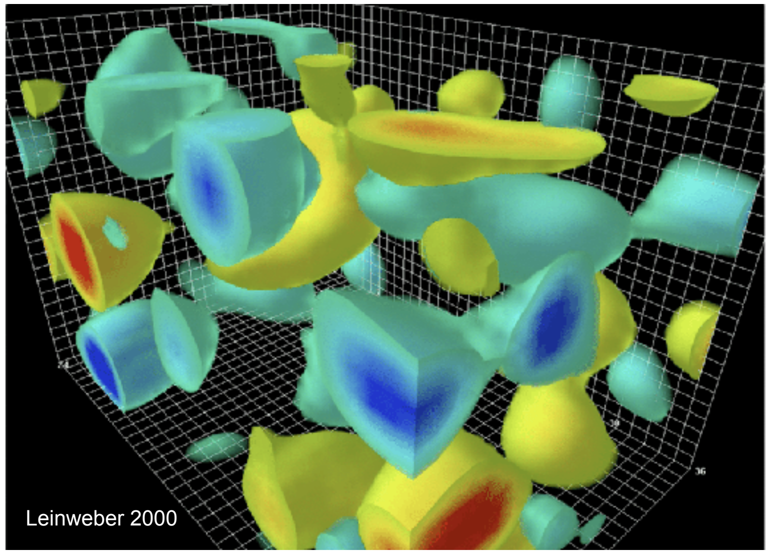
Topological gauge fields and partonic structure

C. Weiss (JLab), POETIC XI, 25-Feb-2025

Jefferson Lab



Topological gauge fields

↓

Chiral symmetry breaking

↓

Quark/gluon structure

Liu, Shuryak, Weiss, Zahed, PRD 110, 054021 (2024) [INSPIRE]

Liu, Shuryak, Zahed, 2023+

J-Y Kim, Weiss, PLB 848, 138387 (2024) [INSPIRE]

Diakonov, Polyakov, Weiss, NPB 461, 539 (1996) [INSPIRE]

Topological vacuum fluctuations

Lattice QCD configurations

Chiral symmetry breaking from zero modes

Effective description

Effective dynamics from chiral symmetry breaking

Effective operators for QCD operators

Partonic structure

Twist-2: Quark/antiquark and gluon PDFs/GPDs

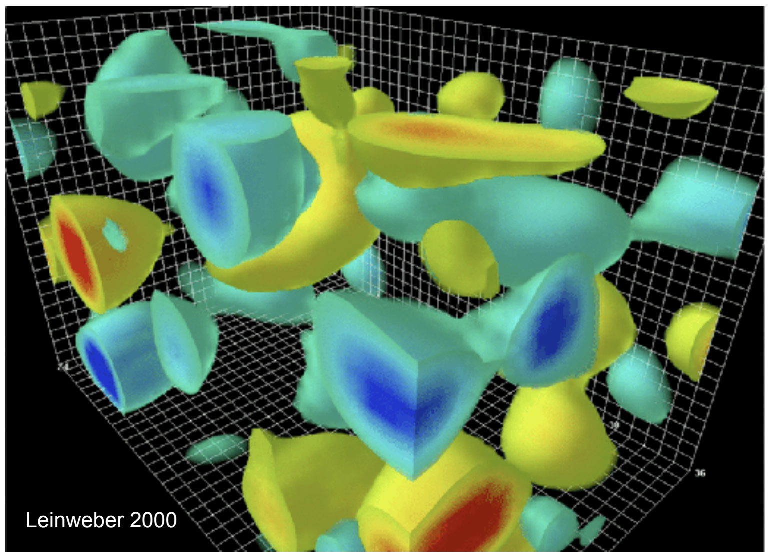
Twist-3: EM tensor and spin-orbit correlations

F^2: Trace anomaly and mass decomposition

Future program

Topological fields: Lattice QCD configurations

Smooth field configurations from LQCD:
"Cooling" of quantum fluctuations here: Leinweber et al. 2000
Gradient flow techniques e.g. Athenodorou et al 2018, Alexandrou et al 2020



Yellow/blue: Top. charge density positive/negative

QCD vacuum populated by localized gauge fields with topological charge $(32\pi^2)^{-1}\int F\tilde{F} \approx \pm 1$

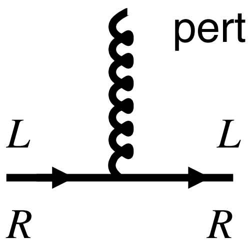
$\leftrightarrow$ Instantons: Classical solutions of YM equations, self-dual fields $\tilde{F} = \pm F$, tunneling trajectories

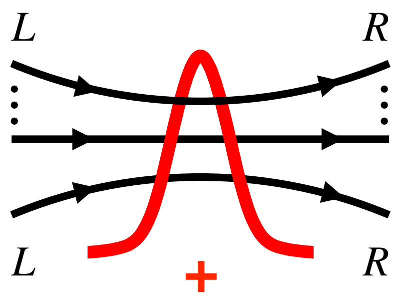
Typical size $\bar{\rho} \approx$ 0.3 fm, separation $\bar{R} \approx$ 1 fm

Fraction of 4D space occupied $\pi^2\bar{\rho}^4/\bar{R}^4 \approx$ 0.1

Strong fields: $(F^2)^{1/2} \approx (32\pi^2/\pi^2\bar{\rho}^4)^{1/2} \sim 2\,\text{GeV}^2$

