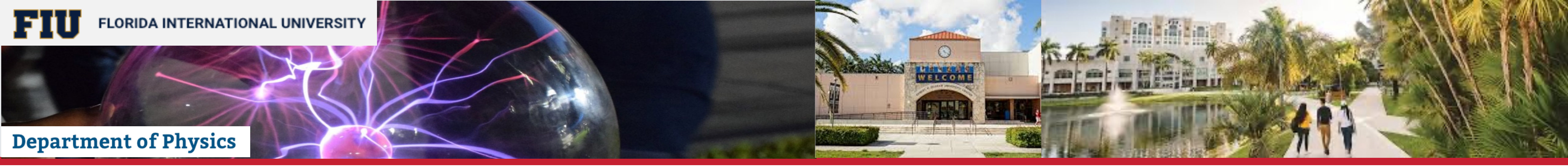


Single Diffractive Hard Exclusive Scattering (SDHEP) for Extracting GPDs

- Explore Hadron's Partonic Structure without Breaking it – GPDs!
- SDHEPs for Extracting GPDs
- QCD Factorization, Angular Modulations, …
- Why GPD's x-dependence is hard to measure?
- Summary and Outlook

In collaboration with N. Sato, Z. Yu, …
See also talk by Z. Yu later today

Jianwei Qiu
Jefferson Lab, Theory Center

Jefferson Lab

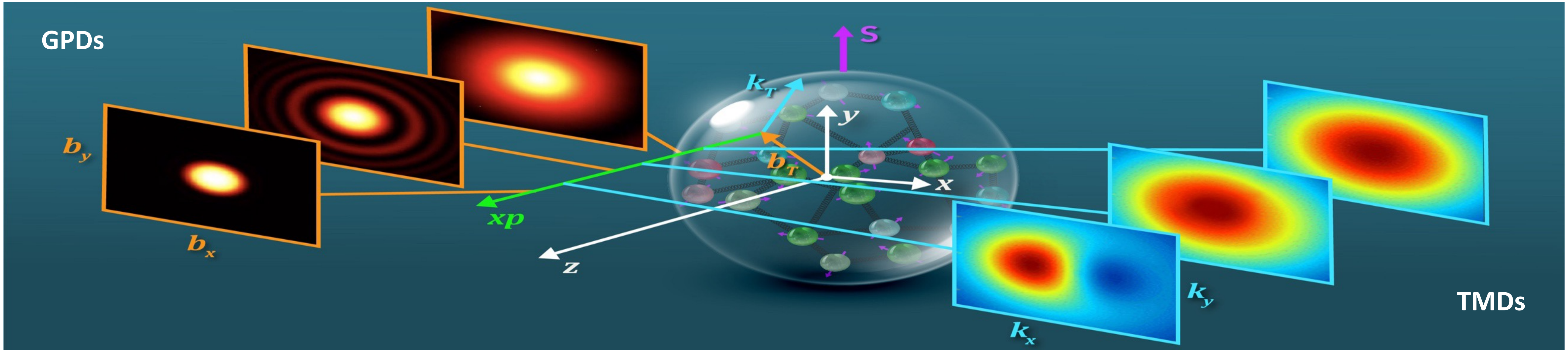
U.S. DEPARTMENT OF ENERGY | Office of Science

JSA

Explore Hadron's Partonic Structure without seeing quarks/gluons directly

- ❑ 3D hadron structure:

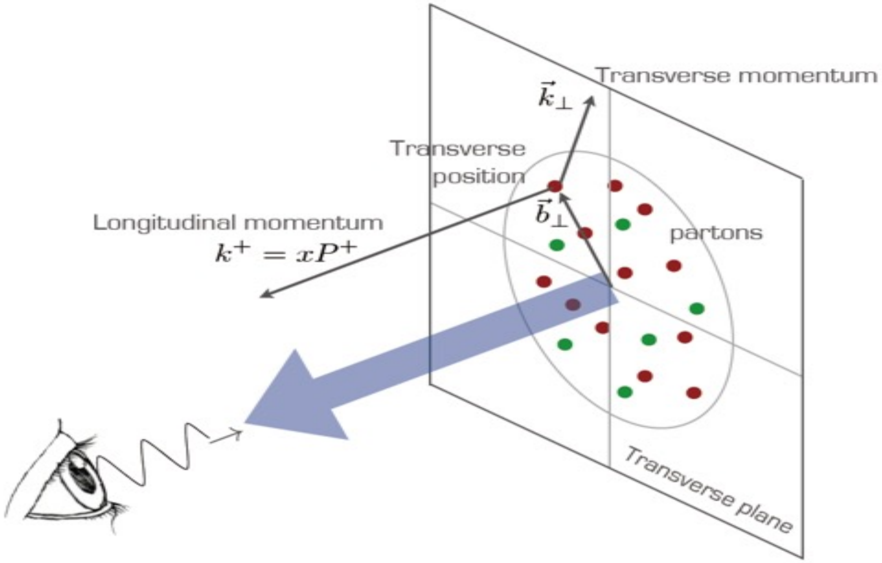
NO quarks and gluons can be seen in isolation!



- ❑ Need new observables with two distinctive scales:

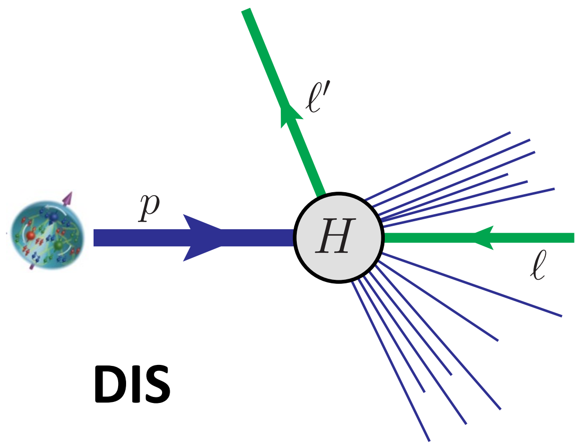
$$Q_1 \gg Q_2 \sim 1/R \sim \Lambda_{\rm QCD}$$

- Hard scale: Q_1 to localize the probe to see the particle nature of quarks/gluons
- "Soft" scale: Q_2 to be more sensitive to the emergent regime of hadron structure ~ 1/fm

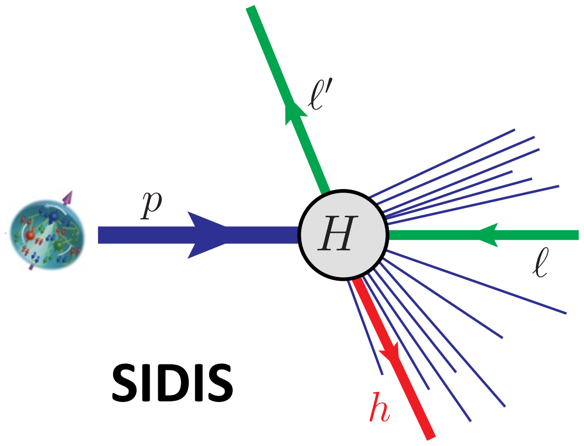


Partonic Structure with or without breaking the hadron

Inclusive scattering

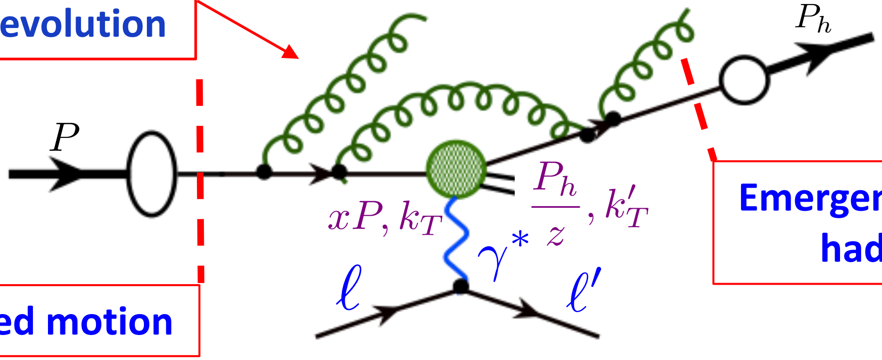


DIS



SIDIS

Gluon shower
– QCD evolution



Confined motion

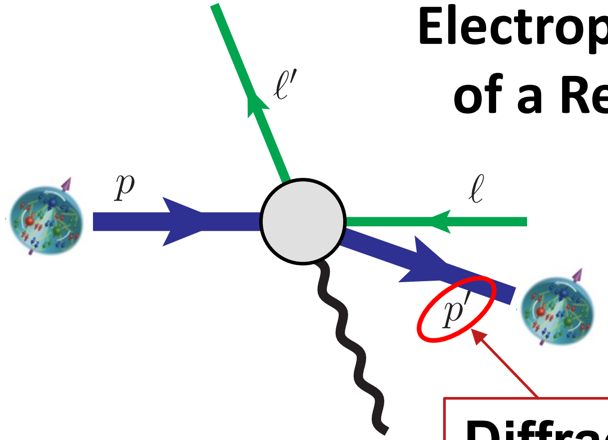
Emergence of a hadron
hadronization

$$e + P \rightarrow e + h + X$$

Measured k_T of TMDs =\= the *confined* motion inside the hadron!

Exclusive diffraction

Electroproduction of a Real Photon

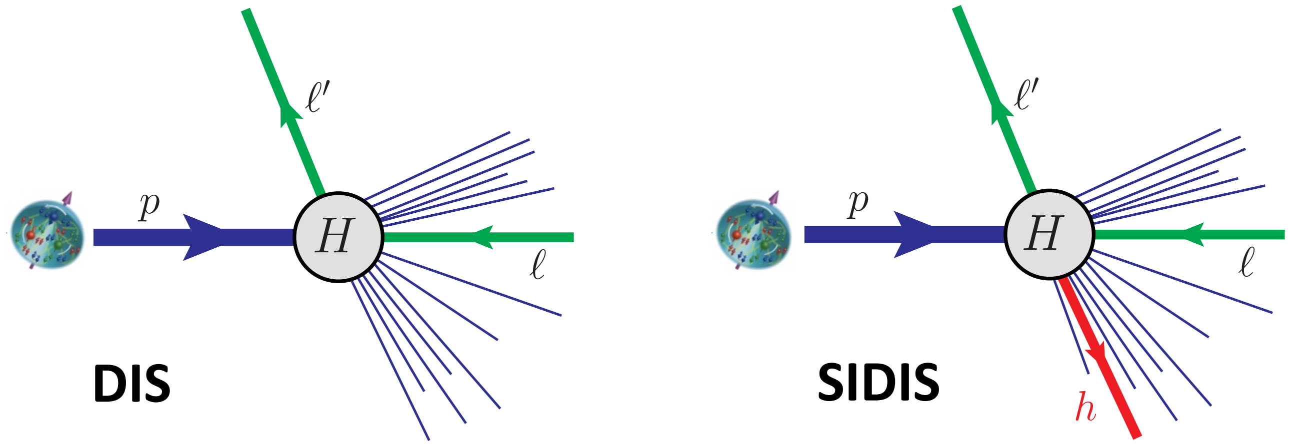


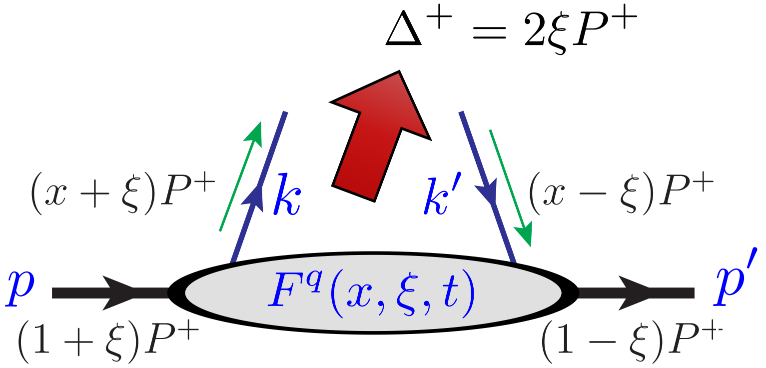
Diffraction

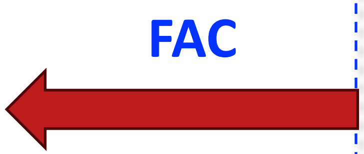
$$Q^2 = -(\ell - \ell')^2 \gg -(p - p')^2 = -t$$

