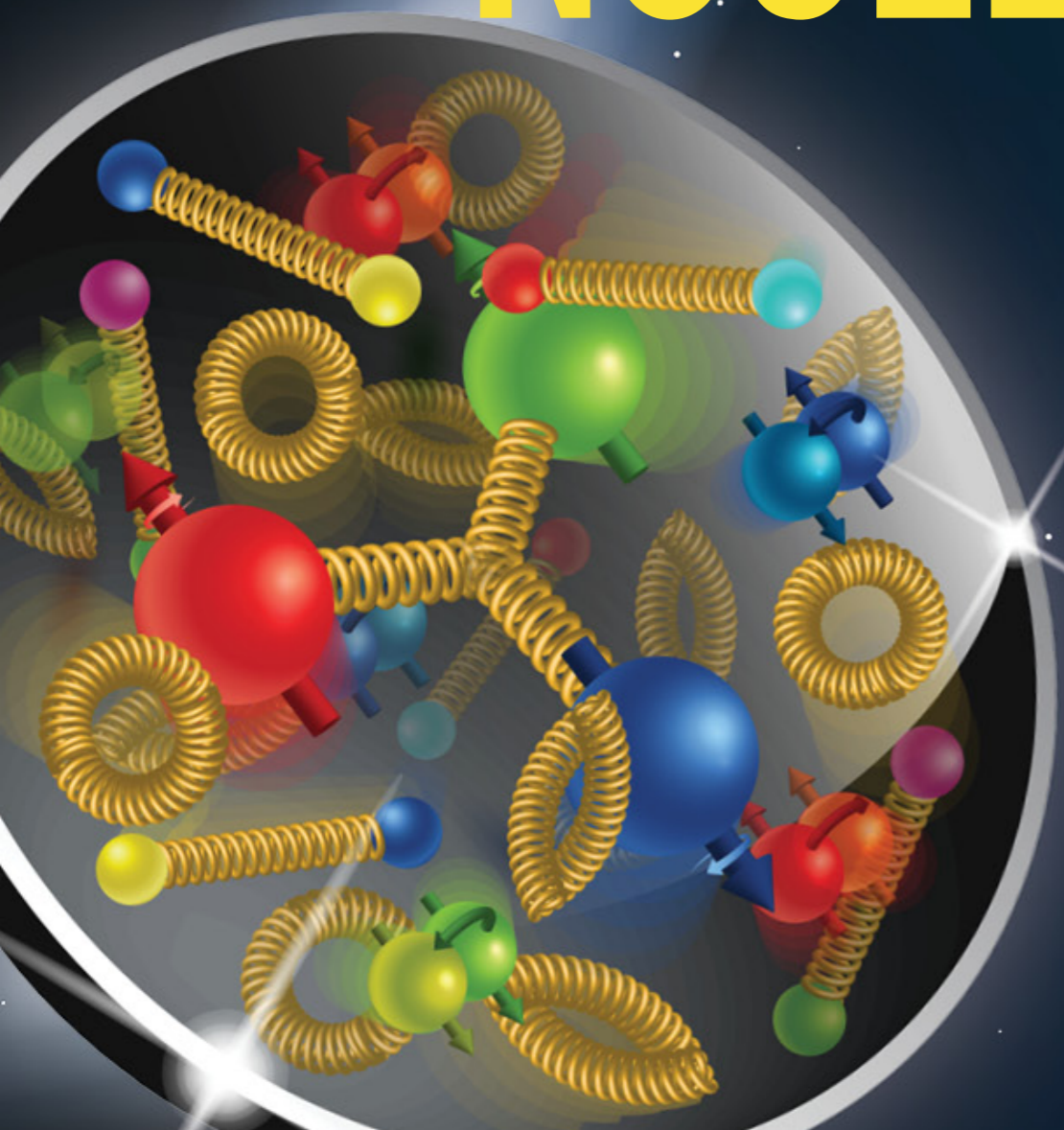




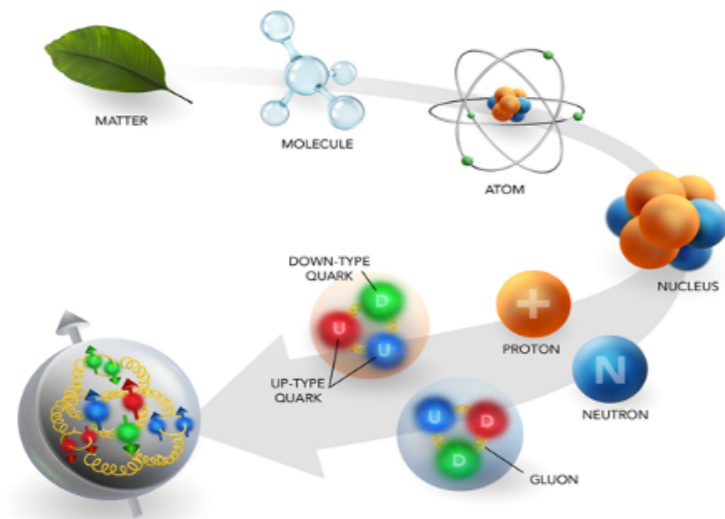
# THREE-DIMENSIONAL NUCLEON STRUCTURE

## AT THE EIC

Alexei Prokudin  
PSU Berks and JLab

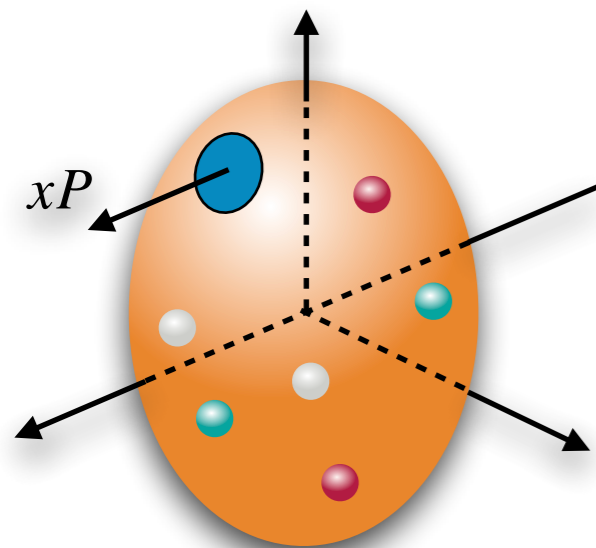


# UNRAVELLING THE MYSTERIES OF RELATIVISTIC HADRONIC BOUND STATES



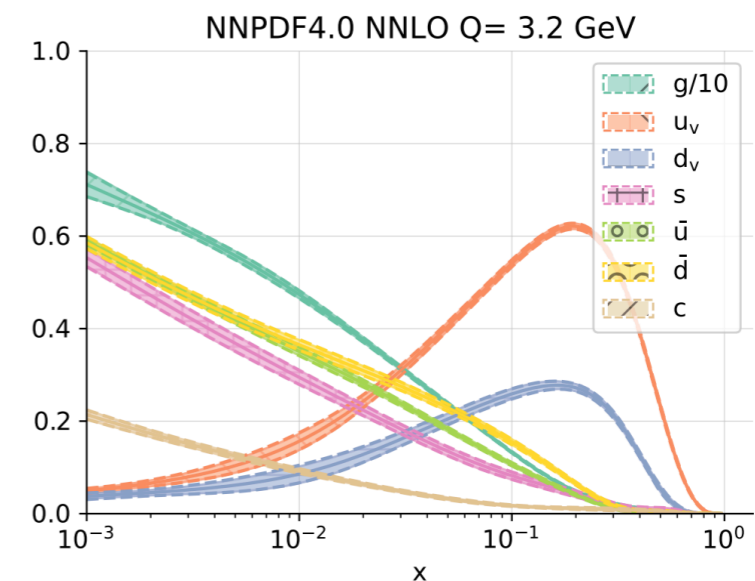
- Nucleons provide 98% of the mass of the visible universe
- One of the goals of the modern nuclear physics is to study details of the structure of the nucleon

