Gravitational form factors in NNLO QCD and hadron mass decompositions

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Symmetric energy-momentum tensor

$$T^{\mu\nu} = \frac{1}{2}\bar{\psi}\gamma^{(\mu} i\overleftrightarrow{D}^{\nu)}\psi \;+\; F^{\mu\rho}F_{\rho}{}^{\nu} + \frac{\eta^{\mu\nu}}{4}F^2$$

$$\left(= \frac{1}{2}\bar{\psi}\gamma^{\mu} i\partial^{\nu}\psi - F^{\mu\rho}\partial^{\nu}A_{\rho} + \frac{\eta^{\mu\nu}}{4}F^2 + \partial_{\lambda}X^{[\lambda\mu]\nu} \right)$$

$$\sum_n \frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi_n)}\,\partial^\nu\phi_n - \eta^{\mu\nu}\mathcal{L}$$

$$T^{\mu\nu} = T^{\nu\mu}$$

$$\partial_{\mu}T^{\mu\nu} = 0$$

Symmetric energy-momentum tensor

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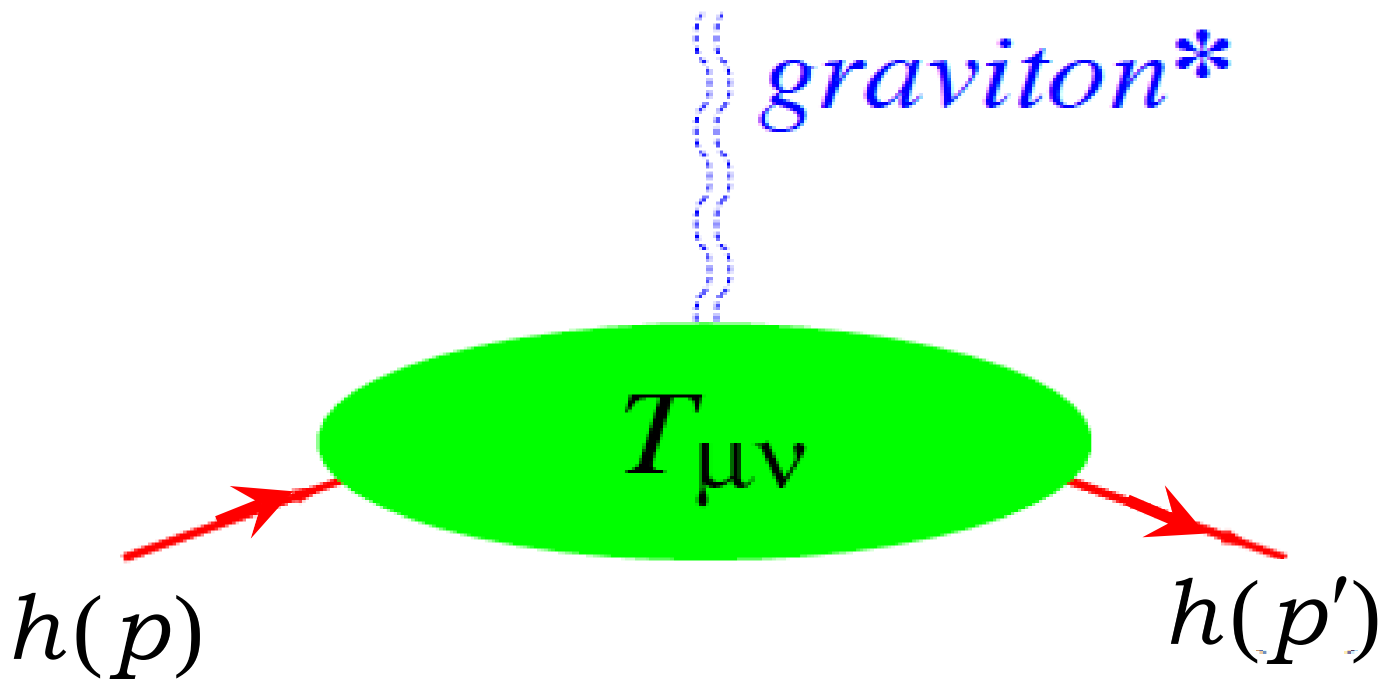
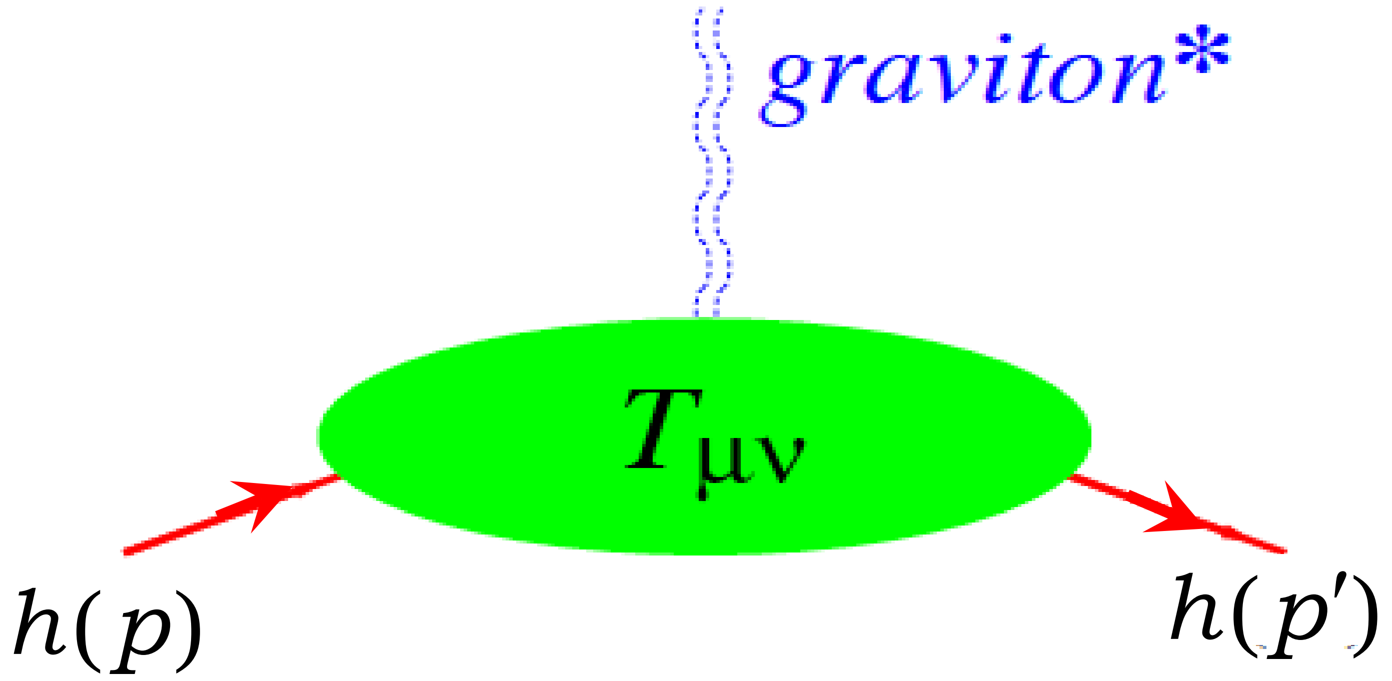
$$\sum_n \frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi_n)}\partial^\nu\phi_n - \eta^{\mu\nu}\mathcal{L}$$

$$T_{\mu\nu}(x) = \frac{2}{\sqrt{-g}}\frac{\delta S}{\delta g^{\mu\nu}(x)}\bigg|_{g^{\mu\nu}\to\eta^{\mu\nu}}$$

$$T^{\mu\nu} = T^{\nu\mu}$$

$$\partial_\mu T^{\mu\nu} = 0$$


graviton*
$T_{\mu\nu}$
$h(p)$
$h(p')$



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$$\equiv \quad T_q^{\mu\nu} \quad + \quad T_g^{\mu\nu}$$

$$T^{\mu\nu} = \frac{1}{2}\bar{\psi}\gamma^{(\mu} i\overleftrightarrow{D}^{\nu)}\psi + F^{\mu\rho}F_{\rho}{}^{\nu} + \frac{\eta^{\mu\nu}}{4}F^2 \equiv T_q^{\mu\nu} + T_g^{\mu\nu}$$

$$\langle N(p')|T^{\mu\nu}_{q,g}|N(p)\rangle = \bar{u}(p')\Big[A_{q,g}(t)\gamma^{(\mu}P^{\nu)} + B_{q,g}(t)\frac{P^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha}{2M} + D_{q,g}(t)\frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{M} + \bar{C}_{q,g}(t)M\eta^{\mu\nu}\Big]u(p)$$

$$P = \frac{p+p'}{2}$$

$$\Delta = p' - p$$

$$t = \Delta^2$$

Ji, PRL78, 610 ('97)
Polyakov, Schweitzer, IJMPA33, 1830025('18)

$$T^{\mu\nu} = \frac{1}{2}\bar{\psi}\gamma^{(\mu} i\overleftrightarrow{D}^{\nu)}\psi + F^{\mu\rho}F_{\rho}{}^{\nu} + \frac{\eta^{\mu\nu}}{4}F^2 \equiv T_q^{\mu\nu} + T_g^{\mu\nu}$$

mass & energy distribution

angular momentum distribution

$$\langle N(p')|T_{q,g}^{\mu\nu}|N(p)\rangle = \bar{u}(p')\Big[A_{q,g}(t)\gamma^{(\mu}P^{\nu)} + B_{q,g}(t)\frac{P^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha}{2M} + D_{q,g}(t)\frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{M} + \bar{C}_{q,g}(t)M\eta^{\mu\nu}\Big]u(p)$$

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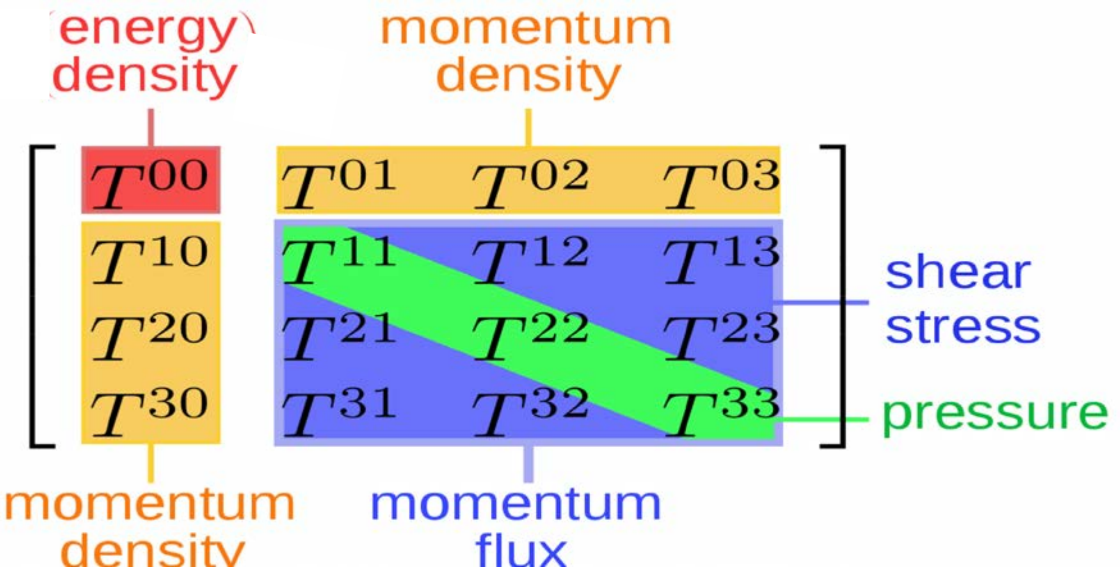
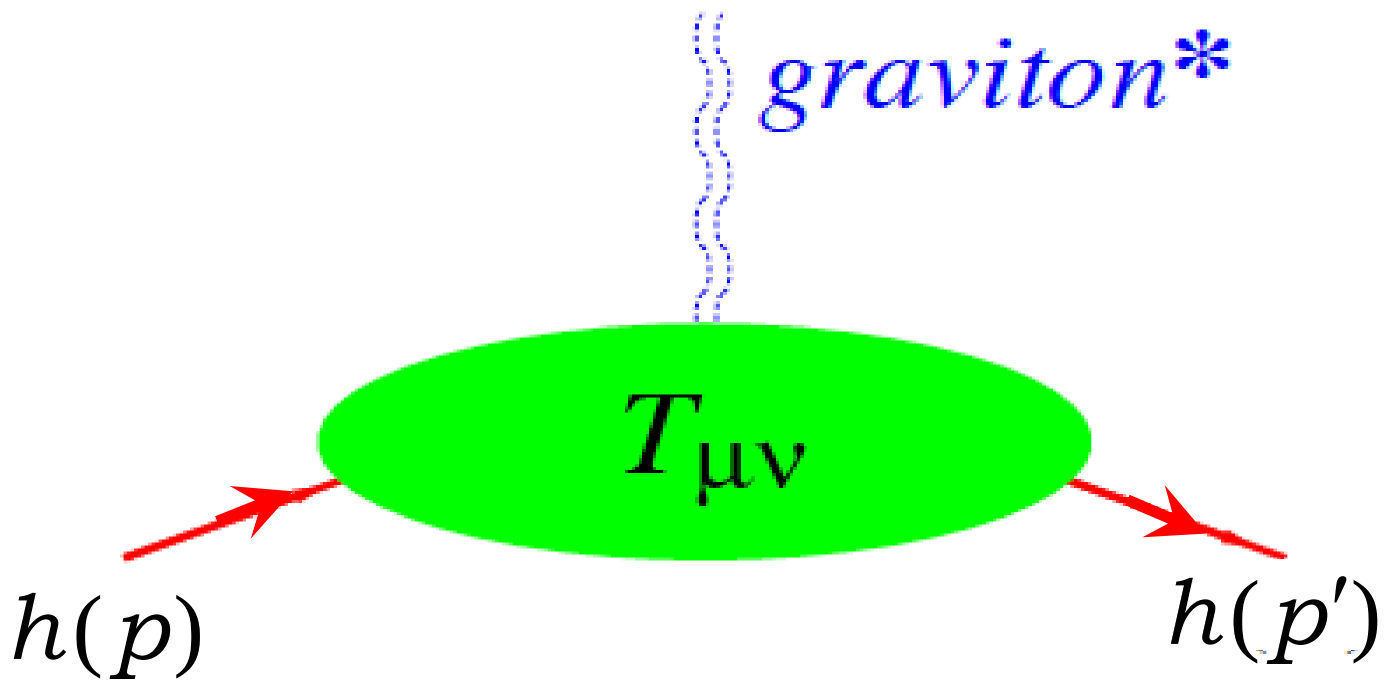
force & pressure distribution