Topological fields: Chiral symmetry breaking

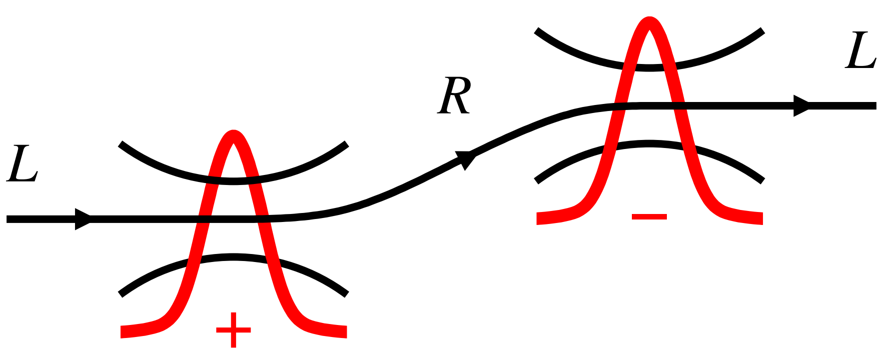




Topological gauge fields induce localized zero mode of fermion fields

$$i\gamma\nabla_{\text{top}\,\pm}\Phi_\pm = \lambda\Phi_\pm \qquad \text{with } \lambda = 0$$

$$\gamma_5\Phi_\pm = \pm\,\Phi_\pm \qquad \text{definite chirality}$$



Interactions flip chirality (not possible in perturb. int.)

Finite density of topological fields: Quarks experience chirality flip at finite rate during propagation

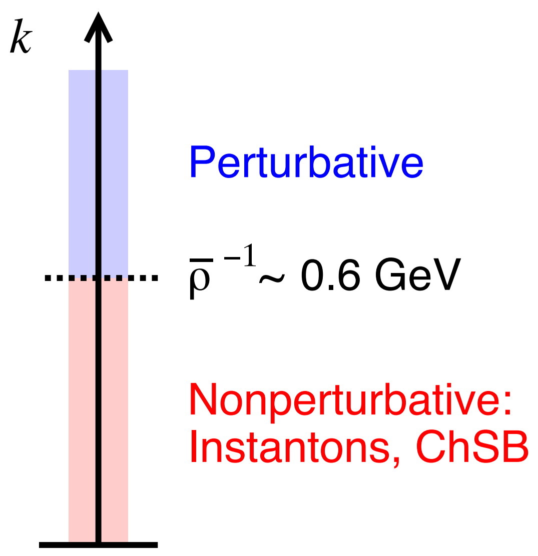
Chiral symmetry breaking

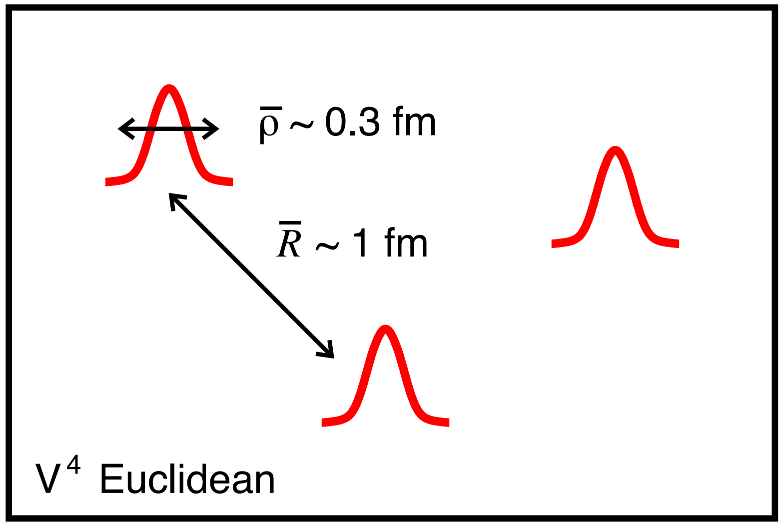
Order parameter: $\langle\bar\psi\psi\rangle \equiv \langle\bar\psi_L\psi_R + \bar\psi_R\psi_L\rangle \neq 0$

Massless bosons: Phase fluctuations in flavor space

Hadron mass generation, interactions

Effective description: Instanton vacuum





Separation of modes

$k > \bar{\rho}^{-1}$: Integrate perturbatively:
Renormalization, $\bar{\rho}^{-2} \gg \Lambda^2_{\rm QCD}$

$k < \bar{\rho}^{-1}$: Integrate nonperturbatively:
Instantons + massive fermions

Instanton ensemble

$$A(x) = \sum_I A_I(x\,|\,z_I, \rho_I, O_I) + \sum_{\bar{I}} A_{\bar{I}}(\,.\,.\,)$$

gauge potential → classical top. fields

$$\int [DA] \rightarrow \int \prod_{I,\bar{I}} dz_I\, d\rho_I\, dO_I\; d_0(\rho_I)$$

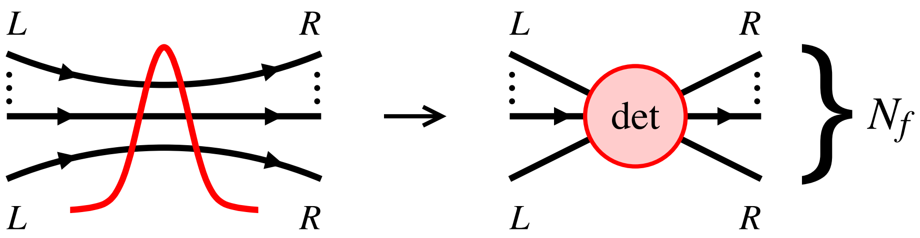
functional integral → collective coordinates

Stable system emerges due to instanton interactions
Implementations: Variational principle Diakonov, Petrov 1984; numerical simulations Shuryak 1988+

Small parameter: Packing fraction $\pi^2 \bar{\rho}^4 / \bar{R}^4 \approx$ 0.1

All dynamical scales "emerge" from $\Lambda_{\rm QCD}$ via instanton density
Preserves renormalization properties of QCD

