Mass dependence of meson distributions and their transverse motion

in collaboration with Fernando Serna, Roberto Correa da Silveira and Gastão Krein

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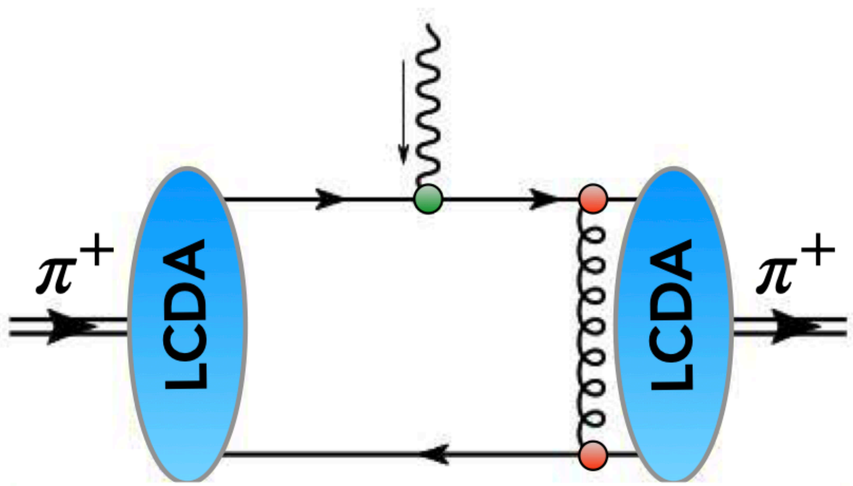
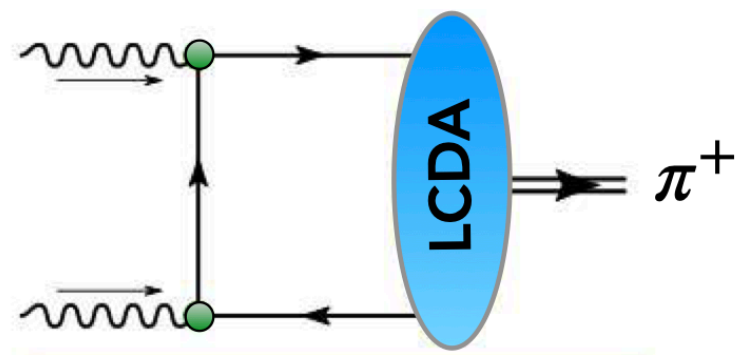
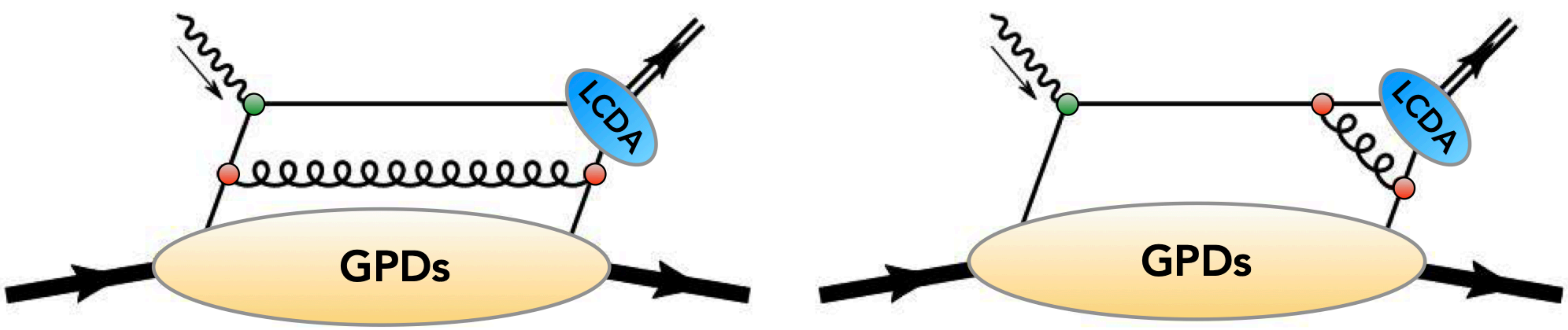
Hard exclusive scattering processes



$$Q^2 F_\pi(Q^2) \to 16\pi f_\pi^2 \alpha_s(Q^2)$$



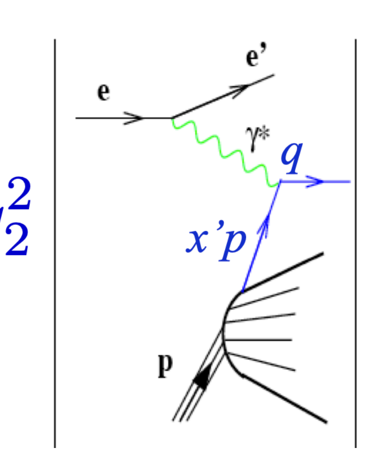
$$Q^2 F_{\gamma^*\gamma\pi}(Q^2) \to 2 f_\pi$$



DIS & Parton Distribution Functions

- Assumes fast moving hadron appears as a jet of partons moving in same direction and sharing its total momentum.
- DIS cross section is an incoherent sum of elastic scattering cross sections off individual partons.
- Parton model should work perfectly for $Q^2 \to \infty$ where coupling constant vanishes.

$$d\sigma \equiv \sum_i \int dx_i q_2^2$$



$$\mathrm{d}\sigma = \sum_i \mathrm{d}\hat{\sigma}(\hat{s}, \hat{t}, \hat{u})\ f_i(x)\mathrm{d}x$$

Define parton momentum distribution: $f_i(x) \equiv \dfrac{dP_i}{dx}$ where $\sum_i \int_0^1 dx f_i(x) = 1$

This defines the probability to find a parton with light-front momentum fraction $x = \dfrac{k^+}{P^+}$ of the hadron. PDF related to structure functions:

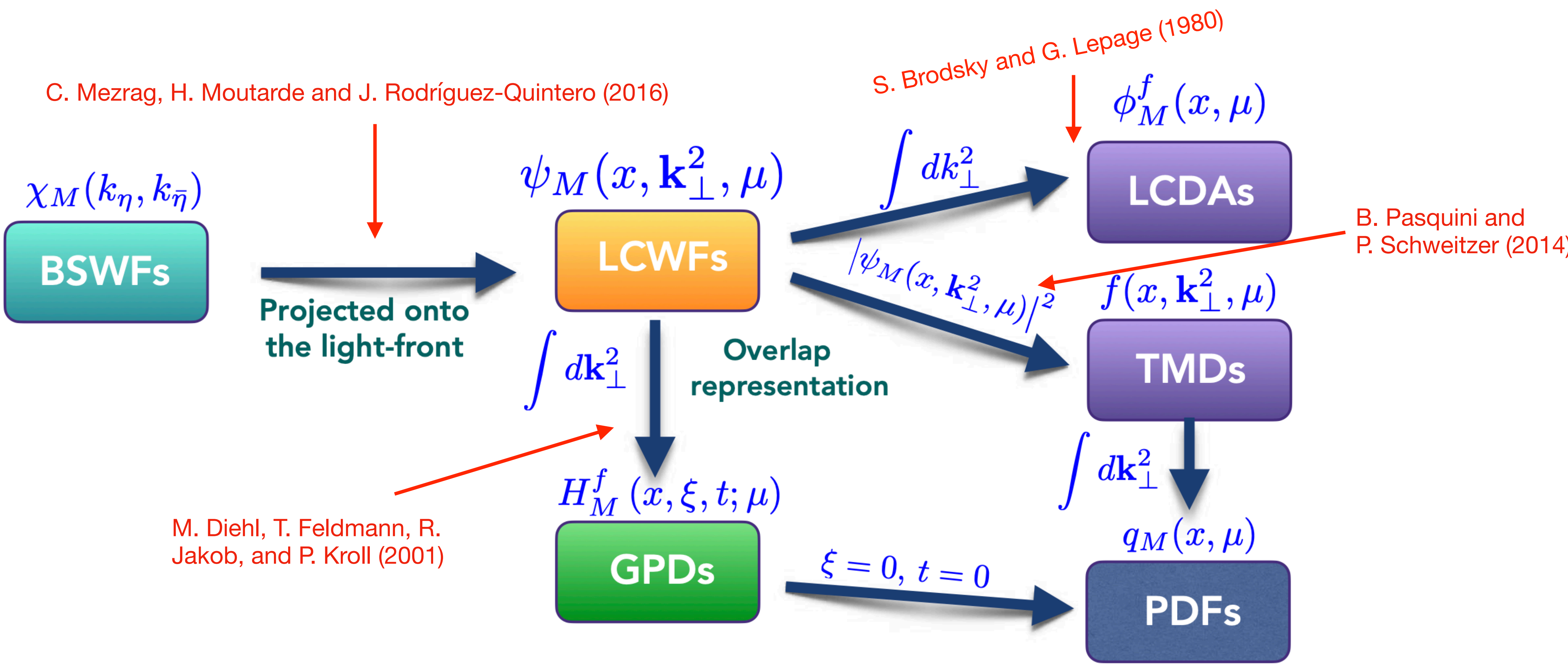
$$F_2(x) = \sum_i q_i^2 x f_i(x)$$

$$F_1(x) = \frac{1}{2x} F_2(x)$$ Callan-Gross relation

Can we obtain these distribution functions from a unified approach ?

Light-Front Wave Functions

With a particular projection of the Bethe-Salpeter wave functions we arrive the light-front wave functions, a more general object to describe probability amplitudes.



Light-Front Wave Functions

- M. Burkardt, X.-D. Ji, and F. Yuan (2002): for pseudoscalar mesons there are two independent light front wave functions for the leading Fock state, with $l_z = 0$ and $l_z = 1$.
- Minimal $\bar{q}q$ Fock-state configuration is given by: $|M\rangle = |M\rangle_{l_z=0} + |M\rangle_{|l_z|=1}$

$$|M\rangle_{l_z=0} = i \int \frac{d^2\boldsymbol{k}_T}{2(2\pi)^3} \frac{dx}{\sqrt{x\bar{x}}} \psi_0\left(x, \boldsymbol{k}_T^2\right) \frac{\delta_{ij}}{\sqrt{3}} \frac{1}{\sqrt{2}} \left[b^\dagger_{f\uparrow i}\left(x, \boldsymbol{k}_T\right) d^\dagger_{h\downarrow j}\left(\bar{x}, \overline{\boldsymbol{k}}_T\right) - b^\dagger_{f\downarrow i}\left(x, \boldsymbol{k}_T\right) d^\dagger_{h\uparrow j}\left(\bar{x}, \overline{\boldsymbol{k}}_T\right)\right] |0\rangle$$

$$|M\rangle_{|l_z|=1} = i \int \frac{d^2\boldsymbol{k}_T}{2(2\pi)^3} \frac{dx}{\sqrt{x\bar{x}}} \psi_1\left(x, \boldsymbol{k}_T^2\right) \frac{\delta_{ij}}{\sqrt{3}} \frac{1}{\sqrt{2}} \left[k_T^- b^\dagger_{f\uparrow i}\left(x, \boldsymbol{k}_T\right) d^\dagger_{h\uparrow j}\left(\bar{x}, \overline{\boldsymbol{k}}_T\right) + k_T^+ b^\dagger_{f\downarrow i}\left(x, \boldsymbol{k}_T\right) d^\dagger_{h\downarrow j}\left(\bar{x}, \overline{\boldsymbol{k}}_T\right)\right] |0\rangle$$

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The LFWFs are obtained from the Bethe-Salpeter wave function via the light front projections:

$$\psi_0\left(x, \mathbf{k}_\perp^2\right) = \sqrt{3}i \int \frac{dk^+ dk^-}{\pi} \delta\left(xP^+ - k^+\right) \mathrm{Tr}_D\left[\gamma^+ \gamma_5 \chi(k, P)\right]$$

$$\psi_1\left(x, \mathbf{k}_\perp^2\right) = -\frac{\sqrt{3}i}{\mathbf{k}_\perp^2} \int \frac{dk^+ dk^-}{\pi} \delta\left(xP^+ - k^+\right) \mathrm{Tr}_D\left[i\sigma_{+i} k_T^i \gamma_5 \chi(k, P)\right]$$

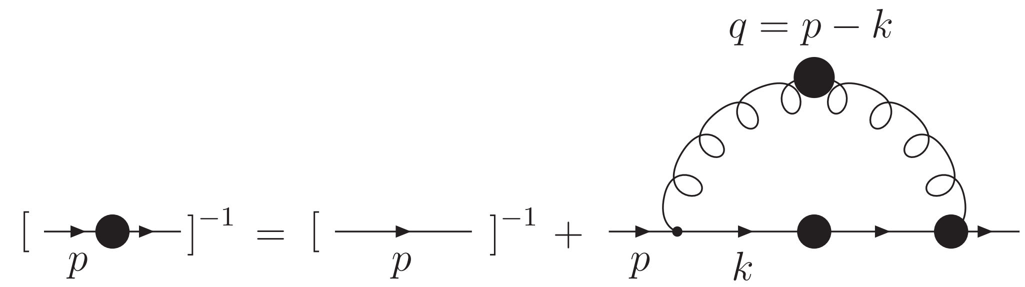
C. Mezrag, H. Moutarde, and J. Rodriguez-Quintero, Few Body Syst. 57 (2016)
C. Shi and I. C. Cloët, Phys. Rev. Lett. 122, 082301 (2019)

$$\left(\Box_x + m^2\right) G(x,y) = -\delta(x-y)$$

Green functions in functional approaches to QCD

Quark-Gap Equation in QCD

The propagator can be obtained from QCD's gap equation: the Dyson-Schwinger equation (DSE) for the dressed-fermion self-energy, which involves the tower of infinitely many coupled equations.



$$S^{-1}(p) = Z_2(i\gamma\cdot p + m^{\rm bm}) + \Sigma(p) := i\gamma\cdot p\, A(p^2) + B(p^2)$$

$$\Sigma(p) = Z_1 \int^{\Lambda} \frac{d^4 q}{(2\pi)^4}\, g^2 D_{\mu\nu}(p-q)\frac{\lambda^a}{2}\gamma_\mu S(q)\Gamma^a_\nu(q,p)$$

with the *running* mass function $M(p^2) = B(p^2)/A(p^2)$.

$D_{\mu\nu}$: dressed-gluon propagator

$\Gamma^a_\nu(q,p)$: dressed quark-gluon vertex

Z_2 : quark wave function renormalization constant

Z_1 : quark-gluon vertex renormalization constant

Each satisfies it's own DSE

$$S^{-1}(p)|_{p^2=\zeta^2} = i\gamma\cdot p + m(\zeta)$$

where ζ is the renormalization point.