Parton Distribution Functions provide fundamental description



$$f_{q/P}(x)$$

longitudinal

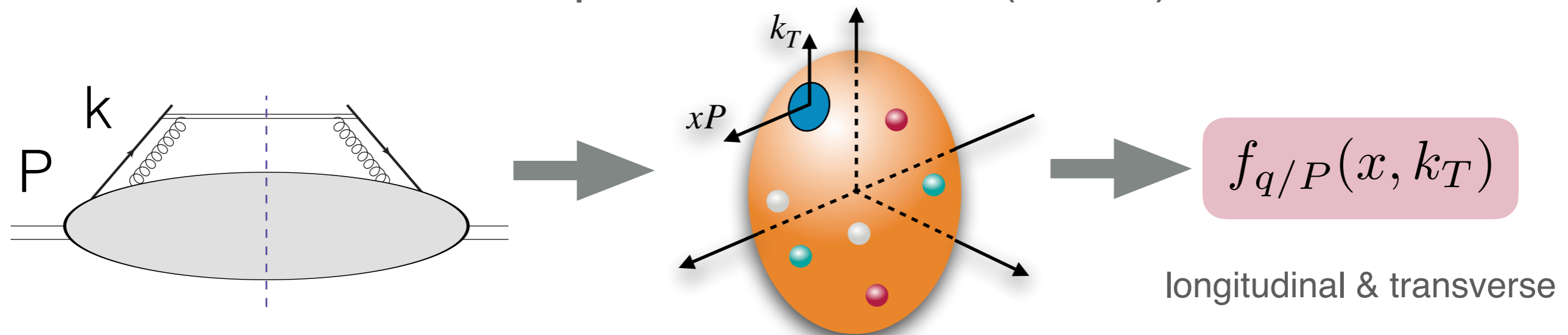


- Probability density to find a quark with a momentum fraction  $x$
- 1D snapshot of fundamental constituents
- Study of confined quarks and gluons

# HADRON'S PARTONIC STRUCTURE

To study the physics of *confined motion of quarks and gluons* inside of the proton one needs a new type “hard probe” with two scales.

Transverse Momentum Dependent functions (TMDs)



- One large scale ( $Q$ ) sensitive to particle nature of quark and gluons
- One small scale ( $k_T$ ) sensitive to *how QCD bounds partons* and to the detailed structure at  $\sim$ fm distances.
- TMDs provide detailed information on the spin structure
- TMDs contain new probes, e.g. qgq operators rather than just qq or gg and thus include correlations
- TMDs encode 3D structure in the momentum space (complementary to GPDs)

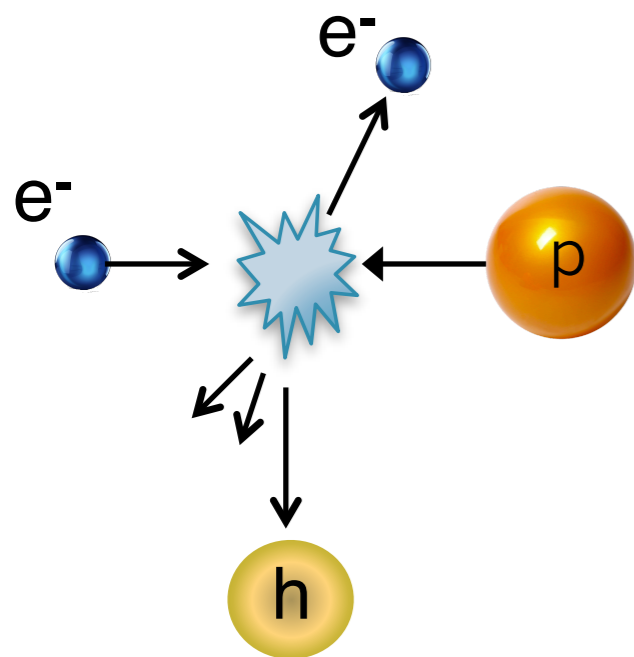
# TRANSVERSE MOMENTUM DEPENDENT FACTORIZATION

Small scale  $\longrightarrow q_T \ll Q \longleftarrow$  Large scale

The confined motion ( $k_T$  dependence) is encoded in TMDs

## Semi-Inclusive DIS

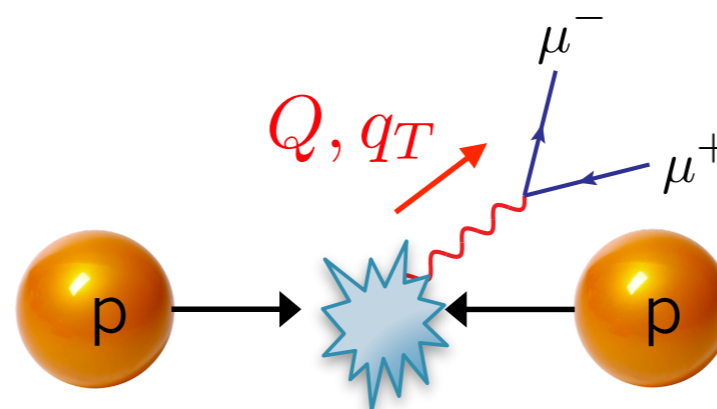
$$\sigma \sim f_{q/P}(x, k_T) D_{h/q}(z, k_T)$$



Meng, Olness, Soper (1992)  
 Ji, Ma, Yuan (2005)  
 Idilbi, Ji, Ma, Yuan (2004)  
 Collins (2011)

## Drell-Yan

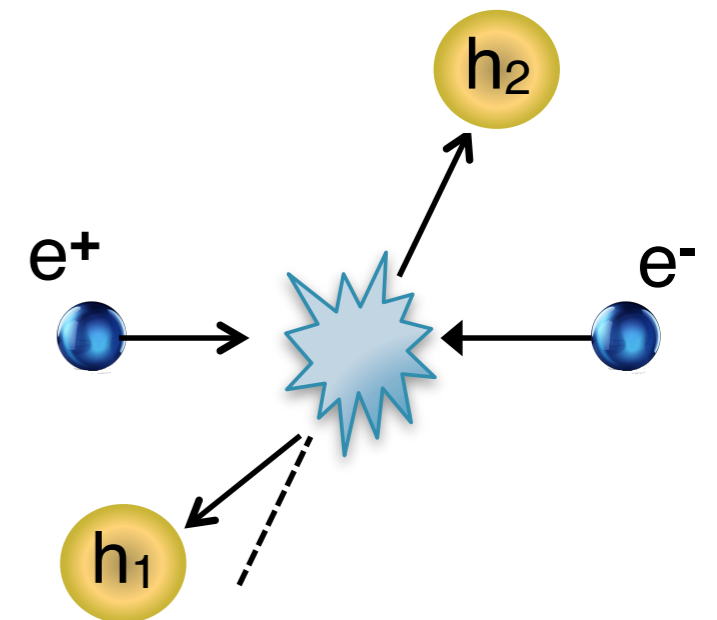
$$\sigma \sim f_{q/P}(x, k_T) f_{q/P}(x, k_T)$$