mass & pressure distribution

$$T^{\mu\nu} =$$





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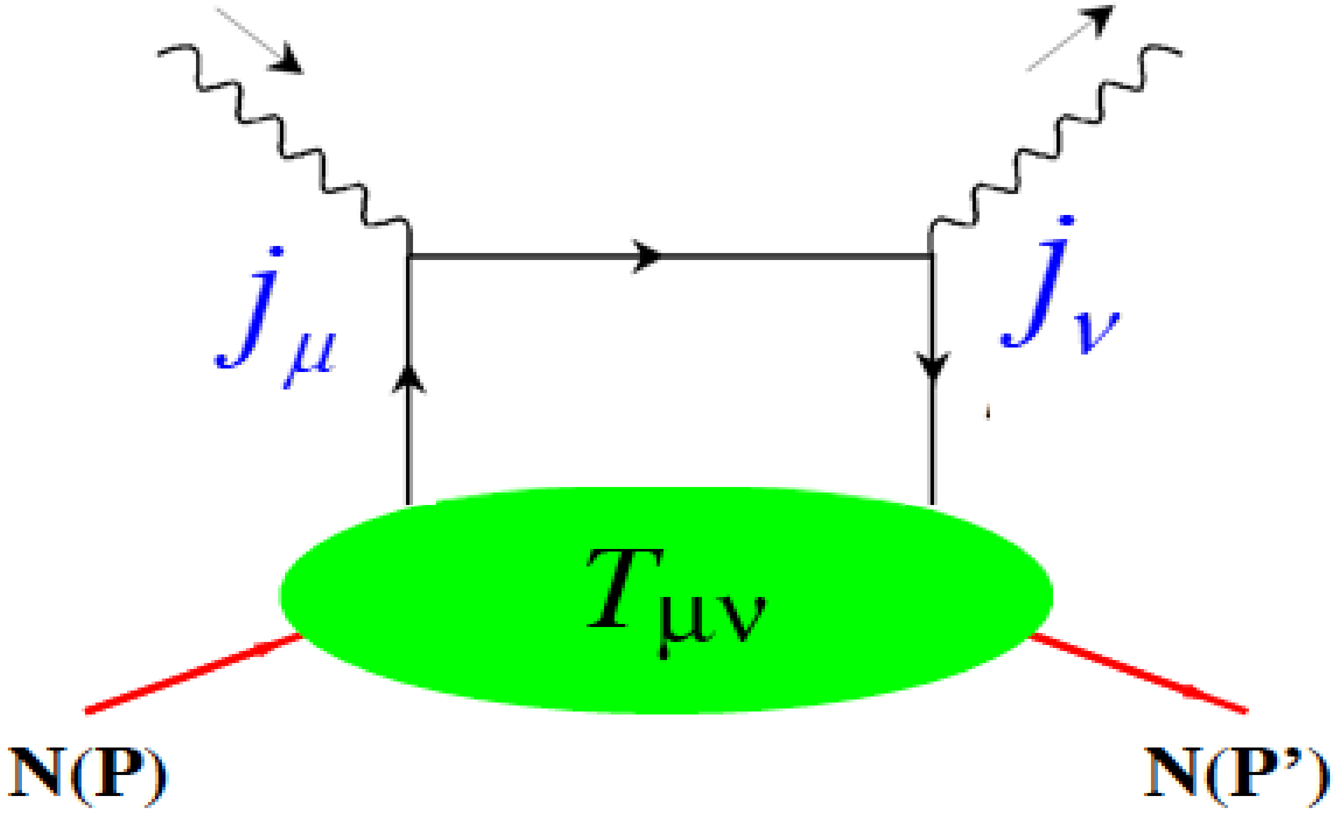

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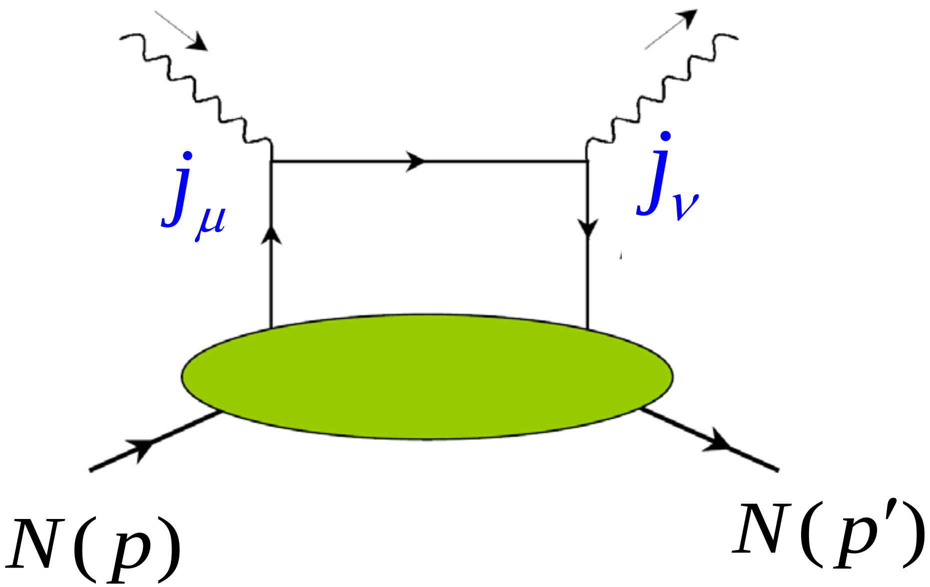
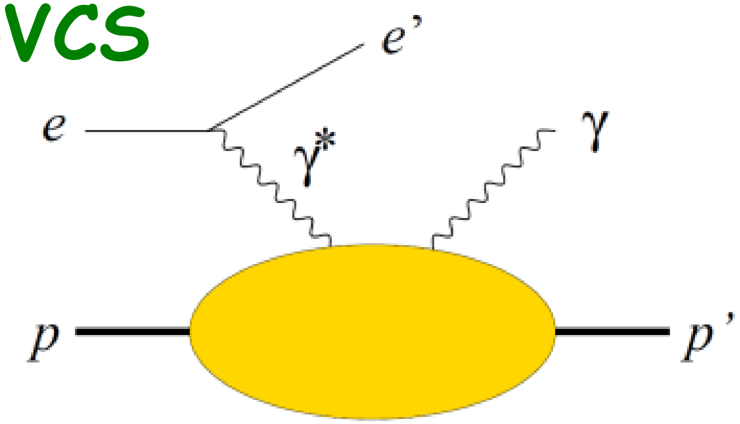


$$j_\mu(x)\, j_\nu(0) \sim \sum_i C_i(x) O_i(0)$$

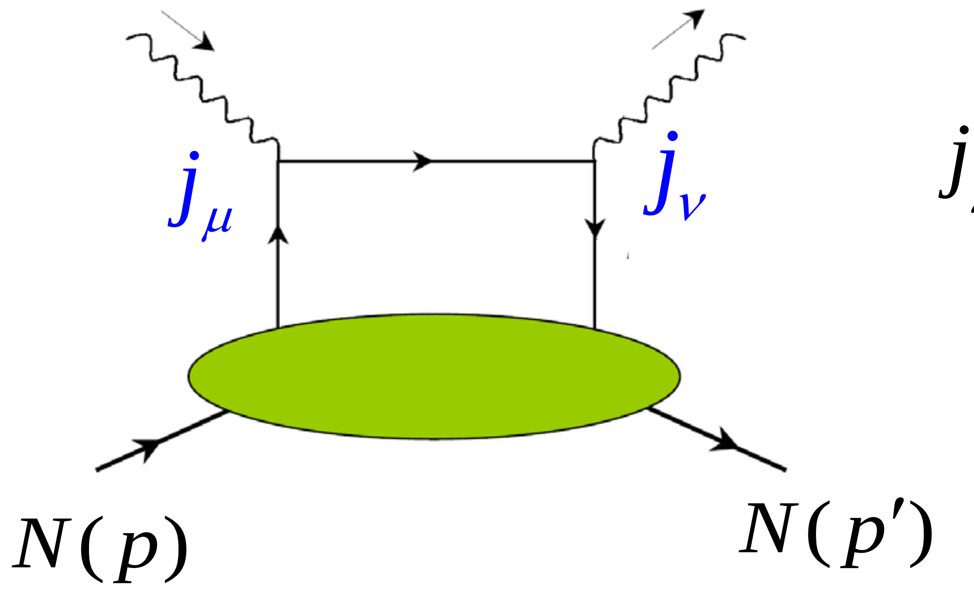
$$C^q_{\mu\nu;\alpha\beta}(x) T_q^{\alpha\beta}$$

$$C^g_{\mu\nu;\alpha\beta}(x) T_g^{\alpha\beta}$$

DVCS



JLab, EIC…



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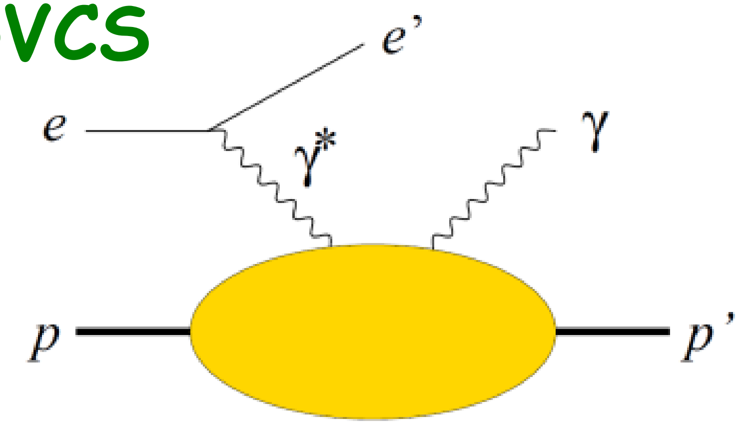

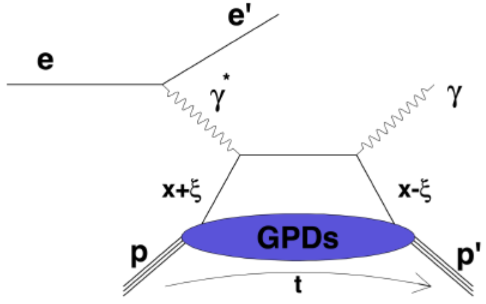

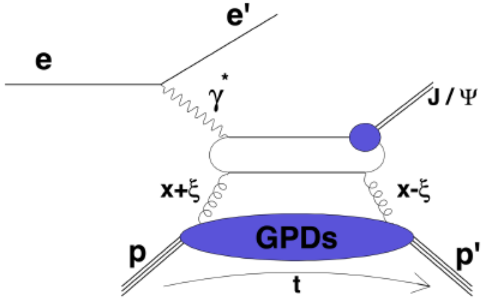

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mass & energy distribution

angular momentum distribution

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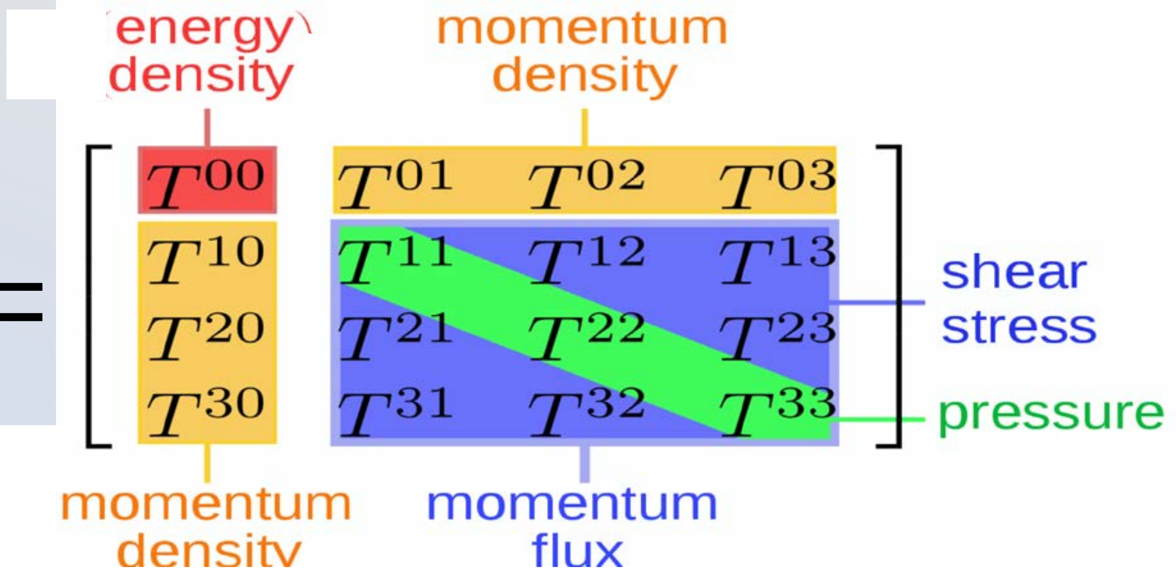
force & pressure distribution

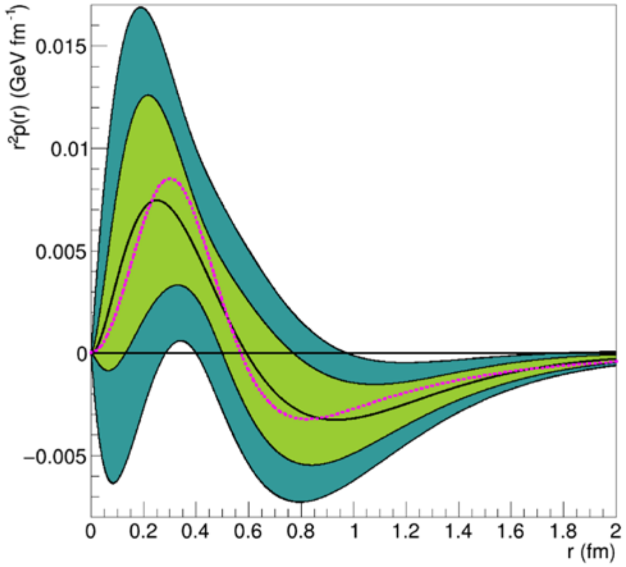
mass & pressure distribution

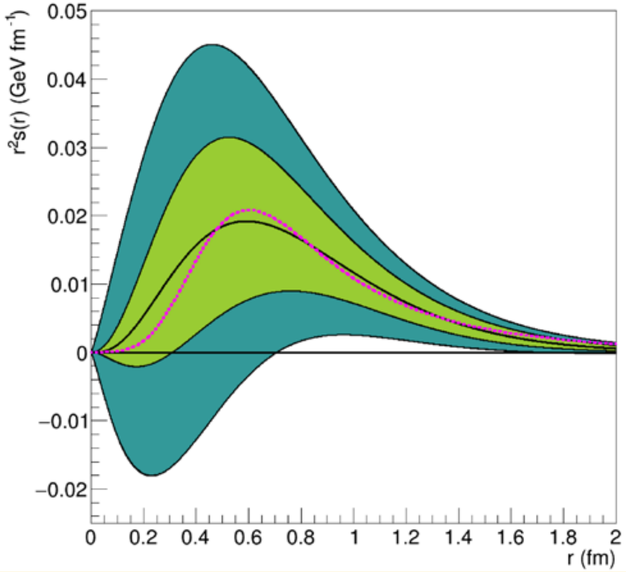
$T^{\mu\nu} =$

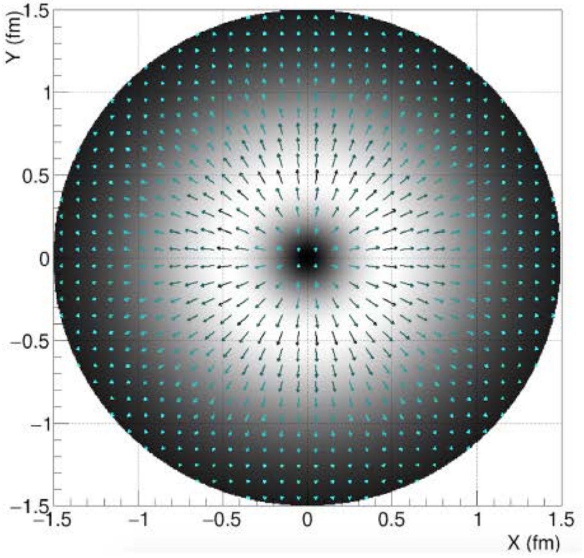


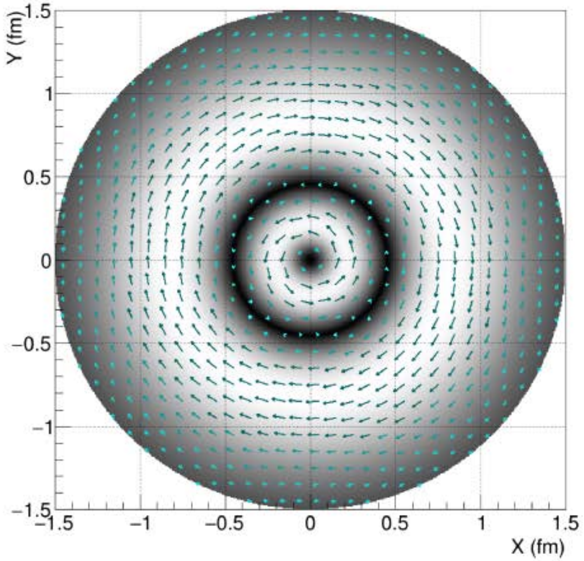
V. D. Burkert et al, Nature 557 ('18) 396