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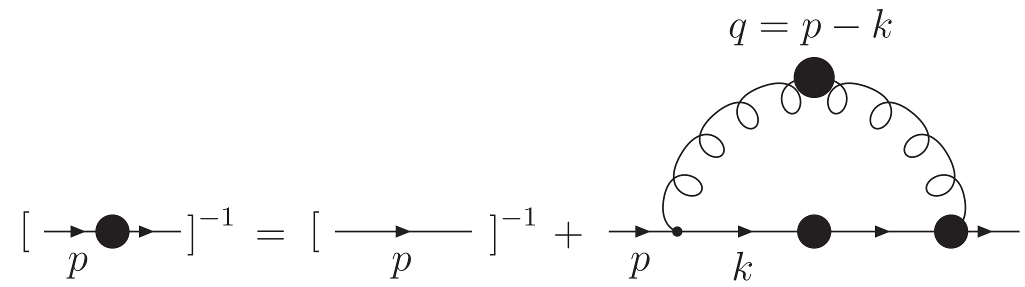

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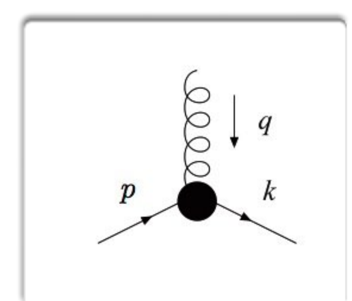

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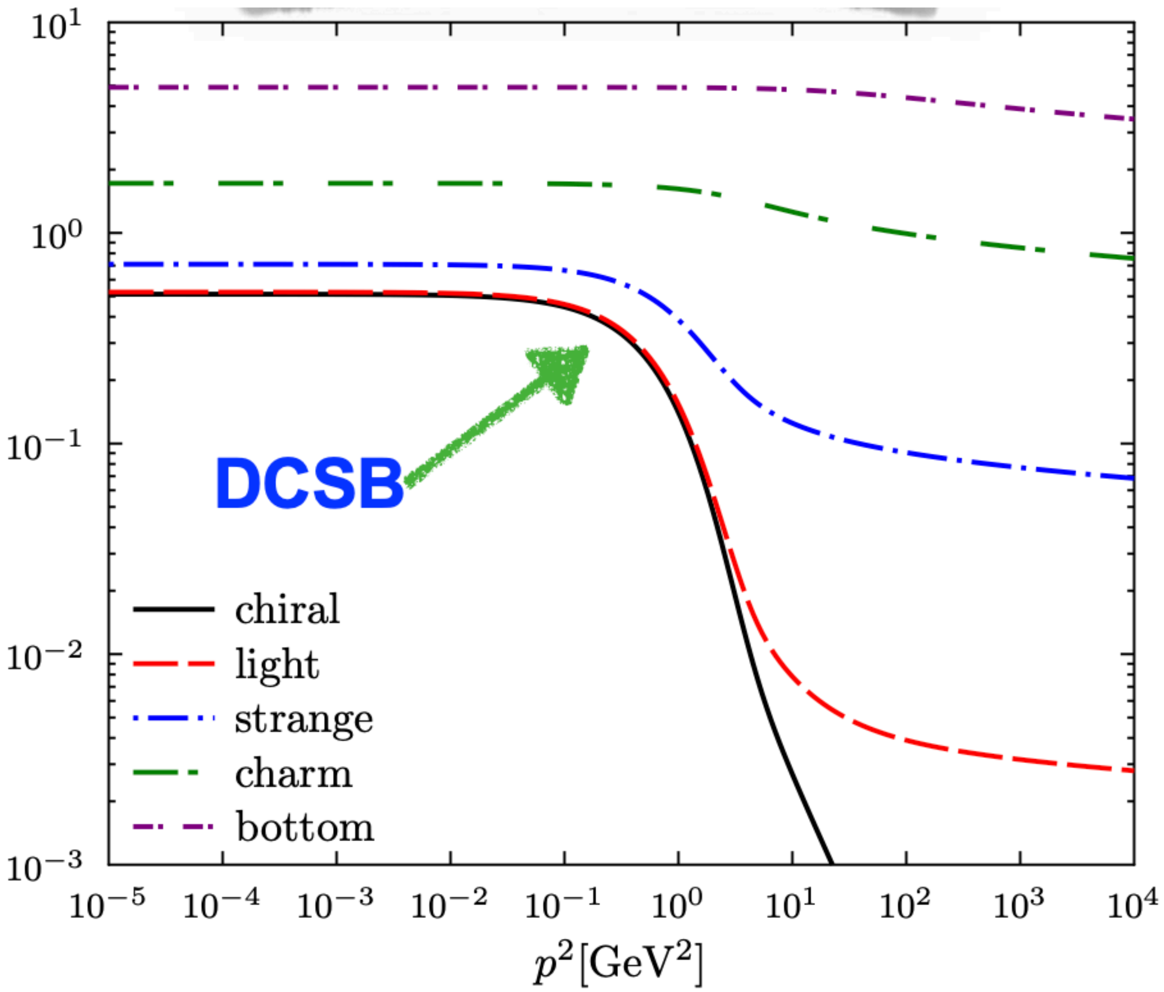
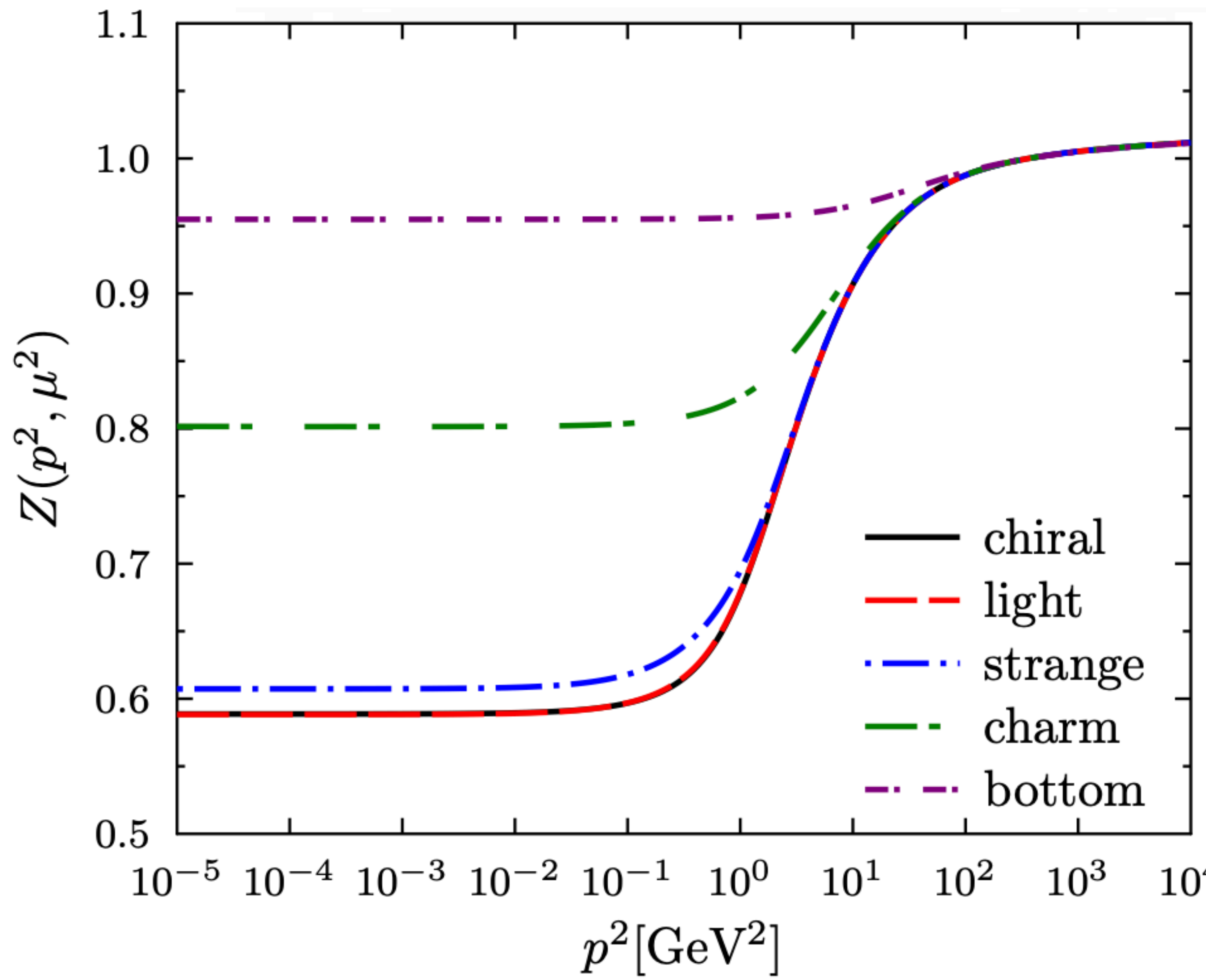
DSE solutions – flavor dependence





$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$

L. Albino, A. Bashir, B.E., L.X. Gutiérrez Guerrero, E. Rojas PRD 100 (2019)

L. Albino, A. Bashir, B.E., E. Rojas, F. E. Serna, R.C. Silveira, JHEP11 (2021)

J. R. Lessa, F. E. Serna, B.E., A. Bashir, O. Oliveira, PRD 107 (2023)

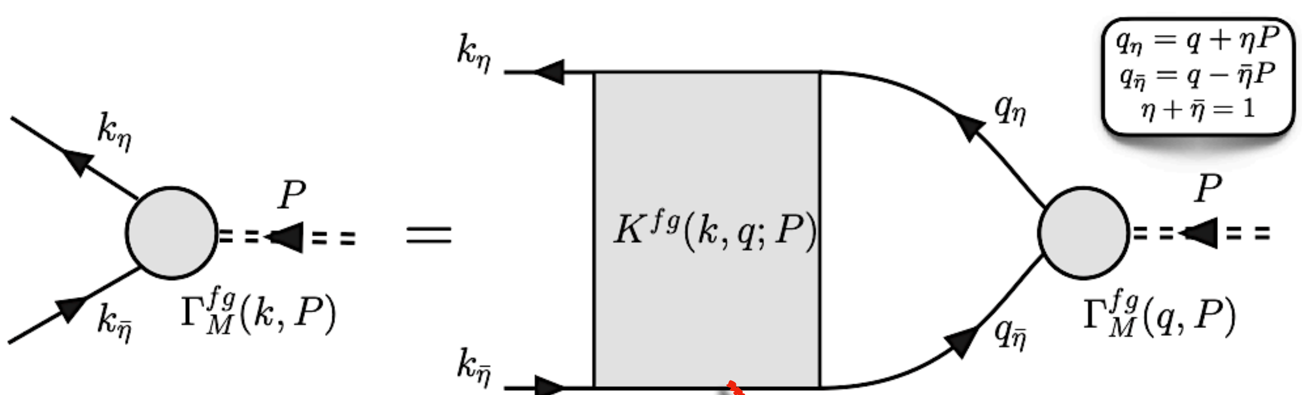
Bethe-Salpeter Equation for QCD Bound States

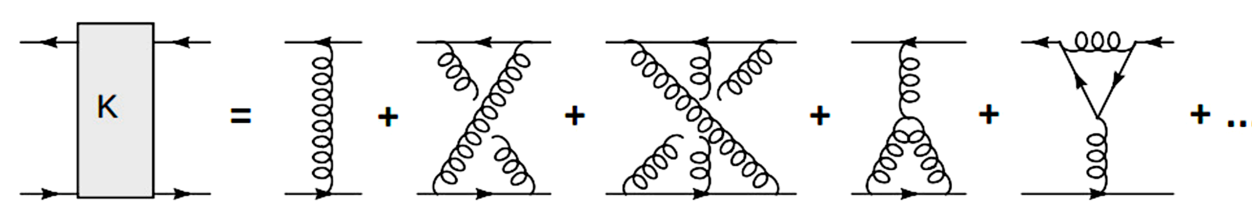
$$\Gamma_M^{fg}(k,P) = \int^{\Lambda} \frac{d^4 q}{(2\pi)^4} K_{fg}(k,q;P) S_f(q_\eta) \Gamma_M^{fg}(q,P) S_g(q_{\bar\eta})$$

$K_{fg}(q,k;P)$ = Quark-antiquark scattering kernel

$S_f(q_\eta)$ = Dressed quark propagator

$\Gamma_M^{fg}(k,P)$ = Meson's Bethe-Salpeter Amplitude (BSA)