Partonic Structure with or without breaking the hadron

Inclusive scattering



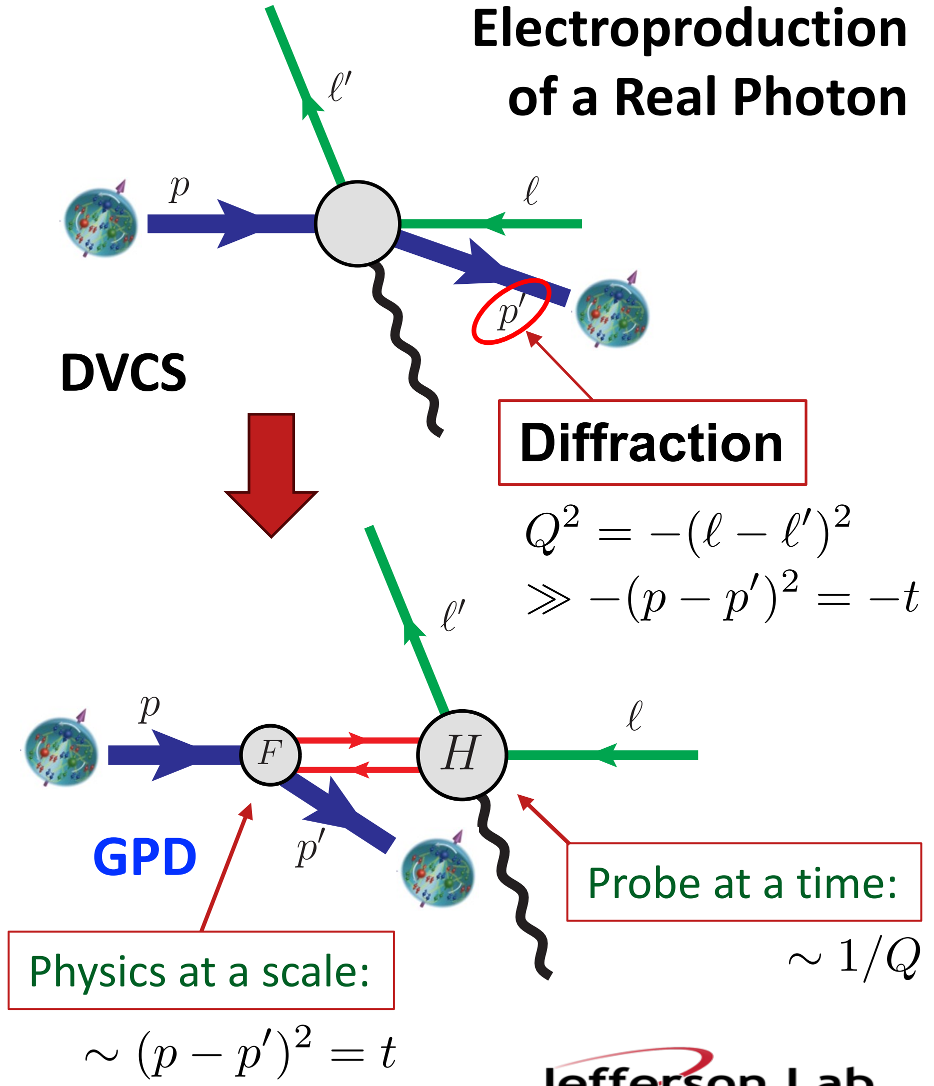




$$F^q(x,\xi,t) = \int \frac{dz^-}{4\pi} e^{-ixP^+z^-} \langle p'|\bar{q}(z^-/2)\gamma^+ q(-z^-/2)|p\rangle$$

$$\widetilde{F}^q(x,\xi,t) = \int \frac{dz^-}{4\pi} e^{-ixP^+z^-} \langle p'|\bar{q}(z^-/2)\gamma^+\gamma_5 q(-z^-/2)|p\rangle$$

Exclusive diffraction



Generalized Parton Distributions (GPDs)

See X. Ji's talk

- **Definition:**

D. Müller, D. Robaschik, B. Geyer, F.-M. Dittes, J. Hořejši, *Fortsch. Phys.* 42 (1994) 101

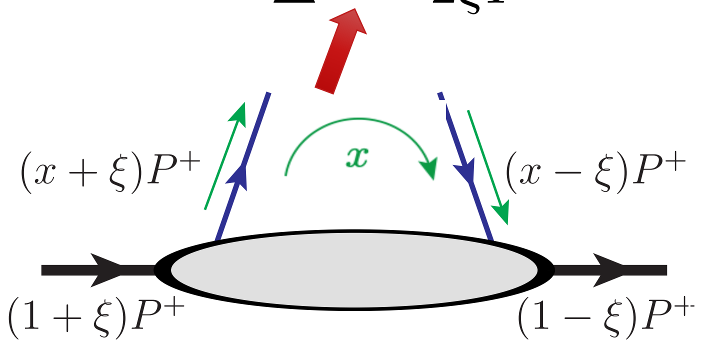
$$F^q(x,\xi,t) = \int \frac{dz^-}{4\pi} e^{-ixP^+z^-} \langle p'|\bar{q}(z^-/2)\gamma^+ q(-z^-/2)|p\rangle$$

$$= \frac{1}{2P^+}\left[H^q(x,\xi,t)\,\bar{u}(p')\,\gamma^+ u(p) - E^q(x,\xi,t)\,\bar{u}(p')\,\frac{i\sigma^{+\alpha}\Delta_\alpha}{2m}u(p)\right],$$

$$\widetilde{F}^q(x,\xi,t) = \int \frac{dz^-}{4\pi} e^{-ixP^+z^-} \langle p'|\bar{q}(z^-/2)\gamma^+\gamma_5 q(-z^-/2)|p\rangle$$

$$= \frac{1}{2P^+}\left[\widetilde{H}^q(x,\xi,t)\,\bar{u}(p')\,\gamma^+\gamma_5 u(p) - \widetilde{E}^q(x,\xi,t)\,\bar{u}(p')\,\frac{\gamma_5\Delta^+}{2m}u(p)\right].$$

$$\Delta^+ = 2\xi P^+$$



$$P^+ = \frac{p^+ + p'^+}{2}$$

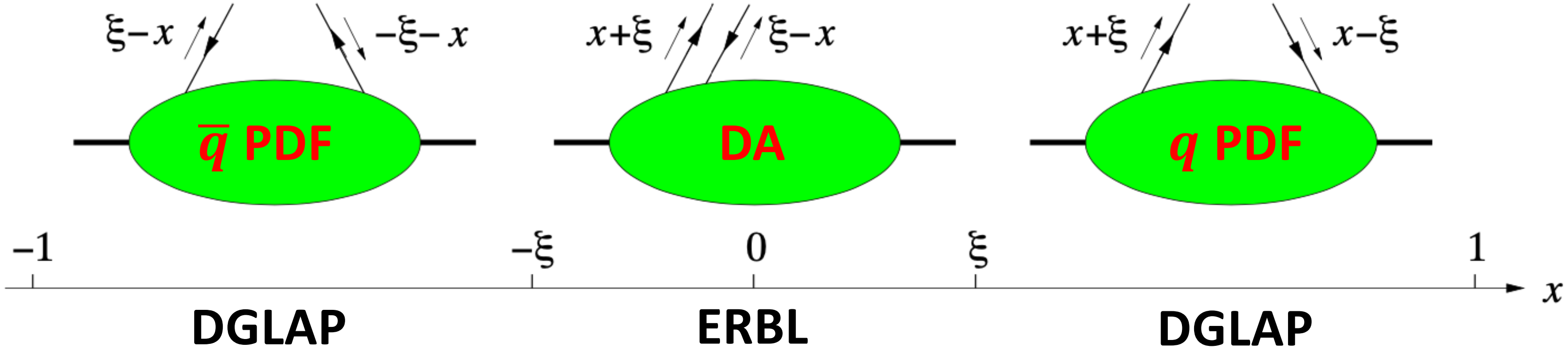
$$\Delta = p - p' \qquad t = \Delta^2$$

- **Combine *PDF* and *Distribution Amplitude (DA):***

Forward limit $\xi = t = 0$ **:** $\quad H^q(x,0,0) = q(x), \quad \tilde{H}^q(x,0,0) = \Delta q(x)$

Similar definition for gluon GPDs

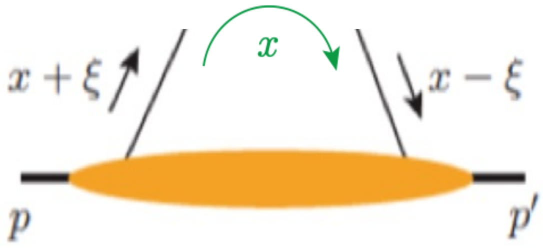
GPDs depend on the choice of "+" component for given p and p' !



Properties of GPDs – Partonic

❑ Impact parameter dependent parton density distribution:

$$q(x, b_\perp, Q) = \int d^2\Delta_\perp e^{-i\Delta_\perp \cdot b_\perp} H_q(x, \xi = 0, t = -\Delta_\perp^2, Q)$$



Measurement of p' fixes (t, ξ)

x = momentum flow between the pair

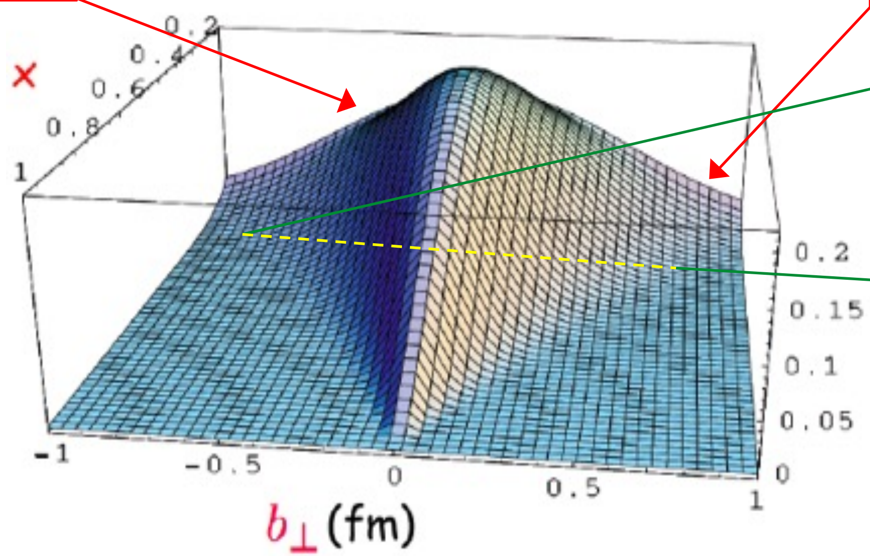
➡ Quark density in $\mathrm{d}x\,\mathrm{d}^2\boldsymbol{b}_T$

How fast does glue density fall?

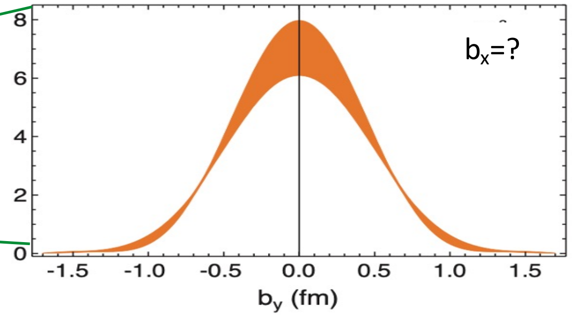
Tomographic image of hadron in slice of x

How far does glue density spread?

➡



Modeled by M. Burkdart, PRD 2000



Slice in (x,Q)

$$\langle q_\perp^N \rangle \equiv \int db_\perp b_\perp^N \, q(x, b_\perp, Q)$$

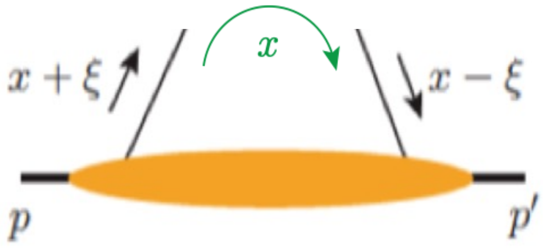
➡ **Proton radii from quark and gluon spatial density distribution, $r_q(x)$ & $r_g(x)$**

Properties of GPDs – Partonic

❑ **Impact parameter dependent parton density distribution:**

$$q(x, b_\perp, Q) = \int d^2\Delta_\perp e^{-i\Delta_\perp \cdot b_\perp} H_q(x, \xi = 0, t = -\Delta_\perp^2, Q)$$

➡ **Quark density in** $\mathrm{d}x\,\mathrm{d}^2\boldsymbol{b}_T$



Measurement of p' fixes (t, ξ)

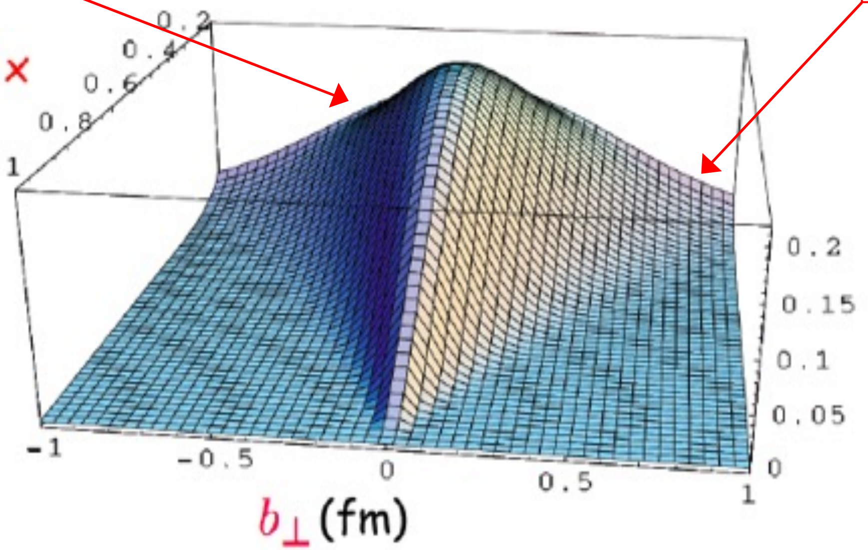
x = momentum flow between the pair

How fast does glue density fall?

Tomographic image of hadron in slice of x

How far does glue density spread?