V. D. Burkert et al, Rev.Mod.Phys. 95 ('23) 041002









$$\langle N(p')|T^{ik}|N(p)\rangle \sim \left(\Delta^i\Delta^k - \delta^{ik}\boldsymbol{\Delta}^2\right)D(t), \qquad \left\langle T^{ij}\right\rangle(r) = \left(\frac{r^i r^j}{r^2} - \frac{1}{3}\delta^{ij}\right)s(r) + \delta^{ij}\,p(r)$$

Spacelike gravitational form factors and radii for pion

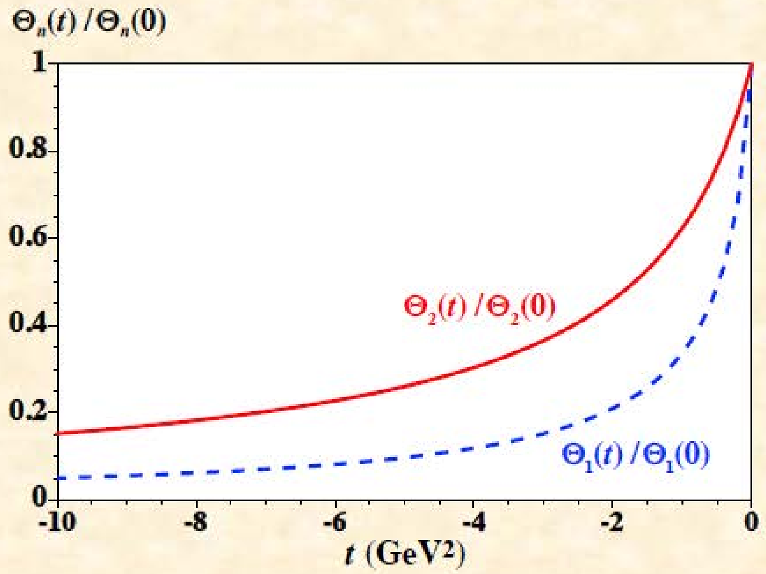
$$F(s)=\Theta_1(s),\ \Theta_1(s),\quad F(t)=\int_{4m_\pi^2}^{\infty} ds \frac{\mathrm{Im}\,F(s)}{\pi(s-t-i\varepsilon)},\quad \rho(r)=\frac{1}{(2\pi)^3}\int d^3q e^{-i\vec{q}\cdot\vec{r}}F(q)=\frac{1}{4\pi^2}\frac{1}{r}\int_{4m_\pi^2}^{\infty} ds\, e^{-\sqrt{s}r}\,\mathrm{Im}\,F(s)$$

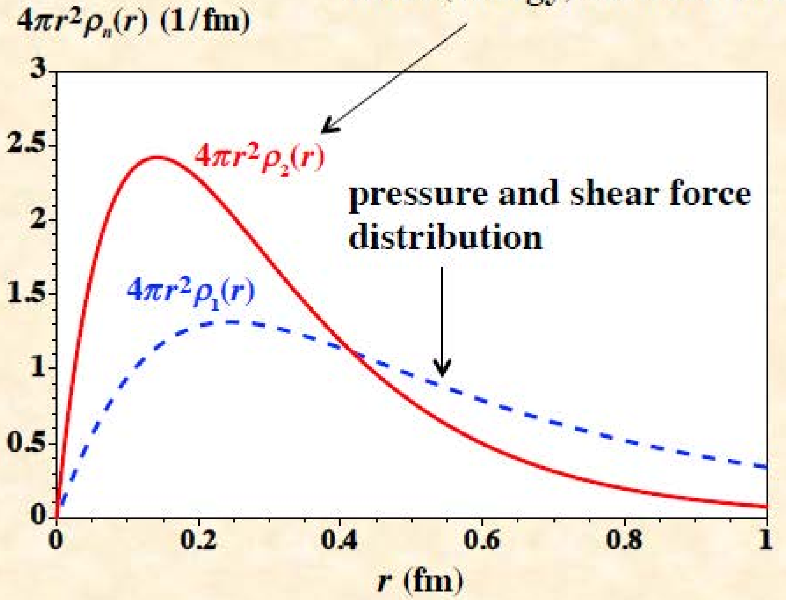
This is the first report on gravitational radii of hadrons from actual experimental measurements.

$$\sqrt{\langle r^2\rangle_{\text{mass}}}=0.32\sim0.39\text{ fm},\quad \sqrt{\langle r^2\rangle_{\text{mech}}}=0.82\sim0.88\text{ fm}$$

First finding on gravitational radius from actual experimental measurements

$$\Leftrightarrow\ \sqrt{\langle r^2\rangle_{\text{charge}}}=0.672\pm0.008\text{ fm}$$





$$\Theta_2(t)=4A^\pi(t),\quad \Theta_1(t)=-D^\pi(t)$$

$$\langle N(p)|T^{00}_{q,g}|N(p)\rangle = \bar{u}(p)\left[A_{q,g}(0)\gamma^{(0}p^{0)} + \bar{C}_{q,g}(0)M\eta^{00}\right]u(p)$$

$$\left\langle \hat{H}_{q,g} \right\rangle = \frac{\langle N|\int d^3x T^{00}_{q,g}|N\rangle}{\langle N|N\rangle}$$

$$\hat{H} = \hat{H}_q + \hat{H}_g = \int d^3x \psi^\dagger\left(-i\boldsymbol{D}\cdot\boldsymbol{\alpha} + m\beta\right)\psi + \int d^3x \frac{1}{2}\left(\boldsymbol{E}^2 + \boldsymbol{B}^2\right)$$

$$M = M_q + M_g$$

$$M_{q,g} = \left(A_{q,g}(0) + \bar{C}_{q,g}(0)\right)M$$

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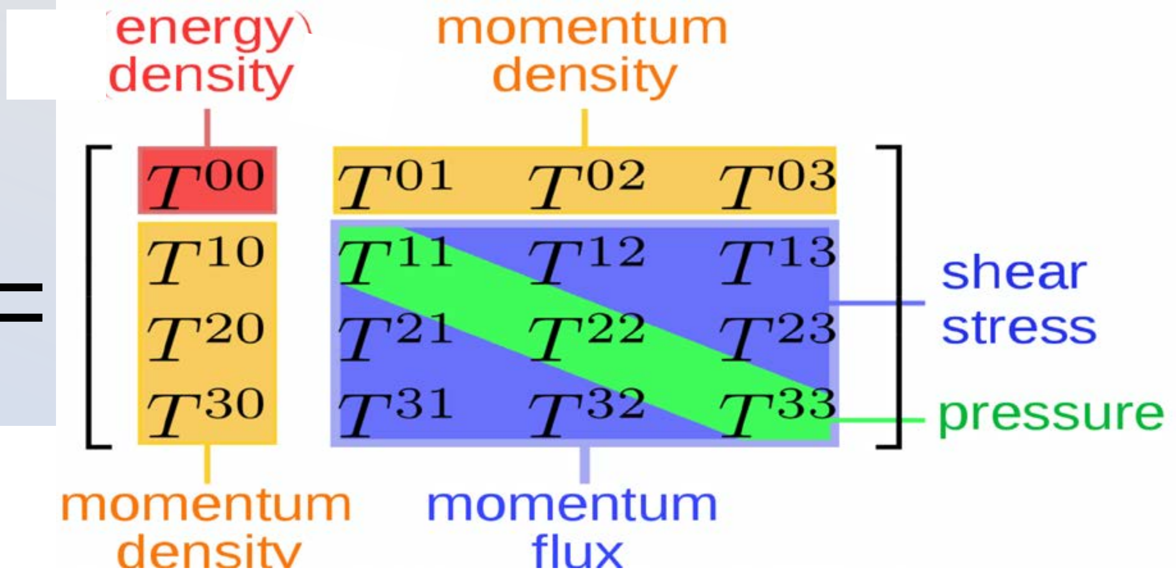

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$$M = M_q + M_g \qquad M_{q,g} = \left(A_{q,g}(0) + \bar{C}_{q,g}(0)\right)M$$

$$\int_0^1 dxx\left[q(x,\mu) + \bar{q}(x,\mu)\right] = \left\langle x_q\right\rangle(\mu) = A_q(t=0,\mu)$$

$A_q\left(t=0,\mu_0=1.3\text{GeV}\right) = 0.613$ **global QCD analysis at NNLO** **(CT18)**

$$A_q(0,\mu) = \frac{n_f}{4C_F+n_f} + \frac{4C_F A_q(0,\mu_0) + n_f\left(A_q(0,\mu_0)-1\right)}{4C_F+n_f}\left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{\frac{8C_F+2n_f}{3\beta_0}} + (2\text{loop}) + (3\text{loop})$$

$\bar{C}_q(0,\mu=1\text{ GeV}) = -\bar{C}_g(0,\mu=1\text{ GeV}) = -0.180\pm0.003$ **NNLO QCD** [KT, JHEP03, 013 ('23)]

$$\bar{C}_q(0,\mu) = -\frac{1}{4}\left(\frac{n_f}{4C_F+n_f}+\frac{2n_f}{3\beta_0}\right)+\frac{1}{4}\left(\frac{2n_f}{3\beta_0}+1\right)\frac{\langle N(p)|\, m\bar{\psi}\psi\, |N(p)\rangle}{2M^2}$$

$$-\frac{4C_F A_q(\mu_0)+n_f\left(A_q(\mu_0)-1\right)}{4(4C_F+n_f)}\left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{\frac{8C_F+2n_f}{3\beta_0}}$$

$$+\frac{\alpha_s(\mu)}{4\pi}\left[\frac{n_f}{4\beta_0}\left(-\frac{34C_A}{27}-\frac{49C_F}{27}\right)+\frac{\beta_1 n_f}{6\beta_0^2}\right]$$

$$+\left[\frac{n_f(34C_A+157C_F)}{108\beta_0}+\frac{C_F}{3}-\frac{\beta_1 n_f}{6\beta_0^2}\right]\frac{\langle N(p)|\, m\bar{\psi}\psi\, |N(p)\rangle}{2M^2}\Bigg)-\frac{1}{4}A_q^{\rm NLO}(\mu)$$

$$+\left(\frac{\alpha_s(\mu)}{4\pi}\right)^2\Bigg(\frac{n_f^2}{\beta_0}\left[\frac{697C_A}{1458}+\frac{169C_F}{2916}\right]+n_f\Bigg[\frac{17\beta_1 C_A}{54\beta_0^2}+\frac{\beta_2}{6\beta_0^2}+\frac{49\beta_1 C_F}{108\beta_0^2}$$

$$+\frac{1}{\beta_0}\left\{\left(\frac{401}{648}-\frac{26\zeta(3)}{9}\right)C_AC_F+\left(2\zeta(3)-\frac{67}{27}\right)C_A^2+\left(\frac{8\zeta(3)}{9}-\frac{2407}{2916}\right)C_F^2\right\}-\frac{\beta_1^2}{6\beta_0^3}\Bigg]$$

$$+\Bigg[-\frac{n_f^2}{\beta_0}\left(\frac{697C_A}{1458}+\frac{1789C_F}{2916}\right)+n_f\Bigg(-\frac{17\beta_1 C_A}{54\beta_0^2}-\frac{\beta_2}{6\beta_0^2}-\frac{157\beta_1 C_F}{108\beta_0^2}+\frac{\beta_1^2}{6\beta_0^3}-\frac{17C_F}{27}\Bigg)$$

$$+\frac{n_f}{\beta_0}\left\{\left(\frac{26\zeta(3)}{9}+\frac{4315}{648}\right)C_AC_F+\left(\frac{67}{27}-2\zeta(3)\right)C_A^2+\left(\frac{11803}{2916}-\frac{8\zeta(3)}{9}\right)C_F^2\right\}$$

$$+\frac{61C_AC_F}{108}-\frac{C_F^2}{27}\Bigg]\frac{\langle N(p)|\, m\bar{\psi}\psi\, |N(p)\rangle}{2M^2}\Bigg)-\frac{1}{4}A_q^{\rm NNLO}(\mu)\,,$$

[KT, JHEP03, 013 ('23)]

$$\bar{C}_q(0,\mu) = -\frac{1}{4}\left(\frac{n_f}{4C_F + n_f} + \frac{2n_f}{3\beta_0}\right) + \frac{1}{4}\left(\frac{2n_f}{3\beta_0} + 1\right)\frac{\langle N(p)|\, m\bar{\psi}\psi \,|N(p)\rangle}{2M^2}$$

$$-\frac{4C_F A_q(\mu_0) + n_f\left(A_q(\mu_0) - 1\right)}{4(4C_F + n_f)}\left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{\frac{8C_F + 2n_f}{3\beta_0}}$$

$$+\frac{\alpha_s(\mu)}{4\pi}\left[\frac{n_f}{4\beta_0}\left(-\frac{34C_A}{27} - \frac{49C_F}{27}\right) + \frac{\beta_1 n_f}{6\beta_0^2}\right]$$

$$+\left[\frac{n_f\,(34C_A + 157C_F)}{108\beta_0} + \frac{C_F}{3} - \frac{\beta_1 n_f}{6\beta_0^2}\right]\frac{\langle N(p)|\, m\bar{\psi}\psi \,|N(p)\rangle}{2M^2}\Bigg) - \frac{1}{4}A_q^{\rm NLO}(\mu)$$

$$+\left(\frac{\alpha_s(\mu)}{4\pi}\right)^2\left(\frac{n_f^2}{\beta_0}\left[\frac{697C_A}{1458} + \frac{169C_F}{2916}\right] + n_f\left[\frac{17\beta_1 C_A}{54\beta_0^2} + \frac{\beta_2}{6\beta_0^2} + \frac{49\beta_1 C_F}{108\beta_0^2}\right.\right.$$

$$\left. + \frac{1}{\beta_0}\left\{\left(\frac{401}{648} - \frac{26\zeta(3)}{9}\right)C_A C_F + \left(2\zeta(3) - \frac{67}{27}\right)C_A^2 + \left(\frac{8\zeta(3)}{9} - \frac{2407}{2916}\right)C_F^2\right\} - \frac{\beta_1^2}{6\beta_0^3}\right]$$

$$+\left[-\frac{n_f^2}{\beta_0}\left(\frac{697C_A}{1458} + \frac{1789C_F}{2916}\right) + n_f\left(-\frac{17\beta_1 C_A}{54\beta_0^2} - \frac{\beta_2}{6\beta_0^2} - \frac{157\beta_1 C_F}{108\beta_0^2} + \frac{\beta_1^2}{6\beta_0^3} - \frac{17C_F}{27}\right)\right.$$

$$+\frac{n_f}{\beta_0}\left\{\left(\frac{26\zeta(3)}{9} + \frac{4315}{648}\right)C_A C_F + \left(\frac{67}{27} - 2\zeta(3)\right)C_A^2 + \left(\frac{11803}{2916} - \frac{8\zeta(3)}{9}\right)C_F^2\right\}$$

$$\left.\left. + \frac{61C_A C_F}{108} - \frac{C_F^2}{27}\right]\frac{\langle N(p)|\, m\bar{\psi}\psi \,|N(p)\rangle}{2M^2}\right) - \frac{1}{4}A_q^{\rm NNLO}(\mu)\ ,$$

