Selected

Recent advances in extracting x-dependent GPDs from lattice QCD

Martha Constantinou

Temple University

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OUTLINE

A. Methods to access GPDs from lattice QCD

B. Selected results for the proton:

- twist-2 GPDs
- twist-3 GPDs

$$f_i = f_i^{(0)} + \frac{f_i^{(1)}}{Q} + \frac{f_i^{(2)}}{Q^2} \cdots$$

C. Synergy with phenomenology

D. Concluding remarks

Twist-2 $(f_i^{(0)})$

Quark / Nucleon	U (γ^+)	L ($\gamma^+\gamma^5$)	T (σ^{+j})
U	$H(x,\xi,t)$ $E(x,\xi,t)$ unpolarized		
L		$\widetilde{H}(x,\xi,t)$ $\widetilde{E}(x,\xi,t)$ helicity	
T			H_T, E_T $\widetilde{H}_T, \widetilde{E}_T$ transversity

(Selected) **Twist-3** $(f_i^{(1)})$

$\mathcal{O}$ / Nucleon	γ^j	$\gamma^j\gamma^5$	σ^{jk}
U	G_1, G_2 G_3, G_4		
L		$\widetilde{G}_1, \widetilde{G}_2$ $\widetilde{G}_3, \widetilde{G}_4$	
T			$H_2'(x,\xi,t)$ $E_2'(x,\xi,t)$

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Theoretical/Technical slide warning

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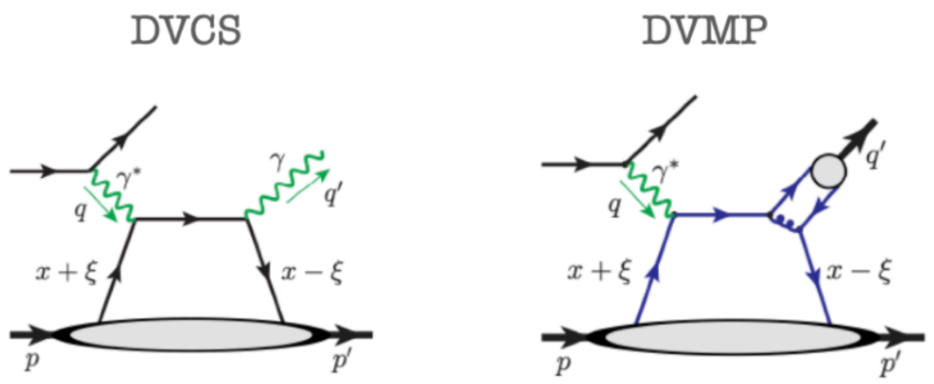
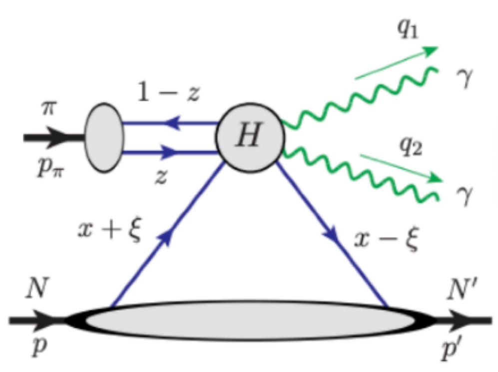
Generalized Parton Distributions

★ GPDs may be accessed via exclusive reactions (DVCS, DVMP)

★ exclusive pion-nucleon diffractive production of a γ pair of high $p_\perp$



[X.-D. Ji, PRD 55, 7114 (1997)]



[J. Qiu et al, arXiv:2205.07846]

Talk by J.W. Qiu

★ GPDs are not well-constrained experimentally:

- x-dependence extraction is not direct. DVCS amplitude: $\mathcal{H} = \int_{-1}^{+1} \frac{H(x, \xi, t)}{x - \xi + i\epsilon} dx$

(SDHEP [J. Qiu et al, arXiv:2205.07846] gives access to x)

- independent measurements to disentangle GPDs
- GPDs phenomenology more complicated than PDFs (multi-dimensionality)
- and more challenges …

Theoretical issues discussed by A. Freese

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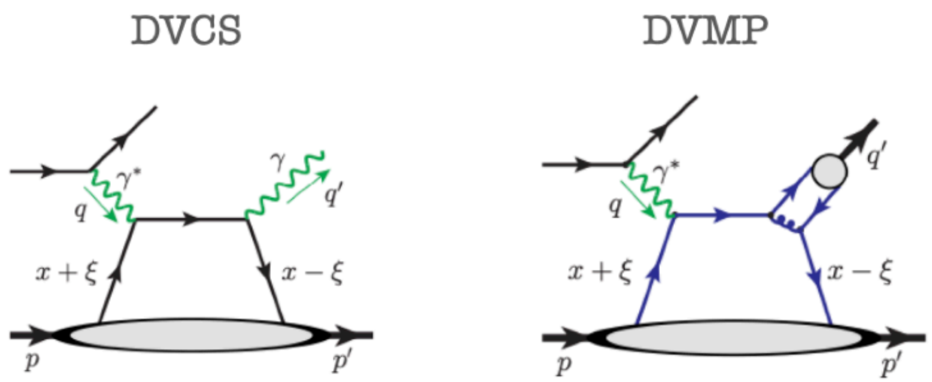

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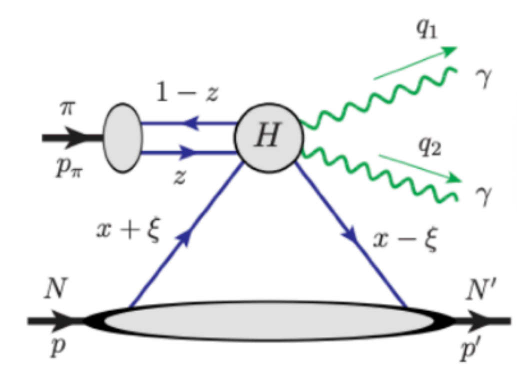

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Theoretical issues discussed by A. Freese

★ Essential to complement the knowledge on GPD from lattice QCD

★ Lattice data may be incorporated in global analysis of experimental data and may influence parametrization of t and ξ dependence

Accessing information on PDFs/GPDs

★PDFs parameterized via matrix elmnts of nonlocal light-cone operators

$$f(x) = \frac{1}{4\pi} \int dy^- e^{-ixP^+y^-} \langle P, S | \bar{\psi}_f \gamma^+ \mathcal{W} \psi_f | P, S \rangle$$

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★ **Mellin moments**

(local OPE expansion)

$$\bar{q}(-\tfrac{1}{2}z)\, \gamma^\sigma W[-\tfrac{1}{2}z, \tfrac{1}{2}z]\, q(\tfrac{1}{2}z) \;=\; \sum_{n=0}^{\infty} \frac{1}{n!}\, z_{\alpha_1} \ldots z_{\alpha_n} \left[\underline{\bar{q} \gamma^\sigma \overleftrightarrow{D}^{\alpha_1} \ldots \overleftrightarrow{D}^{\alpha_n} q}\right]$$

local operators

$$\langle N(P')|\mathcal{O}_V^{\mu\mu_1\cdots\mu_{n-1}}|N(P)\rangle \sim \sum_{\substack{i=0\\ \text{even}}}^{n-1} \left\{ \gamma^{\{\mu} \Delta^{\mu_1} \cdots \Delta^{\mu_i} \overline{P}^{\mu_{i+1}} \cdots \overline{P}^{\mu_{n-1}\}} A_{n,i}(t) - i \frac{\Delta_\alpha \sigma^{\alpha\{\mu}}{2m_N} \Delta^{\mu_1} \cdots \Delta^{\mu_i} \overline{P}^{\mu_{i+1}} \cdots \overline{P}^{\mu_{n-1}\}} B_{n,i}(t) \right\} + \frac{\Delta^\mu \Delta^{\mu_1} \ldots \Delta^{\mu_{n-1}}}{m_N} C_{n,0}(\Delta^2)\Big|_{n \text{ even}} \Big) \Big\} + \frac{\Delta^\mu \Delta^{\mu_1} \ldots \Delta^{\mu_{n-1}}}{m_N} C_{n,0}(\Delta^2)\Big|_{n \text{ even}} \Big] U(P)$$

Reconstruction of PDFs/GPDs very challenging

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Reconstruction of PDFs/GPDs very challenging

★ **Matrix elements of nonlocal operators**

(quasi-GPDs, pseudo-GPDs, …)

$$\langle N(P_f)\,|\,\bar{\Psi}(z)\,\Gamma\,\mathscr{W}(z,0)\Psi(0)\,|\,N(P_i)\rangle_\mu$$

Nonlocal operator with Wilson line

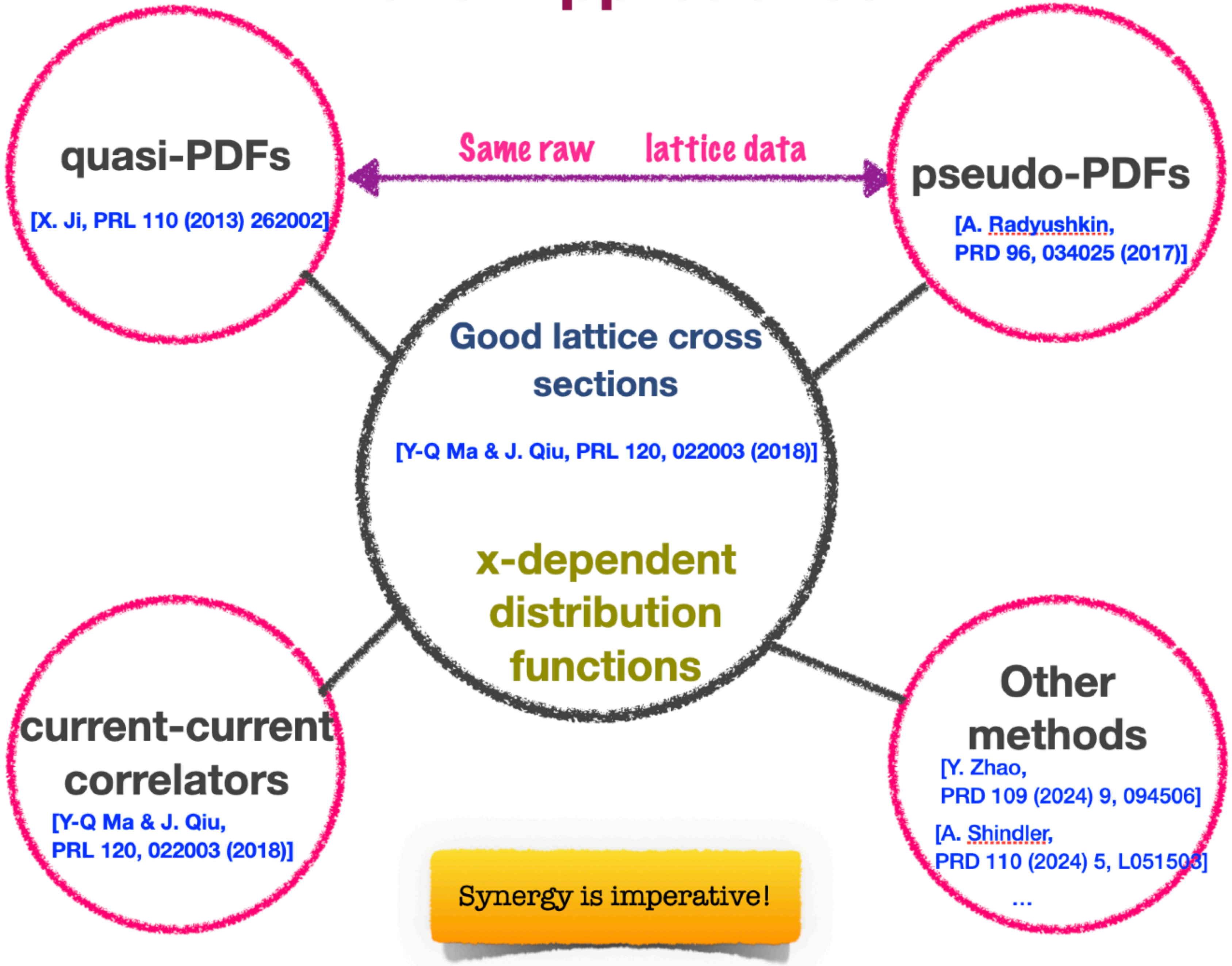
$$\langle N(P')|O_V^\mu(x)|N(P)\rangle = \overline{U}(P')\left\{\gamma^\mu H(x,\xi,t) + \frac{i\sigma^{\mu\nu}\Delta_\nu}{2m_N}E(x,\xi,t)\right\}U(P) + \text{ht}\,,$$

$$\langle N(P')|O_A^\mu(x)|N(P)\rangle = \overline{U}(P')\left\{\gamma^\mu\gamma_5\widetilde{H}(x,\xi,t) + \frac{\gamma_5\Delta^\mu}{2m_N}\widetilde{E}(x,\xi,t)\right\}U(P) + \text{ht}\,,$$

$$\langle N(P')|O_T^{\mu\nu}(x)|N(P)\rangle = \overline{U}(P')\left\{i\sigma^{\mu\nu}H_T(x,\xi,t) + \frac{\gamma^{[\mu}\Delta^{\nu]}}{2m_N}E_T(x,\xi,t) + \frac{\overline{P}^{[\mu}\Delta^{\nu]}}{m_N^2}\widetilde{H}_T(x,\xi,t) + \frac{\gamma^{[\mu}\overline{P}^{\nu]}}{m_N}\widetilde{E}_T(x,\xi,t)\right\}U(P) + \text{ht}$$

This talk

Novel Approaches



Reviews of methods and applications

- ***A guide to light-cone PDFs from Lattice QCD: an overview of approaches, techniques and results***
  K. Cichy & M. Constantinou (invited review) Advances in HEP 2019, 3036904, arXiv:1811.07248
- ***Large Momentum Effective Theory***
  X. Ji, Y.-S. Liu, Y. Liu, J.-H. Zhang, and Y. Zhao (2020), 2004.03543
- ***The x-dependence of hadronic parton distributions: A review on the progress of lattice QCD***
  M. Constantinou (invited review) Eur. Phys. J. A 57 (2021) 2, 77, arXiv:2010.02445

Well-studied “novel” methods for PDFs/GPDs in LQCD

Matrix elements of non-local operators (space-like separated fields) with boosted hadrons

$$\mathcal{M}(P_f, P_i, z) = \langle N(P_f) \,|\, \bar{\Psi}(z)\, \Gamma\, \mathcal{W}(z,0) \Psi(0) \,|\, N(P_i) \rangle_\mu$$

Calculation very taxing!

- length of the Wilson line (z)
- nucleon momentum boost (P_3)

} PDFs, GPDs

- momentum transfer (t)
- skewness (ξ)

} GPDs

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[X. Ji, Phys. Rev. Lett. 110 (2013) 262002]
[X. Ji, Sci. China Phys. M.A. 57 (2014) 1407]

quasi-PDFs

$$\tilde{q}_\Gamma^{\text{GPD}}(x, t, \xi, P_3, \mu) = \int \frac{dz}{4\pi} e^{-i\,x\,P_3\,z} \mathcal{M}(P_f, P_i, z)$$

pseudo-ITD [A. Radyushkin, PRD 96, 034025 (2017)]

$$\mathfrak{M}(\nu, \xi, t; z_3^2) \equiv \frac{\mathcal{M}(\nu, \xi, t; z_3^2)}{\mathcal{M}(0,0,0; z^2)} \qquad (\nu = z \cdot p)$$

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Matching in momentum space
(Large Momentum
Effective Theory)

Matching in ν space

$$Q(\nu, \mu^2) = \int_{-1}^{1} dx\, e^{i\nu x} q(x, \mu^2)$$

Light-cone PDFs & GPDs

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Matching resembles factorization:

$$\sigma_{\mathrm{DIS}}(x, Q^2) = \sum_i \left[H^i_{\mathrm{DIS}} \otimes f_i\right](x, Q^2)$$

$$\tilde{q}^{\mathrm{GPD}}_{\Gamma}(x, t, \xi, P_3, \mu) = \int \frac{dz}{4\pi} e^{-i\,x\,P_3\,z}\, \mathcal{M}(P_f, P_i, z)$$

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GPDs on the lattice

★ **Off-forward matrix elements of non-local light-cone operators**

$$F^{[\gamma^+]}(x,\Delta;\lambda,\lambda') = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ik\cdot z} \langle p';\lambda' | \bar{\psi}(-\tfrac{z}{2})\,\gamma^+ \mathscr{W}(-\tfrac{z}{2},\tfrac{z}{2})\psi(\tfrac{z}{2})\,| p;\lambda\rangle \Big|_{z^+=0,\vec{z}_\perp=\vec{0}_\perp}$$

★ **Parametrization in two leading twist GPDs**

$$F^{[\gamma^+]}(x,\Delta;\lambda,\lambda') = \frac{1}{2P^+}\bar{u}(p',\lambda')\left[\gamma^+ H(x,\xi,t) + \frac{i\sigma^{+\mu}\Delta_\mu}{2M}E(x,\xi,t)\right]u(p,\lambda)$$

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How can one define GPDs on a Euclidean lattice?