➡



Modeled by M. Burkdart, PRD 2000

➡ **Proton radii from quark and gluon spatial density distribution,** $r_q(x)$ **&** $r_g(x)$

- **Should** $r_q(x)$ **>** $r_g(x)$**, or vice versa?**
- **Could** $r_g(x)$ **saturates as** $x \to 0$
- **How do they compare with known radius (EM charge radius, mass radius, …), & why?**
- **How the image correlate to hadron spin, … ?**
- …

Properties of GPDs – Hadronic = Various Moments of GPDs

See X. Ji's talk

Ji, PRL78, 1997
V. D. Burkert, et al. RMP 95 (2023) 041002

❑ QCD energy-momentum tensor:

$$T^{\mu\nu} = \sum_{i=q,g} T_i^{\mu\nu} \quad \textbf{with} \quad T_q^{\mu\nu} = \bar{\psi}_q \, i\gamma^{(\mu} \overleftrightarrow{D}^{\nu)} \, \psi_q - g^{\mu\nu} \bar{\psi}_q \left(i\gamma \cdot \overleftrightarrow{D} - m_q \right) \psi_q \quad \textbf{and} \quad T_g^{\mu\nu} = F^{a,\mu\eta} F^{a,}{}_{\eta}{}^{\nu} + \frac{1}{4} g^{\mu\nu} \left(F_{\rho\eta}^a \right)^2$$

❑ "Gravitational" form factors:

$$\langle p' \, | T_i^{\mu\nu} | \, p \rangle = \bar{u}(p') \left[A_i(t) \frac{P^\mu P^\nu}{m} + J_i(t) \frac{i P^{(\mu} \sigma^{\nu)\Delta}}{2m} + D_i(t) \frac{\Delta^\mu \Delta^\nu - g^{\mu\nu} \Delta^2}{4m} + m \, \bar{c}_i(t) \, g^{\mu\nu} \right] u(p)$$

❑ Connection to GPD moments:

$$\int_{-1}^{1} dx \, x \, F_i(x,\xi,t) \propto \langle p' | T_i^{++} | p \rangle \quad \propto \quad \bar{u}(p') \left[\underbrace{(A_i + \xi^2 D_i)}_{\int_{-1}^{1} dx \, x \, H_i(x,\xi,t)} \gamma^+ + \underbrace{(B_i - \xi^2 D_i)}_{\int_{-1}^{1} dx \, x \, E_i(x,\xi,t)} \frac{i\sigma^{+\Delta}}{2m} \right] u(p)$$

$C_i(t) \leftrightarrow D_i(t)/4$

Related to pressure & stress force inside h

Polyakov, schweitzer, Inntt. J. Mod. Phys. A33, 1830025 (2018)
Burkert, Elouadrhiri , Girod Nature 557, 396 (2018)

❑ Angular momentum sum rule:

$$J_i = \lim_{t \to 0} \int_{-1}^{1} dx \, x \left[H_i(x,\xi,t) + E_i(x,\xi,t) \right] \qquad i = q, g$$

➡ **3D tomography**
Relation to GFFs
Angular Momentum ➡ ***x*-dependence of GPDs!**

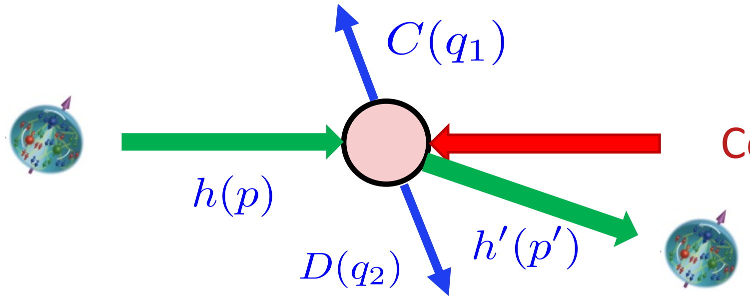
Need to know the x-dependence of GPDs to construct the proper moments!

How to Find Physical Processes to be Sensitive to GPDs?

Qiu & Yu, JHEP 08 (2022) 103
PRD 107 (2023) 014007
PRL 131 (2023) 161902

- **Two-scale exclusive processes – minimal $2 \to 3$ configuration:**

Process: $h(p) + B(p_2) \to h'(p') + C(q_1) + D(q_2)$



Colliding beam: $B(p_2) = e, \gamma, \pi$

Two scales: $Q^2 = -(p_2 - q_1)^2 \gg -(p - p')^2 = -t$

or $q_T \sim q_{1T} \sim q_{2T} \gg \sqrt{-t}$

→ **Single diffractive hard exclusive process**

(SDHEP)

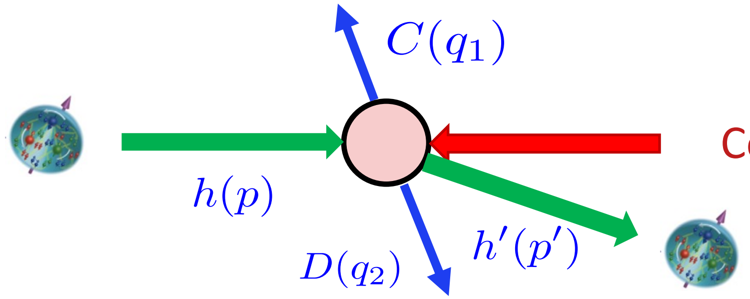
Jefferson Lab

How to Find Physical Processes to be Sensitive to GPDs?

Qiu & Yu, JHEP 08 (2022) 103
PRD 107 (2023) 014007
PRL 131 (2023) 161902

❑ Two-scale exclusive processes – minimal $2 \to 3$ configuration:

Process: $h(p) + B(p_2) \to h'(p') + C(q_1) + D(q_2)$



Colliding beam: $B(p_2) = e, \gamma, \pi$

Two scales: $Q^2 = -(p_2 - q_1)^2 \gg -(p - p')^2 = -t$

or $q_T \sim q_{1T} \sim q_{2T} \gg \sqrt{-t}$

→ **Single diffractive hard exclusive process (SDHEP)**

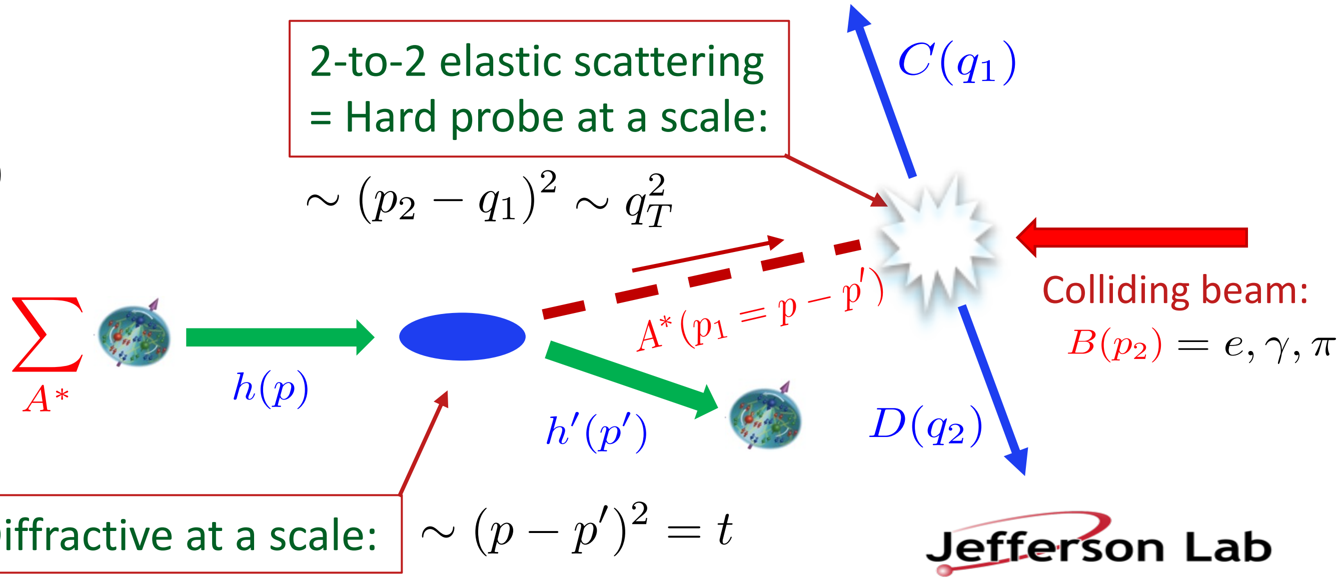
❑ SDHEP – Two-stage paradigm:

Single diffractive: $h(p) \to h'(p') + A^*(p_1 = p - p')$

Necessary **condition:**
$q_T \gg \sqrt{-t} \simeq \Lambda_{\mathrm{QCD}}$

Factorization

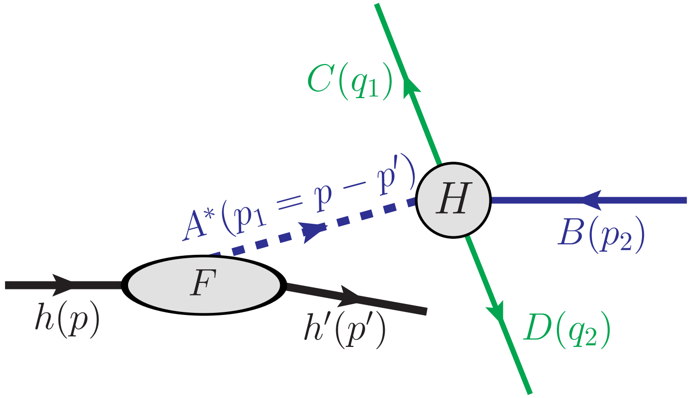
Hard exclusive: $A^*(p_1) + B(p_2) \to C(p_3) + D(p_4)$