$A_q(\mu_0)$

$\langle N(p)|\, m\bar{\psi}\psi \,|N(p)\rangle$

[KT, JHEP03, 013 ('23)]

Nonpert. inputs

$$A_q(\mu_0) = \int_0^1 dx\, x\left[q(x,\mu_0) + \bar{q}(x,\mu_0)\right]$$

global QCD analysis at NNLO

$$A_q\left(\mu_0 = 1.3\,\text{GeV}\right) = 0.613$$

CT18

(MMHT2014, NNPDF)

$$\langle N(p)|\, m\bar{\psi}\psi\,|N(p)\rangle = \langle N(p)|\, m_u\bar{u}u + m_d\bar{d}d + m_s\bar{s}s\,|N(p)\rangle = 2M\left(\sigma_{\pi N} + \sigma_s\right)$$

$$\sigma_{\pi N} = \frac{1}{2M}\langle N(p)|\frac{m_u + m_d}{2}\left(\bar{u}u + \bar{d}d\right)|N(p)\rangle = 59.1 \pm 3.5\,\text{MeV}$$

Hoferichter, Elvira, Kubis, Meißner, PRL115, 092301

$$\sigma_s = \frac{1}{2M}\langle N(p)|\, m_s\bar{s}s\,|N(p)\rangle = 45.6 \pm 6.2\,\text{MeV}$$

Alexandrou, et al., PRD102, 054517

nucleon

$\bar{C}_q(0, \mu \sim 0.4 \text{ GeV}) = 0.25$ **Bag model** [Ji, Melnitchouk, Song, PRD56, 5511 ('97)]

$\bar{C}_q(0, \mu = 2 \text{ GeV}) \approx -0.11$ **Phenomenological** [Lorce, EPJC78, 120 ('18)]

$\bar{C}_q(0, \mu \sim 0.63 \text{ GeV}) = 0.014$ **Instanton** [Polyakov, Son, JHEP09, 156 ('18)]

$\bar{C}_q(0, \mu = 1 \text{ GeV}) = -0.021 \pm 0.008$ **LCSR** [Azizi,Ozdem, EPJC80, 104 ('20)]

$\bar{C}_q(0, \mu \to \infty) \simeq -0.15$ **Trace anomaly** [Hatta, Rajan, KT, JHEP12, 008 ('18)]

$\bar{C}_q(0, \mu = 1 \text{ GeV}) = -0.180 \pm 0.003$
$\bar{C}_q(0, \mu = 2 \text{ GeV}) = -0.163 \pm 0.003$
NNLO QCD [KT, JHEP03, 013 ('23)]

$$\langle N(p)|T^{00}_{q,g}|N(p)\rangle = \bar{u}(p)\left[A_{q,g}(0)\gamma^{(0}p^{0)} + \bar{C}_{q,g}(0)M\eta^{00}\right]u(p)$$

$$\left\langle \hat{H}_{q,g} \right\rangle = \frac{\langle N|\int d^3x T^{00}_{q,g}|N\rangle}{\langle N|N\rangle}$$

$$\hat{H} = \hat{H}_q + \hat{H}_g = \int d^3x \psi^\dagger(-i\boldsymbol{D}\cdot\boldsymbol{\alpha} + m\beta)\psi + \int d^3x \frac{1}{2}\left(\boldsymbol{E}^2 + \boldsymbol{B}^2\right)$$

$$M = M_q + M_g \qquad (M_q: 0.4,\ M_g: 0.6)$$

$$M_{q,g} = \left(A_{q,g}(0) + \bar{C}_{q,g}(0)\right)M$$

$A_{q,g}(0)$: 0.6, 0.4 $\quad \bar{C}_{q,g}(0)$: −0.2, 0.2

$$\int_0^1 dx\, x\left[q(x,\mu) + \bar{q}(x,\mu)\right] = \left\langle x_q \right\rangle(\mu) = A_q(t=0,\mu)$$

$A_q\left(t=0, \mu_0 = 1.3\,\mathrm{GeV}\right) = 0.613$ **global QCD analysis at NNLO (CT18)**

$$A_q(0,\mu) = \frac{n_f}{4C_F + n_f} + \frac{4C_F A_q(0,\mu_0) + n_f\left(A_q(0,\mu_0) - 1\right)}{4C_F + n_f}\left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{\frac{8C_F + 2n_f}{3\beta_0}} + (2\mathrm{loop}) + (3\mathrm{loop})$$

$\bar{C}_q(0, \mu = 1\ \mathrm{GeV}) = -\bar{C}_g(0, \mu = 1\ \mathrm{GeV}) = -0.180 \pm 0.003$ **NNLO QCD** [KT, JHEP03, 013 ('23)]

$$\langle N(p)|T^{00}_{q,g}|N(p)\rangle = \bar{u}(p)\left[A_{q,g}(0)\gamma^{(0}p^{0)} + \bar{C}_{q,g}(0)M\eta^{00}\right]u(p)$$

$$\left\langle \hat{H}_{q,g} \right\rangle = \frac{\langle N|\int d^3x T^{00}_{q,g}|N\rangle}{\langle N|N\rangle}$$

$$\hat{H} = \hat{H}_q + \hat{H}_g = \int d^3x \psi^\dagger \left(-i\boldsymbol{D}\cdot\boldsymbol{\alpha} + m\beta\right)\psi + \int d^3x \frac{1}{2}\left(\boldsymbol{E}^2 + \boldsymbol{B}^2\right)$$

$$M = M_q + M_g$$

0.4   0.6

$$M_{q,g} = \left(A_{q,g}(0) + \bar{C}_{q,g}(0)\right)M$$

0.6, 0.4   −0.2, 0.2

$$M = \left(M_q - M_m\right) + M_g + M_m$$

$$M_m = \left\langle \int d^3x\, m\bar{\psi}\psi \right\rangle = \frac{\sigma_{\pi N} + \sigma_s}{M} M$$

$$\langle N(p)|T^{00}_{q,g}|N(p)\rangle = \bar{u}(p)\left[A_{q,g}(0)\gamma^{(0}p^{0)} + \bar{C}_{q,g}(0)M\eta^{00}\right]u(p)$$

$$\left\langle \hat{H}_{q,g} \right\rangle = \frac{\langle N|\int d^3x T^{00}_{q,g}|N\rangle}{\langle N|N\rangle}$$

$$\hat{H} = \hat{H}_q + \hat{H}_g = \int d^3x \psi^\dagger(-i\boldsymbol{D}\cdot\boldsymbol{\alpha} + m\beta)\psi + \int d^3x \frac{1}{2}(\boldsymbol{E}^2 + \boldsymbol{B}^2)$$

$$M = M_q + M_g$$

M_q: 0.4, M_g: 0.6

$$M_{q,g} = \left(A_{q,g}(0) + \bar{C}_{q,g}(0)\right)M$$

$A_{q,g}(0)$: 0.6, 0.4; $\bar{C}_{q,g}(0)$: −0.2, 0.2

$$M = \left(M_q - M_m\right) + M_g + M_m$$

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M_m: 0.1

$$\langle N(p)|T^{00}_{q,g}|N(p)\rangle = \bar{u}(p)\left[A_{q,g}(0)\gamma^{(0}p^{0)} + \bar{C}_{q,g}(0)M\eta^{00}\right]u(p)$$

$$\left\langle \hat{H}_{q,g} \right\rangle = \frac{\langle N|\int d^3x T^{00}_{q,g}|N\rangle}{\langle N|N\rangle}$$

$$\hat{H} = \hat{H}_q + \hat{H}_g = \int d^3x \psi^\dagger(-i\boldsymbol{D}\cdot\boldsymbol{\alpha} + m\beta)\psi + \int d^3x \frac{1}{2}(\boldsymbol{E}^2 + \boldsymbol{B}^2)$$

$$M = M_q + M_g$$

M_q: 0.4, M_g: 0.6

$$M_{q,g} = \left(A_{q,g}(0) + \bar{C}_{q,g}(0)\right)M$$

$A_{q,g}(0)$: 0.6, 0.4; $\bar{C}_{q,g}(0)$: –0.2, 0.2

$$M = (M_q - M_m) + M_g + M_m$$

$M_q - M_m$: 0.3, M_g: 0.6, M_m: 0.1

$$M_m = \left\langle \int d^3x\, m\bar{\psi}\psi \right\rangle = \frac{\sigma_{\pi N} + \sigma_s}{M} M$$

M_m: 0.1

$$\langle N(p)|T^{00}_{q,g}|N(p)\rangle = \bar{u}(p)\left[A_{q,g}(0)\gamma^{(0}p^{0)} + \bar{C}_{q,g}(0)M\eta^{00}\right]u(p)$$

$$\left\langle \hat{H}_{q,g} \right\rangle = \frac{\langle N|\int d^3x T^{00}_{q,g}|N\rangle}{\langle N|N\rangle} = M\left(A_{q,g}(0) + \bar{C}_{q,g}(0)\right)$$

$$\hat{H} = \hat{H}_q + \hat{H}_g = \int d^3x \psi^\dagger(-i\boldsymbol{D}\cdot\boldsymbol{\alpha} + m\beta)\psi + \int d^3x \frac{1}{2}\left(\boldsymbol{E}^2 + \boldsymbol{B}^2\right)$$

$$M = M_q + M_g$$

0.4 0.6

$$M_{q,g} = \left(A_{q,g}(0) + \bar{C}_{q,g}(0)\right)M$$

0.6, 0.4 −0.2, 0.2

$$M = \left(M_q - M_m\right) + M_g + M_m$$

0.3 0.6 0.1

$$M_m = \left\langle \int d^3x\, m\bar{\psi}\psi \right\rangle = \frac{\sigma_{\pi N} + \sigma_s}{M} M$$

0.1

$$M = \left(M_q - M_m - \Delta M_q\right) + \left(M_g - \Delta M_g\right) + M_m + \left(\Delta M_q + \Delta M_g\right)$$

$$\Delta M_q + \Delta M_g = \frac{1}{4}\left\langle \int d^3x \left(\frac{\beta(g)}{2g}F^2 + \gamma_m(g)m\bar{\psi}\psi\right)\right\rangle = \frac{1}{4}\left(M - M_m\right)$$

$$2M^2 = \langle N | \left(\frac{\beta(g)}{2g} F^2 + \left(1 + \gamma_m(g)\right) m\bar{\psi}\psi \right) | N \rangle$$

$$\langle N(p)|T^{00}_{q,g}|N(p)\rangle = \bar{u}(p)\left[A_{q,g}(0)\gamma^{(0}p^{0)} + \bar{C}_{q,g}(0)M\eta^{00}\right]u(p)$$

$$\left\langle \hat{H}_{q,g}\right\rangle = \frac{\langle N|\int d^3x T^{00}_{q,g}|N\rangle}{\langle N|N\rangle}$$

$$\hat{H} = \hat{H}_q + \hat{H}_g = \boxed{\int d^3x \psi^\dagger(-i\boldsymbol{D}\cdot\boldsymbol{\alpha} + m\beta)\psi} + \boxed{\int d^3x \frac{1}{2}(\boldsymbol{E}^2 + \boldsymbol{B}^2)}$$

$$M = M_q + M_g$$

M_q: 0.4, M_g: 0.6

$$M_{q,g} = \left(A_{q,g}(0) + \bar{C}_{q,g}(0)\right)M$$

$A_{q,g}(0)$: 0.6, 0.4; $\bar{C}_{q,g}(0)$: –0.2, 0.2

$$M = (M_q - M_m) + M_g + M_m$$

$M_q - M_m$: 0.3, M_g: 0.6, M_m: 0.1

$$M_m = \left\langle \int d^3x\, m\bar{\psi}\psi \right\rangle = \frac{\sigma_{\pi N} + \sigma_s}{M} M$$

M_m: 0.1

$$M = (M_q - M_m - \Delta M_q) + (M_g - \Delta M_g) + M_m + (\Delta M_q + \Delta M_g)$$

$$\Delta M_q + \Delta M_g = \frac{1}{4}\left\langle \int d^3x \left(\frac{\beta(g)}{2g}F^2 + \gamma_m(g)m\bar{\psi}\psi\right)\right\rangle = \frac{1}{4}(M - M_m)$$

$\Delta M_q + \Delta M_g$: 0.2

$$\langle N(p)|T^{00}_{q,g}|N(p)\rangle = \bar{u}(p)\left[A_{q,g}(0)\gamma^{(0}p^{0)} + \bar{C}_{q,g}(0)M\eta^{00}\right]u(p)$$

$$\left\langle \hat{H}_{q,g} \right\rangle = \frac{\langle N|\int d^3x T^{00}_{q,g}|N\rangle}{\langle N|N\rangle}$$

$$\hat{H} = \hat{H}_q + \hat{H}_g = \int d^3x \psi^\dagger(-i\boldsymbol{D}\cdot\boldsymbol{\alpha} + m\beta)\psi + \int d^3x \frac{1}{2}(\boldsymbol{E}^2 + \boldsymbol{B}^2)$$

$$M = M_q + M_g$$

M_q: 0.4, M_g: 0.6

$$M_{q,g} = \left(A_{q,g}(0) + \bar{C}_{q,g}(0)\right)M$$

$A_{q,g}(0)$: 0.6, 0.4; $\bar{C}_{q,g}(0)$: −0.2, 0.2

$$M = (M_q - M_m) + M_g + M_m$$

$M_q - M_m$: 0.3, M_g: 0.6, M_m: 0.1

$$M_m = \left\langle \int d^3x\, m\bar{\psi}\psi \right\rangle = \frac{\sigma_{\pi N} + \sigma_s}{M} M$$

M_m: 0.1

$$M = (M_q - M_m - \Delta M_q) + (M_g - \Delta M_g) + M_m + (\Delta M_q + \Delta M_g)$$

