POETIC XI, Feb 24–28, 2025, Miami, Florida International University

EMT properties of proton and neutron, and electromagnetic effects

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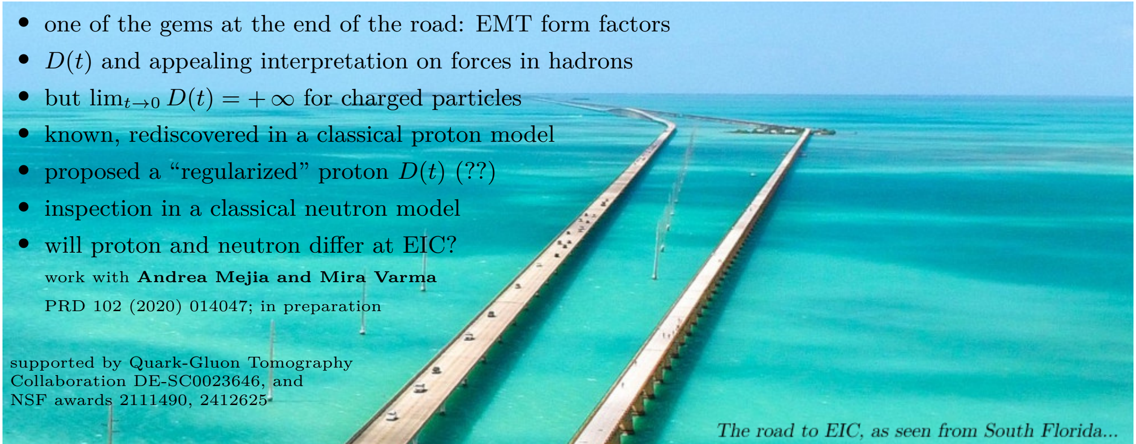
Overview:



- one of the gems at the end of the road: EMT form factors
- $D(t)$ and appealing interpretation on forces in hadrons
- but $\lim_{t\to 0} D(t) = +\infty$ for charged particles
- known, rediscovered in a classical proton model
- proposed a "regularized" proton $D(t)$ (??)
- inspection in a classical neutron model
- will proton and neutron differ at EIC?

work with **Andrea Mejia and Mira Varma**

PRD 102 (2020) 014047; in preparation

supported by Quark-Gluon Tomography
Collaboration DE-SC0023646, and
NSF awards 2111490, 2412625

The road to EIC, as seen from South Florida...

nucleon form factors of symmetric EMT (Kobzarev & Okun 1962, Pagels 1966)

$$\langle p'|\hat{T}^a_{\mu\nu}|p\rangle = \bar{u}(p')\left[\frac{\gamma_\mu P_\nu + \gamma_\nu P_\mu}{2}\, A^a(t,\mu^2) + \frac{i(P_\mu\sigma_{\nu\rho} + P_\nu\sigma_{\mu\rho})\Delta^\rho}{4M}\, B^a(t,\mu^2) + g_{\mu\nu}\,\bar{c}^a(t,\mu^2) + \frac{\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2}{4M}\, D^a(t,\mu^2)\right]u(p)$$

- EMT conserved, $A(t) = \sum_a A^a(t,\mu^2)$, $B(t)$, $D(t)$ scale invariant, $\sum_a \bar{c}^a(t,\mu^2) = 0$
- constraints: **mass** $\Leftrightarrow$ $A(0) = 1$ $\Leftrightarrow$ quarks + gluons carry 100 % of nucleon momentum

  spin $\Leftrightarrow$ $B(0) = 0$ $\Leftrightarrow$ $J(0) = \frac{1}{2}$ spin (and mass) decomposition(s) Ji, PRL 78 (1997) 610

  D-term $\Leftrightarrow$ $D(0) \equiv D$ $\Leftrightarrow$ unconstrained! Polyakov, Weiss PRD 60 (1999) 114017

$P = \frac{1}{2}(p' + p)$, $\Delta = (p' - p)$, $t = \Delta^2$
other EMT form factor notations exist
e.g. $2J^a(t) = A^a(t) + B^a(t)$, $C^a(t) = D^a(t)$
EMT form factors aka gravitational form factors

last global unknown: How do we learn about hadrons?

