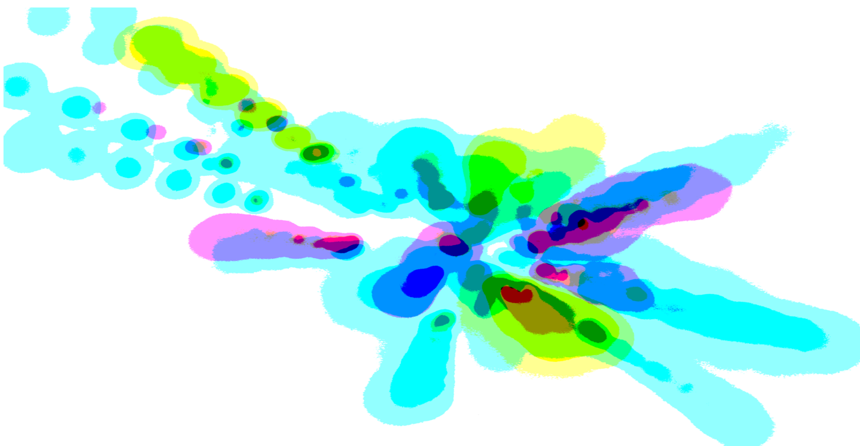
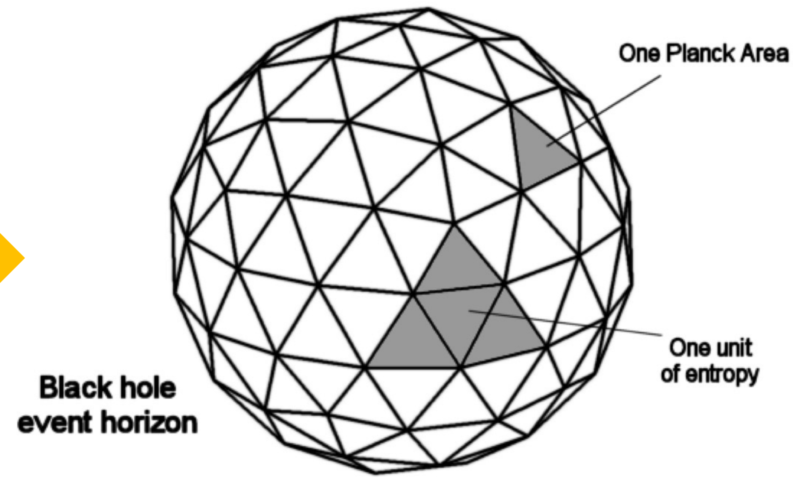
QCD at high energies : some concrete interdisciplinary connections



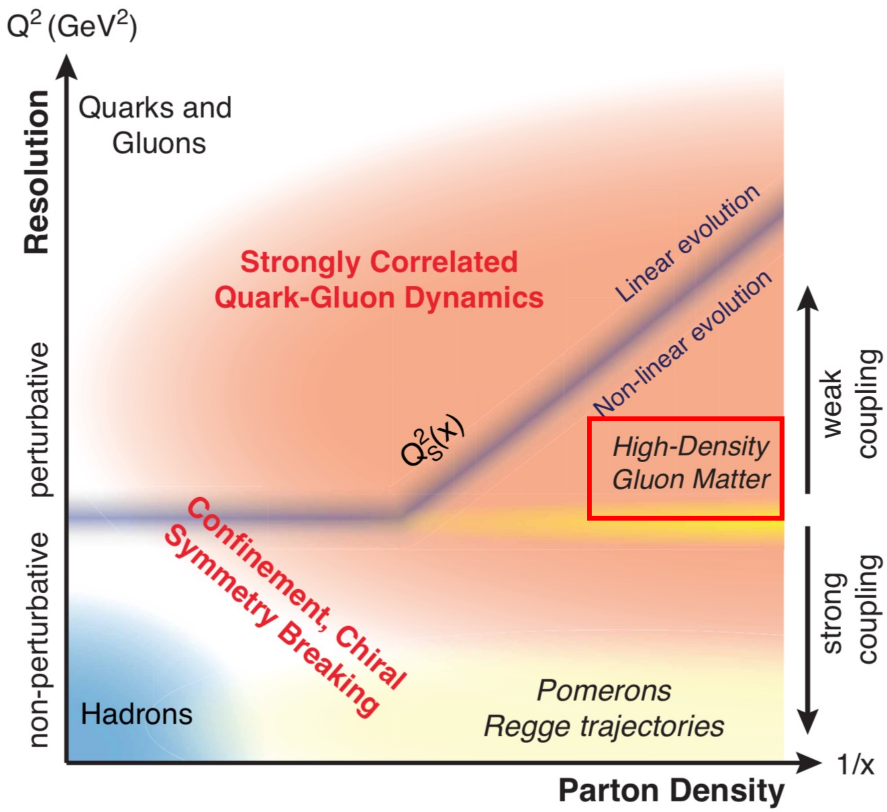


Raju Venugopalan

BNL & CFNS, Stony Brook

POETIC XI, February 24-28, 2025

Mapping out terra incognita in the QCD landscape

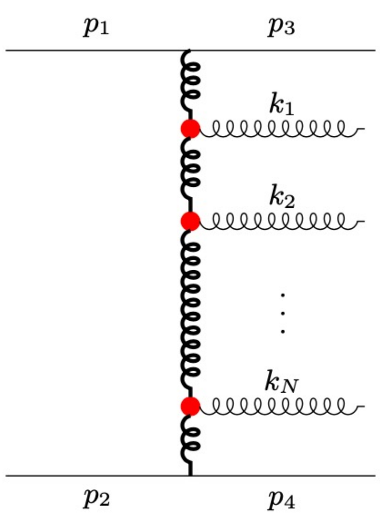


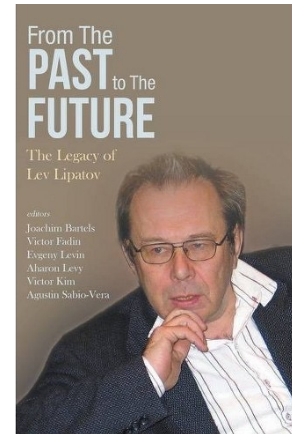
Aschenauer et al., arXiv:1708.01527
Rep.Prog. Phys. 82, 024301 (2019)

DIS off nuclei adds another axis to this plot

Many open questions that will be discussed at this workshop:
3-D quark-gluon structure of the proton, spin and orbital dynamics, many-body correlations, multi-particle production...

