

Studying hadronization at Belle II for the EIC

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on behalf of the Belle II collaboration

POETIC XI

February 24, 2025



Belle → Belle II

Belle at KEKB (1999 - 2010) → Belle II at SuperKEKB (2019 - present)

- B factory at Tsukuba, Japan
- Asymmetric e^+e^- collider at collision energies at or near $\Upsilon(4S)$

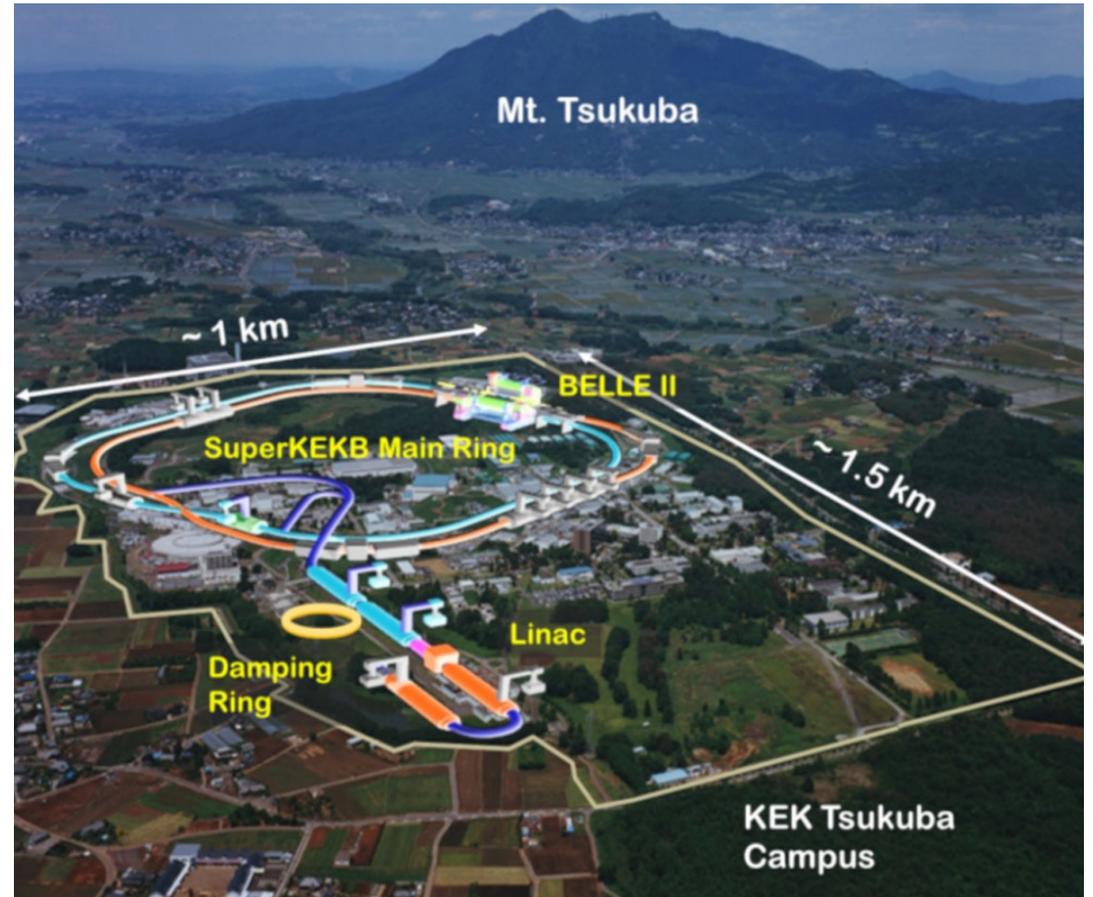
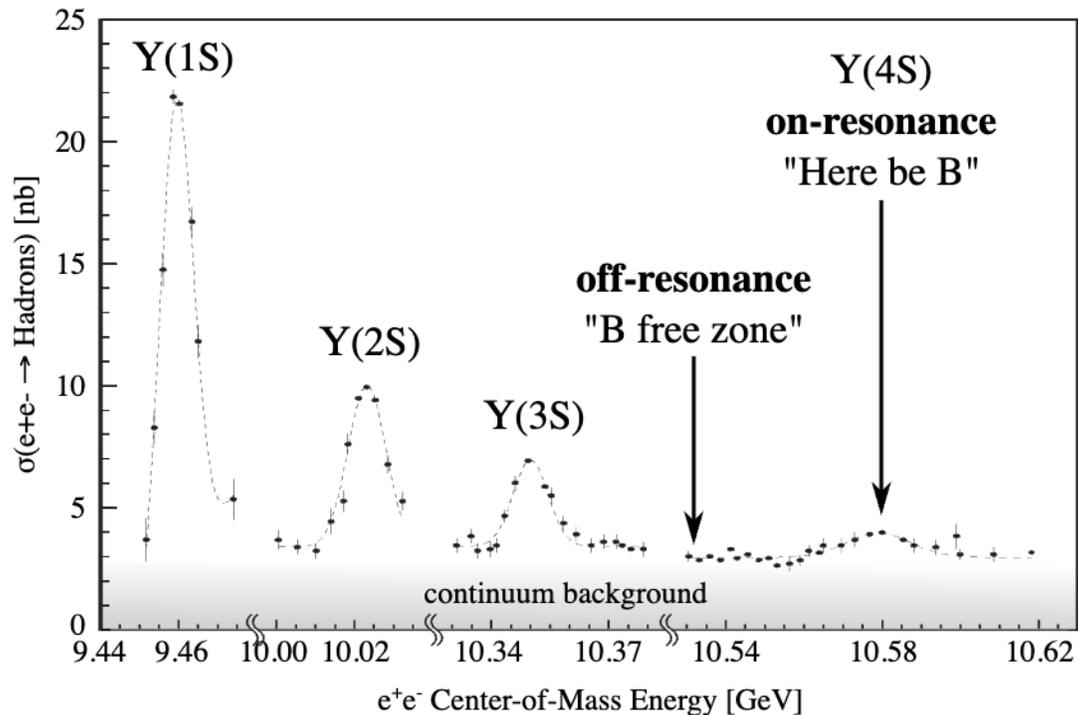
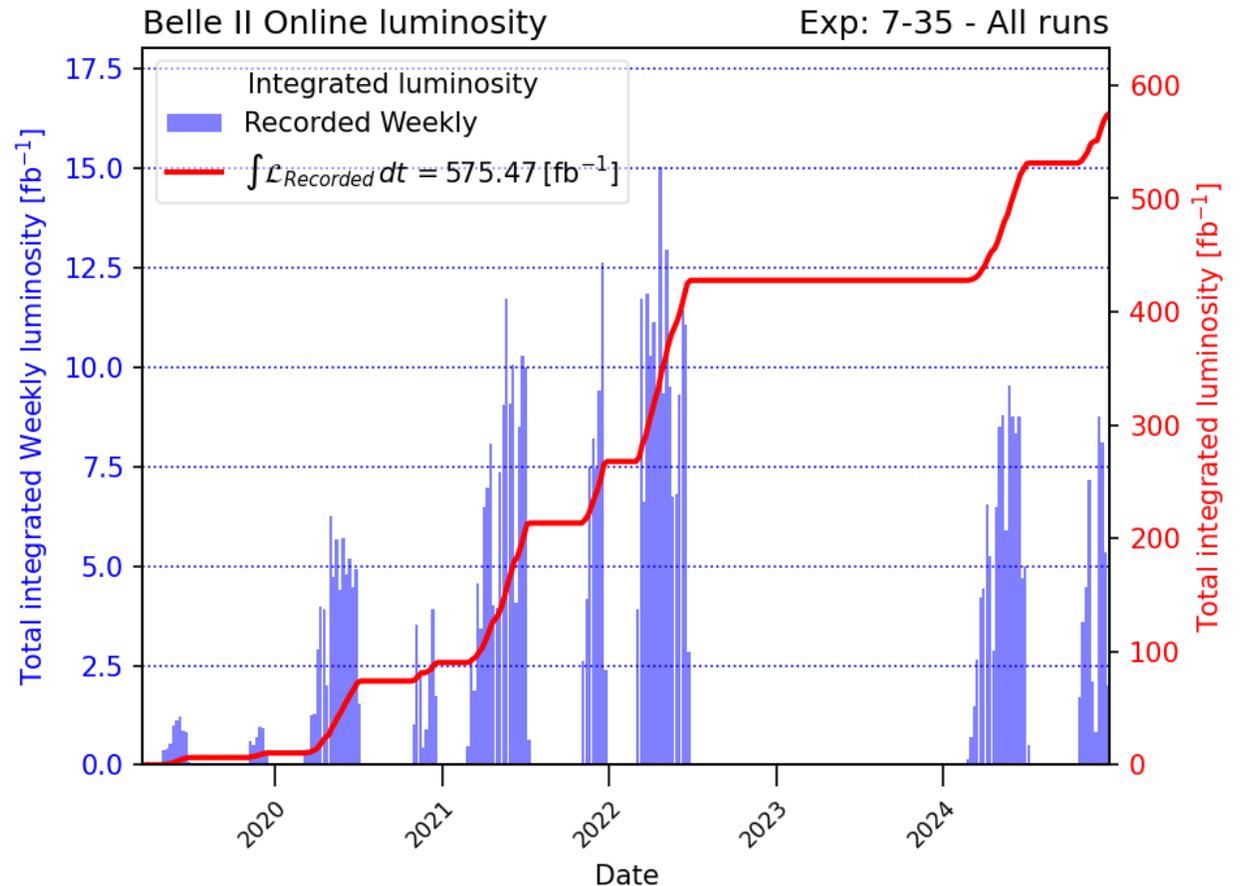


Image from: [Phys. Rev. Accel. Beams 26, 013201 \(2023\)](https://doi.org/10.1103/PhysRevAccelBeams.26.013201)

Belle → Belle II

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- B factory at Tsukuba, Japan
- Asymmetric e^+e^- collider at collision energies at or near $\Upsilon(4S)$
- **Belle**
 - Collected about $\int \mathcal{L}dt = 980 \text{ fb}^{-1}$
- **Belle II**
 - Run 1 (2019-2022)
 - $\int \mathcal{L}dt = 424 \text{ fb}^{-1}$
 - Run 2 (2024-present)
 - In Dec. 2024, achieved luminosity of $5.1 \times 10^{34} \text{ cm}^{-2} \text{ s}^{-1}$

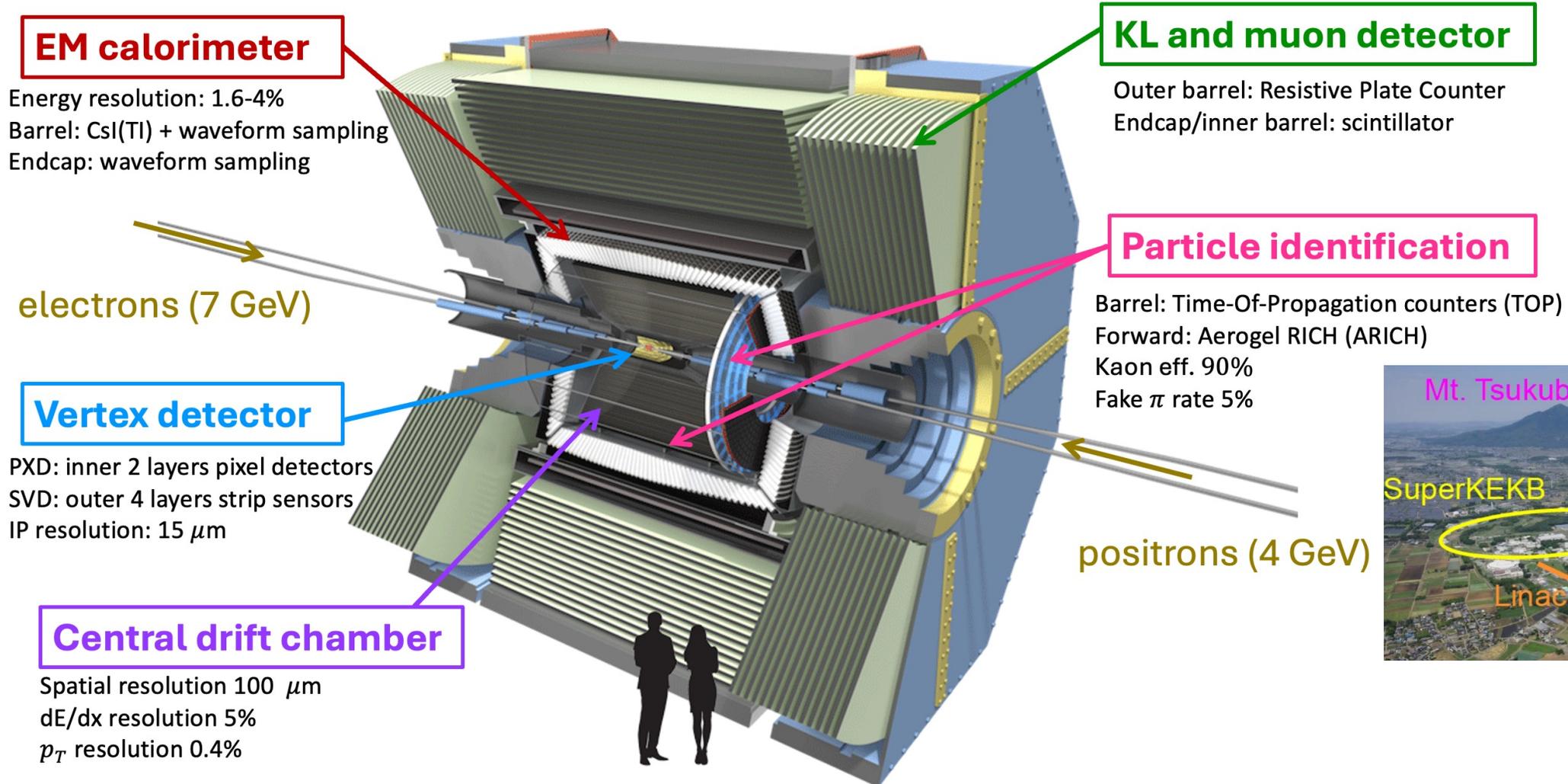


Updated on 2025/01/06 16:16 JST

Belle II Detector @ SuperKEKB

- Large acceptance with good vertexing, PID, and tracking

[KEK Report 2010-1 \[arXiv:1011.0352\]](#)