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$$F^{[\gamma^3]}(x,\Delta;\lambda,\lambda';P^3) = \frac{1}{2P^0}\bar{u}(p',\lambda')\left[\gamma^3 H_{Q(0)}(x,\xi,t;P^3) + \frac{i\sigma^{3\mu}\Delta_\mu}{2M}E_{Q(0)}(x,\xi,t;P^3)\right]u(p,\lambda)$$

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→ reduction of power corrections in fwd limit [Radyushkin, PLB 767, 314, 2017]

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γ^0 ideal for PDFs

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γ^0 ideal for PDFs

γ^0 parametrization is prohibitively expensive

GPDs on the lattice

★ Lorentz invariant parametrization

$$F^{\mu}_{\lambda,\lambda'} = \bar{u}(p',\lambda')\left[\frac{P^{\mu}}{M}A_1 + z^{\mu}\,MA_2 + \frac{\Delta^{\mu}}{M}A_3 + i\sigma^{\mu z}\,MA_4 + \frac{i\sigma^{\mu\Delta}}{M}A_5 + \frac{P^{\mu}i\sigma^{z\Delta}}{M}A_6 + \frac{z^{\mu}i\sigma^{z\Delta}}{M}A_7 + \frac{\Delta^{\mu}i\sigma^{z\Delta}}{M}A_8\right]u(p,\lambda)$$

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★ quasi-GPDs may be redefined (Lorentz covariant) inspired by light-cone

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- symmetric frame: $\vec{p}^{\,s}_f = \vec{P} + \frac{\vec{Q}}{2}, \quad \vec{p}^{\,s}_i = \vec{P} - \frac{\vec{Q}}{2} \qquad -t^s = \vec{Q}^2 = 0.69\, GeV^2$

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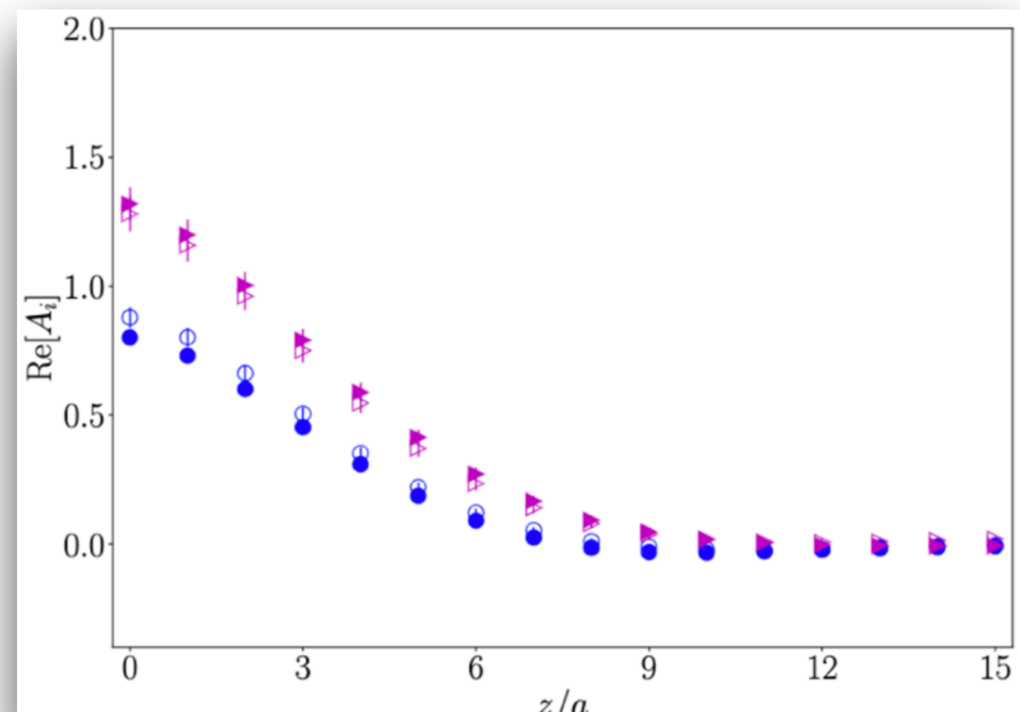
[S. Bhattacharya et al., PRD 106 (2022) 11, 114512]



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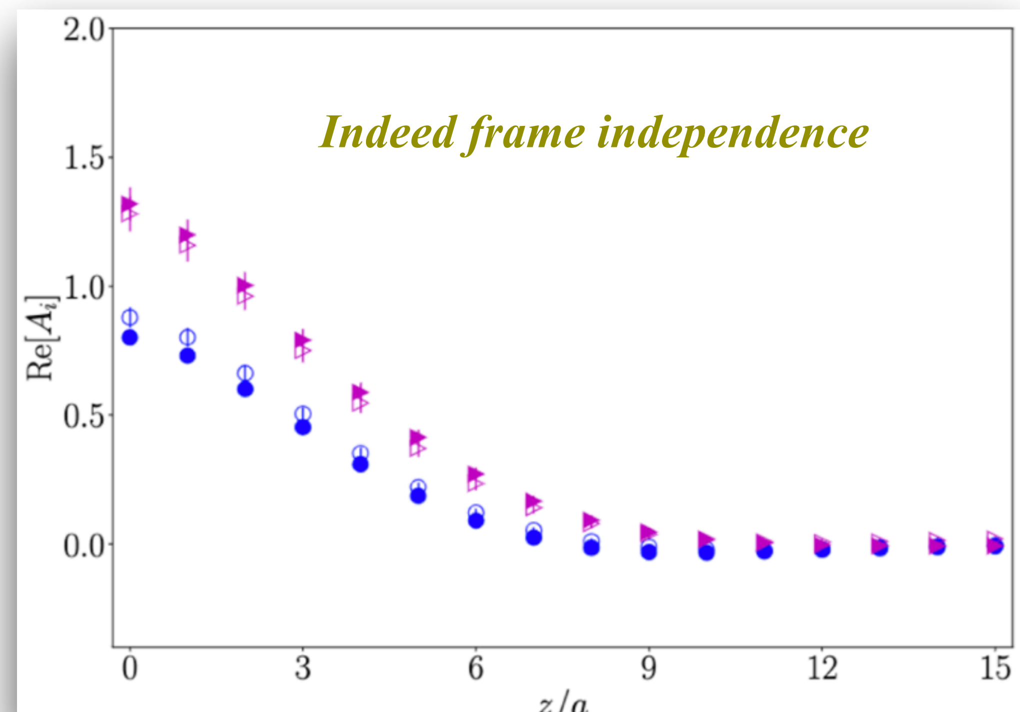
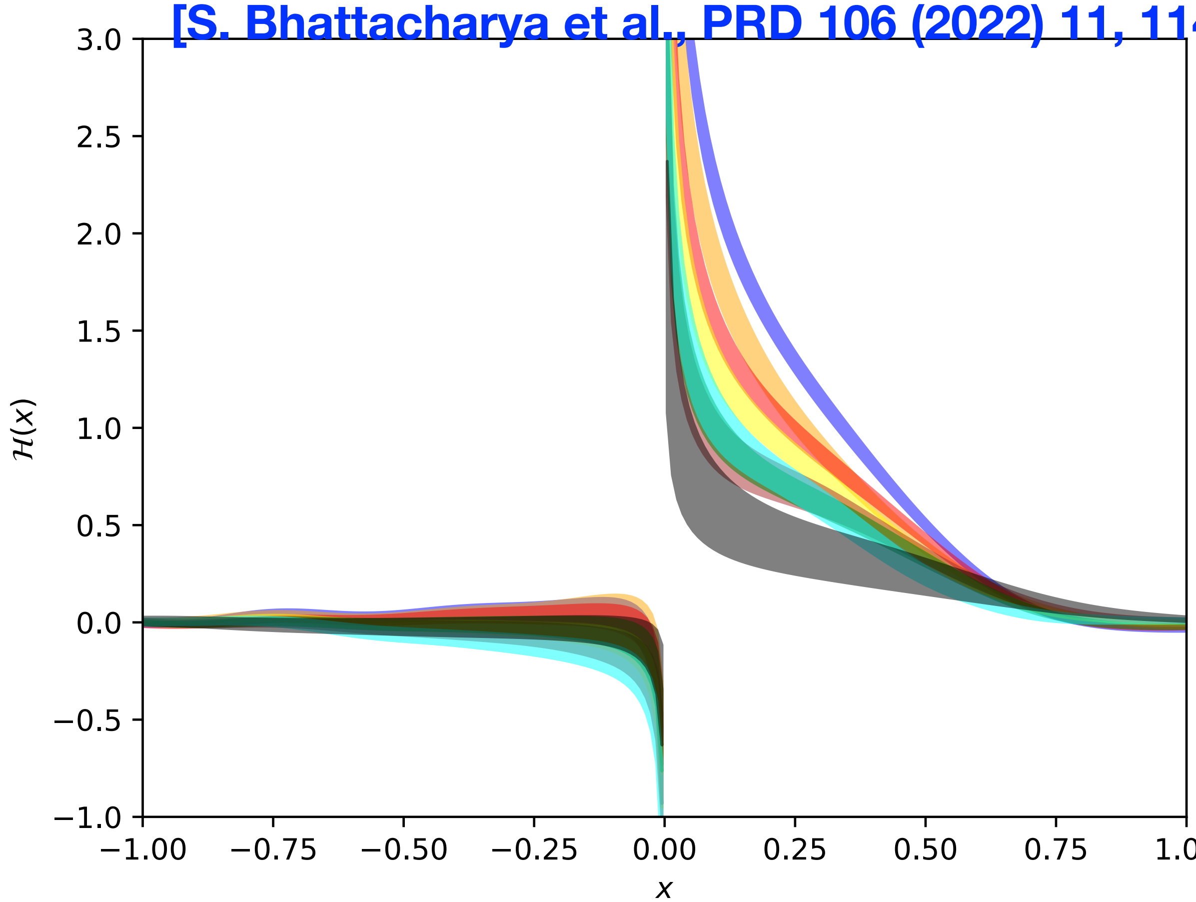
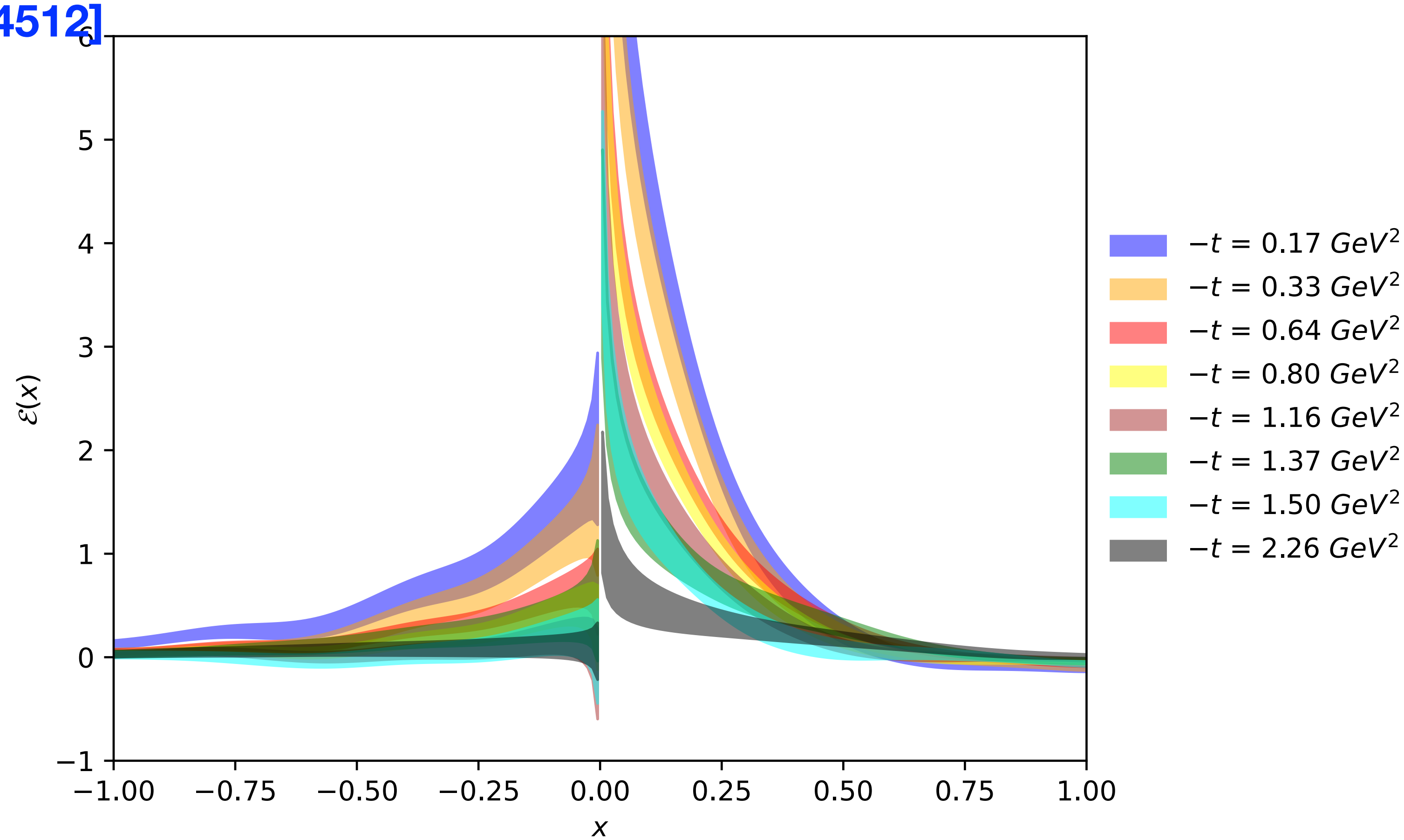
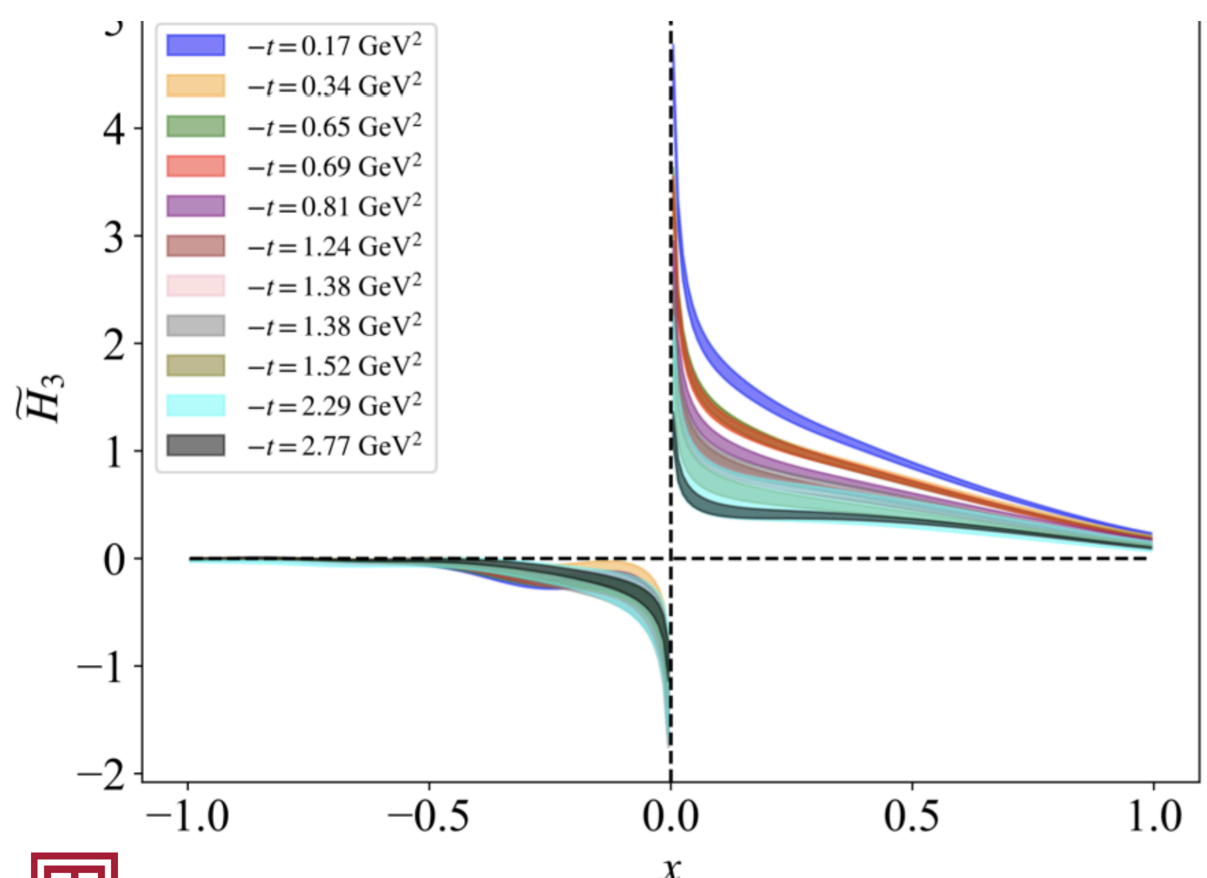