The BFKL paradigm: 2→ N QCD amplitudes in Regge asymptotics*





Compute multiparticle in multi-Regge kinematics of QCD:

$$y_0^+ \gg y_1^+ \gg y_2^+ \gg \cdots \gg y_N^+ \gg y_{N+1}^+ \qquad \text{with} \qquad \boldsymbol{k}_i \simeq \boldsymbol{k}$$

BFKL ladder is ordered in rapidity (or x) . Produced partons are wee in longitudinal momentum(``slow”) but hard in transverse momentum – weak coupling Regge regime

Sophisticated construction involving effective (Lipatov) vertices, ”reggeized” propagators and ”bootstrap” dispersion relations

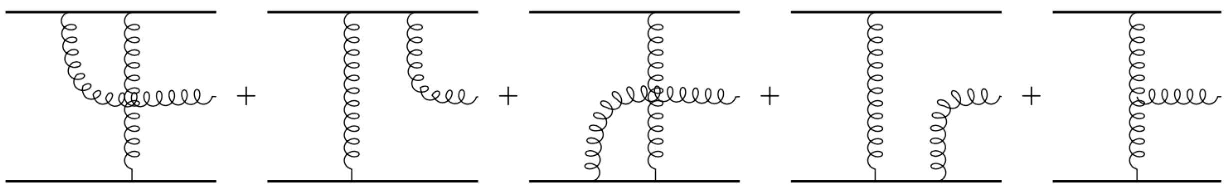
RG description rapidity of evolution given by the BFKL Hamiltonian very rapid growth of the amplitude with energy

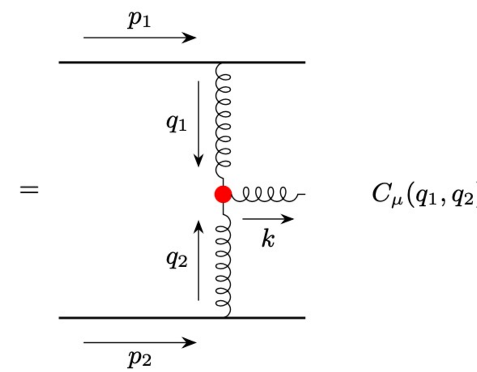
A(s,t) = $s^{\alpha(t)}$ with $\alpha(t) = \alpha_0 + \alpha' \, |t|$ BFKL pomeron

* Asymptotics is the calculus of approximations. It is used to solve hard problems that cannot be solved exactly and to provide simpler forms of complicated results

BFKL: Building blocks

Lipatov effective vertex:

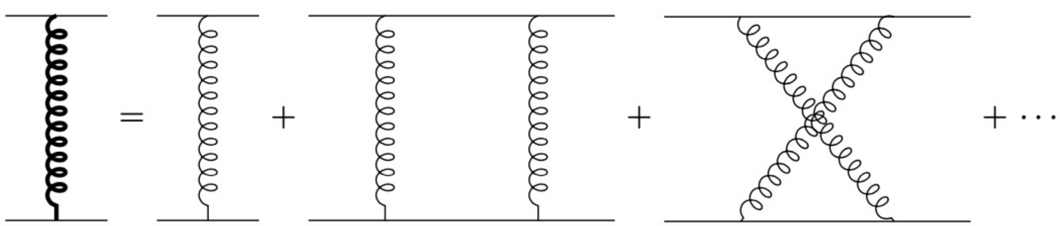




$$C_\mu(\boldsymbol{q}_1, \boldsymbol{q}_2) \simeq -\boldsymbol{q}_{1\mu} + \boldsymbol{q}_{2\mu} + p_{1\mu}\left(\frac{p_2 \cdot k}{p_1 \cdot p_2} - \frac{\boldsymbol{q}_1^2}{p_1 \cdot k}\right) - p_{2\mu}\left(\frac{p_1 \cdot k}{p_1 \cdot p_2} - \frac{\boldsymbol{q}_2^2}{p_2 \cdot k}\right)$$

Gauge covariant, satisfies $k_\mu C^\mu = 0$

Reggeized gluon:



$$\frac{1}{t_i} \to \frac{1}{t_i} e^{\alpha(t_i)(y_{i-1} - y_i)}$$

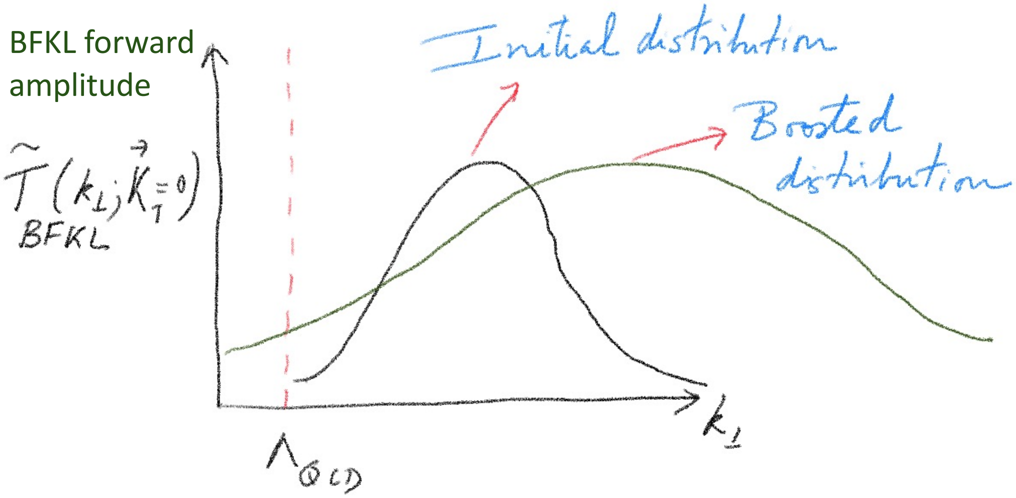
$$\alpha(t) = \alpha_s N_c \, t \int \frac{d\boldsymbol{k}}{(2\pi)^2} \frac{1}{\boldsymbol{k}^2 (\boldsymbol{q} - \boldsymbol{k})^2}, \qquad t = -\boldsymbol{q}^2$$

Leading log x BFKL: Systematic application of multi-Regge kinematics, and dispersive techniques

- ladder structure representing enormous number of Feynman diagrams - to all orders in perturbation theory

Fadin, Kuraev, Lipatov (1976, 1977); Balitsky, Lipatov (1978). Reviews: Forshaw & Ross; Kovchegov & Levin; Raj & RV, in preparation

BFKL: infrared diffusion and gluon saturation



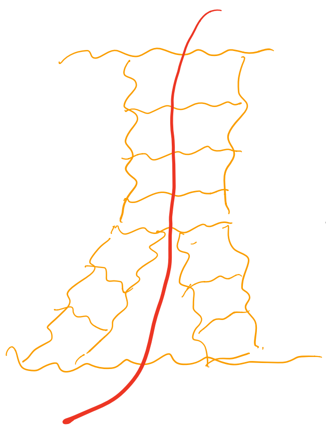
For a fixed large Q^2 there is an $x_0(Q^2)$ such that below x_0 the OPE breaks down…

significant nonperturbative corrections in the leading twist coefficient and anomalous dimension functions due to diffusion of gluons to small values of transverse momentum.