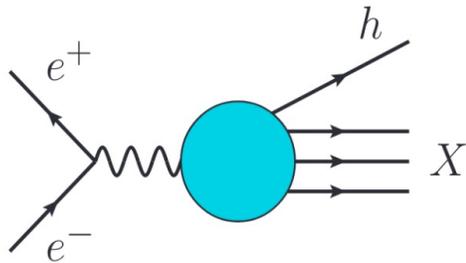


Hadronization at Belle II

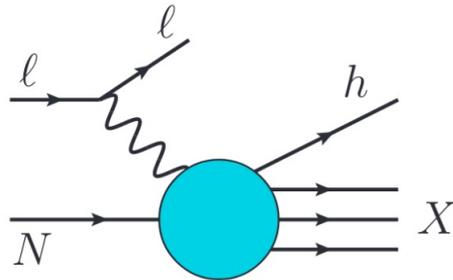
- **Hadronization:** how particular hadrons are formed from scattered quarks and gluons (partons)
- **Fragmentation Functions (FF):** probability distribution of a parton fragmenting into a specific hadron
- **Transverse momentum dependent (TMD):** spin-momentum correlations

Important processes in studying hadron formation

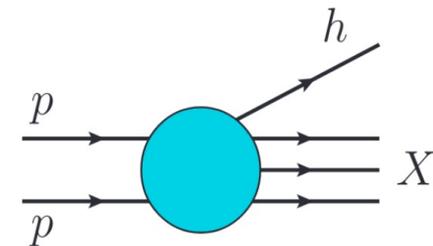
$$\sigma^{e^+e^- \rightarrow hX} = \hat{\sigma} \otimes FF$$



$$\sigma^{lN \rightarrow lhX} = PDF \otimes \hat{\sigma} \otimes FF$$



$$\sigma^{pp \rightarrow hX} = PDF \otimes PDF \otimes \hat{\sigma} \otimes FF$$



Leading Quark TMDFFs  Hadron Spin  Quark Spin

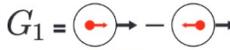
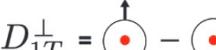
		Quark Polarization		
		Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Unpolarized (or Spin 0) Hadrons		$D_1 = \text{Unpolarized}$ 		$H_1^\perp = \text{Collins}$ 
	L		$G_1 = \text{Helicity}$ 	H_{1L}^\perp 
Polarized Hadrons	T	$D_{1T}^\perp = \text{Polarizing FF}$ 	G_{1T}^\perp 	$H_1 = \text{Transversity}$  H_{1T}^\perp 

Image from [arXiv:2304.03302v1](https://arxiv.org/abs/2304.03302v1)

Progress in Particle and Nuclear Physics (2016) pp. 136-202

Hadronization at Belle

Belle measurements

sensitive to:

- Collins FF
- Di-hadron FF
- Polarizing FF

...

Azimuthal asymmetries in inclusive production of hadron

Phys. Rev. Lett. 96, 232002 (2006) Phys. Rev. D 78, 032011 (2008) [Phys.Rev.D 86, 039905 (2012)]

Transverse polarization asymmetries of charged pion pairs

Phys. Rev. Lett. 107, 072004 (2011)

Inclusive cross sections for pairs of identified light charged hadrons and for single

Phys. Rev. D 92, 092007 (2015)

Invariant-mass and fractional-energy dependence of inclusive production of di-hadrons

Phys. Rev. D 96, 032005 (2017)

Production cross sections of hyperons and charmed baryons

Phys. Rev. D 97, 072005 (2018)

Transverse $\Lambda/\bar{\Lambda}$ Hyperon

Phys. Rev. Lett. 122, 042001 (2019)

Transverse momentum dependent production cross sections of charged pions, kaons and protons

Phys. Rev. D 99, 112006 (2019)

Inclusive cross sections of single and pairs of identified light charged hadrons

Phys. Rev. D 101, 092004 (2020)

Production cross section of light and charmed mesons

Belle preprint 2024-09, KEK Preprint 2024-30, submitted to PRD

Recent measurement

Hadronization at Belle

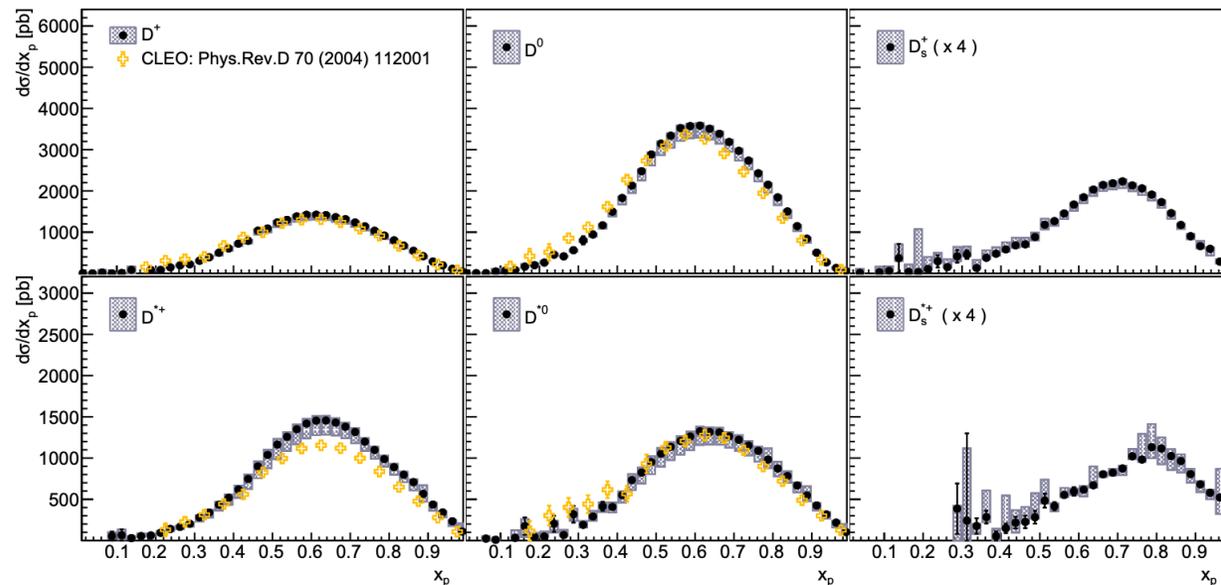
$$x_p = p_h / p_{max}$$

R. Seidl, "Production cross sections of light and charmed mesons in e^+e^- annihilation near 10.58 GeV"

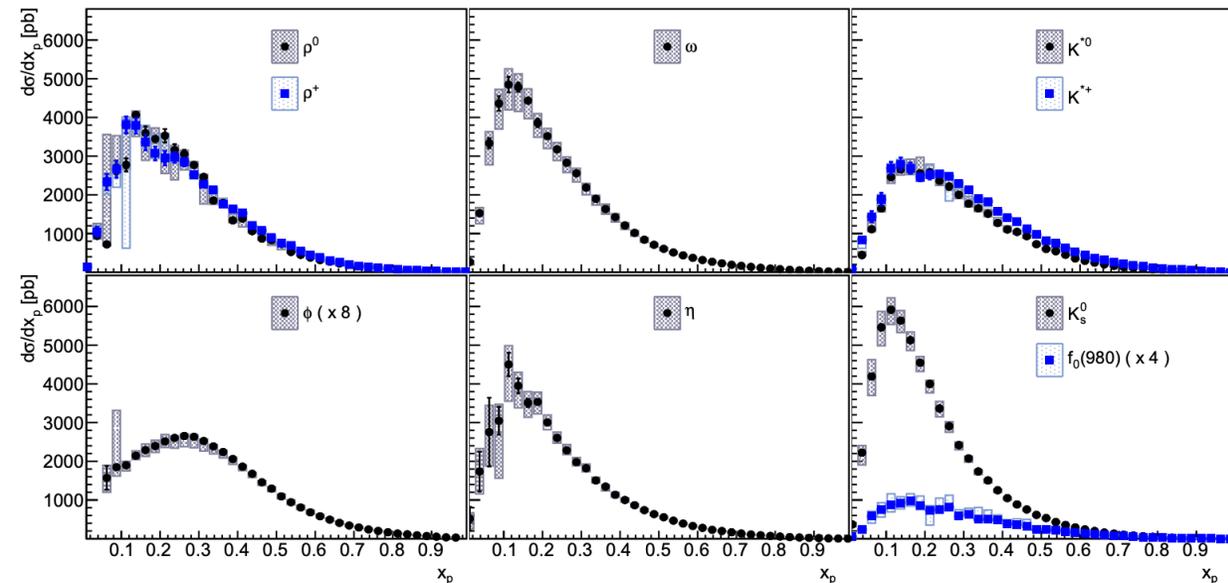
[Belle preprint 2024-09](#), [KEK Preprint 2024-30](#), submitted to PRD

- Comprehensive study of production cross section of light and charmed mesons
- Improved ISR corrections for D-mesons, and detailed comparison with various MC tunes
- Important for future SIDIS measurements at the EIC

Charmed mesons



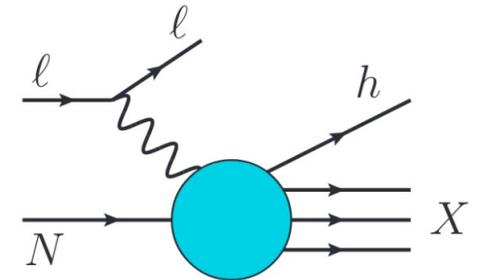
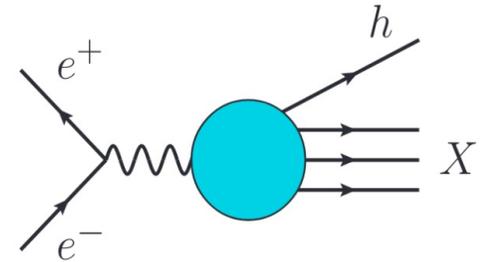
Light mesons



Hadronization at Belle II and for the EIC

- **Belle II can offer high precision, comprehensive measurements essential for the EIC**
 - Clean environment for detailed studies of hadronic final states
 - Multi-dimensional analyses of FFs, correlations, heavy flavor, and hadronization effects in jets
 - Essential for understanding transverse momentum of partons in measurements of PDFs and spin-structure of nucleon at the EIC
- + . . .

$$\sigma^{e^+e^- \rightarrow hX} = \hat{\sigma} \otimes FF$$



$$\sigma^{lN \rightarrow lhX} = PDF \otimes \hat{\sigma} \otimes FF$$

See Snowmass whitepaper [arXiv:2204.02280](https://arxiv.org/abs/2204.02280)