Collins, Soper, Sterman (1985)  
 Ji, Ma, Yuan (2004)  
 Collins (2011)

## Dihadron in e<sup>+</sup>e<sup>-</sup>

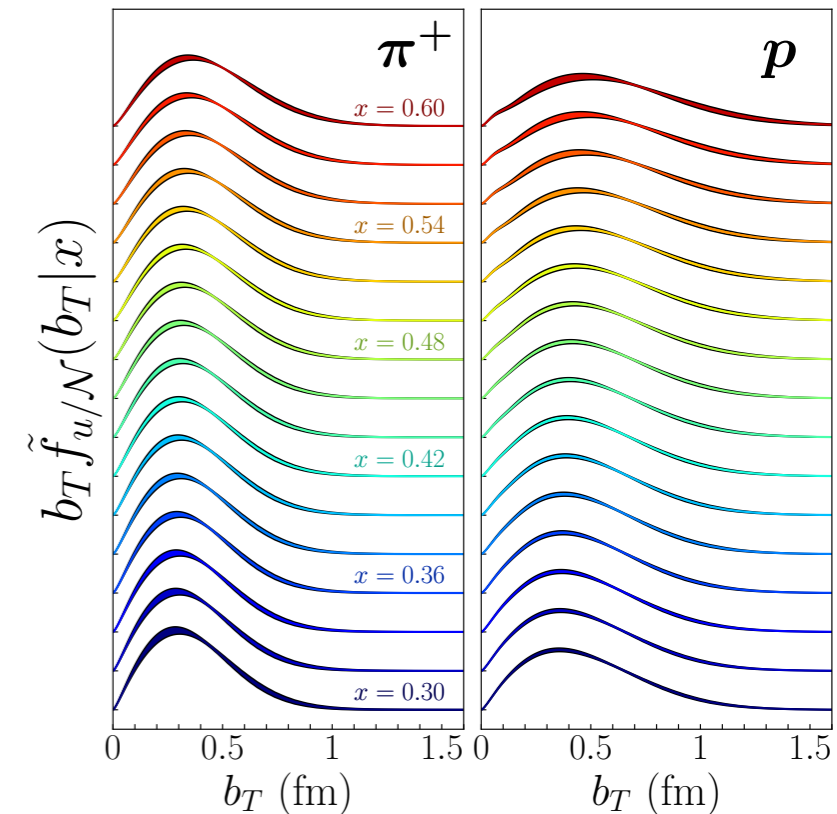
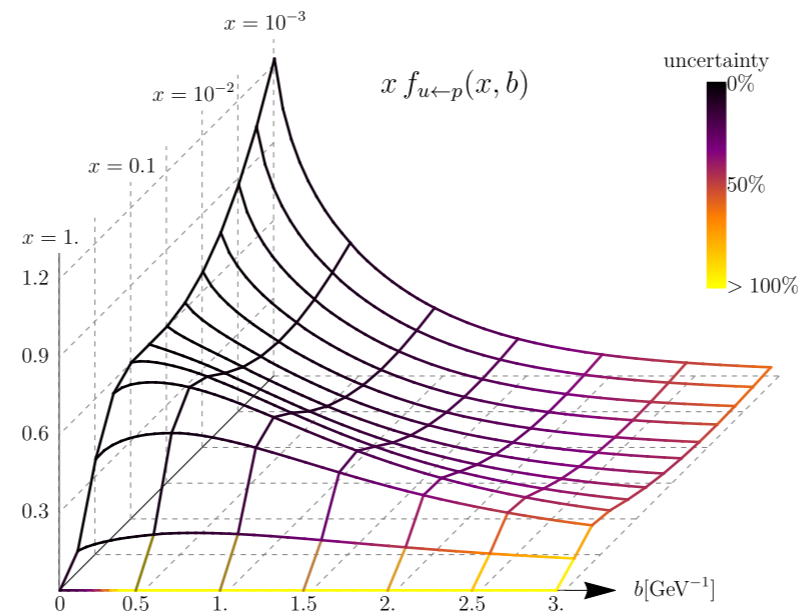
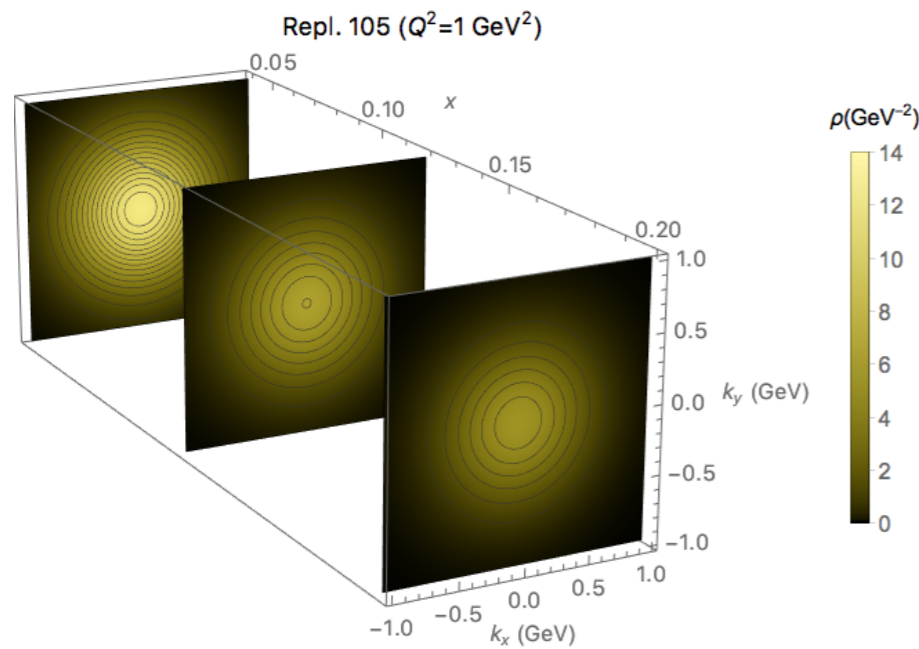
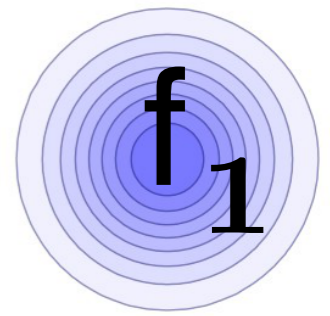
$$\sigma \sim D_{h_1/P}(z, k_T) D_{h_2/q}(z, k_T)$$



Collins, Soper (1983)  
 Collins (2011)

# UNPOLARIZED STRUCTURE OF THE NUCLEON

# UNPOLARIZED TMD MEASUREMENTS



Bacchetta, Delcarro, Pisano, Radici, Signori, arXiv:1703.10157

V. Moos, I. Scimemi, A. Vladimirov, P. Zurita arXiv:2305.07473

P. Barry, L. Gamberg, W. Melnitchouk, E. Moffat, D. Pitonyak, AP, N. Sato Phys.Rev.D 108 (2023) 9, L091504

- Addresses the question of partonic confined motion
- Evolution with  $x$  and  $Q^2$
- Flavor dependence of unpolarized TMDs
- Interplay with collinear QCD at large  $q_T$