A. H. Mueller, PLB 396 (1997) 251

NLL BFKL does not cure infrared diffusion

Gluon saturation cures infrared diffusion

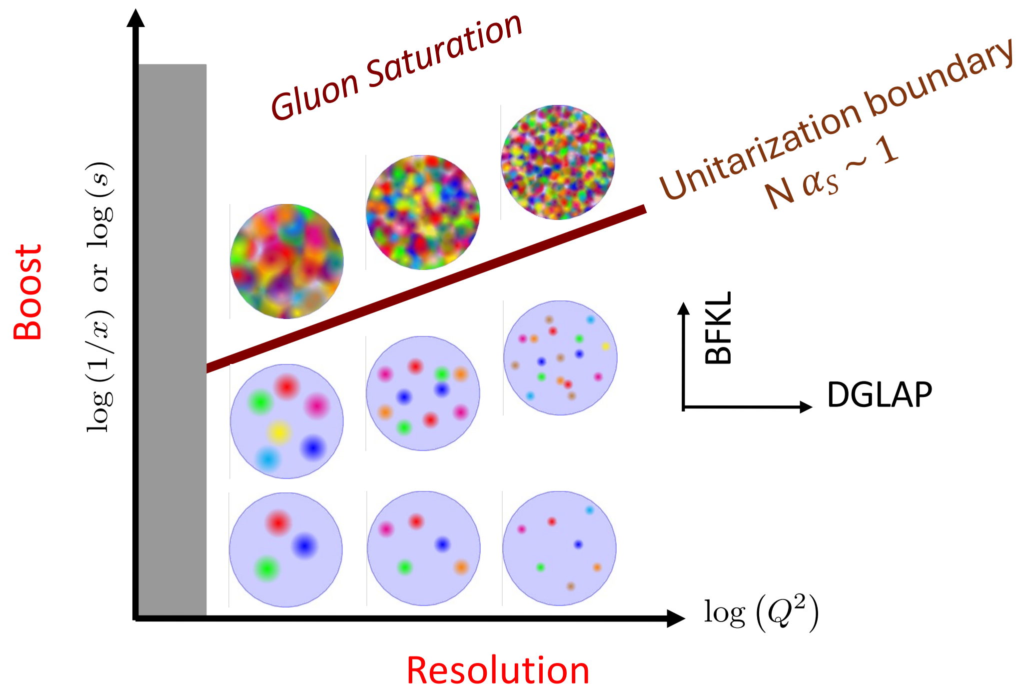


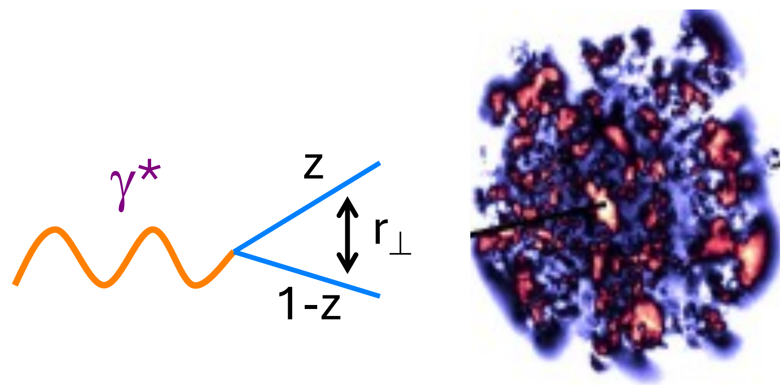
+ other higher twist cuts of O(1) when gluon occupancy

$$N \equiv \frac{xG_A(x, Q_S^2)}{2(N_c^2 - 1)\pi R_A^2 Q_S^2} = \frac{1}{\alpha_S(Q_S)}$$

Classicalization when $\alpha_S(Q_S) \ll 1$ for saturation scale $Q_S \gg \Lambda_{QCD}$

Maximal packing of gluons: Gluon saturation





Wee partons color screened:
live on surface of sphere of radius
$1/\,k^+ \sim 1/k_\perp \sim 1/Q_S$ (x, b)

The emergence of a large saturation scale Q_S(x,b) enables us – due to asymptotic freedom!
- to compute dynamics in this strong field regime of QCD for hadrons and nuclei in one framework

Can we understand gluon saturation quantitatively and interface with experiment ?
Several talks at this workshop addressing various aspects of these issues

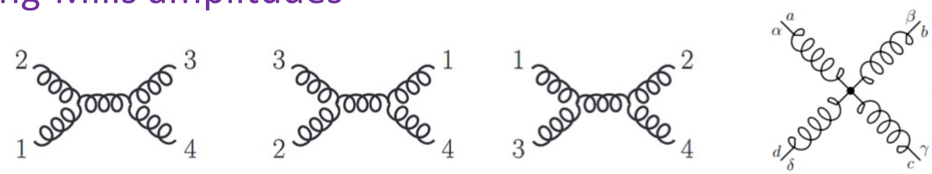
Gauge-Gravity correspondence

Double copy between QCD and Gravity amplitudes

Old idea (Kawai-Lewellyn-Tye, 1986) based on relations between closed and open string amplitudes – in "low energy" limit between Einstein & Yang-Mills amplitudes

$$M_4^{\rm tree}(1,2,3,4) = \left(\frac{\kappa}{2}\right)^2 s\, A_4^{\rm tree}(1,2,3,4)\, A_4^{\rm tree}(1,2,4,3)$$

$$\kappa = 32\,\pi^2\, G_N$$



Remarkable "BCJ" color-kinematics duality

Bern, Carrasco,Johansson,arXiv:0805.3993

Tree level $gg \to gg$ amplitudes (with on shell legs) can be written as

$$i\mathcal{A}_4^{\rm tree} = g^2\left(\frac{n_s c_s}{s} + \frac{n_t c_t}{t} + \frac{n_u c_u}{u}\right)$$

with the s channel color factor $c_s = -2 f^{a_1 a_2 b} f^{b a_3 a_4}$

kinematic factor $n_s = -\frac{1}{2}\Big\{\Big[(\epsilon_1.\epsilon_2)p_1^\mu + 2(\epsilon_1.p_2)\epsilon_2^\mu - (1\leftrightarrow 2)\Big]\Big[(\epsilon_3.\epsilon_4)p_3^\mu + 2(\epsilon_3.p_4)\epsilon_4^\mu - (3\leftrightarrow 4)\Big] + s\Big[(\epsilon_1.\epsilon_3)(\epsilon_2.\epsilon_4) - (\epsilon_1.\epsilon_4)(\epsilon_2.\epsilon_3)\Big]\Big\}$

Tree level gravity amplitude obtained by replacing color factors by kinematic factors

$$i\mathcal{A}_4^{\rm tree}|_{c_i\to n_i,\, g\to\kappa/2} = i\mathcal{M}_4^{\rm tree} = \left(\frac{\kappa}{2}\right)^2\left(\frac{n_s^2}{s} + \frac{n_t^2}{t} + \frac{n_u^2}{u}\right)$$

Significant on-going work on extension to loop amplitudes

Review: Bern et al., arXiv: 1909.01358

Double Copy: gluon → gravitational radiation in shockwave collisions

QCD at high occupancy ⟵ perturbative QCD

Strong field semi-classical double copy ↓   ↓ BCJ double copy

Gravity at high occupancy ⟵ perturbative gravity

Monteiro,O'Connell,White, arXIv:1410.0239
Goldberger, Ridgeway, arXiv:1611.03493

Bern, Carrasco, Johannson,
arXiv: 1004.0476

The BCJ double copy has been remarkably successful in computing the inspiral potential of binary black holes to high powers in a post-Minkowskian expansion in GR – up to $O(G^4)$

Bern et al, PRL 126 (2021) 17, 171601

Can we anticipate the same for ultrarelativistic regimes of BH mergers or close BH encounters?

As emphasized by Weinberg, QFT/EFT ideas are powerful in thinking about gravity
– and this is true for understanding the high occupancy (BH) regime as well…

$2 \to N + 2$ amplitudes in trans-Planckian gravitation scattering: from wee partons to Black Holes

HIGH-ENERGY SCATTERING IN QCD AND IN QUANTUM GRAVITY AND TWO-DIMENSIONAL FIELD THEORIES

L.N. LIPATOV*

We construct effective actions describing high-energy processes in QCD and in quantum gravity with intermediate particles (gluons and gravitons) having the multi-Regge kinematics. The *S*-matrix for these effective scalar field models contains the results of the leading logarithmic approximation and is unitary. It can be expressed in terms of correlation functions for two field theories acting in longitudinal and transverse two-dimensional subspaces.