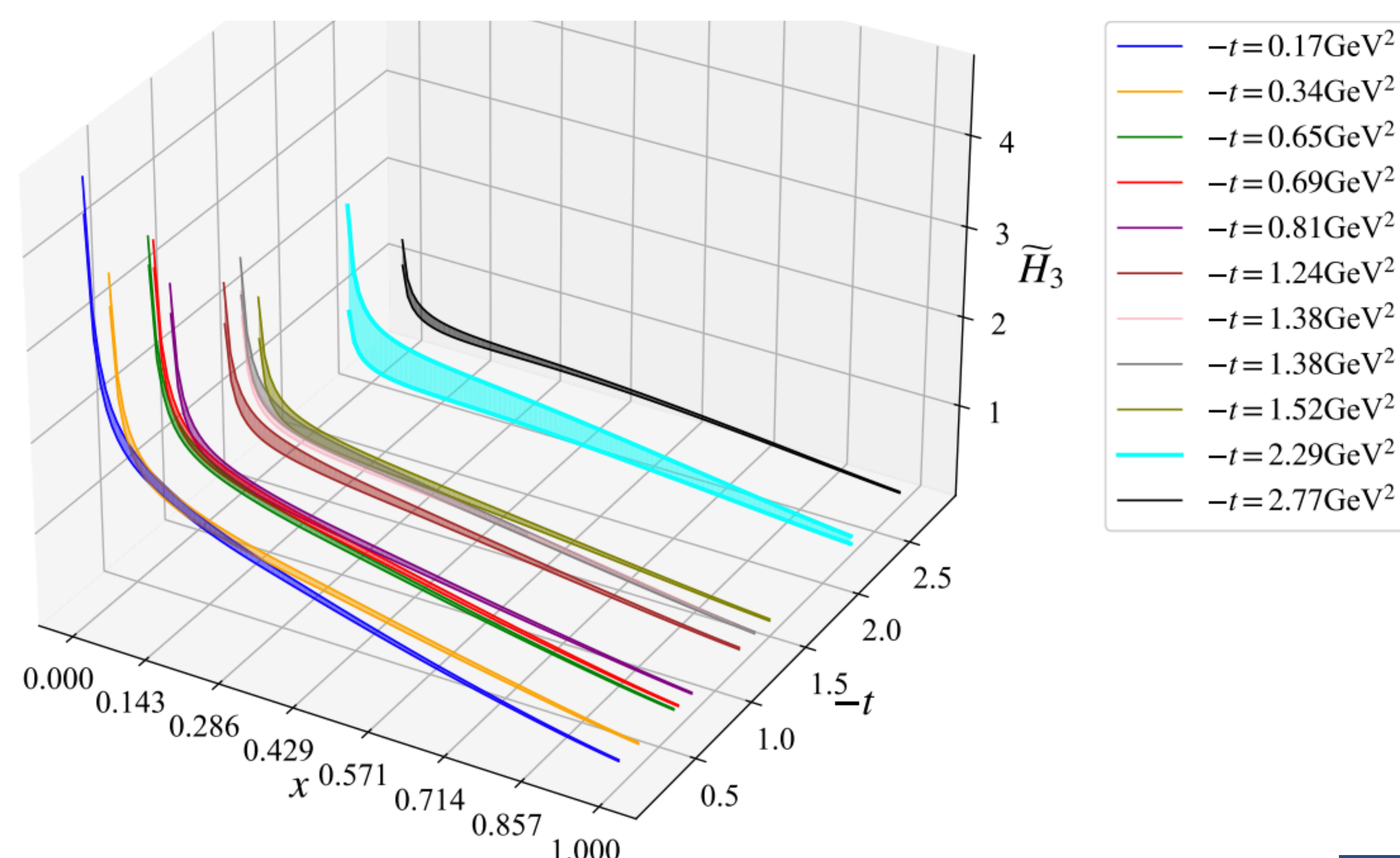
Results: unpolarized, helicity

[S. Bhattacharya et al., PRD 106 (2022) 11, 114512]

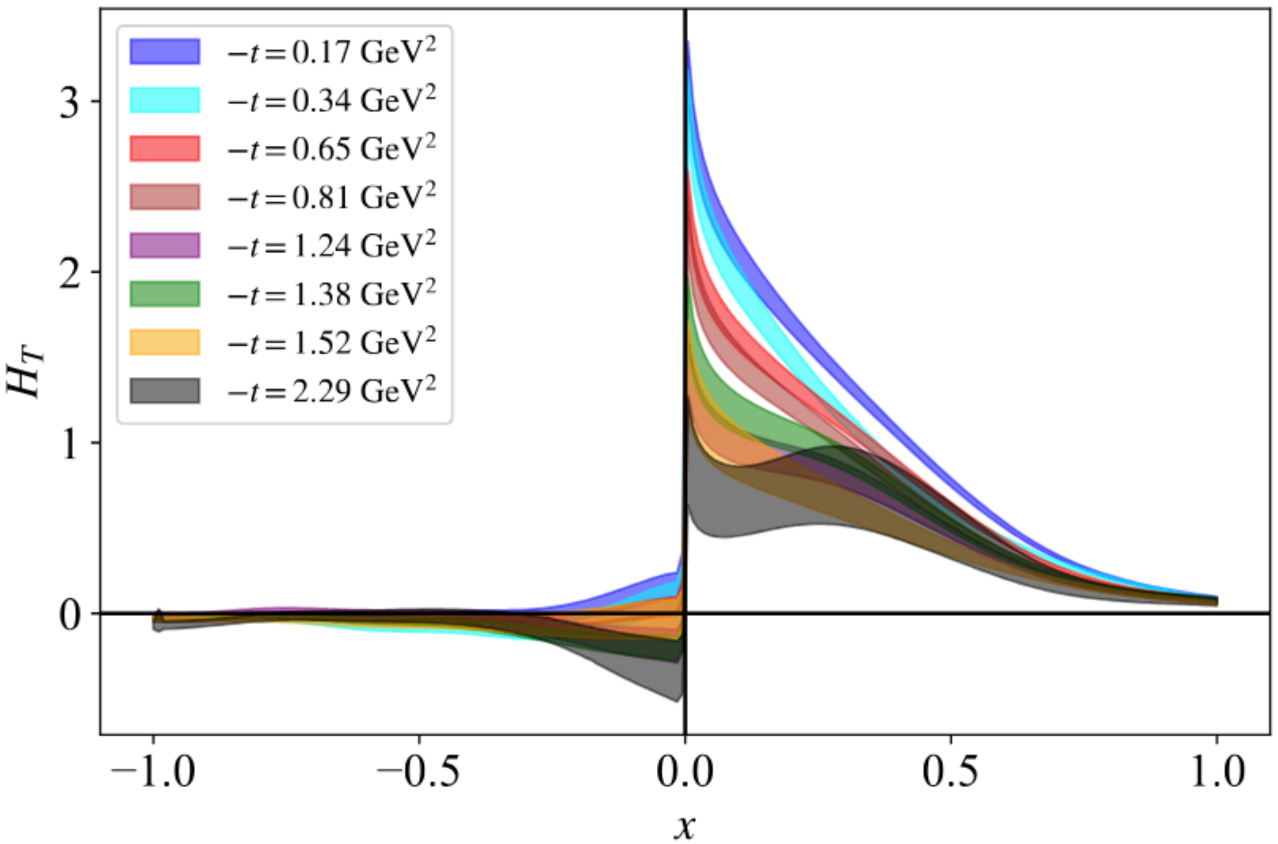


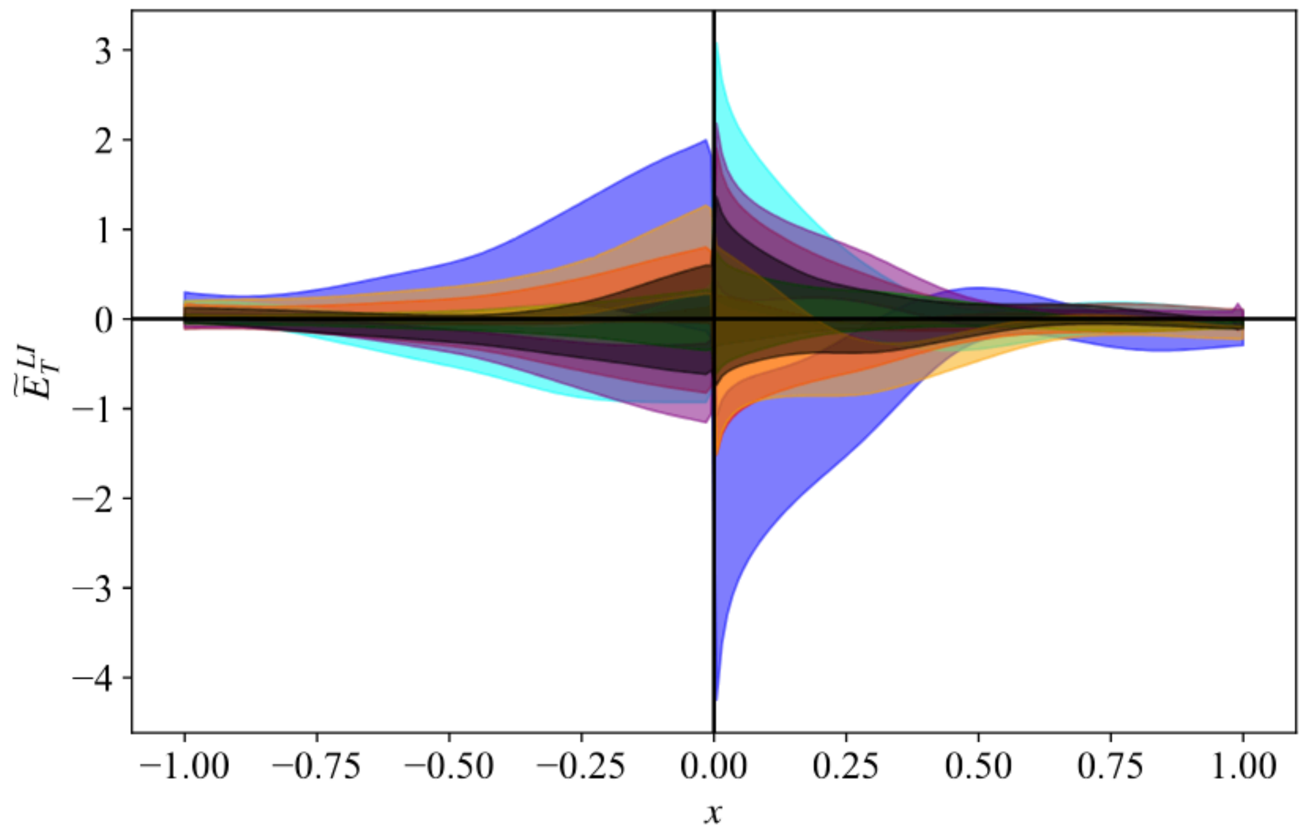


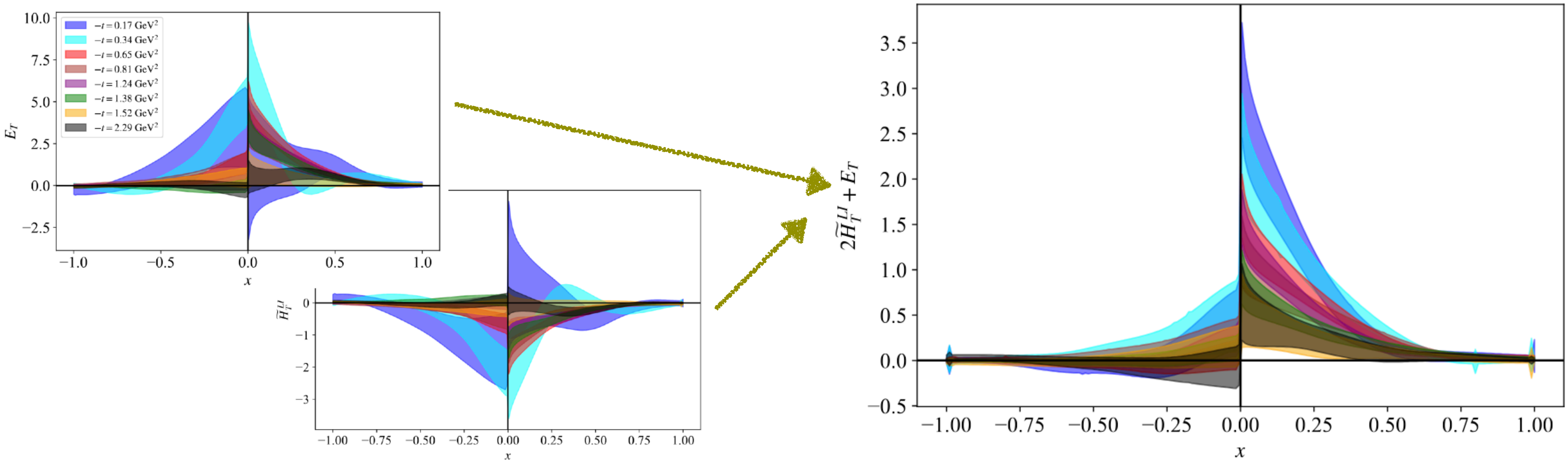




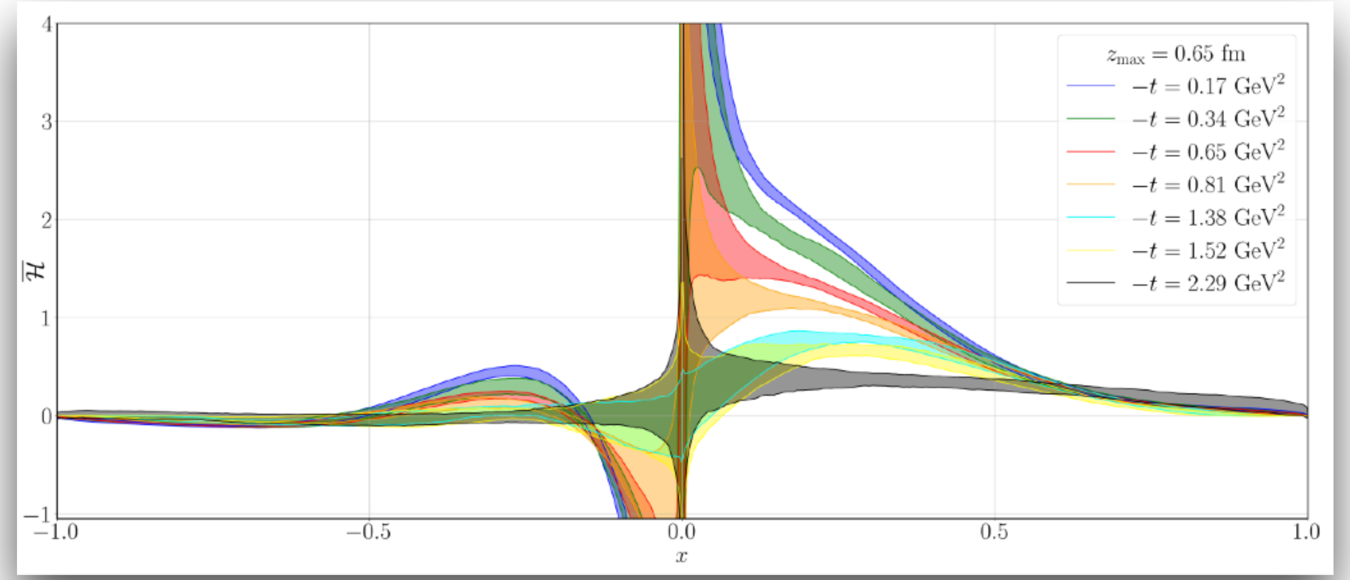
Results: Transversity GPDs







Alternative approach: pseudo-ITD



[Battacharya et al., PRD 110 (2024) 5, 054502]

Different steps between approaches:

- renormalization

- x-dependence reconstruction

- matching formalism

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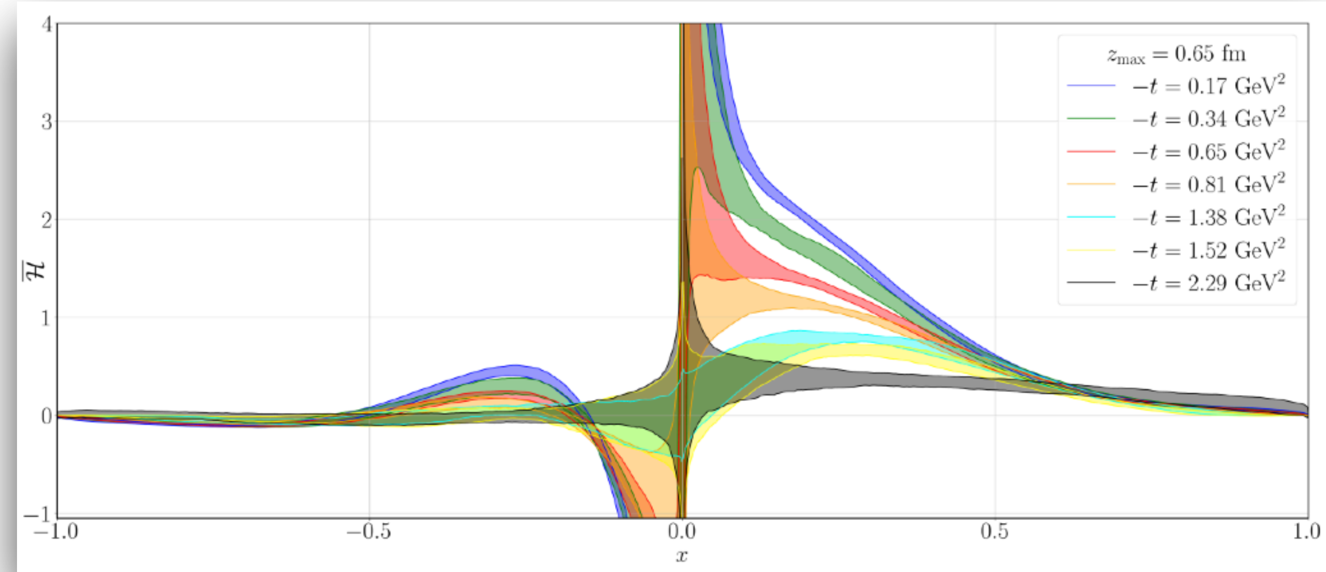

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Different steps between approaches:
- renormalization
- x-dependence reconstruction
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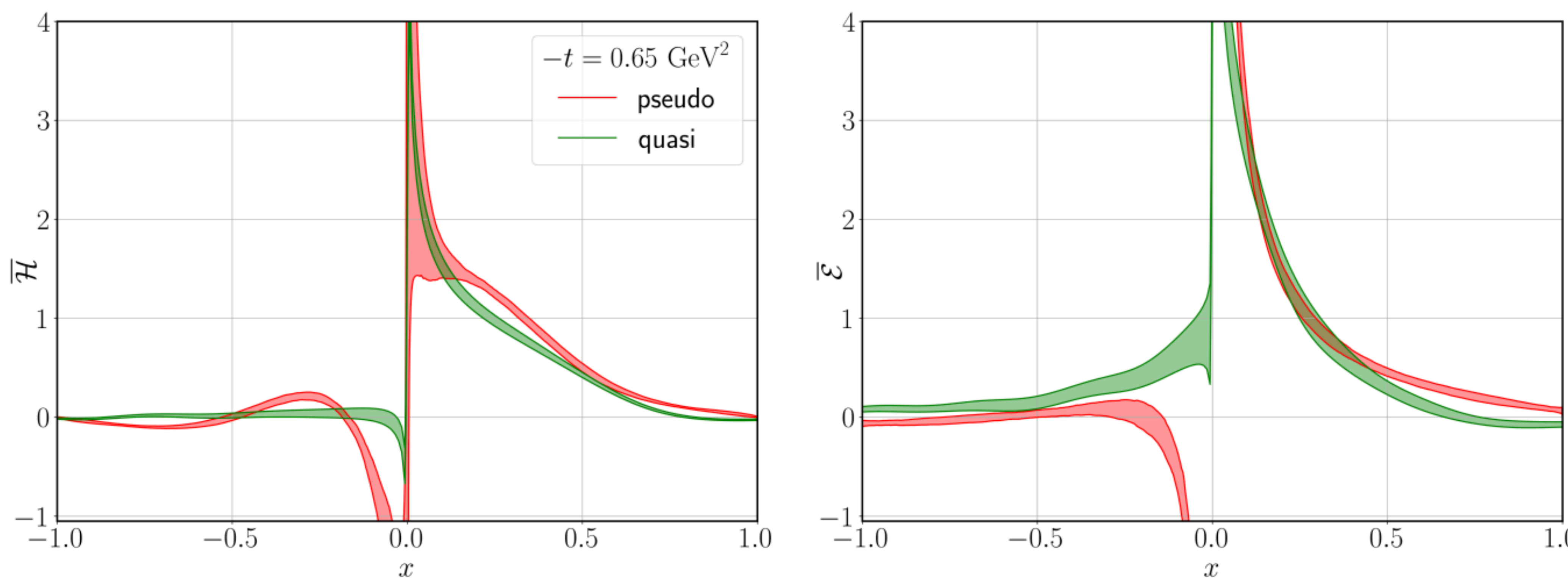
★ Comparison between methods helps assess systematic effects



★ x<0 and small-x regions susceptible to systematic effects

★ Comparison only includes systematic uncertainties

Mellin moments from non-local operators

[S. Bhattacharya et al., PRD 108 (2023) 1, 014507; arXiv:2410.03539]

- ★ Leading-twist factorization formula

$$\mathscr{M}(z,P,\Delta) \equiv \frac{\mathscr{F}(z,P,\Delta)}{\mathscr{F}(z,P=0,\Delta=0)} = \sum_{n=0} \frac{(-izP)^n}{n!} \frac{C_n^{\overline{\mathrm{MS}}}(\mu^2 z^2)}{C_0^{\overline{\mathrm{MS}}}(\mu^2 z^2)} \langle x^n \rangle + \mathscr{O}(\Lambda_{\mathrm{QCD}}^2 z^2)$$

- ★ Avoid power-divergent mixing of multi-derivative operators
- ★ Wilson coefficients known to NLO (or NNLO)
- ★ Both isovector and isoscalar (ignores disconnected; found to be tiny)

[C. Alexandrou et al., PRD 104 (2021) 5, 054503]

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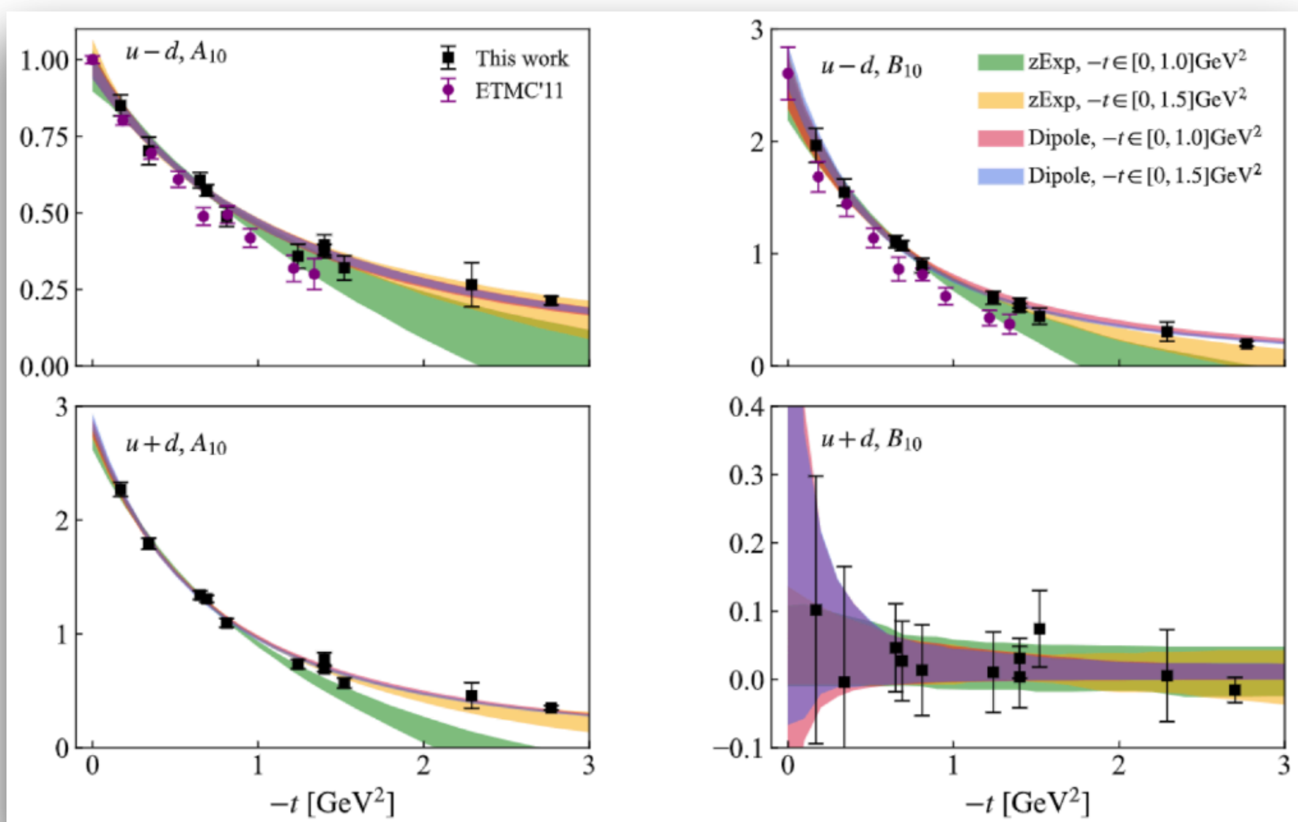

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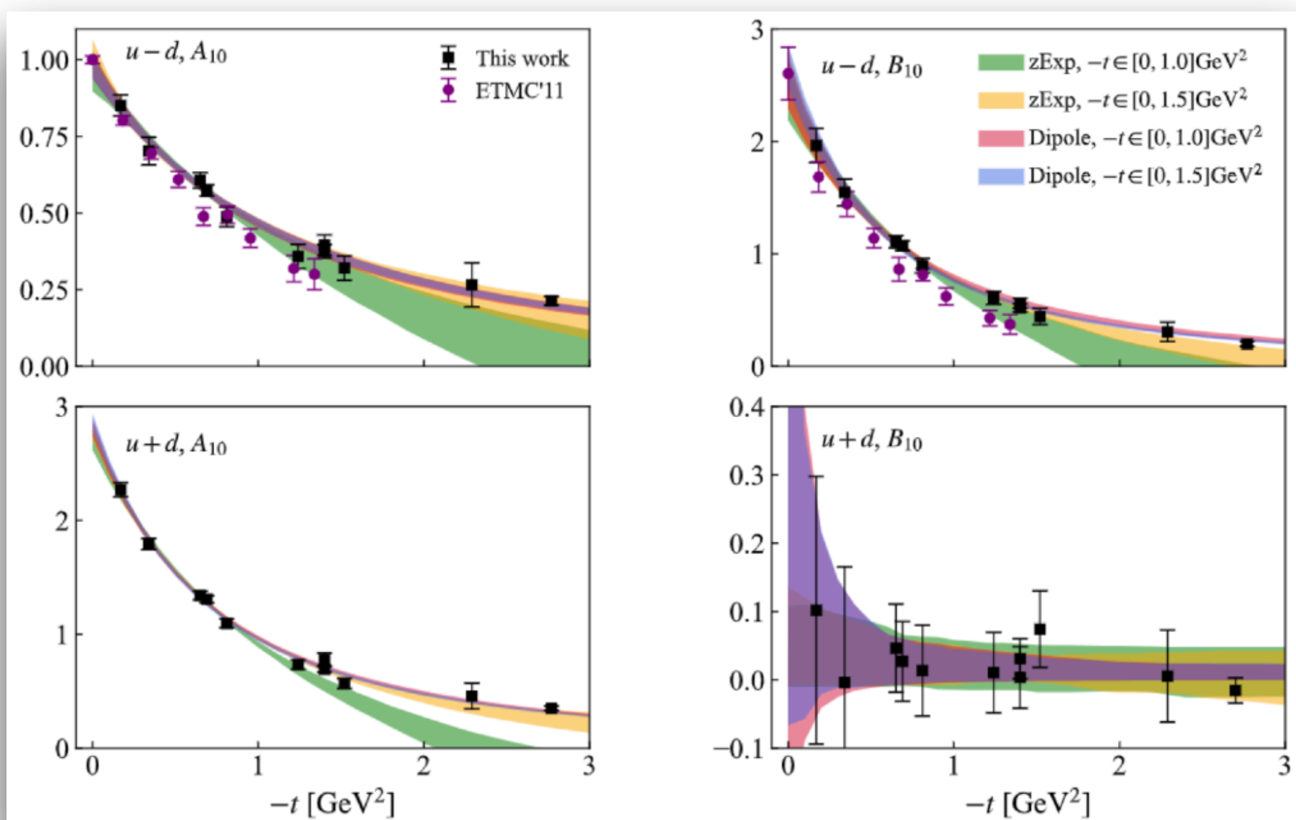

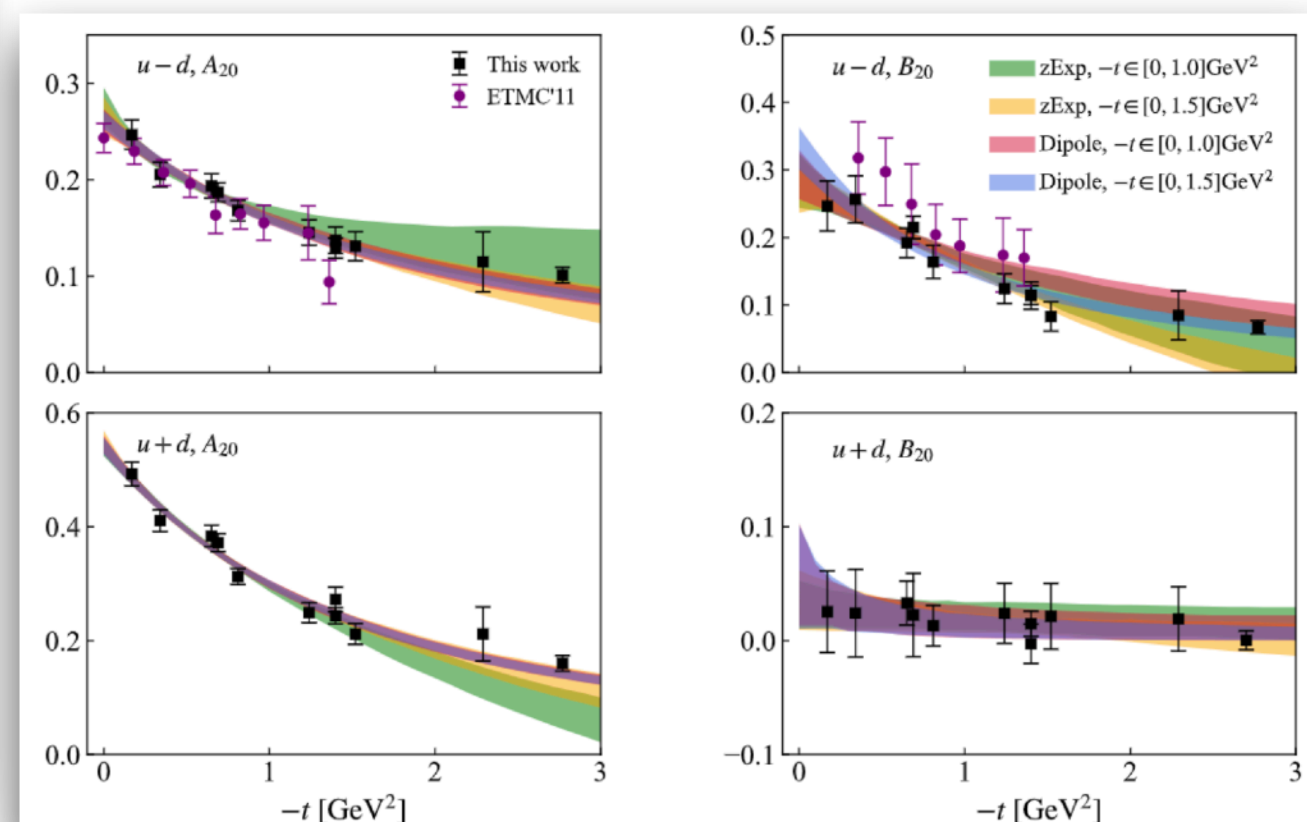

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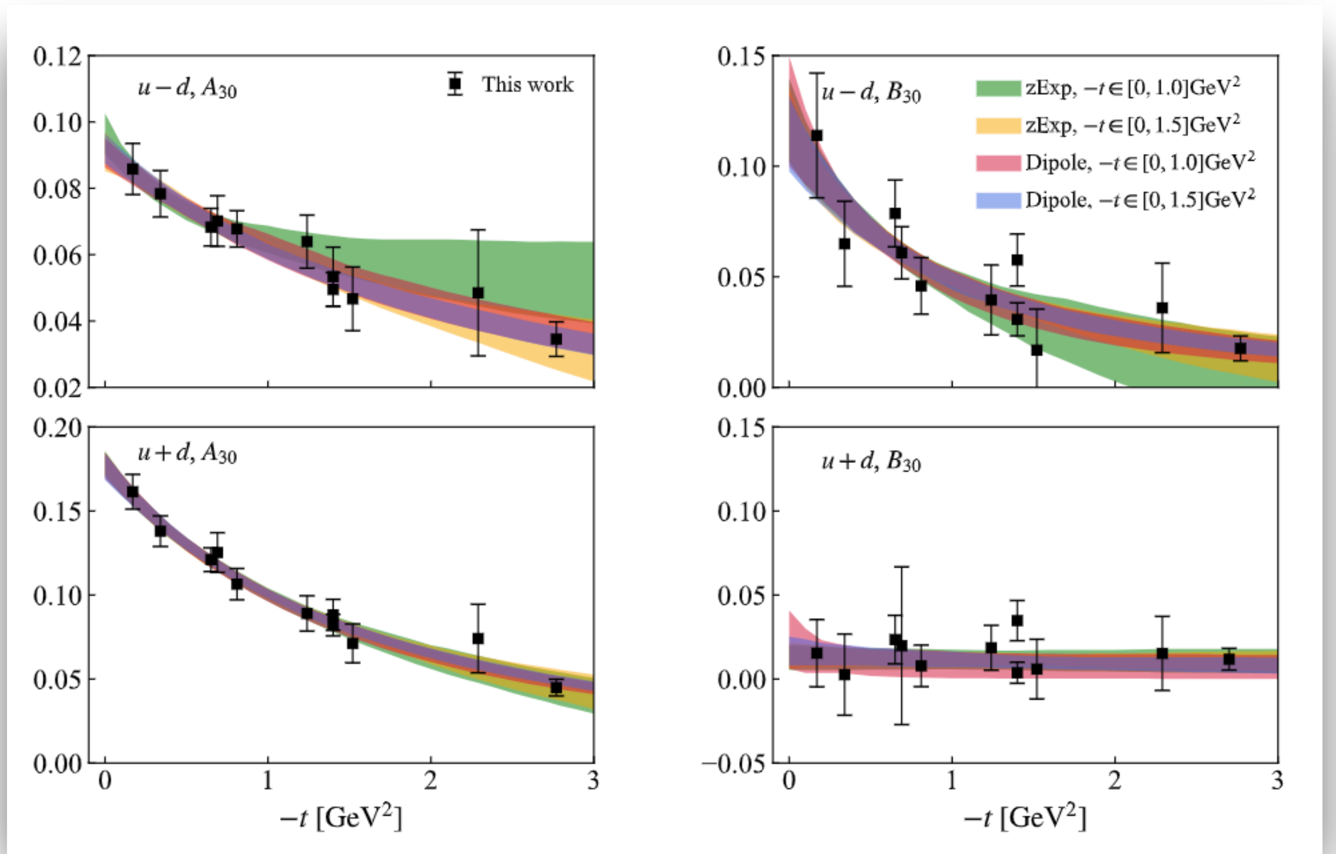

