Deeply virtual ϕ-meson production near threshold

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Based on [Hatta, Klest, Passek-K., Schoenleber, 2501.12343] and [Hatta, Schoenleber, 2502.12061]

POETIC XI

February 24, 2025

- Proton gravitational form factors (GFFs) encode information about the matrix elements of the QCD energy momentum tensor.

$$\langle p'|T_{q,g}^{\alpha\beta}|p\rangle = \bar{u}(p')\Bigg[A_{q,g}(t)\gamma^{(\alpha}P^{\beta)} + B_{q,g}(t)\frac{P^{(\alpha}i\sigma^{\beta)\lambda}\Delta_\lambda}{2M} + D_{q,g}(t)\frac{\Delta^\alpha\Delta^\beta - g^{\alpha\beta}\Delta^2}{4M} + \bar{C}_{q,g}(t)Mg^{\alpha\beta}\Bigg]u(p)$$

- $P^\mu = \frac{p^\mu + p'^\mu}{2}, \qquad \Delta^\mu = p'^\mu - p^\mu, \qquad t = \Delta^2$
- GFFs encode **partonic** information about spin, pressure distribution, mass & mechanical radii, normal & shear forces

- GFFs are x-moments (i.e. removing the 1D momentum information) of GPDs

$$\int_{-1}^{1} dx\, x H_q(x,\xi,t) = A_q(t) + \xi^2 D_q(t)$$
$$\int_{-1}^{1} dx\, x E_q(x,\xi,t) = B_q(t) - \xi^2 D_q(t)$$
$$\int_{0}^{1} dx\, H_g(x,\xi,t) = A_g(t) + \xi^2 D_g(t)$$
$$\int_{0}^{1} dx\, E_g(x,\xi,t) = B_g(t) - \xi^2 D_g(t)$$

- Skewness $\xi = -\frac{\Delta^+}{2P^+}$, where $+$ denotes the large light-cone component under the large boost
- GFFs can be calculated from lattice QCD, see e.g. [Hackett, Pefkou, Shanahan, 2310.08484], but also want to determine experimentally

- Extraction of GPDs from experiment is very challenging due to a variety of problems, such as small cross sections, large perturbative and power corrections

- Probably the most difficult one is the **deconvolution problem**: GPDs only enter DVCS and DVMP observables through convolution (with perturbatively calculable hard kernels)

$$\int_{-1}^{1} dx\, C_a(x, \xi, \alpha_s) H_a(x, \xi, t),$$

  which is not invertible

- Some processes give limited information beyond convolution level [Qiu, Yu, 2210.07995]

- Eventually, we want to obtain GPDs. **Question:** Until then, can we take a short cut to the GFFs by looking at certain observables?

$\Rightarrow$ **Threshold approximation (TA)** is the truncation of the conformal partial wave expansion of the GPD after the first term

$$H_q^{(+)}(x,\xi) \approx \frac{5}{2\xi^2}(A_q + \xi^2 D_q)\frac{3x}{\xi}\left(1 - \frac{x^2}{\xi^2}\right)\theta(\xi - |x|),$$

$$H^g(x,\xi) \approx \frac{15}{8\xi}(A_g + \xi^2 D_g)\left(1 - \frac{x^2}{\xi^2}\right)^2\theta(\xi - |x|),$$

where $H_q^{(+)}(x,\xi) = H_q(x,\xi) - H_q(-x,\xi)$.

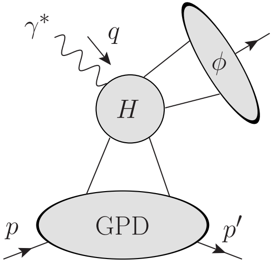
- TA neglects DGLAP region $|x| > \xi$. Decent accuracy requires **"large ξ"**. This is usually achieved in processes when a particle is produced **close to threshold** [Hatta, Strikman, 2102.12631] [Guo, Ji, Liu, 2103.11506] [Guo, Ji, Yuan, 2308.13006]

- Significant effort recently towards photoproduction of J/ψ at threshold [A. Ali et al. (GlueX), 1905.10811] [B. Duran et al., 2207.05212], which gives access to gluon GFFs A_g, D_g
- It is necessary to extend this to additional channels and improve theoretical precision (so far only LO in α_s)
- We consider deeply virtual ϕ production at threshold at NLO [Hatta, Klest, Passek-K., Schoenleber, 2501.12343], which is also sensitive to the strangeness D-term D_s. First time threshold approximation + NLO! See also [Guo, Yuan, Zhao, 2501.10532]
- We also consider deeply virtual ϕ production at threshold off the pion at NLO [Hatta, Schoenleber, 2502.12061] through Sullivan process

- Large ξ means that we need to be close to the threshold

$$(m_N + m_\phi)^2 \approx (p+q)^2 = \frac{1-\xi}{2\xi} Q^2 + \mathcal{O}(Q^0)$$

- However, $|t| \geq \frac{4\xi^2 m_N^2}{1-\xi^2}$ and there are $\sim |t|/Q^2$ power corrections



- Furthermore, if $\frac{1-\xi}{2\xi} Q^2 \sim m_N^2, |t|$ the outgoing proton becomes soft, which is also problematic for factorization.