Effective action and all-order gravitational eikonal at planckian energies

AMATI,CIAFALONI,VENEZIANO **NPB403 (1993)707**

Building on previous work by us and by Lipatov, we present an effective action approach to the resummation of all semiclassical (i.e. $O(\hbar^{-1})$) contributions to the scattering phase arising in high-energy gravitational collisions. By using an infrared-safe expression for Lipatov's effective action, we derive an eikonal form of the scattering matrix and check that the superstring amplitude result is reproduced at first order in the expansion parameter R^2/b^2, where R, b are the gravitational radius and the impact parameter, respectively. If rescattering of produced gravitons is neglected, the longitudinal coordinate dependence can be explicitly factored out and exhibits the characteristics of a shock-wave metric while the transverse dynamics is described by a reduced two-dimensional effective action. Singular behaviours in the latter, signalling black hole formation, can be looked for.

The World as a Hologram

Leonard Susskind

Wee partons, by contrast, are not subject to Lorentz contraction. This implies that in the Feynman Bjorken model, the halo of wee partons eternally "floats" above the horizon at a distance of order $10^{-13}cm$ as it transversley spreads. The remaining valence partons carry the various currents which contract onto the horizon as in the Einstein Lorentz case.

By contrast, both the holographic theory and string theory require all partons to be wee. No Lorentz contraction takes place and the entire structure of the string floats on the stretched horizon. I have explained in previous articles how this behavior prevents the accumulation of arbitrarily large quantities of information near the horizon of a black hole. Thus we are led full circle back to Bekenstein's principle that black holes bound the entropy of a region of space to be proportional to its area.

J.Math.Phys. 36 (1995) 6377; 4018 cites !

In Acknowledgements:

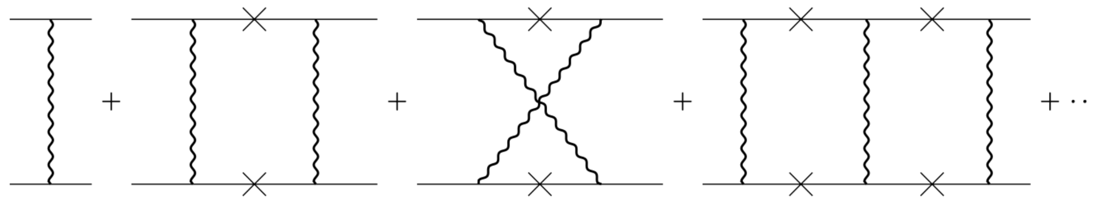
Finally I benefitted from discussions with Kenneth Wilson and Robert Perry, about boosts and renormalization fixed points in light front quantum mechanics and Lev Lipatov about high energy scattering.

30+ years of work by ACV et al. exploring gravitational shockwave collisions in 2-D EFT

Summarized in Di Vecchia, Heissenberg, Russo, Veneziano, *Phys.Rept.* 1083 (2024) 1

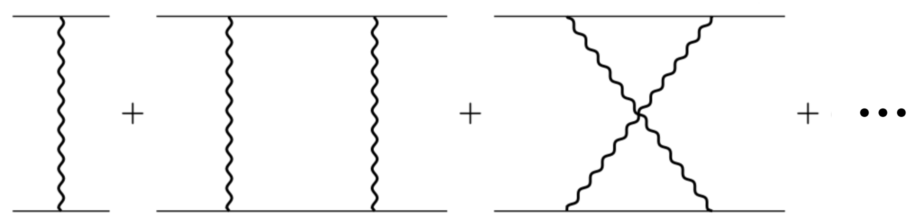
From QCD to gravity in Regge asymptotics: reggeization

In GR, at large impact parameters, dominant contribution is eikonal multiple scattering (Einstein deflection)



$$i\mathcal{M}_{\rm Eik} = 2s \int d^2\boldsymbol{b}\, e^{-i\mathbf{q}\cdot\mathbf{b}} \left(e^{i\chi(\boldsymbol{b},s)} - 1\right) \qquad \text{with} \quad \chi(\boldsymbol{b},s) = \frac{\kappa^2 s}{2} \int \frac{d^2\boldsymbol{k}}{(2\pi)^2} \frac{1}{\boldsymbol{k}^2} e^{i\boldsymbol{b}\cdot\boldsymbol{k}}$$

Reggeized (semi-classical) contributions formally suppressed by R_S^2/b^2



$$\mathcal{M}^{(1)} \sim \frac{\kappa^2}{8\pi^2}\left(-i\pi s \log\left(\frac{-t}{\Lambda^2}\right) + t \log\left(\frac{s}{-t}\right) \log\left(\frac{-t}{\Lambda^2}\right)\right)$$

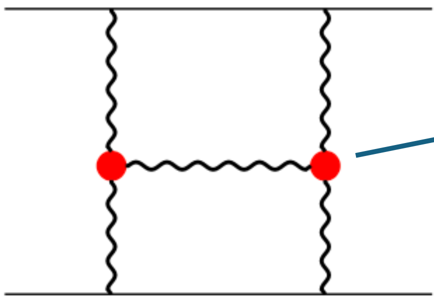
Eikonal Loop

Graviton Regge trajectory: $\alpha(t) = -\kappa^2 t \int \frac{d^2\boldsymbol{k}}{(2\pi)^2} \frac{1}{\boldsymbol{k}^2 (\boldsymbol{q}-\boldsymbol{k})^2} \left[(\boldsymbol{k}\cdot(\boldsymbol{q}-\boldsymbol{k}))^2 \left(\frac{1}{\boldsymbol{k}^2} + \frac{1}{(\boldsymbol{q}-\boldsymbol{k})^2}\right) - \boldsymbol{q}^2\right] , \qquad \boldsymbol{q}^2 = -t$

The IR virtual divergence cancels in the inclusive cross-section

Lipatov, PLB 116B (1982); JETP 82 (1982)

From QCD to gravity in Regge asymptotics: Lipatov vertex



H-diagram of Amati, Ciafaloni, Veneziano

Gravitational Lipatov vertex:

$$\Gamma_{\mu\nu}(\boldsymbol{q}_1, \boldsymbol{q}_2) \equiv \frac{1}{2} C_\mu(\boldsymbol{q}_1, \boldsymbol{q}_2) C_\nu(\boldsymbol{q}_1, \boldsymbol{q}_2) - \frac{1}{2} N_\mu(\boldsymbol{q}_1, \boldsymbol{q}_2) N_\nu(\boldsymbol{q}_1, \boldsymbol{q}_2)$$

Double copy of QCD Lipatov vertex

Double copy of QED Bremsstrahlung vertex

$$N_\mu(\boldsymbol{q}_1, \boldsymbol{q}_2) = \sqrt{\boldsymbol{q}_1^2 \boldsymbol{q}_2^2} \left(\frac{p_{1\mu}}{p_1 \cdot k} - \frac{p_{2\mu}}{p_2 \cdot k} \right)$$

S-matrix power counting a la ACV:

$$S = e^{2i(\delta_0 + \delta_1 + \delta_2 + \cdots)}$$

$$\delta_0 = Gs \log\left(\frac{L}{b}\right), \quad \delta_1 = \frac{6G^2 s}{\pi b^2} \log s, \quad \delta_2 = \frac{2G^3 s^2}{b^2}\left[1 + \frac{i}{\pi} \log s \left(\log \frac{L^2}{b^2} + 2\right)\right]$$