Effective description: Chiral symmetry breaking

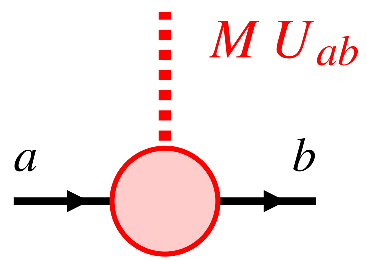


Instanton zero mode induces multifermion vertex
$\det_{ab}\, \bar{\psi}^a_L \psi^b_R \;\times$ form factor

Finite density: Dynamical quark mass $M \approx$ 0.3-0.4 GeV

Active for momenta $k \lesssim \bar{\rho}^{-1} \approx$ 0.6 GeV

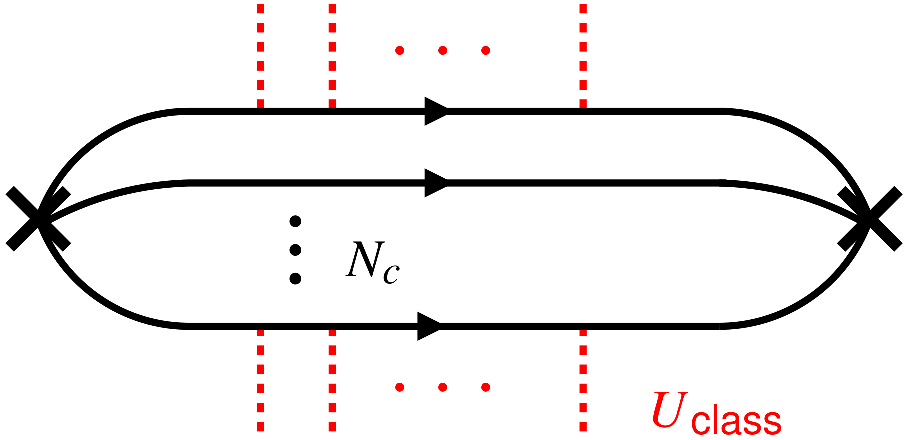
$1/N_c$ expansion, saddle point approximation
Diakonov, Petrov 1986; Pobylitsa 1989; Nowak, Verbaarschot, Zahed 1989; …



Bosonization

$M\, \bar{\psi}_a U_{ab} \psi_b \;\times$ form factor — chiral meson field

Spin-flavor interactions

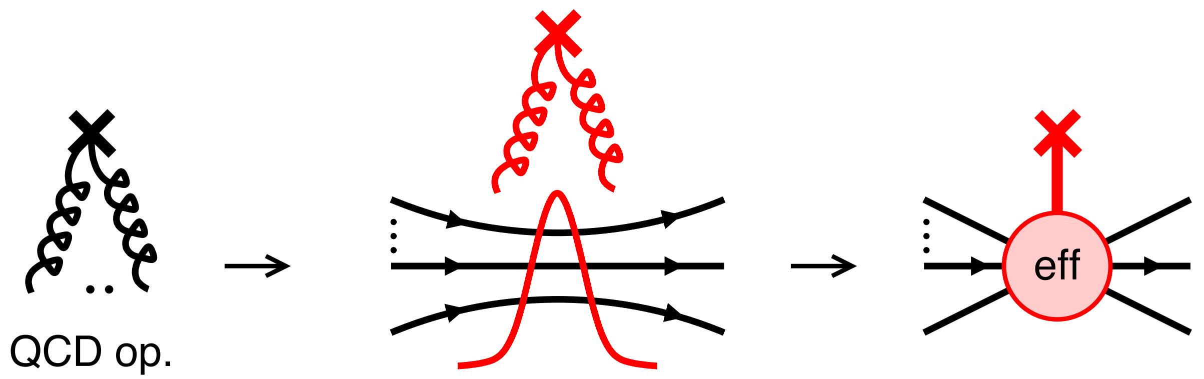


Correlation functions

Meson: Pion pole, η' mass

Baryons: Classical chiral field (“soliton”), N and Δ

Effective description: QCD operators



Gluon operator

$\mathscr{F}[A]$ local QCD gluon operator

Normalized at scale $\mu = \bar{\rho}^{-1}$

Gluon operator in instanton vacuum correlation functions

$$\mathscr{F}[A] \to \sum_{I+\bar{I}} \mathscr{F}[A_I] \; + \; \mathcal{O}(\rho^4/R^4)$$

Evaluated in gluon field of single $I(\bar{I})$

$$\left\langle \ldots \mathscr{F}[A] \; \ldots \right\rangle_{\text{inst}} \; \to \; \left\langle \ldots \mathscr{F}_{\text{eff}}[\bar{\psi}, \psi] \ldots \right\rangle_{\text{eff}}$$

Converted to effective fermionic operator in effective theory of massive fermions

$$\mathscr{F}_{\text{eff}}[\bar{\psi}, \psi] \;=\; \mathscr{N} \times \mathscr{F}[A_I] \times \text{Projector on zero mode}$$

Coupling through zero modes
↔ ‘tHooft vertex

Systematic construction in saddle-point approximation $1/N_c$ [Diakonov, Polyakov, Weiss, 1995]

Advantages of effective operator representation:
Universal, relations between hadronic matrix elements, insight into origin of gluonic structure

Twist-2: Quarks and gluon distributions

$$O^q_{\alpha_1 \dots \alpha_n} = \bar\psi \gamma_{\{\alpha_1} \nabla_{\alpha_2} \dots \nabla_{\alpha_n\}} \psi - \text{traces}$$

Twist-2 spin-n QCD operators, scale $\mu = \bar\rho^{-1}$

$$O^g_{\alpha_1 \dots \alpha_n} = F_{\beta\{\alpha_1} D_{\alpha_2} \dots D_{\alpha_{n-1}} F_{\alpha_n\}\beta} - \text{traces}$$