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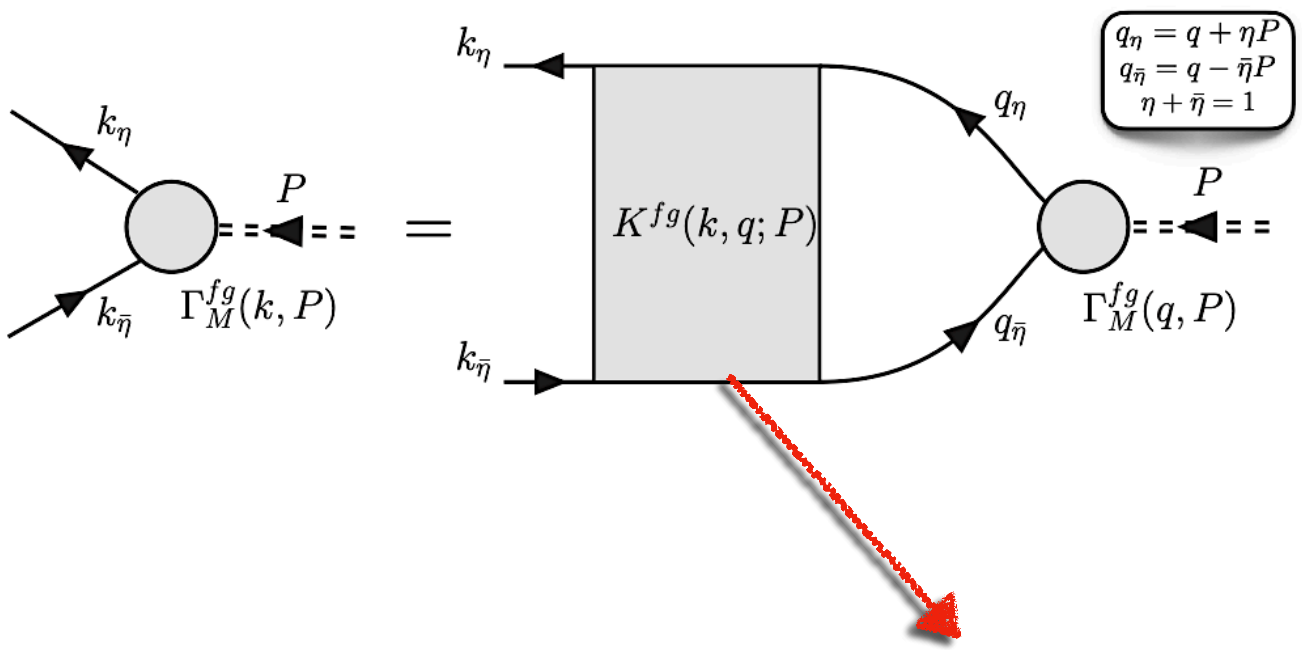
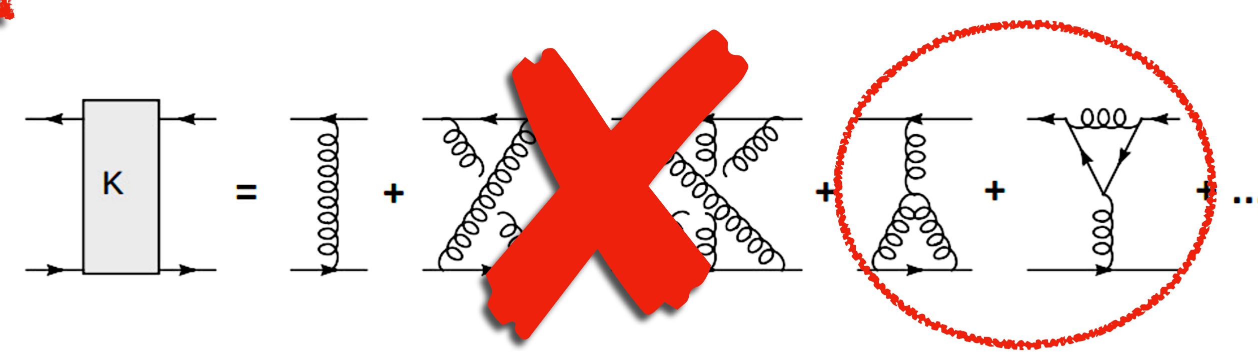


Treated with kernel beyond RL
introducing flavor dependence



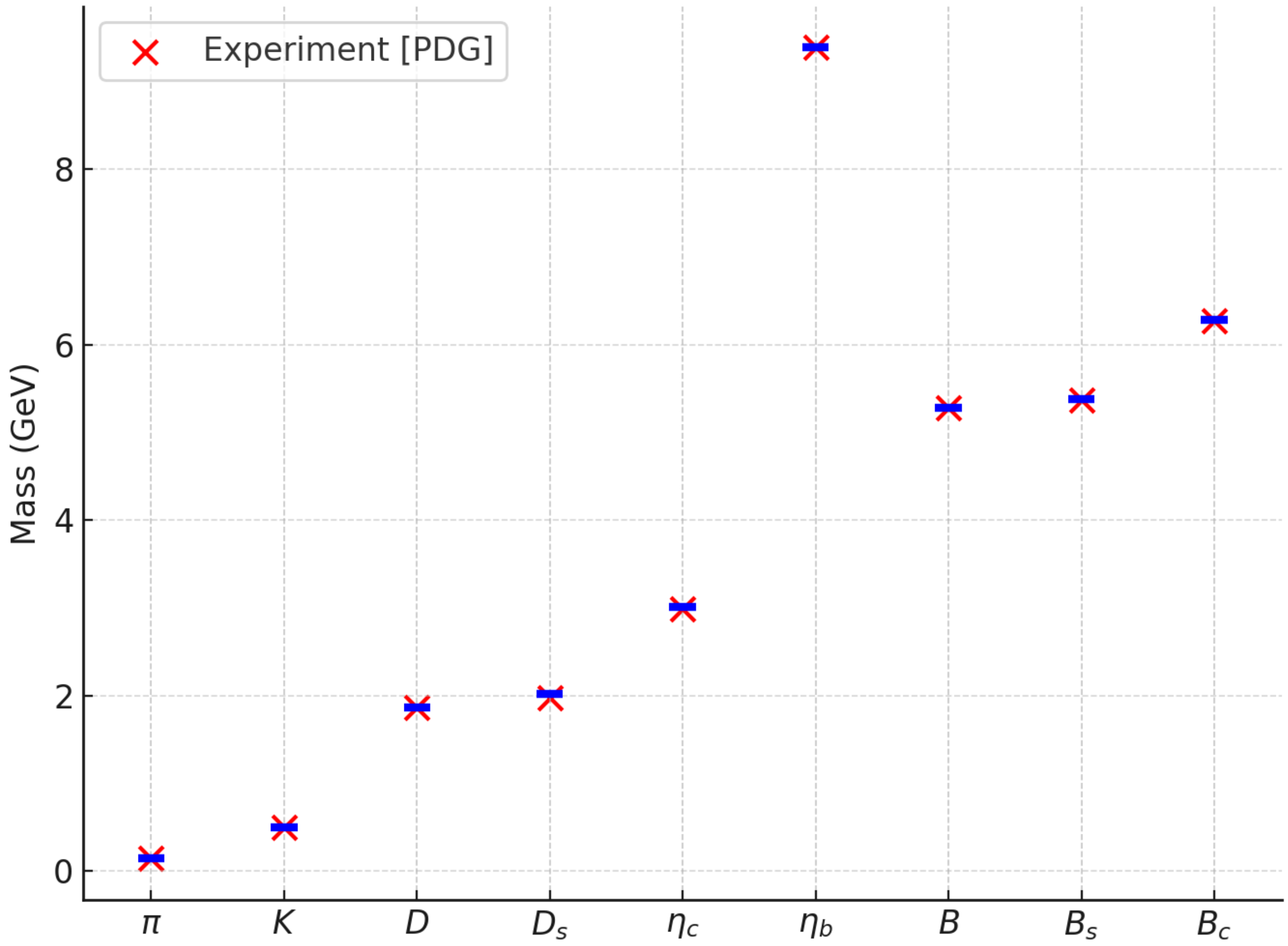
Bethe-Salpeter Amplitudes

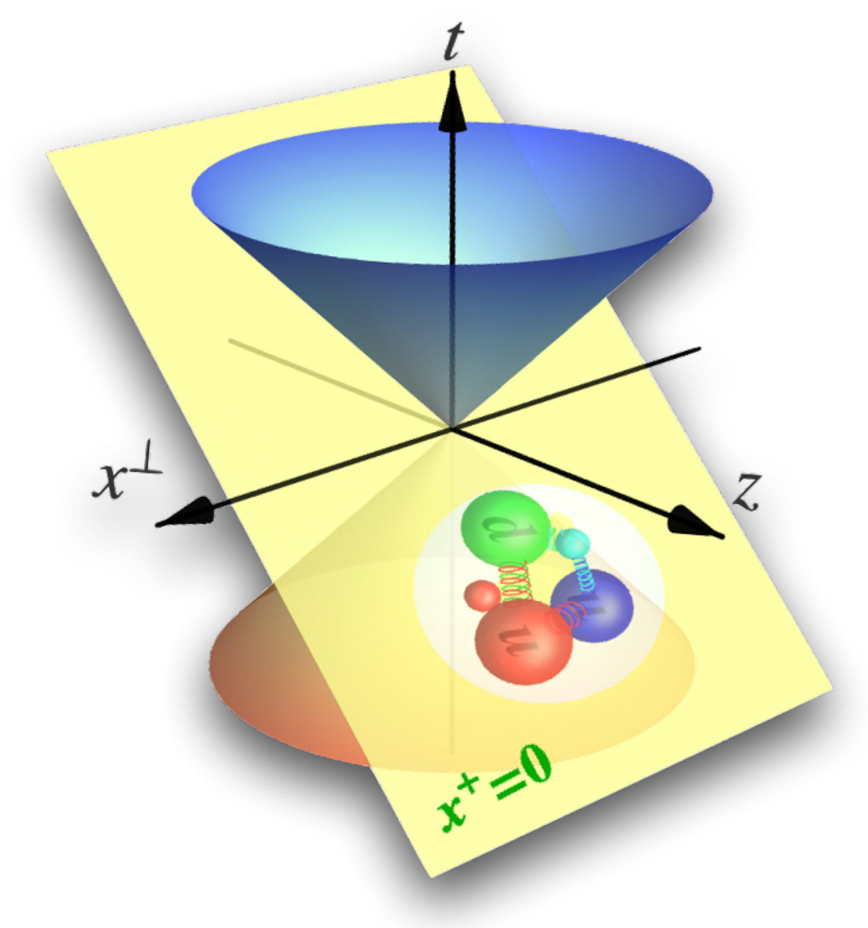
- The general form of $\Gamma_M(k;P)$ is given by

$$\Gamma(k;P) = \sum_{i=1}^{N} \mathcal{T}^i(k,P)\mathcal{F}_i(k^2, z_k, P^2), \;\; z_k = k.P/|k||P|,$$

- where $\mathcal{T}^i$(k,P) are Dirac's covariants;
- $\mathcal{F}_i(k^2, z_k, P^2)$ are Lorentz invariant amplitudes;
- N denotes the number of covariants which are different for different meson's channel.
- For the case of pseudoscalar mesons we have $N = 4$ and for vector mesons one has $N = 8$

Pseudoscalar Meson Spectrum





Light-Front Wave Functions

Light-Front Wave Functions

- The **LCWFs** are obtained from the **BSWF** via the light front projections:

$$\psi_0\left(x,\mathbf{k}_\perp^2\right) = \sqrt{3}i\int\frac{dk^+dk^-}{\pi}\delta\left(xP^+ - k^+\right)\mathrm{Tr}_D\left[\gamma^+\gamma_5\chi(k,P)\right]$$

$$\psi_1\left(x,\mathbf{k}_\perp^2\right) = -\frac{\sqrt{3}i}{\mathbf{k}_\perp^2}\int\frac{dk^+dk^-}{\pi}\delta\left(xP^+ - k^+\right)\mathrm{Tr}_D\left[i\sigma_{+i}k_T^i\gamma_5\chi(k,P)\right]$$

- With the **LCWF** one can readily derive two distributions:
  - The leading-twist **TMD** [B. Pasquini and P. Schweitzer (2014)]

$$f_M\left(x,\mathbf{k}_\perp^2,\mu\right) = \frac{1}{(2\pi)^3}\left|\psi_M^0\left(x,\mathbf{k}_\perp^2,\mu\right) + \mathbf{k}_\perp^2\psi_M^1\left(x,\mathbf{k}_\perp^2,\mu\right)\right|^2$$

  - The **PDF**

$$q_M(x,\mu) = \int d\mathbf{k}_\perp^2 f_M(x,\mathbf{k}_\perp^2,\mu)$$

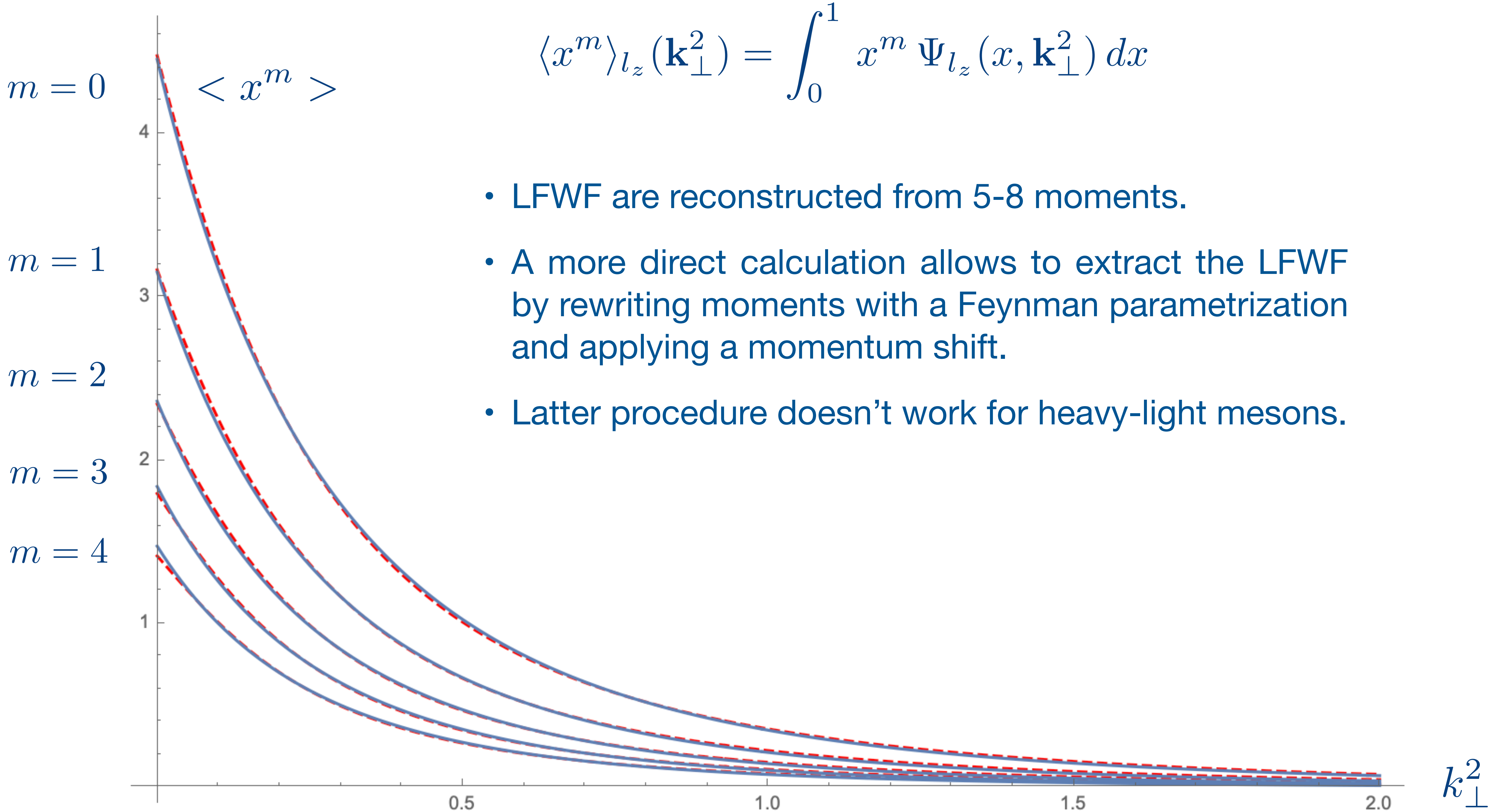
Normalization: $$\int_0^1 dx\, q_M\left(x,\mu\right) = 1.$$