Leading Eikonal term (real)

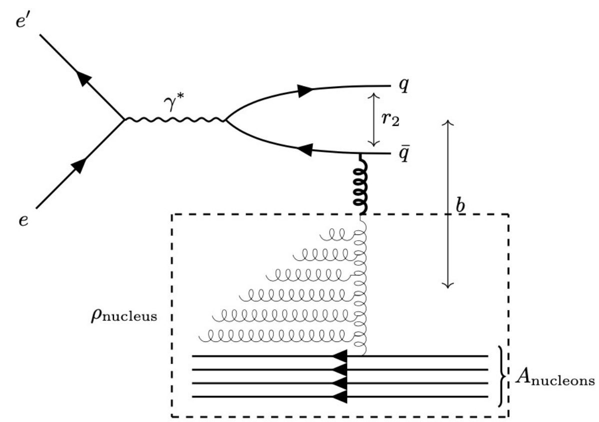
Sub-leading quantum gravity correction $\sim \frac{l_P^2}{b^2}$

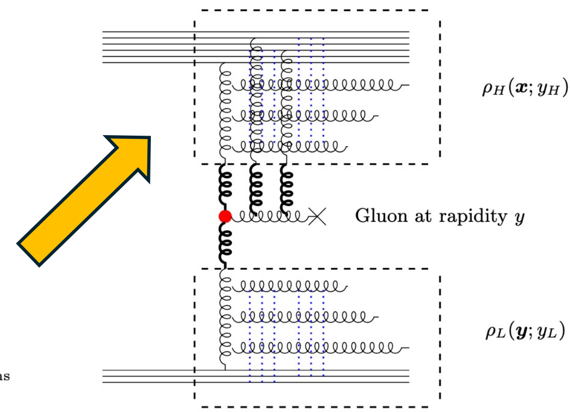
Sub-leading loop contribution $\sim \frac{R_S^2}{b^2}$ - includes absorptive piece

$\delta_2 \gg \delta_1$ for $R_S \gg l_P$

From amplitudes to shockwaves in QCD and GR

As one approaches the dense field regime
In QCD, the shockwave/CGC formalism
is more efficient (and nearly as accurate)
as the amplitude formalism in
computing inclusive final states





Clear map to reggeized propagators and
Lipatov vertices

Hentschinski, 1802.06755

The same may be true in general relativity

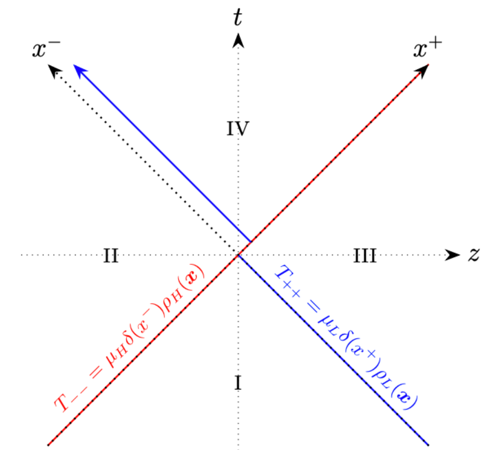
Raj, RV: arXiv, 2311.03463, 2312.035407, 2312.11652, 2406.10483,
Raj, Stasto, RV, in preparation

Shockwave collisions in general relativity: single shock background

Aichelburg-Sexl shockwave metric of a shockwave

$$ds^2 = 2dx^+dx^- - \delta_{ij}dx^i dx^j + f(x^-, \boldsymbol{x})\,(dx^-)^2$$

with $f(x^-, \boldsymbol{x}) = 2\kappa^2 \mu_H \delta(x^-) \dfrac{\rho_H(\boldsymbol{x})}{\Box_\perp} = \dfrac{\kappa^2}{\pi} \mu_H \delta(x^-) \displaystyle\int d^2\boldsymbol{y}\ \ln \Lambda|\boldsymbol{x} - \boldsymbol{y}|\rho_H(\boldsymbol{y})$



$\mu_H = m_H\,\gamma$ = fixed for $\gamma \to \infty$

$\kappa^2 = 8\,\pi\,G$

Linearizing around the metric $g_{\mu\nu} = \bar{g}_{\mu\nu} + \kappa\, h_{\mu\nu}$

Fix light cone gauge $h_{\mu+}$=0. Find solution: $h_{ij}(x^+, x^-, \boldsymbol{x}) = V(x^-, \boldsymbol{x}) h_{ij}(x^+, x^- = x_0^-, \boldsymbol{x})$

with the gravitational Wilson line $V(x^-, \boldsymbol{x}) \equiv \exp\left(\dfrac{1}{2}\displaystyle\int_{x_0^-}^{x^-} dz^- \bar{g}_{--}(z^-, \boldsymbol{x})\,\partial_+\right)$

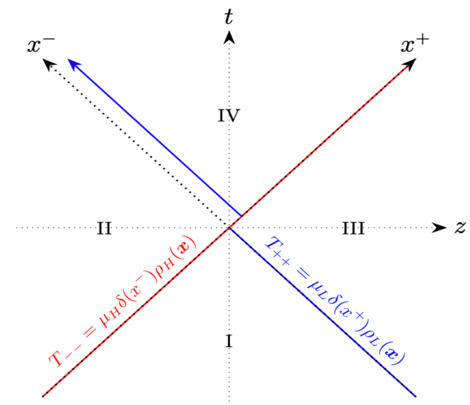
Exactly analogous to the QCD case with $A_- \to\ g_{--}$ and $T^a \to \partial_+$

Shapiro time-delay

Melville,Nachulich,Schnitzer,White,
arXiv:1306.6019

Shockwave collisions in general relativity: dilute-dilute approximation

Now consider interaction of two shockwaves: ρ_L with ρ_H



$$T_{\mu\nu} = \delta_{\mu-}\delta_{\nu-}\mu_H \delta(x^-)\,\rho_H(\boldsymbol{x}) + \delta_{\mu+}\delta_{\nu+}\mu_L \delta\left(x^+\right)\rho_L(\boldsymbol{x})$$

Solve for metric in region IV – forward lightcone around ρ_H

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu} \qquad \bar{g}_{--} = 2\kappa\mu_H \delta(x^-)\,\frac{\rho_H(\boldsymbol{x})}{\Box_\perp}$$

$\mu_H = m_H\,\gamma$ = fixed for $\gamma \to \infty$

$\kappa^2 = 8\,\pi\,G$

We decompose the perturbation $h_{\mu\nu}$ due to ρ_L into a term linear in ρ_L and one bi-linear in $\rho_L \rho_H$

Linearized Einstein's equations in light-cone gauge ($h_{+\mu}$=0) take the form

$$\bar{g}_{--}\partial_+^2 \tilde{h}_{ij} - \Box \tilde{h}_{ij} = \kappa^2 \left[\left(2\partial_i\partial_j - \Box_\perp \delta_{ij}\right) \frac{1}{\partial_+^2} T_{++} + 2T_{ij} - \delta_{ij} T - \frac{2}{\partial_+} \left(\partial_i T_{+j} + \partial_j T_{+i} - \delta_{ij}\partial_k T_{+k}\right) \right]$$

$$\tilde{h}_{ij} \equiv h_{ij} - \tfrac{1}{2}\delta_{ij} h \text{ where } h = \delta_{ij} h_{ij}$$