$M_q - M_m - \Delta M_q$: 0.4, $M_g - \Delta M_g$: 0.3, M_m: 0.1, $\Delta M_q + \Delta M_g$: 0.2

$$\Delta M_q + \Delta M_g = \frac{1}{4}\left\langle \int d^3x \left(\frac{\beta(g)}{2g}F^2 + \gamma_m(g)m\bar{\psi}\psi\right)\right\rangle = \frac{1}{4}(M - M_m)$$

$\Delta M_q + \Delta M_g$: 0.2

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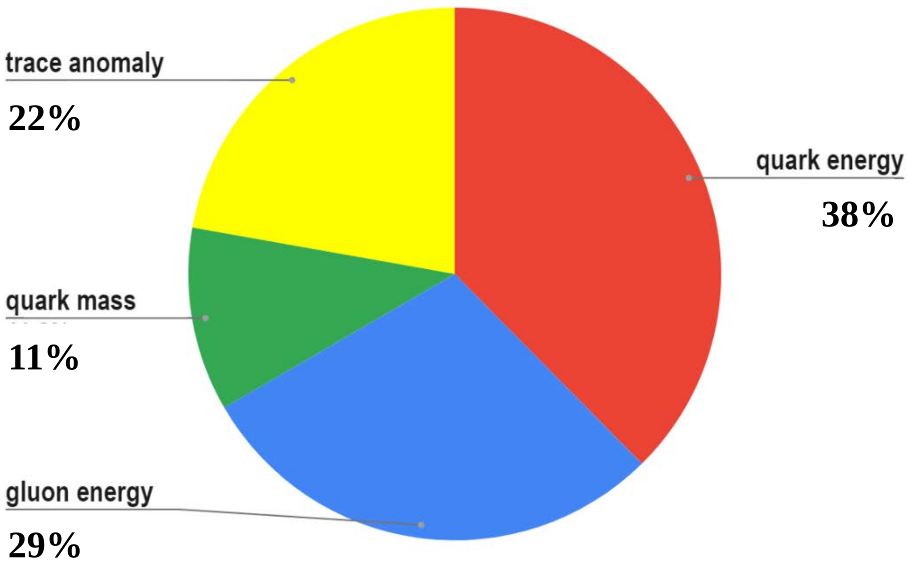
“Ji’s decomposition” in NNLO QCD

proton mass



$$\mu = 1.3 \text{ GeV}$$

$$\langle N(p)|T^{00}_{q,g}|N(p)\rangle = \bar{u}(p)\left[A_{q,g}(0)\gamma^{(0}p^{0)} + \bar{C}_{q,g}(0)M\eta^{00}\right]u(p)$$

$$\left\langle \hat{H}_{q,g} \right\rangle = \frac{\langle N|\int d^3x T^{00}_{q,g}|N\rangle}{\langle N|N\rangle}$$

$$\hat{H} = \hat{H}_q + \hat{H}_g = \boxed{\int d^3x \psi^\dagger(-i\boldsymbol{D}\cdot\boldsymbol{\alpha} + m\beta)\psi} + \boxed{\int d^3x \frac{1}{2}(\boldsymbol{E}^2 + \boldsymbol{B}^2)}$$

$$M = M_q + M_g$$

M_q: 0.4, M_g: 0.6

$$M_{q,g} = \left(A_{q,g}(0) + \bar{C}_{q,g}(0)\right)M$$

$A_{q,g}(0)$: 0.6, 0.4; $\bar{C}_{q,g}(0)$: –0.2, 0.2

$$M = \left(M_q - M_m\right) + M_g + M_m$$

$M_q - M_m$: 0.3, M_g: 0.6, M_m: 0.1

$$M_m = \left\langle \int d^3x\, m\bar{\psi}\psi \right\rangle = \frac{\sigma_{\pi N} + \sigma_s}{M} M$$

M_m: 0.1

$$M = \left(M_q - M_m - \Delta M_q\right) + \left(M_g - \Delta M_g\right) + M_m + \left(\Delta M_q + \Delta M_g\right)$$

$$\Delta M_q + \Delta M_g = \frac{1}{4}\left\langle \int d^3x \left(\frac{\beta(g)}{2g}F^2 + \gamma_m(g)m\bar{\psi}\psi\right)\right\rangle = \frac{1}{4}(M - M_m)$$

$\Delta M_q + \Delta M_g$: 0.2

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$$\langle \pi(p) | T^{00}_{q,g} | \pi(p) \rangle = 2A^{\pi}_{q,g}(0) p^0 p^0 + 2\left(M^{\pi}\right)^2 \bar{C}^{\pi}_{q,g}(0)\eta^{00}$$

$$\left\langle \hat{H}_{q,g} \right\rangle = \frac{\langle \pi | \int d^3x T^{00}_{q,g} | \pi \rangle}{\langle \pi | \pi \rangle}$$

$$\hat{H} = \hat{H}_q + \hat{H}_g = \int d^3x \psi^{\dagger}\left(-i\boldsymbol{D}\cdot\boldsymbol{\alpha} + m\beta\right)\psi + \int d^3x \frac{1}{2}\left(\boldsymbol{E}^2 + \boldsymbol{B}^2\right)$$

$$M^{\pi} = M^{\pi}_q + M^{\pi}_g \qquad M^{\pi}_{q,g} = \left(A^{\pi}_{q,g}(0) + \bar{C}^{\pi}_{q,g}(0)\right) M^{\pi}$$

$$M^{\pi} = \left(M^{\pi}_q - M^{\pi}_m\right) + M^{\pi}_g + M^{\pi}_m \qquad M^{\pi}_m = \left\langle \int d^3x \, m\bar{\psi}\psi \right\rangle$$

$$M^{\pi} = \left(M^{\pi}_q - M^{\pi}_m - \Delta M^{\pi}_q\right) + \left(M^{\pi}_g - \Delta M^{\pi}_g\right) + M^{\pi}_m + \left(\Delta M^{\pi}_q + \Delta M^{\pi}_g\right)$$

$$\Delta M^{\pi}_q + \Delta M^{\pi}_g = \frac{1}{4}\left\langle \int d^3x \left(\frac{\beta(g)}{2g} F^2 + \gamma_m(g) m\bar{\psi}\psi \right) \right\rangle = \frac{1}{4}\left(M^{\pi} - M^{\pi}_m\right)$$

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$$A_q^\pi(\mu_0) = \int_0^1 dx\, x\left[q^\pi(x,\mu_0) + \bar{q}^\pi(x,\mu_0)\right]$$ global QCD analysis at NLO

$$A_q^\pi\left(\mu_0 = 1.3\,\mathrm{GeV}\right) = \begin{cases} 0.70 \pm 0.02 \\ 0.81 \pm 0.16 \\ 0.61 \pm 0.08 \end{cases}$$

JAM ('18)

xFitter ('20)

JAM ('21)

$$\bar{C}_q^\pi(0, \mu = 1\,\mathrm{GeV}) = -0.04 \pm 0.02$$ NNLO QCD with NLO input [KT, JHEP03, 013 ('23)]

nucleon

$\bar{C}_q(0, \mu \sim 0.4\ \mathrm{GeV}) = 0.25$ **Bag model** [Ji, Melnitchouk, Song, PRD56, 5511 ('97)]

$\bar{C}_q(0, \mu = 2\ \mathrm{GeV}) \approx -0.11$ **Phenomenological** [Lorce, EPJC78, 120 ('18)]

$\bar{C}_q(0, \mu \sim 0.63\ \mathrm{GeV}) = 0.014$ **Instanton** [Polyakov, Son, JHEP09, 156 ('18)]

$\bar{C}_q(0, \mu = 1\ \mathrm{GeV}) = -0.021 \pm 0.008$ **LCSR** [Azizi,Ozdem, EPJC80, 104 ('20)]

$\bar{C}_q(0, \mu \to \infty) \simeq -0.15$ **Trace anomaly** [Hatta, Rajan, KT, JHEP12, 008 ('18)]

$\bar{C}_q(0, \mu = 1\ \mathrm{GeV}) = -0.180 \pm 0.003$
$\bar{C}_q(0, \mu = 2\ \mathrm{GeV}) = -0.163 \pm 0.003$ **NNLO QCD** [KT, JHEP03, 013 ('23)]

pion

$\bar{C}_q^{\pi}(0, \mu = 1\ \mathrm{GeV}) = -0.04 \pm 0.02$ **NNLO QCD with NLO input** [KT, JHEP03, 013 ('23)]

$$A_q^\pi(\mu_0) = \int_0^1 dx\, x\left[q^\pi(x,\mu_0) + \bar{q}^\pi(x,\mu_0)\right]$$ global QCD analysis at NLO

$$A_q^\pi\left(\mu_0 = 1.3\,\text{GeV}\right) = \begin{cases} 0.70 \pm 0.02 \\ 0.81 \pm 0.16 \\ 0.61 \pm 0.08 \end{cases}$$

JAM ('18)
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JAM ('18)
xFitter ('20)
JAM ('21)

$$\bar{C}_q^\pi(0, \mu = 1\ \mathrm{GeV}) = -0.04 \pm 0.02$$ NNLO QCD with NLO input [KT, JHEP03, 013 ('23)]

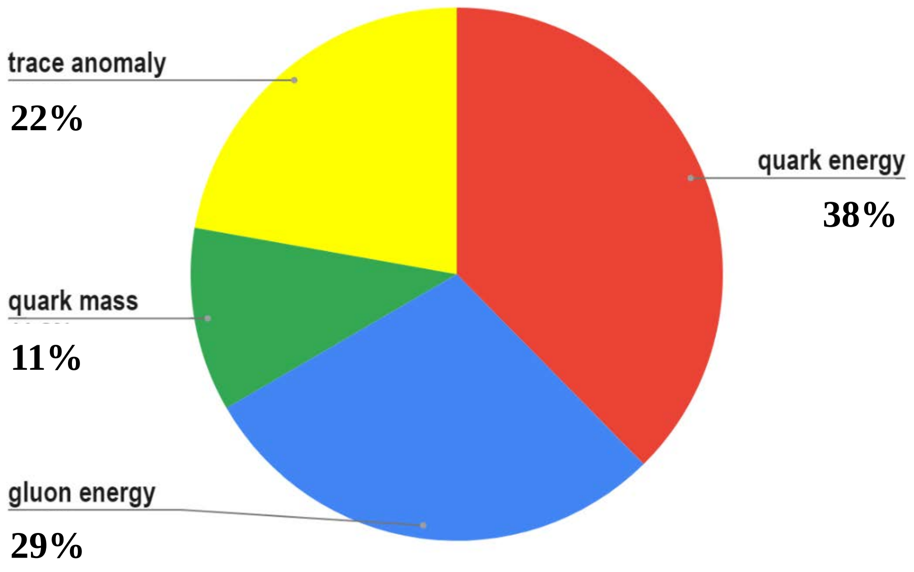
$$M_m^\pi = \left\langle \int d^3x\ m\bar{\psi}\psi \right\rangle = \left(\frac{1}{2} + O(6\%) \right) M^\pi$$ χPT

Gasser, Leutwyler, Annals Phys. 158, 142

Colangelo, Gasser, Leutwyler, PRL86, 5008

"Ji's decomposition" in NNLO QCD

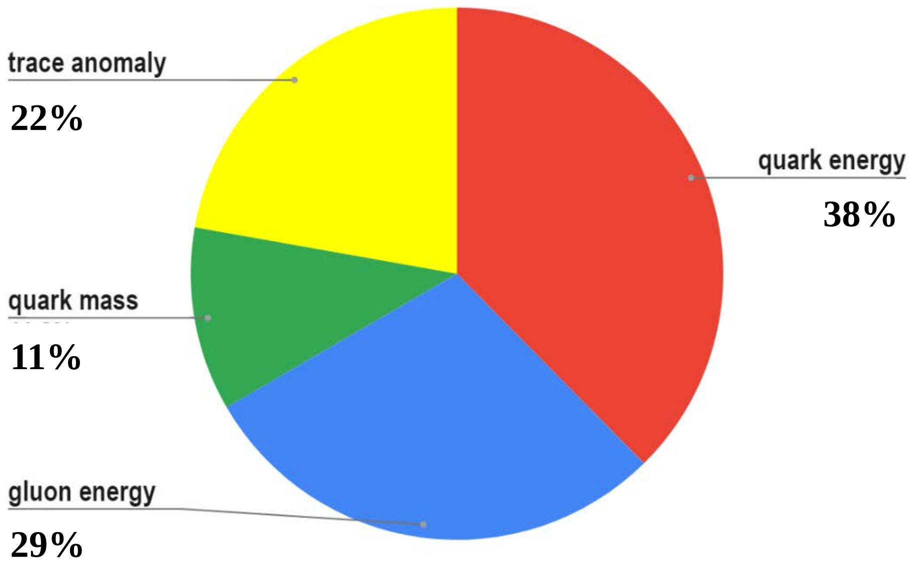
proton mass



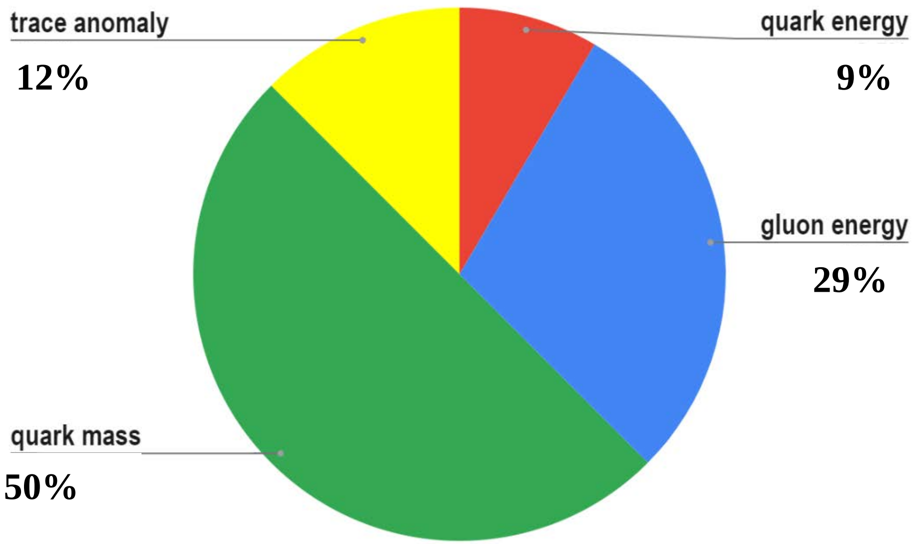
$$\mu = 1.3 \text{ GeV}$$

“Ji’s decomposition” in NNLO QCD

proton mass



pion mass



$$\mu = 1.3\ \text{GeV}$$

$$\langle N(p)|T^{00}_{q,g}|N(p)\rangle = \bar{u}(p)\left[A_{q,g}(0)\gamma^{(0}p^{0)} + \bar{C}_{q,g}(0)M\eta^{00}\right]u(p)$$

$$\left\langle \hat{H}_{q,g} \right\rangle = \frac{\langle N|\int d^3x T^{00}_{q,g}|N\rangle}{\langle N|N\rangle}$$

$$\hat{H} = \hat{H}_q + \hat{H}_g = \int d^3x \psi^\dagger(-i\boldsymbol{D}\cdot\boldsymbol{\alpha} + m\beta)\psi + \int d^3x \frac{1}{2}\left(\boldsymbol{E}^2 + \boldsymbol{B}^2\right)$$

$$M = M_q + M_g \qquad M_{q,g} = \left(A_{q,g}(0) + \bar{C}_{q,g}(0)\right)M$$

$$\int_0^1 dx x\left[q(x,\mu) + \bar{q}(x,\mu)\right] = \left\langle x_q \right\rangle(\mu) = A_q(t=0,\mu)$$