Light-Front Wave Functions

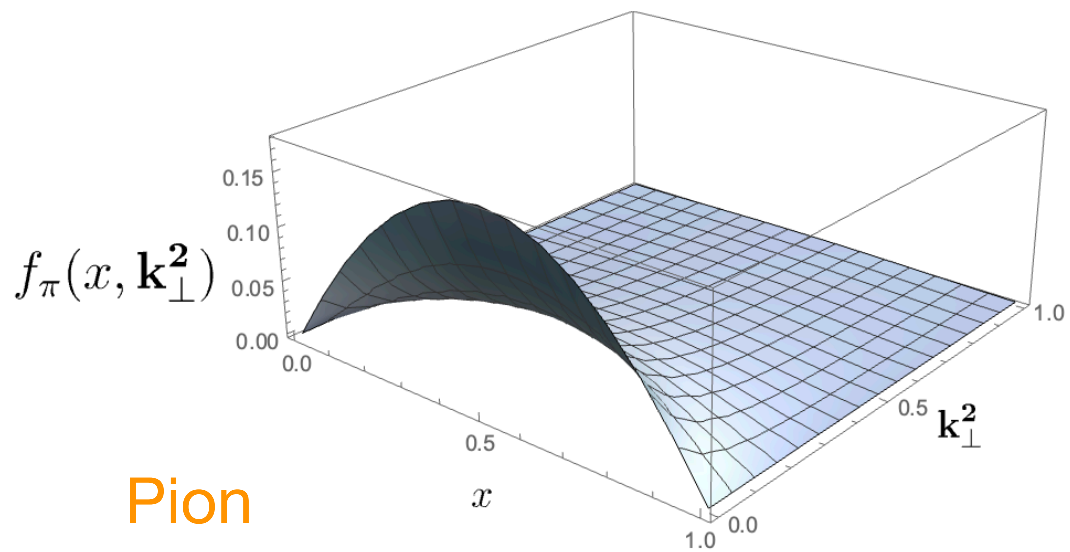
We calculate transverse momentum dependent moments:

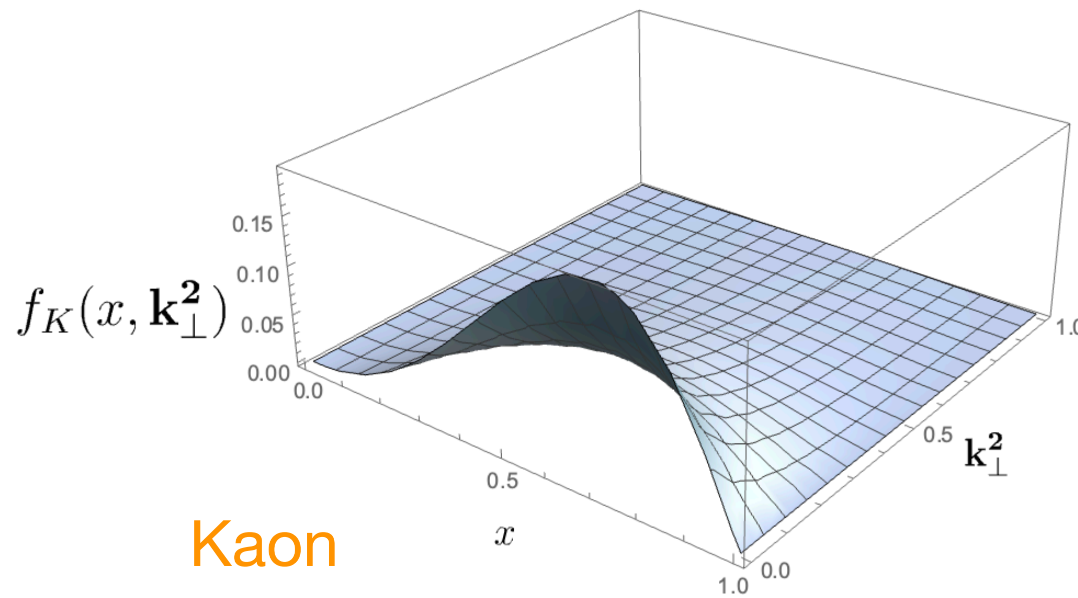
$$\langle x^m \rangle_{l_z}(\mathbf{k}_\perp^2) = \int_0^1 x^m \, \Psi_{l_z}(x, \mathbf{k}_\perp^2) \, dx$$

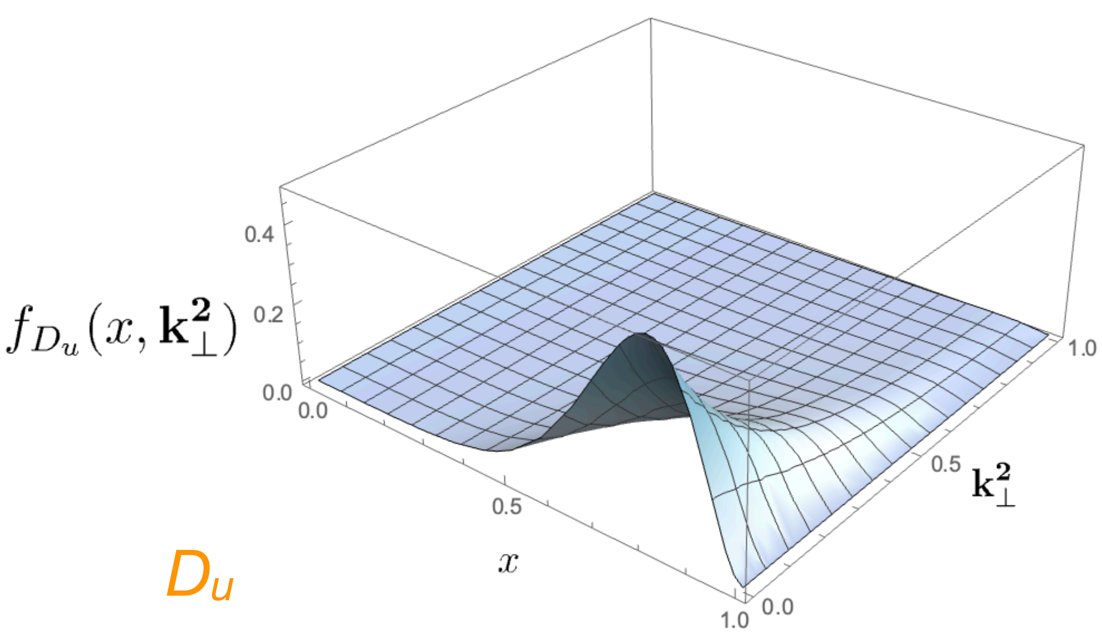
- LFWF are reconstructed from 5-8 moments.
- A more direct calculation allows to extract the LFWF by rewriting moments with a Feynman parametrization and applying a momentum shift.
- Latter procedure doesn't work for heavy-light mesons.