Matrix elements = moments of PDFs/GPDs

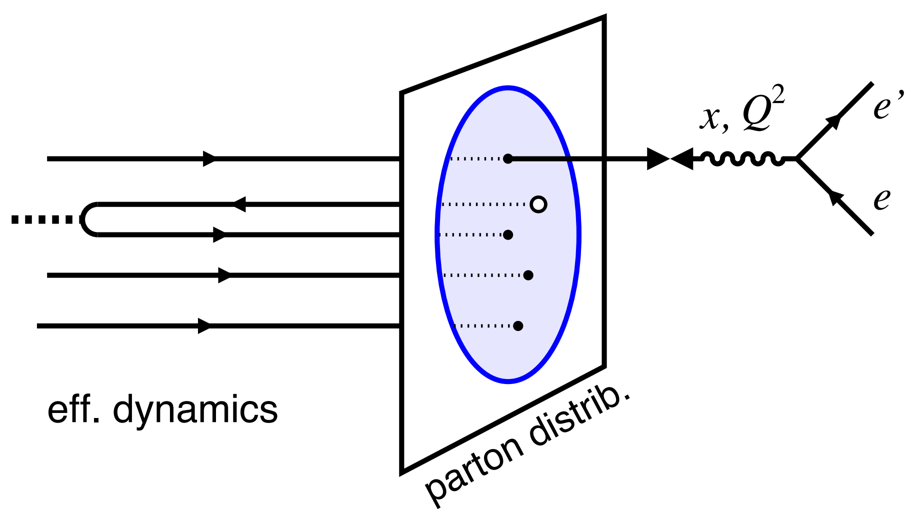
↓ effective theory

$$(O^q_{\text{eff}})_{\alpha_1 \dots \alpha_n} = \bar\psi \gamma_{\{\alpha_1} \partial_{\alpha_2} \dots \partial_{\alpha_n\}} \psi + \mathcal{O}(\rho^4/R^4)$$

Twist-2 quark/antiquark operators $\mathcal{O}(1)$

$$(O^g_{\text{eff}})_{\alpha_1 \dots \alpha_n} = 0 + \mathcal{O}(\rho^4/R^4)$$

Twist-2 gluon operators suppressed in ρ/R



Reason: O(4) symmetry of instanton field

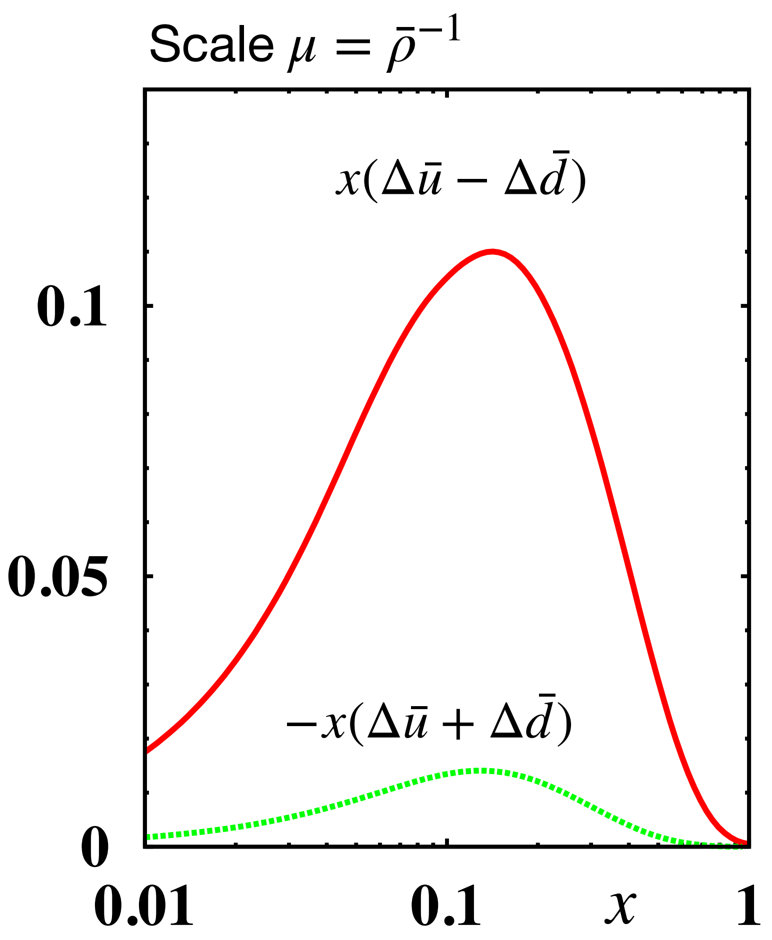
$\int dx\, x\, [q + \bar q](x)$ = 1: Momentum sum rule saturated by quarks + antiquarks at LO in ρ/R

