

# Quantum anomalies & Generalized Parton Distributions



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26 February 2025

Based on:

**Arxiv: 2210.13419, 2305.09431**

with: **Yoshitaka Hatta (BNL)**  
**Werner Vogelsang (Tubingen U.)**

**Arxiv: 2411.07024**

with: **Yoshitaka Hatta (BNL)**  
**Jakob Schoenleber (BNL)**

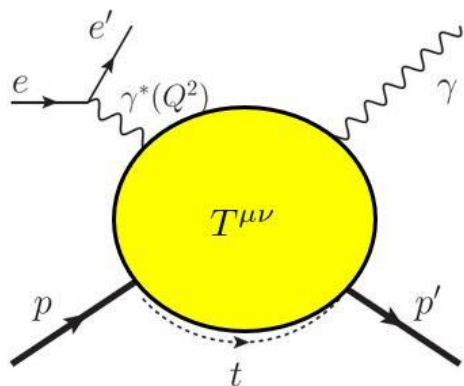
## Physics Opportunities at an Electron-Ion Collider XI



Florida International University

# Outline

- **Motivation: Chiral & trace anomalies & GPDs**

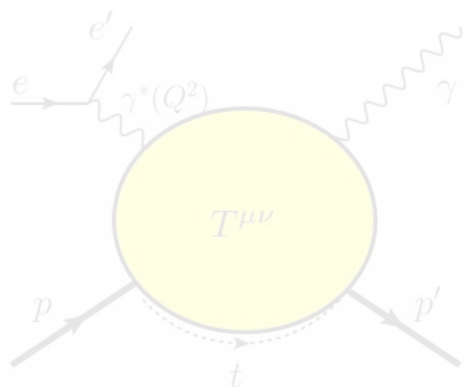


- **Perturbative calculations of box diagrams in DVCS**

- **Non-perturbative relations between GPDs mediated by anomalies**

# Outline

- **Motivation: Chiral & trace anomalies & GPDs**



- Perturbative calculations of box diagrams in DVCS
- Non-perturbative relations between GPDs mediated by anomalies

# Anomalies in QCD



## Chiral anomaly

### Classical:

- **U(1) axial symmetry:** QCD Lagrangian invariant under global chiral rotation of fermionic fields
- **Quantity conserved:** Axial-vector current  $J_5^\mu = \sum_f \bar{\psi}_f \gamma^\mu \gamma_5 \psi_f$

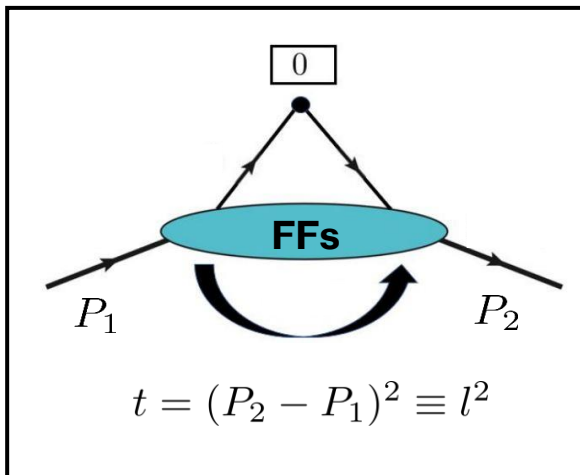
# Anomalies in QCD

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### Consequence:



$$\langle P_2 | J_5^\mu | P_1 \rangle = \bar{u}(P_2) \left[ \gamma^\mu \gamma_5 g_A(t) + \frac{l^\mu \gamma_5}{2M} g_P(t) \right] u(P_1)$$

Current conservation leads to (take  $\partial_\mu J_5^\mu = 0$ ):

$$\frac{g_P(t)}{2M} \approx -\frac{2M g_A(0)}{t} = -\frac{2M \Delta \Sigma}{t}$$

**Pole at  $t=0$  from massless  $\eta_0$  exchange**

# Anomalies in QCD



## Chiral anomaly

Quantum mechanical:

U(1) axial symmetry is explicitly broken by chiral anomaly

**Chiral anomaly equation:**

$$\partial_\mu J_5^\mu = -\frac{n_f \alpha_s}{4\pi} F^{\mu\nu} \tilde{F}_{\mu\nu}$$

$$\tilde{F}^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}$$



# Anomalies in QCD

## Chiral anomaly

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$$\partial_\mu J_5^\mu = -\frac{n_f \alpha_s}{4\pi} F^{\mu\nu} \tilde{F}_{\mu\nu} \quad \tilde{F}^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}$$

Consequence: In real QCD, there is no massless pole in form factor due to pole cancellation

$$\frac{g_P(t)}{2M} = \frac{1}{t} \left[ i \frac{\langle P_2 | \frac{n_f \alpha_s}{4\pi} F \tilde{F} | P_1 \rangle}{\bar{u}(P_2) \gamma_5 u(P_1)} - 2M g_A(t) \right] \sim \frac{1}{t - m_{\eta'}^2}$$

↑  
Anomaly pole

↑  
 $\eta_0$  pole

**Eta-meson mass generation**

(Witten-Veneziano, 1979)

(See Tarasov's talk)

# Anomalies in QCD



## Trace anomaly

### Classical:

- **Conformal symmetry:** QCD Lagrangian invariant under scale transformation
- **Quantity conserved:** Dilatation current  $D^\mu = \Theta^{\mu\nu} x_\nu$        $\Theta^{\mu\nu}$ : Energy Momentum Tensor (EMT)



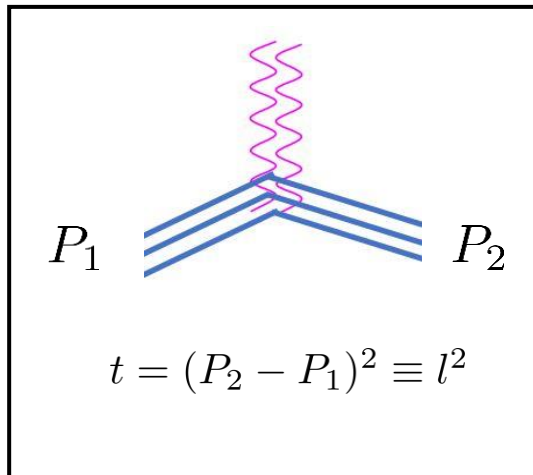
# Anomalies in QCD

## Trace anomaly

### Classical:

- **Conformal symmetry:** QCD Lagrangian invariant under scale transformation
- **Quantity conserved:** Dilatation current  $D^\mu = \Theta^{\mu\nu} x_\nu$        $\Theta^{\mu\nu}$ : Energy Momentum Tensor (EMT)

### Consequence:



$$\langle P_2 | \Theta^{\alpha\beta} | P_1 \rangle = \bar{u}(P_2) \left[ A(t) \frac{P^\alpha P^\beta}{M} + (A(t) + B(t)) \frac{P^{(\alpha} i \sigma^{\beta)\lambda} l_\lambda}{2M} + D(t) \frac{l^\alpha l^\beta - g^{\alpha\beta} t}{4M} \right] u(P_1)$$

Current conservation leads to traceless EMT ( $\Theta^\alpha_\alpha = 0$ ):

$$\frac{3D(t)}{4} \approx \frac{M^2}{t}$$

**Pole at t=0 from massless particle exchange**

# Anomalies in QCD



## Trace anomaly

Quantum mechanical:

Conformal symmetry is explicitly broken by trace anomaly

Trace anomaly equation:

$$\Theta_{\mu}^{\mu} = \frac{\beta(g)}{2g} F^{\mu\nu} F_{\mu\nu}$$

# Anomalies in QCD

## Trace anomaly

Quantum mechanical:

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**Consequence:** In real QCD, there is no massless pole in form factor due to pole cancellation

$$\frac{3D(t)}{4} \approx \frac{M^2}{t} \left( A(t) - \frac{\langle P_2 | \frac{\beta(g)}{2g} F^2 | P_1 \rangle}{M \bar{u}(P_2) u(P_1)} \right) \sim \frac{1}{t - m_G^2}$$

↑  
Anomaly pole

↑  
Massless pole

**Glueball mass generation**

(Mamo, Zahed, 2021/ Fujita et al, 2022)

# Anomalies in QCD



## Trace anomaly

### Main message:

Anomalies constrain Form Factors

Form Factors are moments of GPDs:

$$g_P(t) = \int_{-1}^1 dx \tilde{E}(x, \xi, t)$$

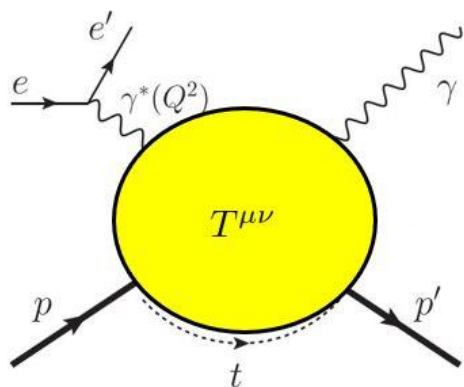
$$A(t) + \xi^2 D(t) = \int_{-1}^1 dx x H(x, \xi, t)$$

**Anomalies constrain GPDs**

eration  
a et al, 2022)

# Outline

- Motivation: Chiral & trace anomalies & GPDs



- Perturbative calculations of box diagrams in DVCS**

- Non-perturbative relations between GPDs mediated by anomalies

# Historical significance of the anomaly problem



## THE ANOMALOUS GLUON CONTRIBUTION TO POLARIZED LEPTOPRODUCTION

G. ALTARELLI and G.G. ROSS <sup>1</sup>

*CERN, CH-1211 Geneva 23, Switzerland*

Received 29 June 1988

## THE ROLE OF THE AXIAL ANOMALY IN MEASURING SPIN-DEPENDENT PARTON DISTRIBUTIONS

R.D. CARLITZ

*Department of Physics and Astronomy, University of Pittsburgh, Pittsburgh, PA 15260, USA*