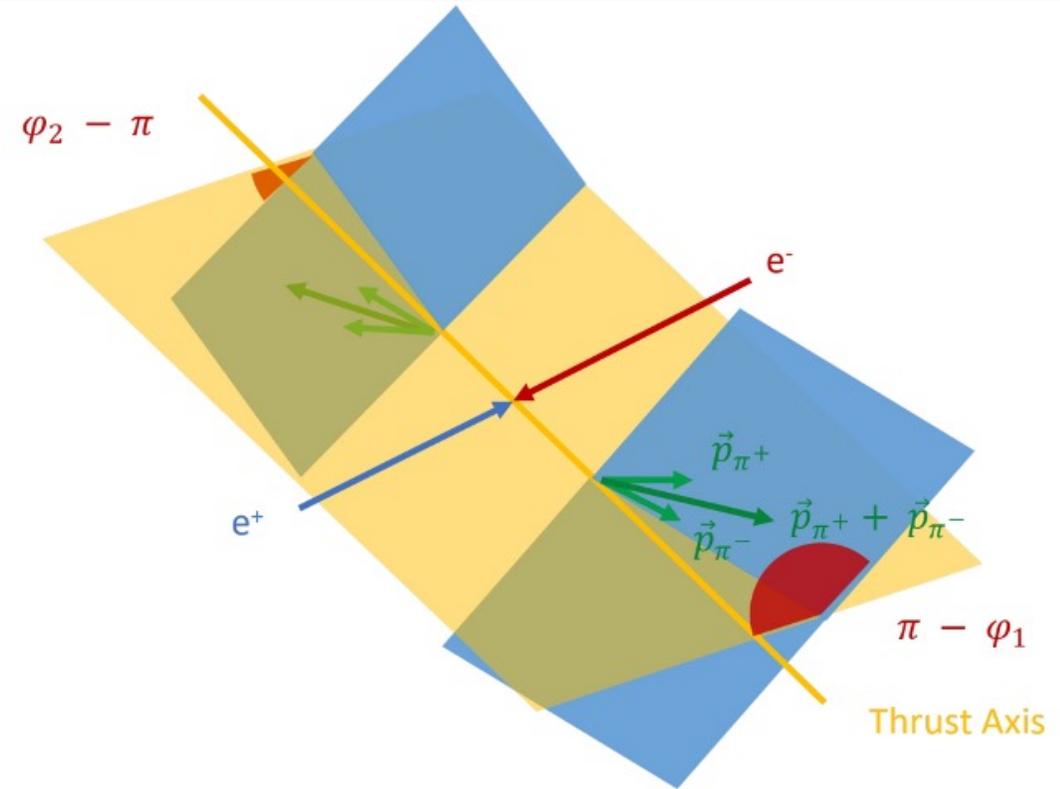
Current ongoing analyses at Belle II

1. Di-hadron Fragmentation Functions
2. Λ Polarization
3. TMD Jet Functions

Di-hadron Fragmentation Functions (DiFF)

- $H_1^{\chi}(z, M, P_h, \theta)$ FF describe fragmentation of polarized quark into pair of spin-0 hadrons
- Spin correlation between the $q\bar{q}$ pair results in correlating between the azimuthal angles of dihadron pairs produced
- Belle measured the azimuthal asymmetries for dihadrons measured as a function of z_h and m_h

[Phys. Rev. Lett. 107, 072004 \(2011\)](#)



Thrust:

$$T = \max \frac{\sum_h |P_h^{CMS} \cdot \hat{T}|}{\sum_h |P_h^{CMS}|}$$

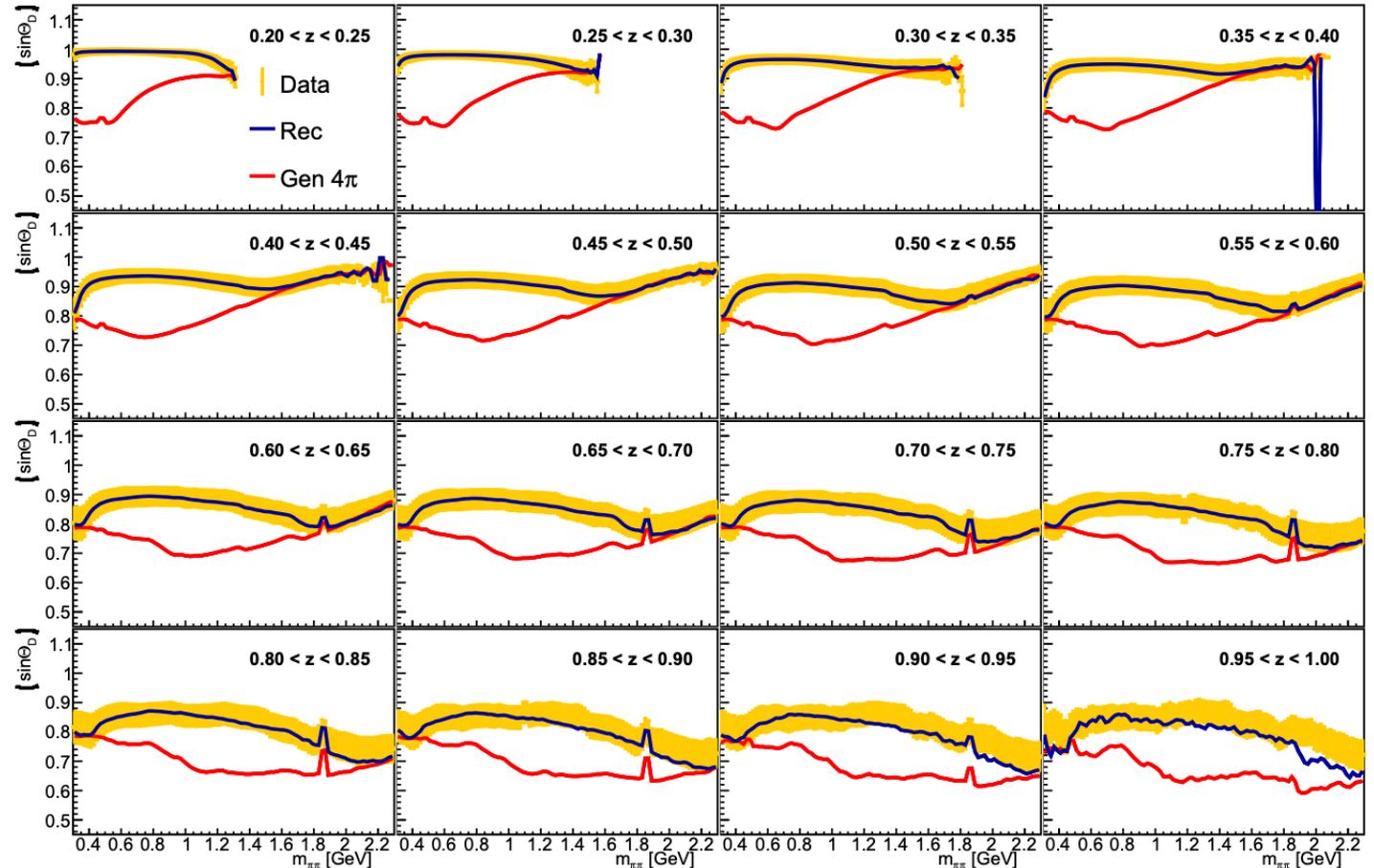
Cuts on thrust provide clean $q\bar{q}$ ($q \in u, d, s, c$) event sample

Di-hadron Fragmentation Functions (DiFF)

Phys. Rev. D 96, 032005 (2017)

Partial wave expansion

- More complex partial wave contribution to transverse polarization dependent DiFF
- Dependence on m , z , p_t , θ , ϕ
- Important to understanding production at the EIC
- Belle II statistics enable multidimensional analysis



[JPS Conf.Proc. 37 \(2022\) 020109](#)

$\sin \theta_D$ decay moment for $\pi^+\pi^-$ pairs; Belle results (655 fb^{-1})

Di-hadron Fragmentation Functions (DiFF)

- **Kaon inclusive**

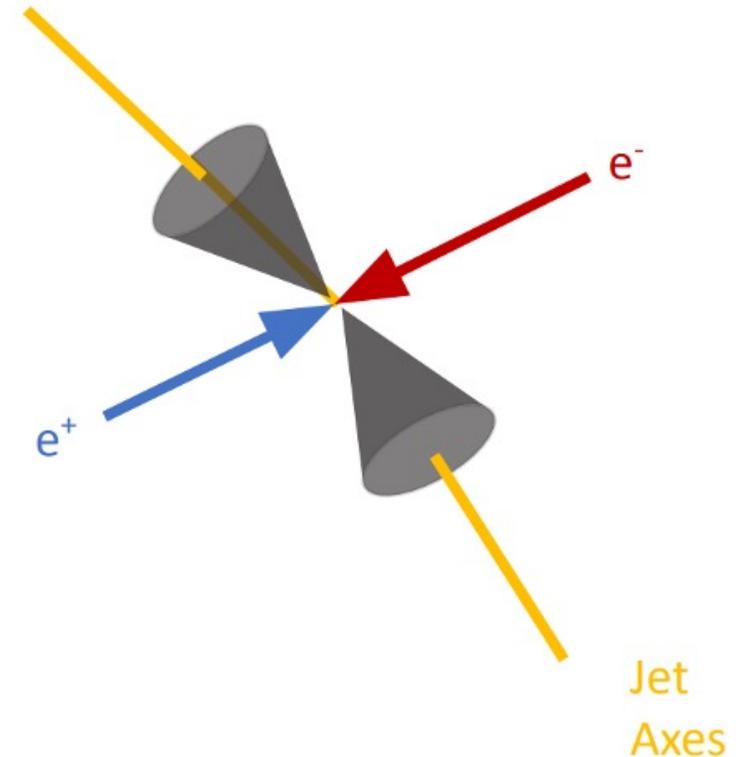
- Measurement with K^+K^- , $K^+\pi^-$, or π^+K^- pairs
- Results of H_1^{χ} can be used to describe strange quark distribution in the nucleon

- **Jet axis**

- Using jets axis instead of $q\bar{q}$ thrust axis
- Results link FFs in e^+e^- to SIDIS

New measurement important for upcoming experiments at JLab and the EIC

Jets: collimated spray of particles originating from partons in collision



Transverse Λ Polarization at Belle

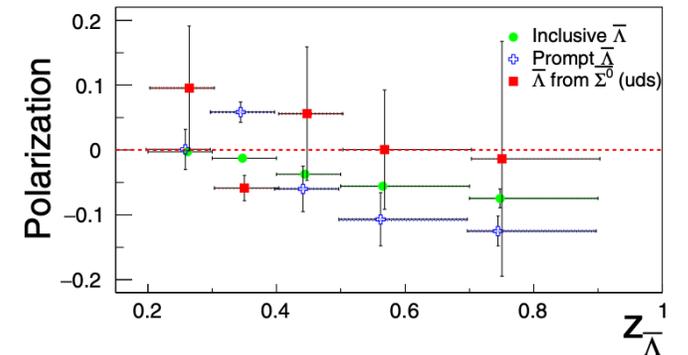
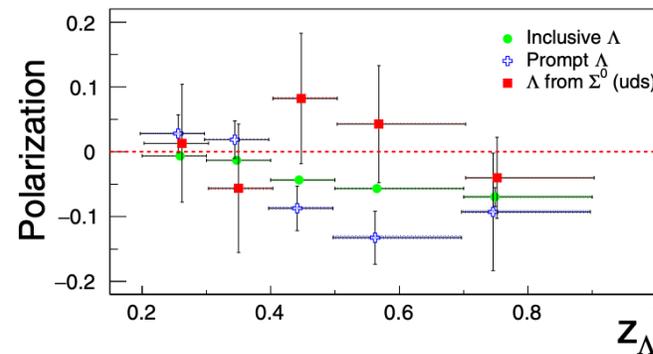
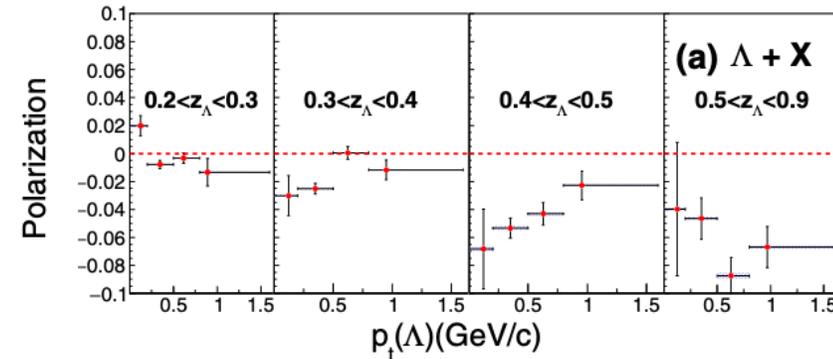
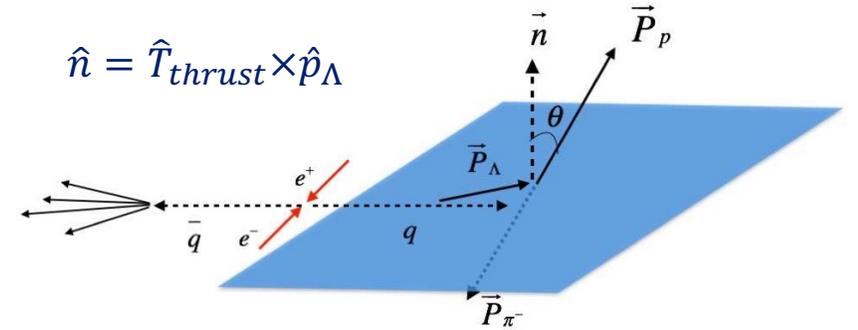
$$z_h = 2E_h/\sqrt{s}$$

- $\Lambda \rightarrow p\pi^-$ self analyzing decay

$$\frac{1}{N} \frac{dN}{d\cos\theta^*} = (1 + \alpha_\Lambda P \cos\theta^*)$$

$$\alpha_\Lambda = 0.748 \pm 0.007 \text{ (PDG 2023)}$$

- Nonzero transverse polarization observed for Λ and $\bar{\Lambda}$ as function of z and p_T
- Investigate feed-down contributions from Σ^0 and charm decays