Twist-2: Instanton field subsumed in massive quarks/antiquarks, no direct effect

Twist-2: Polarized antiquarks in nucleon

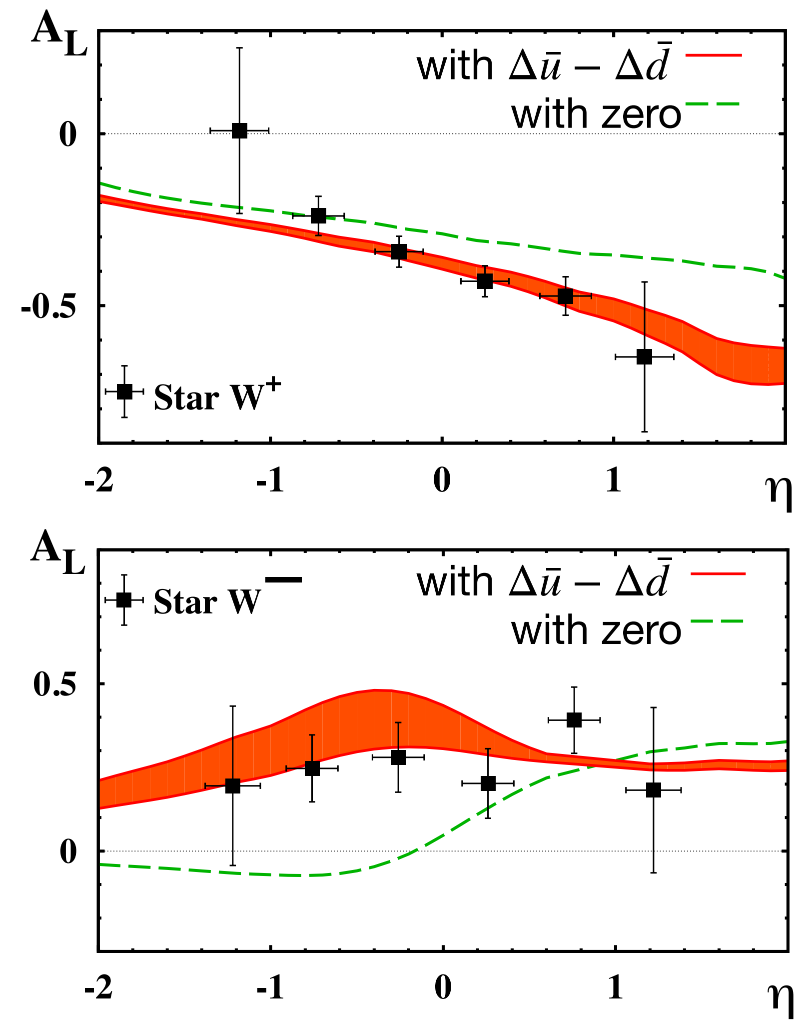
Compute quark/antiquark PDFs/GPDs using effective operators

Nucleon's antiquark content $\mathcal{O}(1)$, exhibits rich spin-flavor dependence



Large polarized flavor asymmetry
$\Delta\bar{u} - \Delta\bar{d}$ predicted $(1/N_c$ expansion)

[Diakonov, Petrov, Pobylitsa, Polyakov, Weiss, 1996]



RHIC $W^{\pm}$ production data
with effect of $\Delta\bar{u} - \Delta\bar{d}$

[Adamczyk et al (STAR) 2014+, Adare et al. (PHENIX) 2016+]

Twist-3: Instanton-induced interactions

$$O^{\alpha\beta}(x) \;=\; \bar{\psi}(x)\,\gamma^{[\alpha}\,\nabla^{\beta]}\tau\,\psi(x)$$

Twist-3 QCD operator, quark spin density in EMT
$\nabla \equiv \partial - iA$ contains gauge potential

↓ effective theory

$$O^{\alpha\beta}_{\text{eff}}(x) \;=\; \bar{\psi}(x)\,\left(\gamma^{[\alpha}\partial^{\beta]}\tau + \tfrac{i}{4}M\sigma^{\alpha\beta}[U^{\gamma_5}(x),\tau]\right)\psi(x)$$

Effective operator (bosonized form).
Instantons induce spin-flavor dependent interaction of massive quarks with chiral field
J-Y Kim, Weiss 2024

Also operator $\gamma^{\alpha}\gamma_5$, quark spin-orbit correlations

QCD equations of motion

QCD operator relation from QCD EOM $\hat{\nabla}\psi = 0$

$$O^{\alpha\beta}(x) \;=\; -\tfrac{1}{4}\,\epsilon^{\alpha\beta\gamma\delta}\,\partial_{\gamma}[\bar{\psi}(x)\,\gamma_{\delta}\gamma_5\tau\,\psi(x)]$$

Effective operator obeys same relation because of effective EOM $[\hat{\partial} - MU^{\gamma_5}]\,\psi = 0$

Consistency effective dynamics ↔ effective operators

Numerical results → discussion

Twist-3 non-forward: Instanton field induces spin-flavor dependent chiral interactions, $\mathcal{O}(1)$ dynamical effect on quark spin density and spin-orbit correlations

Twist-4: F^2 and trace anomaly

$$\langle N' | F^2(0) | N \rangle = C(t)\, m_N \bar{u}' u$$

Nucleon form factor of F^2

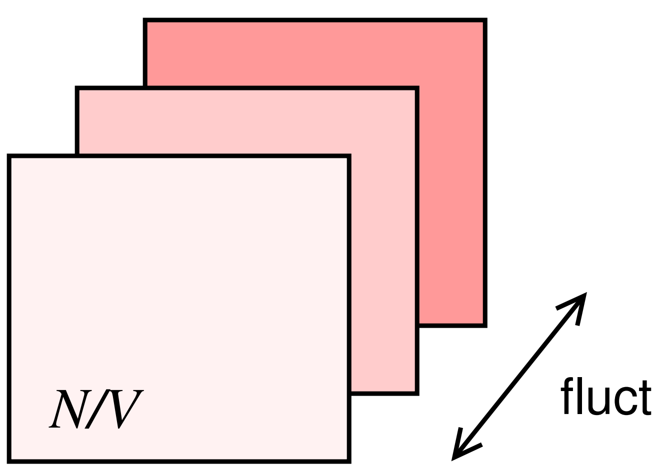
$$C(0) = -\frac{32\pi^2}{b}$$

Relation from QCD trace anomaly $T_{\alpha\alpha}(x) = -\frac{b}{32\pi^2} F^2(x)$

$b = \frac{11}{3} N_c - \frac{2}{3} N_f$ QCD beta function coefficient

[Shifman, Vainshtein, Zakharov 1978; Novikov SVZ 1981]