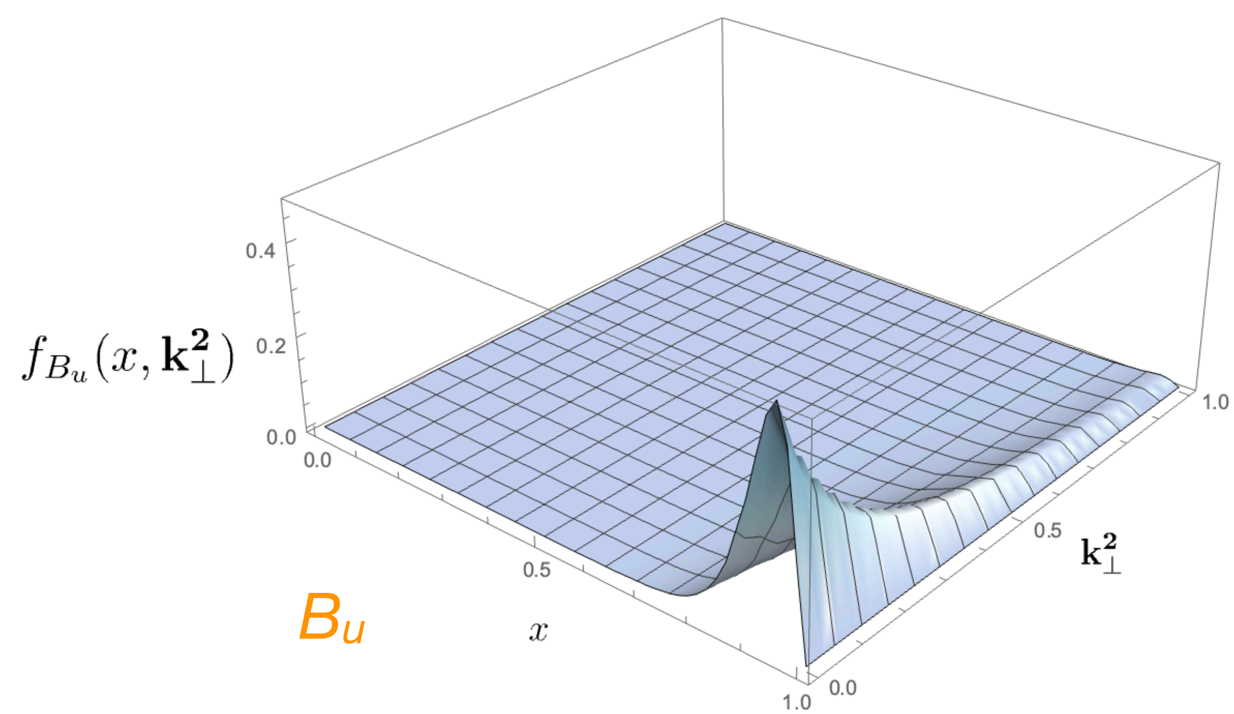


Transverse Momentum Distributions

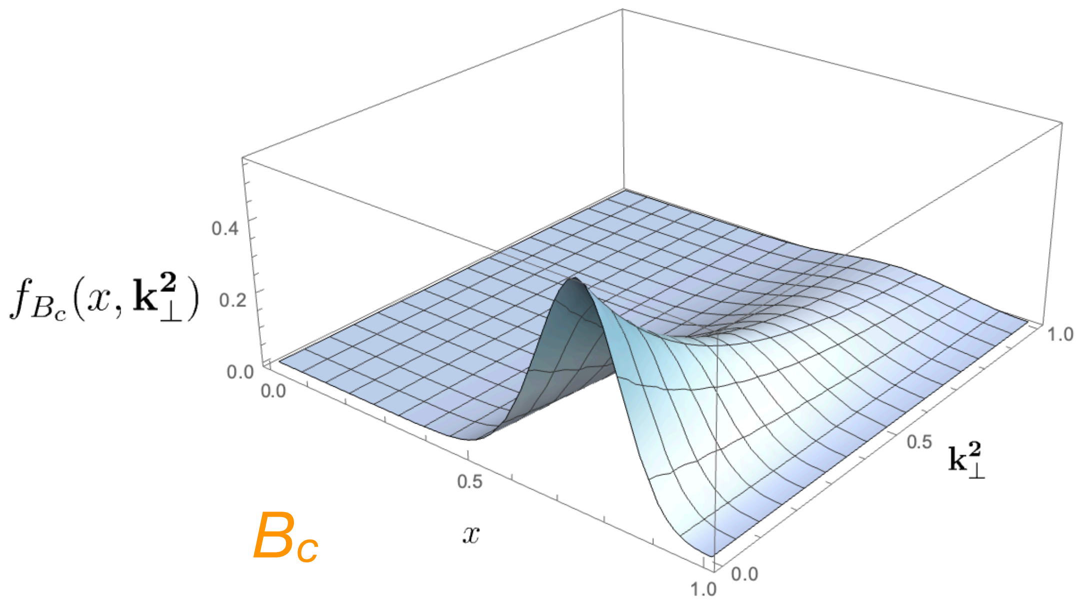




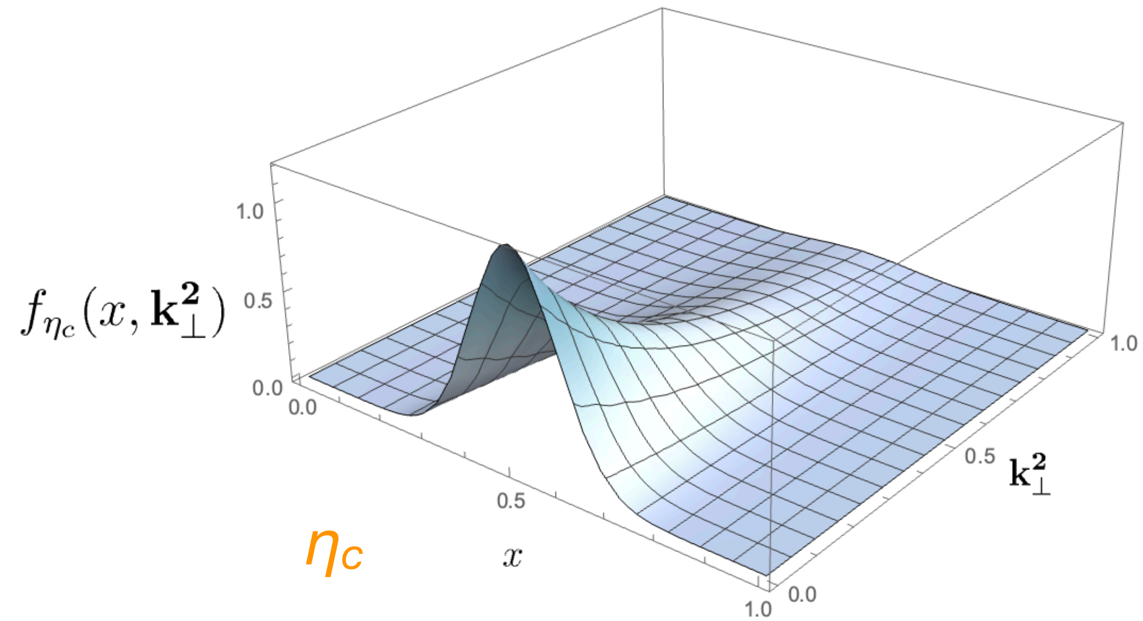




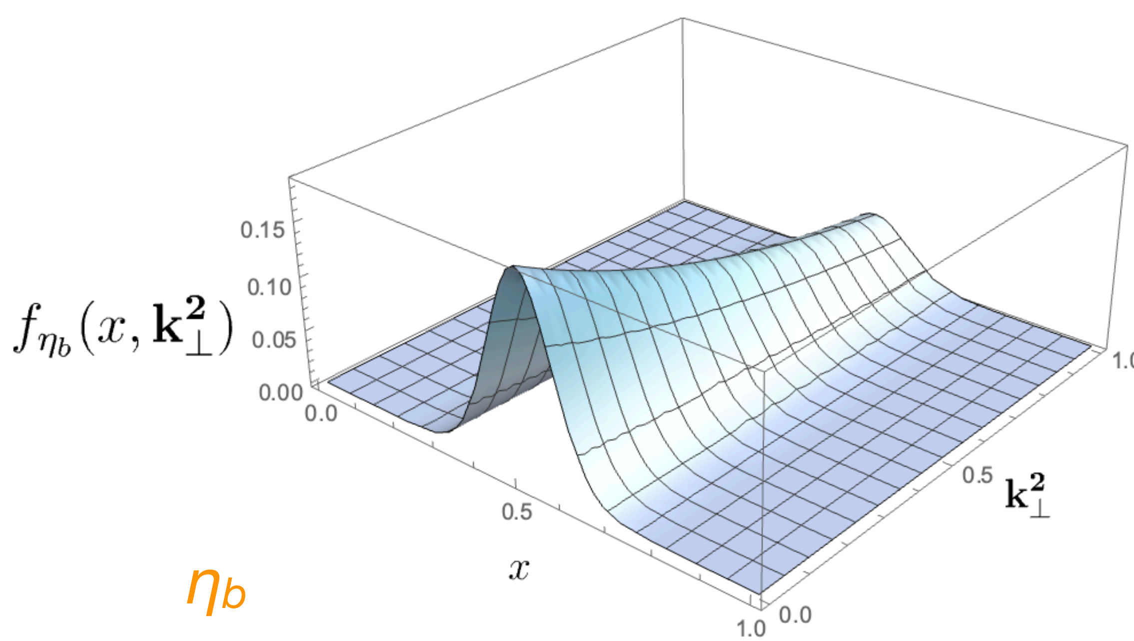
Transverse Momentum Distributions



B_c

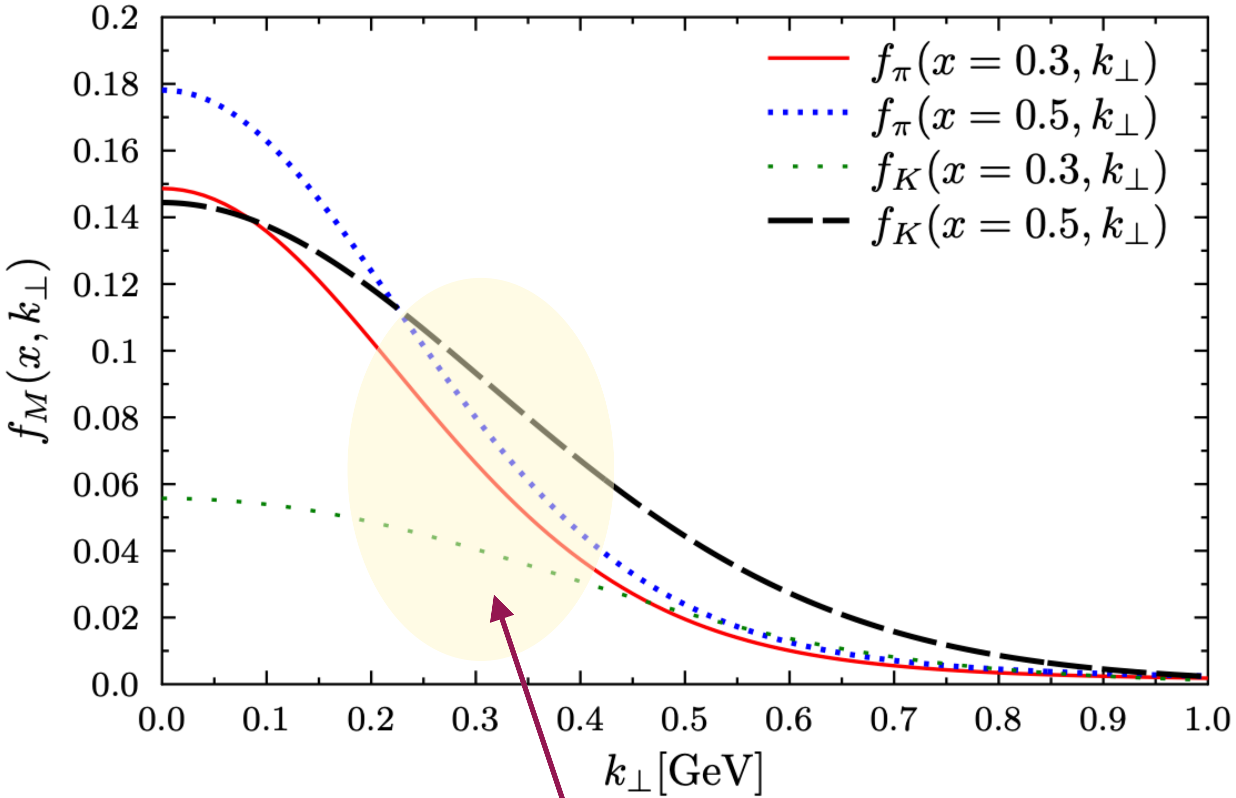


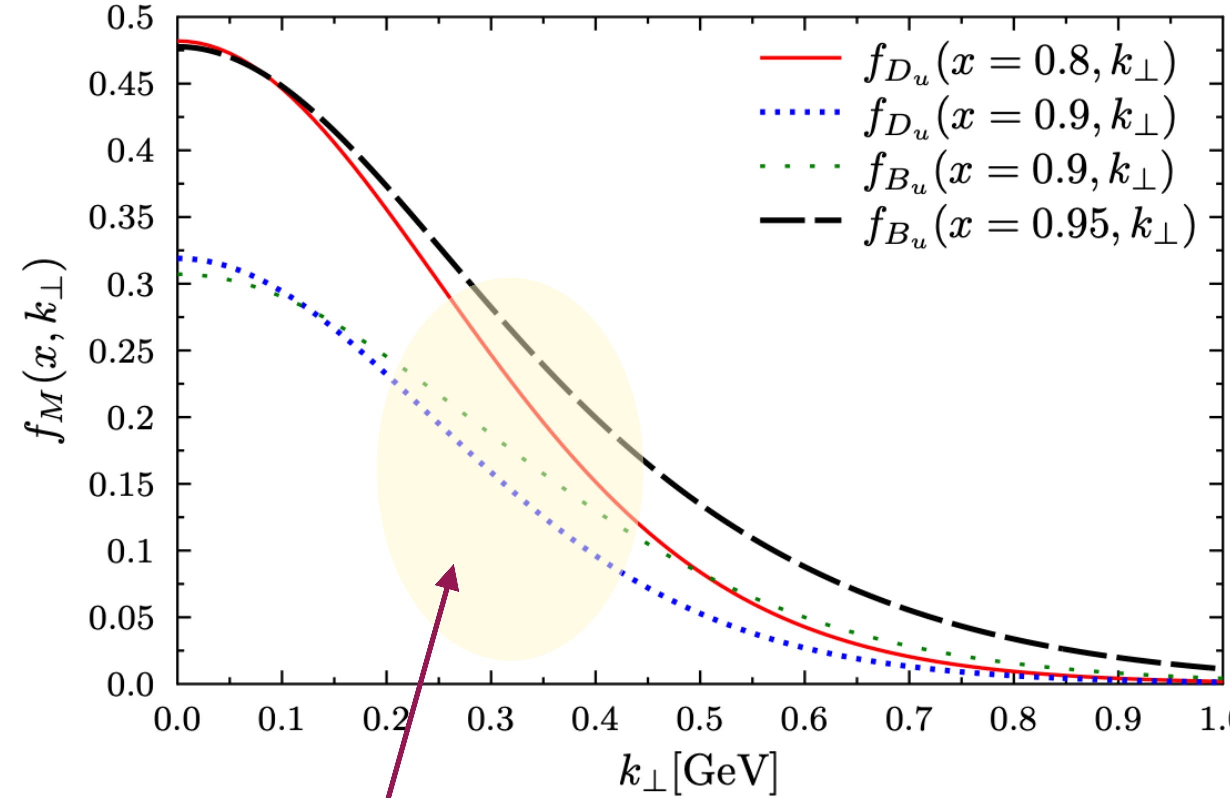
η_c



η_b

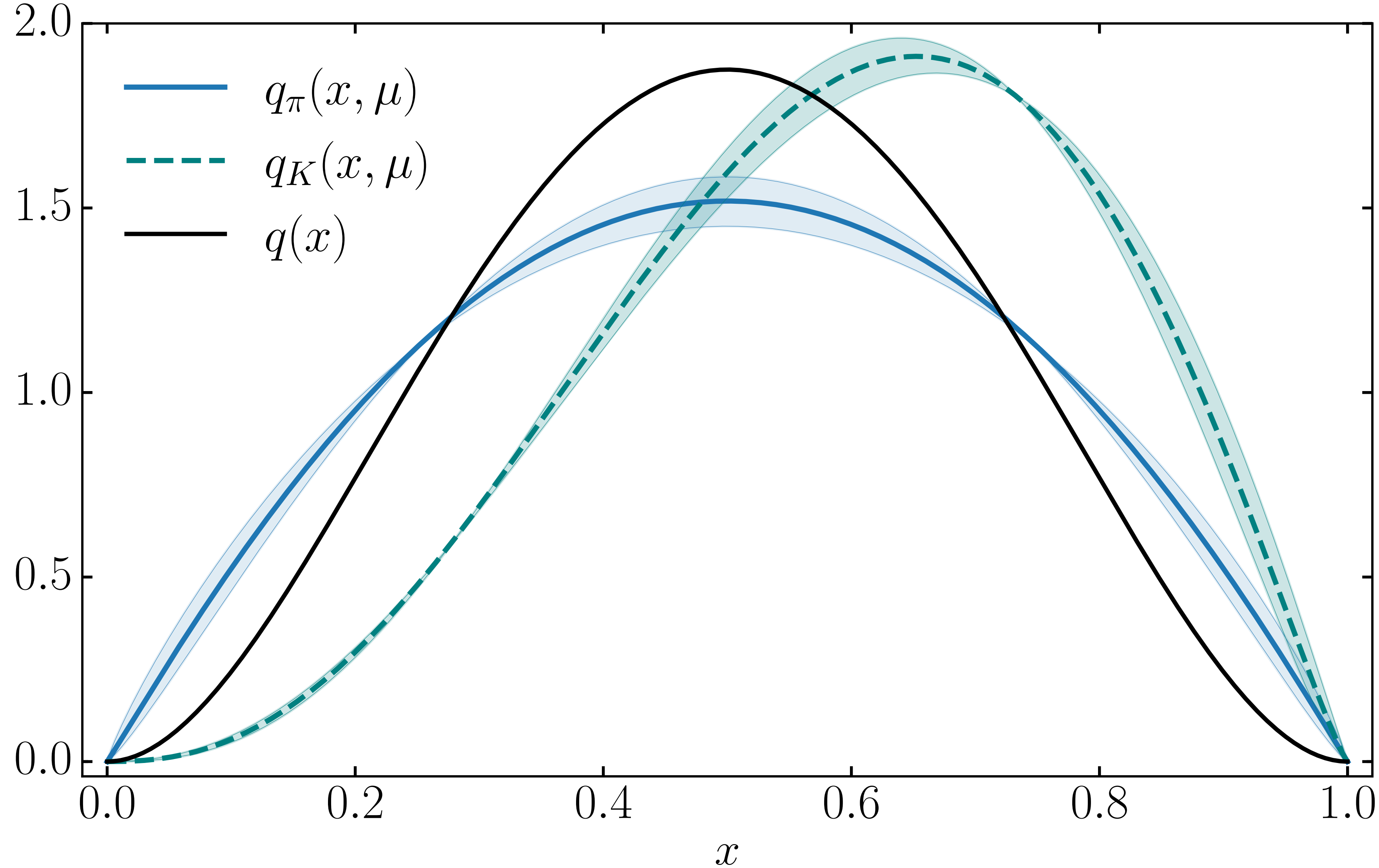
Transverse Momentum Distributions





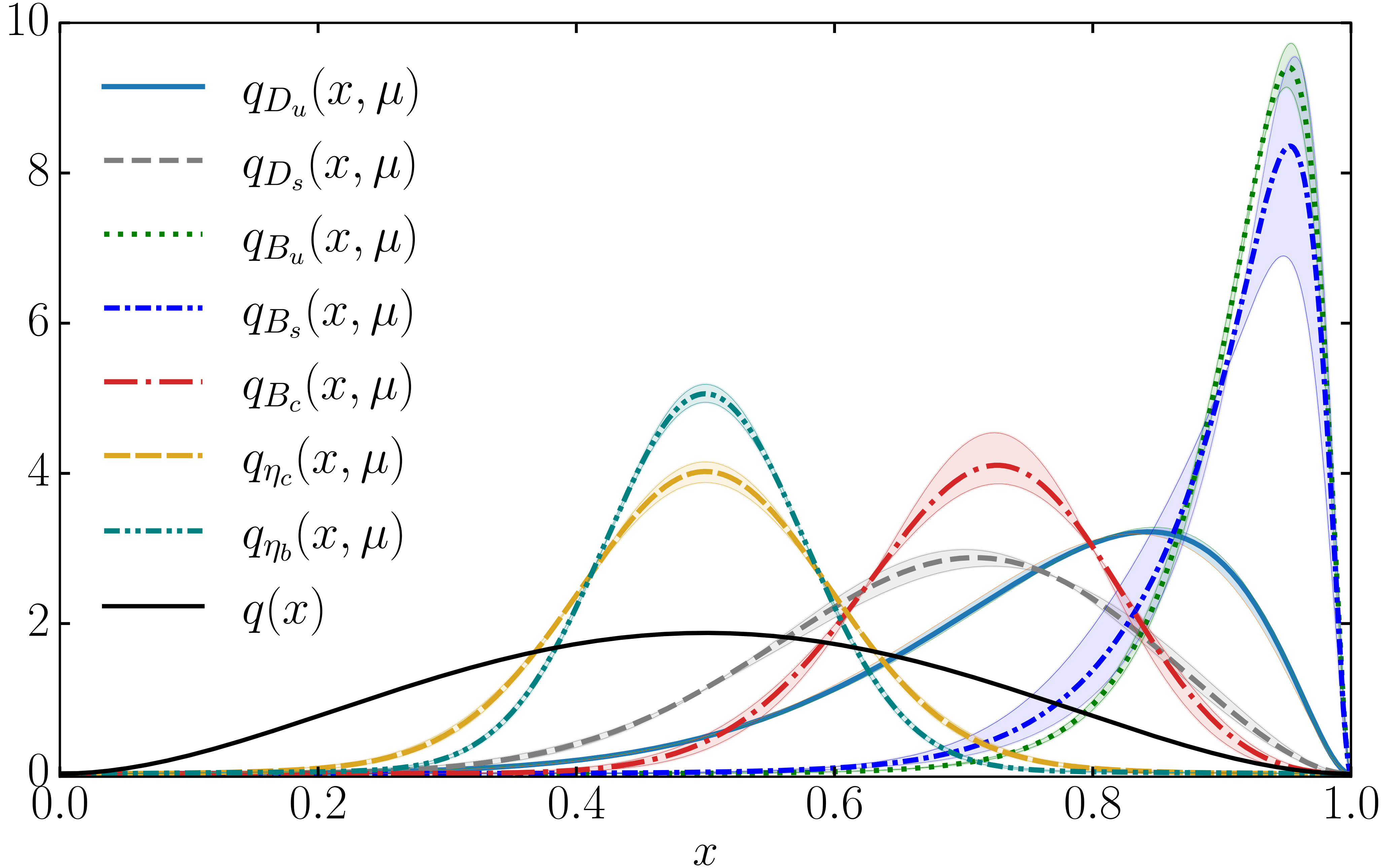
An inflection point from a concave to a convex function generally occurs at about $k_\perp \approx 0.3$ GeV