# TMD FITS OF UNPOLARIZED DATA

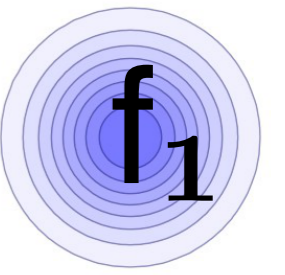
	Framework	W+Y	HERMES	COMPASS	DY	Z boson	W boson	N of points
KN 2006 <a href="#">hep-ph/0506225</a>	LO-NLL	W	✗	✗	✓	✓	✗	98
QZ 2001 <a href="#">hep-ph/0506225</a>	NLO-NLL	W+Y	✗	✗	✓	✓	✗	28 (?)
RESBOS <a href="#">resbos@msu</a>	NLO-NNLL	W+Y	✗	✗	✓	✓	✗	>100 (?)
Pavia 2013 <a href="#">arXiv:1309.3507</a>	LO-PM	W	✓	✗	✗	✗	✗	1538
Torino 2014 <a href="#">arXiv:1312.6261</a>	LO-PM	W	✓ (separately)	✓ (separately)	✗	✗	✗	576 (H) 6284 (C)
DEMS 2014 <a href="#">arXiv:1407.3311</a>	NLO-NNLL	W	✗	✗	✓	✓	✗	223
EIKV 2014 <a href="#">arXiv:1401.5078</a>	LO-NLL	W	1 (x,Q <sup>2</sup> ) bin	1 (x,Q <sup>2</sup> ) bin	✓	✓	✗	500 (?)
SIYY 2014 <a href="#">arXiv:1406.3073</a>	NLO-NLL	W+Y	✗	✓	✓	✓	✗	200 (?)
Pavia 2017 <a href="#">arXiv:1703.10157</a>	LO-NLL	W	✓	✓	✓	✓	✗	8059
SV 2017 <a href="#">arXiv:1706.01473</a>	NNLO-NNLL	W	✗	✗	✓	✓	✗	309
BSV 2019 <a href="#">arXiv:1902.08474</a>	NNLO-NNLL	W	✗	✗	✓	✓	✗	457
Pavia 2019 <a href="#">arXiv:1912.07550</a>	NNLO-N3LL	W	✗	✗	✓	✓	✗	353
SV 2019 <a href="#">arXiv:1912.06532</a>	NNLO-N3LL	W	✓	✓	✓	✓	✗	1039
MAP pion 2022 <a href="#">arXiv:2210.01733</a>	NLO-N3LL	W	✗	✗	✓	✗	✗	138
MAP 2022 <a href="#">arXiv:2206.07598</a>	NNLO-N3LL-	W	✓	✓	✓	✓	✗	2031
JAM 2023 <a href="#">arXiv:2302.01192</a>	NLO-NNLL	W	✗	✗	✓	✗	✗	608
ART 2023 <a href="#">arXiv:2305.07473</a>	N3LO-N4LL	W	✗	✗	✓	✓	✓	627
Aslan et al 2024 <a href="#">arXiv:2401.14266</a>	NLO-NLL	W	✗	✗	✓	✗	✗	130
MAP 2025 <a href="#">arXiv:2502.04166</a>	NNLO-N3LL	W	✗	✗	✓	✓	✗	482

# TMD ANALYSES

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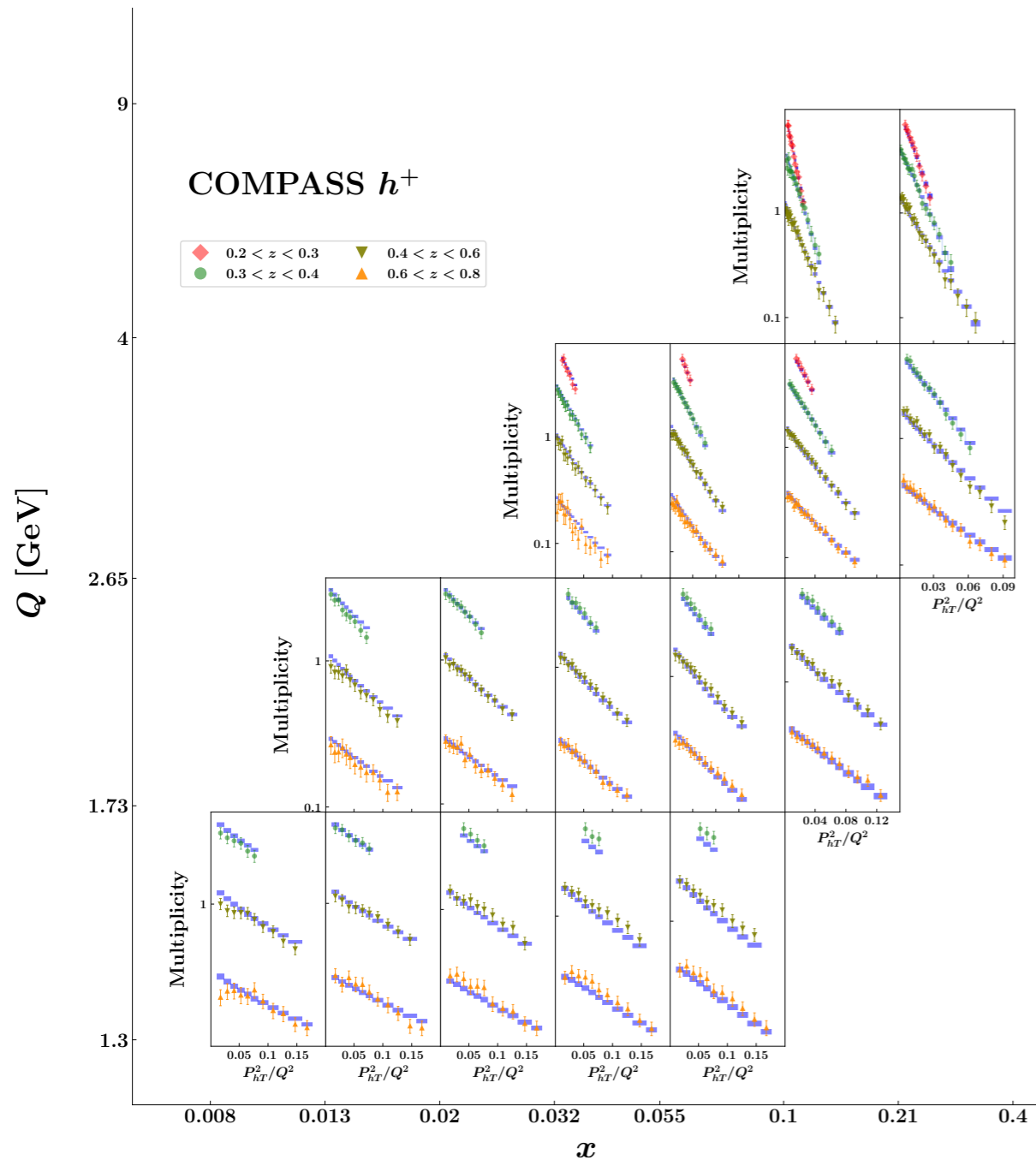
- Usually implement the data cut  $q_T/Q < 0.2 \div 0.25$  to minimize power corrections (W term only)
- High perturbative accuracy and OPE matching to collinear PDFs. Good perturbative convergence
- Neglecting small higher twist contributions (i.e. Boer-Mulders)
- Non perturbative TMD behavior in  $b_T$  and  $x$  - dependent, either flavor dependent or not. Usage of NN in the latest analysis
- Some differences in solutions of evolution equations and separation of perturbative and non perturbative contributions