- Threshold production with pQCD applicable has

$$|t|, m_N^2 \ll \frac{1-\xi}{2\xi} Q^2 \ll Q^2$$

- This covers roughly the range $0.4 \lessapprox \xi \lessapprox 0.6$

- Cross section for longitudinal virtual photon polarization $\frac{d\sigma_L}{dt} \propto |\mathcal{H}|^2 + \ldots$
- Get simple expression at NLO in TA

$$\mathcal{H}_{\rm trunc} = \frac{15}{\xi^2}\Bigg[\underbrace{\left\{\alpha_s + \frac{\alpha_s^2}{2\pi}\left(25.73 - 2n_f + \left(-\frac{131}{18} + \frac{n_f}{3}\right)\ln\frac{Q^2}{\mu^2}\right)\right\}\left(A_s(t,\mu) + \xi^2 D_s(t,\mu)\right)}_{\mathcal{H}^s_{\rm trunc}}$$

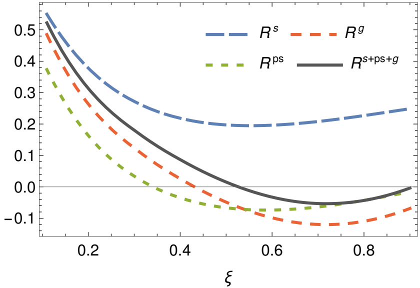
$$+ \underbrace{\frac{\alpha_s^2}{2\pi}\left(-2.39 + \frac{2}{3}\ln\frac{Q^2}{\mu^2}\right)\sum_q \left(A_q(t,\mu) + \xi^2 D_q(t,\mu)\right)}_{\mathcal{H}^{ps}_{\rm trunc}}$$

$$+ \underbrace{\frac{3}{8}\left\{\alpha_s + \frac{\alpha_s^2}{2\pi}\left(13.87 - \frac{83}{18}\ln\frac{Q^2}{\mu^2}\right)\right\}\left(A_g(t,\mu) + \xi^2 D_g(t,\mu)\right)\Bigg]}_{\mathcal{H}^g_{\rm trunc}} + \mathcal{O}(\alpha_s^3)$$

- We have used the simple ϕ meson DA $\phi(u) = 6u(1-u)$
- We compare $\mathcal{H}_{\rm trunc}$ with the amplitude $\mathcal{H}$, which was computed numerically in x-space using the Goloskokov-Kroll (GK) model [Goloskokov, Kroll, 0611290] and NLO hard kernel [Kumericki, Mueller, Passek-K., 0703179]

How good is the threshold approximation?

- Relative error $R^a(\xi) = 1 - \frac{|\mathcal{H}^a(\xi)|}{|\mathcal{H}^a_{\mathrm{trunc}}(\xi)|}$



- TA is good for pure singlet, gluon and total while suboptimal for non-singlet (strange quark) contribution
- There is a cancellation between the pure-singlet and non-singlet contribution $\mathcal{H}^s \approx -\mathcal{H}^{ps}$
- Error for $\mathcal{H}$ in the relevant ξ region is below 10%, which is roughly the size of the expected NNLO α_s or power corrections

- Consider some predictions
- Use dipole and tripole parametrizations for t dependence

$$A_{q,g}(t,\mu) = \frac{A_{q,g}(\mu)}{(1-t/m_A^2)^2}, \quad D_{q,g}(t,\mu) = \frac{D_{q,g}(\mu)}{(1-t/m_D^2)^3}$$

with $m_A = 1.6\,\text{GeV}$ and $m_D = 1.1\,\text{GeV}$ for all parton species

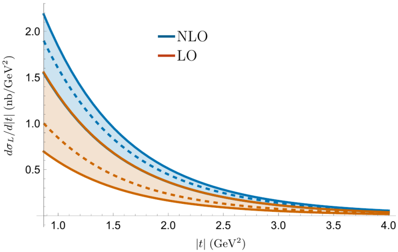
- Use LO evolution from reference scale $\mu_0 = 2\,\text{GeV}$ and initial conditions

$$A_s(\mu_0) = 0.03, \quad A_g(\mu_0) = 0.42, \quad A_{u+d+c}(\mu_0) = 1 - A_s(\mu_0) - A_g(\mu_0)$$
$$D_{u+d+c}(\mu_0) = -1.2$$