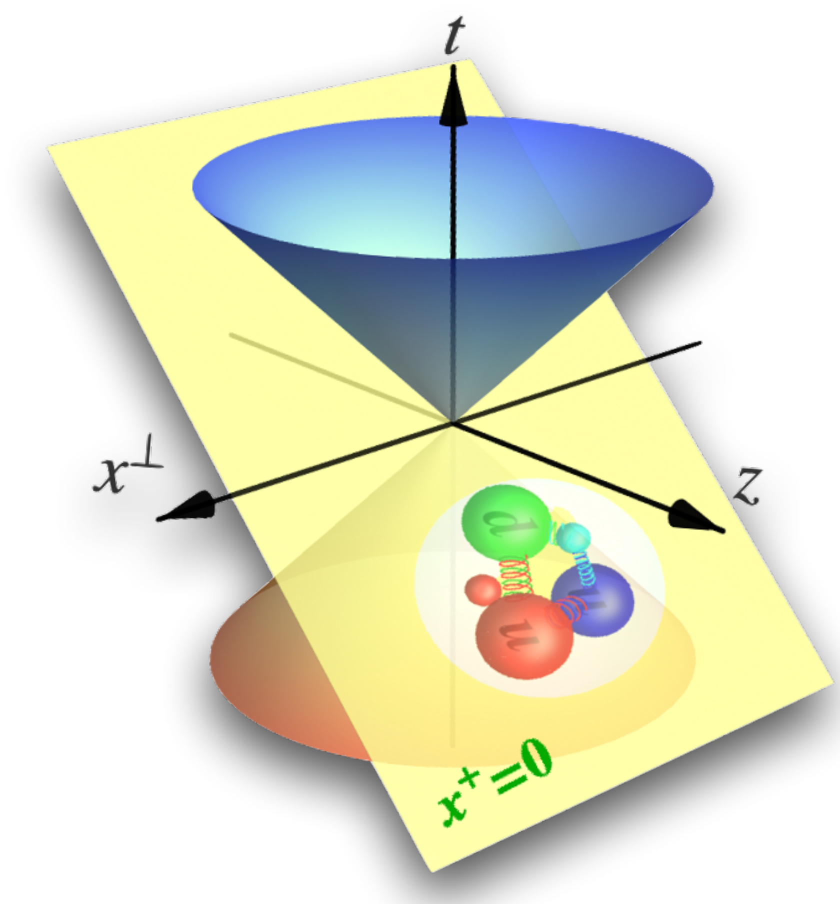
Parton Distribution Functions



The PDFs can be parametrized by: $q(x;\mu_0) = 30[x(1-x)]^2 \left[1 + \sum_{j=1}^{j_m} a_j C_j^{5/2}(2x-1)\right]$

Parton Distribution Functions





Vector Meson Distribution Amplitudes

Vector Meson Distribution Amplitudes

- Vector mesons are described by two distribution amplitudes $\phi_V^{\|}(x;\mu)$ and $\phi_V^{\perp}(x;\mu)$
- Twist-2 distribution amplitudes are obtained with the projections:

$$(n\cdot P) f_V \phi_V^{\|}(x;\mu) = \frac{m_V N_c \mathcal{Z}_2}{\sqrt{2}} \mathrm{Tr}_D \int^{\Lambda} \frac{d^4k}{(2\pi)^4} \delta(n\cdot k_\eta - x\, n\cdot P) \gamma\cdot n\, n_\nu \chi_{V\nu}^{fg}(k;P),$$

$$f_V^{\perp} \phi_V^{\perp}(x;\mu) = -\frac{N_c \mathcal{Z}_T}{2\sqrt{2}} \mathrm{Tr}_D \int^{\Lambda} \frac{d^4k}{(2\pi)^4} \delta(n\cdot k_\eta - x\, n\cdot P) n_\mu \sigma_{\mu\rho} \mathcal{O}_{\rho\nu}^{\perp} \chi_{V\nu}^{fg}(k;P),$$

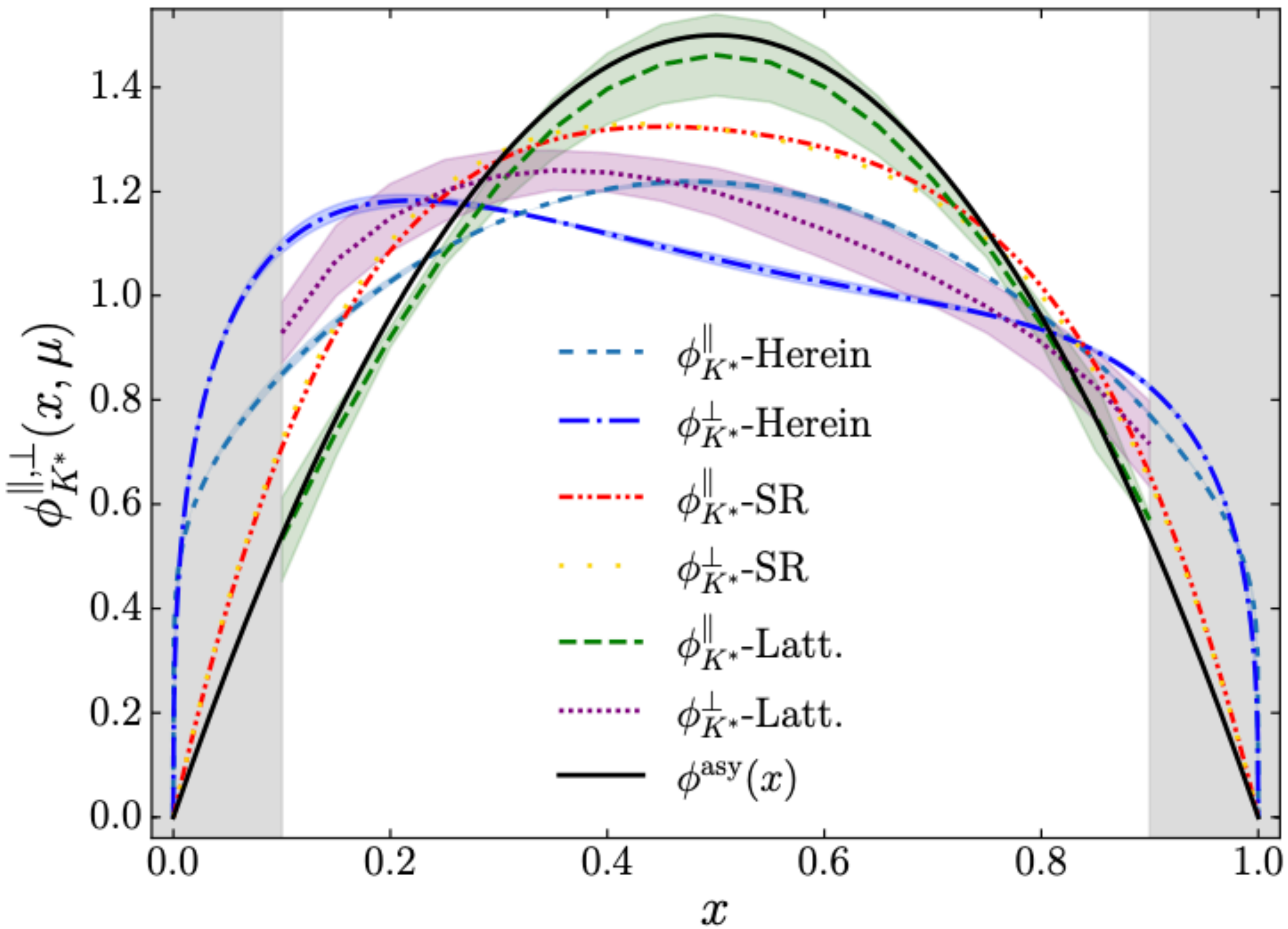
with

$$\chi_{V\nu}^{fg}(k;P) = S_f(k_\eta) \Gamma_{V\nu}^{fg}(k;P) S_g(k_{\bar{\eta}}), \quad \text{and} \quad \mathcal{O}_{\rho\nu}^{\perp} = \delta_{\rho\nu} + n_\rho \bar{n}_\nu + \bar{n}_\rho n_\nu,$$

$n^2 = 0$; $P^2 = -m_V^2$ and $n\cdot P = -m_V$; $\bar{n}$ is a conjugate light-like four-vector, $\bar{n}^2 = 0$, $n\cdot \bar{n} = -1$, $\bar{n}\cdot P = -m_V/2$

Vector Meson Distribution Amplitudes

*K** meson longitudinal and transverse LCDA



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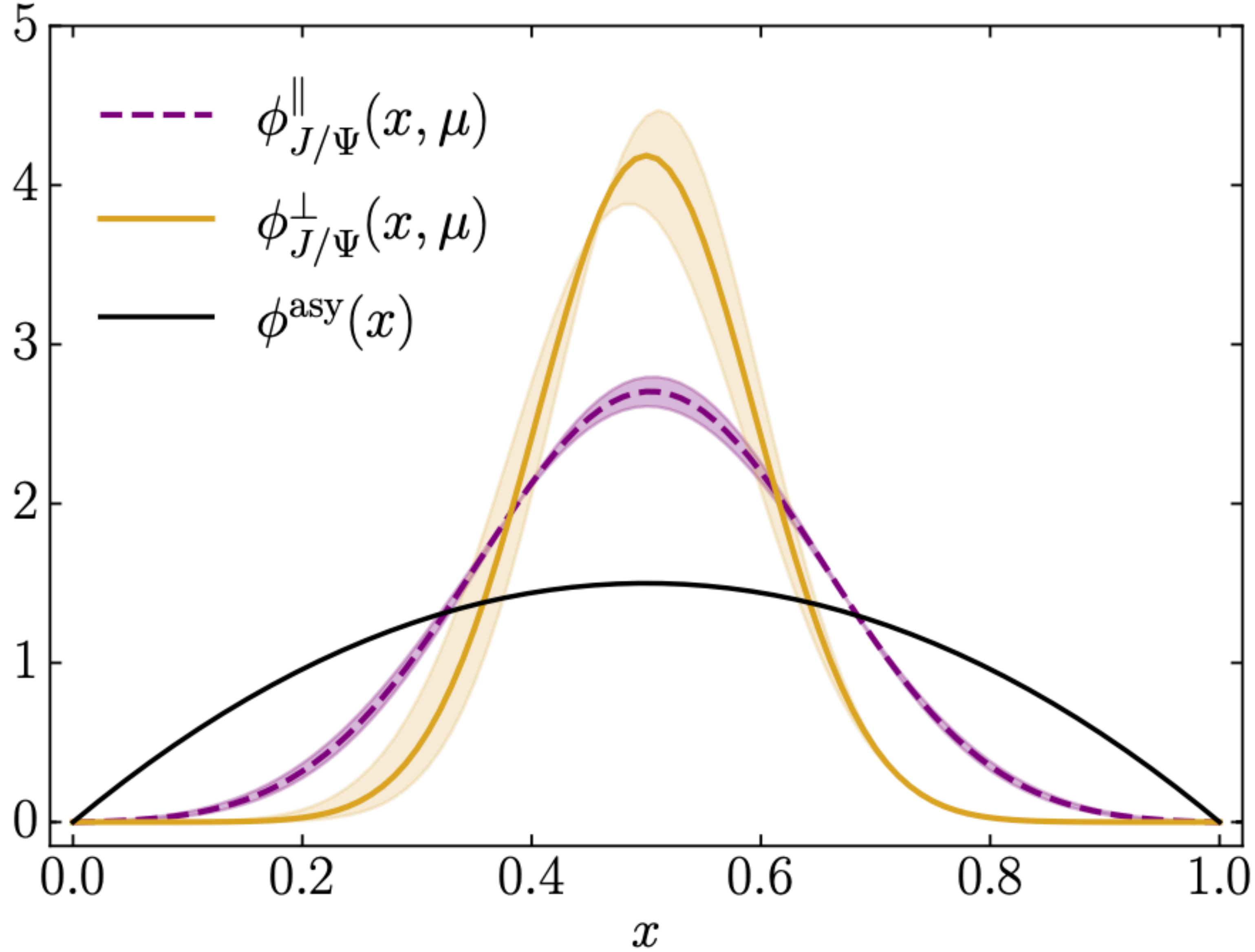
Vector Meson Distribution Amplitudes