# UNPOLARIZED SIDIS TMD MEASUREMENTS

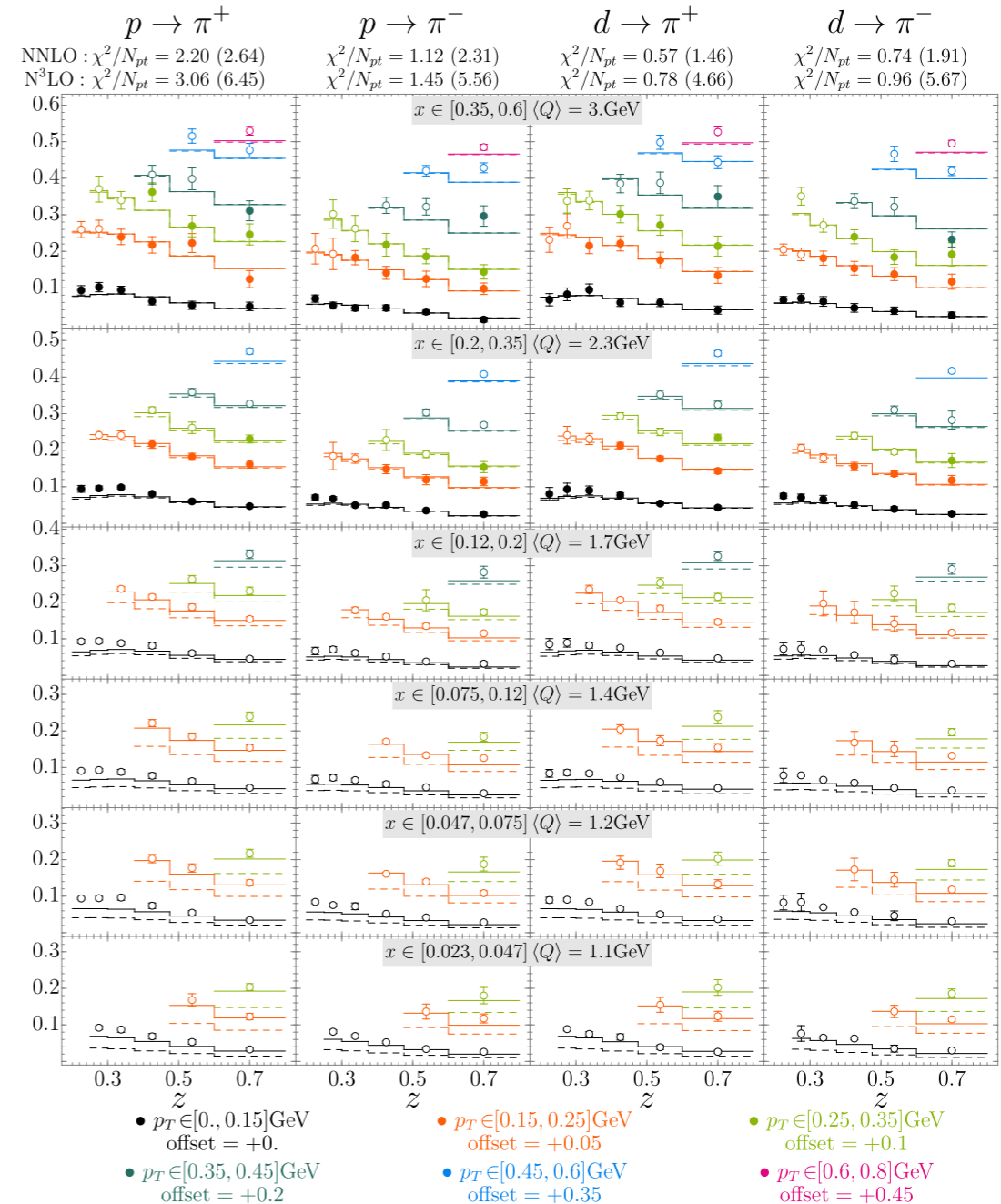
Combination of various processes is important for the tests of universality



MAP22: Bacchetta et al, JHEP 10 (2022) 127

$$\frac{d\sigma_{\omega}^{\text{SIDIS}}}{dx dz d|\mathbf{q}_T| dQ} = \omega(x, z, Q) \frac{d\sigma^{\text{SIDIS}}}{dx dz d|\mathbf{q}_T| dQ}$$

## HERMES



Ignazio Scimemi, Alexey Vladimirov JHEP 06 (2020) 137

# CHALLENGES

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Low  $q_T$  SIDIS data requires normalization of the theory (at least MAP22)

*MAP22: Bacchetta et al, JHEP 10 (2022) 127*

$$\omega(x, z, Q) = \frac{d\sigma^{\text{nomix}}}{dx dz dQ} / \int d^2\mathbf{q}_T W$$

At NLL,  $\omega(x, z, Q) = 1$ . Beyond NLL, the prefactor becomes larger than one and guarantees that the integral of the TMD part of the cross section reproduces most of the collinear cross section, as suggested by the data.

Hard factor to blame?

*MAP22: Bacchetta et al, JHEP 10 (2022) 127*

Should we use a different way of matching to collinear fixed order results?

*Aslan et al Phys.Rev.D 110 (2024) 7, 074016*

Should we understand better SIDIS?

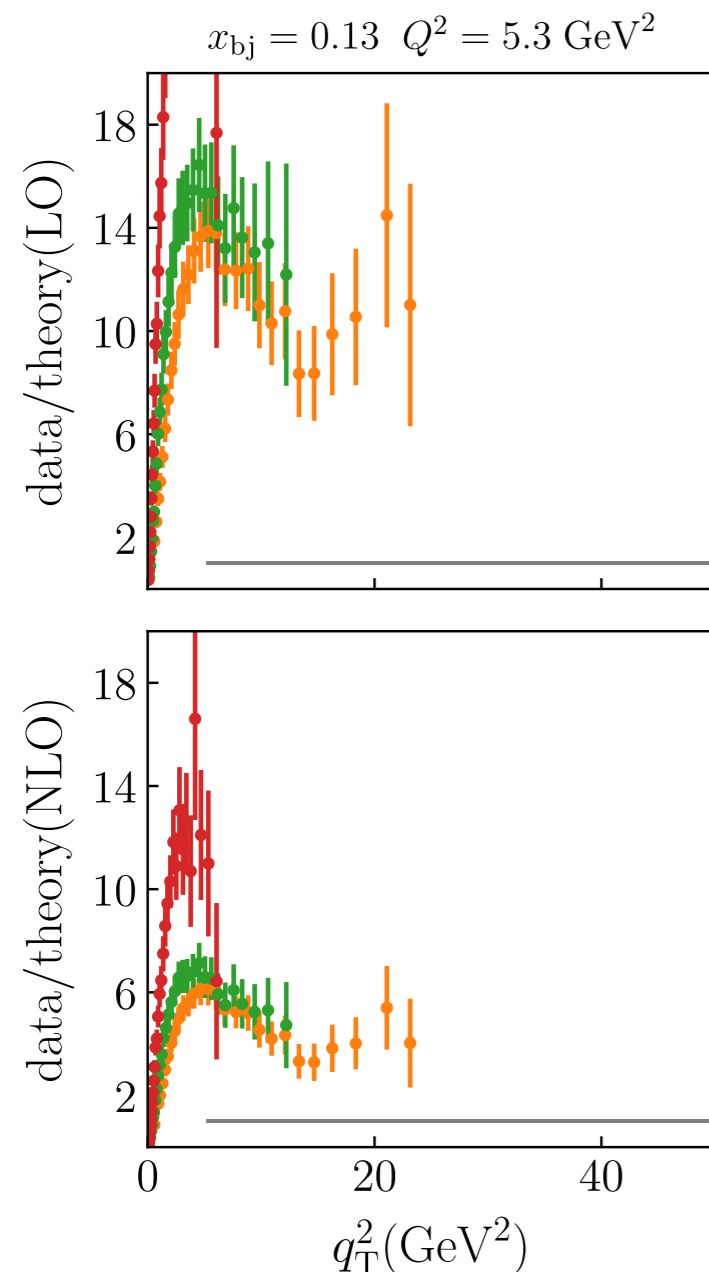
*Jefferson Lab Angular Momentum (JAM), M. Boglione et al JHEP 04 (2022) 084  
M. Boglione et al, Phys.Lett.B 766 (2017) 245-25*

# CHALLENGES

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Large  $q_T$  SIDIS data are in tension with the NLO calculations

*Gonzalez-Hernandez et al Phys.Rev.D 98 (2018) 11, 114005*



The ratio of the data/theory improves from LO to NLO, but still large

Power corrections?