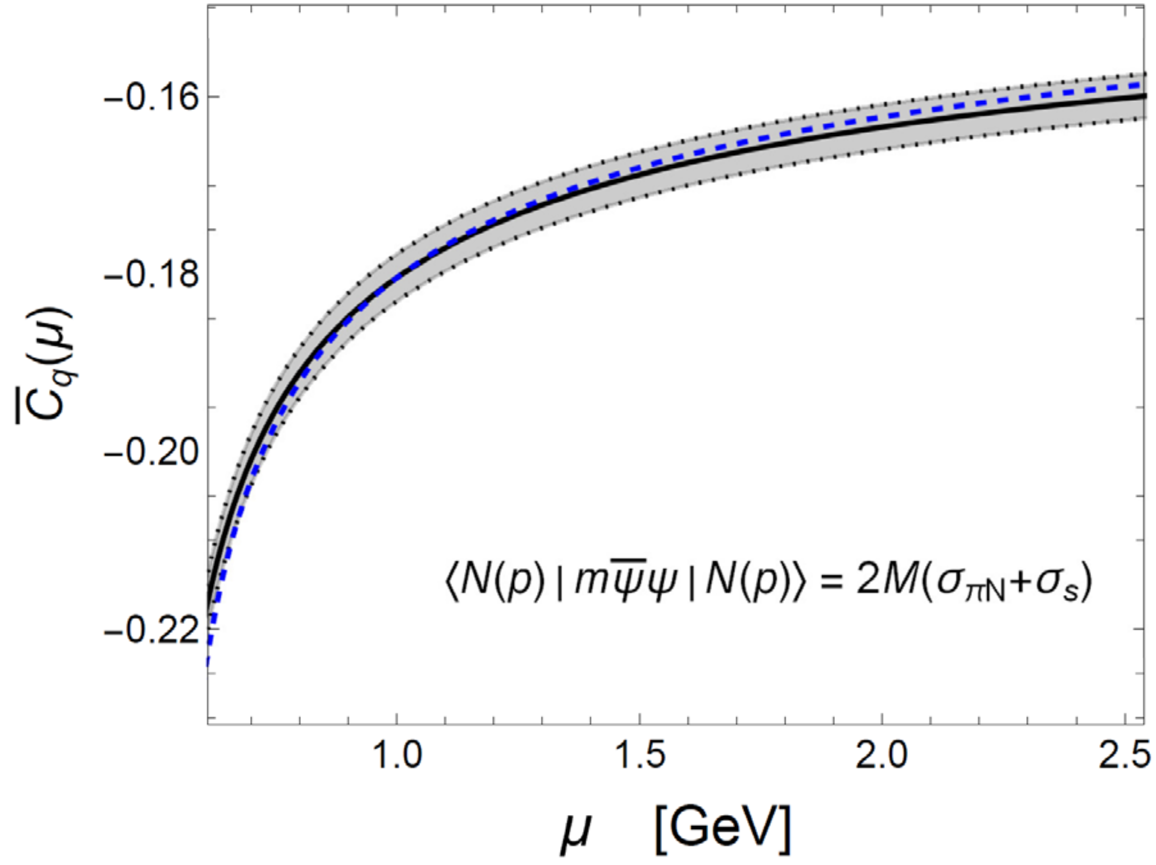
$A_q\left(t=0, \mu_0 = 1.3\mathrm{GeV}\right) = 0.613$ **global QCD analysis at NNLO (CT18)**

$$A_q(0,\mu) = \frac{n_f}{4C_F + n_f} + \frac{4C_F A_q(0,\mu_0) + n_f\left(A_q(0,\mu_0) - 1\right)}{4C_F + n_f}\left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{\frac{8C_F + 2n_f}{3\beta_0}} + (2\mathrm{loop}) + (3\mathrm{loop})$$

$\bar{C}_q(0, \mu = 1\ \mathrm{GeV}) = -\bar{C}_g(0, \mu = 1\ \mathrm{GeV}) = -0.180 \pm 0.003$ **NNLO QCD** [KT, JHEP03, 013 ('23)]

NNLO QCD

[KT, JHEP03, 013 ('23)]



$$\bar{C}_q(\mu = 0.7\ \mathrm{GeV})\big|_{n_f=3} = -0.201 \pm 0.003$$

$$\bar{C}_q(\mu = 1\ \mathrm{GeV})\big|_{n_f=3} = -0.180 \pm 0.003$$

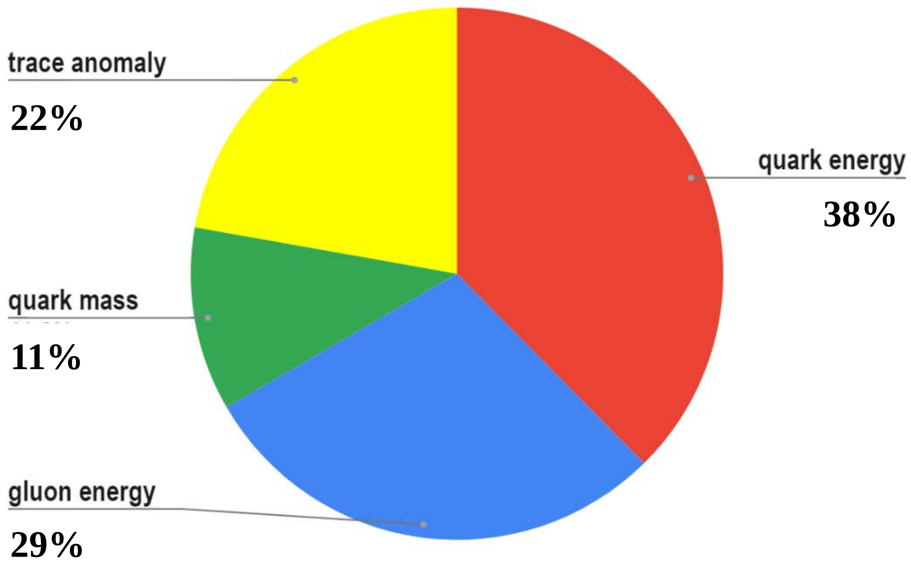
$$\bar{C}_q(\mu = 2\ \mathrm{GeV})\big|_{n_f=3} = -0.163 \pm 0.003$$

$\overline{\mathrm{MS}}$ scheme

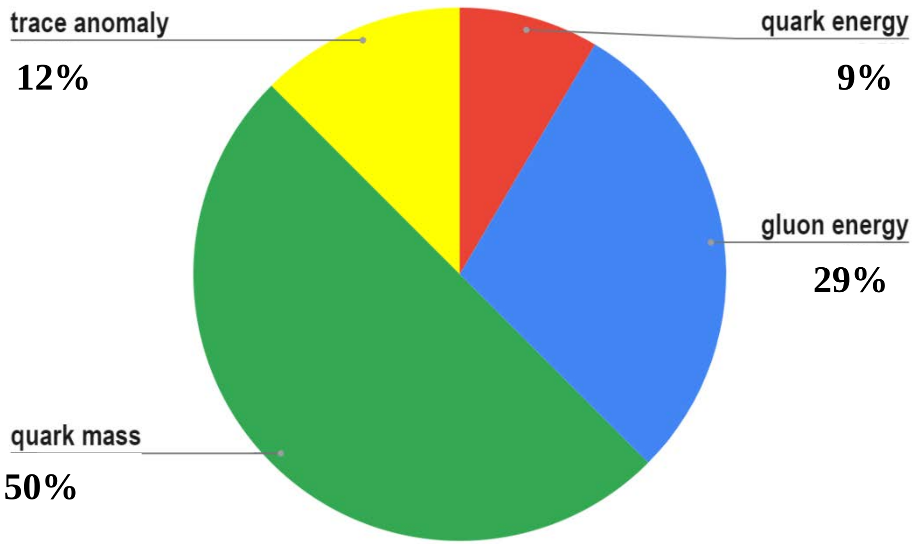
$$\bar{C}_q(\mu)\big|_{n_f=3} \simeq -0.108 - 0.114\,[\alpha_s(\mu)]^{\frac{50}{81}}$$

"Ji's decomposition" in NNLO QCD

proton mass

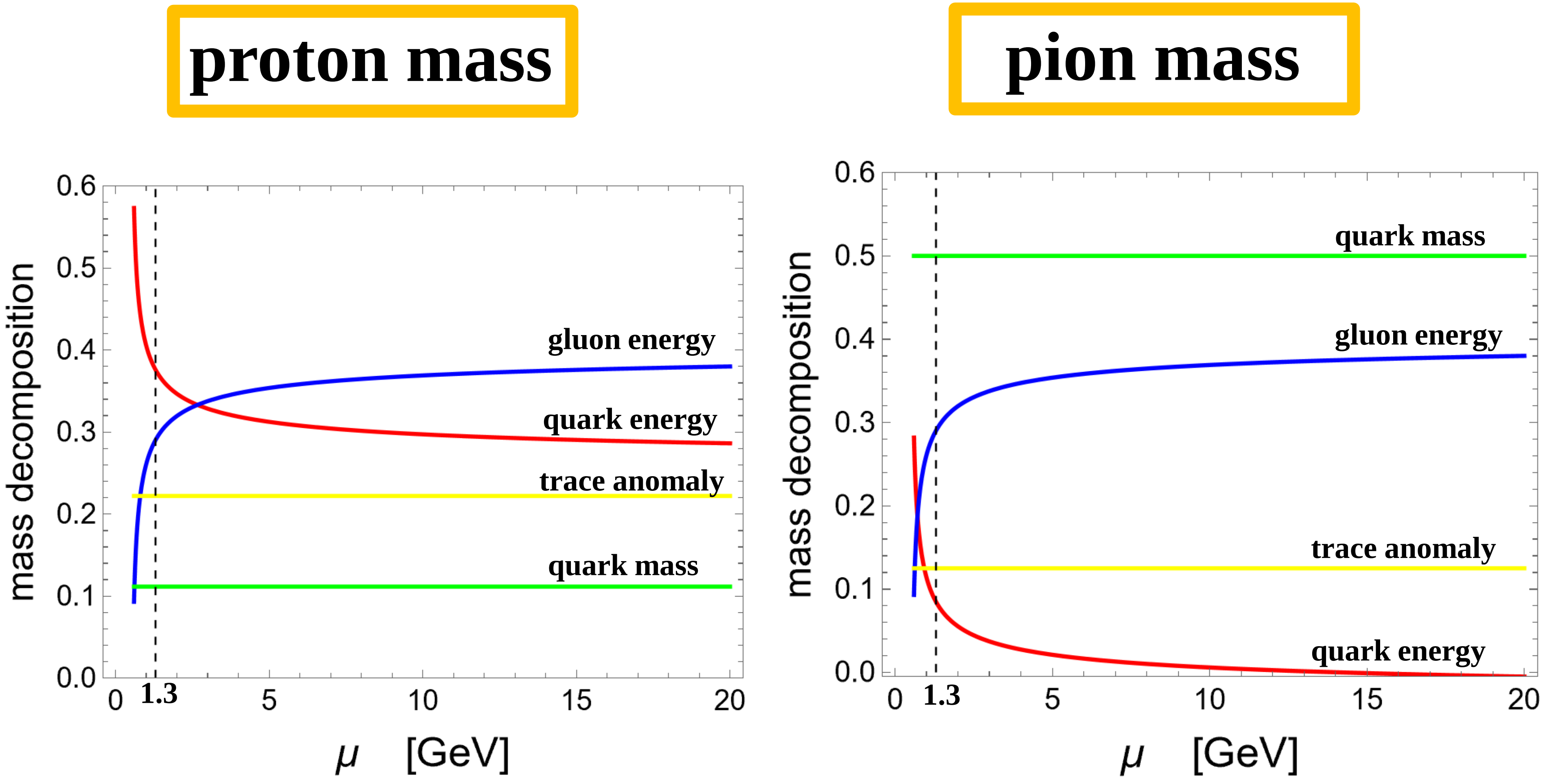


pion mass



$$\mu = 1.3 \text{ GeV}$$

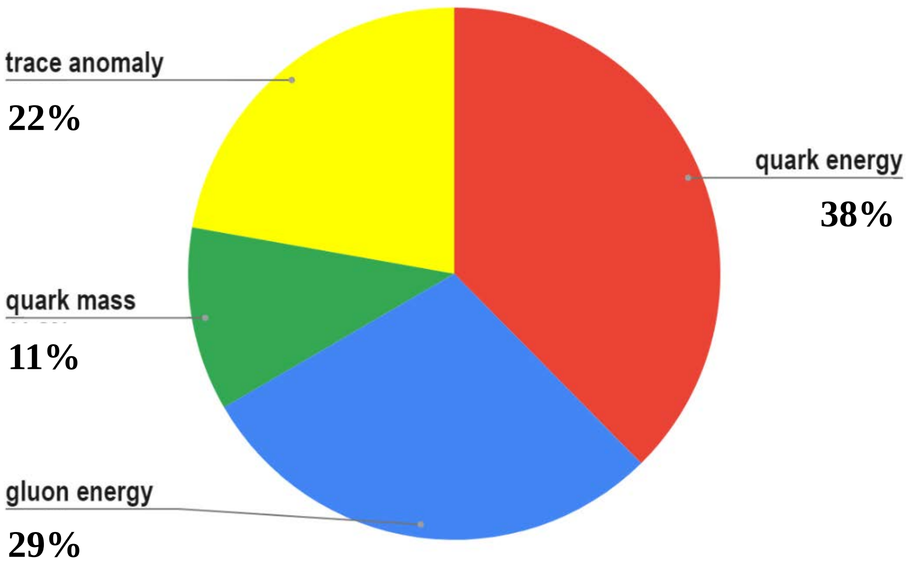
"Ji's decomposition" in NNLO QCD



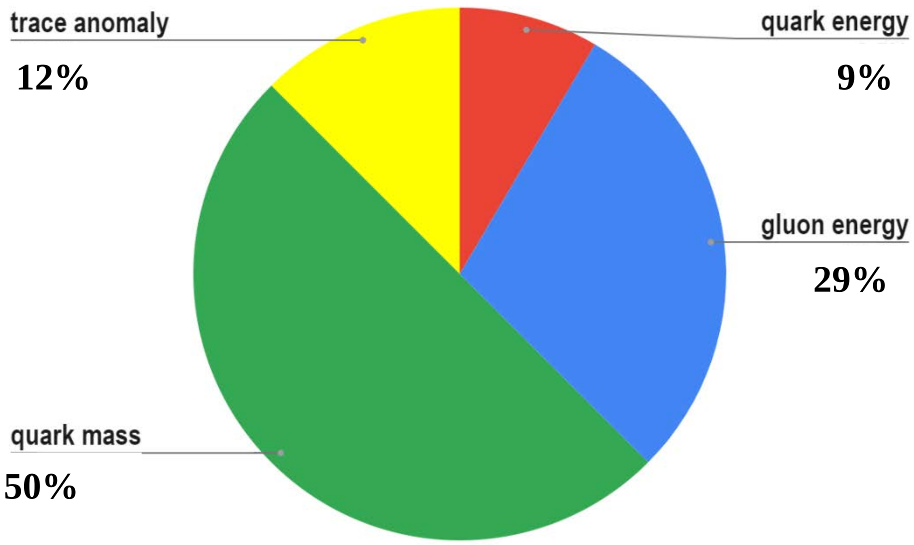
$\overline{\text{MS}}$ scheme

“Ji’s decomposition” in NNLO QCD

proton mass



pion mass



$$\mu = 1.3 \text{ GeV}$$

$$2M^2 = \langle N | \left(\frac{\beta(g)}{2g} F^2 + \left(1 + \gamma_m(g)\right) m\bar{\psi}\psi \right) | N \rangle$$