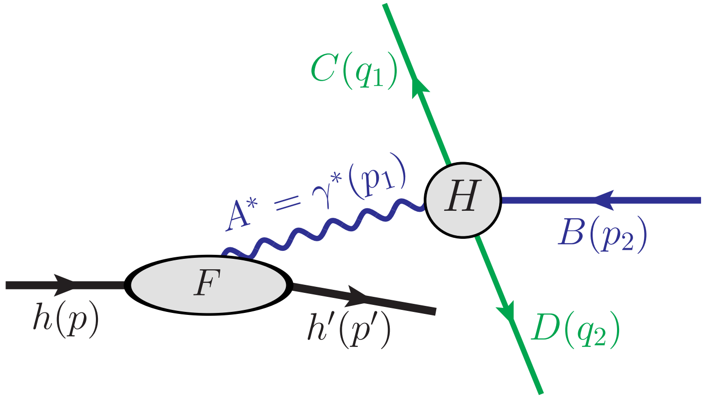
Jefferson Lab

SDHEP: Two-stage Paradigm plus Power Expansion in $\sqrt{-t}/q_T$

$\sum_{A^*}$

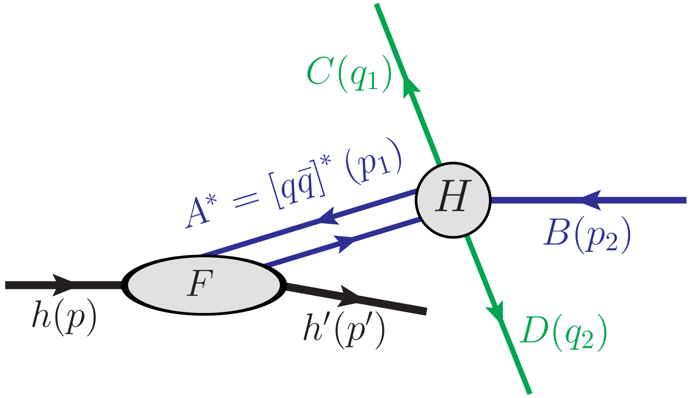


$=$

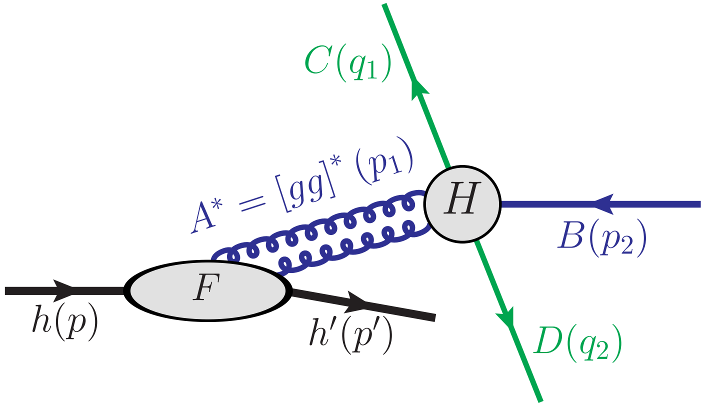


Leading Power in $\frac{\sqrt{-t}}{Q}$

$+$



$+$

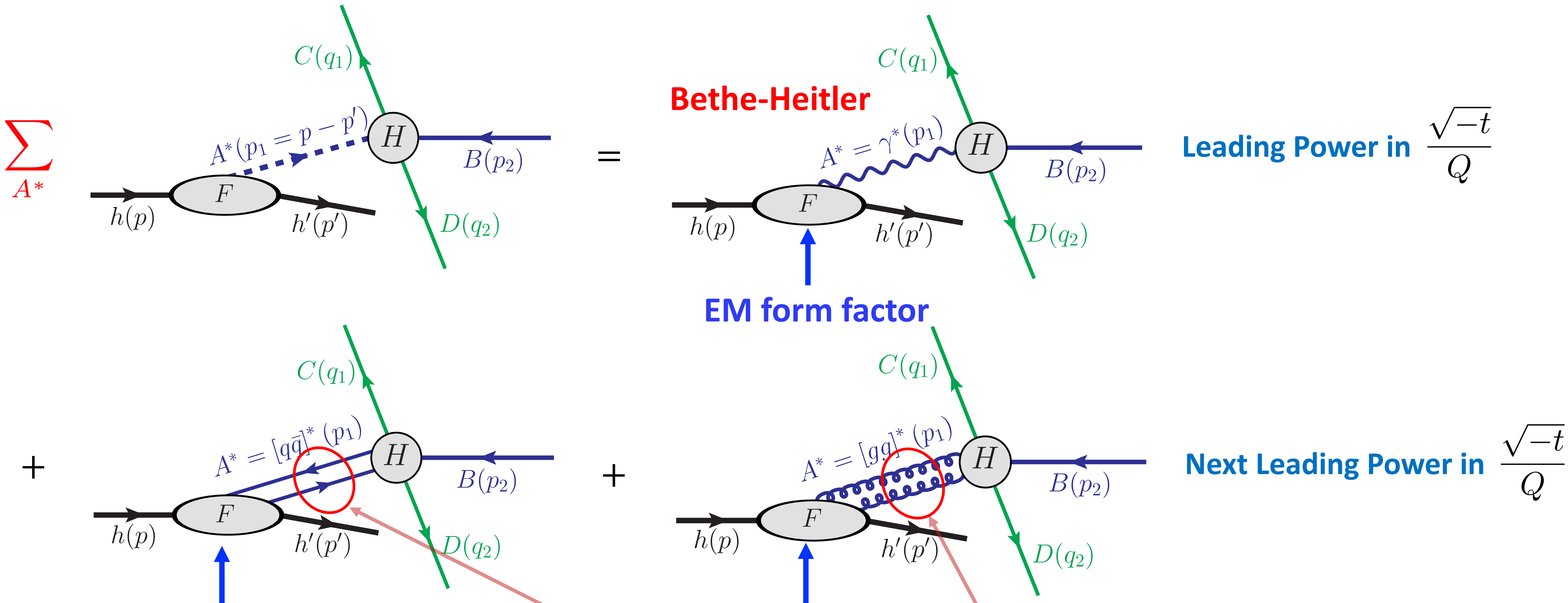


Next Leading Power in $\frac{\sqrt{-t}}{Q}$

$+\cdots$

$\geq$ 3 parton connection:
further Power suppressed

SDHEP: Two-stage Paradigm plus Power Expansion in $\sqrt{-t}/q_T$



$+ \cdots$

$\geq$ 3 parton connection:
further Power suppressed

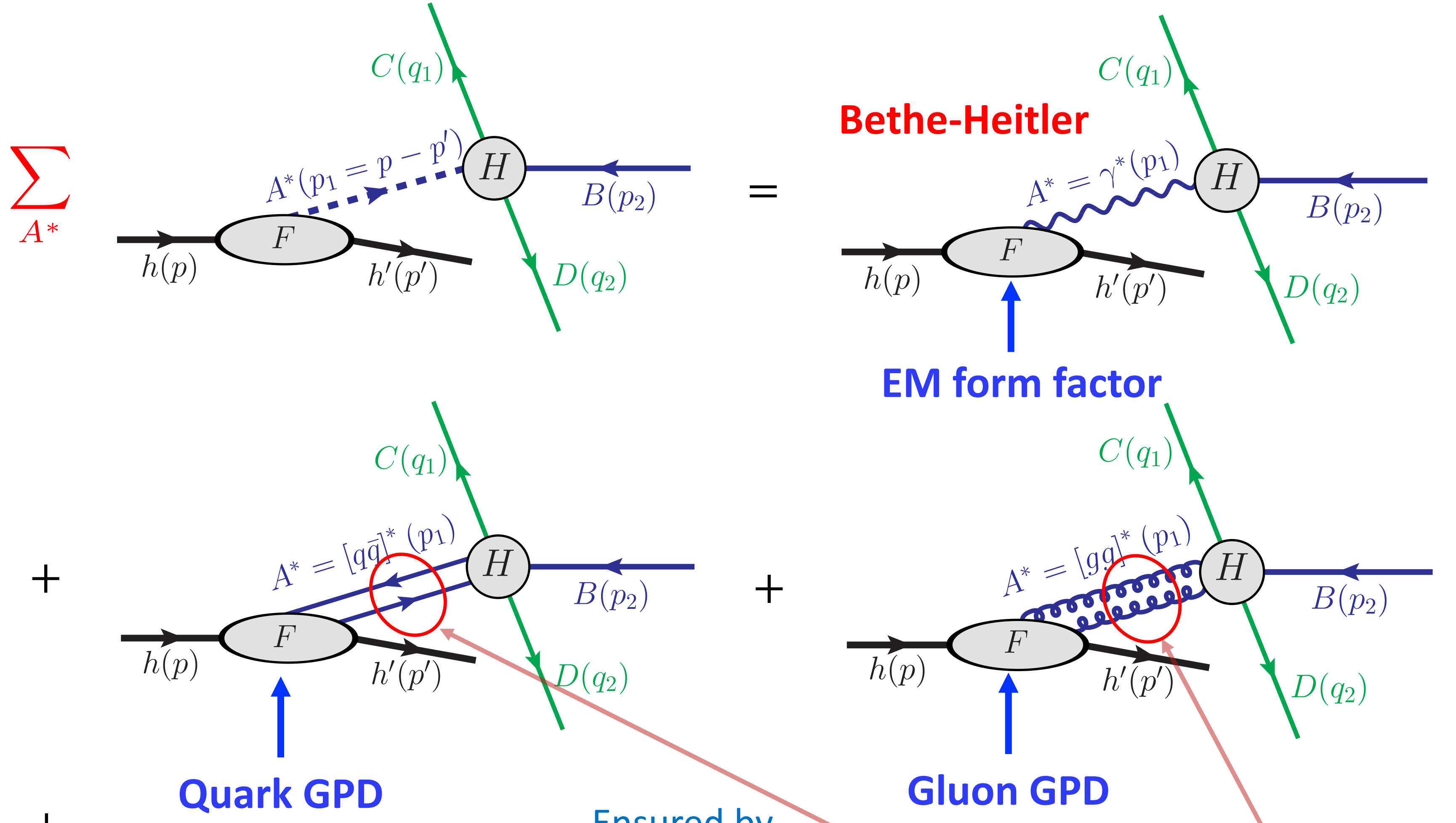
Ensured by
$\sqrt{-t}/q_T \ll 1$

Energy $\gg$ virtuality

Factorization into GPDs

Jefferson Lab

SDHEP: Two-stage Paradigm plus Power Expansion in $\sqrt{-t}/q_T$



Leading Power in $\frac{\sqrt{-t}}{Q}$

Next Leading Power in $\frac{\sqrt{-t}}{Q}$

$+ \cdots$

$\geq$ 3 parton connection: further Power suppressed

Ensured by $\sqrt{-t}/q_T \ll 1$

Energy $\gg$ virtuality

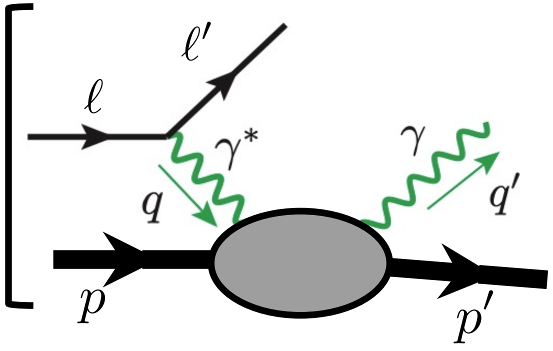
Factorization into GPDs

GPDs are defined by the quantum states of *A**

Exclusive $2 \to 3$ Electroproduction

- **Exclusive electroproduction of a real photon:** $e(\ell) + h(p) \to e(\ell') + h(p') + \gamma(q')$

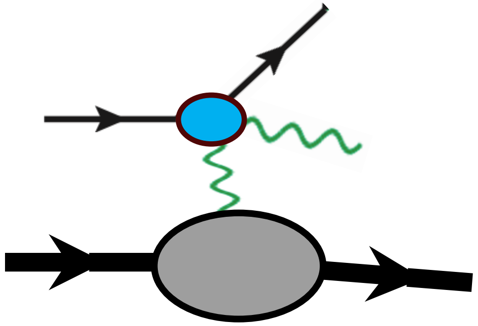
Traditional representation (LO in QED) – Breit frame:



18 scalar functions

DVCS: in terms of Compton Scattering Amplitude

+

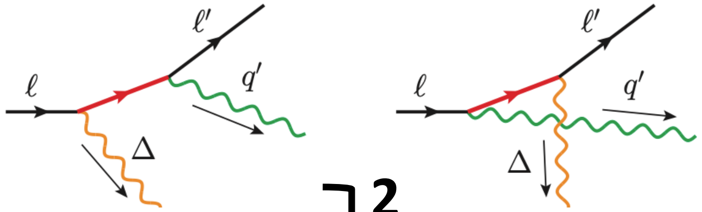


2 EM form factors

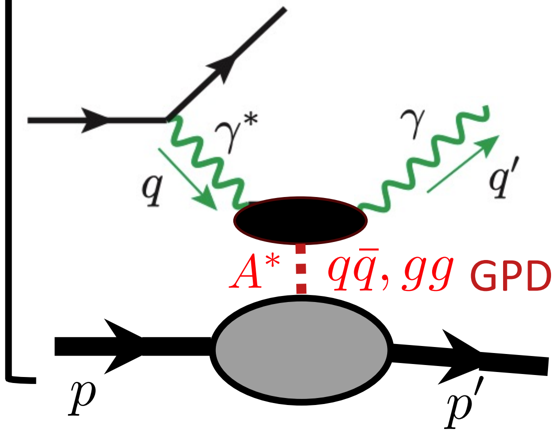
Bethe-Heitler subprocess in terms of EM form factors

$]^2$

Real photon $\gamma(q')$ is in Hadronic plan

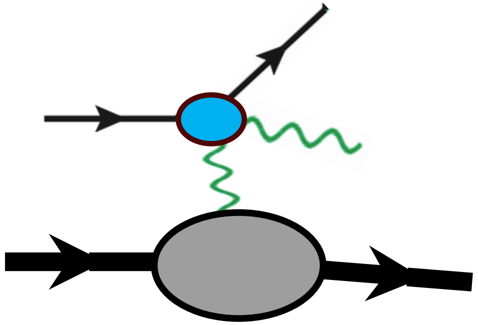


Factorization – Leading power Approximation:



DVCS: in terms of factorized GPDs

+



2 EM form factors

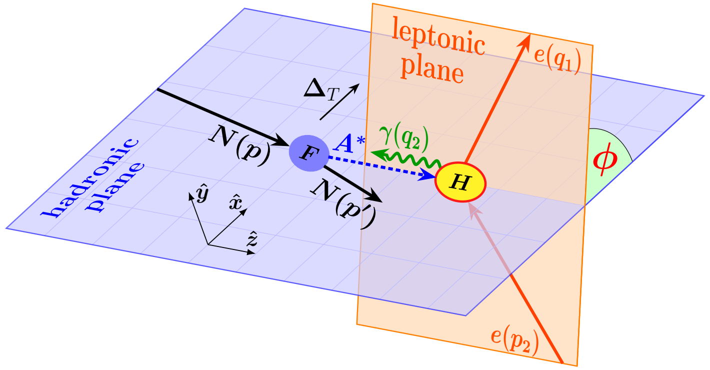
Bethe-Heitler subprocess in terms of EM form factors

$]^2$

Separation of GPDs: angle distribution between leptonic ($\ell \to \ell'$) and hadronic ($p \to p'$) planes!

Angular Modulations – Separation of Different GPDs & Global Analyses

- **Experimental Breit frame is not ideal:** $e(\ell) + h(p) \to e(\ell') + h(p') + \gamma(q_2)$



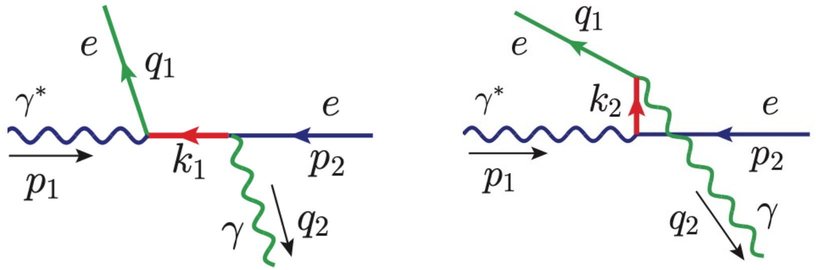
DVCS"

$$e(\ell) \to e(\ell') + \gamma^*(q)$$

$$\gamma^*(q) + h(p) \to h(p') + \gamma(q_2)$$

Out-going photon is in the hadronic plane

BH is not a "t"-channel process:

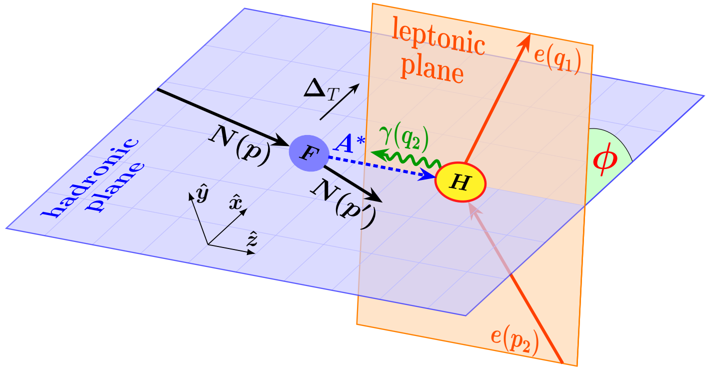


Angular modulation between "leptonic" and "hadronic" planes do not necessarily select the definite spin-state of A* - different GPDs!

Propagators of k_1 & k_2 have different ϕ-dependence!

Angular Modulations – Separation of Different GPDs & Global Analyses

- **Experimental Breit frame is not ideal:** $e(\ell) + h(p) \to e(\ell') + h(p') + \gamma(q_2)$



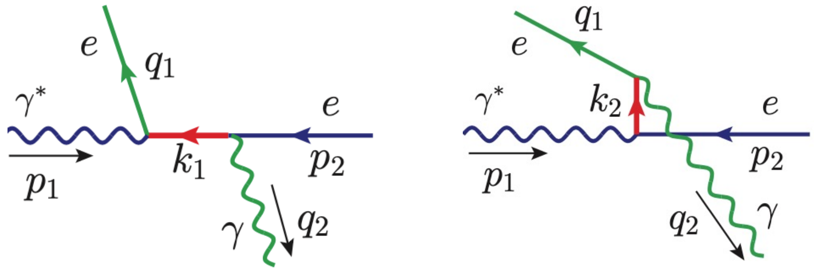
DVCS"

$e(\ell) \to e(\ell') + \gamma^*(q)$

$\gamma^*(q) + h(p) \to h(p') + \gamma(q_2)$

Out-going photon is in the hadronic plane

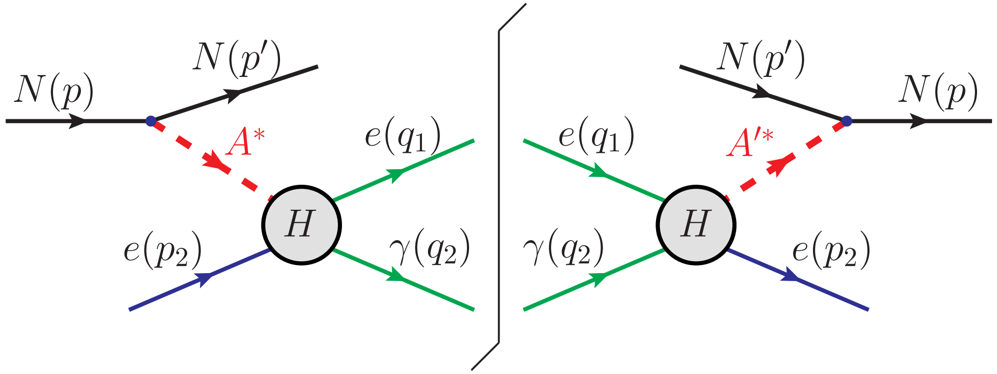
BH is not a "t"-channel process:



Angular modulation between "leptonic" and "hadronic" planes do not necessarily select the definite spin-state of A* - different GPDs!