*Tianbo Liu, Jian-Wei Qiu Phys.Rev.D 101 (2020)*

Incorporation of QCD and QED?

*T.Liu, W. Melnitchouk, Jian-Wei Qiu, N. Sato JHEP 11 (2021) 15*

Should we understand better SIDIS?

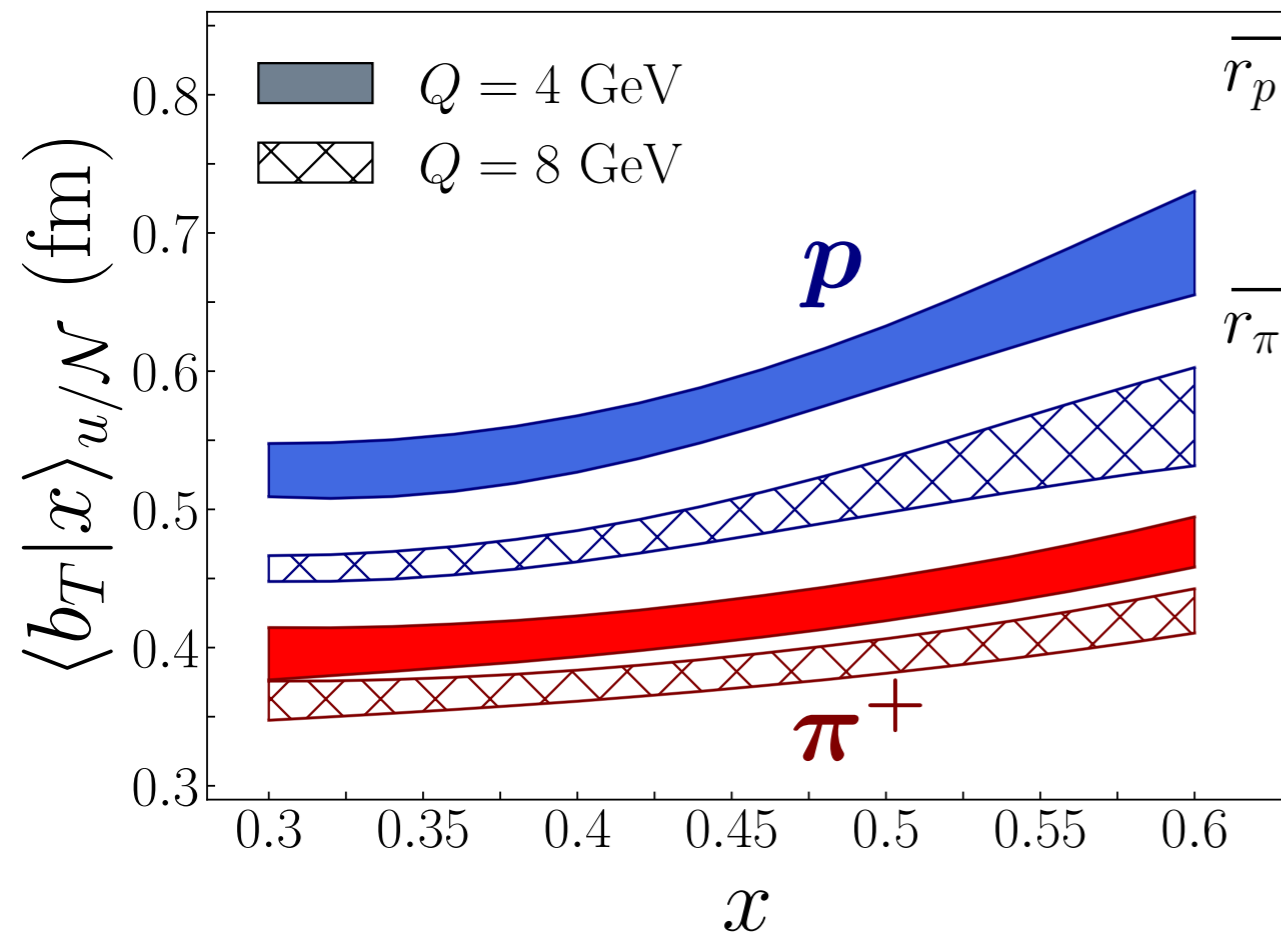
*Jefferson Lab Angular Momentum (JAM), M. Boglione et al JHEP 04 (2022) 084  
M. Boglione et al, Phys.Lett.B 766 (2017) 245-25*

# RECENT ADVANCES: TOMOGRAPHY OF PIONS AND PROTONS



The first simultaneous analysis of collinear and TMD distributions

$$\langle b_T | x \rangle_{q/\mathcal{N}} = \int d^2 \mathbf{b}_T b_T \tilde{f}_{q/\mathcal{N}}(b_T | x; Q, Q^2)$$



Resulting widths at  $Q = 4$  and  $8$  (GeV)

Q dependence is well consistent with widening due to TMD evolution

Pion's width is smaller than that of the proton at  $5.3 - 7.5 \sigma$  confidence level

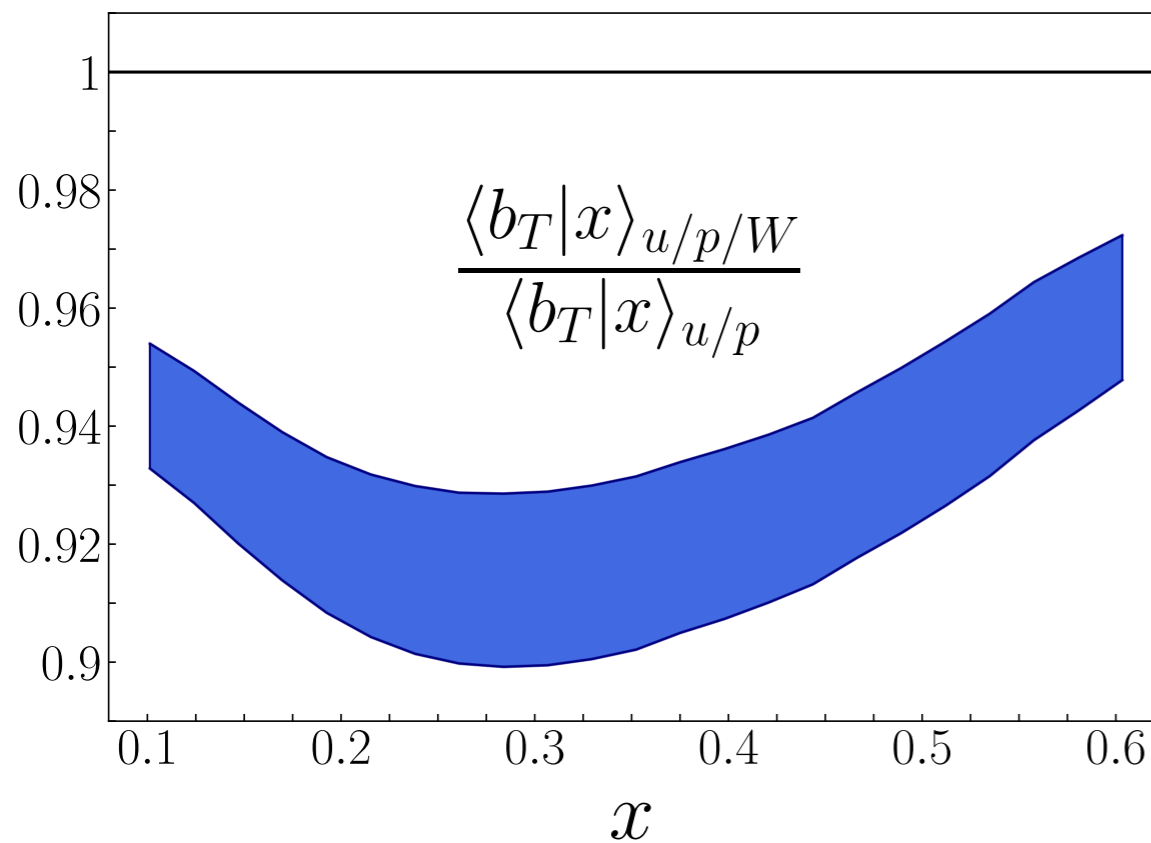
Both decrease as  $x$  decreases, consistent with the emergence of  $q\bar{q}$  condensate characterized by a scale  $\rho \sim 0.3$  (fm)