Instanton number fluctuations



Trace anomaly expressed in instanton number fluctuations:
Grand canonical ensemble [Diakonov, Polyakov, Weiss, 1995]

$$\frac{\overline{(N-\bar{N})^2}}{\bar{N}} = \frac{4}{b}$$

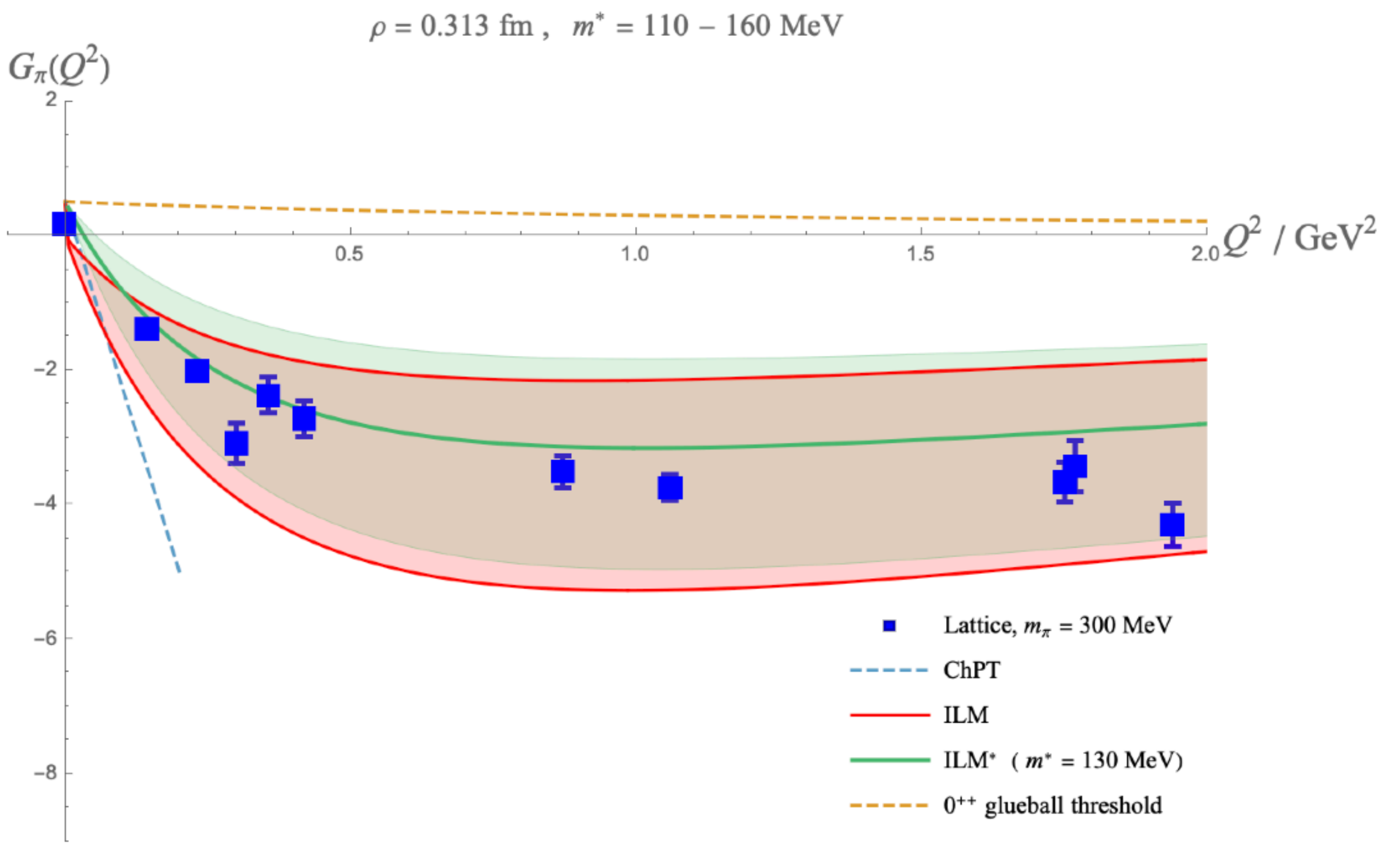
Variance controlled by QCD beta function
"Vacuum compressibility"

$F^2 \propto N$

Effective operator couples to fluctuations
→ nucleon matrix element $\sim 1/b$

Instanton vacuum realizes low-energy theorems from trace anomaly.
Suitable for analyzing hadron mass decomposition, form factors of F^2

Twist-4: Scalar gluon form factor



Pion scalar gluon form factor $\langle \pi(p') \,|\, F^2(0) \,|\, \pi(p) \rangle = -\dfrac{32\pi^2}{b}\,(2M_\pi^2)\; G_\pi(Q^2)$

Covers momentum transfers $Q \ll \bar{\rho}^{-1}$ and $Q \sim \bar{\rho}^{-1}$ Liu, Shuryak, Weiss, Zahed, PRD 110, 054021 (2024)

Includes mixing of scalar gluonic and quark-antiquark modes

Good agreement with LQCD results χQCD Collab: B. Wang et al., Phys.Rev.D 109 (2024) 094504

see also Hackett et al. 2023

Summary

Gluonic structure of nucleon is central to EIC physics program… need to invest in analyzing/interpreting/understanding it based on concepts of nonperturbative dynamics

Effective description of QCD vacuum based on topological gauge fields

Parametric ordering of quark/gluon structure

Analytic approach, synergistic to LQCD

Results can be validated by gradient-flow lattice QCD calculations (future possibilities)

Dynamical effects of topological gauge fields

Twist-3: Spin-flavor interactions in nonforward matrix elements - EMT, spin-orbit correlations

Twist-4: Trace anomaly in hadrons realized/explained by ``topological vacuum compressibility”

Recent advances Shuryak, Zahed, Liu 2023+

Include semiclassical gauge fields beyond well-separated instantons: $I\bar{I}$ molecules.
Twist-2 gluonic operators $\neq 0$