J.C. COLLINS

*Department of Physics, Illinois Institute of Technology, Chicago, IL 60616, USA*

and

A.H. MUELLER

*Department of Physics, Columbia University, New York, NY 10027, USA*

Received 22 August 1988

## Gluonic contribution to $g_1$ and its relationship to the spin-dependent parton distributions

Geoffrey T. Bodwin and Jianwei Qiu\*

**The role of chiral anomaly in polarized DIS is a well-known old story**

See  
Tarasov's  
talk

## THE $g_1$ PROBLEM: DEEP INELASTIC ELECTRON SCATTERING AND THE SPIN OF THE PROTON\*

R.L. JAFFE and Aneesh MANOHAR\*\*

# Recent developments



See  
Tarasov's  
talk

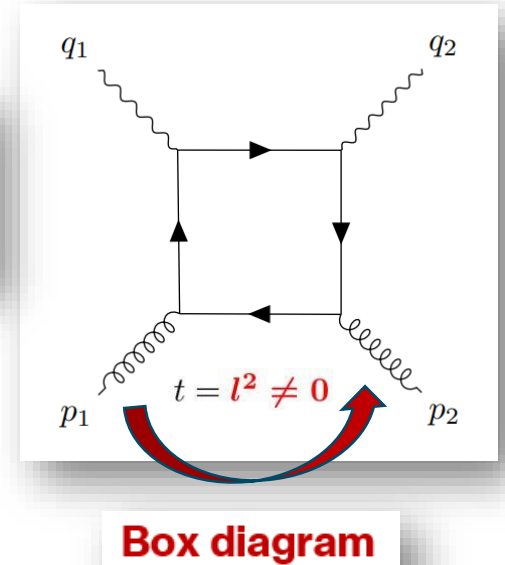
**First calculation of box diagram with  $t = l^2 \neq 0$ :**

The role of the chiral anomaly in polarized deeply inelastic scattering I: Finding the triangle graph inside the box diagram in Bjorken and Regge asymptotics

Andrey Tarasov<sup>1,2</sup> and Raju Venugopalan<sup>3</sup>

The role of the chiral anomaly in polarized deeply inelastic scattering II: Topological screening and transitions from emergent axion-like dynamics

Andrey Tarasov<sup>1,2</sup> and Raju Venugopalan<sup>3</sup>



**They computed polarized DIS using  $t = l^2 \neq 0$   
as a collinear regulator and revealed anomaly pole**

# Imprint of Anomalies in QCD Compton scattering

Chiral and trace anomalies in Deeply Virtual Compton Scattering II:  
QCD factorization and beyond

Shohini Bhattacharya,<sup>1,\*</sup> Yoshitaka Hatta,<sup>2,1,†</sup> and Werner Vogelsang<sup>3,‡</sup>

First calculation of

We explored the physics of anomaly in DVCS using Feynman-diagram approach

Andrey Tarasov<sup>1,2</sup> and Raju Venugopalan<sup>3</sup>

See Tarasov's talk

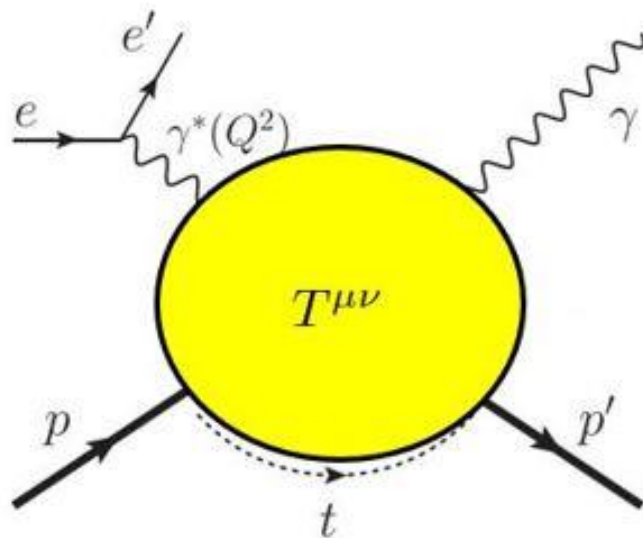
The role of the chiral anomaly  
Topological screening and tri

Andrey Ta

The role of the chiral  
Wess-Zumino-Witten conti

A

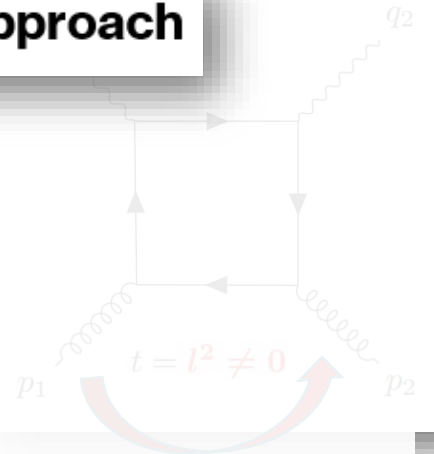
Andrey & Raju demonstrate



:

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te quark mass

re the physics of anomaly



Box diagram

Deeply Virtual Compton Scattering (DVCS): Golden process to access GPDs



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**We explored the physics of anomaly in DVCS using Feynman-diagram approach**

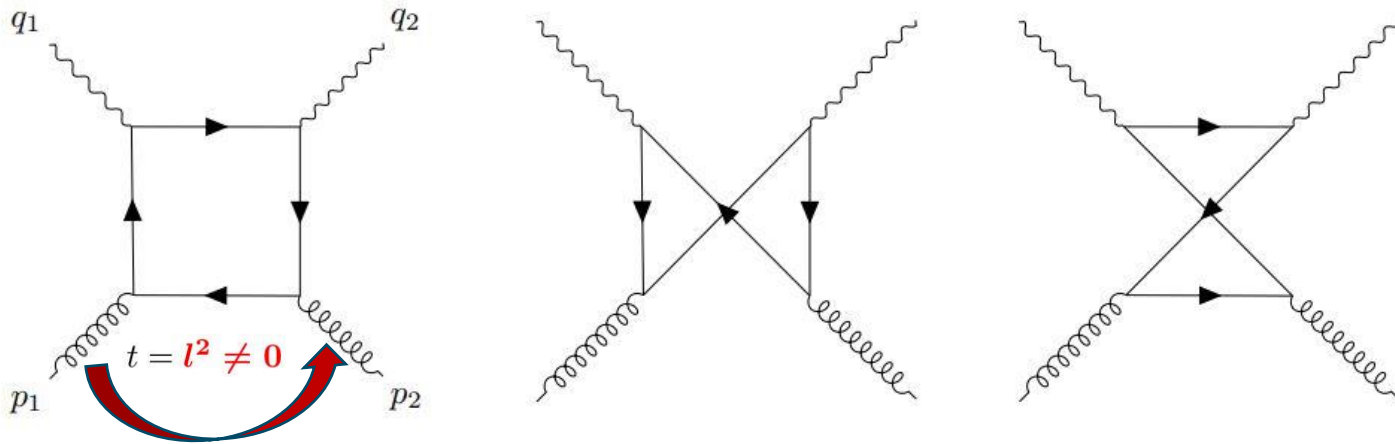


FIG. 1: Diagrams for the subprocess  $\gamma^* g \rightarrow \gamma^* g$  in Compton scattering.

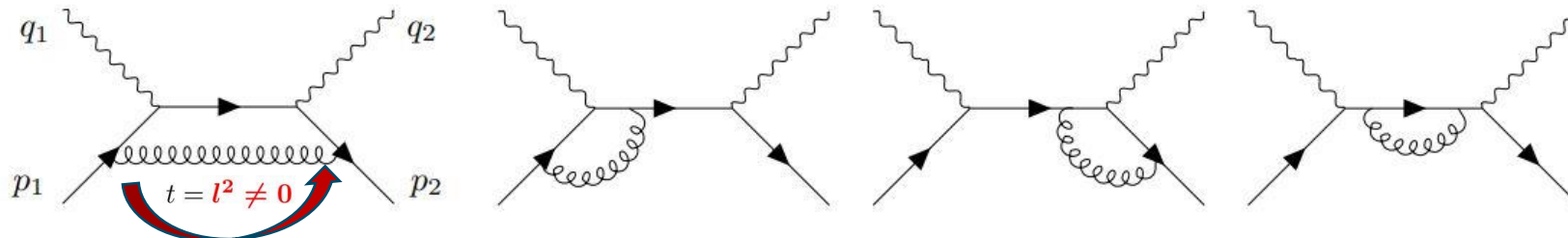


FIG. 2: Diagrams for the subprocess  $\gamma^* q \rightarrow \gamma^* q$  in Compton scattering. Diagrams with photon lines crossed are not shown.