Raj, RV: arXiv, 2311.03463

Shockwave collisions in general relativity: geodesics

Unlike the QCD case, the sub-Eikonal contributions T_{+i}, T_{ij} are required for consistency of equations of motion

Since these are not uniquely fixed by energy-momentum conservation, the dynamics of the sources is needed to fix this. In the point particle approximation,

$$T^{\mu\nu}(x) = \frac{\mu_L}{\sqrt{-\bar{g}}} \int_{-\infty}^{\infty} d\lambda \; \dot{X}^\mu \dot{X}^\nu \; \delta^{(4)}(x - X(\lambda))$$

The solution of the corresponding null geodesic equations $\ddot{X}^\mu + \Gamma^\mu_{\nu\rho} \dot{X}^\nu \dot{X}^\rho = 0 \; , \qquad g_{\nu\rho} \dot{X}^\nu \dot{X}^\rho = 0$

In the shockwave background, given by $X^- = \lambda \; , \quad X^i = b^i - \kappa^2 \mu_H X^- \Theta(X^-) \frac{\partial_i \rho_H(\boldsymbol{b})}{\Box_\perp}$

$$X^+ = -\kappa^2 \mu_H \Theta(X^-) \frac{\rho_H(\boldsymbol{b})}{\Box_\perp} + \frac{\kappa^4 \mu_H^2}{2} X^- \Theta(X^-) \left(\frac{\partial_i \rho_H(\boldsymbol{b})}{\Box_\perp} \right)^2$$

From the geodesic solutions, we can reconstruct the required components of the stress-energy tensor

Shockwave collisions in general relativity: Lipatov vertex

Solving eqns of motion, taking the Fourier transform, and putting the graviton momenta on-shell, one obtains

Gravitational radiational field

$$\tilde{h}^{(2)}_{ij}(k) = \frac{2\kappa^3 \mu_H \mu_L}{k^2 + i\epsilon k^-} \int \frac{d^2\boldsymbol{q}_2}{(2\pi)^2} \Gamma_{ij}(\boldsymbol{q}_1, \boldsymbol{q}_2) \frac{\rho_H}{\boldsymbol{q}_1^2} \frac{\rho_L}{\boldsymbol{q}_2^2}$$

Gravitational Lipatov vertex

recovering Lipatov's result! $\Gamma_{\mu\nu}(\boldsymbol{q}_1, \boldsymbol{q}_2) \equiv \frac{1}{2} C_\mu(\boldsymbol{q}_1, \boldsymbol{q}_2) C_\nu(\boldsymbol{q}_1, \boldsymbol{q}_2) - \frac{1}{2} N_\mu(\boldsymbol{q}_1, \boldsymbol{q}_2) N_\nu(\boldsymbol{q}_1, \boldsymbol{q}_2)$

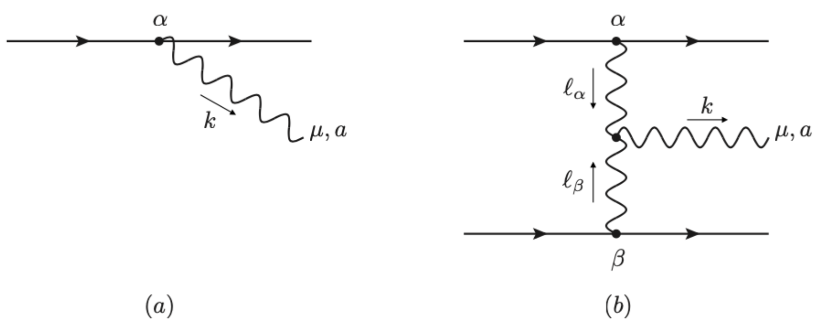
Compare to gauge theory radiation field

$$a_i(k) = \frac{g^3}{k^2 + i\epsilon k^-} \int \frac{d^2\boldsymbol{q}_2}{(2\pi)^2} C_i(\boldsymbol{q}_1, \boldsymbol{q}_2) \frac{\rho_H \cdot T}{\boldsymbol{q}_1^2} \frac{\rho_L}{\boldsymbol{q}_2^2}$$

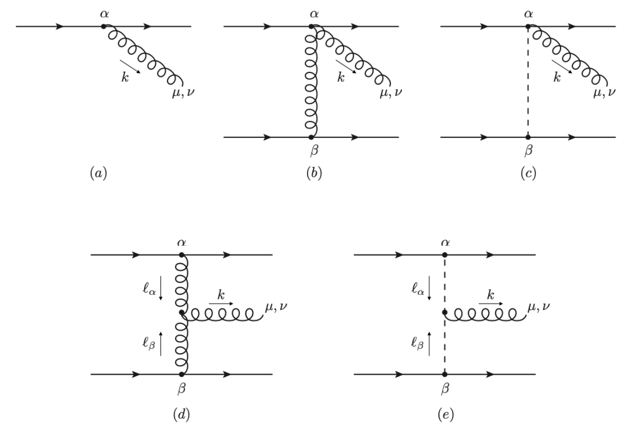
$-if^{abc} T_b T_c C_\mu(\boldsymbol{q}_1, \boldsymbol{q}_2)$ ⟷ Is there a CK relation? ⟷ $s\Gamma_{\mu\nu}(\boldsymbol{q}_1, \boldsymbol{q}_2)$

H.Johansson, A.Sabio Vera, E.Serna Campillo, and M.Vaszquez-Mozo, JHEP10,215(2013),arXiv:1307.3106 [hep-th]

Classical color-kinematic duality-I



From Goldberger, Ridgway
arXiv:1611.03493



A color-kinematic duality exists but it requires one include sub-eikonal corrections to the QCD Lipatov vertex

For this, require a detailed theory of sources: Yang-Mills+Wong equations for classical color sources c^a :

$$D_\mu F_a^{\mu\nu} = g J_a^\nu \qquad J_a^\mu(x) = \sum_{\alpha=1,2} \int d\tau\, c_\alpha^a(\tau) v_\alpha^\mu(\tau) \delta^d\left(x - x_\alpha(\tau)\right)$$

$$\frac{dc^a}{d\tau} = g f^{abc} v^\mu A_\mu^b(x(\tau)) c^c(\tau) \qquad \frac{dp^\mu}{d\tau} = g c^a F^\mu_{a\nu} v^\nu$$

Classical color-kinematic duality-II

Ultrarelativistic limit of Goldberger-Ridgway solution

$$A^{\mu,a}(k) = -\frac{g^3}{k^2}\int \frac{d^2\boldsymbol{q}_2}{(2\pi)^2}\frac{e^{-i\boldsymbol{q}_1\cdot\boldsymbol{b}_1}}{\boldsymbol{q}_1^2}\frac{e^{-i\boldsymbol{q}_2\cdot\boldsymbol{b}_2}}{\boldsymbol{q}_2^2}\Bigg[if^{abc}c_1^b c_2^c\left(-\boldsymbol{q}_1^\mu+\boldsymbol{q}_2^\mu+p_1^\mu\left(\frac{p_2\cdot k}{p_1\cdot p_2}-\frac{\boldsymbol{q}_1^2}{p_1\cdot k}\right)-p_2^\mu\left(\frac{p_1\cdot k}{p_1\cdot p_2}-\frac{\boldsymbol{q}_2^2}{p_2\cdot k}\right)\right)$$