$|N\rangle$ = **strong** interaction particle. Use other forces to probe

em:	$\partial_\mu J^\mu_{\text{em}} = 0$	$\langle N'\|J^\mu_{\text{em}}\|N\rangle$	$\longrightarrow$	$Q,\ \mu,\ \dots$
weak:	PCAC	$\langle N'\|J^\mu_{\text{weak}}\|N\rangle$	$\longrightarrow$	$g_A,\ g_p,\ \dots$
gravity:	$\partial_\mu T^{\mu\nu}_{\text{grav}} = 0$	$\langle N'\|T^{\mu\nu}_{\text{grav}}\|N\rangle$	$\longrightarrow$	$M,\ J,\ D,\ \dots$

global properties:

$$
\begin{aligned}
Q_{\text{prot}} &= 1.602176487(40) \times 10^{-19}\text{C} \\
\mu_{\text{prot}} &= 2.792847356(23)\,\mu_N \\
g_A &= 1.2694(28) \\
g_p &= 8.06(0.55) \\
M &= 938.272013(23)\ \text{MeV} \\
J &= \tfrac{1}{2} \\
D &= \ ??
\end{aligned}
$$

$\hookrightarrow$

***D* = "last" global unknown**
lives in ERBL-region
Polyakov, Weiss 1999
which value does it have?
models, lattice QCD: $D < 0$ for hadrons
Kumano, Song, Teryaev PRD (2018) for π^0
Burkert, Elouadrhiri, Girod, Nature (2018) for proton

interpretation M.V.Polyakov, PLB 555 (2003) 57

- define static EMT: $\boldsymbol{T_{\mu\nu}(\vec{r})} = \int \frac{d^3\vec{\Delta}}{2E(2\pi)^3}\, e^{-i\vec{\Delta}\,\vec{r}}\, \langle P'|\hat{T}_{\mu\nu}|P\rangle$ in Breit frame with $\Delta^\mu = (0,\vec{\Delta})$

- interpretation requires $R_{\rm system} \gg \lambda_{\rm Compton}$
  for nuclei fine; nucleon at the edge Hudson, PS PRD **96** (2017) 114013

- "If one considered the nucleon as a continuous medium, then $\boldsymbol{T_{ij}(\vec{r})}$
  would characterize the force experienced by partons" Polyakov, PLB 555 (2003) 57

- general **stress tensor** decomposition: $\boldsymbol{T_{ij}(\vec{r}) = s(r)\left(\frac{r_i r_j}{r^2} - \frac{1}{3}\,\delta_{ij}\right) + p(r)\,\delta_{ij}}$

$\boldsymbol{s(r)}$ distribution of *shear*
$\boldsymbol{p(r)}$ distribution of *pressure*
} $\longrightarrow$ **"mechanical properties of hadrons"**

$$D_s = -\tfrac{2}{5}M\int d^3r\,T_{ij}\,\left(r^i r^j - \tfrac{1}{3}\delta^{ij}r^2\right) = -\tfrac{4}{15}M\int d^3r\,r^2 s(r)$$
$$D_p = M\int d^3r\,T_{ij}\,\left(\tfrac{1}{3}\delta^{ij}r^2\right) = M\int d^3r\,r^2 p(r) \qquad \boldsymbol{D = D_p = D_s}$$

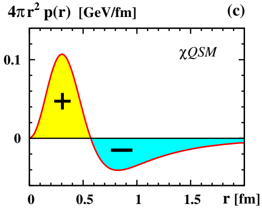
reviews: Polyakov, PS, Int. J. Mod. Phys. A **33** (2018) 1830025, Burkert et al, Rev. Mod. Phys. **95** (2023) 041002

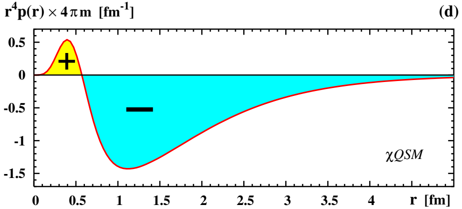
no interpretation, but exact in large N_c limit: $\langle p'|\hat{T}_{\mu\nu}|p\rangle = 2M\int d^3\vec{r}\; T_{\mu\nu}(\vec{r})\; e^{i\vec{\Delta}\cdot\vec{r}}$

chiral quark soliton model

Goeke, Grabis, Ossmann, Polyakov, PS, Silva, Urbano, PRD **75**, 094021 (2007)