Gluon channel  
Diagrams

Quark channel  
Diagrams

# Imprint of Anomalies in QCD Compton scattering

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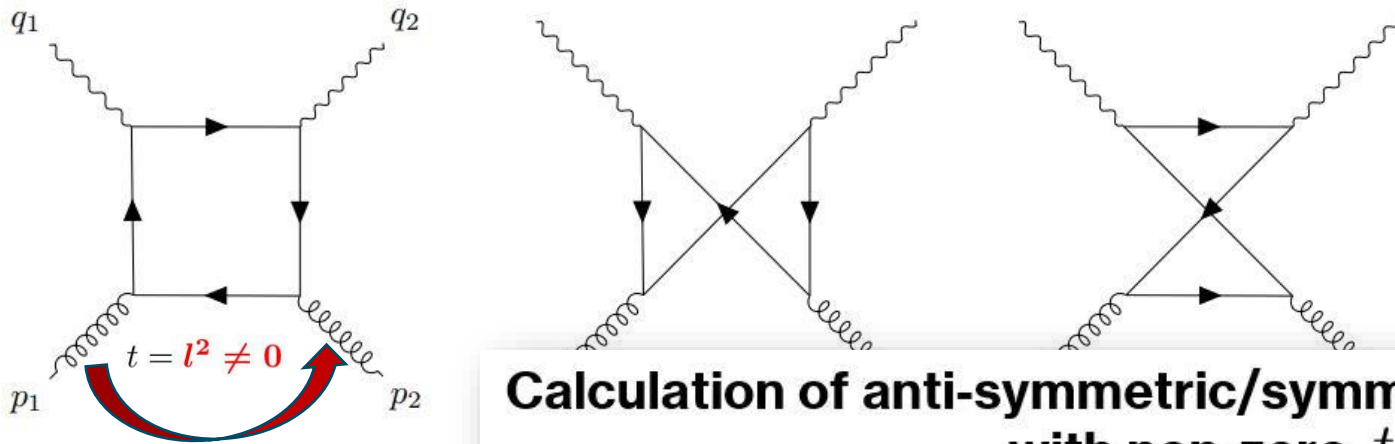


FIG. 1: Diagrams for

**Calculation of anti-symmetric/symmetric  $(\mu, \nu)$  of Compton amplitude with non-zero  $t$**

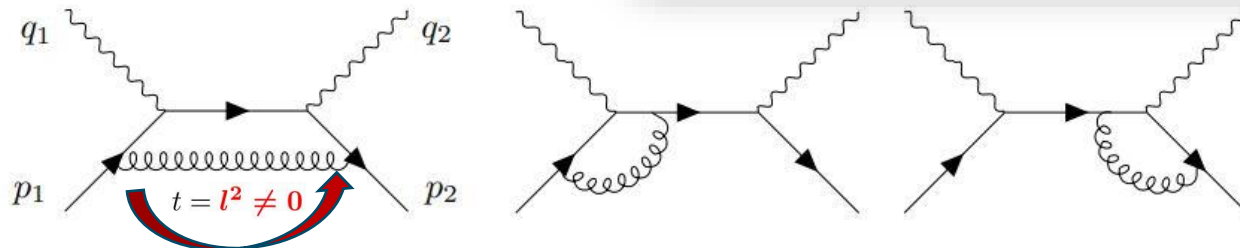
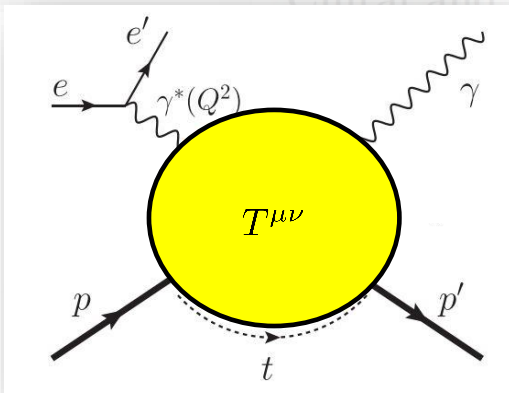


FIG. 2: Diagrams for the subprocess  $\gamma^* q \rightarrow \gamma^* q$  in Compton scattering. Diagrams with photon

**Our calculation covers regimes:**  
 $t = \Lambda_{\text{QCD}}^2$  &  $\Lambda_{\text{QCD}}^2 \ll t \ll Q^2$

# Remarks on DVCS:

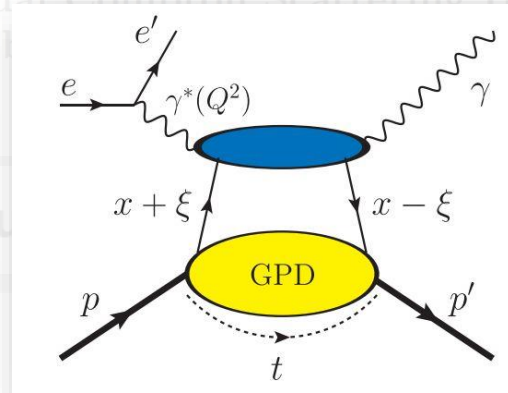
## Importance of Anomalies in QCD Compton scattering



**Bjorken limit**



$$t \sim \Lambda_{\text{QCD}}^2 \ll Q^2$$



The QCD factorization theorem: **Collins, Freund; Ji, Osborne (1998)**

$$T^{\mu\nu} = \sum_{a=q,g} \int \frac{dx}{x} C_a^{\mu\nu} \left( \frac{x_B}{x}, \frac{\xi}{x} \right) f_a(x, \xi, t) + \mathcal{O}(1/Q^2)$$

- Original proofs implicitly assumed  $t = \Lambda_{\text{QCD}}^2$
- When  $\Lambda_{\text{QCD}}^2 \ll t \ll Q^2$ , there are anomaly poles and Sudakov double logarithms that need to be addressed

**Our calculation covers regimes:**

$$t = \Lambda_{\text{QCD}}^2 \quad \& \quad \Lambda_{\text{QCD}}^2 \ll t \ll Q^2$$

# Imprint of Anomalies in QCD Compton scattering



## Perturbative calculations of box diagrams

### Main findings:

We computed one-loop Compton amplitude in all channels (quark/gluon, polarized/unpolarized) using  $\epsilon$  as a regulator and find:

- Collinear logs (complete GPD evolution kernel)
- Anomaly poles
- Sudakov double poles

Demonstrated that all the singularities can be absorbed into GPDs by infrared matching

**Beyond factorization: What happens to the anomaly poles within GPDs?**

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We explored the physics of anomaly in DVCS using Feynman-diagram approach

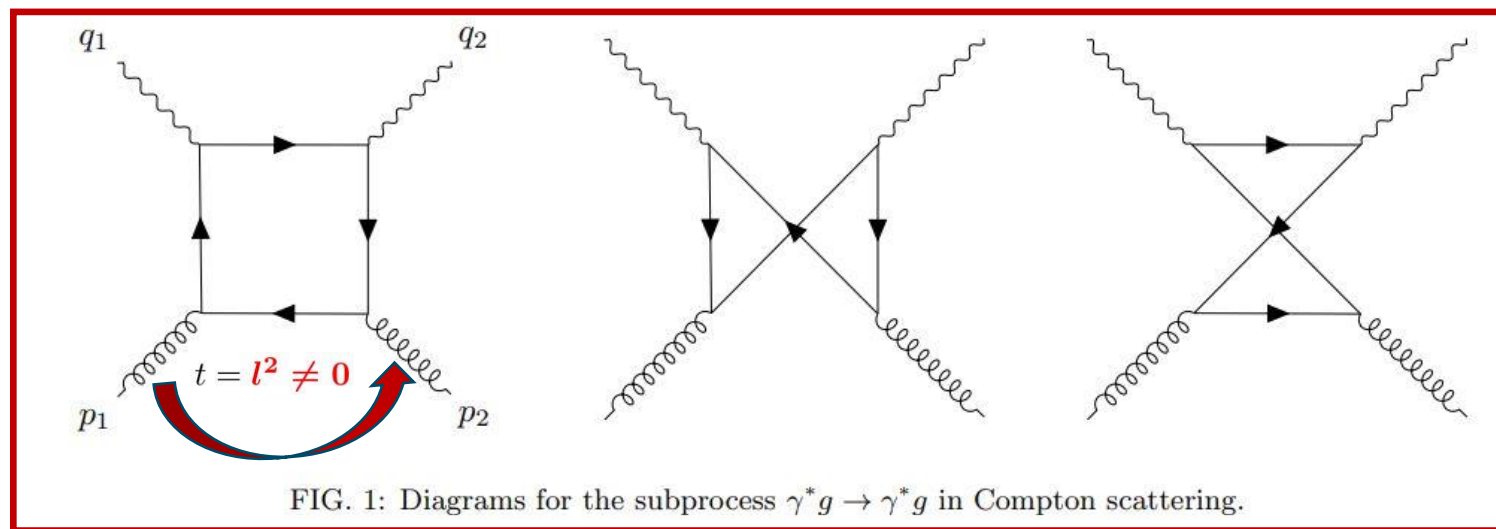


FIG. 1: Diagrams for the subprocess  $\gamma^* g \rightarrow \gamma^* g$  in Compton scattering.

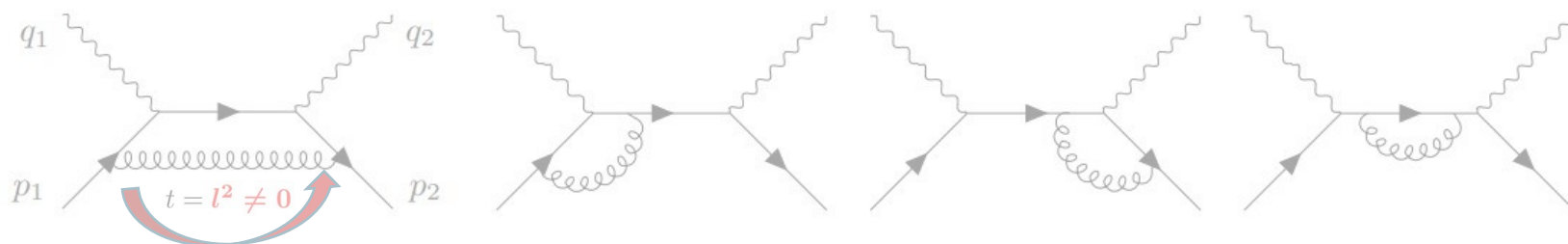


FIG. 2: Diagrams for the subprocess  $\gamma^* q \rightarrow \gamma^* q$  in Compton scattering. Diagrams with photon lines crossed are not shown.