# RECENT ADVANCES: TOMOGRAPHY OF PIONS AND PROTONS



The first simultaneous analysis of collinear and TMD distributions

$$\langle b_T | x \rangle_{q/\mathcal{N}} = \int d^2 \mathbf{b}_T b_T \tilde{f}_{q/\mathcal{N}}(b_T | x; Q, Q^2)$$



Hints of the nuclear dependence of TMDs

Consistent with findings of

*M. Alrashed, D. Anderle, Z. Kang, J. Terry, H. Xing, Phys.Rev.Lett. 129 (2022)*

# RECENT ADVANCES

Oscar del Rio, Alexei Prokudin, Ignazio Scimemi, Alexey Vladimirov *Phys.Rev.D* 110 (2024)

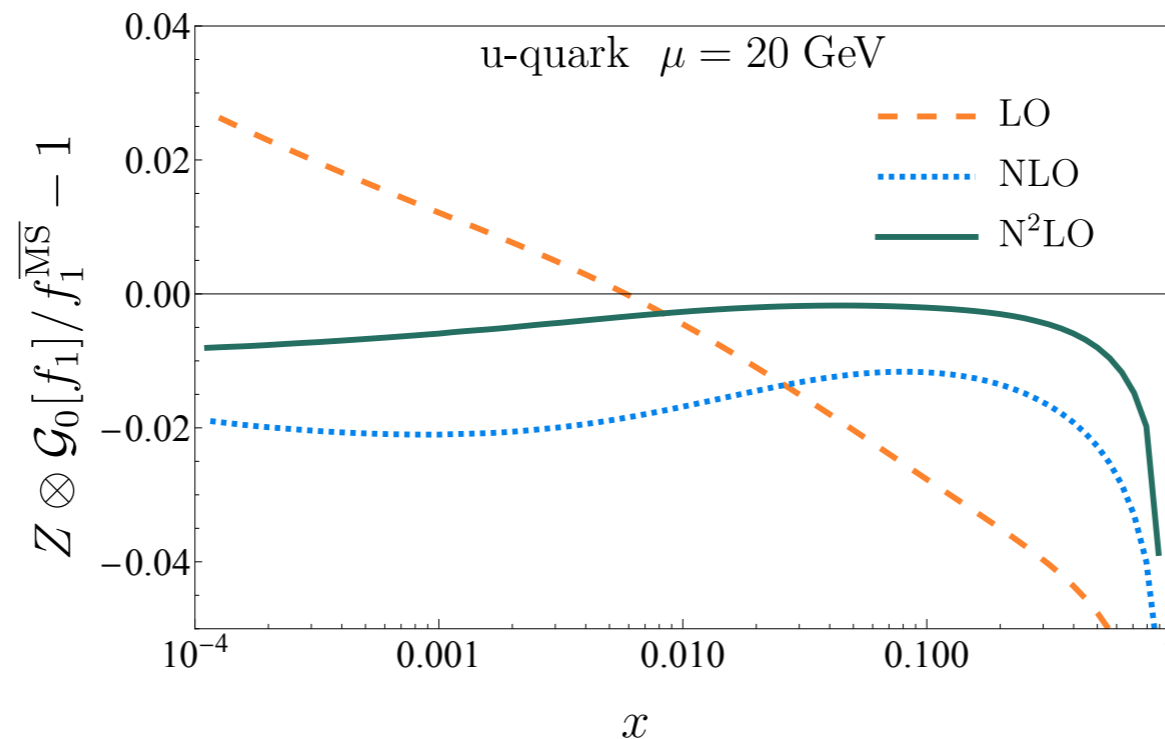
We established a robust relationship of the Transverse Momentum Moments (weighted integrals of TMDs with an upper cut-off) and collinear distributions, consistent with previous studies

*M. A. Ebert, J. K. L. Michel, I. W. Stewart and Z. Sun, JHEP 07 (2022) 129*

*J. O. Gonzalez-Hernandez, T. Rainaldi, T. C. Rogers Phys.Rev.D 107 (2023) 9, 094029*

$$\int^{\mu} d^2 k_T f(x, k_T; \mu, \zeta) = f(x; \mu)$$

We demonstrated these relations are very precise and extended them to higher moments



The usage of TMMs will be useful in the future theoretical and phenomenological studies, as well as in lattice QCD studies. They also provide a foundation of relation of the collinear QCD and TMD physics

# RECENT ADVANCES

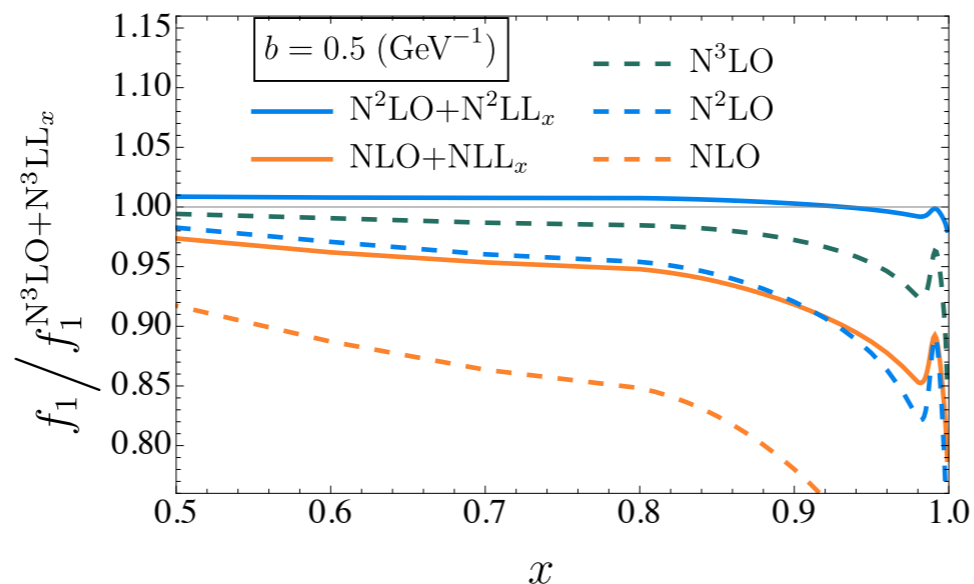
Large logarithms arise at order  $\alpha_s^n$  in the threshold regime  $x \rightarrow 1$  in the form of the “plus” distributions, up to  $\mathcal{L}_m \equiv [\ln^{m-1}(1-x)/(1-x)]_+$ , ( $m \leq 2n$ )