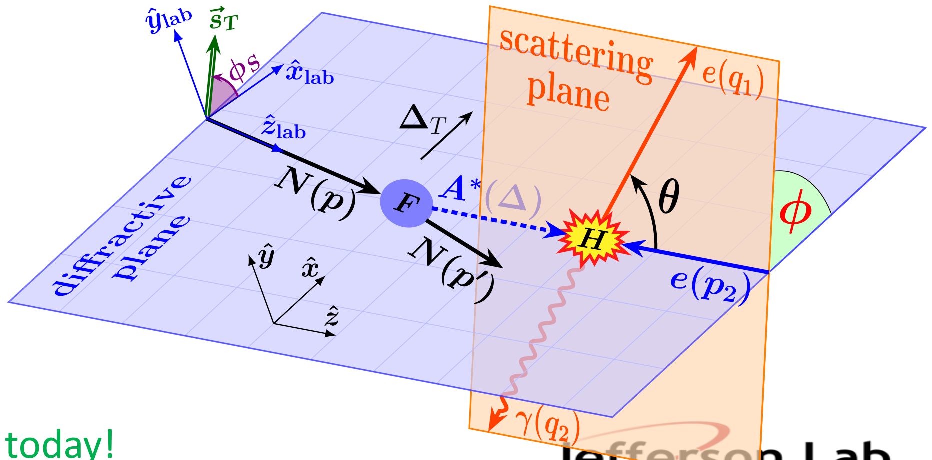
Propagators of k_1 & k_2 have different ϕ-dependence!

- **SDHEP frame = A* - lepton frame (*switch the role of lepton and hadron in the Breit frame*):**



$$\cos[(\Delta\lambda_A)\phi]$$
$$\sin[(\Delta\lambda_A)\phi]$$

$$\Delta\lambda_A = \lambda_A - \lambda'_A$$



Angular modulation between "diffractive" and "scattering" planes to select the spin-state of A* - different GPDs

See the talk by Z. Yu later today!

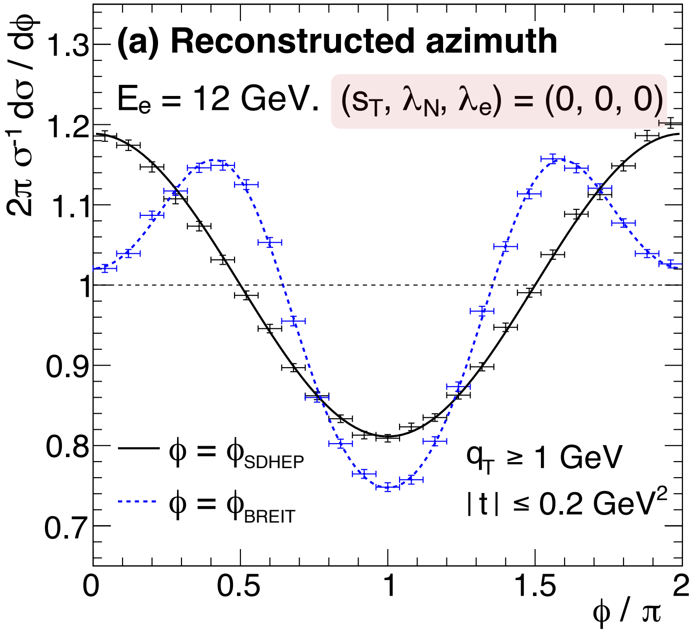
Simple Numerical Examples for Angular Distribution/Modulation

See talk by Z. Yu

$$\frac{d\sigma}{dt\,d\xi\,d\phi_S\,d\cos\theta\,d\phi} = \frac{1}{(2\pi)^2}\frac{d\sigma^{\text{unpol}}}{dt\,d\xi\,d\cos\theta}\cdot\Big[1+\lambda_e\lambda_N A_{LL}^{\text{LP}} + \lambda_e s_T A_{TL}^{\text{LP}}\cos\phi_S$$
$$+\left(A_{UU}^{\text{NLP}} + \lambda_e\lambda_N A_{LL}^{\text{NLP}}\right)\cos\phi + \left(\lambda_e A_{UL}^{\text{NLP}} + \lambda_N A_{LU}^{\text{NLP}}\right)\sin\phi$$
$$+ s_T\left(A_{TU,1}^{\text{NLP}}\cos\phi_S\sin\phi + A_{TU,2}^{\text{NLP}}\sin\phi_S\cos\phi\right)$$
$$+\lambda_e s_T\left(A_{TL,1}^{\text{NLP}}\cos\phi_S\cos\phi + A_{TL,2}^{\text{NLP}}\sin\phi_S\sin\phi\right)\Big]$$

Generate 10^6 events and reconstruct

unpolarized



(normalized)

SDHEP fit to $1.00 + 0.190\cos\phi$ → $\langle A_{UU}^{\text{NLP}}\rangle = 0.190$

Breit fit to $1.00 + 0.15\cos\phi - 0.12\cos 2\phi - 0.01\cos 3\phi + 0.01\cos 4\phi$

- **No straightforward interpretation of the coefficients.**
- **Need to introduce more gears in GPD extraction.**

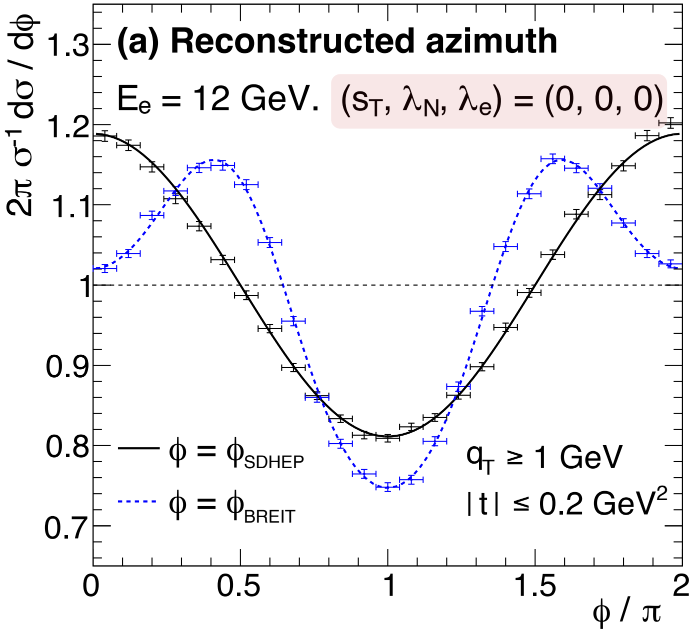
[A.V. Belitsky et al., 2002]
[B. Kriesten et al., 2020, 2022]
[Y. Guo et al., 2021, 2022]
…

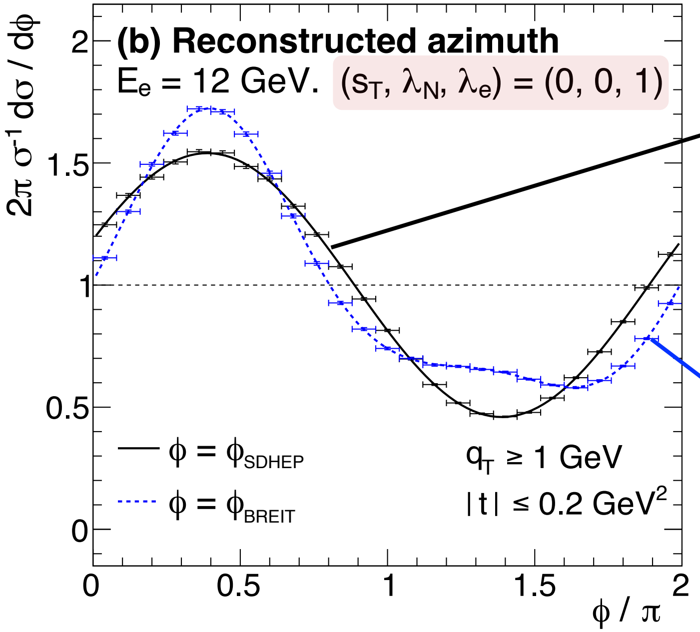
Simple Numerical Examples for Angular Distribution/Modulation

See talk by Z. Yu

$$\frac{d\sigma}{dt\, d\xi\, d\phi_S\, d\cos\theta\, d\phi} = \frac{1}{(2\pi)^2}\frac{d\sigma^{\rm unpol}}{dt\, d\xi\, d\cos\theta}\cdot\Big[1 + \lambda_e\lambda_N A_{LL}^{\rm LP} + \lambda_e s_T A_{TL}^{\rm LP}\cos\phi_S + \left(A_{UU}^{\rm NLP} + \lambda_e\lambda_N A_{LL}^{\rm NLP}\right)\cos\phi + \left(\lambda_e A_{UL}^{\rm NLP} + \lambda_N A_{LU}^{\rm NLP}\right)\sin\phi + s_T\left(A_{TU,1}^{\rm NLP}\cos\phi_S\sin\phi + A_{TU,2}^{\rm NLP}\sin\phi_S\cos\phi\right) + \lambda_e s_T\left(A_{TL,1}^{\rm NLP}\cos\phi_S\cos\phi + A_{TL,2}^{\rm NLP}\sin\phi_S\sin\phi\right)\Big]$$

Generate 10^6 events and reconstruct





$$1.00 + 0.19\cos\phi + 0.51\sin\phi$$

$$\langle A_{UU}^{\rm NLP}\rangle = 0.19$$

$$\langle A_{UL}^{\rm NLP}\rangle = 0.51$$

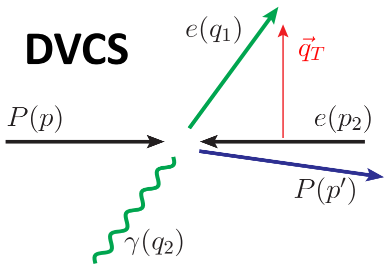
?

single electron polarization

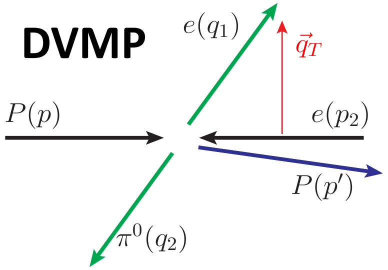
Classification of SDHEPs – Known processes for extracting GPDs

- **Electro-production** (JLab, EIC, …)

DVCS

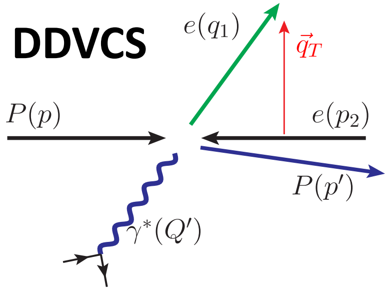


DVMP



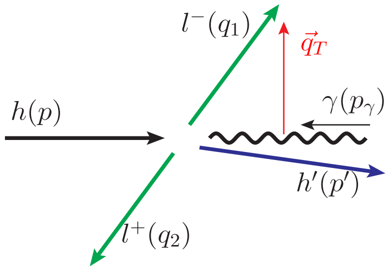
add virtuality

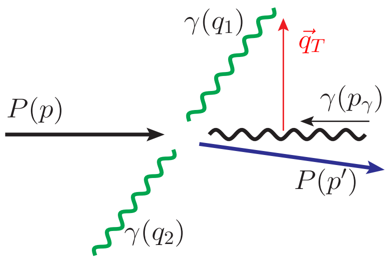
DDVCS

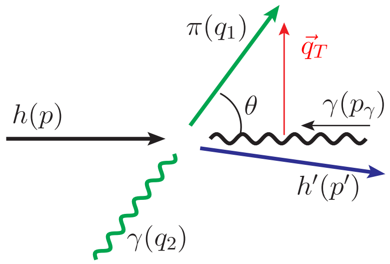


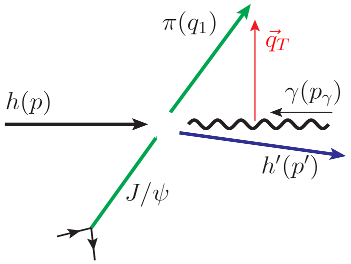
DHQP…

- **Photo-production** (JLab, EIC, …)

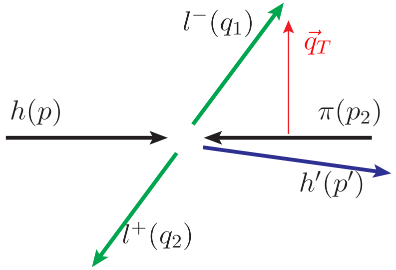


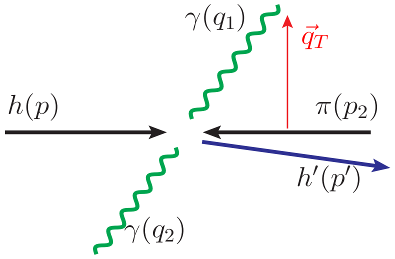






- **Meso-production** (AMBER, J-PARC, …)



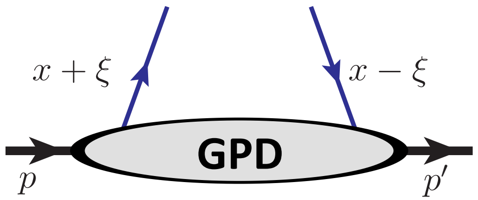


…

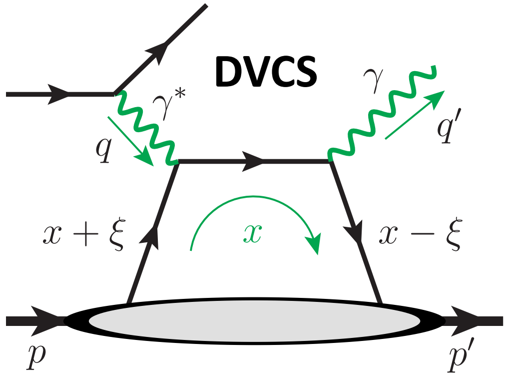
In the SDHEP frame, all GPDs are defined with the same choice of "+" component – defined by the colliding beam of momentum p_2 – good for Global analyses, …

Why is the GPD's *x*-dependence so *difficult* to measure?