QCD Lipatov vertex

$$+\,c_1\cdot c_2\left\{\frac{q_1^2c_1^a}{p_1\cdot k}\left(-q_2^\mu+\frac{k\cdot q_2}{k\cdot p_1}p_1^\mu+\frac{k\cdot p_1}{p_1\cdot p_2}p_2^\mu-\frac{k\cdot p_2}{p_1\cdot p_2}p_1^\mu\right)+\frac{q_2^2c_2^a}{p_2\cdot k}\left(-q_1^\mu+\frac{k\cdot q_1}{k\cdot p_2}p_2^\mu+\frac{k\cdot p_2}{p_1\cdot p_2}p_1^\mu-\frac{k\cdot p_1}{p_1\cdot p_2}p_2^\mu\right)\right\}\Bigg]$$

sub-eikonal correction

↑ $1/p_1^+$ ↑ $1/p_2^-$

Sub-Eikonal contributions are not universal – for instance, they depend on the spin of the particles

Classical color-kinematic replacement rule:

$$c_\alpha^a \to p_\alpha^\mu\,,$$

$$if^{a_1a_2a_3} \to \Gamma^{\nu_1\nu_2\nu_3}(q_1,q_2,q_3) = -\frac{1}{2}\left(\eta^{\nu_1\nu_3}(q_1-q_3)^{\nu_2}+\eta^{\nu_1\nu_2}(q_2-q_1)^{\nu_3}+\eta^{\nu_2\nu_3}(q_3-q_2)^{\nu_1}\right)$$

$$g \to \kappa\,,$$

Gluon 3-pt vertex with f^{abc} stripped off

Classical color-kinematic duality-III

Ultrarelativistic limit of Goldberger-Ridgway solution

$$A^{\mu,a}(k) = -\frac{g^3}{k^2}\int \frac{d^2\boldsymbol{q}_2}{(2\pi)^2}\frac{e^{-i\boldsymbol{q}_1\cdot\boldsymbol{b}_1}}{\boldsymbol{q}_1^2}\frac{e^{-i\boldsymbol{q}_2\cdot\boldsymbol{b}_2}}{\boldsymbol{q}_2^2}\Bigg[if^{abc}c_1^b c_2^c\left(-\boldsymbol{q}_1^\mu+\boldsymbol{q}_2^\mu+p_1^\mu\left(\frac{p_2\cdot k}{p_1\cdot p_2}-\frac{\boldsymbol{q}_1^2}{p_1\cdot k}\right)-p_2^\mu\left(\frac{p_1\cdot k}{p_1\cdot p_2}-\frac{\boldsymbol{q}_2^2}{p_2\cdot k}\right)\right)$$

QCD Lipatov vertex

$$+c_1\cdot c_2\left\{\frac{q_1^2c_1^a}{p_1\cdot k}\left(-q_2^\mu+\frac{k\cdot q_2}{k\cdot p_1}p_1^\mu+\frac{k\cdot p_1}{p_1\cdot p_2}p_2^\mu-\frac{k\cdot p_2}{p_1\cdot p_2}p_1^\mu\right)+\frac{q_2^2c_2^a}{p_2\cdot k}\left(-q_1^\mu+\frac{k\cdot q_1}{k\cdot p_2}p_2^\mu+\frac{k\cdot p_2}{p_1\cdot p_2}p_1^\mu-\frac{k\cdot p_1}{p_1\cdot p_2}p_2^\mu\right)\right\}\Bigg]$$

sub-eikonal correction

$1/p_1^+$ $\qquad$ $1/p_2^-$

Sub-Eikonal contributions are not universal – for instance, they depend on the spin of the particles

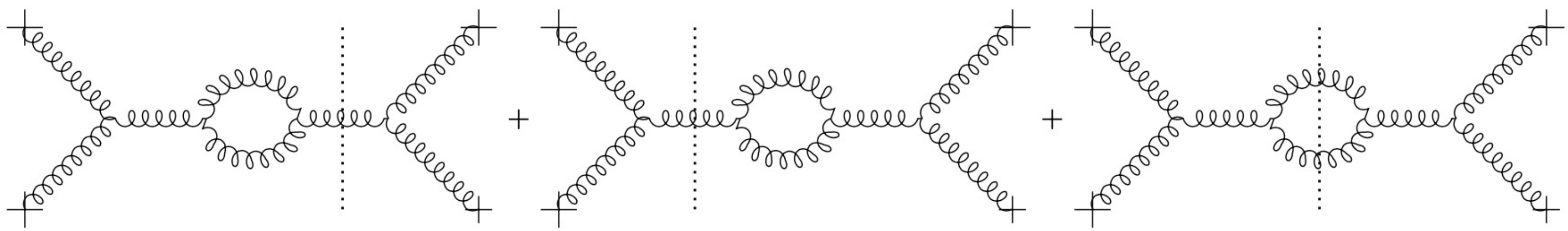
Performing the substitution, one finds the result we obtained by direct computation!

$$A^{\mu\nu}(k) = \frac{\kappa^3 s}{2\,k^2}\int\frac{d^2\boldsymbol{q}_2}{(2\pi)^2}\frac{e^{-i\boldsymbol{q}_1\cdot\boldsymbol{b}_1}}{\boldsymbol{q}_1^2}\frac{e^{-i\boldsymbol{q}_2\cdot\boldsymbol{b}_2}}{\boldsymbol{q}_2^2}\frac{1}{2}\left[C^\mu C^\nu - N^\mu N^\nu + k^\mu\left(\frac{p_1^\nu}{p_1\cdot k}\boldsymbol{q}_1^2+\frac{p_2^\nu}{p_2\cdot k}\boldsymbol{q}_2^2\right)\right]$$

Unphysical – drops out when contracted with the gravitational polarization tensor

Raj, RV, arXiv: 2312.035407

Shockwave propagators in GR-I



NLO corrections (absorptive piece of three loop diagram) in strong background field

Rederive Lipatov's GR "BFKL equation" in shockwave language - in preparation

Key ingredients are retarded shockwave propagators

Shockwave propagators-II

Shockwave propagators
(graviton-reggeized graviton-graviton propagator)

Raj, RV, arXiv:2406.10483

GR Wilson line

$$G_{\mu\nu\rho\sigma}(x,y) = -\int \frac{d^4k}{(2\pi)^4}\frac{1}{k^2+i\epsilon k^-}\sum_{\lambda\lambda'} h^{(\lambda\lambda')}_{\mu\nu,k}(x)h^{*(\lambda\lambda')}_{\rho\sigma,k}(y)$$

Soln. of small fluctuation equation: $h^{(\lambda\lambda')}_{\mu\nu,k}(x^-,x^+,\boldsymbol{x}) = e^{(\lambda\lambda')}_{\mu\nu}(k)\left[\Theta(-x^-)\,e^{-ikx} + \Theta(x^-)\,e^{-ikx}U_k(\boldsymbol{x})\right]$

$$\tilde{G}_{\mu\nu\rho\sigma}(p,p') = \tilde{G}^0_{\mu\nu\rho\sigma}(p)(2\pi)^4\delta^{(4)}(p-p') + \tilde{G}^0_{\mu\nu\alpha\beta}(p)\mathcal{T}^{\alpha\beta\gamma\delta}(p,p')\tilde{G}^0_{\gamma\delta\rho\sigma}(p')$$

where $\widetilde{G^0}$ is the free propagator and the shockwave effective vertex is