Trace anomaly decomposition in NNLO QCD

$$\langle N(p)|T^{\mu\nu}_{q,g}|N(p)\rangle = \bar{u}(p)\left[A_{q,g}(0)\gamma^{(\mu}p^{\nu)} + \bar{C}_{q,g}(0)M\eta^{\mu\nu}\right]u(p)$$

$$\eta_{\mu\nu}\langle N(p)|T^{\mu\nu}_{q,g}|N(p)\rangle = 2M^2\left(A_{q,g}(0) + 4\bar{C}_{q,g}(0)\right)$$

$$2M^2 = \eta_{\mu\nu}\langle N|T^{\mu\nu}|N\rangle = \langle N|\left(\frac{\beta(g)}{2g}F^2 + \left(1+\gamma_m(g)\right)m\bar{\psi}\psi\right)|N\rangle$$

$$M = \tilde{M}_q + \tilde{M}_g \qquad \tilde{M}_{q,g} = \left(A_{q,g}(0) + 4\bar{C}_{q,g}(0)\right)M$$

Trace anomaly decomposition in NNLO QCD

$$\langle N(p)|T^{\mu\nu}_{q,g}|N(p)\rangle = \bar{u}(p)\left[A_{q,g}(0)\gamma^{(\mu}p^{\nu)} + \bar{C}_{q,g}(0)M\eta^{\mu\nu}\right]u(p)$$

$$\eta_{\mu\nu}\langle N(p)|T^{\mu\nu}_{q,g}|N(p)\rangle = 2M^2\left(A_{q,g}(0) + 4\bar{C}_{q,g}(0)\right)$$

$$2M^2 = \eta_{\mu\nu}\langle N|T^{\mu\nu}|N\rangle = \langle N|\left(\frac{\beta(g)}{2g}F^2 + \left(1+\gamma_m(g)\right)m\bar{\psi}\psi\right)|N\rangle$$

$$M = \tilde{M}_q + \tilde{M}_g \qquad \tilde{M}_{q,g} = \left(A_{q,g}(0) + 4\bar{C}_{q,g}(0)\right)M$$

$$\int_0^1 dxx\left[q(x,\mu) + \bar{q}(x,\mu)\right] = \left\langle x_q\right\rangle(\mu) = A_q(t=0,\mu)$$

$A_q\left(t=0, \mu_0 = 1.3\text{GeV}\right) = 0.613$ **global QCD analysis at NNLO (CT18)**

$$A_q(0,\mu) = \frac{n_f}{4C_F + n_f} + \frac{4C_F A_q(0,\mu_0) + n_f\left(A_q(0,\mu_0) - 1\right)}{4C_F + n_f}\left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{\frac{8C_F + 2n_f}{3\beta_0}} + (2\text{loop}) + (3\text{loop})$$

$\bar{C}_q(0,\mu = 1\text{ GeV}) = -\bar{C}_g(0,\mu = 1\text{ GeV}) = -0.180 \pm 0.003$ **NNLO QCD** [KT, JHEP03, 013 ('23)]

JUNTENDO UNIVERSITY TOKYO 1838

Trace anomaly decomposition in NNLO QCD

$$\langle N(p)|T^{\mu\nu}_{q,g}|N(p)\rangle = \bar{u}(p)\left[A_{q,g}(0)\gamma^{(\mu}p^{\nu)} + \bar{C}_{q,g}(0)M\eta^{\mu\nu}\right]u(p)$$

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$$M = \tilde{M}_q + \tilde{M}_g \qquad \tilde{M}_{q,g} = \left(A_{q,g}(0) + 4\bar{C}_{q,g}(0)\right)M$$

$$\int_0^1 dx\,x\left[q(x,\mu) + \bar{q}(x,\mu)\right] = \left\langle x_q \right\rangle(\mu) = A_q(t=0,\mu)$$

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$\bar{C}_q(0,\mu=1\,\mathrm{GeV}) = -\bar{C}_g(0,\mu=1\,\mathrm{GeV}) = -0.180\pm0.003$ **NNLO QCD** [KT, JHEP03, 013 ('23)]

$\bar{C}^{\pi}_q(0,\mu=1\,\mathrm{GeV}) = -0.04\pm0.02$ **NNLO QCD with NLO input** [KT, JHEP03, 013 ('23)]

Trace anomaly decomposition in NNLO QCD

$$\eta_{\mu\nu}\langle N(p)|T^{\mu\nu}_{q,g}|N(p)\rangle = 2M^2\left(A_{q,g}(0)+4\bar{C}_{q,g}(0)\right)$$

$$2M^2 = \eta_{\mu\nu}\langle N|T^{\mu\nu}|N\rangle = \langle N|\left(\frac{\beta(g)}{2g}F^2+\left(1+\gamma_m(g)\right)m\bar{\psi}\psi\right)|N\rangle$$

$$M = \tilde{M}_q + \tilde{M}_g \qquad \tilde{M}_{q,g} = \left(A_{q,g}(0)+4\bar{C}_{q,g}(0)\right)M$$

	$\tilde{M}_q$	$\tilde{M}_g$	$A_{q,g}(0)$	$\bar{C}_{q,g}(0)$
proton	–0.1	1.1	0.6, 0.4	–0.18, 0.18
pion	0.4	0.6	0.6, 0.4	–0.04, 0.04

$\mu = 1.3$ GeV

Trace anomaly decomposition in NNLO QCD

$$\eta_{\mu\nu}\langle N(p)|T^{\mu\nu}_{q,g}|N(p)\rangle = 2M^2\left(A_{q,g}(0)+4\bar{C}_{q,g}(0)\right)$$

$$2M^2 = \eta_{\mu\nu}\langle N|T^{\mu\nu}|N\rangle = \langle N|\left(\frac{\beta(g)}{2g}F^2+(1+\gamma_m(g))m\bar{\psi}\psi\right)|N\rangle$$

$$M = \tilde{M}_q + \tilde{M}_g \qquad \tilde{M}_{q,g} = \left(A_{q,g}(0)+4\bar{C}_{q,g}(0)\right)M$$

	$\tilde{M}_q$	$\tilde{M}_g$	$A_{q,g}(0)$	$\bar{C}_{q,g}(0)$
proton	-0.1	1.1	$0.6,\ 0.4$	$-0.18,\ 0.18$
pion	0.4	0.6	$0.6,\ 0.4$	$-0.04,\ 0.04$

$$\mu = 1.3\ \text{GeV}$$

$$\hat{H} = \hat{H}_q + \hat{H}_g = \int d^3x\,\psi^\dagger(-i\boldsymbol{D}\cdot\boldsymbol{\alpha}+m\beta)\psi + \int d^3x\frac{1}{2}\left(\boldsymbol{E}^2+\boldsymbol{B}^2\right) \qquad \int d^3x T^{00}_{q,g} = \hat{H}_{q,g}$$

$$M = M_q + M_g \qquad M_{q,g} = \left(A_{q,g}(0)+\bar{C}_{q,g}(0)\right)M$$

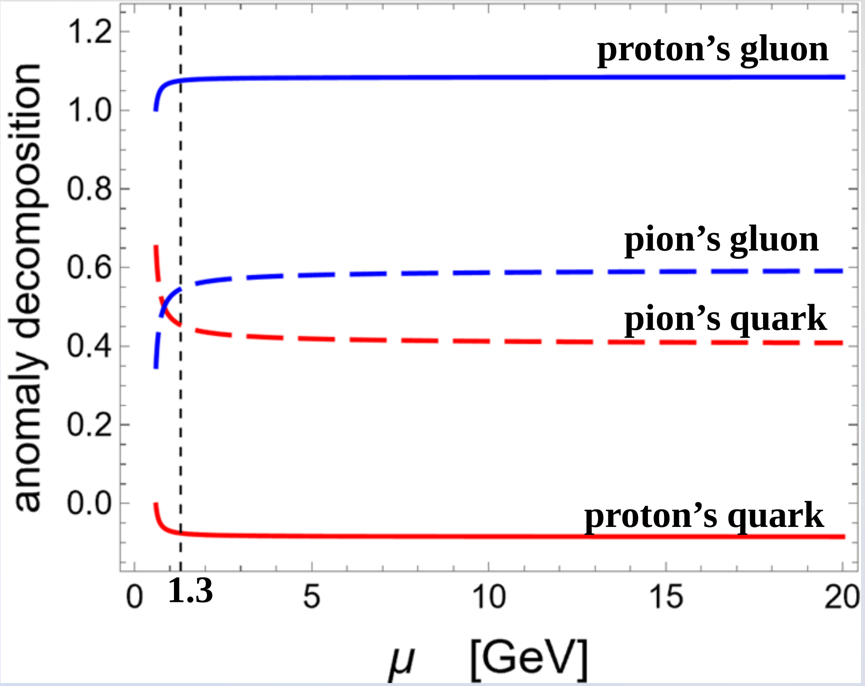
	M_q	M_g	$A_{q,g}(0)$	$\bar{C}_{q,g}(0)$
proton	0.4	0.6	$0.6,\ 0.4$	$-0.18,\ 0.18$
pion	0.6	0.4	$0.6,\ 0.4$	$-0.04,\ 0.04$

Trace anomaly decomposition in NNLO QCD

$$2M^2 = \eta_{\mu\nu}\langle N | T^{\mu\nu} | N\rangle = \langle N | \left(\frac{\beta(g)}{2g} F^2 + \left(1+\gamma_m(g)\right) m\bar{\psi}\psi \right) | N\rangle$$

$$M = \tilde{M}_q + \tilde{M}_g \qquad \tilde{M}_{q,g} = \left(A_{q,g}(0) + 4\bar{C}_{q,g}(0) \right) M$$

$\tilde{M}_q$	$\tilde{M}_g$	$A_{q,g}(0)$	$\bar{C}_{q,g}(0)$	
–0.1	1.1	0.6, 0.4	–0.18, 0.18	proton
0.4	0.6	0.6, 0.4	–0.04, 0.04	pion



$\overline{\text{MS}}$ scheme

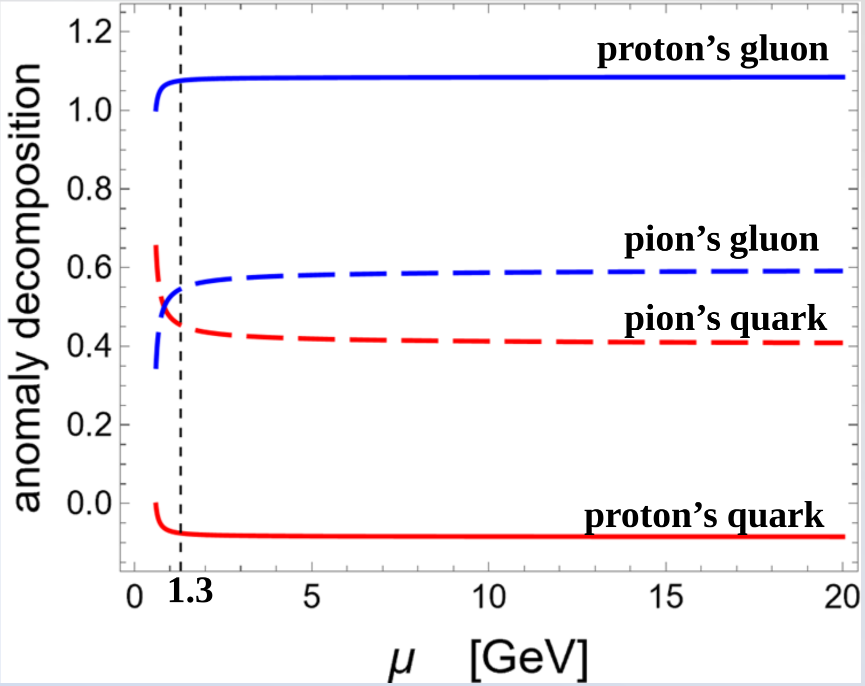
Trace anomaly decomposition in NNLO QCD

$$2M^2 = \eta_{\mu\nu}\langle N | T^{\mu\nu} | N\rangle = \langle N | \left(\frac{\beta(g)}{2g} F^2 + (1+\gamma_m(g)) m\bar{\psi}\psi \right) | N\rangle$$

$$M = \tilde{M}_q + \tilde{M}_g$$

$$\tilde{M}_{q,g} = \left(A_{q,g}(0) + 4\bar{C}_{q,g}(0) \right) M$$

	$\tilde{M}_q$	$\tilde{M}_g$	$A_{q,g}(0)$	$4\bar{C}_{q,g}(0)$
proton	-0.1	1.1	$0.6,\ 0.4$	$-0.18,\ 0.18$
pion	0.4	0.6	$0.6,\ 0.4$	$-0.04,\ 0.04$



$$T^{\mu\nu} = \frac{1}{2}\bar{\psi}\gamma^{(\mu} i\overleftrightarrow{D}^{\nu)}\psi + F^{\mu\rho}F_\rho{}^\nu + \frac{\eta^{\mu\nu}}{4}F^2$$

$$= T_q^{\mu\nu} + T_g^{\mu\nu}$$

$$A_{q,g}(0) + 4\bar{C}_{q,g}(0) = \frac{\langle N(p) | \eta_{\mu\nu} T_{q,g}^{\mu\nu} | N(p)\rangle}{2M^2}$$

$\overline{\text{MS}}$ scheme

Hatta, Rajan, KT, JHEP 12 ('18) 008

KT, JHEP 01 ('19) 120

trace anomaly separately for q, g $\overline{\text{MS}}$ scheme KT, JHEP01 ('19)120

$$\eta_{\mu\nu}T_q^{\mu\nu} = m\bar{\psi}\psi + \frac{\alpha_s}{4\pi}\left(\frac{4}{3}C_F m\bar{\psi}\psi + \frac{1}{3}n_f F^2\right) + \left(\frac{\alpha_s}{4\pi}\right)^2\left[\left(C_F\left(\frac{61C_A}{27} - \frac{68n_f}{27}\right) - \frac{4C_F^2}{27}\right)m\bar{\psi}\psi + \left(\frac{17C_A n_f}{27} + \frac{49C_F n_f}{54}\right)F^2\right]$$

$$+\left(\frac{\alpha_s}{4\pi}\right)^3\left[\left\{n_f\left(\left(\frac{64\zeta(3)}{9} - \frac{8305}{729}\right)C_F^2 - \frac{2}{243}(864\zeta(3)+1079)C_A C_F\right) - \frac{8}{729}(972\zeta(3)+143)C_A C_F^2\right.\right.$$

$$\left. + \left(\frac{32\zeta(3)}{9} + \frac{6611}{729}\right)C_A^2 C_F - \frac{76}{243}C_F n_f^2 + \frac{8}{729}(648\zeta(3)-125)C_F^3\right\}m\bar{\psi}\psi$$

$$\left. + \left\{n_f\left(\left(\frac{52\zeta(3)}{9} - \frac{401}{324}\right)C_A C_F + \left(\frac{134}{27} - 4\zeta(3)\right)C_A^2 + \left(\frac{2407}{1458} - \frac{16\zeta(3)}{9}\right)C_F^2\right) + n_f^2\left(-\frac{697C_A}{729} - \frac{169C_F}{1458}\right)\right\}F^2\right]$$

$$\eta_{\mu\nu}T_g^{\mu\nu} = \frac{\alpha_s}{4\pi}\left(\frac{14}{3}C_F m\bar{\psi}\psi - \frac{11}{6}C_A F^2\right) + \left(\frac{\alpha_s}{4\pi}\right)^2\left[\left(C_F\left(\frac{812C_A}{27} - \frac{22n_f}{27}\right) + \frac{85C_F^2}{27}\right)m\bar{\psi}\psi + \left(\frac{28C_A n_f}{27} - \frac{17C_A^2}{3} + \frac{5C_F n_f}{54}\right)F^2\right]$$