[Phys. Rev. Lett. 122, 042001 \(2019\)](#)

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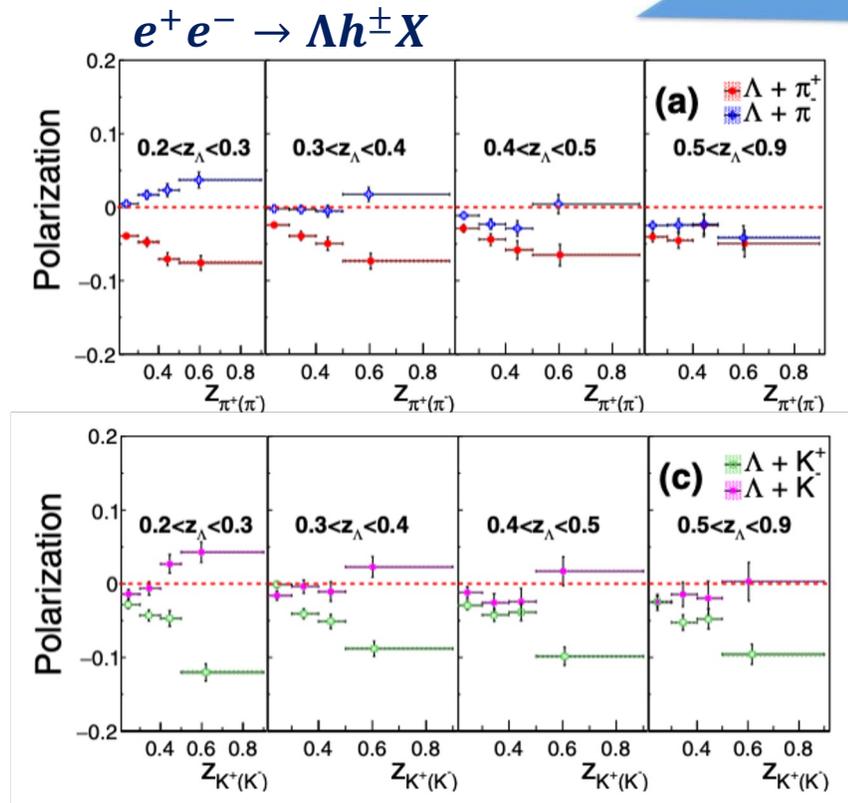
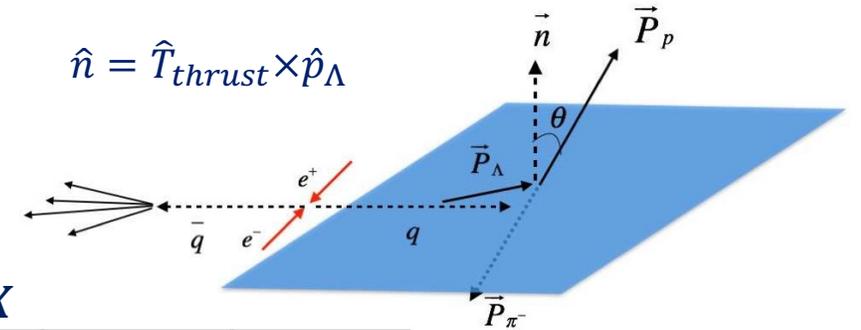
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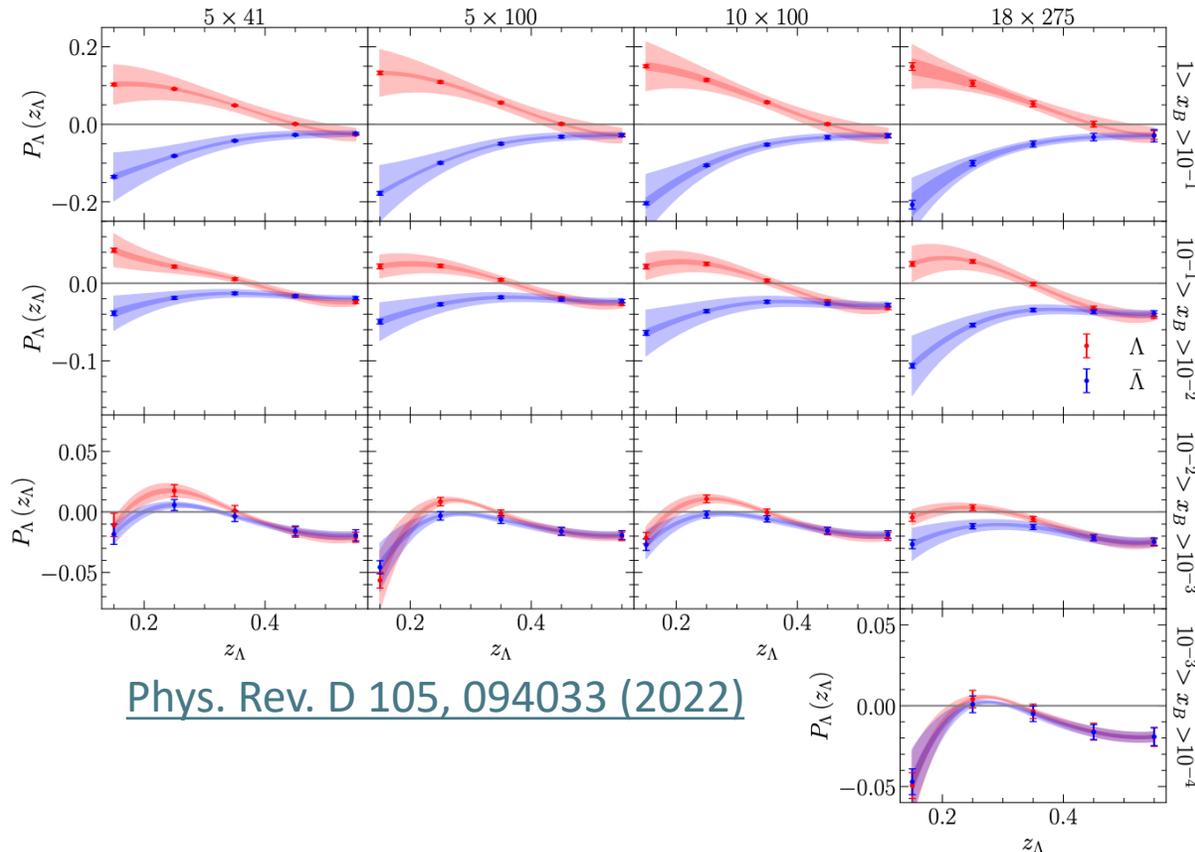
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- Investigate feed-down contributions from Σ^0 and charm decays
- Polarization measurement also with respect to hadron in opposite hemisphere



[Phys. Rev. Lett. 122, 042001 \(2019\)](#)

Transverse Λ Polarization

- Belle measurement data accurate enough for phenomenological studies
- Used for extractions of polarizing FF and Λ polarization predictions in $ep \rightarrow \Lambda X$; for example:



Leading Quark TMDFFs  Hadron Spin  Quark Spin

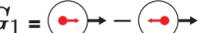
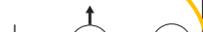
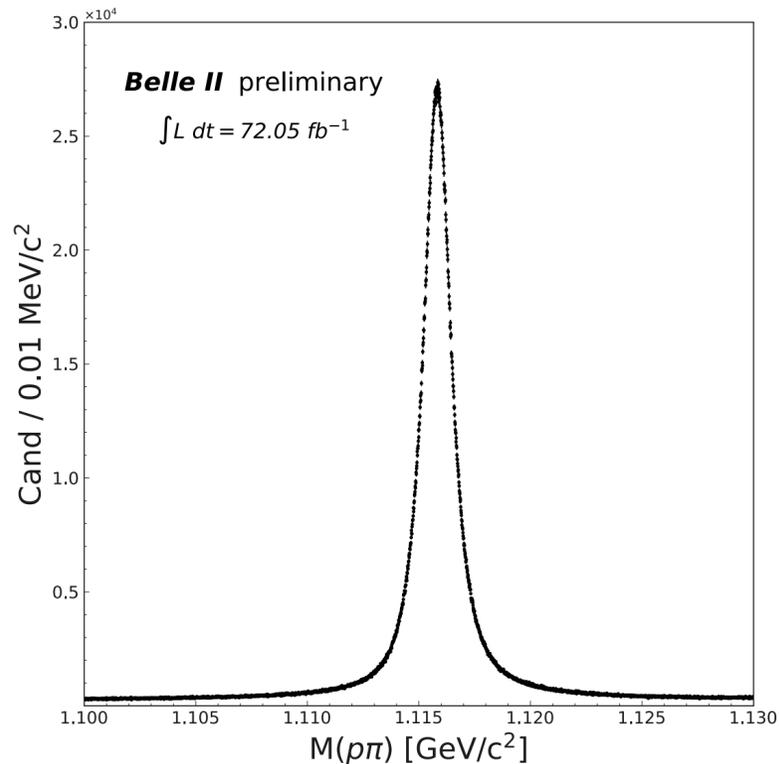
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Image from [arXiv:2304.03302v1](https://arxiv.org/abs/2304.03302v1)

Transverse Λ Polarization

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Measurements at Belle II:

- Reduce uncertainties from feed-down and the prompt Λ
- Λ polarization with respect to the plane spanned by beam axis and Λ momentum

Λ Spin Correlation

[Phys. Rev. D 106, L031501 \(2022\)](#)

[Phys. Rev. D 109, 116003 \(2024\)](#)

- Entanglement as a probe to hadronization
 - Spin correlation extracted from the correlation of relative spin projections
 - $N \propto 1 + \alpha^2 P_{\Lambda, \Lambda} \cos(n\theta_{ab})$
 - Get expected zero result in simulation

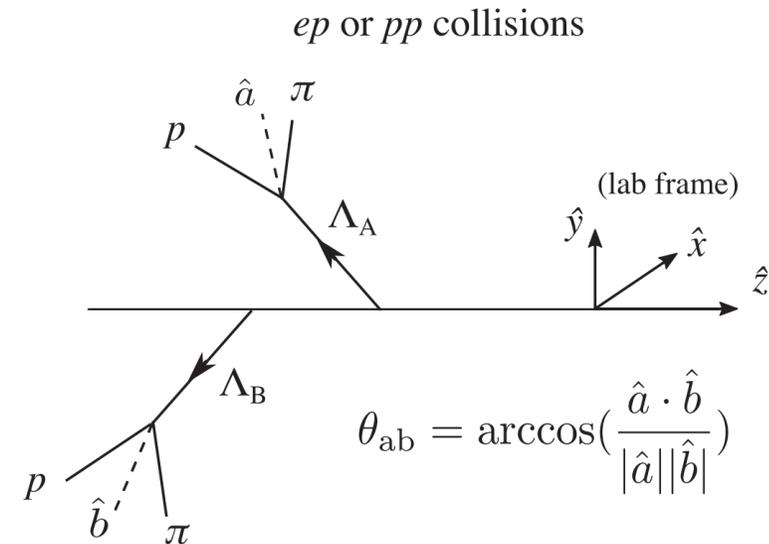
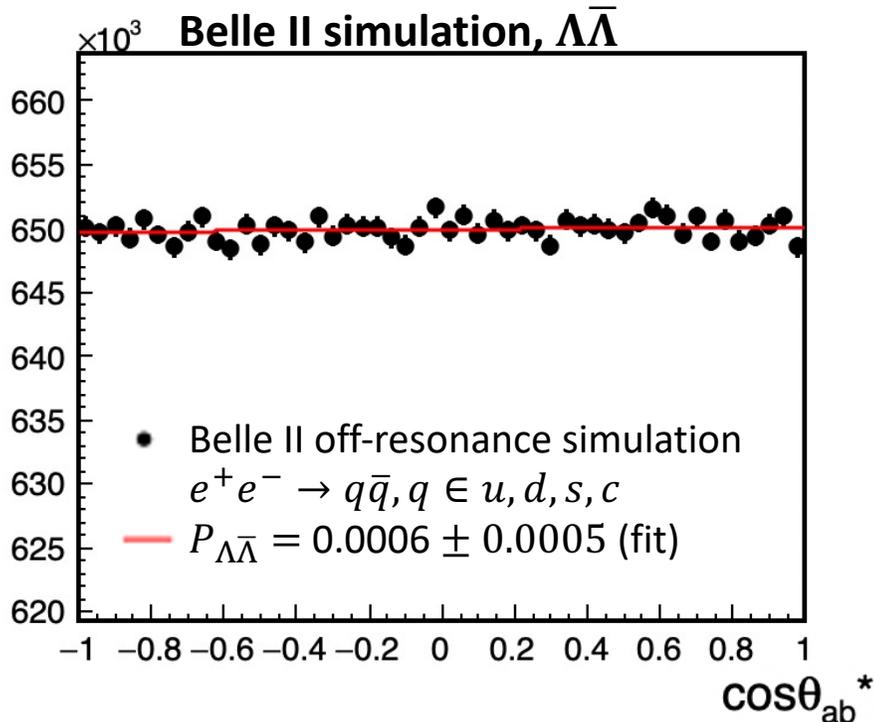


FIG. 3. Illustration of double Λ polarization; here \hat{a} (\hat{b}) denotes the momentum direction of Λ_A (Λ_B) daughter particle in the Λ_A (Λ_B) rest frame.