Gluon channel  
Diagrams

Box diagram

kinematics ( $l = p_2 - p_1$ ):

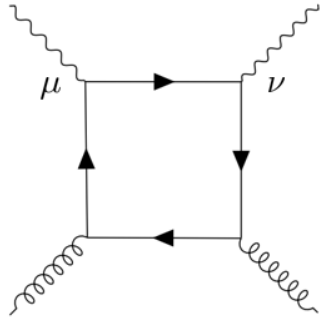
$F_{\alpha\beta}^a |p_1\rangle$

by infra-red pole

# Imprint of Anomalies in QCD Compton scattering



## Polarized DVCS & chiral anomaly



Antisymmetric part of Compton amplitude:

Collinear singularity regularized by  $l^2$

$$-\epsilon^{\alpha\beta\mu\nu} P_\beta \times T_{\mu\nu}^{\text{asym}} \approx \frac{1}{2} \frac{\alpha_s}{2\pi} \left( \sum_f e_f^2 \right) \bar{u}(P_2) \left[ \left( \tilde{\kappa}_{qg} \ln \frac{Q^2}{-l^2} + \delta\tilde{C}_g^{\text{off}} \right) \otimes \langle F^{+\mu} \tilde{F}_\mu^+ \rangle \gamma^\alpha \gamma_5 + \frac{l^\alpha}{l^2} \tilde{A}_g^{\text{anom}} \otimes \tilde{\mathcal{F}}(x_B) \gamma_5 \right] u(P_1)$$

Reproduced the GPD evolution kernel

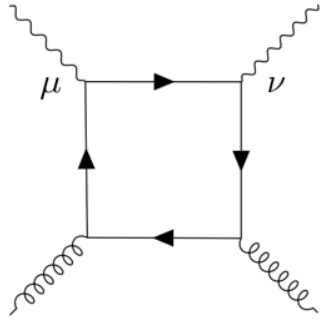
$$\tilde{\kappa}_{qg}(\hat{x}, \hat{\xi}) = \frac{2\hat{x} - 1 - \hat{\xi}^2}{(1 - \hat{\xi}^2)^2} \ln \frac{\hat{x} - 1}{\hat{x}} - 2 \frac{\hat{x} - \hat{\xi}}{(1 - \hat{\xi}^2)^2} \ln \frac{\hat{x} - \hat{\xi}}{\hat{x}} - (\hat{x} \rightarrow -\hat{x})$$

Ji, Osborne; Belitsky, Mueller

# Imprint of Anomalies in QCD Compton scattering



## Polarized DVCS & chiral anomaly



Antisymmetric part of Compton amplitude:

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Anomalous coefficient function

Twist 4 GPD

$$\tilde{\mathcal{F}}(x, \xi, t) = \frac{iP^+}{M} \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \frac{\langle P_2 | F^{\mu\nu}(-z^-/2) W \tilde{F}_{\mu\nu}(z^-/2) | P_1 \rangle}{u(P_2) \gamma_5 u(P_1)}$$

(Non-local) chiral anomaly manifests itself in high-energy scattering amplitude

Imaginary part of pole term ( $\xi = 0$ ) in agreement with Andrey & Raju

Anomalous coefficient function:

$$\tilde{A}_g^{\text{anom}} = \frac{8T_R}{x} \frac{(1 - \hat{x}) \ln \frac{\hat{x}-1}{x} + (\hat{x} - \hat{\xi}) \ln \frac{\hat{x}-\hat{\xi}}{\hat{x}} - (\hat{x} \rightarrow -\hat{x})}{1 - \hat{\xi}^2}$$



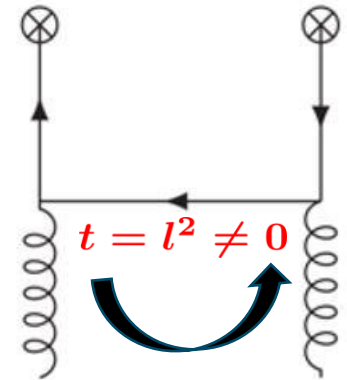
# Imprint of Anomalies in QCD Compton scattering



## Justifying factorization: “Infrared subtraction”

### GPD at one loop:

We computed quark GPD of a gluon & found the same pole structure in one-loop calculation:



$$\begin{aligned}
 & \frac{\alpha_s T_R}{2\pi} \left[ (1-\xi^2) i\epsilon^{+p\epsilon_2^*\epsilon_1} \left( \frac{2x-1-\xi^2}{(1-\xi^2)^2} \left( \frac{\tilde{\mu}^2}{-l^2} \right)^\epsilon - \ln \frac{(1-x)^2}{1-\xi^2} \right) - 2 \frac{1-x}{(1-\xi^2)^2} \right] + \boxed{\frac{2il^+ \epsilon^{\epsilon_1 \epsilon_2^* l p}}{l^2} \frac{1-x}{1-\xi^2}} & x > \xi \\
 = & \left\{ \frac{\alpha_s T_R}{2\pi} \left[ (1-\xi^2) i\epsilon^{+p\epsilon_2^*\epsilon_1} \frac{\left( \frac{\tilde{\mu}^2}{-l^2} \right)^\epsilon}{\epsilon_{UV}} \frac{-1}{(1+\xi)^2} + \boxed{\frac{2il^+ \epsilon^{\epsilon_1 \epsilon_2^* l p}}{l^2} \frac{1}{1+\xi}} \right] \right. & \text{The pole “belongs” to GPD} & x < \xi \\
 & \left. (1-\xi^2) i\epsilon^{+p\epsilon_2^*\epsilon_1} \frac{1}{(1-\xi^2)^2} \left[ -2\xi \ln(\xi^2-x^2) + (1+\xi^2) \ln(1-x^2) - 2x \ln \frac{(1-x)(x+\xi)}{(1+x)(\xi-x)} - 2(1+\xi^2) \ln(1+\xi) + 4\xi \ln(2\xi) + 2\xi - 2 \right] \right]
 \end{aligned}$$

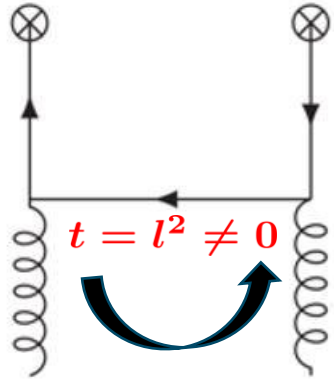
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$x > \xi$

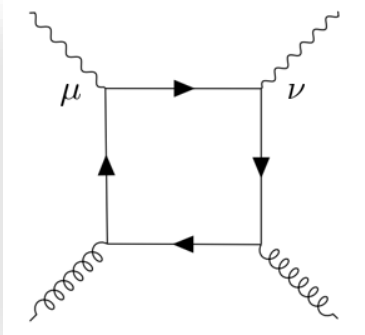
$x < \xi$

**The pole "belongs" to GPD**

**Absorb the 1/t poles of Compton amplitude into twist-2 GPDs**

**Factorization restored**

# Comment on equivalence with $\overline{\text{MS}}$ scheme



## Antisymmetric part of Compton amplitude

$$-\epsilon^{\alpha\beta\mu\nu} P_\beta \times T_{\mu\nu}^{\text{asym}} \approx \frac{1}{2} \frac{\alpha_s}{2\pi} \left( \sum_f e_f^2 \right) \bar{u}(P_2) \left[ \left( \tilde{\kappa}_{qg} \ln \frac{Q^2}{-l^2} + \delta\tilde{C}_g^{\text{off}} \right) \otimes \langle F^{+\mu} \tilde{F}_\mu^+ \rangle \gamma^\alpha \gamma_5 + \frac{l^\alpha}{l^2} \tilde{A}_g^{\text{anom}} \otimes \tilde{\mathcal{F}}(x_B) \gamma_5 \right] u(P_1)$$

Coefficient function

$$\begin{aligned} \delta\tilde{C}_1^g(\hat{x}, \hat{\xi}) = & -\frac{2\hat{x}-1-\hat{\xi}^2}{(1-\hat{\xi}^2)^2} \ln \frac{\hat{x}-1}{\hat{x}} + 2\frac{\hat{x}-\hat{\xi}}{(1-\hat{\xi}^2)^2} \ln \frac{\hat{x}-\hat{\xi}}{\hat{x}} \\ & + \frac{2\hat{x}-1-\hat{\xi}^2}{2(1-\hat{\xi}^2)^2} \ln^2 \frac{\hat{x}-1}{\hat{x}} + \frac{\hat{\xi}}{1-\hat{\xi}^2} \ln^2 \frac{\hat{x}-\hat{\xi}}{\hat{x}} - \frac{\hat{x}}{(1-\hat{\xi}^2)^2} \ln \frac{\hat{x}-\hat{\xi}}{\hat{x}} \ln \frac{\hat{x}+\hat{\xi}}{\hat{x}} \\ & + \frac{2\hat{\xi}}{(1-\hat{\xi}^2)^2} \text{Li}_2 \frac{-2\hat{\xi}}{\hat{x}-\hat{\xi}} + \frac{2\hat{x}-1-\hat{\xi}^2}{(1-\hat{\xi}^2)^2} \left( \text{Li}_2 \frac{1-\hat{\xi}}{1-\hat{x}} + \text{Li}_2 \frac{1+\hat{\xi}}{1-\hat{x}} \right) - (\hat{x} \rightarrow -\hat{x}) \end{aligned}$$

After subtracting IR singularities and finite terms,  $t \neq 0$  regularization is equivalent to  $\overline{\text{MS}}$  scheme

This means that the result can be smoothly connected to the regime  $t \sim \Lambda_{\text{QCD}}^2$  as considered in the works by Collins, Freund; Ji, Osborne

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Demonstrated that all the singularities can be absorbed into GPDs by infrared matching

**Beyond factorization: What happens to the anomaly poles within GPDs?**

# Imprint of Anomalies in QCD Compton scattering



## Fate of anomaly pole

Chiral anomaly pole in GPD  $\tilde{E}$  :

$$\tilde{E}_f(x_B, l^2) + \tilde{E}_f(-x_B, l^2) = \frac{\alpha_s}{2\pi} \frac{2M}{l^2} \tilde{A}_g^{\text{anom}} \otimes \tilde{\mathcal{F}}(x_B, l^2)$$

↑  
Anomaly pole

Remarks:

- We absorbed the  $\frac{1}{t}$  pole in GPD  $\tilde{E}$ . What does this mean physically?

The GPD  $\tilde{E}$  **cannot** have  $\frac{1}{t}$  pole; instead, it **should have**  $\frac{1}{t - m_{\eta'}^2}$ . This shift  $\frac{1}{t} \rightarrow \frac{1}{t - m_{\eta'}^2}$  is well-known to occur in form factor  $g_P(t)$ .

Can we discuss the same thing for GPD?