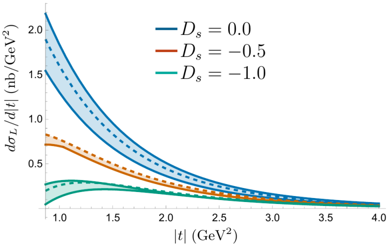
- Vary $D_s(\mu_0)$ and $D_g(\mu_0)$ to estimate sensitivity
- Consider $Q = 2.5\,\text{GeV}$ and vary μ in the range $Q/2 < \mu < 2Q$ to estimate perturbative uncertainty

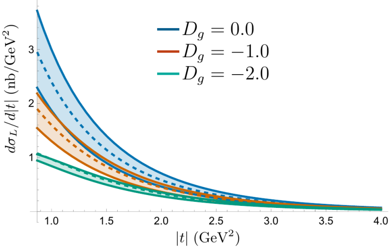
Results

- Predictions for cross section using $D_s(\mu_0) = 0$ and $D_g(\mu_0) = -1$. NLO corrections are large.

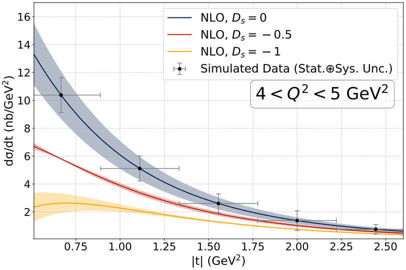


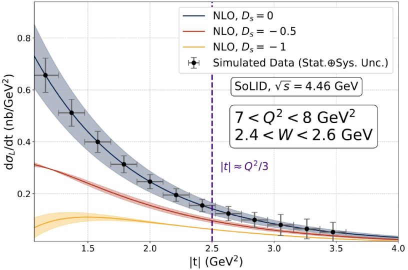
- Cross section (NLO) is sensitive to strange and gluon D-terms





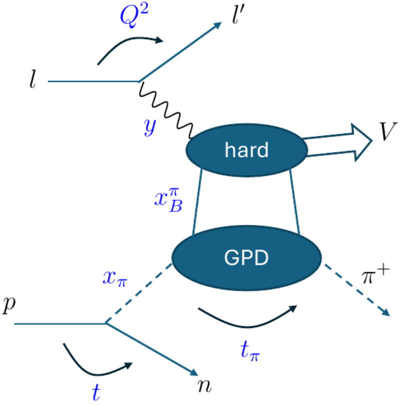
- Projections using event generator `lAger`. Left: ePIC (EIC), Right: SoLID (Jlab)



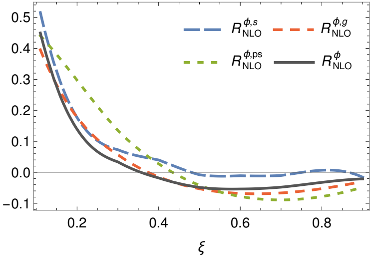


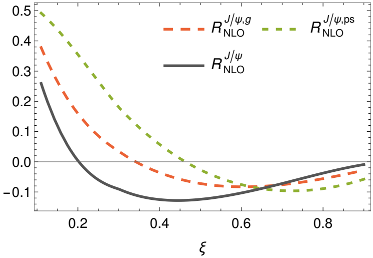
- Error bars are smaller than changes due to varying D_s. This is similar for D_g (not shown here)
- This is true in both cases, but SoLID is better

- Sullivan process can be used to access the π^+ GPD [Amrath, Diehl, Lansberg, 0807.4474]
- Due to the small pion mass $|t_\pi| \geq \frac{4\xi^2 m_\pi^2}{1-\xi^2}$ is much less restrictive
- Can go to up to $\xi \approx 0.8$ while keeping t_π small
- We considered both deeply virtual ϕ production and J/ψ photoproduction at NLO
- Can be used to measure pion GFFs!



- TA accuracy for meson production off the pion using the pion GPD model from [Chavez et al., 2110.06052] at $\mu^2 = 10\,\text{GeV}^2$ and NLO accuracy. Left: deeply virtual ϕ production, Right: photoproduction of J/ψ





- Similarly good accuracy as for meson production of the proton

- Left: Longitudinal DVϕP cross section at $x_B = \frac{Q^2}{2p\cdot q} = 0.2$, $x_\pi = \frac{p_\pi \cdot l}{p\cdot l} = 0.3$, $Q^2 = 10\,\text{GeV}^2$. Upper plot (blue bands): NLO, Lower plot (orange bands): LO
- Right: J/ψ photoproduction cross section integrated over t_π as a function of x_π
- Bands from scale variation