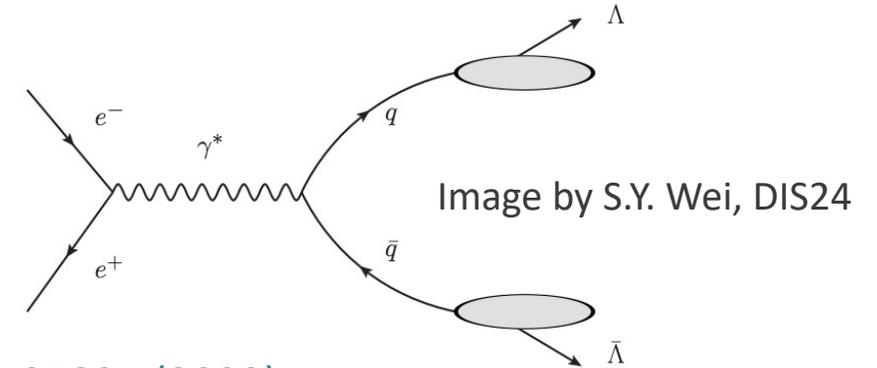
Longitudinal spin transfer via dihadron polarization

- Helicity correlation of two produced partons
- Alternative approach to traditional methods using polarized beams and targets

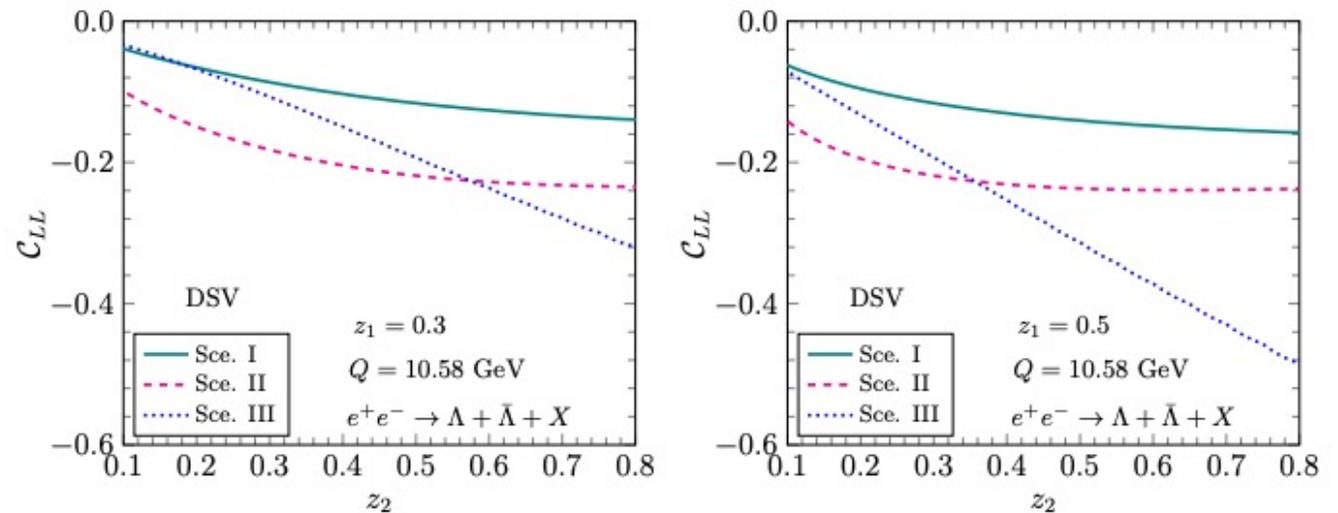
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Image from [arXiv:2304.03302v1](https://arxiv.org/abs/2304.03302v1)



[Phys.Lett.B 839, 137821 \(2023\)](https://arxiv.org/abs/2304.03302v1)



$$\frac{1}{N} \frac{dN}{d \cos \theta_1^* d \cos \theta_2^*} = \frac{1}{4} + P_L^\Lambda \frac{1}{4} \alpha \cos \theta_1^* + P_L^{\bar{\Lambda}} \frac{1}{4} \alpha \cos \theta_2^* + C_{LL} \frac{1}{4} \alpha^2 \cos \theta_1^* \cos \theta_2^*,$$

TMD Jet Functions

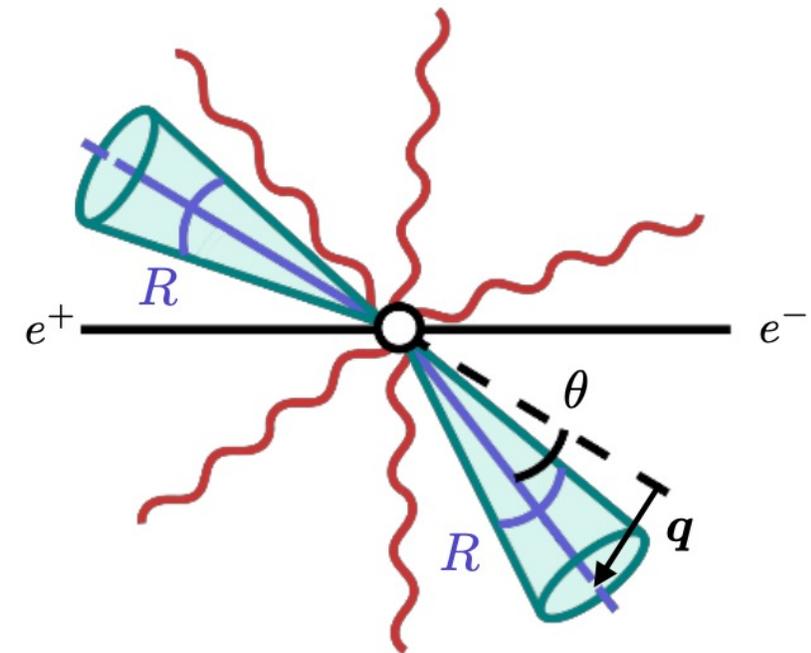
- **TMD FF \rightarrow TMD Jet Functions**

- Use jets (instead of hadrons) in final state
 - Jet momentum is perturbatively calculable
 - Reduce uncertainty and improve sensitivity to PDFs in SIDIDS
- Measuring the jet q_T spectrum:

$$\mathbf{q} = \frac{\mathbf{p}_1}{z_1} + \frac{\mathbf{p}_2}{z_2}$$

Require decorrelation to be small: $q_T \equiv |\mathbf{q}| \ll \frac{\sqrt{s}}{2}$

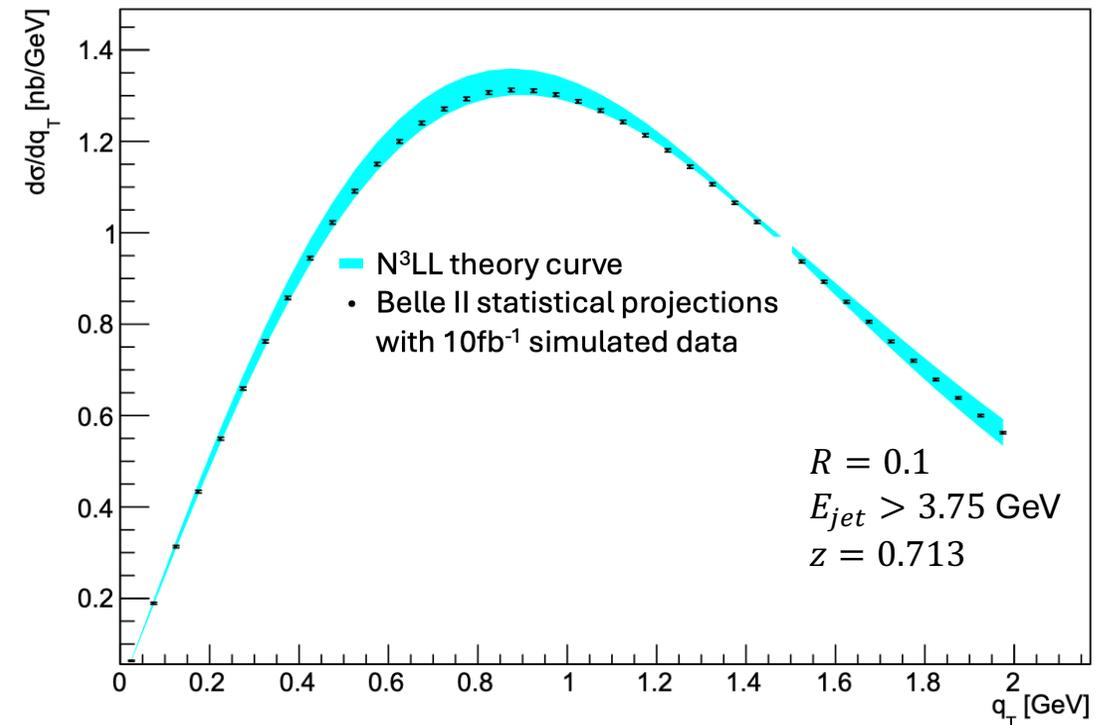
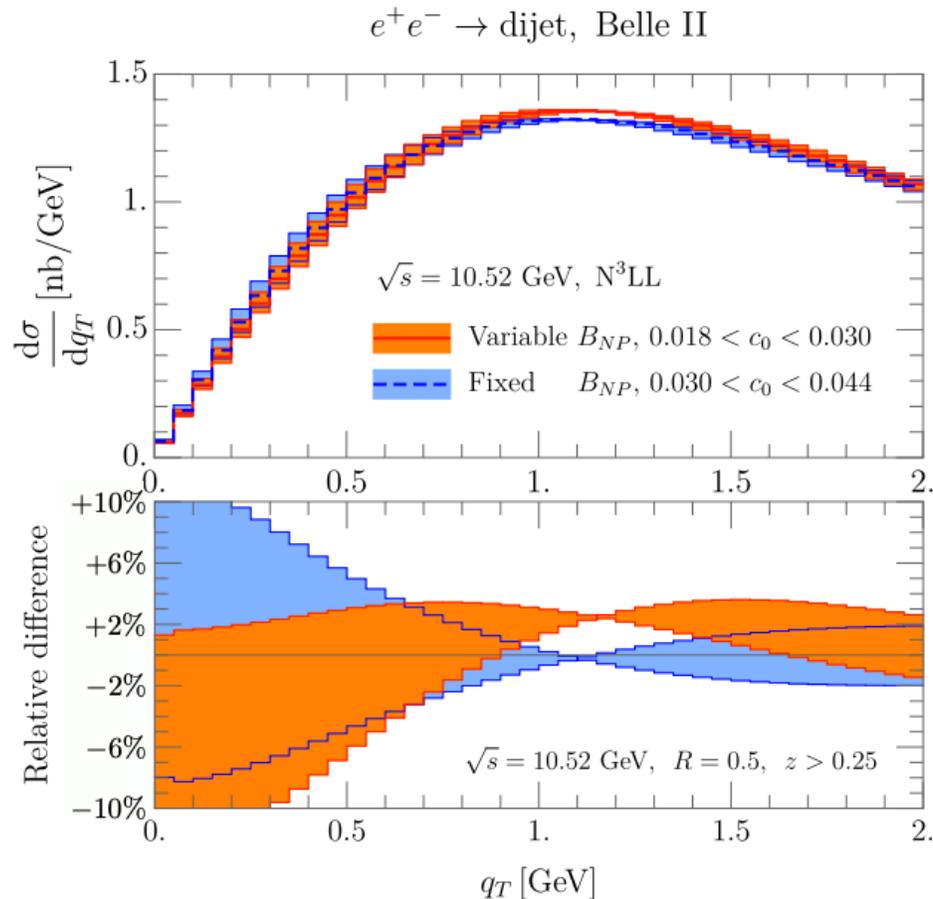
[Phys. Rev. Lett. 121, 162001 \(2018\)](#)
[J. High Energ. Phys. 2019, 31 \(2019\)](#)



TMD Jet Functions – q_T Spectrum

Theoretical predictions for q_T predictions

[JHEP10\(2019\)031](#)
[arXiv:2204.02280v2](#)