# Imprint of Anomalies in QCD Compton scattering



## Fate of anomaly pole

Chiral anomaly pole in GPD  $\tilde{E}$  :

$$\tilde{E}_f(x_B, l^2) + \tilde{E}_f(-x_B, l^2) = \frac{\alpha_s}{2\pi} \frac{2M}{l^2} \tilde{A}_g^{\text{anom}} \otimes \tilde{\mathcal{F}}(x_B, l^2)$$

↑  
Anomaly pole

Remarks:

- The GPD  $\tilde{E}$  **cannot** have  $\frac{1}{t}$  pole; instead, it **should have**  $\frac{1}{t - m_{\eta'}^2}$ .
- Unlike the anomaly pole, the  $\eta_0$  pole does not appear in perturbation theory. Instead, it can be addressed through an effective action approach (see Tarasov's talk), but later **we will explicitly derive the  $\eta_0$  pole and find that its residue consists of the polarized GPD  $\tilde{H}$  and twist-4 GPDs.**



# Imprint of Anomalies in QCD Compton scattering



## Fate of anomaly pole

Chiral anomaly pole in GPD  $\tilde{E}$  :

$$\tilde{E}_f(x_B, l^2) + \tilde{E}_f(-x_B, l^2) = \tilde{E}_f^{\text{c.t.}}(x_B, l^2) + \tilde{E}_f^{\text{c.t.}}(-x_B, l^2) + \frac{\alpha_s}{2\pi} \frac{2M}{l^2} \tilde{A}_g^{\text{anom}} \otimes \tilde{\mathcal{F}}(x_B, l^2) \quad \sim \frac{1}{l^2 - m_{\eta'}^2}$$

$\uparrow$   
 $\eta_0$  pole
 $\uparrow$   
Anomaly pole

Connection between twist 2 & twist 4 GPDs due to anomaly

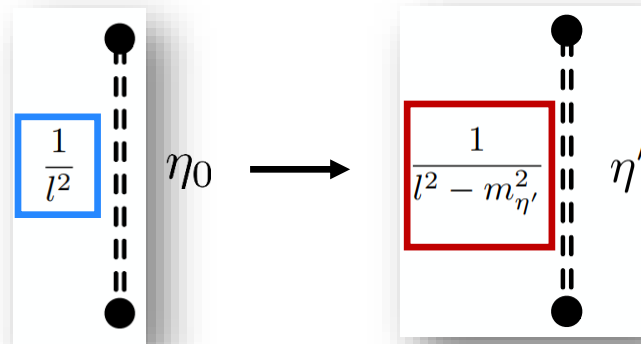
### Fate of anomaly pole:

Cancellation with non-perturbative pole arising due to  $\eta_0$  exchange

$$\tilde{E}_f^{\text{c.t.}}(x_B, l^2) + \tilde{E}_f^{\text{c.t.}}(-x_B, l^2) \approx -\frac{\alpha_s}{2\pi} \frac{2M}{l^2} \tilde{A}_g^{\text{anom}} \otimes \tilde{\mathcal{F}}(x_B, l^2 = 0)$$

(Witten-Veneziano scenario at the GPD level)

Eta-meson mass generation:



# Imprint of Anomalies in QCD Compton scattering



## Fate of anomaly pole

Chiral anomaly pole in GPD  $\tilde{E}$  :

$$\tilde{E}_f(x_B, l^2) + \tilde{E}_f(-x_B, l^2) = \tilde{E}_f^{\text{c.t.}}(x_B, l^2) + \tilde{E}_f^{\text{c.t.}}(-x_B, l^2) + \frac{\alpha_s}{2\pi} \frac{2M}{l^2} \tilde{A}_g^{\text{anom}} \otimes \tilde{\mathcal{F}}(x_B, l^2) \sim \frac{1}{l^2 - m_{\eta'}^2}$$

$\uparrow$   $\eta_0$  pole                       $\uparrow$  Anomaly pole

$\downarrow \int dx$

$$\frac{g_P(l^2)}{2M} = \frac{1}{l^2} \left[ i \frac{\langle P_2 | \frac{n_f \alpha_s}{4\pi} F \tilde{F} | P_1 \rangle}{\bar{u}(P_2) \gamma_5 u(P_1)} - 2M g_A(l^2) \right]$$

Upon integrating, we exactly reproduce the pole-cancellation expression for the form factor

# Imprint of Anomalies in QCD Compton scattering



## Fate of anomaly pole

Chiral anomaly pole in GPD  $\tilde{E}$  :

$$\tilde{E}_f(x_B, l^2) + \tilde{E}_f(-x_B, l^2) = \tilde{E}_f^{\text{c.t.}}(x_B, l^2) + \tilde{E}_f^{\text{c.t.}}(-x_B, l^2) + \frac{\alpha_s}{2\pi} \frac{2M}{l^2} \tilde{A}_g^{\text{anom}} \otimes \tilde{\mathcal{F}}(x_B, l^2) \sim \frac{1}{l^2 - m_{\eta'}^2}$$

$\uparrow$   $\eta_0$  pole                       $\uparrow$  Anomaly pole

The physics of anomalies present at the level of Form Factor is established for the first time at the level of GPDs

# Imprint of Anomalies in QCD Compton scattering



## Unpolarized DVCS & trace anomaly

Trace anomaly pole in GPDs  $H$  &  $E$  :

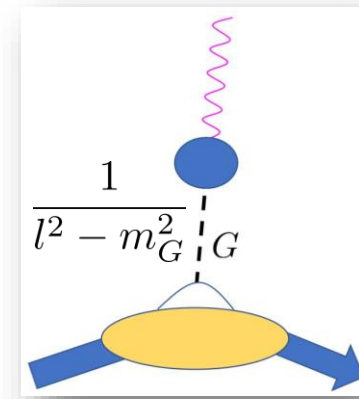
$$H \sim -E \sim \frac{\alpha_s}{l^2} A \otimes \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \frac{\langle P_2 | F^{\mu\nu}(-z^-/2) W F_{\mu\nu}(z^-/2) | P_1 \rangle}{\bar{u}(P_2)u(P_1)}$$



**(Non-local) trace anomaly manifests itself in the GPD**

**Fate of anomaly pole:**

Pole cancellation results in the generation of glueball mass



# Imprint of Anomalies in QCD Compton scattering

First calculation of

Chiral and trace anomalies in Deeply Virtual Compton Scattering :  
QCD factorization and beyond

Shohini Bhattacharya,<sup>1,\*</sup> Yoshitaka Hatta,<sup>2,1,†</sup> and Werner Vogelsang<sup>3,‡</sup>

We explored the physics of anomaly in DVCS using Feynman-diagram approach

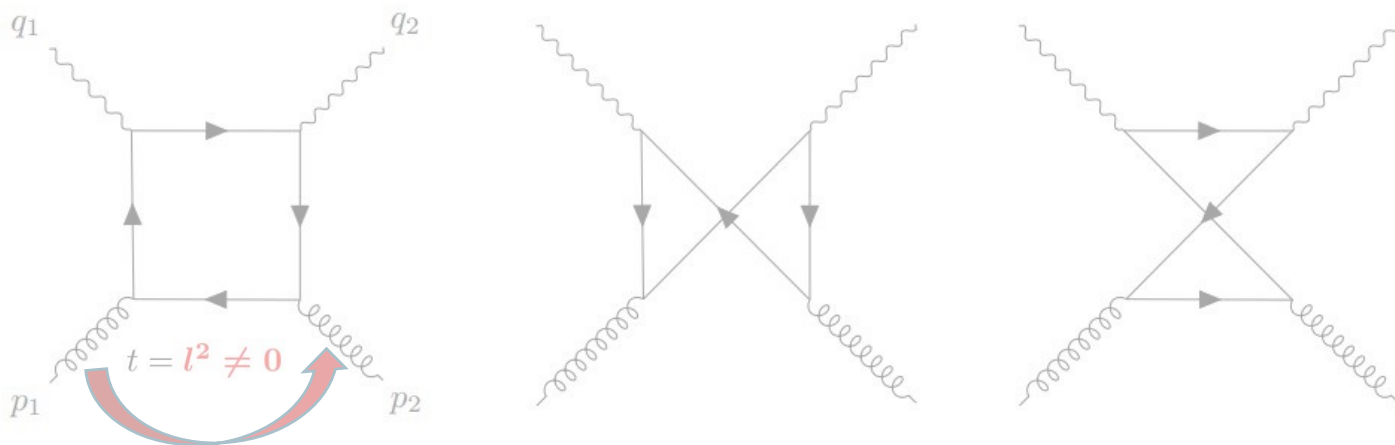


FIG. 1: Diagrams for the subprocess  $\gamma^* g \rightarrow \gamma^* g$  in Compton scattering.

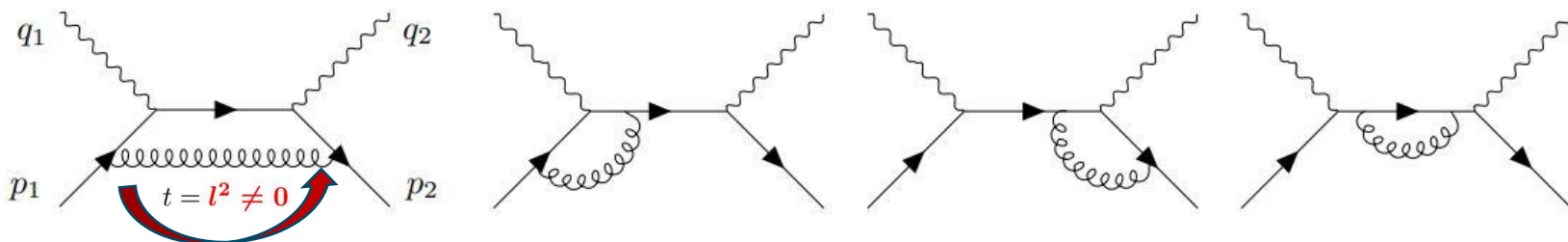
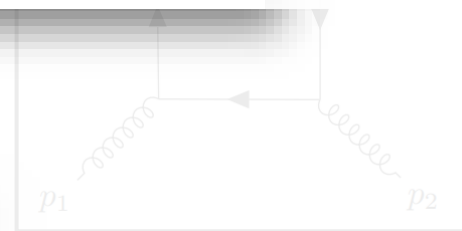


FIG. 2: Diagrams for the subprocess  $\gamma^* q \rightarrow \gamma^* q$  in Compton scattering. Diagrams with photon lines crossed are not shown.



