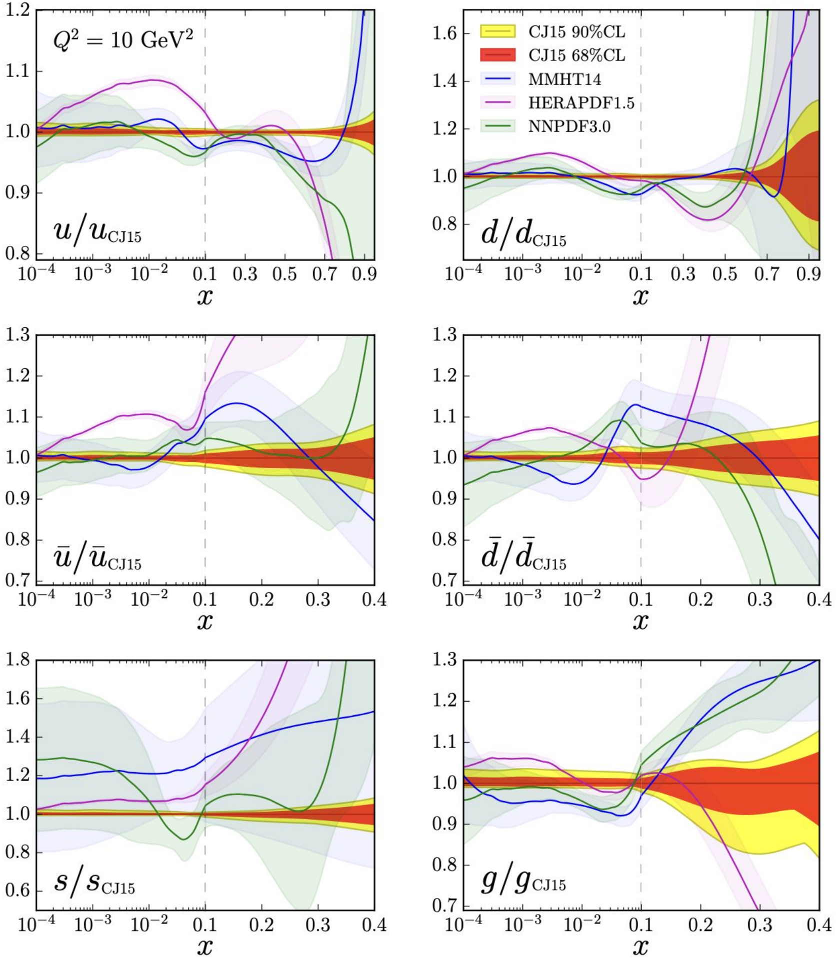
uncorrelated

Proton



Deuteron





$Q^2 = 10$ GeV^2
CJ15 90%CL
CJ15 68%CL
MMHT14
HERAPDF1.5
NNPDF3.0
$u/u_{\rm CJ15}$
$d/d_{\rm CJ15}$
$\bar{u}/\bar{u}_{\rm CJ15}$
$\bar{d}/\bar{d}_{\rm CJ15}$
$s/s_{\rm CJ15}$
$g/g_{\rm CJ15}$
x