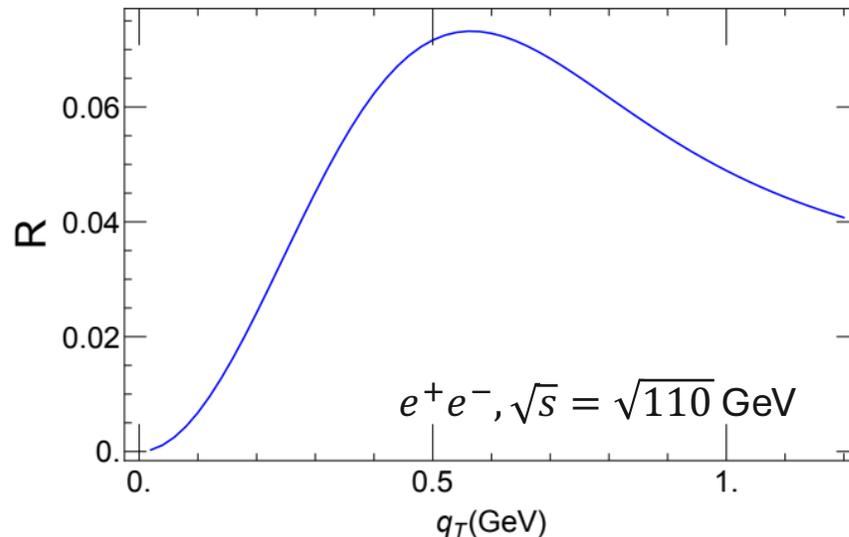
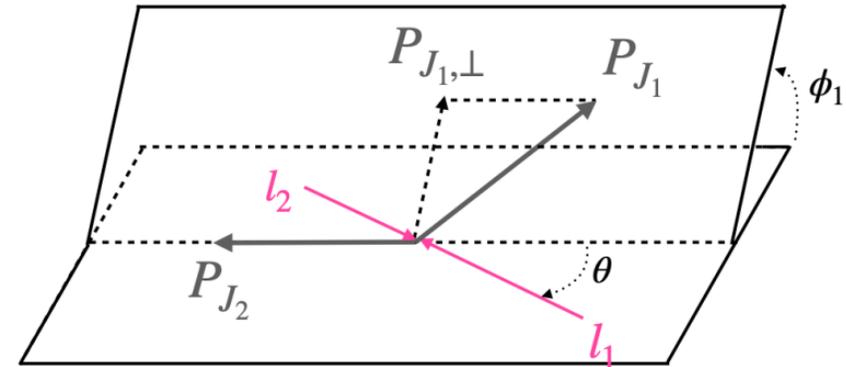
- Sensitivity of the TMD to nonperturbative effects

- Statistical projections with Belle II simulation

TMD Jet Functions – T-odd side of jets

- T-odd jet components:
 - Recently found to survive due to non-perturbative effects
 - Important to access nucleon spin structure
- T-odd component can couple to the proton transversity at the EIC

arXiv:2104.03328 [hep-ph]



Azimuthal asymmetry

$$R^{J_1 J_2} = 1 + \cos(2\phi_1) \frac{\sin^2 \theta}{1 + \cos^2 \theta} \frac{F_T(q_T)}{F_U(q_T)}$$

$$R = 2 \int d \cos \theta \frac{d\phi_1}{\pi} \cos(2\phi_1) R^{J_1 J_2}$$

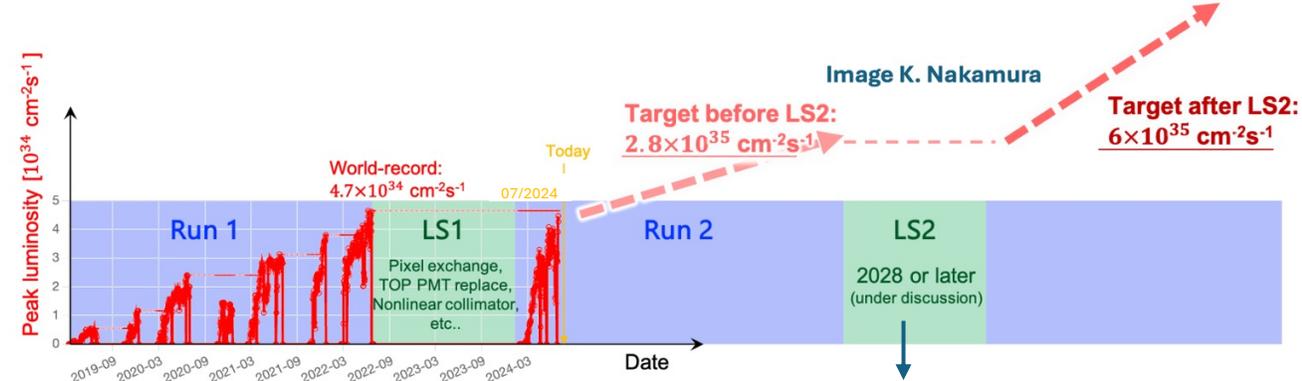
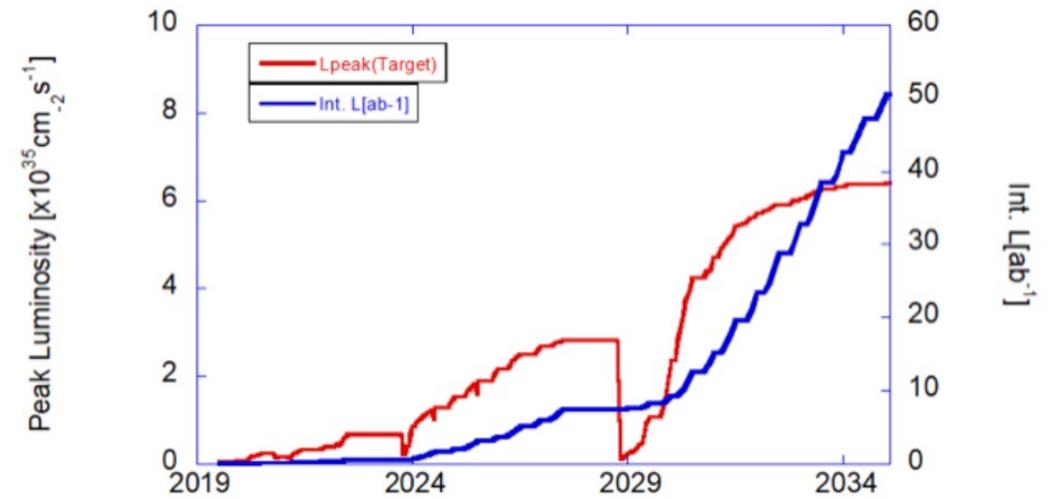
Summary

- Belle II is currently collecting data during Run 2
- Belle and Belle II play an important role in understanding hadronization dynamics

[arXiv:2204.02280](https://arxiv.org/abs/2204.02280)

- Provide key information on hadronization for future EIC measurements
- Lots of measurement opportunities at Belle II, with several current ongoing analyses underway
- Future QCD studies with polarized electron beams at SuperKEKB

Chiral Belle Project: [arXiv:2205.12847v3](https://arxiv.org/abs/2205.12847v3)



Chiral Belle (polarized e^- beams)

Thank You!

Thank you for the help in preparing this presentation to S. Schneider, K. Parham, A. Vossen, and the Belle II collaboration!

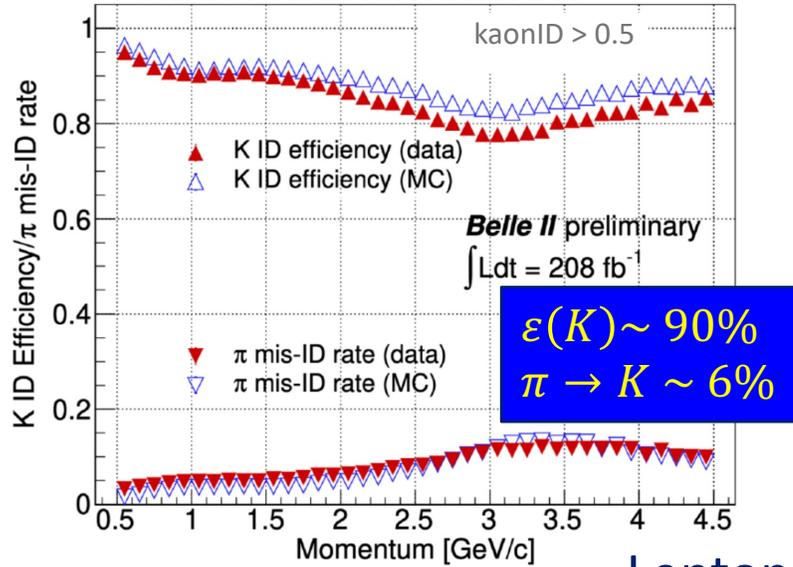
Back up

Belle II Detector @ SuperKEKB

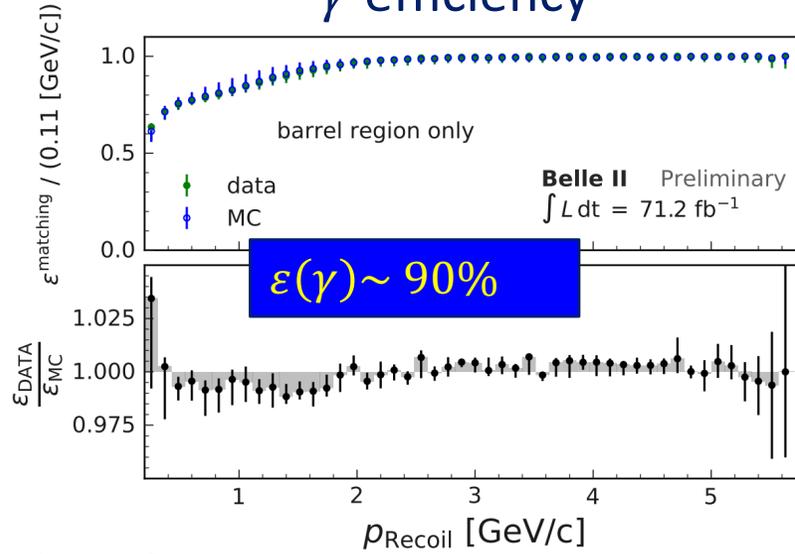
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BELLE2-CONF-PH-2022-003