*G. Sterman, Nucl. Phys. B281 (1987) 310, S. Catani and L. Trentadue, Physics B 327 (1989) 323*

We demonstrated that they can be resummed in the coefficient functions

$$\lim_{b \rightarrow 0} \tilde{f}_q(x, b) = \sum_{q'} C_{qq'}(x, b) \otimes f_q(x)$$

and the result is universal for *unpolarized, helicity, and transversity* TMD and FFs

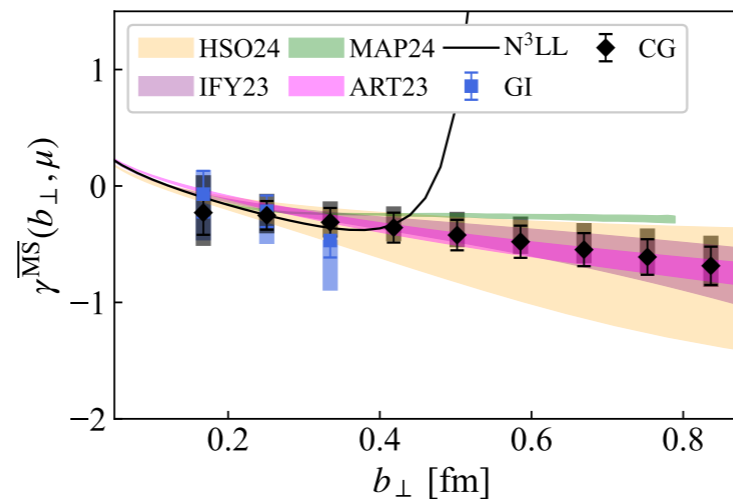
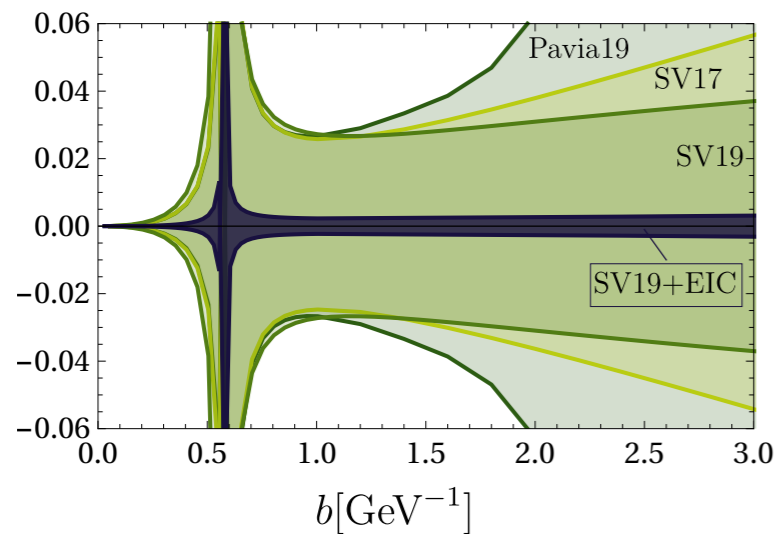


It will be very useful for TMDs where higher orders are not studied and potentially will introduce strong constraints on the non perturbative models for the Collins-Soper kernel

# OPPORTUNITIES AT THE EIC

*EIC Yellow Report, Nucl.Phys.A 1026 (2022) 122447*

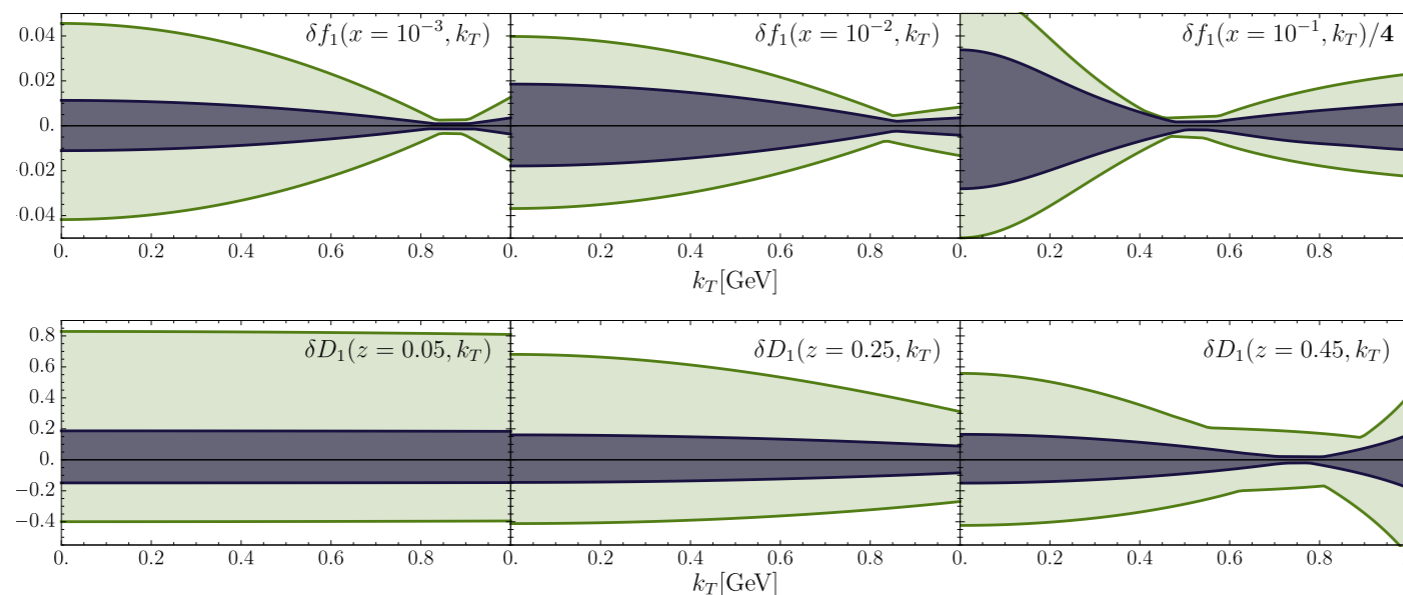
- We expect the EIC to have an impact on the knowledge of the Collins-Soper kernel, the essential ingredient of factorization



Extensively studied by lattice QCD

*A. Avkhadiev, P. Shanahan, M. Wagman, Y. Zhao Phys.Rev.Lett. 132 (2024) 23, 231901*

- We expect the impact on both TMD PDFs and TMD FFs





# CONCLUSIONS

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- TMD physics is a data driven science
- TMD studies have made great progress, they are synergistic with many other areas: lattice QCD, SCET, small-x, jets, etc
- Current: HERMES, COMPASS, JLab 12, BELLE, RHIC spin, and LHC provide great experimental measurements for TMD physics
- Future: Electron-Ion Collider, together with other experiments such as JLab 12, LHC, and BELLE II, will make significant contributions to TMD studies