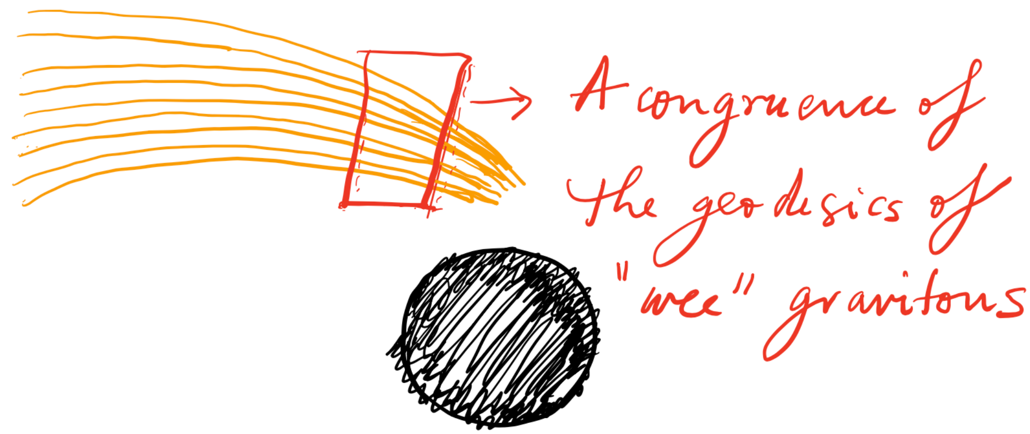
$$\mathcal{T}_{\mu\nu\rho\sigma}(p,p') = -\frac{1}{2}\left(\Lambda_{\mu\rho}\Lambda_{\nu\sigma}+\Lambda_{\mu\sigma}\Lambda_{\nu\rho}-\Lambda_{\mu\nu}\Lambda_{\rho\sigma}\right)4\pi i(p')^-\delta(p^- - (p')^-)\int d^2\boldsymbol{z}\; e^{i(\boldsymbol{p}-\boldsymbol{p}')\cdot\boldsymbol{z}}\left(e^{if_1(\boldsymbol{z})p'_+}-1\right)$$

$$\Lambda_{\mu\nu} = \eta_{\mu\nu} - \frac{n_\mu k_\nu + n_\nu k_\mu}{n\cdot k} \qquad f_1(\boldsymbol{x}) = \kappa^2\mu\frac{\rho(\boldsymbol{x})}{\Box_\perp}$$

Remarkably, they satisfy double-copy relations to the QCD shock wave propagators

Geodesic congruence: the geometry of quantum information

The Raychaudhuri equation
- key in Hawking-Penrose singularity theorems :



Volume change of geodesic convergence

$$\dot{\theta} = -\Omega^i{}_j \Omega^j{}_i + K^i{}_i$$
$$= -\frac{1}{3}\theta^2 - \sigma_{ij}\sigma^{ij} + \omega_{ij}\omega^{ij} + K^i{}_i.$$

Bulk scalar — Shear tensor — Rotation tensor

Includes Ricci curvature + stochastic graviton noise

H.-T. Cho and B.-L Hu, arxiv:2301.06325
M. Parikh, F. Wilczek, G. Zaharaide, PRL (2021)

Remarkably, the Raychaudhuri equation can be rephrased as the Bishop-Gromov upper bound on the "complexity volume in D-1 dimensions" of gate complexity – realizing M. A. Nielsen's geometric picture in quantum information theory?

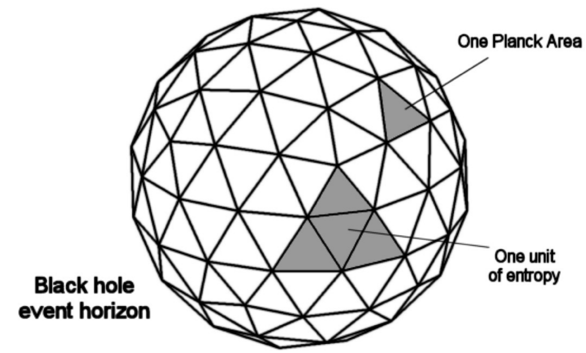
A.R. Brown and L. Susskind, arXiv:1903.12621
A. R. Brown, arXiv:2112.05724

CGC-Black Hole correspondence

Inspired by ideas developed previously by Gia Dvali et al. of BHs
as overoccuped graviton states

Dvali, RV conjecture: CGC-BH graviton states at max. occupancy $N = \frac{1}{\alpha}$

Dvali,RV:arXiv:2106.11989



$S_{Bekenstein} = f_G^2 * \text{Area}$

Micro-states of the CGC and BHs are Goldstone modes
of broken global symmetries (Poincaré+large gauge transformations)

See also, Ball,Pate,Raclariu,Strominger,RV:arXiv:1805.12224

These micro-states saturate the Bekenstein area law
for the maximal information for a state of given energy in a finite size region

The Goldstone decay scale takes universal form $f_G^2 = \frac{Q_S^2}{\alpha}$: in GR, it is Planck mass squared.
In QCD, it is a screening scale of early time QGP evolution– recovering ”bottom-up” thermalization scenario

R. Baier,A.H. Mueller,D. Schiff,D. Son,hep-ph/0009237

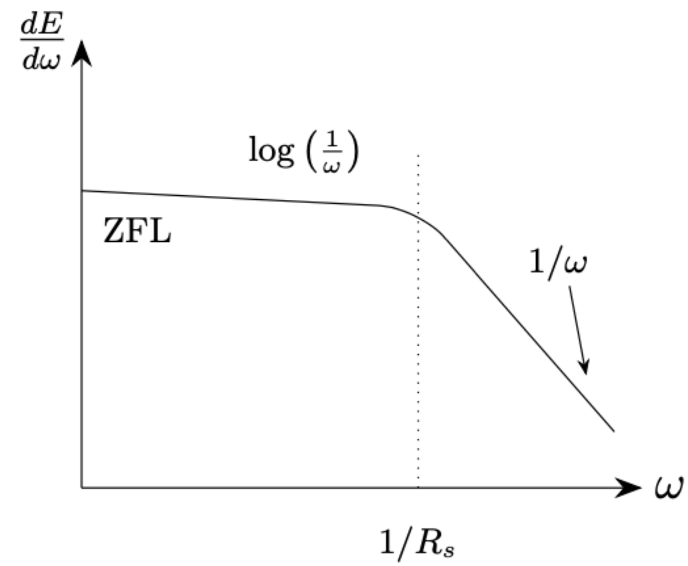
As for the CGC case in the IR, can we understand BH formation as a non-trivial UV fixed point of RG evolution ?

Next steps…

Compute WW spectrum in GR for dilute-dilute and dilute-dense case

$$\frac{dE^{\mathrm{GW}}}{d\omega d\Omega} = \frac{1}{2\pi^2}\omega^2 \sum_\lambda \left| \mathscr{M}^{(\lambda)} \right|^2 \qquad \mathscr{M}^{(\lambda)} = k^2 \tilde{h}^{(2)}_{ij}(k)\varepsilon^{(\lambda)}_{ij}$$

Eg. Gruzinov, Veneziano, arXiv:1409.4555
Ciafaloni,Colferai,Coradeschi,Veneziano, arXiv:1512.00281



What is the shape of the spectrum as the compact objects get close. Solution of the Lipatov equation will tell us the spectrum as a function of impact parameter and rapidity

Amazingly, this has not been done…

Next steps…

Extend analysis to the dilute-dense case to compute coherent multi-gravi-reggeon contributions to the radiation spectrum