BELLE2-NOTE-PL-2021-008
BELLE2-NOTE-PL-2020-031
BELLE2-CONF-PH-2021-002

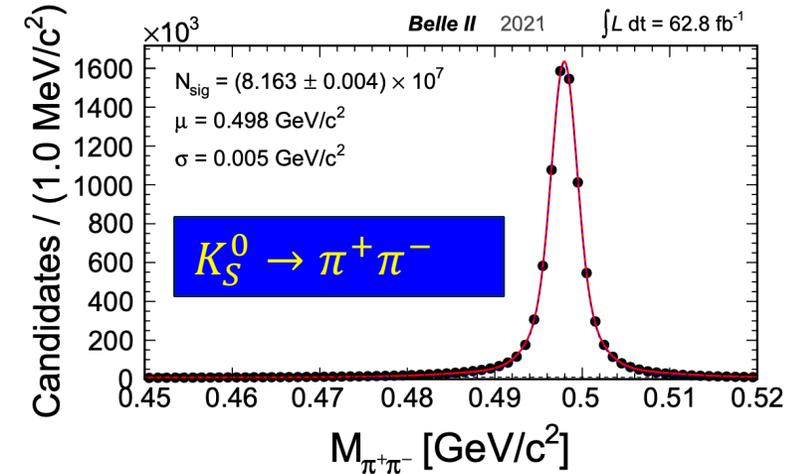
Kaon identification



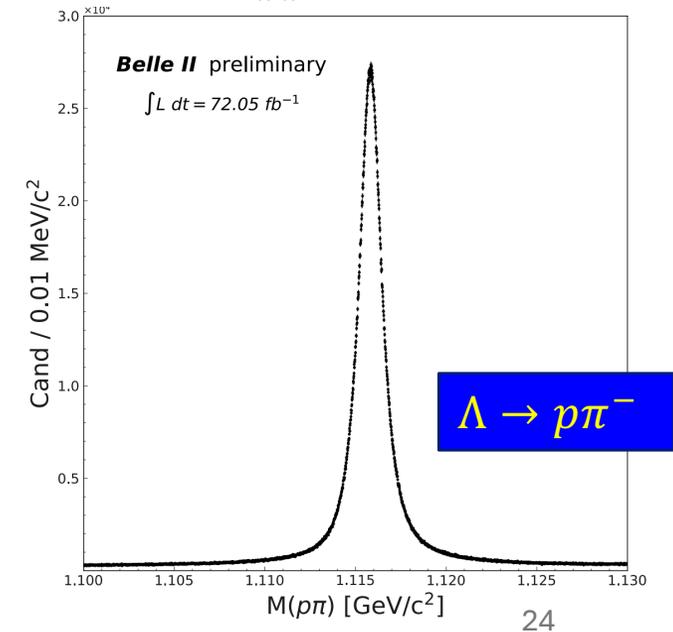
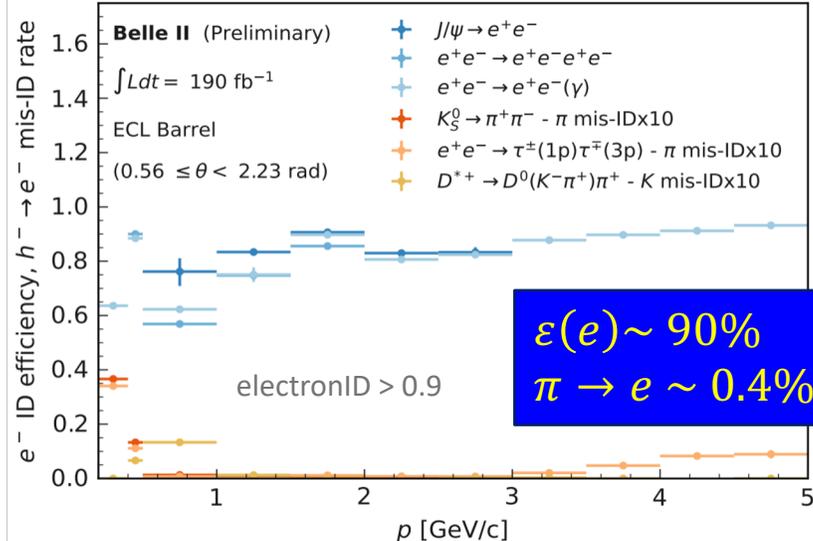
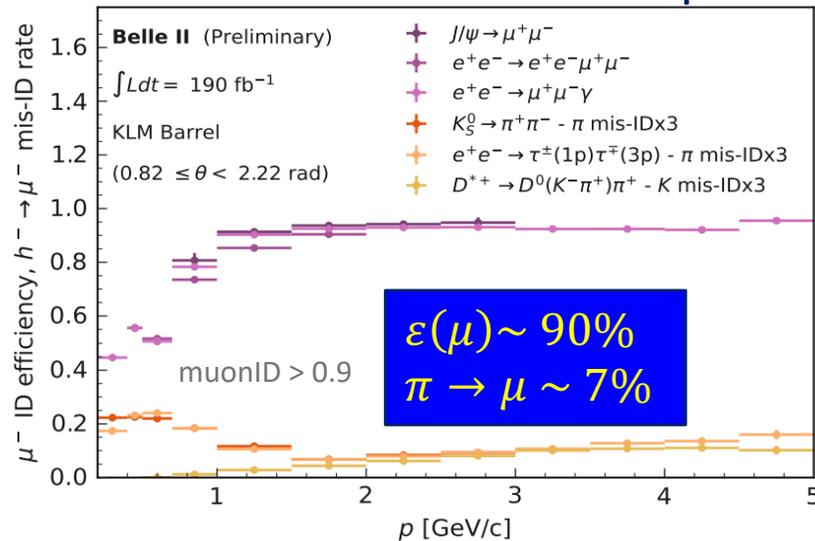
γ efficiency



V^0 Reconstruction



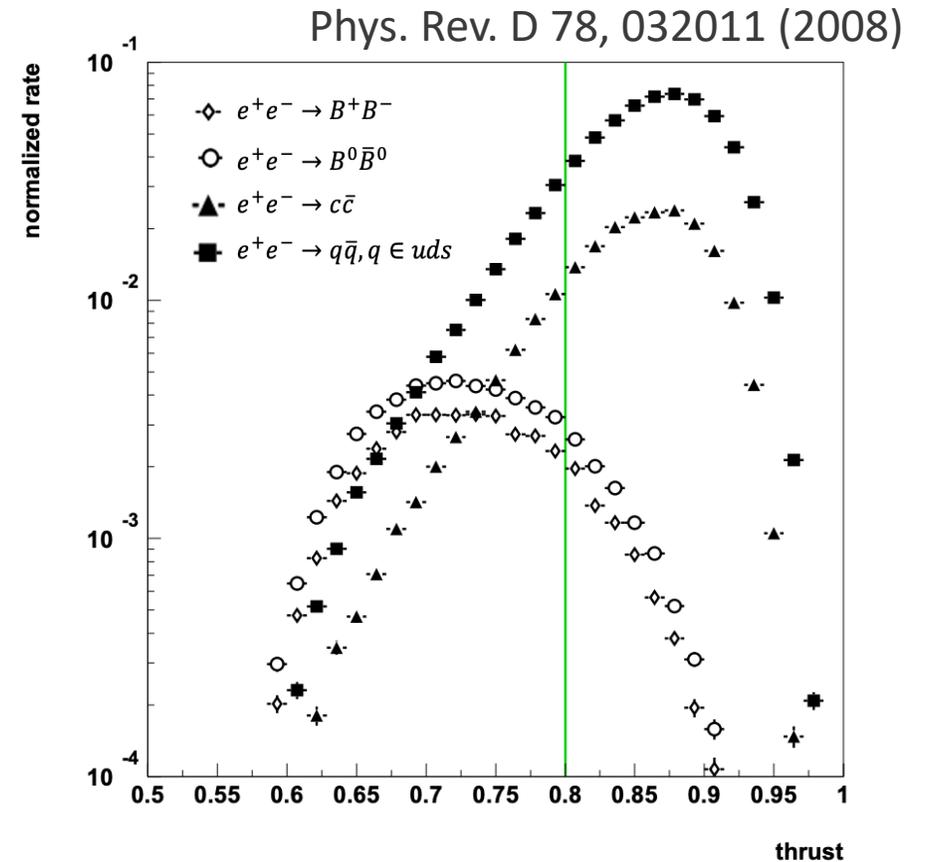
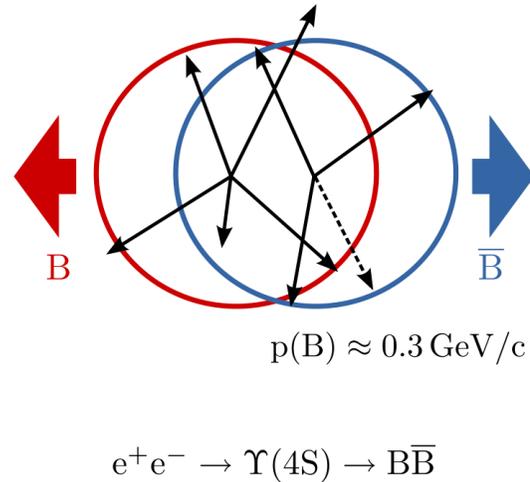
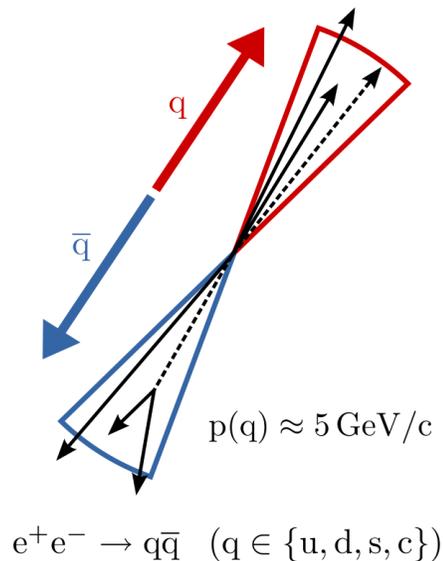
Lepton identification



Belle II event shape: thrust axis

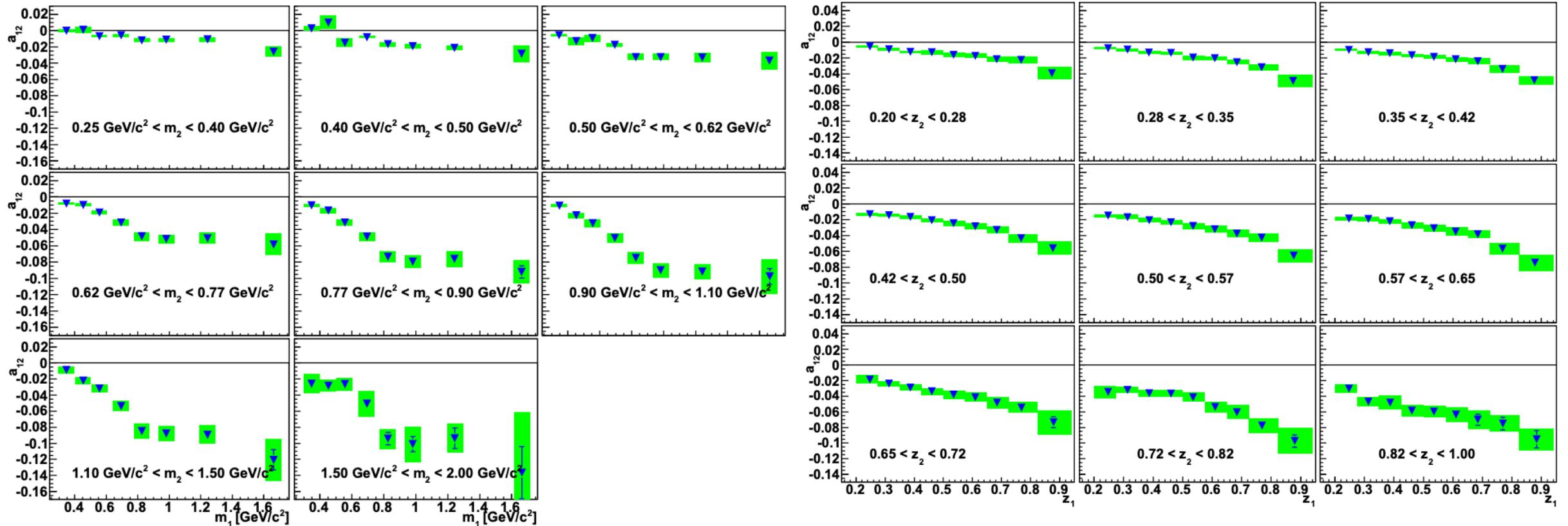
- Using B-factory for hadronization studies
 - Events produced at or near $\Upsilon(4S)$ have different shapes
 - Cuts on thrust provide clean $q\bar{q}$ event sample

$$T = \max \frac{\sum_h |P_h^{CMS} \cdot \hat{T}|}{\sum_h |P_h^{CMS}|}$$



Di-hadron Fragmentation Functions (DiFF)

Azimuthal asymmetries for $e^+e^- \rightarrow (\pi^+\pi^-)(\pi^+\pi^-)$ Belle results (670 fb^{-1})



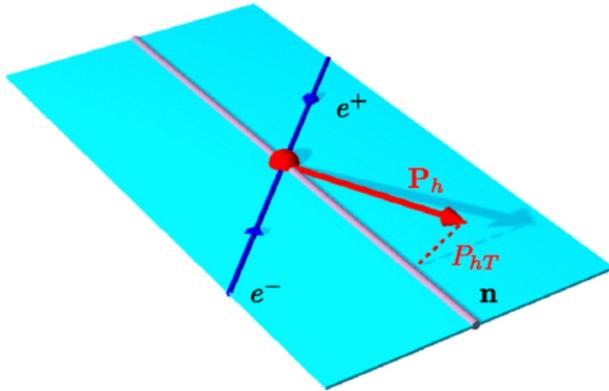
Phys. Rev. Lett. 107, 072004 (2011)

Belle results – a recent review

Belle data provided essential measurements, including recent results:

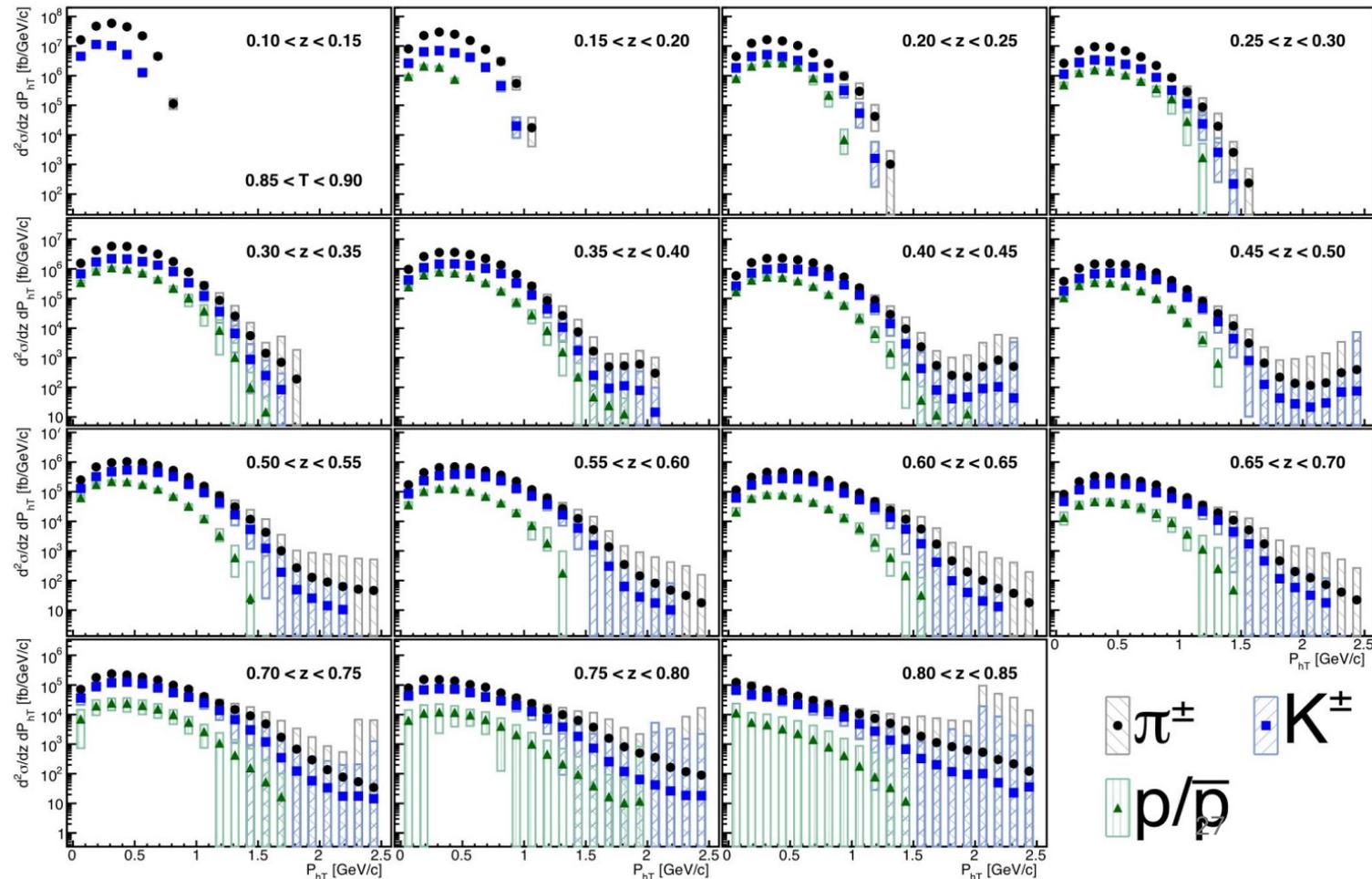
R. Seidl et al., “Transverse momentum dependent production cross sections of charged pions, kaons and protons produced in inclusive e^+e^- annihilation” at $\sqrt{s}=10.58$ GeV