- $p(0) = 0.23\,\text{GeV/fm}^3 = 4 \times 10^{34}\,\text{N/m}^2 \gtrsim$ 10-100×(pressure in center of neutron star)
- at $r_0 = 0.57\,\text{fm}$ change of sign, and $p(r) = -\left(\frac{3g_A^2}{8\pi f_\pi}\right)^2 \frac{1}{r^6}$ at large r in chiral limit





$$\int_0^\infty dr\, \boldsymbol{r^2\, p(r)} = 0$$

necessary (von Laue) stability condition

$$D = 4\pi M \int_0^\infty dr\, \boldsymbol{r^4\, p(r)} < 0$$

proof of negative sign based on mechanical concepts, Perevalova et al,PRD94, 054024

- chiral quark soliton model: first model study of EMT densities
- subsequent studies (Skyrme, solitons, Q-balls, bag, cloudy bag, lattice ...)
- in all studies (until 2020) ⇝ only the **short-range strong forces** considered

what about including electromagnetic (long range forces) ... ?

- do electromagnetic interaction even matter for hadron structure?
- why would one even bother?
- well, let's see

- need a model which
  (i) includes strong **and** em interaction, and is
  (ii) theoretically consistent,
  (iii) insightful,
  (iv) solvable,
  (v) lucid

best (and only) choice: classical model by Białynicki-Birula Phys.Lett.A 182 (1993) 346-352

no interpretation, but exact pressure-$D(t)$-relation in a classical model

Prof. Iwo Białynicki-Birula

- theoretical physicist, one of the most eminent Polish scientists
- graduated and professor at the Faculty of Physics of Warsaw University
- co-founder and long-term director of Center for Theoretical Physics of Polish Academy of Science
- specialisation on fundamental problems in quantum field theory
- scientific father of eighteen PhD students (ten of whom became professors themselves)
- turned 90 in summer 2023 (big symposium in Warsaw) publishes like a postdoc
- multi-disciplinary (fraction of works on spires; on research gate 269 papers with 8,500 citations)
- important achievements include:

  proved Feynman's theorem on gauge invariance of transition amplitudes in QED wrong

  gravitational wave solutions with angular momentum that can concentrate gravitational masses

  uncertainty principle and wave functions of photons in the framework of quantum electrodynamics

  many more (including applications of Wigner functions in QED)

  $\longrightarrow$ first consistent model of finite-size classical charge without ad-hoc "Poincaré stresses" $\longleftarrow$

Why classical model?

1. "I never satisfy myself until I can make a mechanical model of a thing." Lord Kelvin (1884)
2. since 1897 (discovery of electron) many attempts of classical models of extended electrically charged particle
3. many tried: Thomson, Abraham, Lorentz, Poincaré, Einstein, Wien, Planck, Sommerfeld, Langevin, Ehrenfest, Born, Pauli, von Laue, Dirac (1962), Schwinger (1983) $\rightarrow$ in all cases ad-hoc forces imposed to bind electric charge
4. after advent of QM and QFT, interesting theoretical problem for its own sake Białynicki-Birula, Phys.Lett.A 182 (1993) 346
5. last but not least: we can learn insightful lessons from it!

relativistic classical proton model

Białynicki-Birula, Phys.Lett.A 182 (1993) 346

"dust particles" described by phase-space distribution $\Gamma(\vec{r}, \vec{p}, t)$

- attract each other due to **strong massive scalar field** $\boldsymbol{\phi}$
- repel due to **strong massive vector field** $\boldsymbol{V^\mu}$**,** and
- due to **electromagnetic force due to** $\boldsymbol{A^\mu}$

field equations:

$$\left[(m - g_S\phi)(\partial_t + \vec{v}\cdot\vec{\nabla}_r) + m\,\vec{F}\cdot\vec{\nabla}_p\right]\Gamma(\vec{r},\vec{p},t) = 0$$

$$\partial_\alpha G^{\alpha\beta} + m_V^2 V^\beta = g_V\, j^\beta$$

$$(\Box + m_S^2)\phi = g_S\,\rho$$

$$\partial_\alpha F^{\alpha\beta} = e\, j^\beta$$

definitions:

$$\vec{F} = \vec{f}/u^0,$$

$$\rho(\vec{r},t) = \int \frac{d^3p}{E_p}\, m\,\Gamma(\vec{r},\vec{p},t)$$

$$j^\alpha(\vec{r},t) = \int \frac{d^3p}{E_p}\, p^\alpha\,\Gamma(\vec{r},\vec{p},t)$$