Derive light-front structure from instanton vacuum: Potentials, wave functions

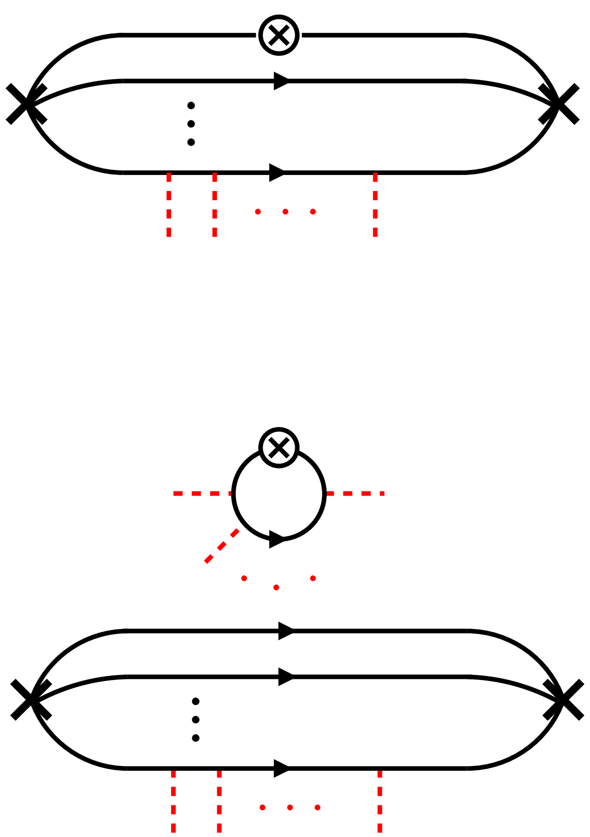
Supplemental material

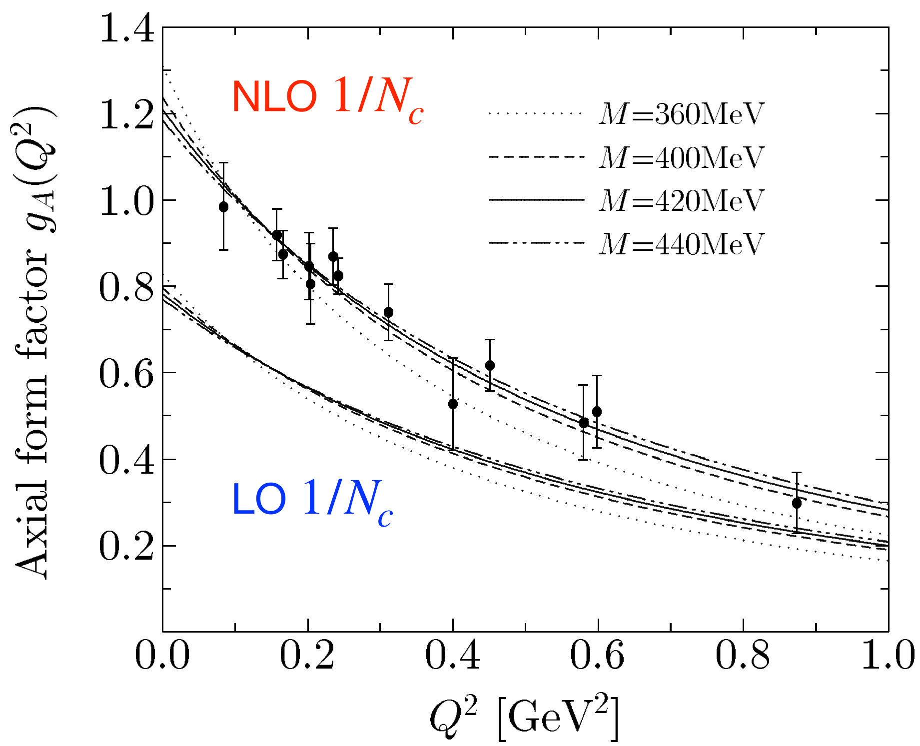
Effective description: Nucleon structure

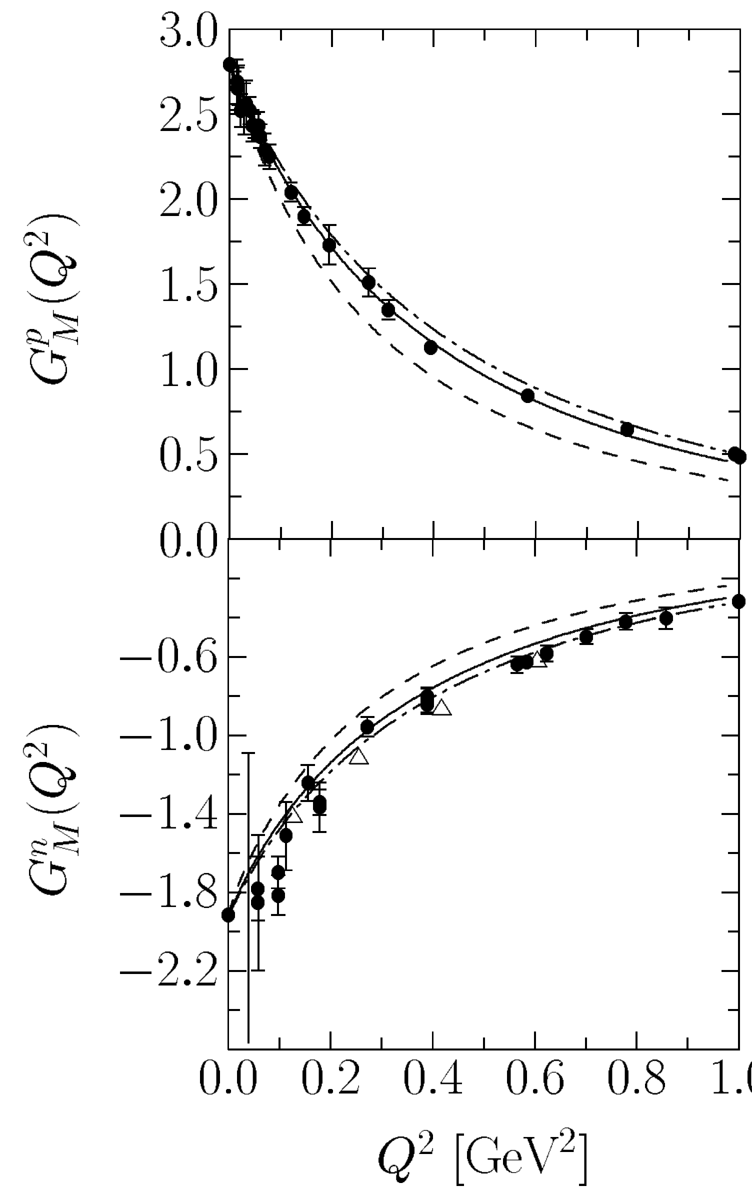
3-point functions include disconnected quark diagrams… connected by chiral field!