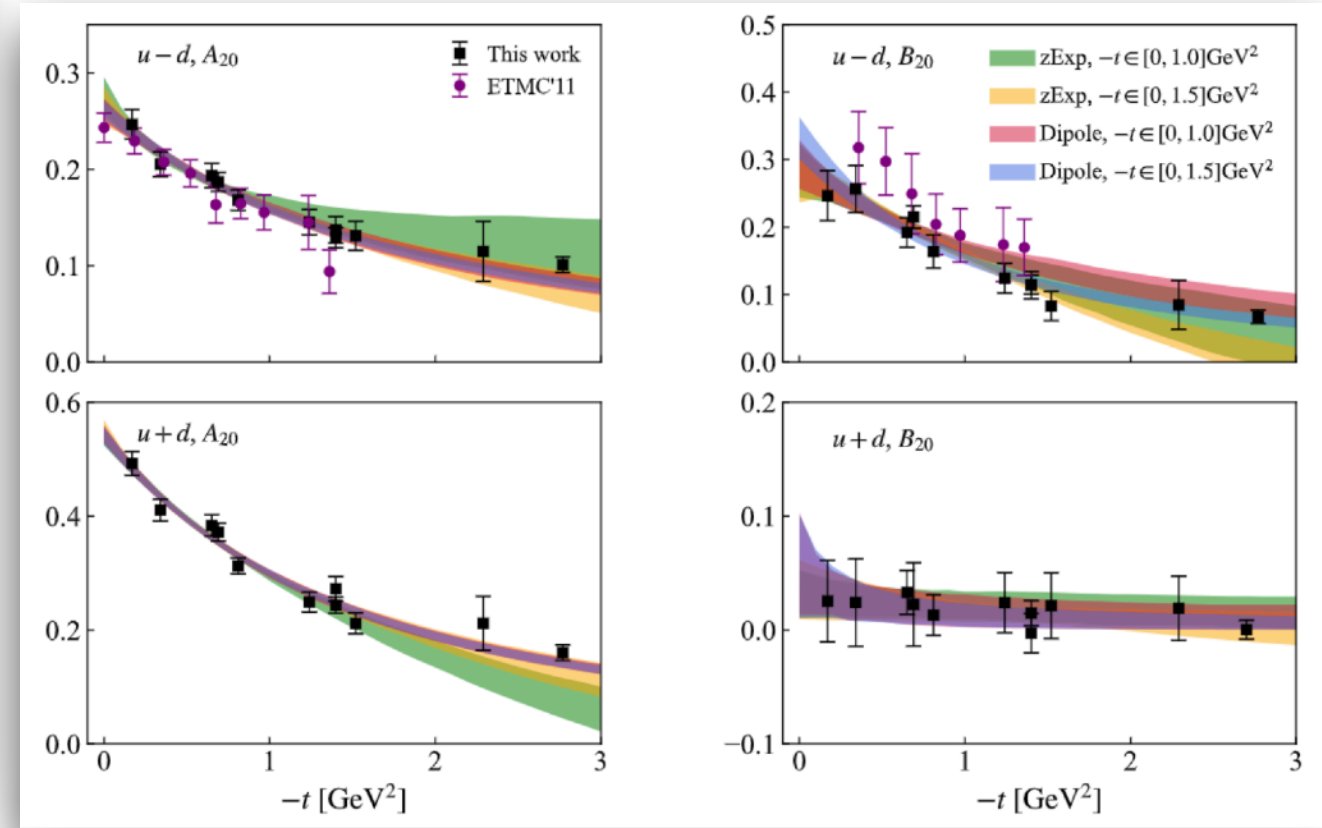

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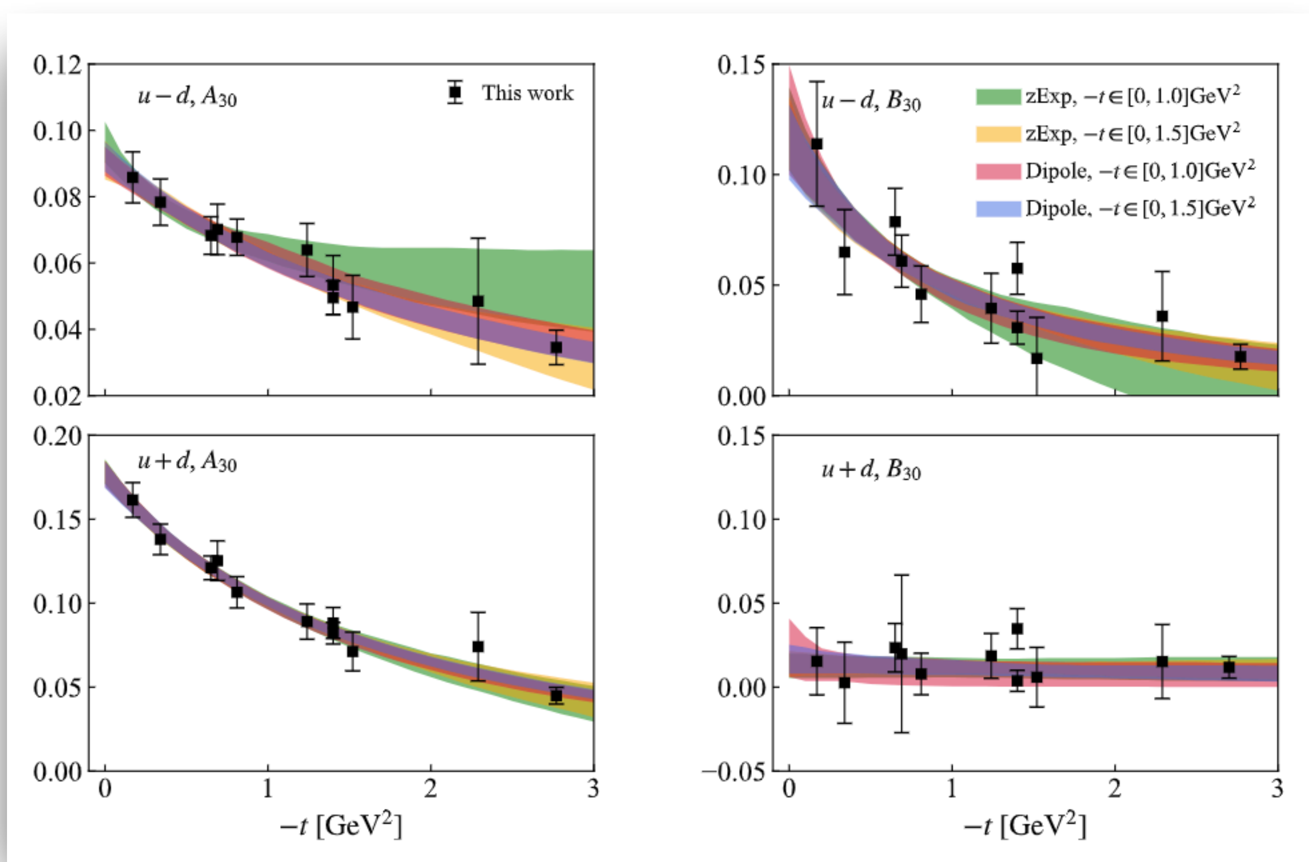

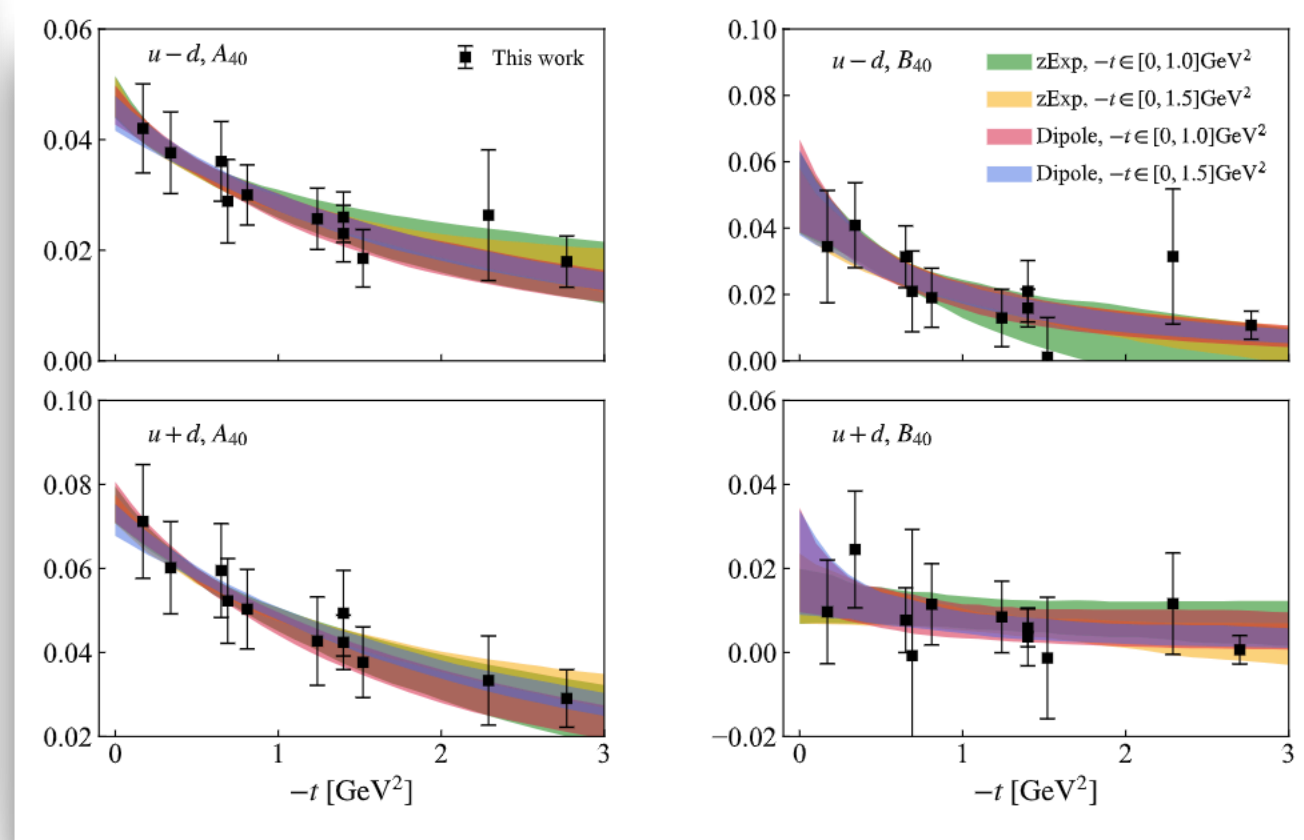

Beyond leading twist

Extraction twist-3 very challenging
both experimentally and theoretically

Theoretical setup

[D. Kiptily and M. Polyakov, Eur. Phys. J. C37 (2004) 105] [F. Aslan et a., Phys. Rev. D 98, 014038 (2018)]

$$F^{[\gamma^\mu\gamma_5]}(x,\Delta;P^3) = \frac{1}{2P^3}\bar{u}(p_f,\lambda')\Big[P^\mu\frac{\gamma^3\gamma_5}{P^0}F_{\widetilde{H}}(x,\xi,t;P^3) + P^\mu\frac{\Delta^3\gamma_5}{2mP^0}F_{\widetilde{E}}(x,\xi,t;P^3)$$
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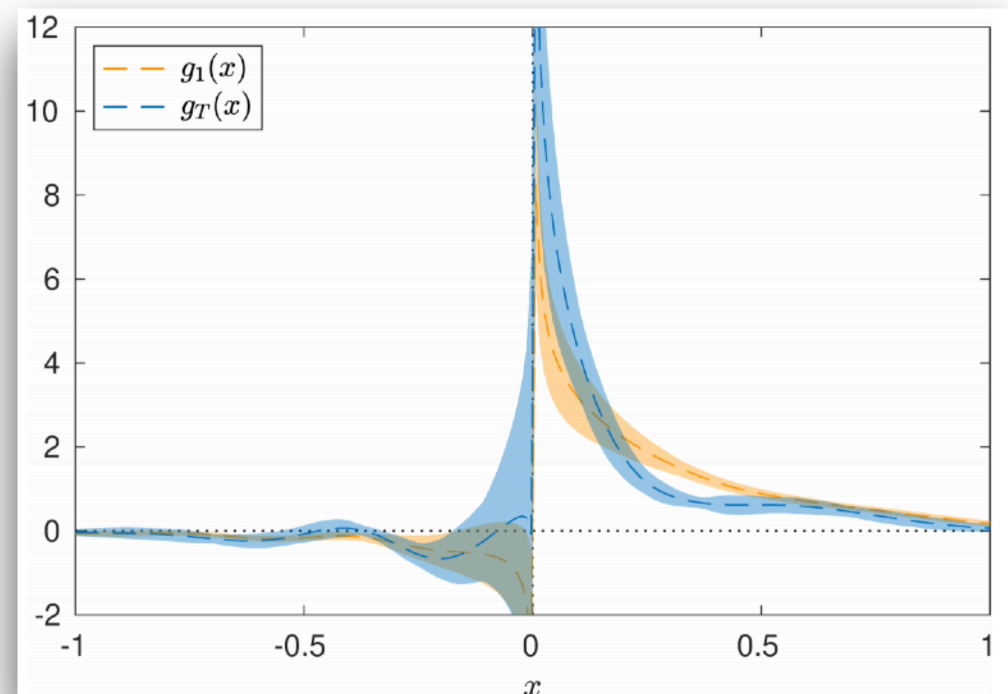
★ **Forward limit for twist-3: only $\widetilde{H}+\widetilde{G}_2 \equiv g_T$ survives**

[S. Bhattacharya et al., PRD 102 (2020) 11 (Editors Selection)]



Twist-3 very important and have physical interpretation:

- as sizable as twist-2

- contain information about quark-gluon correlations inside hadrons

- appear in QCD factorization theorems for various observables (e.g. g_2)