$$F^{\alpha\beta} = \partial^\alpha A^\beta - \partial^\beta A^\alpha, \quad G^{\alpha\beta} = \partial^\alpha V^\beta - \partial^\beta V^\alpha$$

with $p^\alpha = m\,u^\alpha$, $\vec{v} = \vec{u}/u^0$, and $f^\alpha = eF^{\alpha\beta}u_\beta + g_V G^{\alpha\beta}u_\beta - g_S(\partial^\alpha - u^\alpha u^\beta\partial_\beta)\phi$ (4-force acting on dust)

generalization of Vlasov-Maxwell equations in plasma physics, covariant

Białynicki-Birula, Hubbard, Turski, Physica 128A (1984) 504

proton solution

static in rest frame $u^\alpha = (1,0,0,0)$ with $\Gamma(\vec{r},\vec{p},t) = \delta^{(3)}(\vec{p})\,\rho(r)$

$$\begin{aligned}
\rho(r) &= \Big(f_+(r) - f_-(r)\Big)\,\Theta(R_p - r)\,, \\
eA_0(r) &= e^2\left(\frac{f_+(r)}{k_+^2} - \frac{f_-(r)}{k_-^2} + \frac{2E_B}{e^2}\right)\Theta(R_p - r) \;+\; \frac{e^2}{4\pi r}\,\Theta(r - R_p)\,, \\
g_S\phi(r) &= g_S^2\left(\frac{f_+(r)}{k_+^2 + m_S^2} - \frac{f_-(r)}{k_-^2 + m_S^2}\right)\Theta(R_p - r) \;+\; \frac{b_S}{4\pi r}e^{-m_S(r-R_p)}\,\Theta(r - R_p)\,, \\
g_V V_0(r) &= g_V^2\left(\frac{f_+(r)}{k_+^2 + m_V^2} - \frac{f_-(r)}{k_-^2 + m_V^2}\right)\Theta(R_p - r) \;+\; \frac{b_V}{4\pi r}e^{-m_V(r-R_p)}\,\Theta(r - R_p)\,,
\end{aligned}$$

$f_\pm(r) = \frac{d_\pm \sin(k_\pm r)}{4\pi r}$, $k_\pm = \sqrt{\frac{B_p \pm \sqrt{D_p}}{2Q_p^2}}$, $B_p = (g_S^2 - e^2)m_V^2 - (g_V^2 + e^2)m_S^2$, $D_p = B_p^2 - 4e^2Q_p^2 m_S^2 m_V^2$,
$Q_p^2 = g_V^2 + e^2 - g_S^2$ with b_V, b_S, d_+, d_-, $2E_B$, R_p fixed from continuity and differentiability of fields

parameters

$$m = 938\,\mathrm{MeV}, \quad m_S = 550\,\mathrm{MeV}, \quad m_V = 783\,\mathrm{MeV}, \quad \frac{g_S^2}{4\pi\hbar c} = 7.29\,, \quad \frac{g_V^2}{4\pi\hbar c} = 10.8\,, \quad \alpha = \frac{e^2}{4\pi\hbar c} = \frac{1}{137}.$$

from mean field theory of nuclear matter model QHD-I Serot, Walecka, Adv. in Nucl. Phys. Vol. 16 (1986)

mass of the system

mass of dust by definition $m = 938\,\mathrm{MeV}$
mass of bound dust $m + E_B$ with $E_B = -15.71\,\mathrm{MeV}$
compare to bulk binding energy per nucleon in nuclear matter $E_B = -15.75\,\mathrm{MeV}$ Serot, Walecka, op. cit.

proton in classical model (not made of quarks bound by QCD forces but)
made of dust bound by residual nuclear forces

size of the system

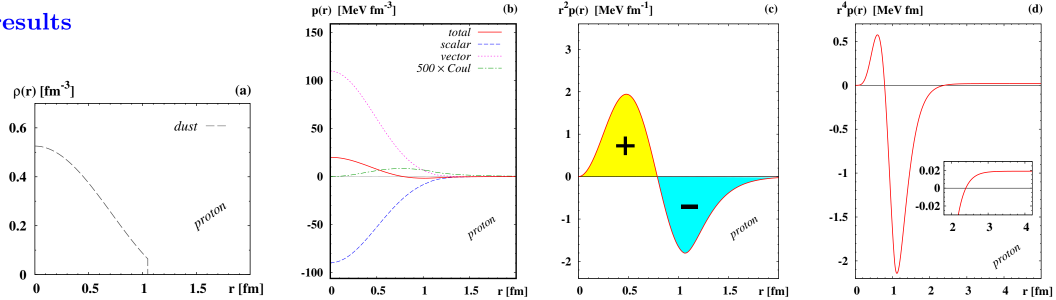
$\langle r^2_{\mathrm{dust}}\rangle^{1/2} = (\int d^3r\ r^2\ \rho_p(r)/\int d^3r\ \rho_p(r))^{1/2} = 0.71\,\mathrm{fm}$
compare to proton charge radius $\langle r_p^2\rangle^{1/2} = 0.84\,\mathrm{fm}$
“not completely out of touch with reality”