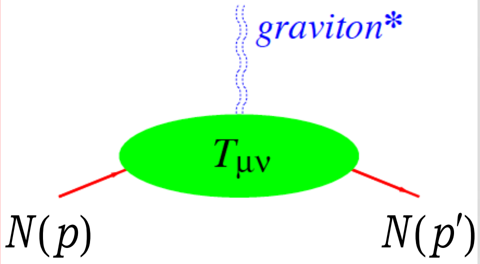
$$+\left(\frac{\alpha_s}{4\pi}\right)^3\left[\left\{n_f\left(\left(\frac{368\zeta(3)}{9} - \frac{25229}{729}\right)C_F^2 - \frac{2}{243}(4968\zeta(3)+1423)C_A C_F\right) + \left(\frac{32\zeta(3)}{3} - \frac{91753}{1458}\right)C_A C_F^2\right.\right.$$

$$\left. + \left(\frac{294929}{1458} - \frac{32\zeta(3)}{9}\right)C_A^2 C_F - \frac{554}{243}C_F n_f^2 + \left(\frac{95041}{729} - \frac{64\zeta(3)}{9}\right)C_F^3\right\}m\bar{\psi}\psi$$

$$\left. + \left\{n_f\left(\left(\frac{1123}{162} - \frac{52\zeta(3)}{9}\right)C_A C_F + \left(4\zeta(3) + \frac{293}{36}\right)C_A^2 + \frac{16}{729}(81\zeta(3)-98)C_F^2\right) + n_f^2\left(\frac{655C_A}{2916} - \frac{361C_F}{729}\right) - \frac{2857C_A^3}{108}\right\}F^2\right]$$

$$\eta_{\mu\nu}T^{\mu\nu} = \frac{\beta(g)}{2g}F^2 + (1+\gamma_m(g))m\bar{\psi}\psi \qquad C_A = N_c\,, \quad C_F = \frac{N_c^2-1}{2N_c}$$

Summary



Gravitational form factors relevant at EIC determine the hadron mass decomposition in QCD

mass & energy distribution; **spin** distribution

$$\langle N(p')|T^{\mu\nu}_{q,g}|N(p)\rangle = \bar{u}(p')\Big[A_{q,g}(t)\gamma^{(\mu}P^{\nu)} + B_{q,g}(t)\frac{P^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha}{2M}$$

$$+D_{q,g}(t)\frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{M} + \bar{C}_{q,g}(t)M\eta^{\mu\nu}\Big]u(p)$$

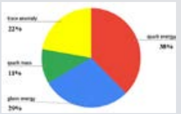
force & pressure distribution; **mass & pressure** distribution

Using recent NNLO values for $A_{q,g}(0), \bar{C}_{q,g}(0),$

$$T^{00} = T^{00}_q + T^{00}_g$$

$$M = M_q + M_g$$

	M_q	M_g
proton	0.4	0.6
pion	0.6	0.4



$$M_{q,g} = \left(A_{q,g}(0) + \bar{C}_{q,g}(0)\right)M$$

strong μ dep.

	$A_{q,g}(0)$	$\bar{C}_{q,g}(0)$
proton	0.6, 0.4	−0.18, 0.18
pion	0.6, 0.4	−0.04, 0.04

$$\eta_{\mu\nu}T^{\mu\nu} = \eta_{\mu\nu}T^{\mu\nu}_q + \eta_{\mu\nu}T^{\mu\nu}_g$$

$$M = \tilde{M}_q + \tilde{M}_g$$

	$\tilde{M}_q$	$\tilde{M}_g$
proton	−0.1	1.1
pion	0.4	0.6

$$\tilde{M}_{q,g} = \left(A_{q,g}(0) + 4\bar{C}_{q,g}(0)\right)M$$

weak μ dep.

	$A_{q,g}(0)$	$\bar{C}_{q,g}(0)$
proton	0.6, 0.4	−0.18, 0.18
pion	0.6, 0.4	−0.04, 0.04

backup

NNLO QCD

[KT, JHEP03, 013 ('23)]

$$\bar{C}_q(0,\mu)\Big|_{n_f=3} = -0.145556 + 0.305556\frac{\langle N(p)|m\bar{\psi}\psi|N(p)\rangle}{2M^2}$$

$$+\left(0.09-0.25A_q(\mu_0)\right)\left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{\frac{50}{81}}$$

$$+\alpha_s(\mu)\Big[0.00553609+0.0803962\frac{\langle N(p)|m\bar{\psi}\psi|N(p)\rangle}{2M^2}$$

$$+\left(0.0127684-0.0354678A_q(\mu_0)\right)\left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{\frac{50}{81}}-\left(0.0279651-0.0354678A_q(\mu_0)\right)\left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{-\frac{31}{81}}\Big]$$

$$+\left(\alpha_s(\mu)\right)^2\Big[0.00174426+0.0312256\frac{\langle N(p)|m\bar{\psi}\psi|N(p)\rangle}{2M^2}$$

$$-\left(0.0059729-0.0165914A_q(\mu_0)\right)\left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{\frac{50}{81}}$$

$$-\left(0.00396745-0.00503187A_q(\mu_0)\right)\left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{-\frac{31}{81}}$$

$$+\left(0.0237481-0.0216233A_q(\mu_0)\right)\left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{-\frac{112}{81}}\Big]$$

$$A_q(\mu_0)=\int_0^1 dx\,x\left[q(x,\mu_0)+\bar{q}(x,\mu_0)\right]$$

$$\langle N(p)|m\bar{\psi}\psi|N(p)\rangle = \langle N(p)|m_u\bar{u}u+m_d\bar{d}d+m_s\bar{s}s|N(p)\rangle$$

① **mass decomposition**

Ji, PRD52 271 ('95)
Lorce,Moutarde,Trawinski, EPJC79, 89 ('19)
Metz, Pasquini, Rodini, PRD102, 114042 ('20)
Ji, Liu, Schafer, NPB971, 115537 ('21)

② **pressure**

$$-\bar{C}_{q,g}\frac{M}{V}$$

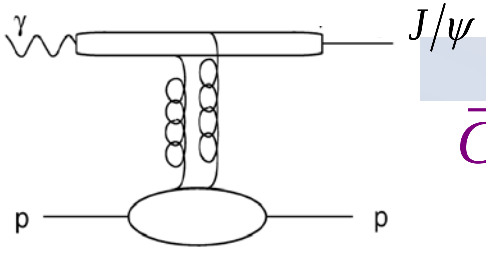
Lorce, EPJC78, 120 ('18)
Liu, PRD104, 076010 ('21)

③ **nucleon's transverse spin sum rule**

Hatta, KT,Yoshida, JHEP 02 ('13) 003

$$J_{q,g}=\frac{1}{2}(A_{q,g}+B_{q,g})+\frac{p^3}{2(p^0+M)}\bar{C}_{q,g}$$

④ $\gamma p \to J/\psi\ p$ **near threshold** JLab, EIC



$\bar{C}_g\ (=-\bar{C}_q)$

Y. Hatta, D. Yang, PRD98, 074003
Y. Hatta, A. Rajan, D. Yang, PRD100, 014032

Studies for $\bar{C}_{q,g}$ themselves

QCD EOMs $(i\not{D}-m)\psi=0, \quad D_\nu F^{\mu\nu}=g\bar{\psi}\gamma^\mu\psi$

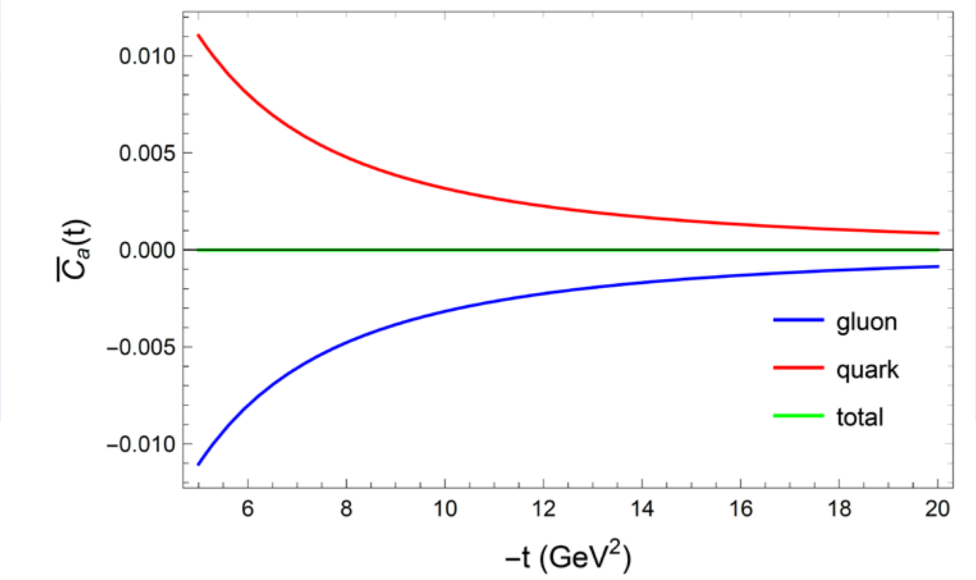
$$\partial_\nu T_q^{\mu\nu}=-\bar{\psi}gF^{\mu\nu}\gamma_\nu\psi, \quad \partial_\nu T_g^{\mu\nu}=-F_a^{\mu\nu}D_{ab}^\rho F_{\rho\nu}^b$$

KT, PRD98, 034009 ('18)

$$\Delta^\mu\bar{u}(p')u(p)M\bar{C}_q(t)=\langle N(p')|\bar{\psi}igF^{\mu\nu}\gamma_\nu\psi|N(p)\rangle$$

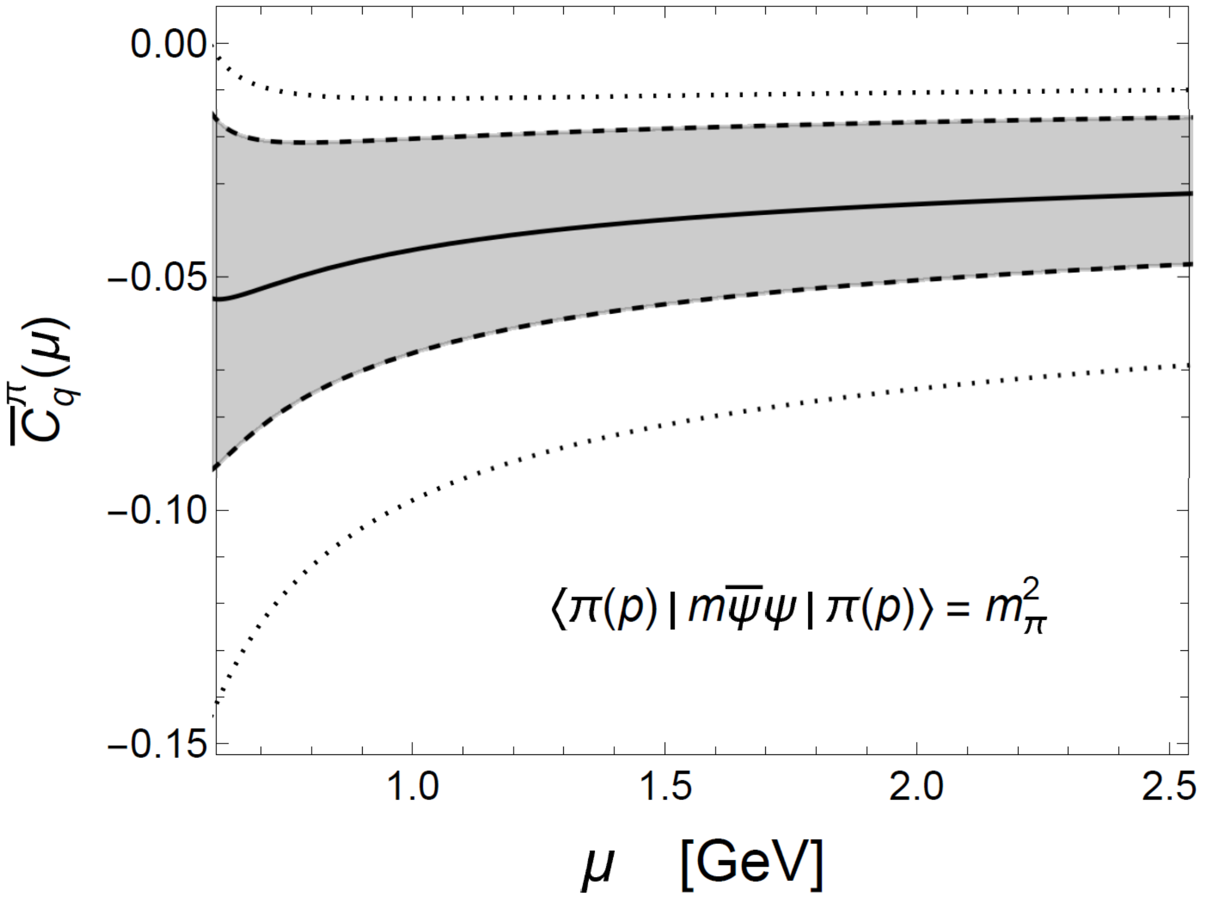
$$\Delta^\mu\bar{u}(p',S')u(p,S)M\bar{C}_g(t)=\langle N(p')|F_a^{\mu\nu}iD_{ab}^\rho F_{\rho\nu}^b|N(p)\rangle$$

pQCD for large t



Tong, Ma, Yuan, PLB823, 136751 ('21)

Tong, Ma, Yuan, JHEP10, 046 ('22)



NNLO QCD

[KT, JHEP03, 013 ('23)]

$$\bar{C}_q^\pi(\mu = 0.7\ \mathrm{GeV})\Big|_{n_f=3} = -0.05 \pm 0.03$$

$$\bar{C}_q^\pi(\mu = 1\ \mathrm{GeV})\Big|_{n_f=3} = -0.04 \pm 0.02$$

$$\bar{C}_q^\pi(\mu = 2\ \mathrm{GeV})\Big|_{n_f=3} = -0.03 \pm 0.02$$

$\overline{\mathrm{MS}}$ scheme