Box diagram

anomaly

kinematics ( $l = p_2 - p_1$ ):

$$F_{\alpha\beta}^a |p_1\rangle$$

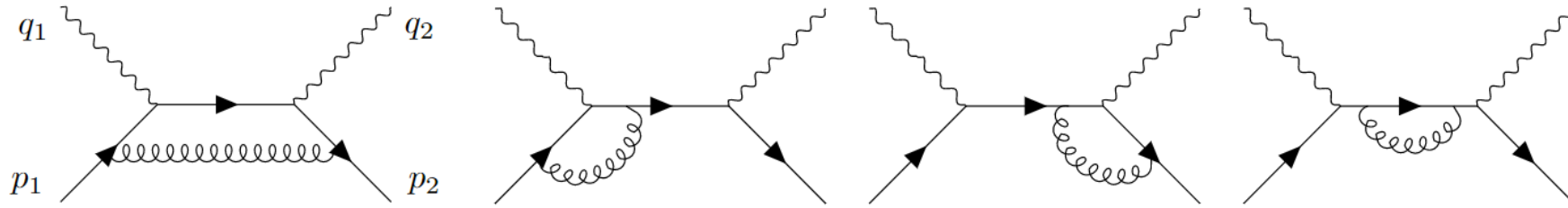
Quark channel  
Diagrams

by a crossed pole

# Imprint of Anomalies in QCD Compton scattering



## Quark-channel diagrams in DVCS



### Example: Antisymmetric case

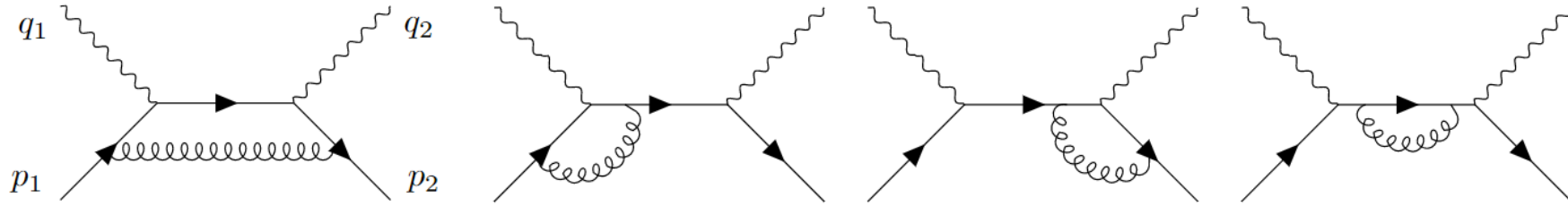
$$\sim \cancel{\frac{1}{l^2}} + \tilde{\kappa}_{qq}(\hat{x}, \hat{\xi}) \ln \frac{Q^2}{-l^2} + \delta \tilde{C}_1^q(\hat{x}, \hat{\xi})$$

**No pole!**

# Imprint of Anomalies in QCD Compton scattering



## Quark-channel diagrams in DVCS



### Example: Antisymmetric case

$$\sim \cancel{\frac{1}{l^2}} + \tilde{\kappa}_{qq}(\hat{x}, \hat{\xi}) \ln \frac{Q^2}{-l^2} + \delta \tilde{C}_1^q(\hat{x}, \hat{\xi})$$

### Reproduced the known logarithms from literature

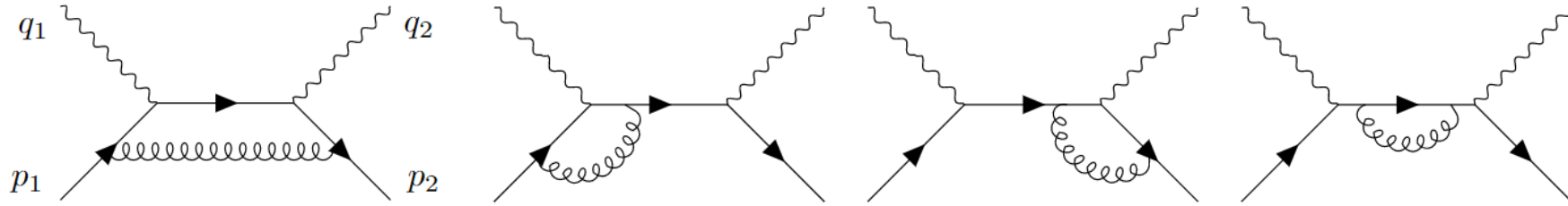
$$\tilde{\kappa}_{qq}(\hat{x}, \hat{\xi}) = \frac{3}{2(1-\hat{x})} + \frac{\hat{x}^2 + 1 - 2\hat{\xi}^2}{(1-\hat{\xi}^2)(1-\hat{x})} \ln \frac{\hat{x}-1}{\hat{x}} - \frac{(\hat{x}-\hat{\xi})(1+\hat{x}^2+2\hat{x}\hat{\xi})}{(1-\hat{x}^2)(1-\hat{\xi}^2)} \ln \frac{\hat{x}-\hat{\xi}}{\hat{x}} - (\hat{x} \rightarrow -\hat{x})$$

**Ji, Osborne; Belitsky, Mueller**

# Imprint of Anomalies in QCD Compton scattering



## Quark-channel diagrams in DVCS



### Example: Antisymmetric case

$$\sim \cancel{\frac{1}{l^2}} + \tilde{\kappa}_{qq}(\hat{x}, \hat{\xi}) \ln \frac{Q^2}{-l^2} + \delta\tilde{C}_1^q(\hat{x}, \hat{\xi})$$

Coefficient function

$$\delta\tilde{C}_1^q(\hat{x}, \hat{\xi}) = -\frac{\left(\frac{Q^2}{-l^2}\right)^{\epsilon_{IR}}}{\epsilon_{IR}^2(1-\hat{x})} - \frac{3\left(\frac{Q^2}{-l^2}\right)^{\epsilon_{IR}}}{2\epsilon_{IR}(1-\hat{x})} + \frac{-1+2\hat{x}-4\hat{x}^2+3\hat{\xi}^2}{2(1-\hat{x})(1-\hat{\xi}^2)} \ln \frac{\hat{x}-1}{\hat{x}} + \frac{(\hat{x}-\hat{\xi})(1+2\hat{x}^2+3\hat{x}\hat{\xi})}{(1-\hat{x}^2)(1-\hat{\xi}^2)} \ln \frac{\hat{x}-\hat{\xi}}{\hat{x}}$$

$$+ \frac{\hat{x}^2-2\hat{\xi}^2}{\hat{x}(1-\hat{\xi}^2)} \ln^2 \frac{\hat{x}-1}{\hat{x}} + \frac{\hat{x}^2-2\hat{\xi}^2}{\hat{x}(1-\hat{\xi}^2)} \ln \frac{\hat{x}-\hat{\xi}}{\hat{x}} \ln \frac{\hat{x}+\hat{\xi}}{\hat{x}} + \frac{\pi^2-54}{12(1-\hat{x})}$$

$$+ \frac{\hat{\xi}}{1-\hat{\xi}^2} \text{Li}_2 \frac{2\hat{\xi}}{\hat{x}-\hat{\xi}} + \frac{1+\hat{x}^2}{(1-\hat{x})}$$

Unexpected double IR pole

Unexpected single IR pole

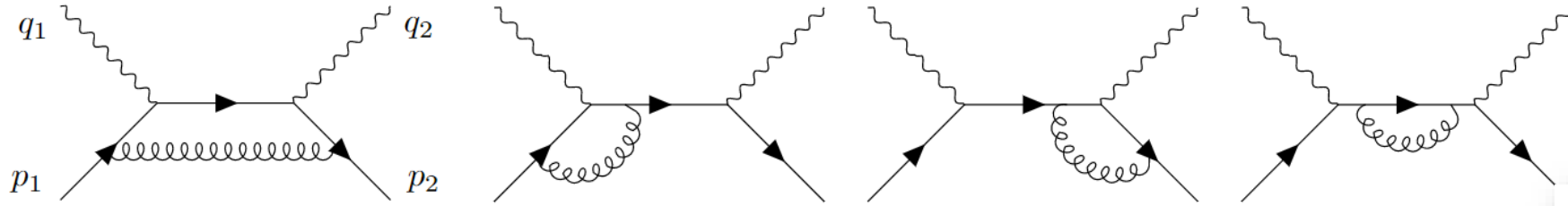
Sudakov logs!  $\ln\left(\frac{Q^2}{-l^2}\right), \ln^2\left(\frac{Q^2}{-l^2}\right)$



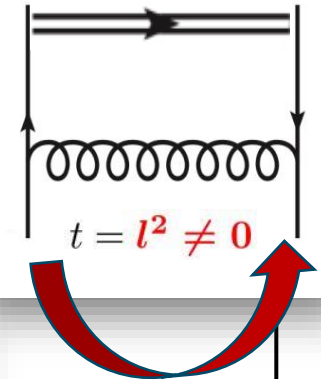
# Imprint of Anomalies in QCD Compton scattering



## Quark-channel diagrams in DVCS



But, when you compute GPD itself, you find the same double, single IR poles!  
 These poles can be systematically absorbed into GPD