Talk by S. Li

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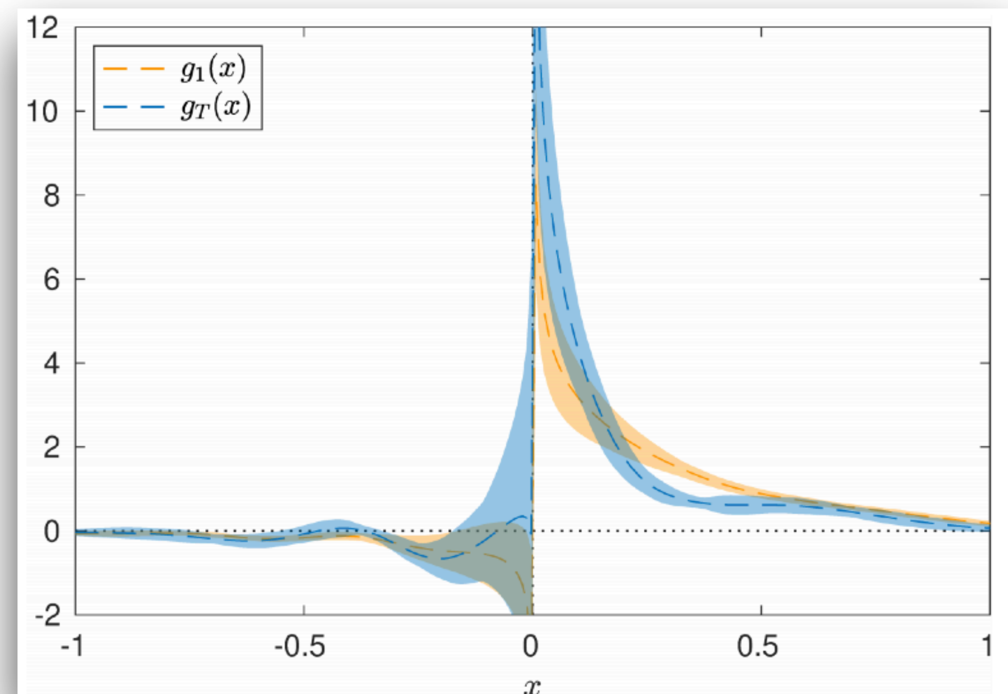

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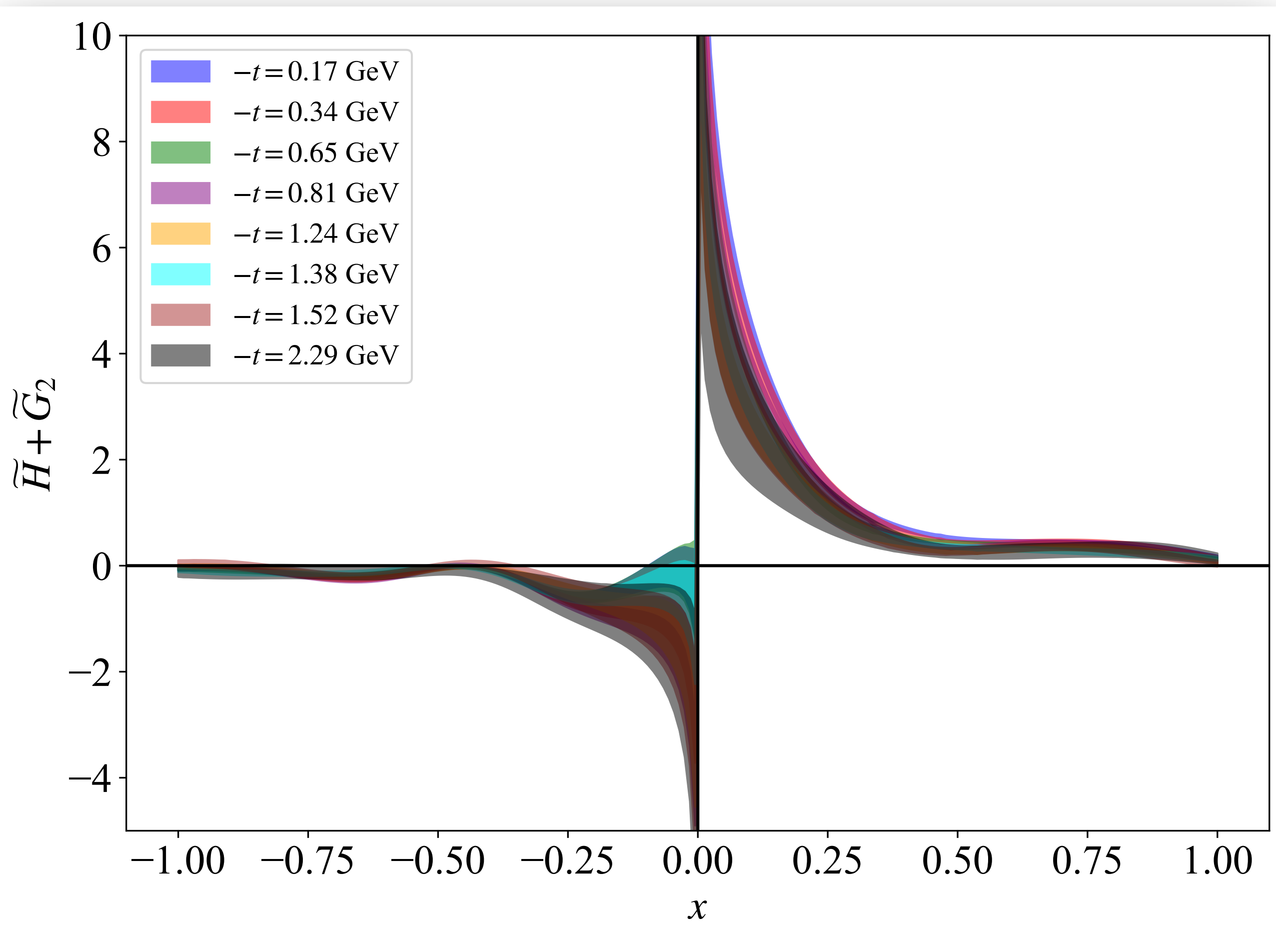
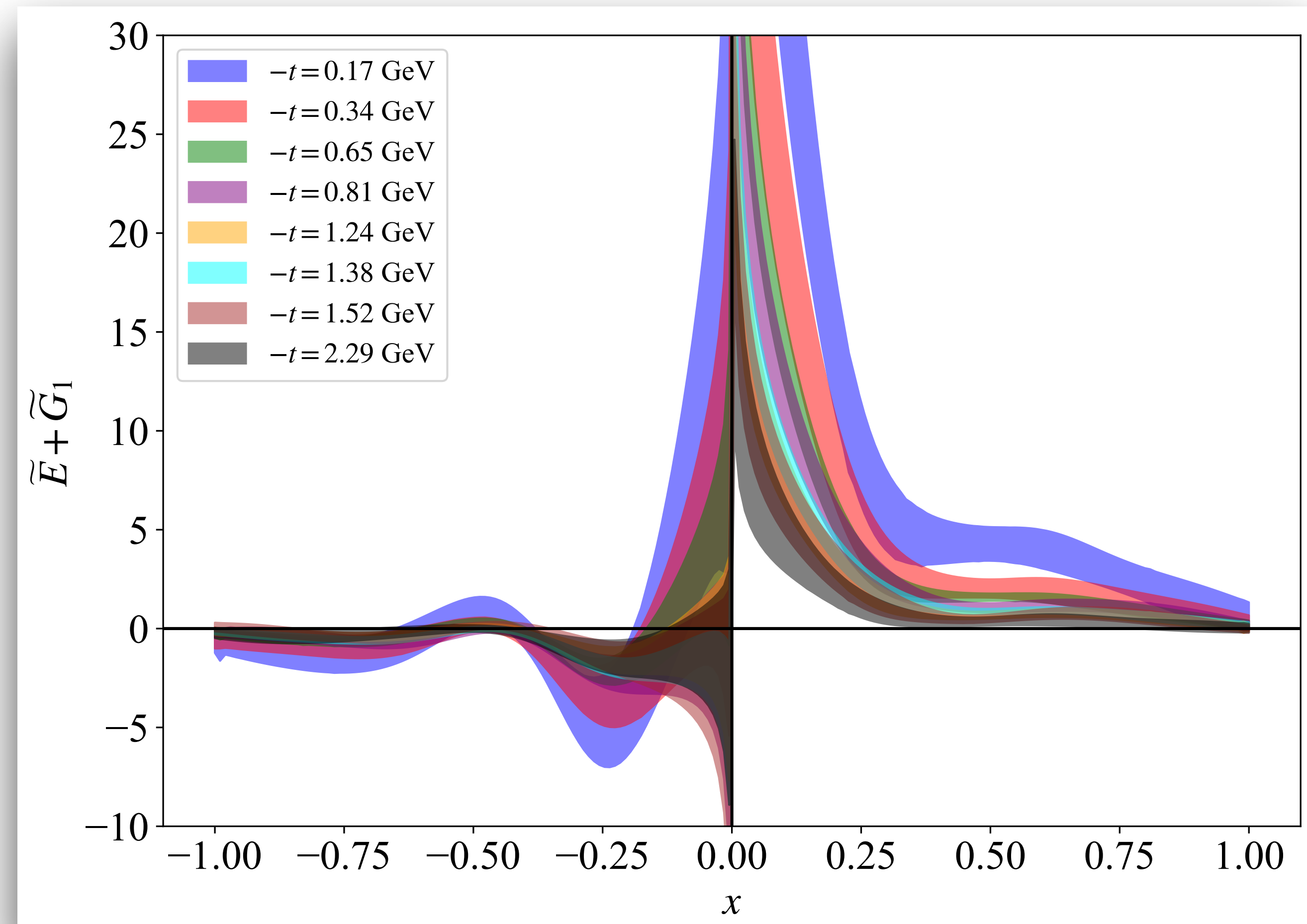
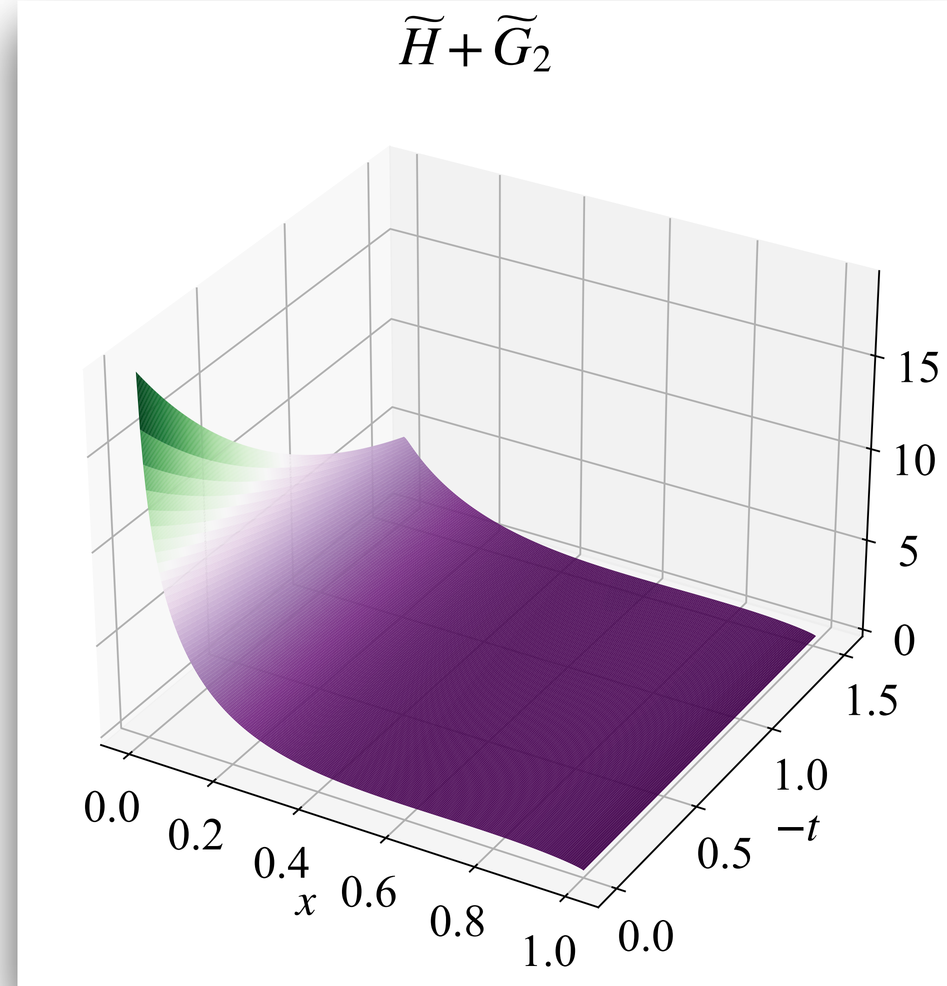


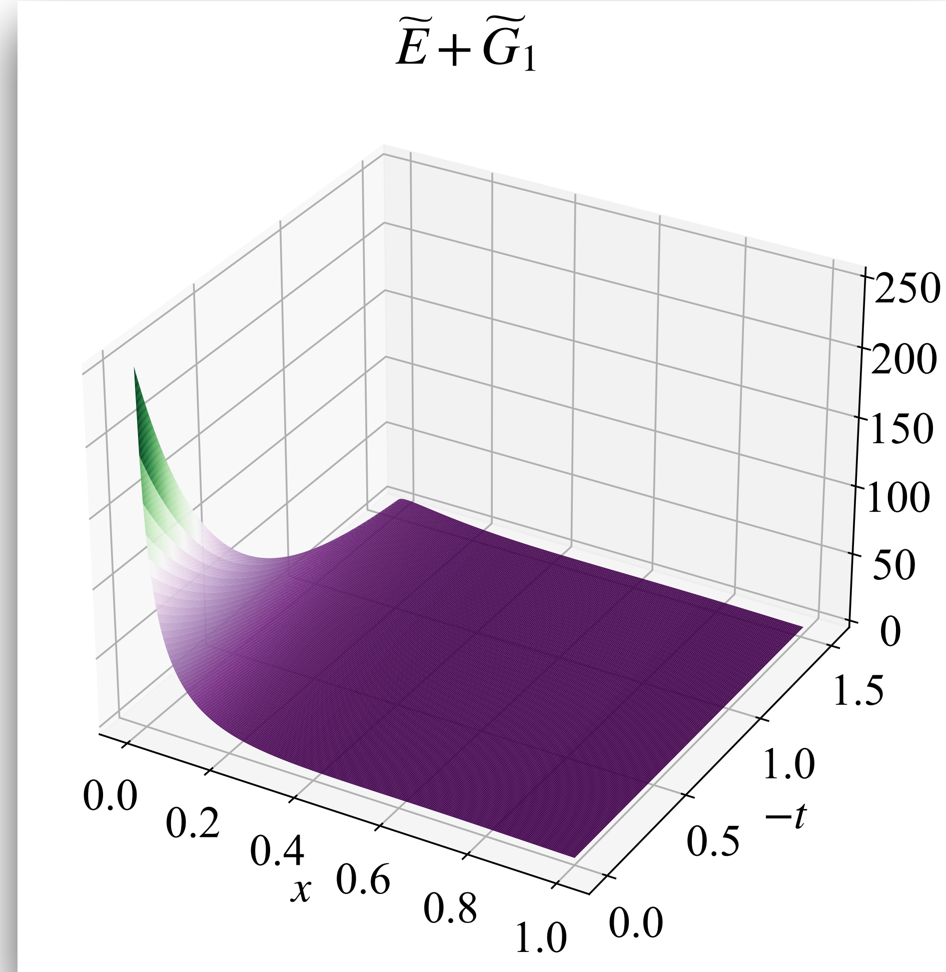


★ Parametrization of -t dependence (preliminary)

$$\text{GPD}(x,-t,0) = A x^{\alpha_0-\alpha_1 t}(1-x)^{\beta}$$

Ademollo & Del Giudice Gatto & Preparata





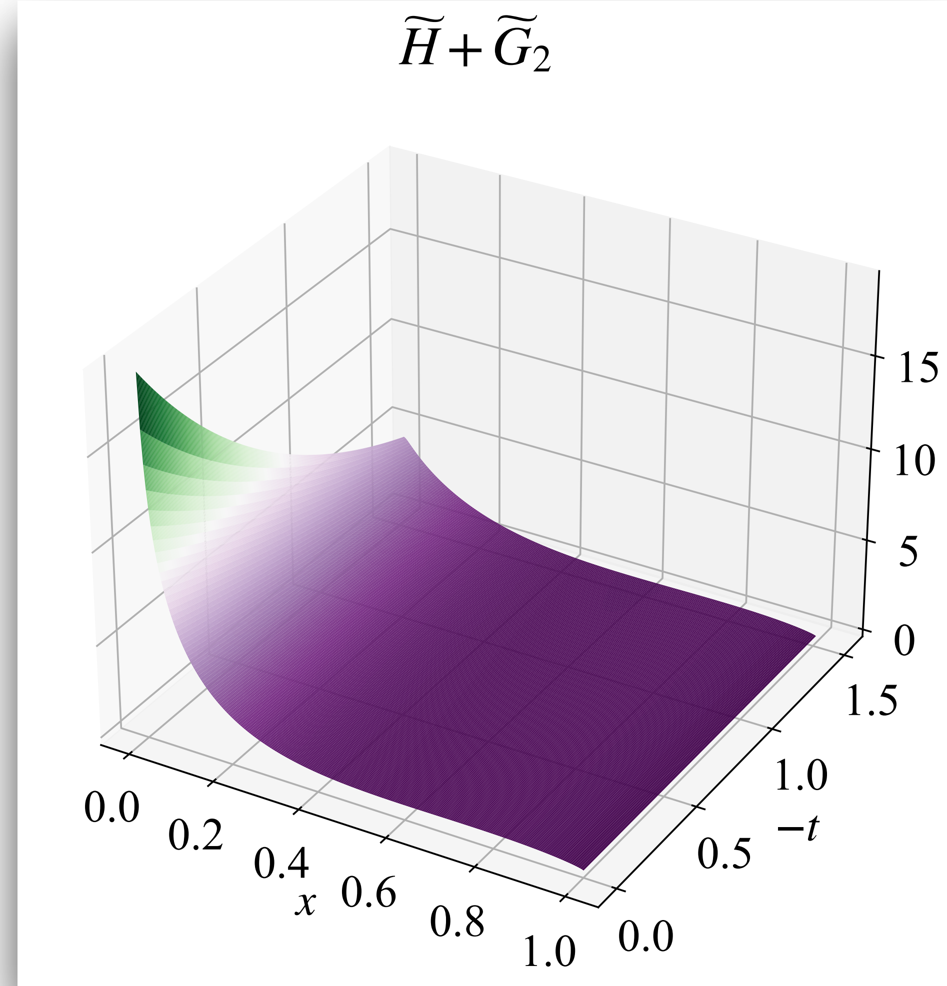
★ Direct access to $\widetilde{E}$-GPD not possible for zero skewness