J/ψ meson longitudinal and transverse LCDA



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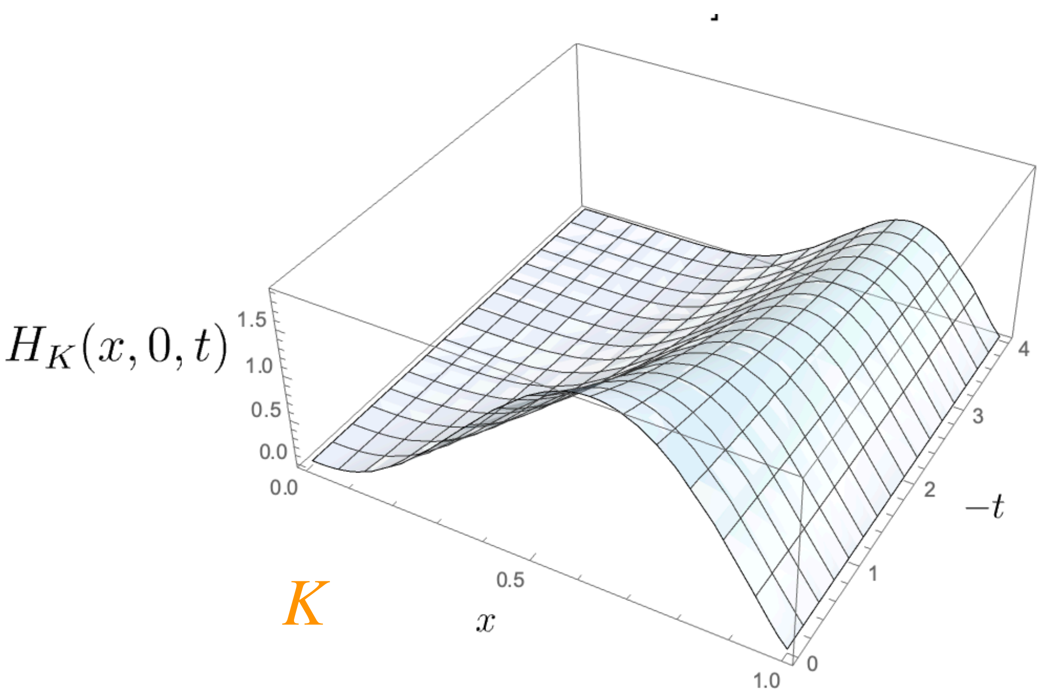
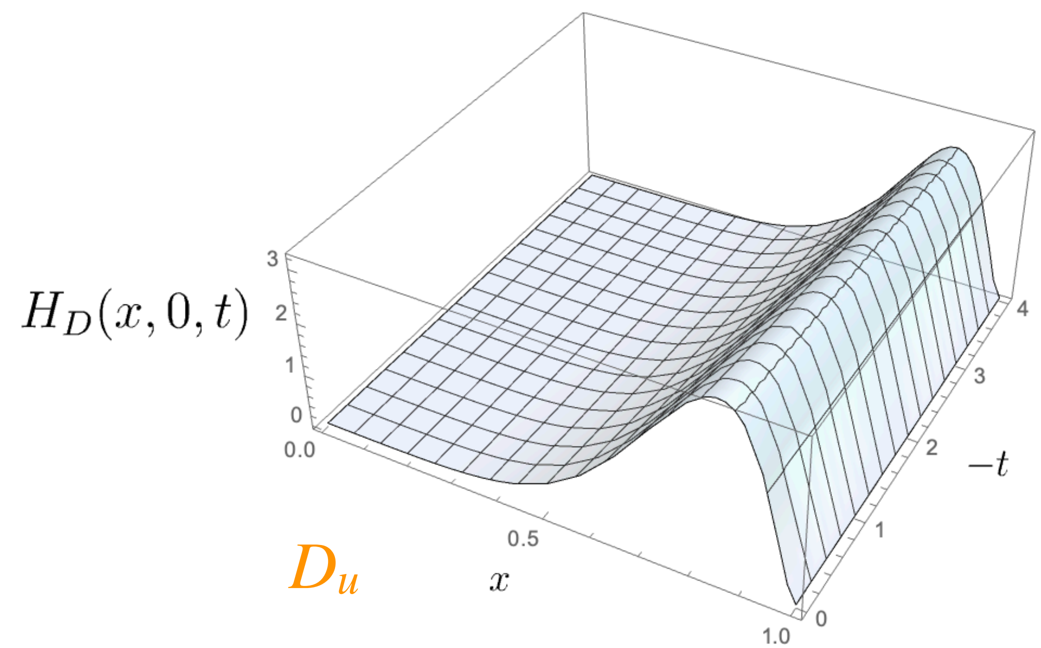
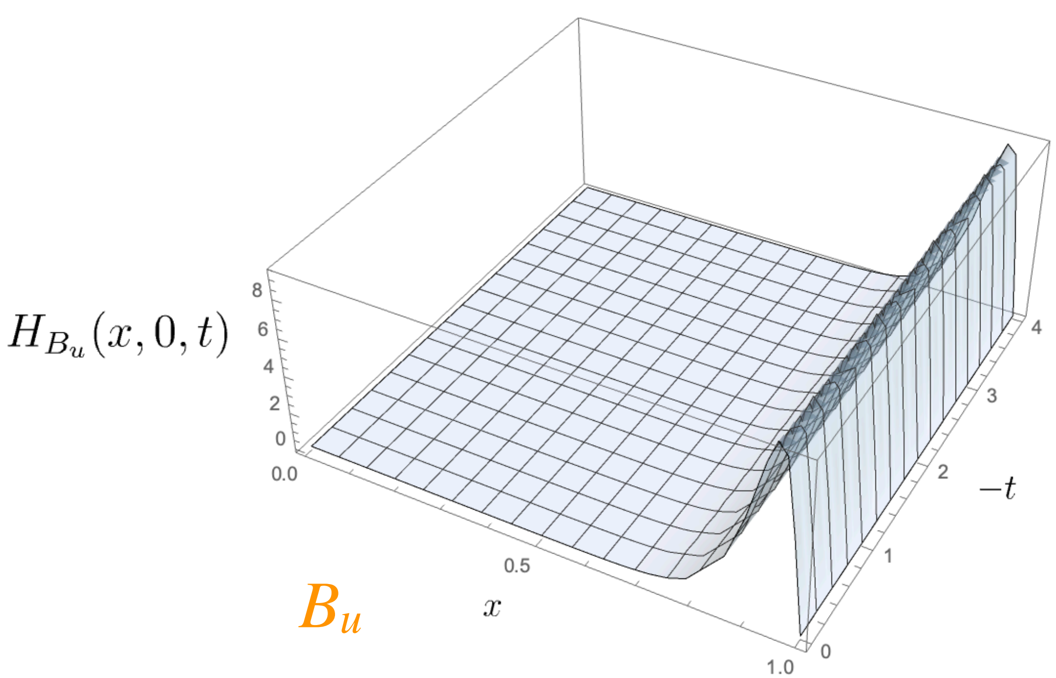
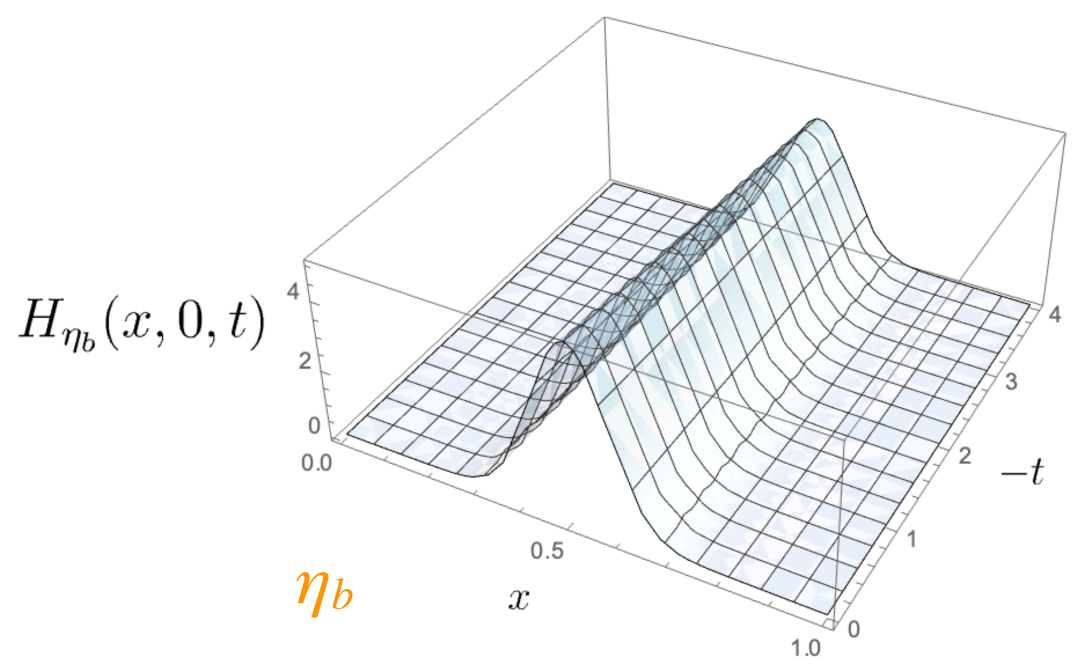
Generalized Parton Distributions ($\xi = 0$)

Obtained from overlap representation in DGLAP region (fulfills polynomiality and positivity).









Future Prospects

- Explore more the case of vector-mesons.
- Improve truncation schemes of gap and bound-state equations.
- Similar approach was also applied to the nucleons.
- Towards a fully interacting 3-quark Faddeev amplitude and from there to TMDs, GPDs …?

Back-up slides

Pseudoscalar meson spectrum

	M_P	$M_P^{\exp}$	ϵ_{M_P} [%]	f_P	$f_P^{\exp/\mathrm{lQCD}}$	ϵ_{f_P} [%]
$\pi(u\bar{d})$	0.140	0.138	1.45	$0.094^{+0.001}_{-0.001}$	0.092(1)	2.17
$K(u\bar{s})$	0.494	0.494	0	$0.110^{+0.001}_{-0.001}$	0.110(2)	0
$D(c\bar{d})$	$1.867^{+0.008}_{-0.004}$	1.864	0.11	$0.144^{+0.001}_{-0.001}$	0.150 (0.5)	4.00
$D_s(c\bar{s})$	$2.015^{+0.021}_{-0.018}$	1.968	2.39	$0.179^{+0.004}_{-0.003}$	0.177(0.4)	1.13
$\eta_c(c\bar{c})$	$3.012^{+0.003}_{-0.039}$	2.984	0.94	$0.270^{+0.002}_{-0.005}$	0.279(17)	3.23
$\eta_b(b\bar{b})$	$9.392^{+0.005}_{-0.004}$	9.398	0.06	$0.491^{+0.009}_{-0.009}$	0.472(4)	4.03
$B(u\bar{b})$	$5.277^{+0.008}_{-0.005}$	5.279	0.04	$0.132^{+0.004}_{-0.002}$	0.134(1)	4.35
$B_s(s\bar{b})$	$5.383^{+0.037}_{-0.039}$	5.367	0.30	$0.128^{+0.002}_{-0.003}$	0.162(1)	20.5
$B_c(c\bar{b})$	$6.282^{+0.020}_{-0.024}$	6.274	0.13	$0.280^{+0.005}_{-0.002}$	0.302(2)	10.17

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Vector meson spectrum

	M_V	M_V^{exp}	ϵ_{M_V} [%]	f_V	$f_V^{\mathrm{exp/lQCD}}$	ϵ_{f_V} [%]
$\rho(u\bar{u})$	0.730	0.775	5.81	0.145	0.153(1)	5.23
$\phi(s\bar{s})$	1.070	1.019	5.20	0.187	0.168(1)	11.31
$K^*(u\bar{s})$	0.942	0.896	5.13	0.177	0.159(1)	11.32
$D^*(c\bar{d})$	2.021	2.009	0.60	0.165	0.158(6)	4.43
$D_s^*(c\bar{s})$	2.169	2.112	2.70	0.205	0.190(5)	7.90
$J/\psi(c\bar{c})$	3.124	3.097	0.87	0.277	0.294(5)	5.78
$\Upsilon(b\bar{b})$	9.411	9.460	0.52	0.594	0.505(4)	17.62