**Factorization restored**

$$\delta\tilde{C}_1^q(\hat{x}, \hat{\xi}) = -\frac{\left(\frac{Q^2}{-l^2}\right)^{\epsilon_{IR}}}{\epsilon_{IR}^2(1-\hat{x})} - \frac{3\left(\frac{Q^2}{-l^2}\right)^{\epsilon_{IR}}}{2\epsilon_{IR}(1-\hat{x})} + \frac{-1+2\hat{x}-4\hat{x}^2+3\hat{\xi}^2}{2(1-\hat{x})(1-\hat{\xi}^2)} \ln \frac{\hat{x}-1}{\hat{x}} + \frac{(\hat{x}-\hat{\xi})(1+2\hat{x}^2+3\hat{x}\hat{\xi})}{(1-\hat{x}^2)(1-\hat{\xi}^2)} \ln \frac{\hat{x}-\hat{\xi}}{\hat{x}}$$

$$+ \frac{\hat{x}^2-2\hat{\xi}^2}{2(1-\hat{x})(1-\hat{\xi}^2)} \ln^2 \frac{\hat{x}-1}{\hat{x}} + \frac{\hat{x}^2-2\hat{\xi}^2}{2(1-\hat{x})(1-\hat{\xi}^2)} \ln \frac{\hat{x}-\hat{\xi}}{\hat{x}} \ln \frac{\hat{x}+\hat{\xi}}{\hat{x}} + \frac{\pi^2-54}{12(1-\hat{x})}$$

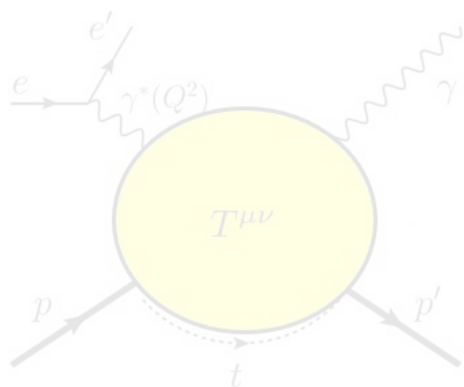
$$+ \frac{\hat{\xi}}{1-\hat{\xi}^2} \text{Li}_2 \frac{-2\hat{\xi}}{\hat{x}-\hat{\xi}} + \frac{1+\hat{x}^2-2\hat{\xi}^2}{(1-\hat{x})(1-\hat{\xi}^2)} \left( \text{Li}_2 \frac{1-\hat{\xi}}{1-\hat{x}} + \text{Li}_2 \frac{1+\hat{\xi}}{1-\hat{x}} \right) - (\hat{x} \rightarrow -\hat{x})$$

Unexpected double IR pole

Unexpected single IR pole

# Outline

- Motivation: Chiral & trace anomalies & GPDs



- Perturbative calculations of box diagrams in DVCS

- **Non-perturbative relations between GPDs mediated by anomalies**

# Anomaly-mediated relations between twist-2,3,4 GPDs



## Nonlocal chiral anomaly and generalized parton distributions

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Yoshitaka Hatta

*Physics Department, Brookhaven National Laboratory, Upton, NY 11973, USA and  
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*RIKEN BNL Research Center, Brookhaven National Laboratory, Upton, NY 11973, USA*

# Anomaly-mediated relations between twist-2,3,4 GPDs



## Main findings:

- Derived  $\eta_0$  pole of nonperturbative origin to cancel the  $1/t$  anomaly pole:

$$\tilde{E}_f(x_B, l^2) + \tilde{E}_f(-x_B, l^2) = \boxed{\tilde{E}_f^{\text{c.t.}}(x_B, l^2) + \tilde{E}_f^{\text{c.t.}}(-x_B, l^2)} + \frac{\alpha_s}{2\pi} \frac{2M}{l^2} \tilde{A}_g^{\text{anom}} \otimes \tilde{\mathcal{F}}(x_B, l^2)$$

$\uparrow$   $\eta_0$  pole                       $\uparrow$  Anomaly pole

# Anomaly-mediated relations between twist-2,3,4 GPDs



## Main findings:

- Derived  $\eta_0$  pole of nonperturbative origin to cancel the  $1/t$  anomaly pole.
- The  $1/t$  poles had been identified in **partonic scattering amplitudes** and GPDs were evaluated using **partonic matrix elements**. It had been argued that they must be 'promoted' to **proton matrix elements** in order to be consistent with form factor relation:

$$g_A(t) + \frac{t}{4M^2} g_P(t) = \frac{i}{2M} \frac{\langle p' | \frac{n_f \alpha_s}{4\pi} F \tilde{F} | p \rangle}{\bar{u}(p') \gamma_5 u(p)}$$

We now derived the distributions as well as the 't' at the proton (not partonic) level.

# Anomaly-mediated relations between twist-2,3,4 GPDs



## Main findings

**Non-local chiral anomaly equation at the operator level:**

$$\mathcal{D}_\mu [\bar{\psi}(z_2^-)W\gamma^\mu\gamma_5\psi(z_1^-)] = O_F(z_2^-, z_1^-) - \frac{n_f\alpha_s}{2\pi} \int_0^1 d\alpha \int_0^{1-\alpha} d\beta F^{\mu\nu}(z_{12}^{\beta-}) \tilde{W} \tilde{F}_{\mu\nu}(z_{21}^{\alpha-}) + \dots$$

↓

Twist-4 GPD:  $O_F(z_2, z_1) \equiv iz^\nu \int_0^1 d\alpha \bar{\psi}(z_2)\gamma^\mu\gamma_5 W g F_{\mu\nu}(z_{21}^\alpha) W \psi(z_1),$

$$z^\mu \equiv z_1^\mu - z_2^\mu, \quad (z_{21}^\alpha)^\mu \equiv \alpha z_2^\mu + (1-\alpha)z_1^\mu$$

- The full non-Abelian contribution of  $F\tilde{F}$  and the Wilson line is included, unlike in perturbative calculations.
- Agreement with Mueller, Teryaev, 1997, but we further clarified the accuracy of this relation (leading log).

# Anomaly-mediated relations between twist-2,3,4 GPDs



## Main findings

**Non-local chiral anomaly equation at the operator level:**

$$\mathcal{D}_\mu [\bar{\psi}(z_2^-) W \gamma^\mu \gamma_5 \psi(z_1^-)] = O_F(z_2^-, z_1^-) - \frac{n_f \alpha_s}{2\pi} \int_0^1 d\alpha \int_0^{1-\alpha} d\beta F^{\mu\nu}(z_{12}^{\beta-}) \tilde{W} \tilde{F}_{\mu\nu}(z_{21}^{\alpha-}) + \dots$$

**Byproduct:**

In the **local limit**, one can make use of the following identity:

$$4z^\nu F_{\nu\mu} \tilde{F}^{\mu\rho} z_\rho = -z^2 F^{\mu\nu} \tilde{F}_{\mu\nu}$$

This showed us that the symmetric limit procedure  $\lim_{x \rightarrow 0} \frac{x^\mu x^\nu}{x^2} \rightarrow \frac{g^{\mu\nu}}{4}$  is actually unnecessary even in Peskin's textbook derivation of the chiral anomaly.

# Anomaly-mediated relations between twist-2,3,4 GPDs



## Main findings

Non-local chiral anomaly equation with proton matrix element:

$$\Delta_{\mu} P^{+} \int \frac{dz^{-}}{2\pi} e^{ixP^{+}z^{-}} \langle p' | \bar{\psi}(-z^{-}/2) W \gamma^{\mu} \gamma_5 \psi(z^{-}/2) | p \rangle = \frac{n_f \alpha_s M}{2\pi} \tilde{C}^{anom} \otimes \tilde{\mathcal{F}}(x, \xi, t) + O_F(x, \xi, t)$$



$$iO_F(x, \xi, t) = P^{+} \int \frac{dz^{-}}{2\pi} e^{ixP^{+}z^{-}} \langle p' | O_F(-z^{-}/2, z^{-}/2) | p \rangle$$



# Anomaly-mediated relations between twist-2,3,4 GPDs



## Main findings

### 1) Relations between the twist-2 and twist-4 GPDs of the proton mediated by the chiral anomaly:

$$\tilde{E} + \tilde{E}_4 = \frac{4M^2}{t} \left( \frac{n_f \alpha_s}{2\pi} \tilde{C}^{anom} \otimes (\tilde{\mathcal{F}}_2 + \tilde{\mathcal{F}}_4) - \tilde{H} - \tilde{H}_4 + O_{2F} + O_{4F} \right), \quad \left( t = \text{Hadron-level Variable} \right)$$

This is the **anomaly pole** that has been identified in the partonic level calculations of the DVCS amplitude & GPDs (SB, Hatta, Vogelsang) & in (Tarasov, Vengugopalan) for polarized DIS

# Anomaly-mediated relations between twist-2,3,4 GPDs



## Main findings

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↓  
 $\eta_0$  pole

$$O_F(x, \xi, t) = \bar{u}(p') \left[ \Delta^- \gamma^+ \gamma_5 O_{F2} + \Delta_i \gamma^i \gamma_5 O_{F3} + \Delta^+ \gamma^- \gamma_5 O_{F4} \right] u(p)$$

Thus, we have explicitly derived the  $\eta_0$  pole and find that its residue consists of the polarized GPD  $\tilde{H}$  and twist-four GPDs  $O_F$

# Anomaly-mediated relations between twist-2,3,4 GPDs



## Main findings

### 1) Relations between the twist-2 and twist-4 GPDs of the proton mediated by the chiral anomaly:

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↓  $\int dx$

$$g_A(t) + \frac{t}{4M^2} g_P(t) = \frac{i}{2M} \frac{\langle p' | \frac{n_f \alpha_s}{4\pi} F \tilde{F} | p \rangle}{\bar{u}(p') \gamma_5 u(p)}$$

Upon integrating, we exactly reproduce the pole-cancellation expression for the form factor

# Anomaly-mediated relations between twist-2,3,4 GPDs



## Main findings

1) Relations between the twist-2 and twist-4 GPDs of the proton mediated by the chiral anomaly:

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2) Relations between the twist-3 and twist-4 GPDs of the proton mediated by the chiral anomaly:

$$\tilde{E}_3 = \frac{4M^2}{t} \left( \frac{n_f \alpha_s}{2\pi} \tilde{C}^{anom} \otimes \tilde{\mathcal{F}}_3 - \tilde{H}_3 + O_{F3} \right), \quad \left( t = \begin{array}{l} \text{Hadron-} \\ \text{level} \\ \text{Variable} \end{array} \right)$$

The result provides a nonperturbative foundation for the anomaly pole previously identified in perturbation theory, further strengthening all our conclusions.