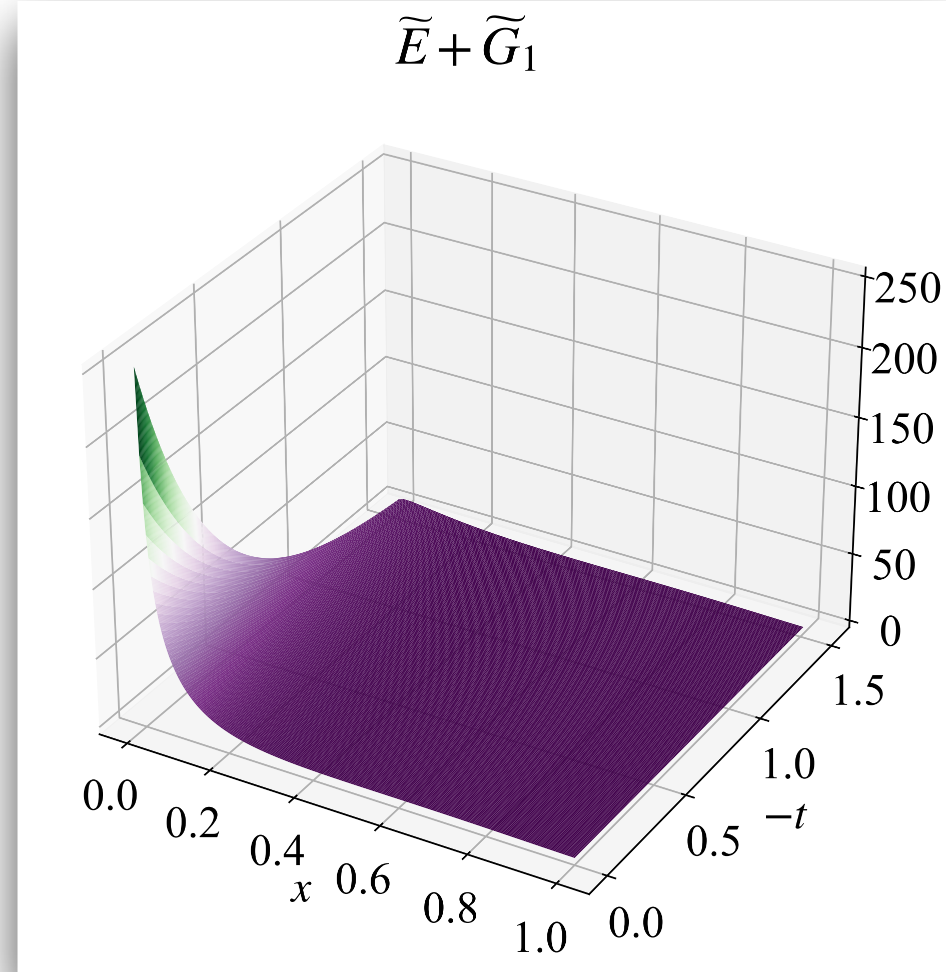
$$P^\mu \frac{\Delta^3 \gamma_5}{2mP^0} F_{\widetilde{E}}(x,\xi,t;P^3)$$

★ Glimpse into $\widetilde{E}$-GPD through twist-3 :

$$\int_{-1}^{1} dx\, \widetilde{E}(x,\xi,t) = G_P(t)$$

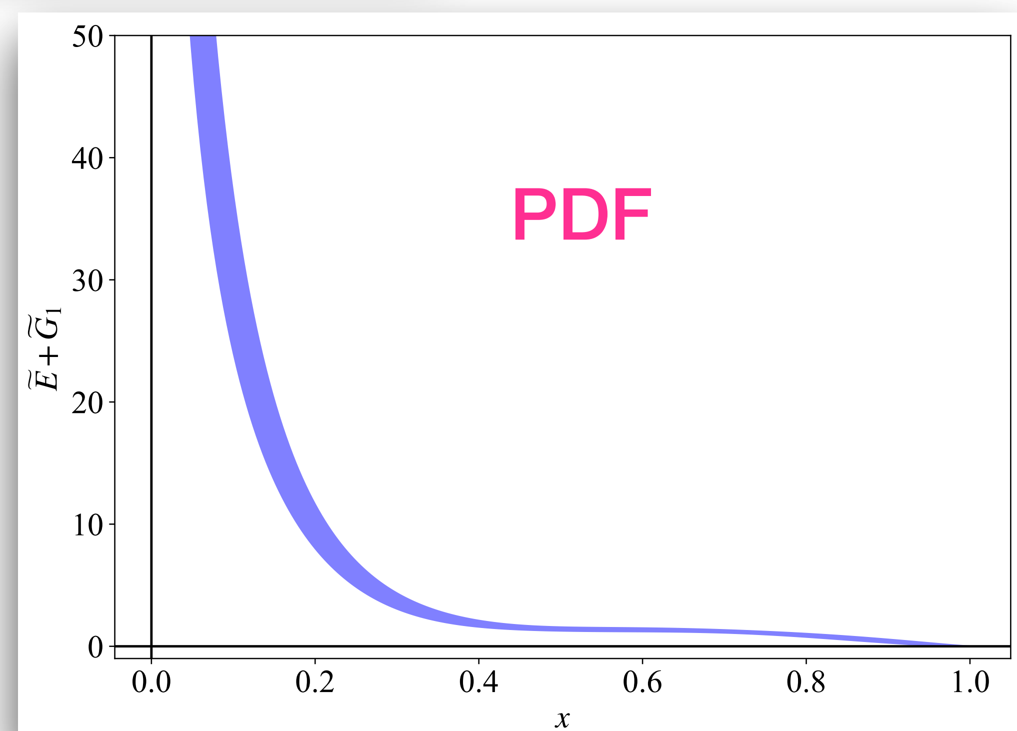
$$\int_{-1}^{1} dx\, \widetilde{G}_i(x,\xi,t) = 0\,, \quad i=1,2,3,4$$





★ Direct access to $\widetilde{E}$-GPD not possible for zero skewness

$$P^\mu \frac{\Delta^3 \gamma_5}{2mP^0} F_{\widetilde{E}}(x,\xi,t;P^3)$$

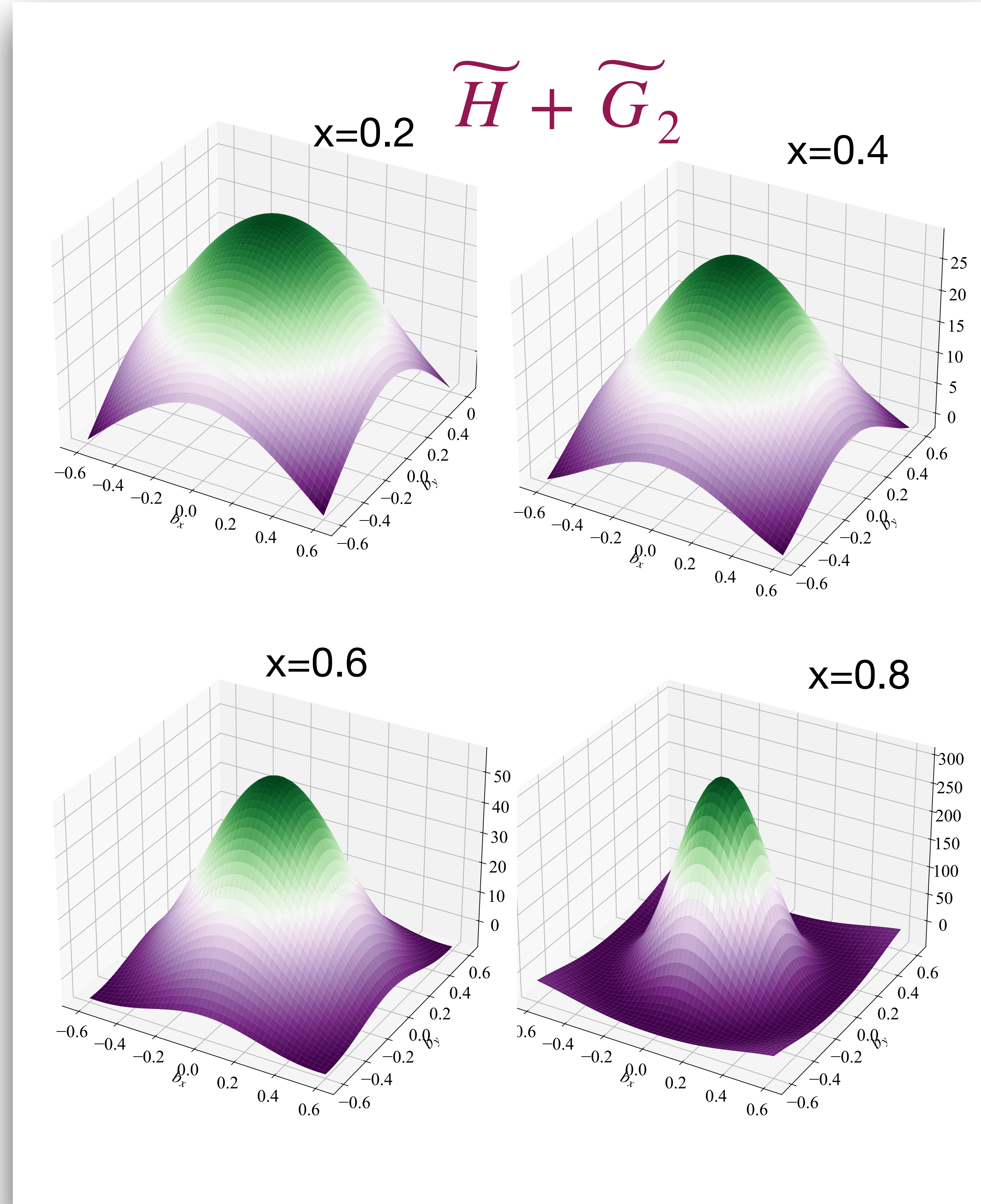


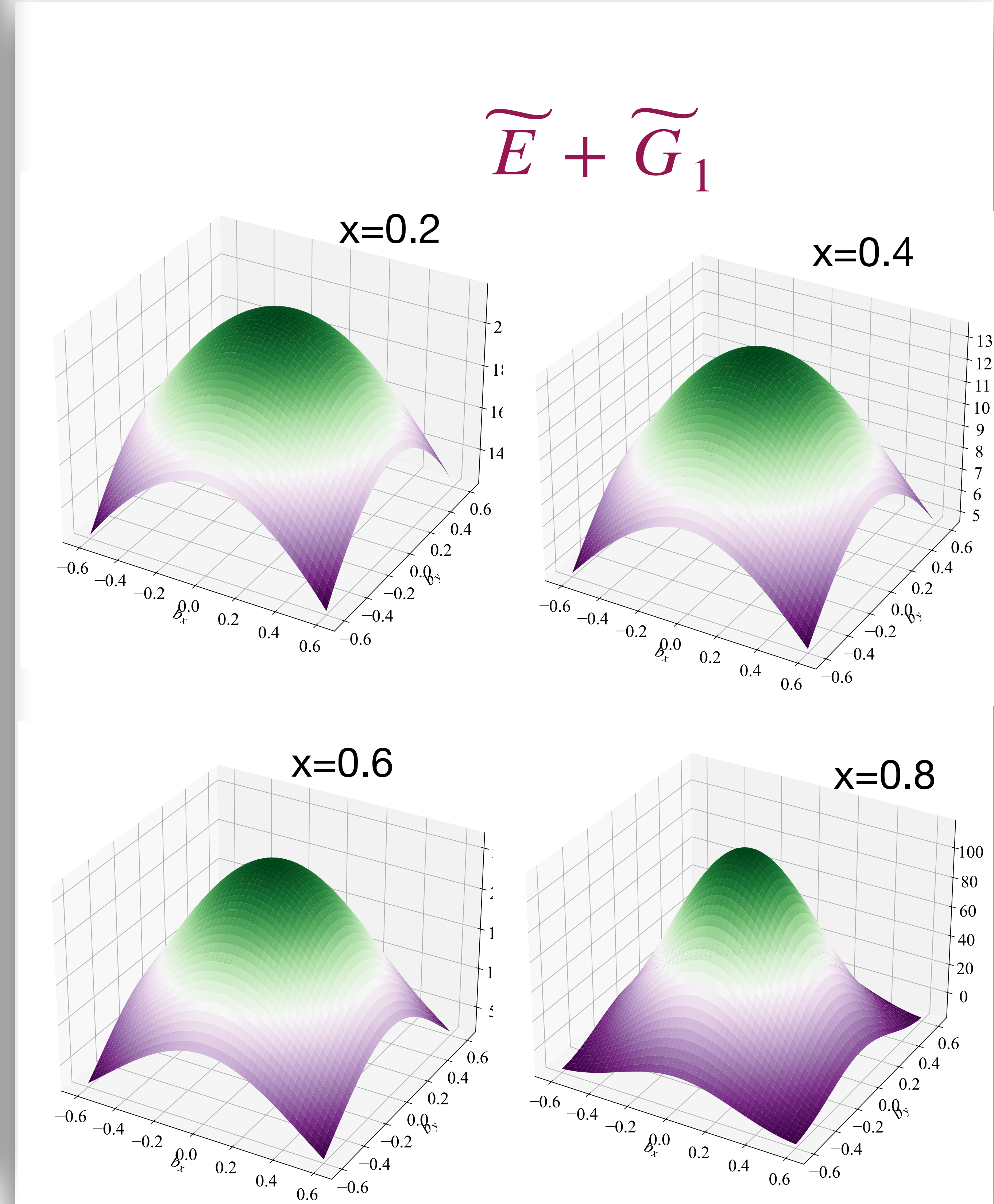
★ Glimpse into $\widetilde{E}$-GPD through twist-3 :

$$\int_{-1}^{1} dx\, \widetilde{E}(x,\xi,t) = G_P(t)$$

$$\int_{-1}^{1} dx\, \widetilde{G}_i(x,\xi,t) = 0\,, \quad i = 1,2,3,4$$

Impact parameter space

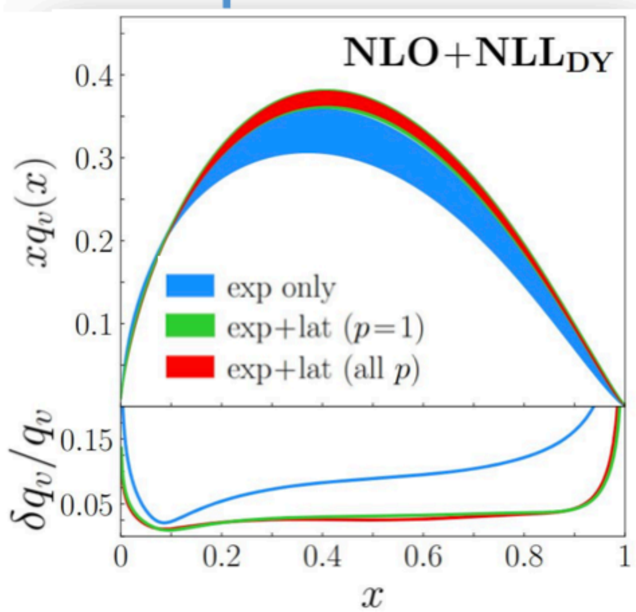




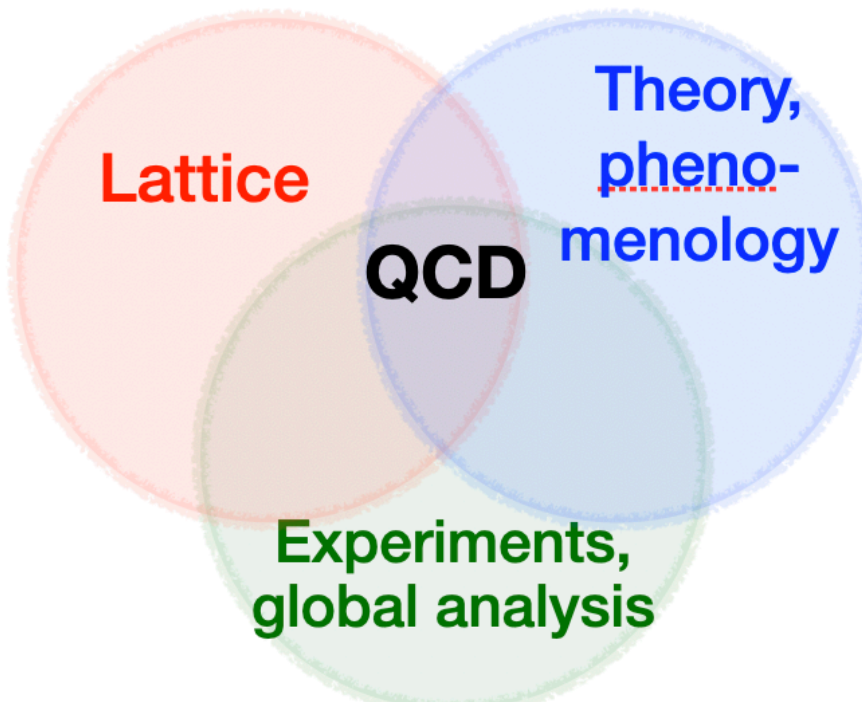
Synergy/Complementarity of lattice and phenomenology