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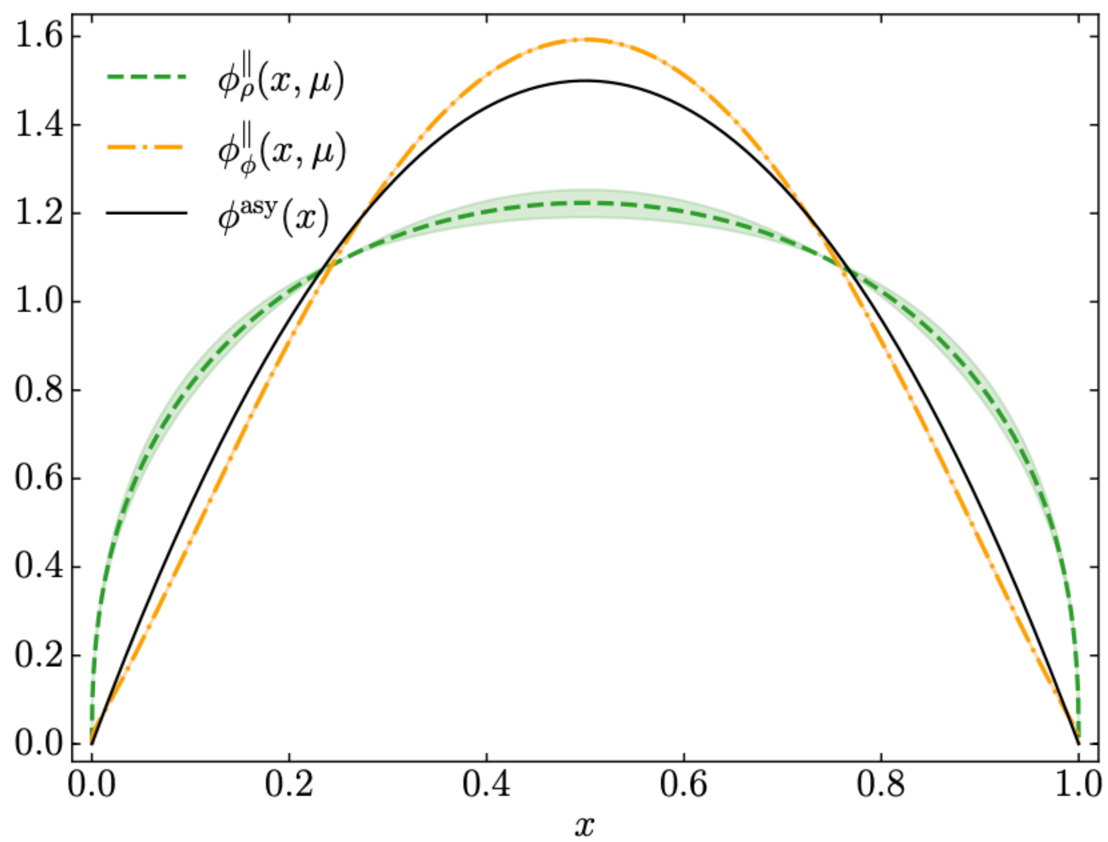
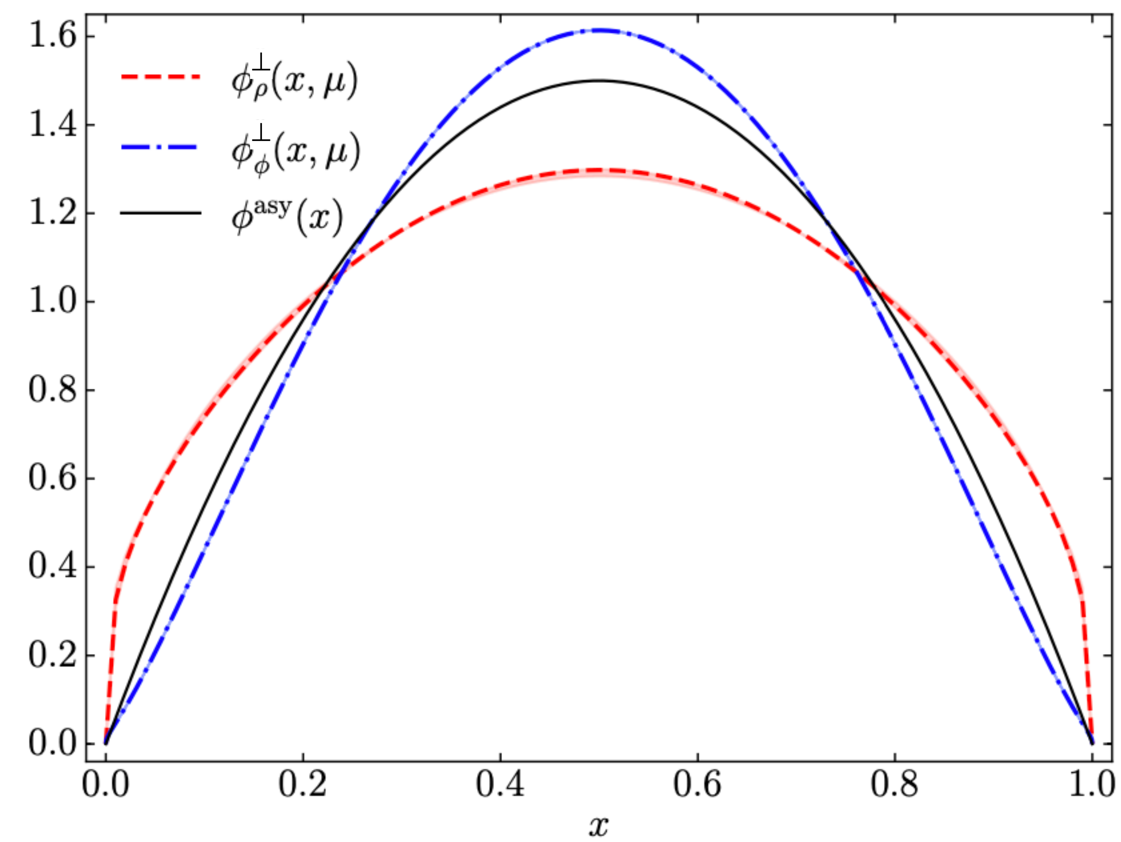
Vector Meson LCDA

ρ and **ϕ** mesons: longitudinal LCDA



ρ and **ϕ** mesons: transverse LCDA



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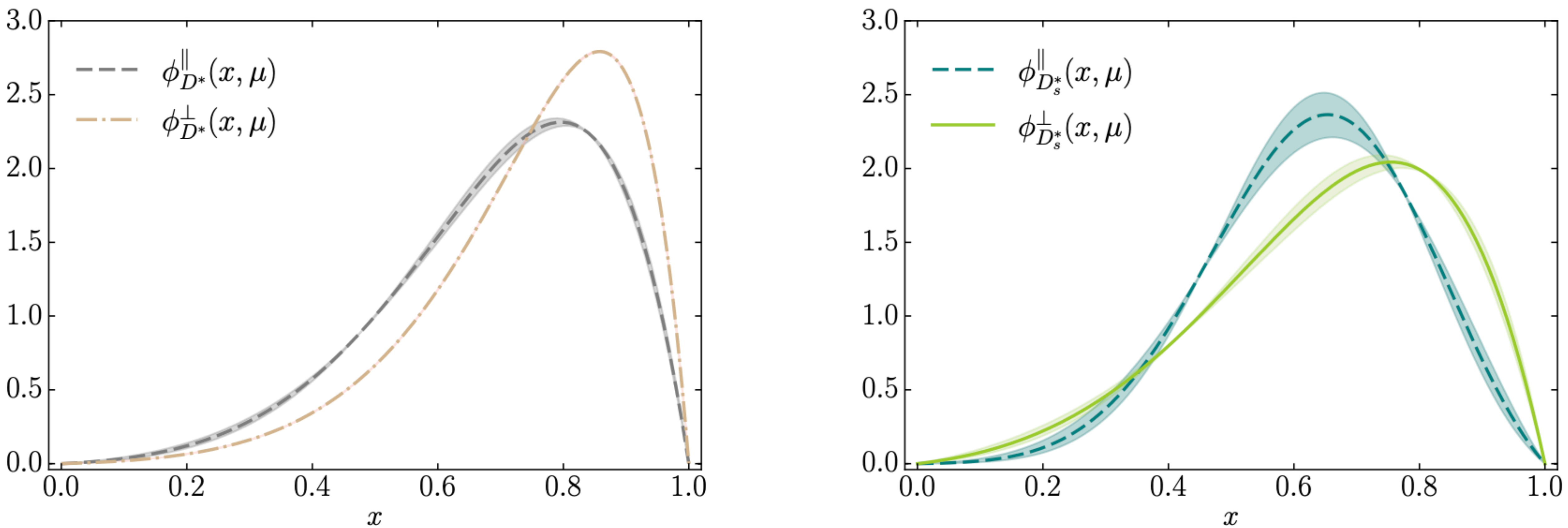
We find: $\phi_\phi^\parallel(x,\mu) \approx \phi_\phi^\perp(x,\mu)$

Light-Cone Distribution Amplitudes

D and D_s** mesons longitudinal and transverse LCDA



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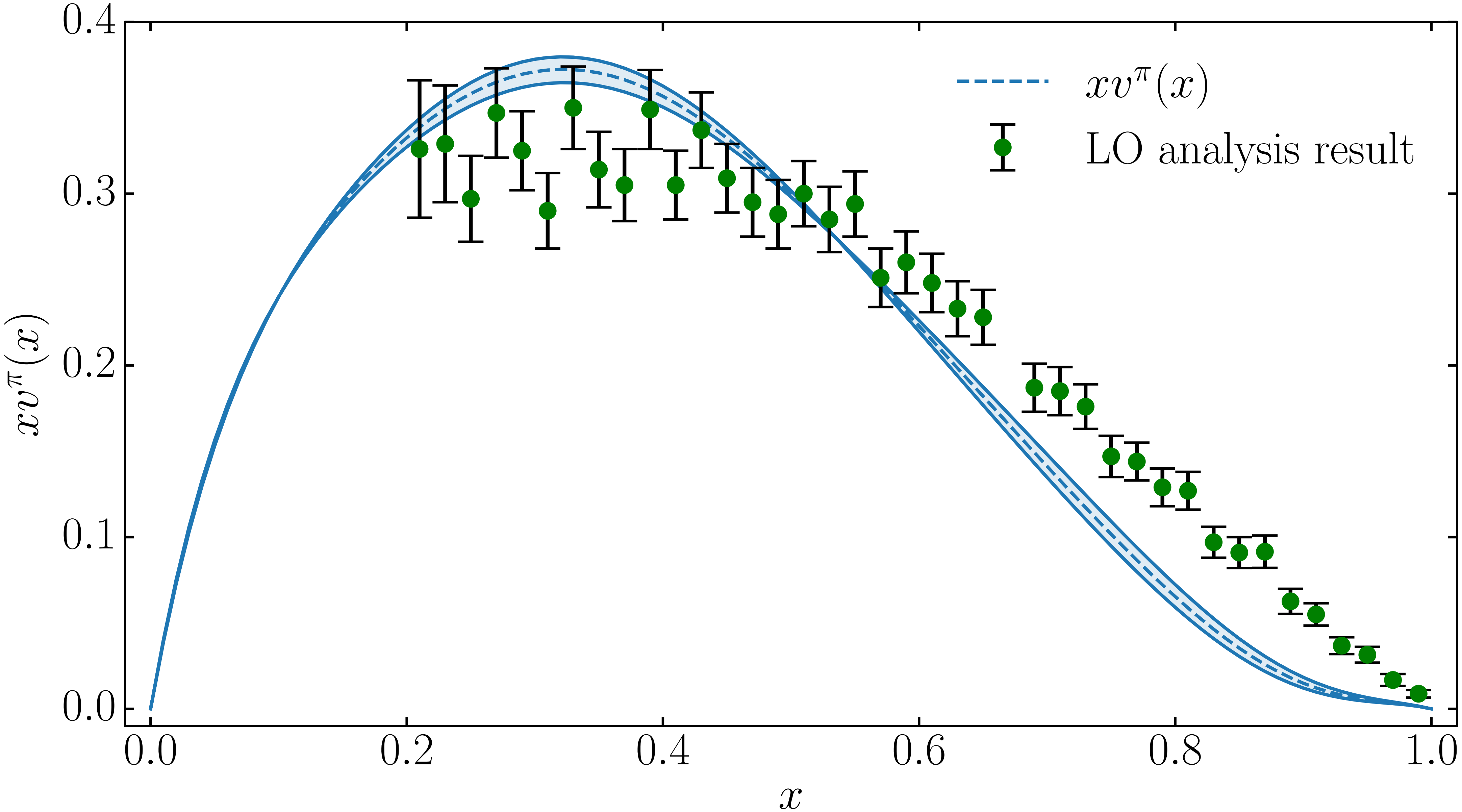
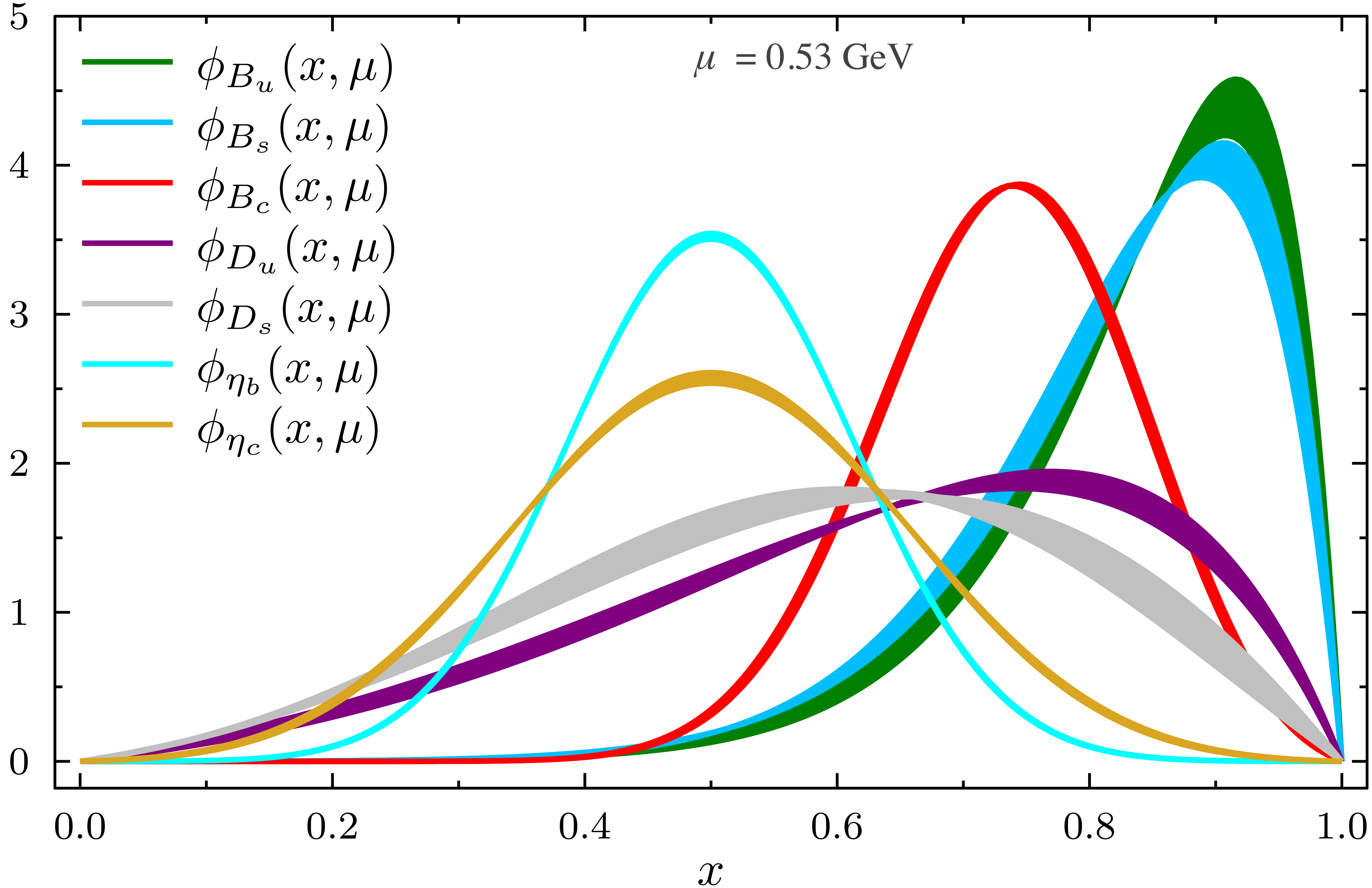
PDF DGLAP evolution to 2 GeV



Light-Cone Distribution Amplitudes



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