Phys. Rev. D 99, 112006 (2019)



$$z = 2E_h/\sqrt{s}$$

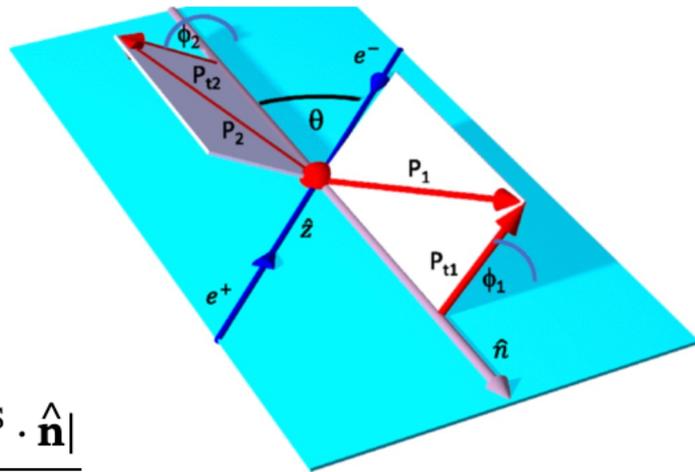
$$T \max \frac{\sum_h |\mathbf{P}_h^{\text{CMS}} \cdot \hat{\mathbf{n}}|}{\sum_h |\mathbf{P}_h^{\text{CMS}}|}$$



Belle results – a recent review

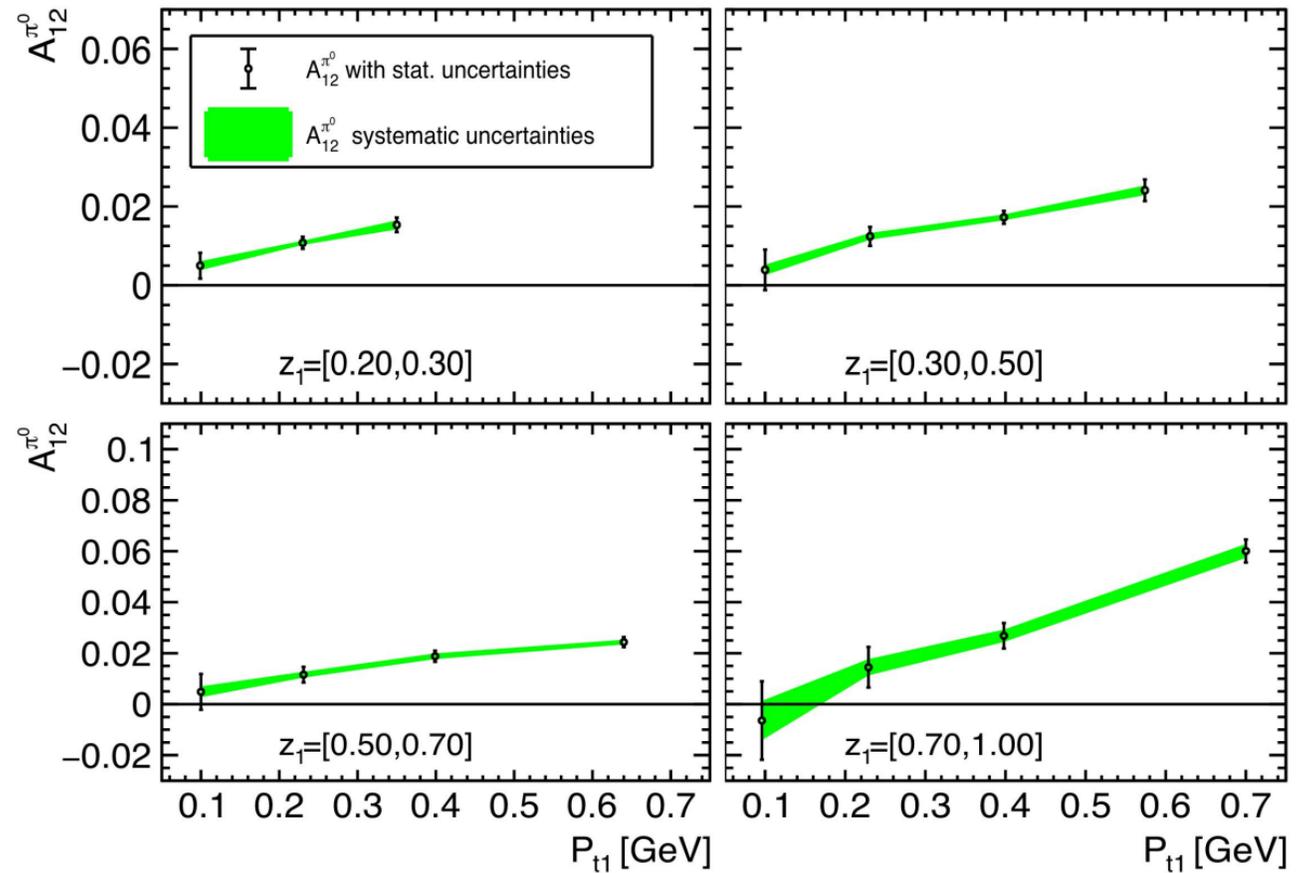
Belle data provided essential measurements, including recent results:

H. Li, A. Vossen, et al., “Azimuthal asymmetries of back-to-back $\pi^\pm - (\pi^0, \eta, \pi^\pm)$ pairs in e^+e^- annihilation” Phys.Rev.D 100 9, 092008 (2019)



$$z = 2E_h/\sqrt{s}$$

$$T \stackrel{\text{max}}{=} \frac{\sum_h |\mathbf{P}_h^{\text{CMS}} \cdot \hat{\mathbf{n}}|}{\sum_h |\mathbf{P}_h^{\text{CMS}}|}$$

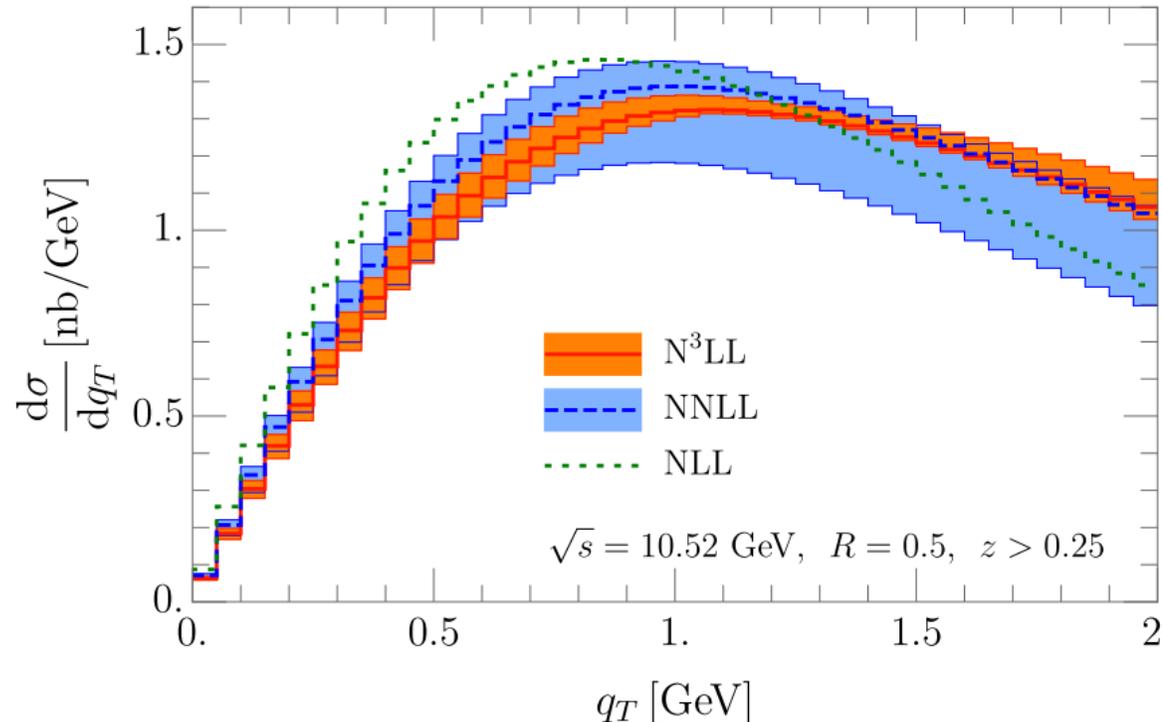


TMD Jet Functions – q_T Spectrum

Theoretical predictions for q_T predictions

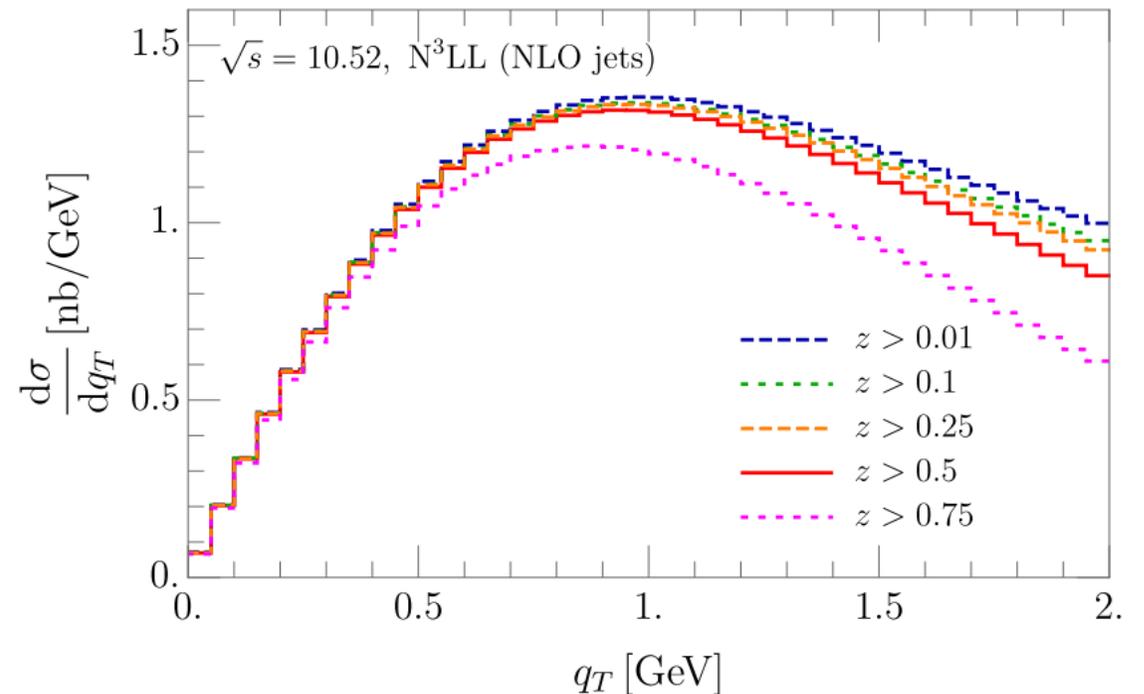
J. High Energ. Phys. 2019, 31 (2019)

$e^+e^- \rightarrow$ dijet, Belle II



- Perturbative convergence of cross-section

$e^+e^- \rightarrow$ dijet, Belle II, $R = 0.7$



- Dependence on z

Belle II upgrades