- **Amplitude nature of the exclusive processes:**



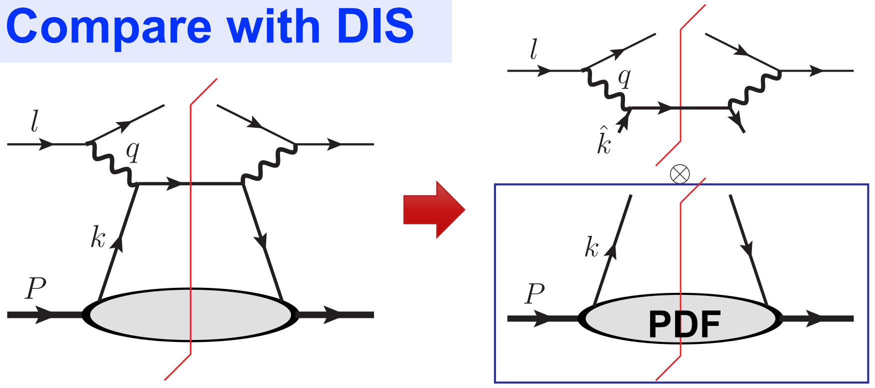
$x \sim$ loop momentum



$$i\mathcal{M} \sim \int_{-1}^{1} \mathrm{d}x\, F(x,\xi,t)\cdot C(x,\xi;Q/\mu)$$

Full range of x, including $x=0; \quad x=\pm\xi$

Compare with DIS



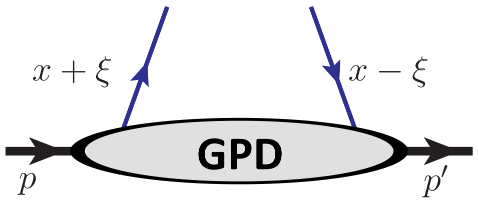
***cross section*: cut diagram**

$$\sigma_{\mathrm{DIS}} \simeq \int_{x_B}^{1} \mathrm{d}x\, f(x)\,\hat{\sigma}(x/x_B)$$

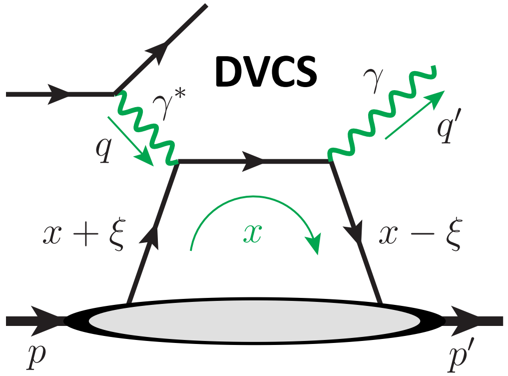
Jefferson Lab

Why is the GPD's *x*-dependence so *difficult* to measure?

- **Amplitude nature of the exclusive processes:**



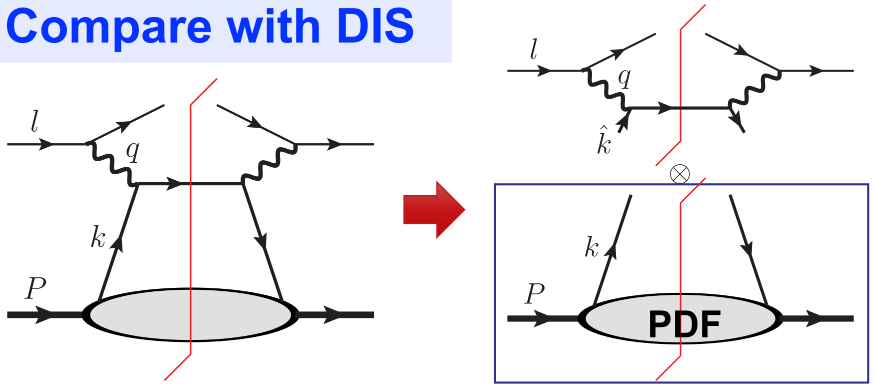
x ~ loop momentum



$$i\mathcal{M} \sim \int_{-1}^{1} dx\, F(x,\xi,t) \cdot C(x,\xi;Q/\mu)$$

Full range of x, including $x = 0; \quad x = \pm\xi$

Compare with DIS



***cross section*: cut diagram**

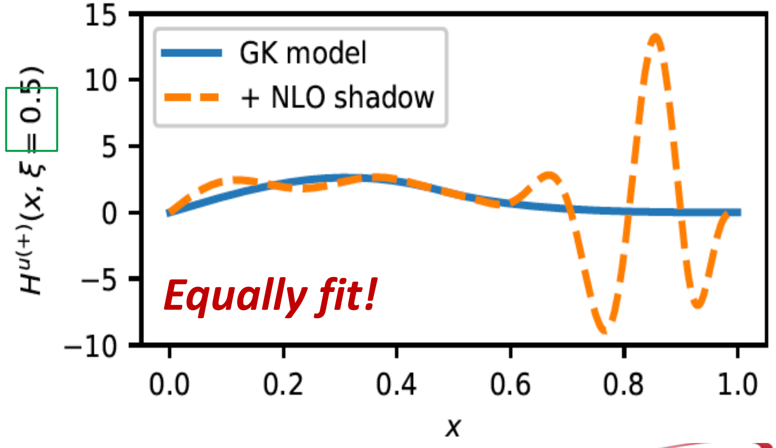
$$\sigma_{\rm DIS} \simeq \int_{x_B}^{1} dx\, f(x)\, \hat{\sigma}(x/x_B)$$

- **Sensitivity to x: comes from $C(x,\xi;Q/\mu)$**

$$C(x,\xi;Q/\mu) = T(Q/\mu)\cdot G(x,\xi) \propto \frac{1}{x-\xi+i\epsilon}\cdots$$

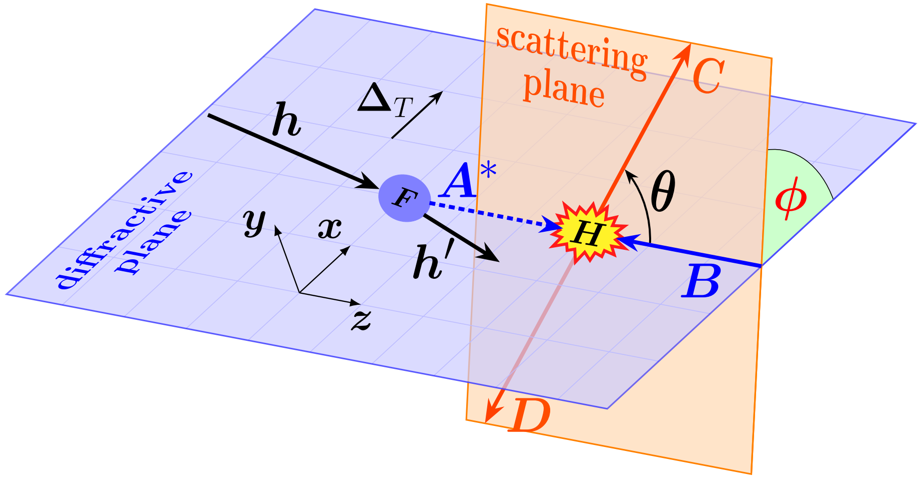
$$\Rightarrow\ i\mathcal{M} \propto \int_{-1}^{1} dx\, \frac{F(x,\xi,t)}{x-\xi+i\varepsilon} \equiv \text{“}F_0(\xi,t)\text{”}$$ **“moment”**

DVCS is an example



[Bertone et al. PRD `21]

Where can the SDHEP get the x-sensitivity?



- x-sensitivity $\Leftrightarrow$ 2 → 2 hard scattering:

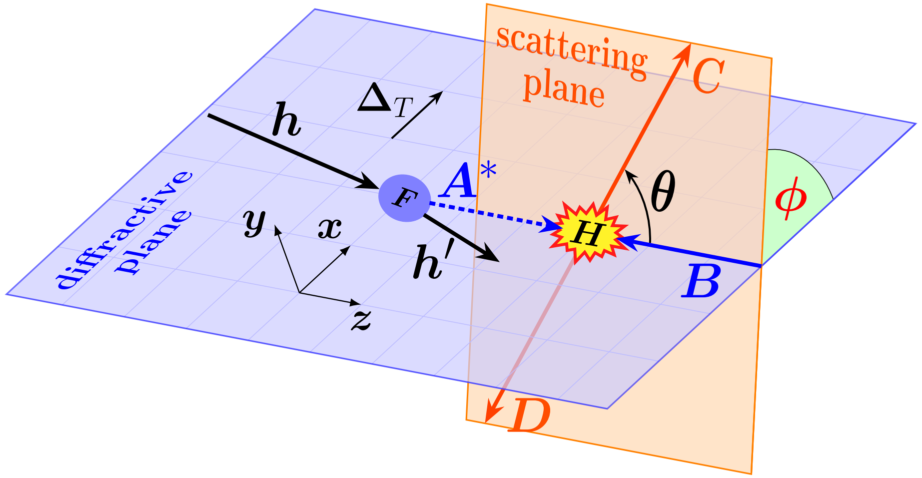
  Kinematics:

  1. $\hat{s} = 2\,\xi\,s\,/\,(1+\xi)$ ⟵ ξ
  2. θ or $q_T = (\sqrt{\hat{s}}/2)\sin\theta$ ⟷ x
  3. ϕ ⟵ (A^*B) spin states

$$\mathcal{M}(Q,\phi) \simeq \sum_A e^{i(\lambda_A-\lambda_B)\phi} \cdot \int_{-1}^{1} dx\, F_A(x)\, C_A(x;Q) \qquad (Q = \theta \text{ or } q_T)$$

[suppressing t and ξ dependence]

Where can the SDHEP get the x-sensitivity?



- x-sensitivity ⇔ 2 → 2 hard scattering:

Kinematics:

1. $\hat{s} = 2\,\xi\,s\,/\,(1+\xi)$ ⬅ ξ
2. θ or $q_T = (\sqrt{\hat{s}}/2)\sin\theta$ ⬌ x
3. ϕ ⬅ (A^*B) spin states

$$\mathcal{M}(Q,\phi) \simeq \sum_A e^{i(\lambda_A-\lambda_B)\phi}\cdot\int_{-1}^{1} dx\, F_A(x)\, C_A(x;Q) \qquad (Q=\theta \text{ or } q_T)$$

[suppressing t and ξ dependence]

- **Moment-type sensitivity:** $C(x;Q) = G(x)\cdot T(Q)$ ➡ $F_G = \int_{-1}^{1} dx\, G(x)\, F(x,\xi,t)$ Independent of Q; Scaling for F_G

➡ Inversion problem: *shadow GPD* $S_G = \int_{-1}^{1} dx\, G(x)\, S(x,\xi) = 0$ [Bertone et al. PRD `21]

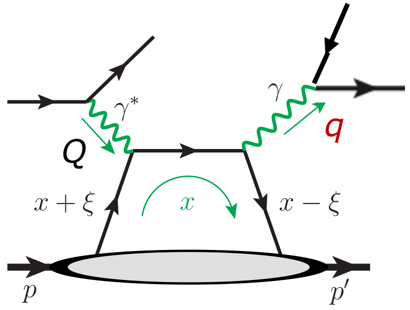
- **Enhanced sensitivity:** $C(x;Q) \neq G(x)\cdot T(Q)$ ➡ $d\sigma/dQ \sim \left|C(x;Q)\otimes_x F(x,\xi,t)\right|^2$

What Kind of Process Could be Sensitive to the x-Dependence?

❑ **Create an entanglement between the internal x and an externally measured variable?**

$$i\mathcal{M} \propto \int_{-1}^{1} \mathrm{d}x \frac{F(x,\xi,t)}{x - x_p(\xi,q) + i\varepsilon}$$

Change external q to sample different part of x.



- **Double DVCS (two scales):**

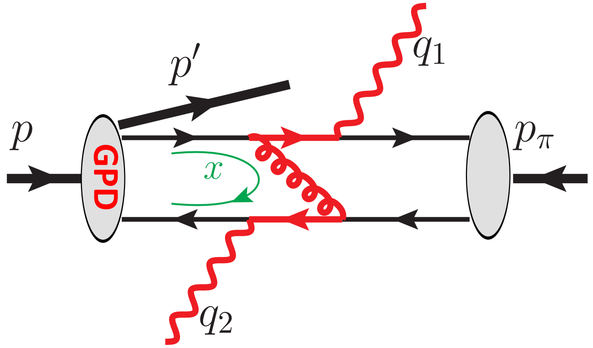
$$x_p(\xi,q) = \xi\left(\frac{1-q^2/Q^2}{1+q^2/Q^2}\right) \to \xi \quad \text{same as} \quad \text{DVCS} \quad \text{if } q \to 0$$

- **Production of two back-to-back high pT particles (say, two photons):**

$$\pi^-(p_\pi) + P(p) \to \gamma(q_1) + \gamma(q_2) + N(p')$$

Qiu & Yu
JHEP 08 (2022) 103

Hard scale: $q_T \gg \Lambda_{\rm QCD}$ Soft scale: $t \sim \Lambda^2_{\rm QCD}$



- **Factorization:**

[*suppressing pion DA factor*]

$$\mathcal{M}(t,\xi,q_T) = \int_{-1}^{1} \mathrm{d}x\, F(x,\xi,t;\mu)\cdot C(x,\xi;q_T/\mu) + \mathcal{O}(\Lambda_{\rm QCD}/q_T)$$