conclusion

not unrealisitc, exactly solvable, consistent, classical model of proton

sufficient for our purposes → get an insight of impact of em interaction!

proton results



- $p(0) = 20\,\text{MeV}$ in proton center (order of magnitude less than χQSM, residual forces)
- balance of very strong opposite forces inside proton $\int_0^\infty dr\; r^2 p(r) = 0$
- but $D_s = -\frac{4}{15}\,M \int d^3r\; r^2 s(r)$ and $D_p = M \int d^3r\; r^2 p(r)$ diverge
- reason $T^{ik}_{\text{Maxwell}} = -\frac{1}{4\pi}(E^i E^k - \frac{1}{2}\delta^{ik}\vec{E}^2)$ (static case, $r > R_p$)

  $\rightarrow\;\; s(r) = -\frac{\alpha}{4\pi}\,\frac{\hbar c}{r^4} + \ldots$ and $p(r) = \frac{\alpha}{24\pi}\,\frac{\hbar c}{r^4} + \ldots$

consequence

$D(t) = \frac{\alpha\pi}{4}\,\frac{M}{\sqrt{-t}} + \ldots$ for $t \to 0$ Kubis, Meissner, NPA 671, 332 (2000); Donoghue et al, PLB 529, 132 (2002)

proton form factor $D(t)$

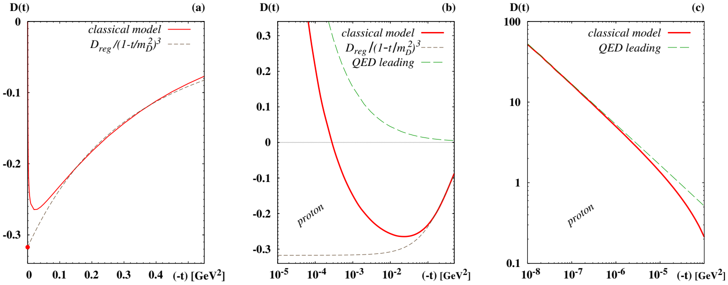
- classical model reproduces QED

  Metz, Pasquini, Rodini, PLB 820 (2021)

- if convergent $D = \zeta\, D_p + (1-\zeta)\, D_s$ same result for any value of ζ

- for proton divergent for all values except $\zeta = \frac{8}{3}$ $\rightarrow$ when $\frac{1}{r^4}$-tails in $p(r)$ and $s(r)$ exactly cancel

- regularized result $D_{\rm prot,\,reg} = -0.317$



is "regularization" sensible at all?

for charged particle $D(t) = \frac{\alpha\pi}{4}\,\frac{M}{\sqrt{-t}} + \ldots$ as $t \to 0$ $\quad\ldots$ can this be measured?

- argument 1, Fig.(a) this slide: $D_{\rm prot,\,reg}$ expected extrapolating from $|t| \gtrsim 0.1\,{\rm GeV}$

- argument 2, Fig.(b) previous slide: removes minuscule em contribution to proton structure outside+inside

- argument 3, see study of neutron — next slides

classical neutron model = $\lim_{e\to 0}$ [**classical proton model**]

Andrea Mejia, PS, forthcoming

Białynicki-Birula, Phys.Lett.A 182 (1993) 346

$$\text{classical neutron model} = \lim_{e\to 0}\left[\text{classical proton model}\right]$$

in principle straightforward, except:

- certain expressions in denominator $\to 0$ for $e \to 0$ in proton model $\Rightarrow$ proceed with care
- proton $\int_0^\infty dr\, r^2 p_i(r) = \begin{cases} -10.916\,\text{MeV} & \text{for} \quad i = \text{scalar} \\ 10.891\,\text{MeV} & \text{for} \quad i = \text{vector} \\ 0.025\,\text{MeV} & \text{for} \quad i = \text{Coulomb} \end{cases}$ $\Rightarrow$ remove em part & recalculate

$$\sum_i \qquad \textbf{zero}$$

neutron results

- **neutron size**

  mean radius of dust distribution in neutron $\langle r^2_{\text{dust}}\rangle^{1/2} = 0.704\,\text{fm}$
  vs proton $0.710\,\text{fm}$ $\rightarrow$ proton swollen due to Coulomb repulsion

- **energy density $T_{00}(r)$**

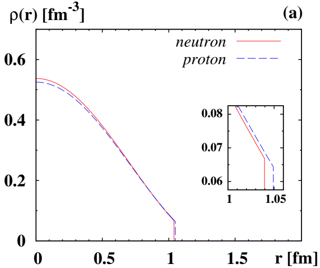
  $T_{00}(0)$ in the center larger for neutron than for proton
  $\rightarrow$ neutron more compact, proton swollen by Coulomb repulsion

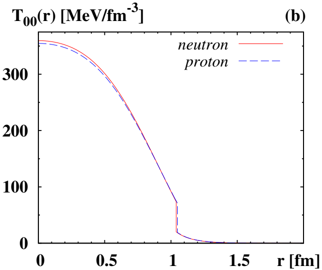
- **electromagnetic mass difference**

  classical model: $(M_p - M_n)_{em} = 0.95\,\text{MeV}$

  vs lattice QCD+QED: $1.00(07)(14)\,\text{MeV}$
  Borsanyi et al, Science 347 (2015) 1452

  in nature $(M_n - M_p) = 1.29333236(46)\,\text{MeV}$
  due to em + isospin breaking effects $m_d > m_u$
  Navas et al. (Particle Data Group), PRD 110, 030001 (2024)
  in classical model no isospin violation implemented

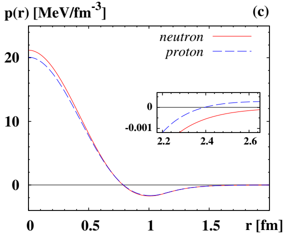




- **pressure**

  $p(0)$ in the center higher in neutron than in proton
  makes sense: higher energy density in the neutron

  $p(r)$ for $r > r_0$ throughout negative(!) in neutron
  (as in all stable systems with short range forces)



- ***D*-term**

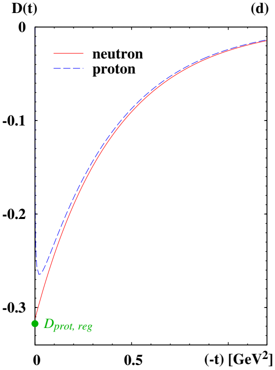
  well-defined finite $D_{\rm neut} = -\frac{4}{15}\,M\int d^3r\; r^2 s(r) = M\int d^3r\; r^2 p(r)$

  $\mathbf{D_{neut} = -0.312}$ nearly identical to $\mathbf{D_{prot,reg} = -0.317}$
  as one would expect for proton and neutron

- **$D(t)$ form factor**

  proton-neutron (em) difference negligible for $(-t) \gtrsim 0.05\,\mathrm{GeV}^2$
  (structure & pressure nearly identical for most of r-range)

  could such a small difference be even seen in experiment?
  $\to$ let's make a rough estimate



- **difference observable in experiment?**

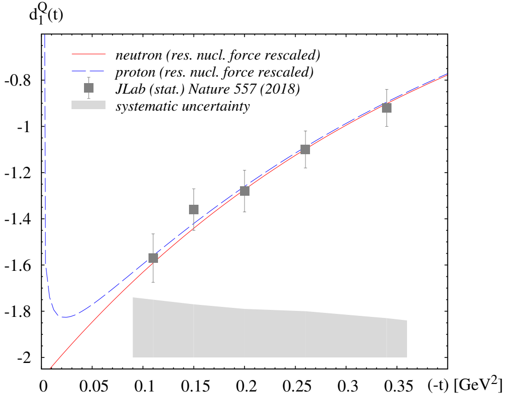
comparison not straightforward

in experiment:
(i) proton in nature bound by QCD
(ii) DVCS gives insight on quark contribution

in classical model:
electromagnetic contribution is about right,
but residual nuclear forces → need to be “rescaled”

rough estimate [rescaling factor ≈ (5.6-5.8)]
cf. first attempt of experimental extraction
Burkert, Elouadrhiri, Girod, Nature 557 (2018) 396-399
difficult extraction, uncertainties might be underestimated
Kumerički, Nature 570 (2019) E1

will it be possible to see a difference between proton and neutron
in the **experimentally accessible t-region**?