Example: Axial and vector and current matrix elements for SU(2) flavor
[Review: Christov et al 1995. Many more results: Sigma term, $N \to \Delta$, SU(3), …]

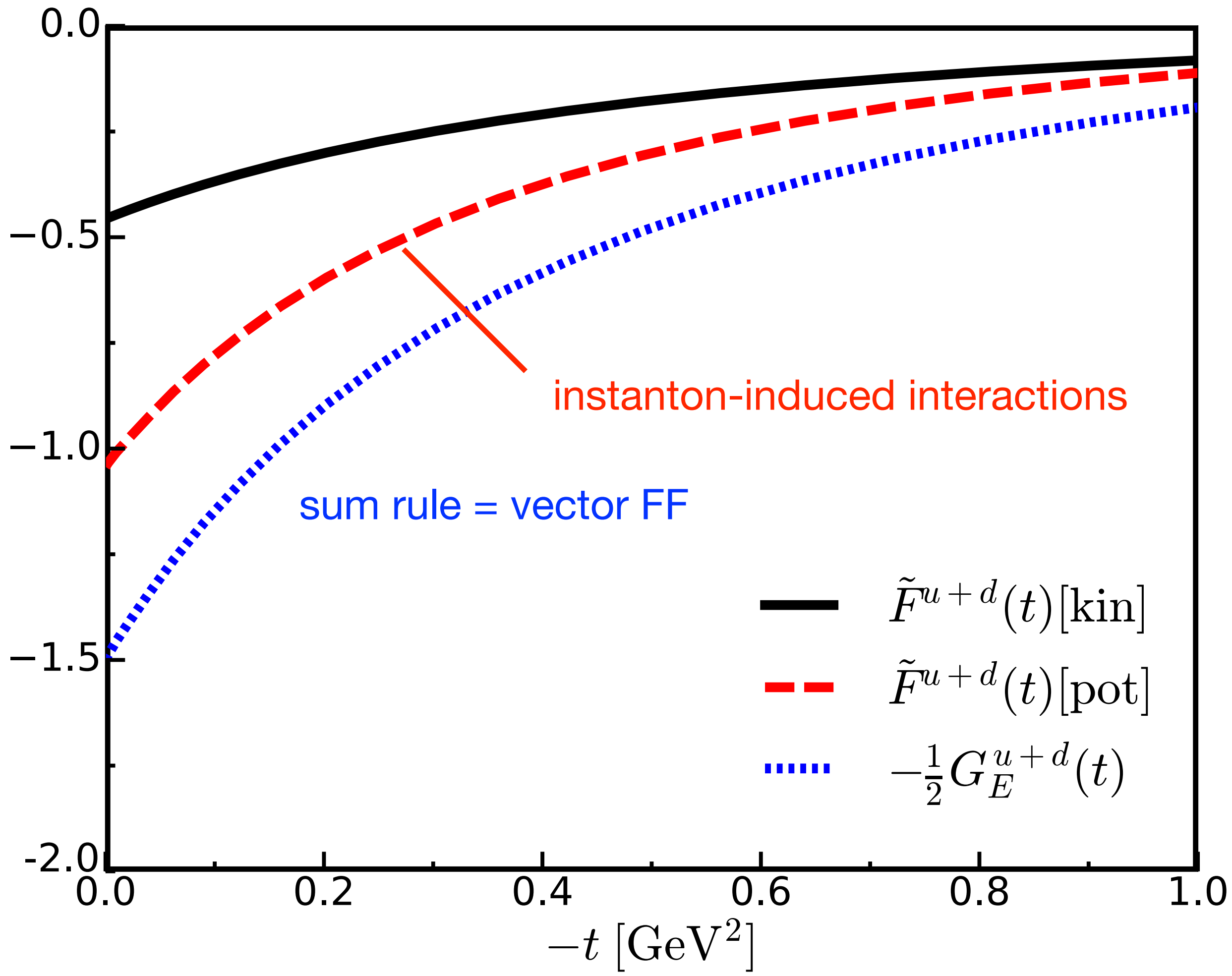
Systematic $1/N_c$ expansion







Twist-3 operators: Spin-orbit correlations



Twist-3 operator $O_5^{\alpha\beta}(x) \;=\; \bar{\psi}(x)\,\gamma^{[\alpha}\gamma_5\nabla^{\beta]}\tau\,\psi(x)$ describing quark spin-orbit correlations

Nucleon form factor $\tilde{F}^{u+d}(t)$

Instanton-induced interaction term make large contribution, ensures sum rule with vector form factor

[J-Y Kim, H-Y Won, H-C Kim, Weiss 2024]