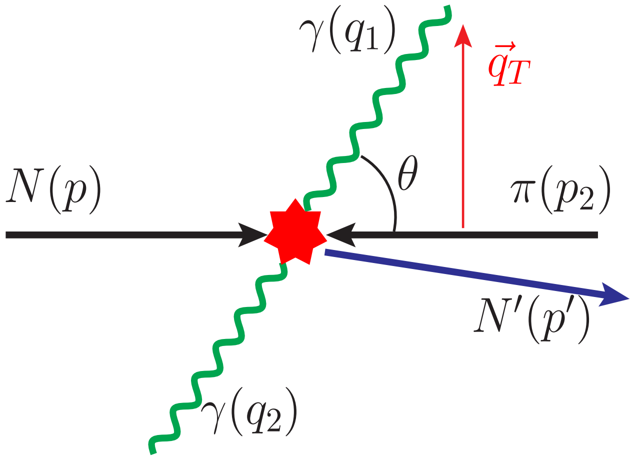
$$\Rightarrow \quad \frac{\mathrm{d}\sigma}{\mathrm{d}t\,\mathrm{d}\xi\,\mathrm{d}q_T} \sim |\mathcal{M}(t,\xi,q_T)|^2$$

q_T distribution is "conjugate" to x distribution

$$x \leftrightarrow q_T$$

Enhanced x-Sensitivity: (1) Diphoton Meso-production

Qiu & Yu, PRD 109 (2024) 074023

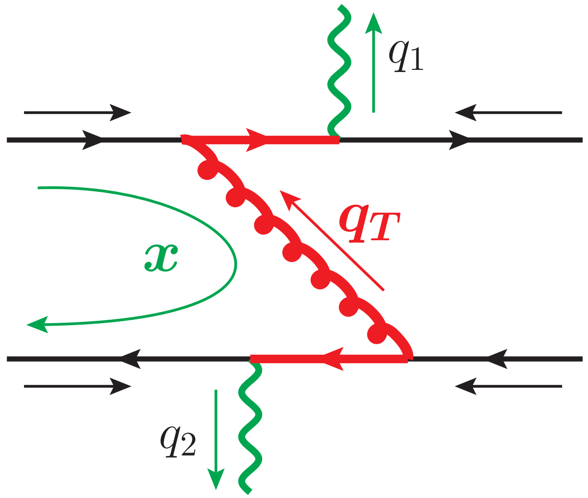


In addition to

$$F_0(\xi,t) = \int_{-1}^{1} \frac{dx\, F(x,\xi,t)}{x-\xi+i\epsilon}$$

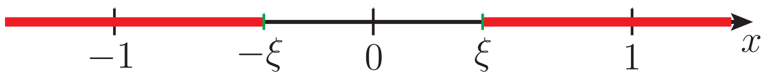
When two photons are radiated from the same charged line

$i\mathcal{M}$ **also contains**

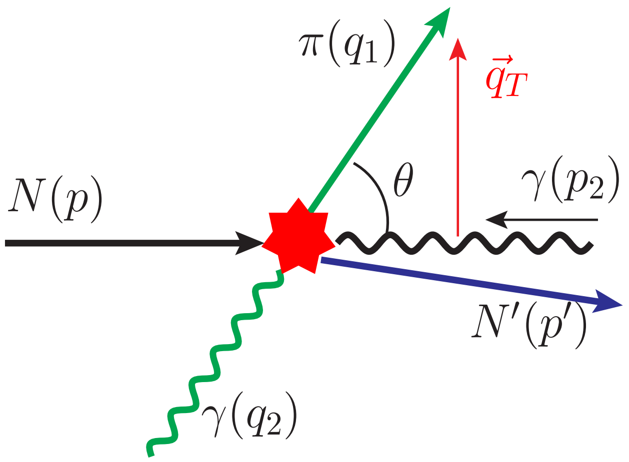


$$I(t,\xi;z,\theta) = \int_{-1}^{1} \frac{dx\, F(x,\xi,t)}{x-\rho(z;\theta)+i\epsilon\,\mathrm{sgn}\left[\cos^2(\theta/2)-z\right]}$$

$$\rho(z;\theta) = \xi \cdot \left[\frac{1-z+\tan^2(\theta/2)\,z}{1-z-\tan^2(\theta/2)\,z}\right] \in (-\infty,-\xi] \cup [\xi,\infty)$$



Enhanced x-Sensitivity: (2) γ-π Pair Photoproduction

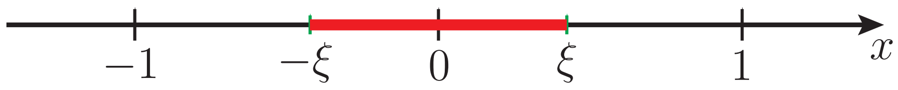


$i\mathcal{M}$ **also contains the special integral:**

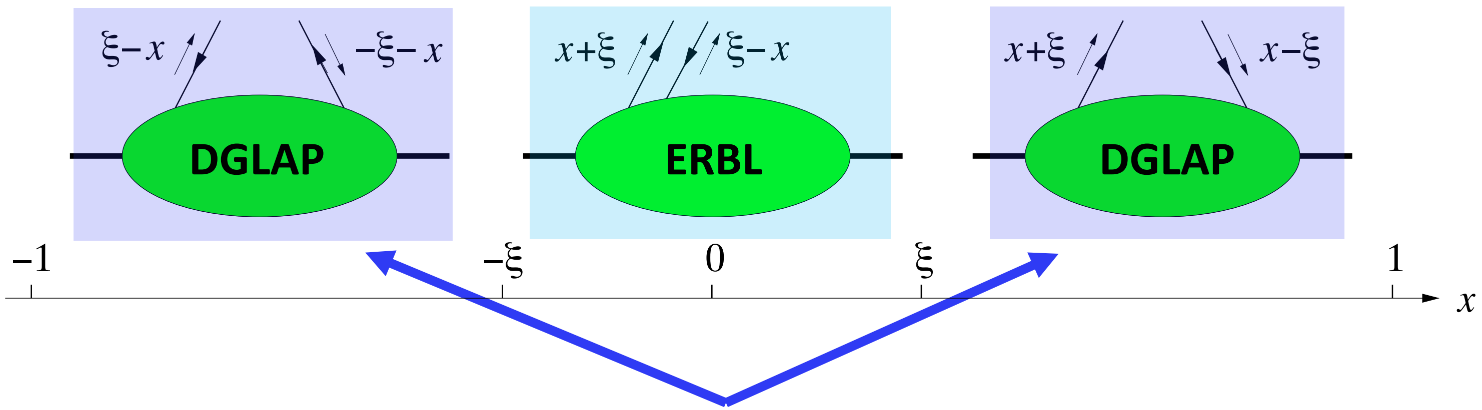
G. Duplancic et al., JHEP 11 (2018) 179
G. Duplancic et al., JHEP 03 (2023) 241
G. Duplancic et al., PRD 107 (2023), 094023
Qiu & Yu, PRL 131 (2023), 161902

$$I'(t,\xi;z,\theta) = \int_{-1}^{1} \frac{dx\, F(x,\xi,t)}{x - \rho'(z;\theta) + i\epsilon}$$

$$\rho'(z;\theta) = \xi \cdot \left[\frac{\cos^2(\theta/2)\,(1-z) - z}{\cos^2(\theta/2)\,(1-z) + z}\right] \in [-\xi, \xi]$$



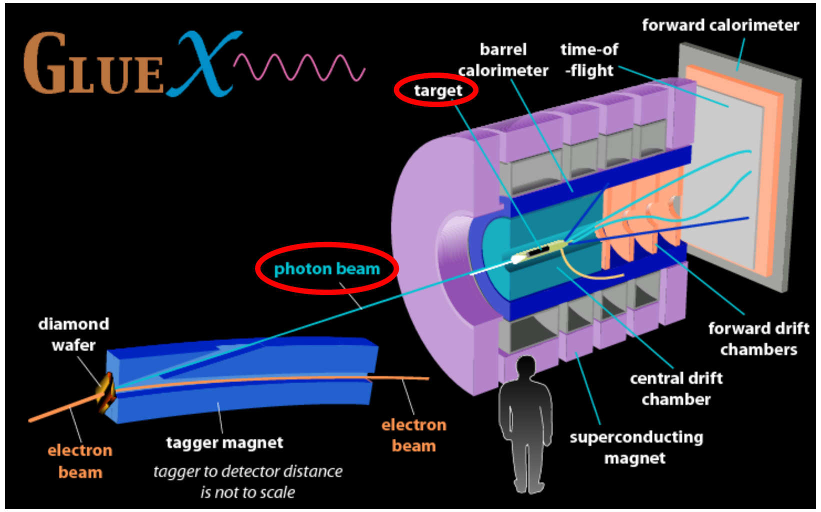
Complementary sensitivity:

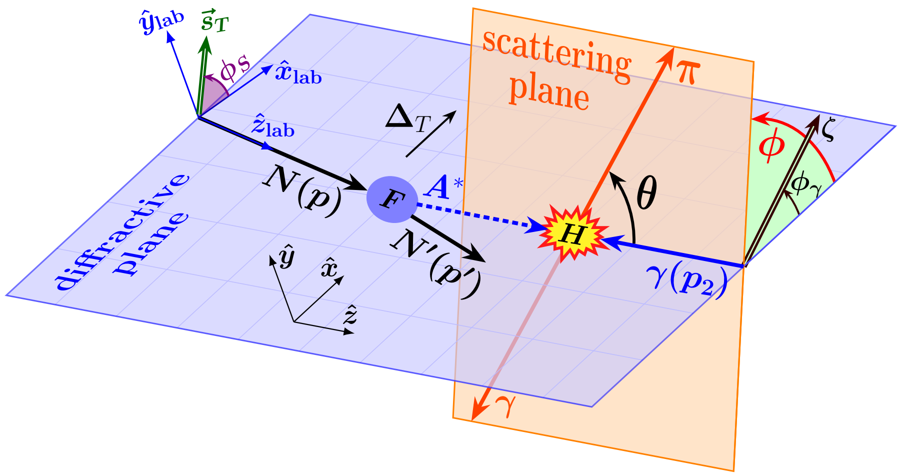


$N\,\pi \to N'\,\gamma\,\gamma$

Enhanced x-Sensitivity: γ-π Pair Photoproduction (at JLab Hall D)

Qiu & Yu, PRL 131 (2023), 161902





❑ Polarization asymmetries:

$$\frac{d\sigma}{d|t|\,d\xi\,d\cos\theta\,d\phi} = \frac{1}{2\pi}\,\frac{d\sigma}{d|t|d\xi d\cos\theta}\cdot\left[1+\lambda_N\lambda_\gamma A_{LL} + \zeta A_{UT}\cos 2(\phi-\phi_\gamma) + \lambda_N\zeta A_{LT}\sin 2(\phi-\phi_\gamma)\right]$$

$$\frac{d\sigma}{d|t|\,d\xi\,d\cos\theta} = \pi\,(\alpha_e\alpha_s)^2\left(\frac{C_F}{N_c}\right)^2\frac{1-\xi^2}{\xi^2 s^3}\Sigma_{UU}$$

$$\Sigma_{UU} = |\mathcal{M}_+^{[\widetilde{H}]}|^2 + |\mathcal{M}_-^{[\widetilde{H}]}|^2 + |\widetilde{\mathcal{M}}_+^{[H]}|^2 + |\widetilde{\mathcal{M}}_-^{[H]}|^2,$$

$$A_{LL} = 2\,\Sigma_{UU}^{-1}\,\mathrm{Re}\left[\mathcal{M}_+^{[\widetilde{H}]}\,\widetilde{\mathcal{M}}_+^{[H]*} + \mathcal{M}_-^{[\widetilde{H}]}\,\widetilde{\mathcal{M}}_-^{[H]*}\right],$$

$$A_{UT} = 2\,\Sigma_{UU}^{-1}\,\mathrm{Re}\left[\widetilde{\mathcal{M}}_+^{[H]}\,\widetilde{\mathcal{M}}_-^{[H]*} - \mathcal{M}_+^{[\widetilde{H}]}\,\mathcal{M}_-^{[\widetilde{H}]*}\right],$$

$$A_{LT} = 2\,\Sigma_{UU}^{-1}\,\mathrm{Im}\left[\mathcal{M}_+^{[\widetilde{H}]}\,\widetilde{\mathcal{M}}_-^{[H]*} + \mathcal{M}_-^{[\widetilde{H}]}\,\widetilde{\mathcal{M}}_+^{[H]*}\right].$$

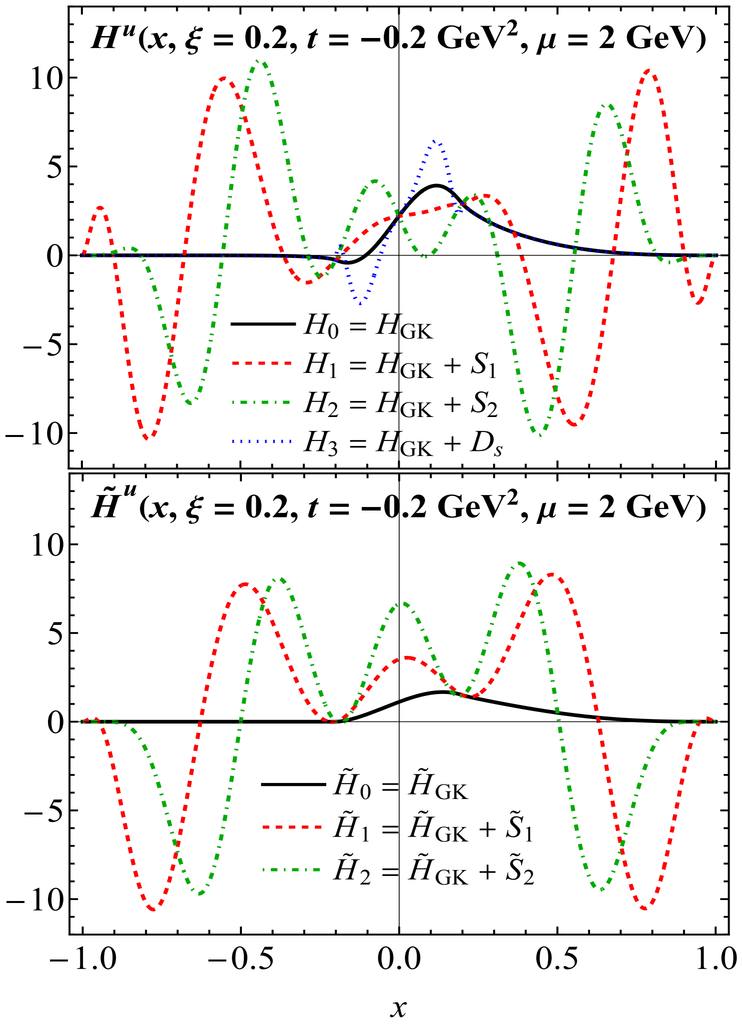
Neglecting: (1) E and $\widetilde{E}$; (2) gluon channel