notation: $d_1(t) = \frac{4}{5}\,D(t)$

Conclusions

- for charged particles $D(t)$ divergent for $t \to 0$ Kubis, Meissner (2000), Donoghue et al (2002)
  known fact "rediscovered" in a classical proton model Mira Varma, PS, PRD 102 (2020) 014047

- **classical proton model** Białynicki-Birula (1993)
  realistic description of long-range electromagnetic contribution
  strong interaction simulated by short-range residual nuclear forces
  $\Rightarrow$ em contribution to proton structure minuscule
  not unreasonable to "regularize it away" Mira Varma, PS (2020)

- **natural expectation** if we refrain from em effects (and isospin breaking)
  we get the same properties for proton and neutron including $D(t)$

- **classical neutron model**
  realistic description of $(M_p - M_n)_{em}$ and physically appealing
  confirmation of findings in classical neutron model study

- based on model results: difference between $D(t)_{\rm prot}$ and $D(t)_{\rm neut}$
  negligibly small in experimentally observable range of t
  practically the same $D(t)_{\rm prot}$ and $D(t)_{\rm neut}$ after all?

Conclusions

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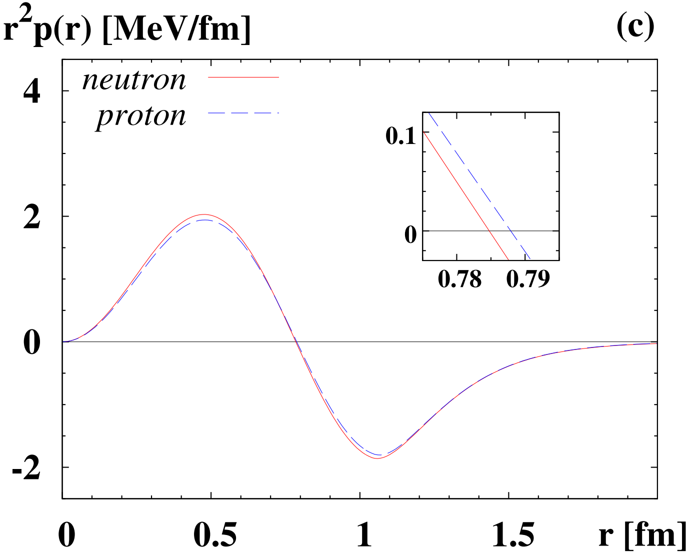
Thank you!

Support slides

in all respects:

- neutron slightly denser and slightly smaller
- proton a bit swollen due to repulsive Coulomb forces
- e.g. $r_{0,\mathrm{neut}} = 0.785\,\mathrm{fm}$ vs $r_{0,\mathrm{prot}} = 0.788\,\mathrm{fm}$



remark on D-term in atomic systems

D-term of H-atom computed and found positive, defying connection to stability (dictates $D < 0$)
Ji, Liu, PRD 106 (2022) 034028 (2022)

At first glance, unrelated to our topic (H-atom neutral, $D(t)$ well-defined for $t \to 0$ albeit opposite sign)
But undeniable common theme: long-range character of em interaction. So some first thoughts:

- D-term of photon (QED state) is also positive Polyakov, Sun, PRD100 (2019) 036003, Freese, Cosyn, PRD 106 (2022) 114014
- interpretation and relation to mechanical property requires to consider "nucleon as a continuous medium" (Polyakov, PLB 555 (2003) 57)
- "medium" in H-atom → one electron mass m_e distributed over a volume of $\mathcal{O}(1\,\text{Å}^3)$
- "medium" in nucleon case → one nucleon mass distributed over a volume of $\mathcal{O}(1\,\text{fm}^3)$.
- average densities of the "media" in atoms vs hadrons compare like $\langle\rho\rangle_{\text{atom}} : \langle\rho\rangle_{\text{hadron}} \approx 1 : 10^{18}$.

Could continuum mechanics concepts be inappropriate for atoms (dilute system due to long-range forces) and still work hadrons (dense systems bound by short-range forces)....?
see Burkert et al, Rev. Mod. Phys. 95 (2023) 041002; Lorcé and PS, arXiv:2501.04622

very interesting! More work needed!