# Anomaly-mediated relations between twist-2,3,4 GPDs



**How does turning on quark masses modify the results?**

# Anomaly-mediated relations between twist-2,3,4 GPDs



## How does turning on quark masses modify the results?

### Remarks:

- As discussed, in perturbative calculations with **massless fermions**, the **anomaly manifests** as a pole at  $t = 0$ .
- When the **fermion has a finite mass**, the **anomaly pole disappears**. Instead of a pole, the singularity is replaced by a **branch cut** in the time like region ( $t > 0$ ). This corresponds to the threshold for real fermion-antifermion pair production. (This is **relevant for QED**; see Adler, Bardeen, 69; Castelli, Freese, Lorcé, Metz, Pasquini, Rodini, 24.)

# Anomaly-mediated relations between twist-2,3,4 GPDs



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- In **QCD**, a similar **threshold** for quark-antiquark pair production at  $t > 4m_q^2$  **cannot exist** because of confinement.

# Anomaly-mediated relations between twist-2,3,4 GPDs



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- In **QCD**, a similar **threshold** for quark-antiquark pair production at  $t > 4m_q^2$  **cannot exist** because of confinement.
- So, what happens?

**Nucleon isovector axial form factor & GPD:**  
(Nambu, 1960; Penttinen, Polyakov, Goeke, 2000)

$$g_P^{(3)}(t) \sim \frac{1}{t} \xrightarrow[\substack{\text{quark mass} \\ (m_q \neq 0)}]{\text{switch on}} \frac{1}{t - m_\pi^2}$$
$$\tilde{E}^{(3)}(t) \sim \frac{1}{t - m_\pi^2}$$

**Non-perturbative  
shift in pole:**

$$m_\pi^2 \propto m_q$$

**Still a pole!**



# Anomaly-mediated relations between twist-2,3,4 GPDs



## How does turning on quark masses modify the results?

### Remarks:

- As discussed, in perturbative calculations with **massless fermions**, the **anomaly manifests** as a pole at  $t = 0$ .
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- In **QCD**, a similar **threshold** for quark-antiquark pair production at  $t > 4m_q^2$  **cannot exist** because of confinement.
- So, what happens?

### Nucleon singlet axial form factor & GPD:

(Witten Veneziano 1979;  
SB, Hatta, Schoenleber, 2024;  
Tarasov, Venugopalan, 2025)

$$g_P(t) \sim \frac{1}{t} \xrightarrow{(m_q \neq 0)} \frac{1}{t - m_\pi^2}$$

**Still an anomaly pole!**

# Anomaly-mediated relations between twist-2,3,4 GPDs

## How does turning on quark masses modify the results?

### Remarks:

- As discussed, in perturbative calculations with **massless fermions**, the **anomaly manifests** as a pole at  $t = 0$ .
- When the **fermion has a finite mass**, the **anomaly pole disappears**. Instead of a pole, the singularity is replaced by a **branch cut** in the time like region ( $t > 0$ ). This corresponds to the threshold for real fermion-antifermion pair production. (This is **relevant for QED**.)
- In **QCD**, a similar **threshold** for quark-antiquark pair production at  $t > 4m_q^2$  **cannot exist** because of confinement.
- So, what happens?

### Nucleon singlet axial form factor & GPD:

(Witten Veneziano 1979;  
SB, Hatta, Schoenleber, 2024;  
Tarasov, Venugopalan, 2025)

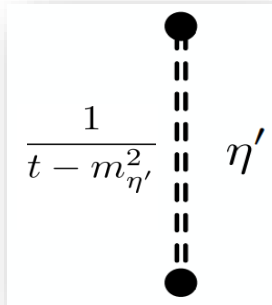
$$g_P(t) \sim \frac{1}{t} \xrightarrow{(m_q \neq 0)} \frac{1}{t - m_\pi^2} \xrightarrow{\text{Resummation}} \frac{1}{t - m_\pi^2 + \frac{4n_f}{f_{\eta'}^2} \chi} = \frac{1}{t - m_{\eta'}^2}$$

$$\tilde{E}(t) \sim \frac{1}{t - m_{\eta'}^2} \quad \text{Still an anomaly pole!} \quad \text{Topological susceptibility}$$

# Summary

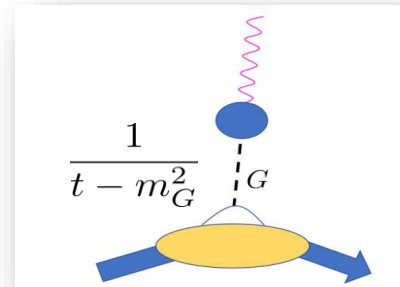


- 1) We calculated one-loop Compton amplitude in all channels (quark/gluon, polarized/unpolarized) using  $t$  as a regulator and demonstrated **factorization**
- 2) The physics of anomalies present at the level of Form Factor is established for the first time at the level of GPDs



**Eta meson mass generation:**

$$\tilde{E}(x) \sim \frac{1}{t - m_{\eta'}^2}$$



**Glueball mass generation:**

$$H(x), E(x) \sim \frac{1}{t - m_G^2}$$

**GPDs encode profound aspects of QCD such as symmetry breakings and mass generations:**

Reach out to a broader QCD community

# Backup slides

# Imprint of Anomalies in QCD Compton scattering



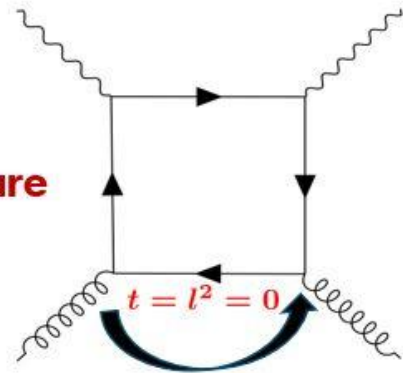
## Elusive pole

### Box diagram in off-forward kinematics

Setup of calculation:

Calculated box diagrams with  $t = l^2 \neq 0$  whereas all existing works in the literature set  $t = 0$  ( $\xi \neq 0$ )

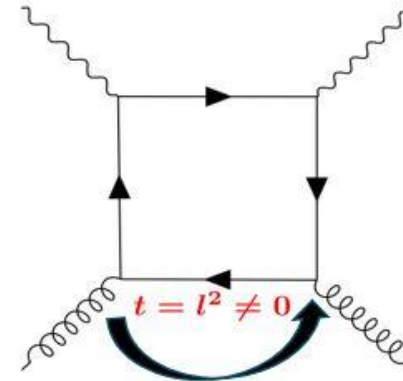
**GPD literature**



$$p_1 = p - \frac{l}{2} \approx p(1 + \xi)$$

Only longitudinal momentum transfer

**This work**



Transverse momentum transfer essential to capture physics of anomalies

$$\begin{aligned}
 \Delta^- P^+ \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle p' | \bar{\psi}(-z/2) \gamma^+ \gamma_5 \psi(z/2) | p \rangle &= \Delta^- \bar{u}(p') \left[ \tilde{H} \gamma^+ \gamma_5 + \tilde{E} \frac{\Delta^+}{2M} \gamma_5 \right] u(p), \\
 \Delta_i P^+ \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle p' | \bar{\psi}(-z/2) \gamma^i \gamma_5 \psi(z/2) | p \rangle \\
 &= \Delta_i \bar{u}(p') \left[ \tilde{H}_3 \gamma^i \gamma_5 + \tilde{E}_3 \frac{\Delta^i}{2M} \gamma_5 + \tilde{G}_3 \frac{\Delta^i}{P^+} \gamma^+ \gamma_5 + i \tilde{G}'_3 \epsilon^{ij} \frac{\Delta^j}{P^+} \gamma^+ \right] u(p), \\
 \Delta^+ P^+ \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle p' | \bar{\psi}(-z/2) \gamma^- \gamma_5 \psi(z/2) | p \rangle &= \Delta^+ \bar{u}(p') \left[ \tilde{H}_4 \gamma^- \gamma_5 + \tilde{E}_4 \frac{\Delta^-}{2M} \gamma_5 \right] u(p).
 \end{aligned} \tag{33}$$

All the GPDs are functions of  $x, \xi$  and  $t$  (and the renormalization scale),  $\tilde{H} = \tilde{H}(x, \xi, t)$ , etc. Also, the summation over quark flavors is implied,  $\tilde{H} = \sum_q \tilde{H}_q$ , etc. The twist-3 GPDs are from [24] where we redefined  $\tilde{H} + \tilde{G}_2 \rightarrow \tilde{H}_3$  and  $\tilde{E} + \tilde{G}_1 \rightarrow \tilde{E}_3$ . (We also redefined  $\tilde{G}_4 \rightarrow \tilde{G}'_3$ .) The twist-4 GPDs are parametrized differently from [25] but the two parametrizations are equivalent in the present frame  $P^i = 0$ . On the right hand side, the twist-4 pseudoscalar GPD (25) is parametrized as [11]

$$\tilde{\mathcal{F}}(x, \xi, t) = \frac{1}{M} \bar{u}(p') \left[ \Delta^- \gamma^+ \gamma_5 \tilde{\mathcal{F}}_2 + \Delta_i \gamma^i \gamma_5 \tilde{\mathcal{F}}_3 + \Delta^+ \gamma^- \gamma_5 \tilde{\mathcal{F}}_4 \right] u(p). \tag{34}$$

For phenomenological purposes, one may implement various approximations. If one ignores the differences due to different twists, namely  $\tilde{H}_{3,4} \approx \tilde{H}$ ,  $\tilde{\mathcal{F}}_{3,4} \approx \tilde{\mathcal{F}}_2$ , etc., from (32) one immediately obtains

$$\tilde{E}(x, \xi, t) \approx \frac{4M^2}{t} \left( \frac{n_f \alpha_s}{2\pi} \tilde{C}^{anom} \otimes \tilde{\mathcal{F}}_2 - \tilde{H} + O_{F2} \right). \tag{49}$$