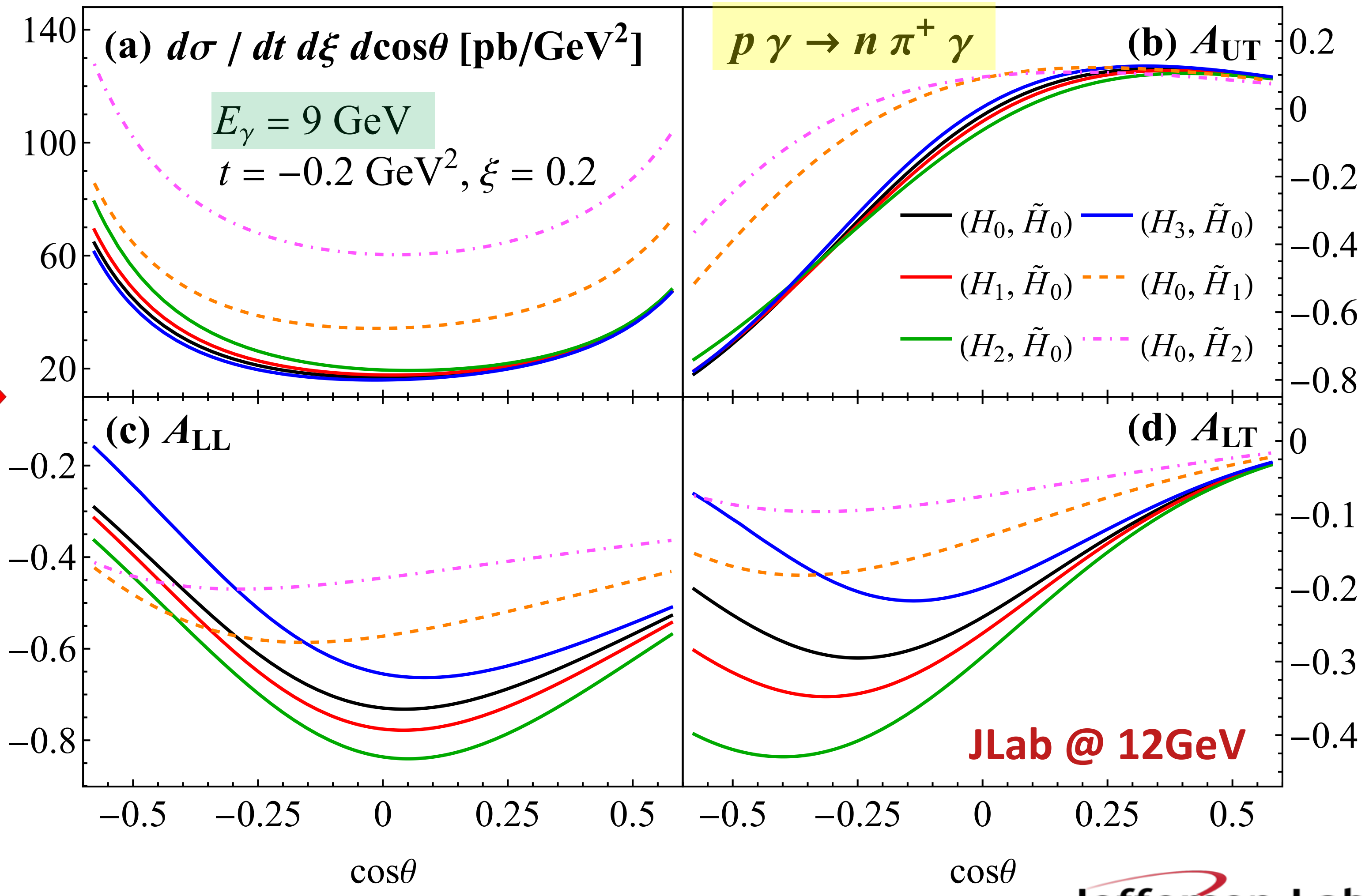
Enhanced x-sensitivity: (2) γ-π pair photoproduction (at JLab Hall D)

Goloskokov, Kroll, `05, `07, `09
Bertone et al. `21
Moffat et al. `23

GPD models = GK model + shadow GPDs

$$\int_{-1}^{1} \frac{dx\, S(x,\xi)}{x-\xi \pm i\epsilon} = 0$$



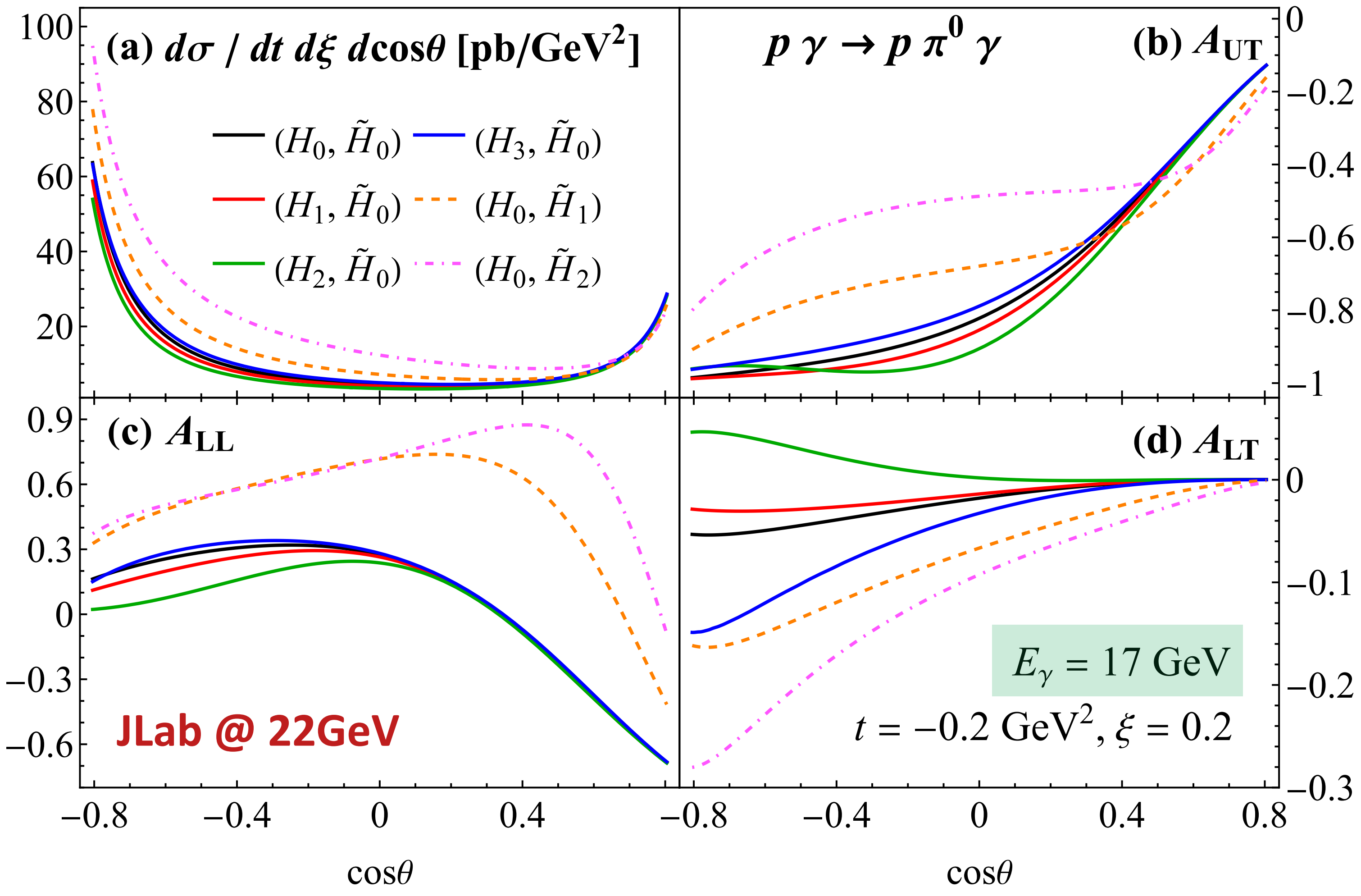


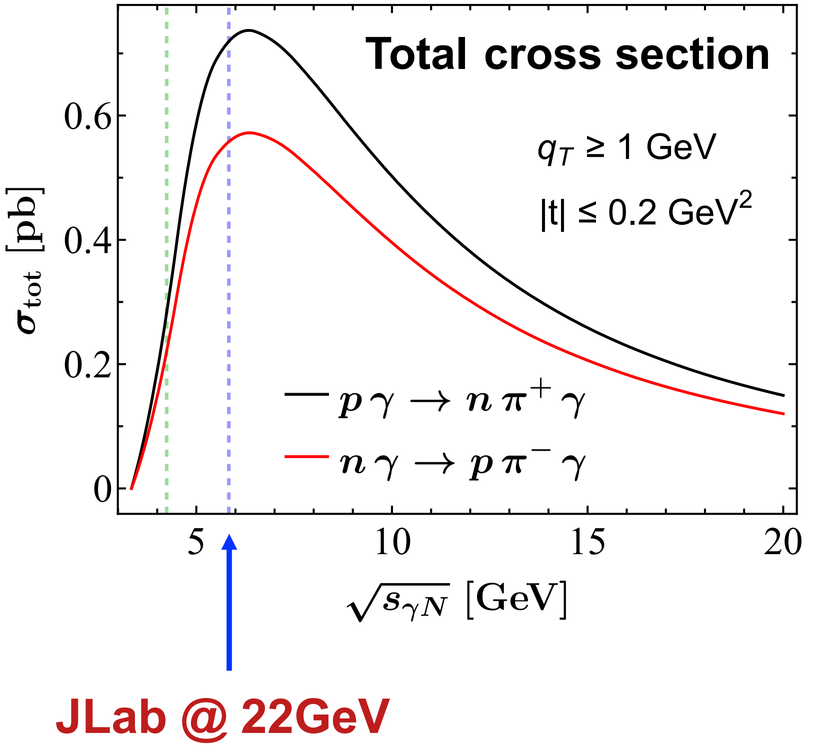
Jefferson Lab

Enhanced x-sensitivity: (2) γ-π pair photoproduction (at upgraded energy)

GPD models = GK model + shadow GPDs ⟸ $$\int_{-1}^{1} \frac{dx\, S(x,\xi)}{x-\xi \pm i\epsilon} = 0$$

Goloskokov, Kroll, \`05, \`07, \`09
Bertone et al. \`21
Moffat et al. \`23
Qiu & Yu, \`23





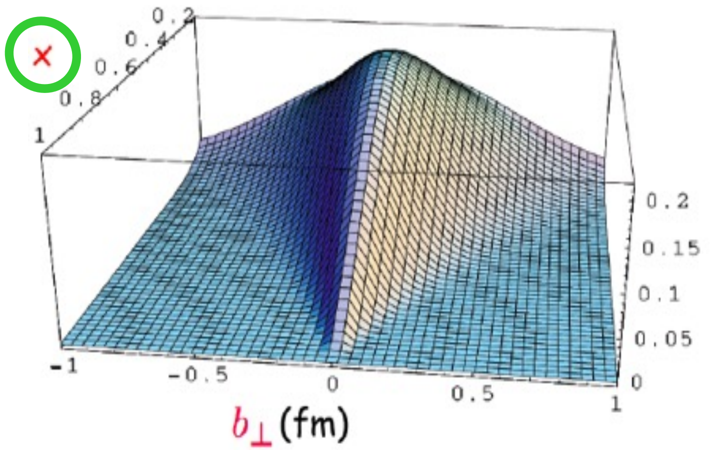
A. Accardi et al. [arXiv:2306.09360]

Summary and Outlook

- GPDs are fundamental, carrying rich information on:
  - Tomographic images of confined quarks and gluons
  - Underline dynamics of hadronic properties
- The $2 \to 3$ SDHEPs are necessary physical processes for extracting of GPDs
  - SDHEP frame is the right one for evaluating angular modulations
  - Need SDHEPs with ***x*** of GPDs entangled with measured hard scales!
- QCD Global analyses to extract GPDs:
  - With $p \neq p'$, the choice of "+" component is not unique
  - SDHEP frame for all known SDHEPs provides a unique way to define the GPDs, necessary for Global analyses
  - Need to identify more factorizable SDHEPs for extracting GPDs through Global analyses
  - …

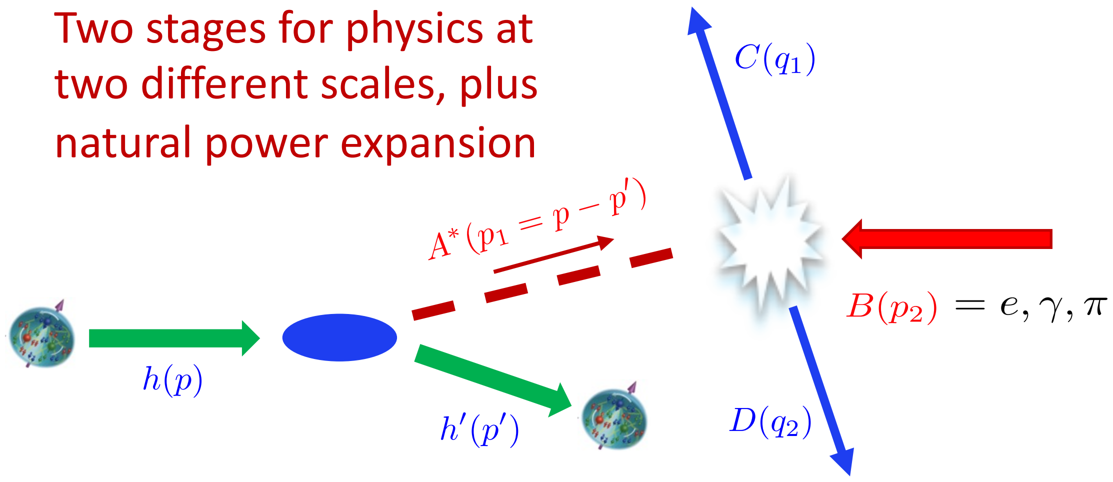
A long but challenging & exciting way to go!

3D image



SDHEPs:

Two stages for physics at two different scales, plus natural power expansion



Thanks!