Synergies: constraints & predictive power of lattice QCD

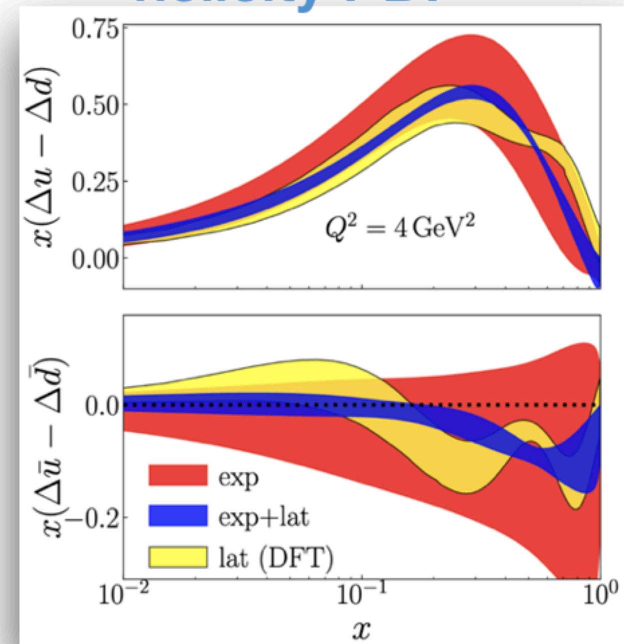
pion PDF



[JAM/HadStruc, PRD105 (2022) 114051]

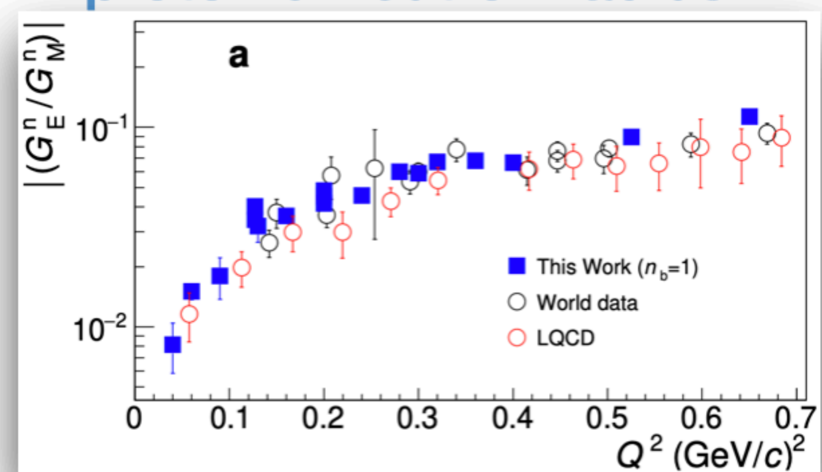


helicity PDF



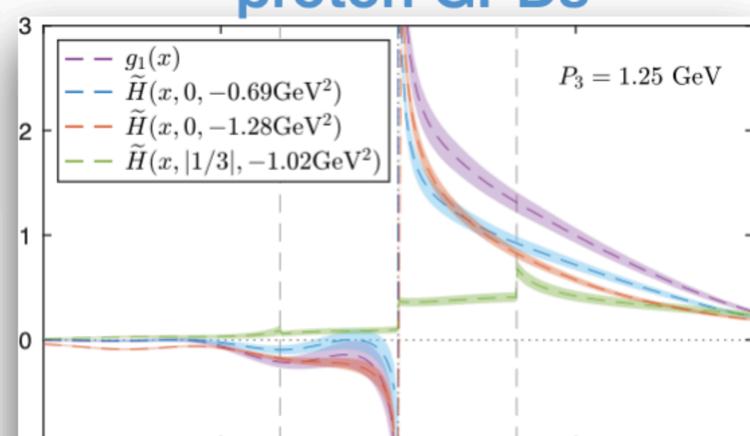
[JAM & ETMC, PRD 103 (2021) 016003]

proton & neutron radius



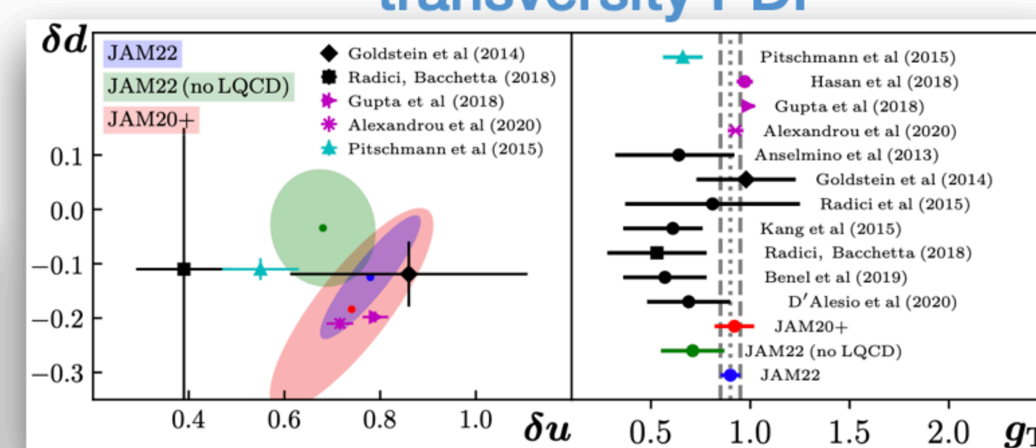
[Atac et al., Nature Comm. 12, 1759 (2021)]

proton GPDs



[ETMC, PRL 125 (2020) 262001]

transversity PDF



[JAM, PRD 106 (2022) 3, 034014]

And many more!

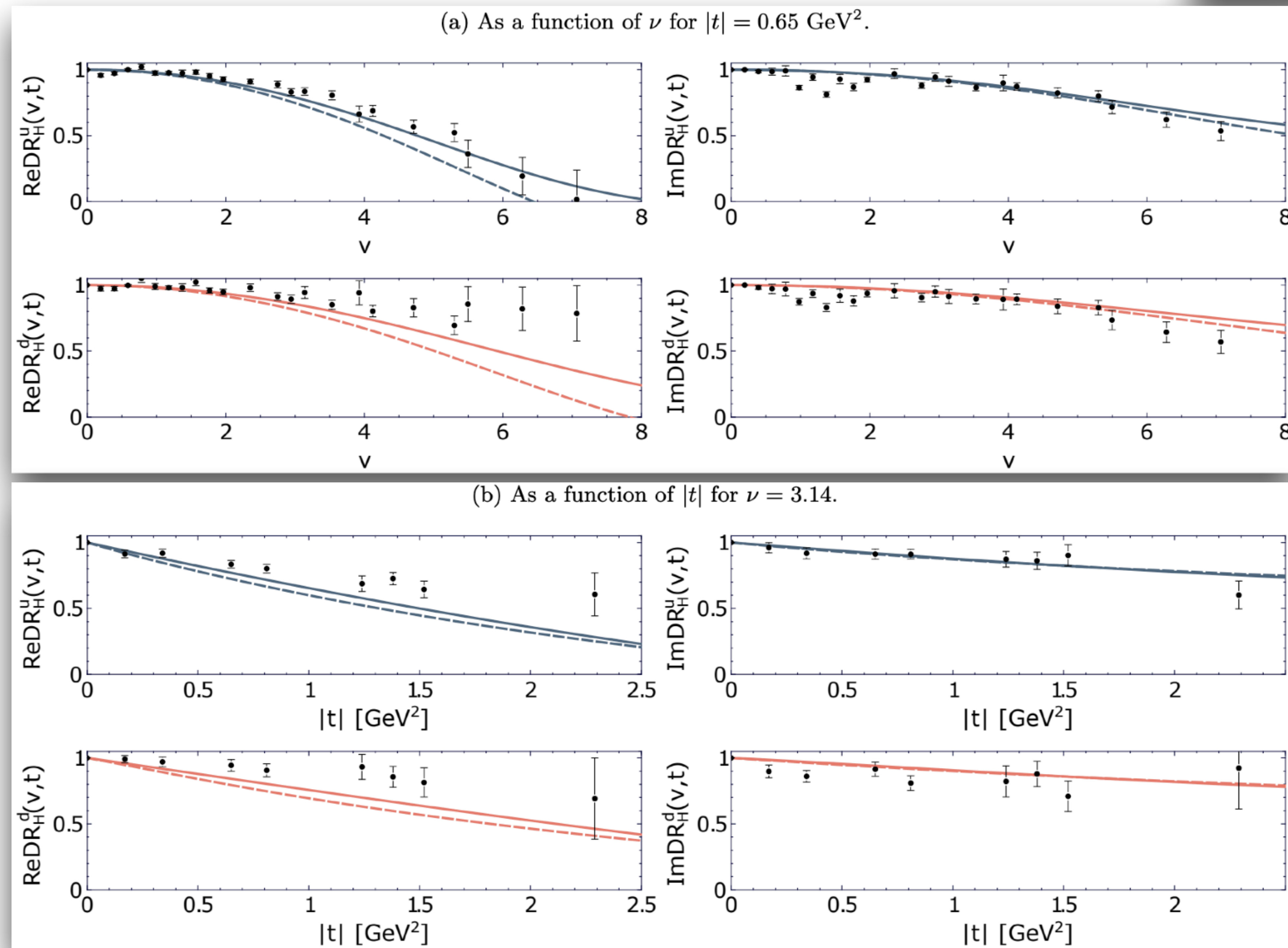
Toward synergy for GPDs

★ Forming ratios of GPDs seems to suppress systematic uncertainties

[K. Cichy et al., PRD 110 (2024) 11, 114025]

$$\mathrm{DR}^{\hat{H}^q}_{\mathrm{Re}}(\nu,t) = \frac{\mathrm{Re}\hat{H}^q(\nu,t)}{\mathrm{Re}\hat{H}^q(\nu,0)}\,\frac{\mathrm{Re}\hat{H}^q(0,0)}{\mathrm{Re}\hat{H}^q(0,t)}\,,$$

$$\mathrm{DR}^{\hat{H}^q}_{\mathrm{Im}}(\nu,t) = \lim_{\nu'\to 0}\frac{\mathrm{Im}\hat{H}^q(\nu,t)}{\mathrm{Im}\hat{H}^q(\nu,0)}\,\frac{\mathrm{Im}\hat{H}^q(\nu',0)}{\mathrm{Im}\hat{H}^q(\nu',t)}$$

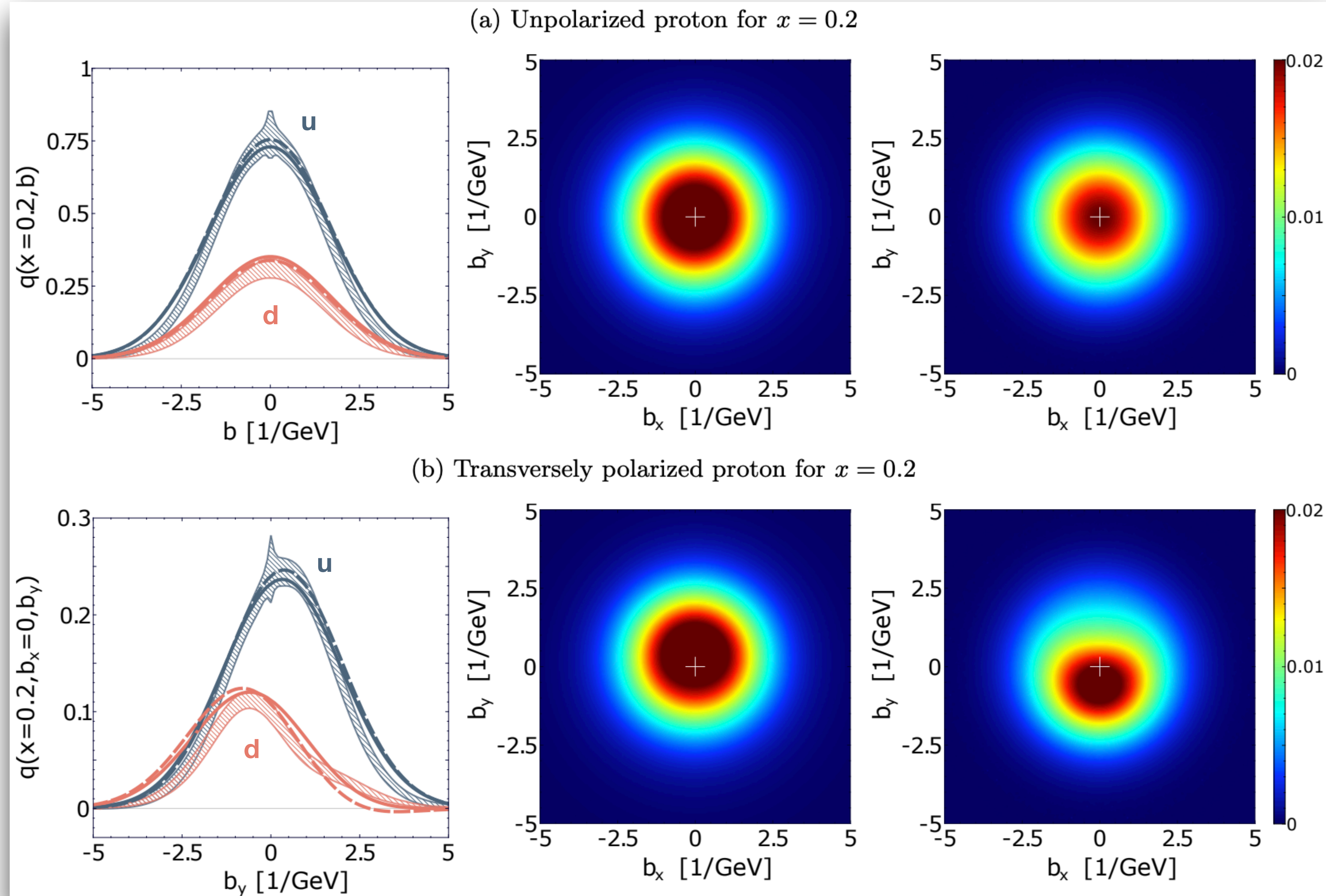


- GK (solid curve)
- VGG (dashed curve)

- Good agreement for up quark
- Reasonable agreement for down quark

- Further study needed on how to combine lattice results with data

Tomographic Images



- GK (solid line), VGG (dashed line)

[K. Cichy et al., PRD 110 (2024) 11, 114025]

How to lattice QCD data fit into the overall effort for hadron tomography

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★ Lattice data may be incorporated in global analysis of experimental data and may influence parametrization of t and ξ dependence

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PHENOMENOLOGY
LATTICE QCD
THEORY

QUARK-GLUON TOMOGRAPHY COLLABORATION

U.S. DEPARTMENT OF ENERGY | Office of Science

Award Number:
DE-SC0023646

1. **Theoretical studies** of high-momentum transfer processes using perturbative QCD methods and study of GPDs properties

2. **Lattice QCD** calculations of GPDs and related structures

3. **Global analysis** of GPDs based on experimental data using modern data analysis techniques for inference and uncertainty quantification

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Other GPD global analysis efforts:

- Gepard [https://gepard.phy.hr/]
- PARTONS [https://partons.cea.fr]
- EXCLAIM [https://exclaimcollab.github.io/web.github.io/#/]

See talk by S. Liuti

Concluding Remarks

★ Impressive progress in the extraction of PDFs from Lattice QCD

★ Extensive programs in Gluon PDFs

★ New Developments in several promising directions:
DA, GPDs, TMDs

★ Synergy with phenomenology has the potential to enhance the impact of lattice QCD data and complement data sets

Join us at EINN 2025

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on Electromagnetic Interactions with Nucleons and Nuclei

Main conference:
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Organizers:
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A. Denig (Vice-Chair)
C. Alexandrou
A. Deshpande
B. Pasquini

https://2025.einnconference.org/

Pre-conference: 26 - 27 October, 2025
Frontiers and Careers Workshops – skill development and talks for students

Thank you

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DOE Early Career Award (NP)
Grant No. DE-SC0020405
& Grant No. DE-SC0025218

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LATTICE QCD PHENOMENOLOGY THEORY

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ASCR Leadership Computing Challenge ALCC